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A chance constrained programming approach for HazMat capacitated vehicle routing problem in Type-2 fuzzy environment

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ABSTRACT

This work focuses on a HazMat capacitated vehicle routing problem (H-CVRP) in type-2 fuzzy environment, which aims to determine a set of routes with the minimum transportation risk. Since uncertainty can lead to significant differences in transportation risk, we propose a H-CVRP model with the objective function involving trapezoidal interval type-2 fuzzy variables (IT2-FVs). Based on the credibility measure, a chance constrained programming (CCP) approach is employed to transform the H-CVRP model into its equivalent deterministic form. A simulated annealing algorithm (SAA) is designed to solve the equivalent deterministic model. The proposed SAA is a global optimization algorithm, which converges to the optimal solution with probability and has high parallelism. To test the performance of the proposed algorithm, the optimal solutions obtained by SAA are compared with the counterparts obtained by genetic algorithm (GA) and tabu search (TS). Experimental results indicate that the proposed SAA is competitive in terms of stability and efficiency. At last, a sensitivity analysis is presented to demonstrate the applicability of the proposed method.

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1. Introduction

With the rapid development of industry, HazMat have become an indispensable part of industrial production. Due to the nature of these HazMat, every activity related to their use may result in severe human casualties and property losses, especially for HazMat transportation (Pradhananga et al., 2009). Currently, the annual production of HazMat is about 400 million tons, of which more than 300 million tons of HazMat are transported by road (Jiaoman Du et al., 2017). HazMat release events caused by traffic accidents may cause enormous damage to the surrounding residents and environment. According to the US Department of Transportation, there are about 1500 HazMat transportation accidents each year, resulting in economic losses of about one billion dollars (U. DOT and U.S, 2016).

As one of the main problems in the field of transportation safety, HazMat transportation problems have received extensive attention. Most of the related studies focus on the HazMat transportation routing problems. As mentioned by Bula et al. (2017), HazMat transportation routing problems can be divided into 2 types: shortest path problems (SPPs) and vehicle routing problems (VRPs). Research on the first type of problems is abundant which usually aims at finding a path with minimum transportation risk for a given origin-destination pair (Holeczek, 2019). The first VRP was developed to determine a set of routes for transporting gasoline, its objective is to minimize the total travel distance (Dantzig and Ramser, 1959). This is not a real HazMat VRP, as it ignores the danger of HazMat. Erkut et al. (2007) separated routing and scheduling problems into scheduling problems with a priori optimization and adaptive routing. Since then, the HazMat VRPs have gradually gained attention. As the transportation risk is the primary ingredient that separates HazMat VRPs from classical VRPs, risk minimization is the primary or only objective of most research. The basic framework for estimating risk is so-called “probability-consequence framework”, which considers both accident probability and consequence (Ingolfsson, 2005). The accident probability represents the likelihood of a HazMat release event, and the consequence measure can be expressed as the population density around the accident site (Kazantzi VGerogiannis, 2011). Androutsopoulos and Zografos (2010) raised a bi-objective HazMat vehicle routing problem with time windows (H-VRPTW) of which the objective function included both risk minimization and cost minimization. A weighted aggregation function was adopted to combine two conflicting objectives into a single objective optimization problem. To solve the problem, a label-setting based route-

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building heuristic was presented. In their latest work (Androutsopoulos and Zografos, 2012), risk values are time-dependent and load-variant. They studied the trend of population density over time. The impact area of a HazMat release event is related to the vehicle load and weather factors. Pradhananga et al. (Yamada, 2010) considered a bi-objective H-VRPTW. A Pareto-based meta-heuristic was presented to solve the problem. They assumed that there were various types of potential HazMat release events and various types of accidents consequences. In their work, the transportation risk is defined as the sum of the risk associated with different situations. J. Du et al. (2017) studied a HazMat multi-depot vehicle routing problem (H-MDVRP). A fuzzy bilevel programming model was proposed to minimize the transportation risk. Based on the risk model proposed by Batta and Chiu (Batta R, 1988), they defined the population density as a Type-1 fuzzy variable (T1-FV). A numerical-integration-based fuzzy simulation algorithm (Li, 2015) was adopted to calculate the fuzzy expected value. Bula et al. (2017) proposed a variable neighborhood search (VNS) algorithm to solve the HazMat heterogeneous vehicle routing problem (H-HVRP). The objective is to obtain a routing decision with minimum transportation risk. In their work, the accident probability on a specific path is determined by the vehicle load, the vehicle type, the HazMat type and the path length. The transportation risk is expressed by an approximated piece-wise linear function. In their follow-up work (Bula et al., 2019), they raised a bi-objective form of the original H-HVRP. To find non-dominated solutions, a multi-objective neighborhood dominance-based algorithm and an ε-constraint heuristic algorithm were proposed.

There is still a gap between current research and practical applications. Due to the lack of available data, it is difficult to obtain accurate values for the relevant risk parameters (P.M.RNancy P. Button, 2000). The transportation risk is often accompanied by high levels of uncertainty (Kazantzi et al., 2011). As reported by various researchers (Holeczek, 2019; Batta R, 1988; Erkut and Verter, 1998), different risk modeling approaches can lead to different routing decisions. Thus, the routing decisions obtained in a deterministic context may not meet the practical application needs. As far as we know, only J. Du et al. (2017) consider the uncertainty of transportation risk in HazMat VRP where the population density is assumed to be a type-1 fuzzy variable (T1-FV). However, the exact membership function (MF) required by the type-1 fuzzy set (T1-FS) is usually difficult to construct, as it depends on data sources used and assumptions made (Kundu et al., 2019).

Therefore, inspired by T1-FS, type-2 fuzzy set (T2-FS) was developed by Zadeh (1975). The MF of T2-FS is also a fuzzy set. Recently, T2-FS theory has been applied to many decision-making problems. Due to the high computational complexity for processing T2-FSs, most studies usually perform type reduction on T2-FSs before defuzzification (Pramanik et al., 2015), and the interval type-2 fuzzy sets (IT2-FSs) are widely used (Kundu et al., 2019; MJ.RI and L. F, 2006; Wu and Mendel, 2007). To handle a data envelopment analysis problem with T2-FVs, Qin et al. (2011) presented 3 type reduction methods based on 3 kinds of critical values (CV) respectively (optimistic CV, pessimistic CV and regular CV). Pramanik et al. (2015) raised a two-stage supply chain fixed-charge transportation problem of which the uncertain parameters were represented by a gaussian type-2 fuzzy variable (GT2-FV). A CV-based reduction method (Qin et al., 2011) was adopted for type reduction. The fuzzy models are converted to their equivalent deterministic forms by credibility measure (Liu and Liu, 2010) and centroid method (Yang et al., 2015). Then, 2 meta-heuristic algorithms (GA and PSO) are developed to solve the equivalent deterministic models. Kundu et al. (2019) proposed a CCP method to solve the linear programming problems with IT2-FVs. The proposed method was also be applied to deal with the SPPs, the TPs and the minimum spanning tree problems. Vasant (Vasant and Barsoum, 2009) provided a comprehensive review of those meta-heuristic algorithms for solving fuzzy optimization problems and demonstrated the availability of these methods in handling practical industrial production planning problems. These applications suggest a promising future for applying T2-FS theory in solving the HazMat VRPs.

This work considers a H-CVRP in type-2 fuzzy environment. CVRP is an extension of the classical VRP in which each vehicle has its own unique capacity limit (M et al., 2014). Based on the risk modeling approach proposed by Erkut et al. (Erkut and Verter, 1998), the population density near the accident site is expressed as a trapezoidal IT2-FV. Compared to T1-FV, T2-FV can provide additional degrees of freedom for modeling uncertainty (Kundu et al., 2019). According to the lower and upper MFs of trapezoidal IT2-FV, two CCP models are developed by using credibility measure. The linear weighting method is adopted to combine the two CCP models into the equivalent deterministic form of H-CVRP model. The deterministic model is solved by 3 meta-heuristic algorithms (GA (Wang and Lu, 2009), TS (P et al., 1998) and SAA). The optimal solutions obtained by 3 algorithms are compared. At last, a sensitivity analysis is presented to demonstrate the applicability of the proposed method.

The rest of this paper is stated as follows. The mathematical formulation of the risk model and H-CVRP model are described in Section 2. Section 3 introduces the proposed SAA. Section 4 provides some numerical experiments to test the performance of the proposed algorithm. A sensitivity analysis is also given in Section 4. At last, the work is concluded in Section 5. The relevant knowledge of T1-FS and T2-FS is given in Appendix. A and Appendix. B.

2. Problem formulation

2.1. Background

The H-CVRP is defined on an undirected network $G = (N, L)$, where the node set $N = \{0, 1, 2\cdots, n\}$ consists of a depot node 0 and a customer set $C = \{1, 2, \ldots, n\}$. Each node is associated with a fixed demand q_i. Any pair of nodes $i, j \in \mathrm{N}$ is connected by an arc $arc_{ij} \in L$. Each arc has a fixed length d_{ij}. $K = \{k_1, k_2, \ldots, k_{|K|}\}$ is the transportation fleet. Each vehicle $k \in K$ has a capacity limit Q_k. A solution χ is composed of a set of routes, $\{\vartheta_1, \vartheta_2, \ldots, \vartheta_{|\chi|}\}$. Each route $\vartheta_r \in \chi$ is represented by a sequence of nodes, $\{n_1^r, n_2^r, \ldots, n_{|\vartheta_r|}^r\}$, where n_2^r is the second node visited in this route. The H-CVRP aims at determining an optimal solution $\chi^* \in \mathscr{FS}$ with minimum transportation risk as well as the following constraints are met. $\mathscr{FS}$ is the feasible set.

- Each vehicle starts and ends at the depot node;
- All customer demands must be satisfied;
- Each customer can only be served once;
- Vehicles cannot be overloaded;
- The number of vehicles used cannot exceed $|K|$

2.2. Risk model formulation

According to “probability-consequence framework”, the transportation risk is defined as the product of accident consequence and probability, which is given as follows:

$$R_{ij} = P_{ij} \times Cs_{ij}, i, j \in \mathrm{N} \tag{1}$$

where R_{ij} is the risk of transporting HazMat on $arc_{ij} \in L$; P_{ij} is the accident probability on arc_{ij};Cs_{ij} represents the accident consequence on $arc_{ij} \in L$.

The accident probability on arc_{ij} is estimated as follows (Erkut and Verter, 1998):

$$P_{ij} = AR_{ij} \times Pr_{ij} \times d_{ij}, i, j \in \mathrm{N} \tag{2}$$

where AR_{ij} is the traffic accident rate on arc_{ij}; Pr_{ij} is the probability of a HazMat release event on arc_{ij}.

The accident consequence on arc_{ij} is measured by the number of exposure people in impact area (Kazantzi VGerogiannis, 2011):

$$Cs_{ij} = pd_{ij} \times \pi \times r_{ij}^2, i, j \in \mathrm{N} \tag{3}$$

where pd_{ij} is the population density near the accident site; r_{ij} is the radius of impact area, usually r_{ij} is set to 1 km (Ingolfsson, 2005).

Due to the mobility of the population, this work considers the population density as a trapezoidal IT2-FV $\widetilde{Pd}_{ij}$. According to the **Definition B.3**, $\widetilde{Pd}_{ij}$ is given as follows:

$$\widetilde{Pd}_{ij} = \left(Pd_{ij}^U, Pd_{ij}^L\right) = \begin{pmatrix} Pd_{ij,1}^U, Pd_{ij,2}^U, Pd_{ij,3}^U, Pd_{ij,4}^U, \omega_{ij}^U \\ Pd_{ij,1}^L, Pd_{ij,2}^L, Pd_{ij,3}^L, Pd_{ij,4}^L, \omega_{ij}^L \end{pmatrix}, i, j \in \mathrm{N} \tag{4}$$

where Pd_{ij}^U and Pd_{ij}^L are T1-FVs defined on upper MF $(Pd_{ij,1}^U, Pd_{ij,2}^U, Pd_{ij,3}^U, Pd_{ij,4}^U, \omega_{ij}^U)$ and lower MF $(Pd_{ij,1}^L, Pd_{ij,2}^L, Pd_{ij,3}^L, Pd_{ij,4}^L, \omega_{ij}^L)$.

Thus, the transportation risk on arc_{ij} can be calculated as follows:

$$\widetilde{R_{ij}} = AR_{ij} \times Pr_{ij} \times d_{ij} \times \pi \times r_{ij}^2 \times \widetilde{Pd}_{ij}, i, j \in \mathrm{N} \tag{5}$$

2.3. HazMat capacitated vehicle routing problem formulation

This section introduces a H-CVRP model based on the above risk model. This model requires a decision variable x_{ijk}.

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ goes through } arc_{ij} \\ 0, & \text{otherwise} \end{cases}, k \in K, i, j \in N \tag{6}$$

Formally, the H-CVRP model can be stated as follows:

$$Min\ Z = \sum_{k \in K} \sum_{i,j \in N} x_{ijk} \times \widetilde{R_{ij}} = \sum_{k \in K} \sum_{i,j \in N} x_{ijk} \times AR_{ij} \times Pr_{ij} \times d_{ij} \times \pi \times r_{ij}^2 \times \widetilde{Pd}_{ij} \tag{7}$$

s.t.

$$\sum_{k \in K} \sum_{j \in C} x_{0jk} \le |K| \tag{8}$$

$$\sum_{k \in K} \sum_{j \in C, j \ne i} x_{ijk} = 1, \forall i \in N \tag{9}$$

$$\sum_{i,j \in N, i \ne j} q_i \times x_{ijk} \le Q_k,\ \forall\ k \in K \tag{10}$$

$$\sum_{j \in C} x_{0jk} = 1,\ \forall\ k \in K \tag{11}$$

$$\sum_{i \in C} x_{i0k} = 1,\ \forall\ k \in K \tag{12}$$

$$\sum_{i \in C} x_{ihk} - \sum_{j \in C} x_{hjk} = 0,\ \forall\ k \in K, \forall h \in C \tag{13}$$

where Eq. (8) represents that the number of vehicles used cannot exceed $|K|$; Eq. (9) represents every customer must be satisfied and can only be serviced once; Eq. (10) represents that the total demand handled by any vehicle does not exceed its maximal capacity Q_k; Eq. (11) ~ Eq. (13) represent that each vehicle must depart from the depot node and return to the depot node after visiting a number of non-repetitive customers.

2.4. Chance constrained programming

Based on the boundary MFs of IT2-FV, this work adopts the credibility measure (Liu and Liu, 2010) to transform the H-CVRP model into 2 CCP models.

For the upper MF $(Pd_{ij,1}^U, Pd_{ij,2}^U, Pd_{ij,3}^U, Pd_{ij,4}^U, \omega_{ij}^U)$, the CCP model is stated as follows:

$$\begin{cases} Min\ Z^U \\ s.t.: \\ \quad Cr\left\{\sum_{k \in K} \sum_{i,j \in N} x_{ijk} \times R_{ij}^U \le Z^U\right\} \ge \alpha^U \\ Eq.(8) \sim (13) \end{cases} \tag{14}$$

For the lower MF $(Pd_{ij,1}^L, Pd_{ij,2}^L, Pd_{ij,3}^L, Pd_{ij,4}^L, \omega_{ij}^L)$, the CCP model is stated as follows:

$$\begin{cases} Min\ Z^L \\ s.t.: \\ \quad Cr\left\{\sum_{k \in K} \sum_{i,j \in N} x_{ijk} \times R_{ij}^L \le Z^L\right\} \ge \alpha^L \\ Eq.(8) \sim (13) \end{cases} \tag{15}$$

where α^U and α^L are predefined credibility levels; $R_{ij}^U = AR_{ij} \times Pr_{ij} \times d_{ij} \times \pi \times r_{ij}^2 \times Pd_{ij}^U$; $R_{ij}^L = AR_{ij} \times Pr_{ij} \times d_{ij} \times \pi \times r_{ij}^2 \times Pd_{ij}^L$.

According to **Theorem A.2,** we have the following equivalent constraints.

$$\begin{cases} Cr\left\{\sum_{k \in K} \sum_{i,j \in N} x_{ijk} \times R_{ij}^U \le Z^U\right\} \ge \alpha^U \Rightarrow \sum_{k \in K} \sum_{i,j \in N} x_{ijk} \times H_{ij}^U \le Z^U \\ Cr\left\{\sum_{k \in K} \sum_{i,j \in N} x_{ijk} \times R_{ij}^L \le Z^L\right\} \ge \alpha^L \Rightarrow \sum_{k \in K} \sum_{i,j \in N} x_{ijk} \times H_{ij}^L \le Z^L \end{cases} \tag{16}$$

where

$$H_{ij}^U = \begin{cases} \dfrac{\left(\left(\omega_{ij}^U - 2\alpha^U\right) Pd_{ij,1}^U + 2\alpha^U Pd_{ij,2}^U\right)}{\omega_{ij}^U}, & if\ \alpha^U \le \dfrac{\omega_{ij}^U}{2} \\ \dfrac{\left(2\left(\omega_{ij}^U - 2\alpha^U\right) Pd_{ij,3}^U + \left(2\alpha^U - \omega_{ij}^U\right) Pd_{ij,4}^U\right)}{\omega_{ij}^U}, & if\ \alpha^U > \dfrac{\omega_{ij}^U}{2} \end{cases} \tag{17}$$

$$H_{ij}^{L}=\begin{cases}\dfrac{\left(\left(\omega_{ij}^{L}-2\alpha^{L}\right)Pd_{ij,1}^{L}+2\alpha^{L}Pd_{ij,2}^{L}\right)}{\omega_{ij}^{L}} & ,\ if\ \alpha^{L}\leq\dfrac{\omega_{ij}^{L}}{2}\\ \dfrac{\left(2\left(\omega_{ij}^{L}-2\alpha^{L}\right)Pd_{ij,3}^{L}+\left(2\alpha^{L}-\omega_{ij}^{L}\right)Pd_{ij,4}^{L}\right)}{\omega_{ij}^{L}} & ,\ if\ \alpha^{L}>\dfrac{\omega_{ij}^{L}}{2}\end{cases} \tag{18}$$

The equivalent deterministic forms of CCP models (14) and (15) are stated as follows:

$$\begin{cases}Min\ Z^{U}=\sum_{k\in K}\sum_{i,j\in N}x_{ijk}\times H_{ij}^{U}\\ s.t.:\\ Eq.(8)\sim(13)\end{cases} \tag{19}$$

$$\begin{cases}Min\ Z^{L}=\sum_{k\in K}\sum_{i,j\in N}x_{ijk}\times H_{ij}^{L}\\ s.t.:\\ Eq.(8)\sim(13)\end{cases} \tag{20}$$

Finally, the H-CVRP model is redefined as follows:

$$\begin{cases}Min\ Zr=\dfrac{1}{2}\left(Z^{U}+Z^{L}\right)\\ s.t.:\\ Eq.(8)\sim(13)\end{cases} \tag{21}$$

3. Solution method

According to the characteristics of the CVRP, this section proposes an efficient SAA to solve the equivalent deterministic form of H-CVRP model (22). The general framework of SAA is presented in **Algorithm 1**.

Algorithm 1 Simulated Annealing Algorithm (SAA)

```
Input: Instance data
Output: An optimal solution χ* = {ϑ_1, ϑ_2, ..., ϑ_|χ|}
Initialization:
  Construct an initial solution χ_0;
  Set initial temperature T;
  Set temperature damping rate TDR;
  Set maximum number of inner iterations Inmax;
  Update best solution: χ* = χ_0
While stopping criterion not met Do:
    in = 1
    Update current solution: χ = χ*
    While in ≤ Inmax Do
        χ_new ← Neighborhood operator(χ)
        If Metropolis acceptance criterion met Do
            Update current solution: χ = χ_new
        End If
        If Z*(χ) < Z*(χ*) Do
            Update best solution: χ* = χ
        End If
        in = in + 1
    End While
    Reduce temperature: T = T * TDR
End While
```

The proposed SAA starts with an initial solution χ_0. A new solution χ_{new} is generated by performing neighborhood operators on current solution χ. *Metropolis acceptance criterion* (Schuur, 1997) is used to determine whether a new solution χ_{new} is acceptable. When χ_{new} is accepted, replace χ with χ_{new}. If the current solution χ is superior to the optimal solution χ^*, replace χ^* with χ. So far, an inner iteration of SAA is completed. After achieving the maximum number of inner iterations, the temperature T is reduced according to the temperature damping rate *TDR*. The algorithm will then reiterate through this process until the predefined stopping criterion are met. It's worth mentioning that in order to improve the convergence of the algorithm, the proposed SAA relaxes the vehicle capacity constraints during the iterative process. Therefore, based on the original objective function *Zr*, we introduce a penalty factor to construct a new objective function Z^*. Through the penalty factor, we can turn the constrained problem into an unconstrained problem. Z^* gives a large objective function value for a solution that violates the capacity constraints. According to the *Metropolis acceptance criterion*, the probabilities of accepting new solutions that violate the capacity constraints are much lower than feasible new solutions. Thus, the proposed SAA will force the current solution to move closer to the feasible domain of the original constrained problem during the iterative process. In fact, when the algorithm approaches convergence, the current solution will basically remain moving in the feasible domain of the original constrained problem.

3.1. Initial solution χ_0

The initial solution is generated by a random sequence. A concrete example is used to illustrate the initial solution construction. As shown in Fig. 1, a transportation fleet of three vehicles needs to serve 15 customers, i.e. $K=\{k_1,k_2,k_3\}$, $N=\{0,1,2\cdots,15\}$ and $C=\{1,2\cdots,15\}$. A random sequence $'12,1,5,11,16,2,10,4,9,3,8,17,6,13,14,7,15'$ consists of integers from 1 to $(|K|+|C|-1)$. The initial solution $\chi_0=\{\vartheta_1,\vartheta_2,\vartheta_3\}$, $\vartheta_1=\{0,12,1,5,11,0\}$, $\vartheta_2=\{0,2,10,4,9,3,8,0\}$, $\vartheta_3=\{0,6,13,14,7,15,0\}$ is constructed with numbers greater than $|C|$ as the breakpoints.

3.2. Objective function

The proposed SAA relaxes the vehicle capacity constraints during the iterative process. This indicates that a new solution χ_{new} that violates the capacity constraints is likely to be accepted. However, not all new solutions that violate the capacity constraints are worthy of acceptance. To further distinguish the advantages and disadvantages of new solutions that violate the capacity constraints, a penalty variable δ_k is defined as follows:

$$\delta_k=\begin{cases}\sum_{i,j\in N,i\neq j}q_i\times x_{ijk}-Q_k, & if\ \sum_{i,j\in N,i\neq j}q_i\times x_{ijk}>Q_k\\ 0 & ,otherwise\end{cases} \tag{22}$$

The objective function with penalty factor is defined as follows:

$$Z^*=Zr(1+pc*pf) \tag{23}$$

where $pf=\frac{\sum_{k\in K}\delta_k}{|K|}$ represents the degree to which constraints are violated, pc is the penalty coefficient. A current solution χ with small pf can often generate a feasible new solution through minor adjustments, and its objective function value may drop significantly. This will help the algorithm converge faster. Moreover, Z^* is more realistic in practice, as the overload degree of a vehicle and its transportation risk are often positively correlated (U. DOT and U.S, 2016).

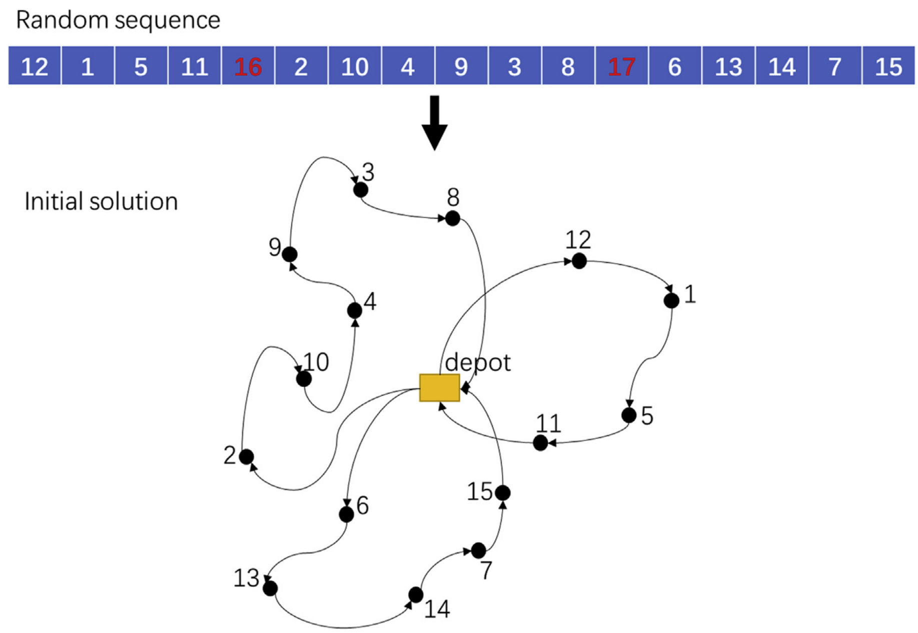

Fig. 1. An illustrative example of initial solution construction.

3.3. *Neighborhood search*

The proposed SAA randomly selects a neighborhood operator to generate a new solution during each inner iteration (Afifi et al., 2013). Three basic neighborhood operators are presented as follows.

3.3.1. *Swap*

Swap operator randomly swaps the positions of two numbers in the corresponding sequence. As shown in Fig. 2, **Swap** operator is performed on a solution $\chi = \{\{0,12,1,5,11,0\}, \{0,2,10,4,9,3,8,0\}, \{0,6,13,14,7,15,0\}\}$, and swaps the positions of '4' and '17' in the corresponding sequence. Thus, $\chi_{new} = \{\{0,12,1,5,11,0\},\{0,2,10,0\},\{0,9,3,8,4,6,13,14,7,15,0\}\}$.

3.3.2. *Reversion*

Reversion operator reverses the order of a random region in the corresponding sequence. As shown in Fig. 3, **Reversion** operator is performed on a solution $\chi = \{\{0,12,1,5,11,0\}, \{0,2,10,4,9,3,8,0\},\{0,6,13,14,7,15,0\}\}$, and reverses the order $'4,9,3,8,17'$ in the corresponding sequence. Thus, $\chi_{new} = \{\{0,12,1,5,11,0\},\{0,2,10,0\},\{0,8,3,9,4,6,13,14,7,15,0\}\}$.

3.3.3. *Insertion*

Insertion operator removes a random number from the corresponding sequence and re-inserts into another position. As shown in Fig. 4, **Insertion** operator is performed on a solution $\chi = \{\{0,12,1,5,11,0\},\{0,2,10,4,9,3,8,0\},\{0,6,13,14,7,15,0\}\}$, and removes '4' from the corresponding sequence and re-inserts into the position back after '17'. Thus, $\chi_{new} = \{\{0,12,1,5,11,0\}, \{0,2,10,9,3,8,0\},\{0,4,6,13,14,7,15,0\}\}$.

3.4. *Metropolis acceptance criterion*

Metropolis acceptance criterion is adopted to determine whether a new solution χ_{new} is acceptable (See **Algorithm 2**). Δ is the objective function increment between the current solution χ and the new solution χ_{new}. For a minimization problem, $\Delta < 0$ indicates that χ_{new} is better than χ, and χ is replaced by χ_{new}. When $\Delta > 0$, the probability of accepting χ_{new} is $exp(-(\Delta/T))$.

Algorithm 2 Metropolis acceptance criterion

Calculate the objective function increment: $\Delta = Z^*(\chi_{new}) - Z^*(\chi)$
If $\Delta < 0$ ***Do***
  Update current solution: $\chi = \chi_{new}$
Else
  Generate a random number: $\varepsilon \in [0,1]$
  If $\varepsilon \le exp(-(\Delta/T))$ ***Do***
    Update current solution: $\chi = \chi_{new}$
  End If
End Else

4. Numerical experiments

4.1. *Problem instances*

In the best of our knowledge, there is no suitable problem instances in the existing studies for the proposed H-CVRP model. Thus, this work develops 3 problem instances to demonstrate the advantages of the proposed method. Information relevant to the 3 problem instances includes the number of customers, the number of vehicles available and the capacities of vehicles, which are stated in Table 1. The positions of customers are randomly distributed. Due to the lack of relevant risk parameters, we construct a set of fictitious traffic accident rate data and release probability data according to the statistical data provided by Vasiliki Kazantzi et al. (Kazantzi VGerogiannis, 2011).

For each $Pd_{ij}^U, i,j \in N$, $Pd_{ij,1}^U, Pd_{ij,2}^U, Pd_{ij,3}^U, Pd_{ij,4}^U$ *and* ω_{ij}^U are randomly assigned values from [75, 94], [96, 117], [147, 162], [165, 175] and [0,1] respectively. For each $Pd_{ij}^L, i,j \in N$, $Pd_{ij,1}^L, Pd_{ij,2}^L, Pd_{ij,3}^L, Pd_{ij,4}^L$ *and* ω_{ij}^L are randomly assigned values from [96, 101], [118, 125], [128, 138], [143, 157] and [0,1] respectively.

4.2. *Optimal results*

To test the performance of the proposed algorithm, the optimal solutions obtained by SAA are compared with the counterparts

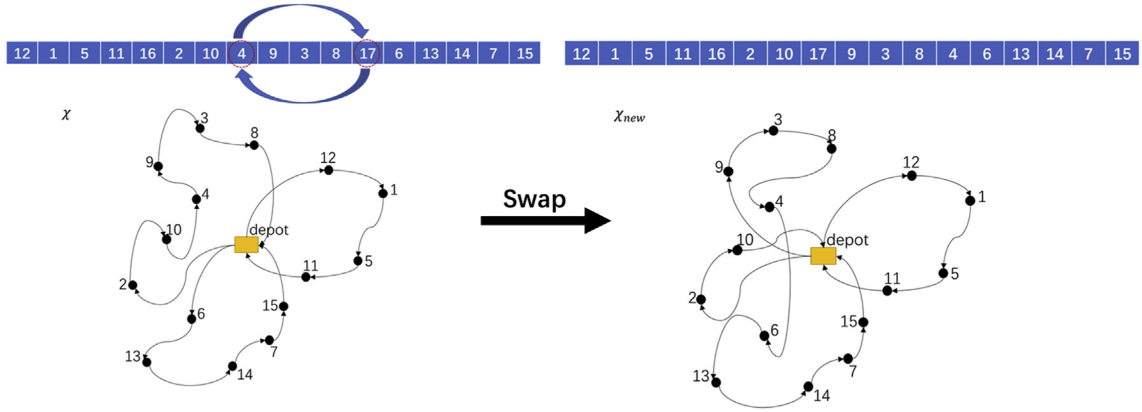

Fig. 2. An illustration of **Swap** operator.

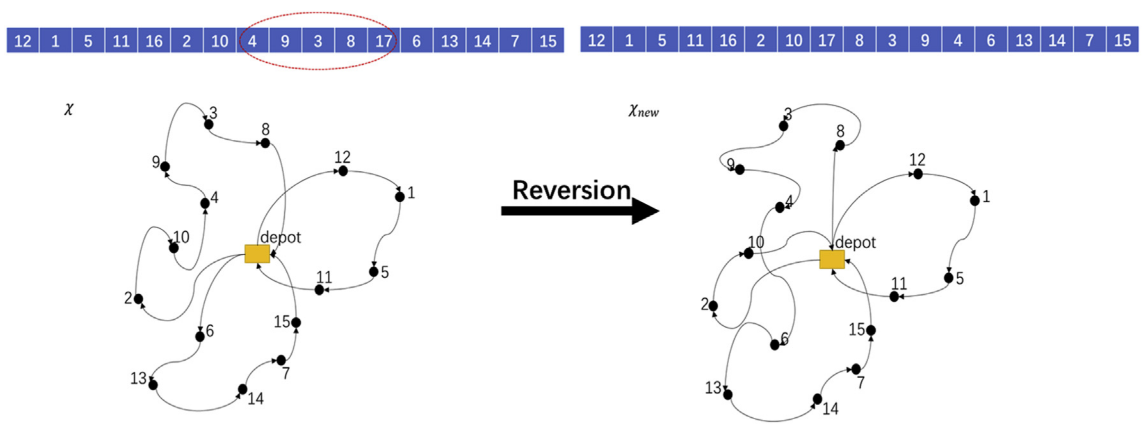

Fig. 3. An illustration of **Reversion** operator.

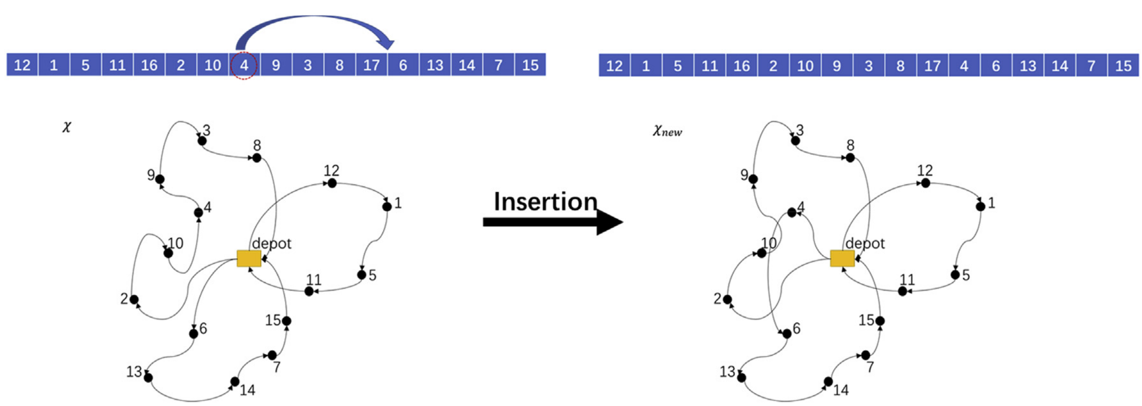

Fig. 4. An illustration of **Insertion** operator.

obtained by GA (Wang and Lu, 2009) and TS (P et al., 1998). Both GA and TS are common meta-heuristic algorithm for solving CVRPs. The predefined credibility levels α^U and α^L are set to 0.05, i.e. $\alpha^U = \alpha^L = 0.05$. All the algorithms run on a computer with Intel (R) Core (TM) i7-7700 CPU @ 3.60 GHz and 32.0 GB RAM. The parameters of 3 algorithm are shown in Table 2[a].

In Table 3, we compare the experimental results of each algorithm after 10 independent runs. The best convergence curves of 3 algorithms are shown in Fig. 5, Fig. 6 and Fig. 7. Due to the stochasticity of the heuristics, the results may be very different when the same problem instance is solved many times (Marinakis et al., 2019). This phenomenon will become more and more obvious as the size of the problem instance increases. Thus, the algorithm performs very well as it can be concluded from the best quality of the results and the average quality of the results.

For small-sized instance C8–K3, we find the true optimal solution ($Z^* = 8.377e+3$) through the traversal search. Experimental results indicate that SAA, GA and TS can all find the true optimal

Table 1
Problem instances description.

Problem instances	Customer number	Transportation fleet size	Vehicle capacity
C8−K3	8	3	(54,46,49)
C25−K5	25	5	(130,139,134,138,135,138,150)
C70−K8	70	8	(167,160,173,162,172,165,166,189)

Table 2
[a]. Parameters of algorithms.

SAA	IN = 1500	$Inmax = 100$	$T = 100$	$TDR = 0.98$	$pc = 5$
GA	IN = 1500	$Npop = 100$	$Cp = 0.8$	$Mp = 0.05$	$pc = 5$
TS	IN = 1500	$NA = S(2S-1), S = \|C\| + \|K\| - 1$		$TS = 0.1{*}NA$	$pc = 5$

[a] MIN is the maximum iteration number; *Inmax* is the maximum inner iteration number; *T* is the initial temperature; *TDR* is the temperature damping rate; *pc* is the penalty coefficient; *Npop* is the population size; *Cp* is the crossover probability; *Mp* is the mutation probability. *NA* is the number of neighborhood structures, which increases with the problem size; *TS* is the tabu length.

Table 3
Experimental results obtained by three algorithms.

No	SAA		GA		TS	
	Z^*	Time(s)	Z^*	Time(s)	Z^*	Time(s)
C8−K3						
1	8.377e+3	36.145	9.797e+3	86.978	8.377e+3	26.116
2	9.444e+3	28.371	8.377e+3	90.644	8.377e+3	25.542
3	8.377e+3	32.331	9.689e+3	96.012	8.377e+3	24.157
4	8.377e+3	30.561	9.687e+3	86.898	8.377e+3	25.788
5	8.377e+3	27.124	9.797e+3	90.124	8.377e+3	24.813
6	8.377e+3	31.365	9.689e+3	86.978	8.377e+3	25.376
7	8.377e+3	33.598	9.797e+3	83.158	8.377e+3	25.778
8	8.377e+3	34.449	9.689e+3	91.271	8.377e+3	26.147
9	8.377e+3	29.981	9.689e+3	92.991	8.377e+3	25.686
10	9.444e+3	31.688	9.797e+3	86.744	8.377e+3	24.836
Avg	8.590e+3	31.561	9.600e+3	89.180	8.377e+3	25.423
Best	8.377e+3	27.124	8.377e+3	83.158	8.377e+3	24.157
C25−K5						
1	1.242e+4	45.925	2.126e+4	151.860	1.485e+4	48.057
2	1.293e+4	41.919	1.774e+4	143.721	1.459e+4	51.369
3	1.228e+4	43.803	2.196e+4	142.075	1.293e+4	50.920
4	1.098e+4	39.982	1.982e+4	144.462	1.464e+4	53.464
5	1.324e+4	45.821	2.196e+4	129.367	1.484e+4	50.125
6	1.337e+4	44.124	1.874e+4	133.741	1.377e+4	53.412
7	1.381e+4	45.187	2.363e+4	142.188	1.451e+4	49.597
8	1.176e+4	42.117	1.982e+4	144.462	1.485e+4	52.123
9	1.327e+4	44.319	1.864e+4	129.367	1.459e+4	51.981
10	1.235e+4	40.447	1.910e+4	143.721	1.311e+4	49.970
Avg	1.244e+4	43.364	2.026e+4	140.496	1.426e+4	51.102
Best	1.092e+4	39.982	1.774e+4	129.367	1.293e+4	48.057
C70−K8						
1	2.205e+4	128.019	3.466e+4	471.224	3.237e+4	133.704
2	2.541e+4	135.735	3.045e+4	519.178	3.126e+4	124.116
3	2.119e+4	135.803	3.901e+4	500.973	3.277e+4	145.837
4	2.551e+4	144.946	3.333e+4	566.329	3.731e+4	156.188
5	2.802e+4	115.624	4.407e+4	526.963	3.377e+4	135.111
6	2.661e+4	145.914	3.812e+4	557.156	3.728e+4	135.788
7	2.418e+4	144.254	3.045e+4	533.264	2.987e+4	151.568
8	2.728e+4	152.997	3.832e+4	526.884	2.811e+4	143.413
9	2.158e+4	123.751	4.236e+4	499.421	2.694e+4	151.523
10	2.677e+4	147.972	4.886e+4	511.774	3.082e+4	143.816
Avg	2.486e+4	137.506	3.796e+4	521.316	3.082e+4	142.106
Best	2.119e+4	115.624	3.045e+4	471.224	2.694e+4	124.116

solution during 10 independent runs. However, 3 algorithms perform differently in terms of stability and efficiency. TS can find the true optimal solution every time it runs. SAA finds the true optimal solution 8 times. GA only finds the optimal solution once. In addition, we find that the average running time of GA is much longer than SAA and TS. This indicates that the three neighborhood operators adopted in this paper are more efficient. As shown in Figs. 5, Figs. 6 and 7, SAA is almost always the first to find the optimal solution during the iterative process. However, in the case of the same number of iterations, the average running time of SAA is 6.318 s longer than the counterpart of TS. For small-sized instance C8−K3, the number of neighborhood structures *NA* is only 190. The tabu list used by TS can largely prevent duplicate searches (Glover et al., 1997).

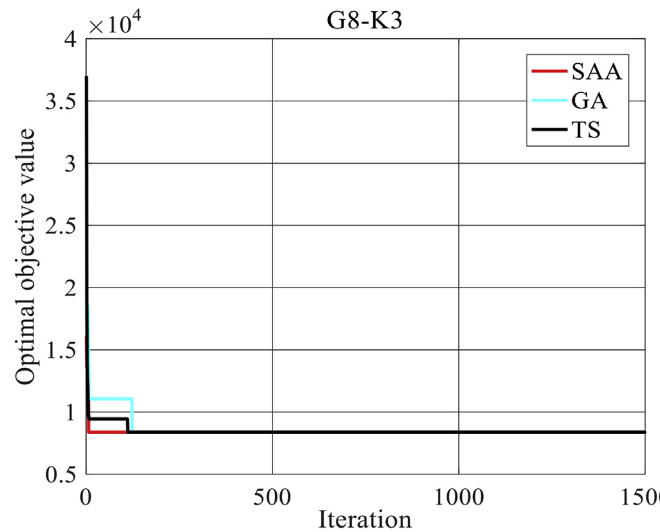

Fig. 5. Algorithm convergence curves for solving C8−K3.

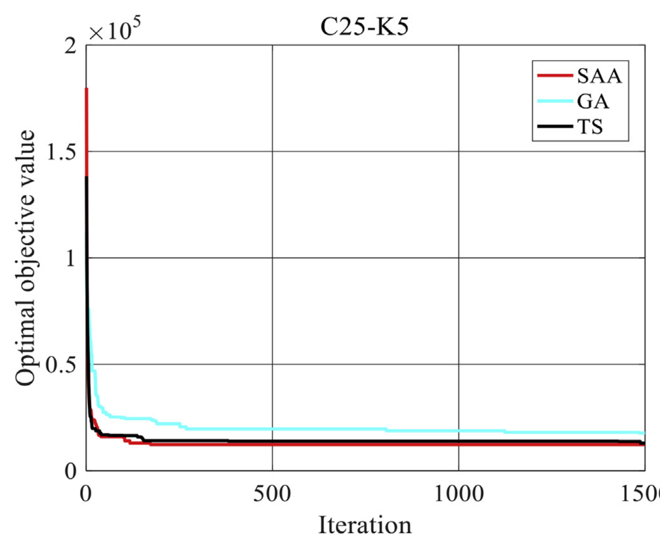

Fig. 6. Algorithm convergence curves for solving C25−K5.

For medium-sized instance C25−K5, the exact algorithm has been unable to solve the model in an acceptable time. Experimental

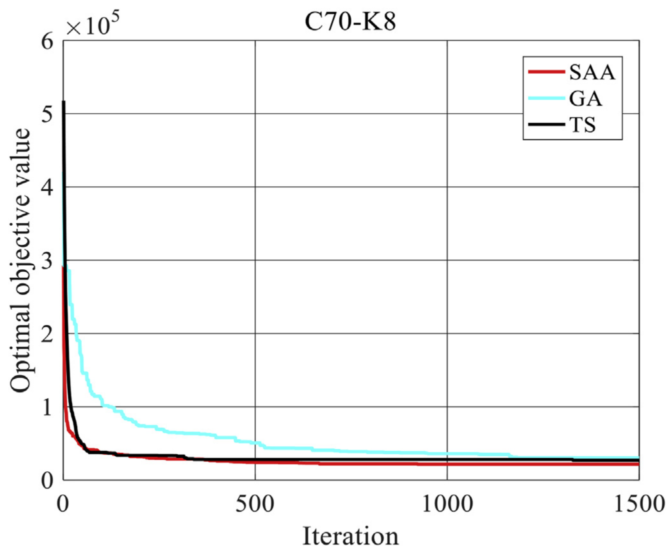

Fig. 7. Algorithm convergence curves for solving C70−K8.

results indicate that SAA performs better than the others. For TS, the number of neighborhood structures increases with the problem size. When the number of neighborhood structures is too large, only a small part of the neighborhood structures are used for each iteration. Thus, the advantage of tabu list is not as obvious as they are in small-sized instance. For large-sized instance C70−K8, the advantage of SAA becomes more obvious. Accepting inferior solutions with controllable probability to escape local minima is more suitable for global optimization of large-scale instances. In conclusion, among the 3 algorithms, SAA performs best overall.

4.3. Sensitivity analysis

A sensitivity analysis is presented to demonstrate the applicability of the proposed method. SAA is adopted as the solution approach for the sensitivity analysis, as it performs best among the three algorithms. Obviously, the objective function varies with the predefined credibility levels α^U, α^L, and the objective values of the same solution are different with different credibility levels α.

For small-sized instance C8−K3, experimental results indicate that the optimal solutions obtained under different predefined credibility levels are the same. As shown in Fig. 8, we find that for $\alpha \in [0, 0.1]$, the transportation risk of this optimal solution decreases as α increases. However, when $\alpha \in [0.1, 0.5]$, the transportation risk increases as α increases. For $\alpha \in [0.5, 1]$, there is a linear relationship between the transportation risk and the credibility level α. As mentioned above $\forall i, j \in N$, $\exists\ \omega_{ij}^U, \omega_{ij}^L \in [0, 1]$, thus when α^U, $\alpha^L > 0.5$, H_{ij}^U and H_{ij}^L are fixed, i.e. $H_{ij}^U = \frac{(2(\omega_{ij}^U - 2\alpha^U)Pd_{ij,3}^U + (2\alpha^U - \omega_{ij}^U)Pd_{ij,4}^U)}{\omega_{ij}^U}$, $H_{ij}^L = \frac{(2(\omega_{ij}^L - 2\alpha^L)Pd_{ij,3}^L + (2\alpha^L - \omega_{ij}^L)Pd_{ij,4}^L)}{\omega_{ij}^L}$.

For medium-sized instance C25−K5, experimental results indicate that the optimal solutions obtained under different predefined credibility levels are different. Similarly, to ensure the reliability of experimental results, 10 independent runs are performed for each predefined credibility level.

As shown in Fig. 9, we adopt the predefined credibility levels, $\alpha^U, \alpha^L = 0.95,\ 0.75,\ 0.5$ to obtain the optimal solutions, and the change of predefined credibility levels has little effect on the optimal solutions. The objective values with different credibility level α are similar to Fig. 8. For $\alpha^U = \alpha^L = 0.45,\ 0.35,\ 0.25,\ 0.2$ shown in Fig. 9, the objective values with other credibility levels $\alpha \neq \alpha^U, \alpha^L$ increase significantly. For $\alpha^U = \alpha^L = 0.1,\ 0.05,\ 0.01$ shown in Fig. 9, the change of the objective values with small credibility levels α becomes insignificant.

The change of average transportation risk with different credibility levels is shown in Fig. 10. Experimental results for large-sized instance C70−K8 are similar to the medium-sized instance C25−K5. In practical application, to minimize the impact of fuzzy parameters on the model, the predefined credibility levels should be set between 0.5 and 1, $\alpha^U, \alpha^L \in [0.5, 1]$. In Table 4 and Fig. 11, the best routing decisions for 3 problem instances are presented in detail ($\alpha^U = \alpha^L = 0.95$).

5. Conclusion

This work considers a CVRP for HazMat transportation. The

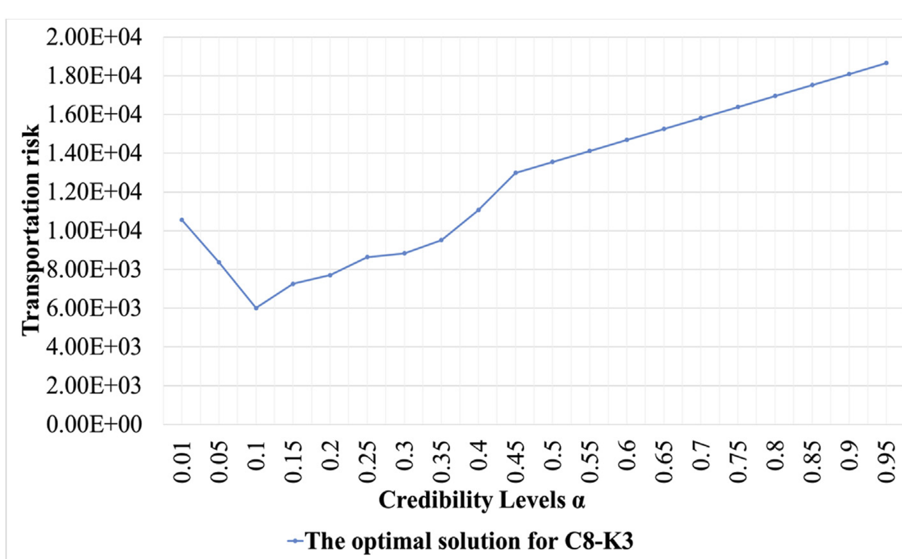

Fig. 8. The change of transportation risk for different credibility levels α (C8−K3).

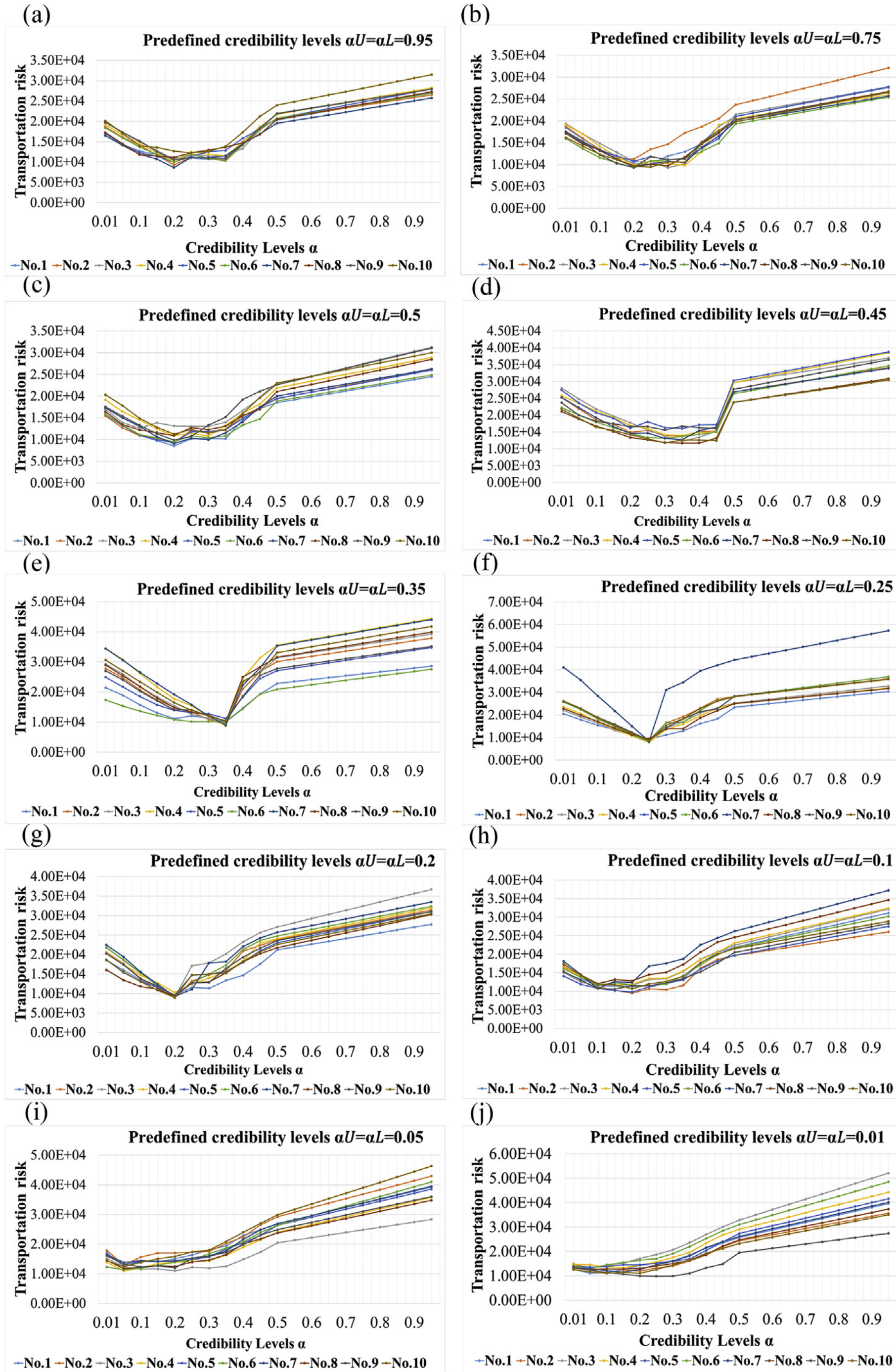

Fig. 9. The change of transportation risk under different credibility levels (C25−K5).

objective is to determine a set of routes with minimum transportation risk. The novelty of this work mainly has two aspects. On the one hand, this work proposes a H-CVRP model with the objective function involving IT2-FVs. Due to the mobility of the population, the population density parameter is defined as a trapezoidal IT2-FV. A CCP approach is adopted to deal with the IT2-FV. On the other hand, a SAA is designed to solve the deterministic model. We test the performance of SAA in problem instances of different sizes. Experimental results show that the proposed SAA is competitive in terms of stability and efficiency. A sensitivity analysis is presented to demonstrate the applicability of the proposed method. Even though HazMat VRP was the main motivation of our work, the proposed method is not limited to this, as many decision-making problems have uncertain attributes.

For future work, we try to consider both uncertain and multi-objective attributes of the HazMat transportation problems. Since economic factors are also very important in practical applications, the transportation risk and cost should be minimized simultaneously.

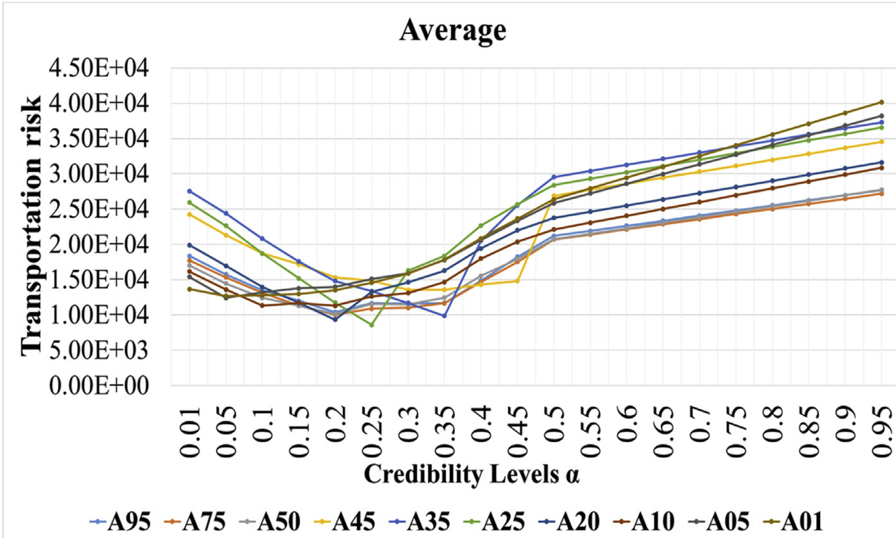

Fig. 10. The change of average transportation risk with different credibility levels (C25–K5).

Table 4
The best solutions for 3 problem instances ($\alpha^U = \alpha^L = 0.95$).

Problem instances	C8–K3	C25–K5	C70–K8
k_1	[0,7,8,3,2,0]	[0,4,6,19,15,24,10,0]	[0,7,26,52,46,63,59,39,49,36,0]
k_2	[0,4,6,0]	[0,25,20,13,18,1,0]	[0,20,18,38,2,40,30,41,3,35,33,0]
k_3	[0,1,5,0]	[0,3,2,17,0]	[0,47,28,9,54,56,51,24,12,44,0]
k_4	~	[0,7,23,11,5,8,0]	[0,11,16,43,14,6,10,53,23,0]
k_5	~	[0,22,9,14,21,16,12,0]	[0,60,21,8,64,69,19,27,68,31,0]
k_6	~	~	[0,25,17,32,50,13,29,48,67,0]
k_7	~	~	[0,70,4,42,65,55,15,66,37,34,0]
k_8	~	~	[0,61,5,45,62,57,1,58,22,0]
Z^*	1.866e+4	2.573e+4	4.982e+4

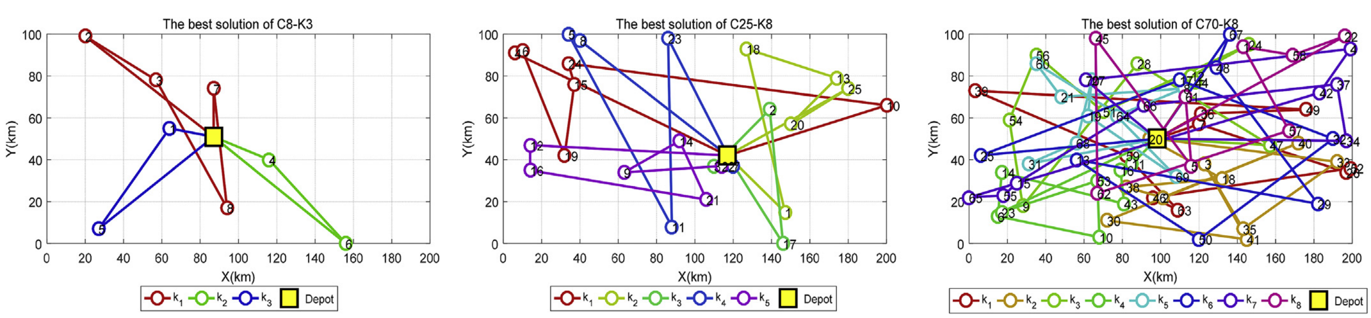

Fig. 11. Routing graphs for 3 problem instances ($\alpha^U = \alpha^L = 0.95$).

Acknowledgements

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Appendix. A Type-1 fuzzy set

Definition A.1. A T1-FS A can be defined as follows:

Ordered Pairs:

$$A = \{(x, \mu_A(x)) : \forall x \in U\} \tag{A.1}$$

Zadeh:

$$A = \begin{cases} \sum_{x \in U} \mu_A(x)/x, U \text{ is finite set} \\ \int_{x \in U} \mu_A(x)/x, U \text{ is infinite set} \end{cases} \quad (A.2)$$

vector:

$$A = \{\mu_A(x) : \forall x \in U\} \quad (A.3)$$

where $\mu_A(x)$ is the *type-1 membership function* (MF); U is the *universe*. It is worth mentioning that $\int and \sum$ represent an union over all admissible x.

Definition A.2. (Nahmias, 1978) Assume that Θ is a *nonempty set*, $\mathscr{P}$ is the *power set* of Θ, *Pos* is a *possibility measure* and $\Re$ is a real number set representing fuzzy cases. Let $(\Theta, \mathscr{P}, Pos)$ be the *possibility space*, a function $\xi : (\Theta, \mathscr{P}, Pos) \to \Re$ is called a *Type-1 fuzzy variable* (T1-FV).

$$Cr\{\xi \in B\} = \frac{1}{2}\left(sup_{x\in\Re}\, \mu_\xi(x) + sup_{x\in B}\, \mu_\xi(x) - sup_{x\in B^c}\, \mu_\xi(x)\right) \quad (A.7)$$

Theorem A.1. (Kundu et al., 2019) *For* $\xi = (a, b, c, d, w)$, $a < b \le c < d, 0 < w \le 1$, $Cr\{\xi \le x\}$ *is given as follows*:

$$Cr\{\xi \le x\} = \frac{1}{2}\left(w + sup_{r \le x}\, \mu_\xi(r) - sup_{r > x}\, \mu_\xi(r)\right) \quad (A.8)$$

where if $x \le a$, then $Cr\{\xi \le x\} = \frac{1}{2}(w + 0 - w) = 0$; if $a \le x \le b$, then $Cr\{\xi \le x\} = \frac{1}{2}\left(w + \frac{w(x-a)}{b-a} - w\right) = \frac{w(x-a)}{2(b-a)}$; if $b \le x \le c$, then $Cr\{\xi \le x\} = \frac{1}{2}(w + w - w) = \frac{w}{2}$; if $c \le x \le d$, then $Cr\{\xi \le x\} = \frac{1}{2}\left(w + w - \frac{w(d-x)}{d-c}\right) = \frac{w(x+d-2c)}{2(d-c)}$; if $x > d$, then $Cr\{\xi \le x\} = \frac{1}{2}(w + w - 0) = w$.

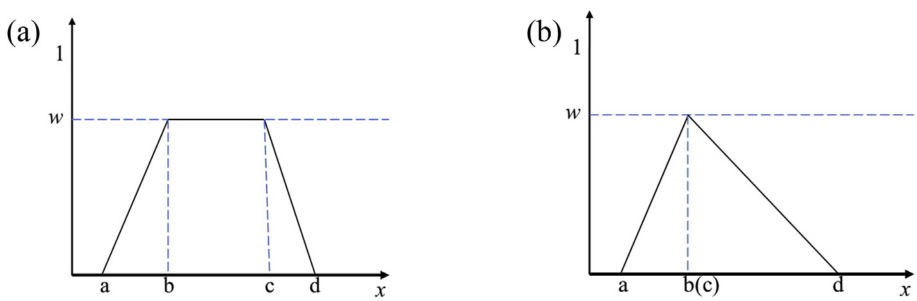

Fig. A.1. . A trapezoidal fuzzy variable ξ

Example A.1. A trapezoidal T1-FV ξ determined by $(a, b, c, d, w), a < b \le c < d, 0 < w \le 1$ is shown in Fig. A. 1(a), its MF is given as follows:

$$\mu_\xi(x) = \begin{cases} \frac{w(x-a)}{b-a}, & if\ a \le x \le b \\ w, & if\ b \le x \le c \\ \frac{w(d-x)}{d-c}, & if\ c \le x \le d \\ 0, & otherwise \end{cases} \quad (A.4)$$

where w is the height of ξ. If ξ is normalized, then $w = 1$, i.e. $sup_{x\in\Re}\, \mu_\xi(x) = 1$. When b = c, ξ is a triangular T1-FV (Fig. A. 1(b)).

Definition A.3. (X, 2013) For a fuzzy event $\{\xi \in B\}, B \subset \Re$, its *possibility measure* $Pos\{\xi \in B\}$ and *necessity measure* $Nec\{\xi \in B\}$ are defined as follows:

$$Pos\{\xi \in B\} = sup_{x\in B}\, \mu_\xi(x) \quad (A.5)$$

$$Nec\{\xi \in B\} = 1 - Pos\{\xi \in B^c\} = 1 - sup_{x\in B^c}\, \mu_\xi(x) \quad (A.6)$$

Definition A.4. (X, 2013) For a fuzzy event $\{\xi \in B\}, B \subset \Re$, its *credibility measure* $Cr\{\xi \in B\}$ is defined as follows:

To sum up,

$$Cr\{\xi \le x\} = \begin{cases} 0, & if\ x \le a \\ \frac{w(x-a)}{2(b-a)}, & if\ a \le x \le b \\ \frac{w}{2}, & if\ b \le x \le c \\ \frac{w(x+d-2c)}{2(d-c)}, & if\ c \le x \le d \\ w, & if\ x > d \end{cases} \quad (A.9)$$

Theorem A.2. (Kundu et al., 2019) *Given a trapezoidal T1-FV* $\xi = (a, b, c, d, w), a < b \le c < d, 0 < w \le 1$ *and a predefined credibility level* $0 < \alpha \le 1$, *we have*:

$$Cr\{\xi \le x\} \ge \alpha \Rightarrow \begin{cases} \frac{((w-2\alpha)a + 2\alpha b)}{w} \le x, & if\ \alpha \le \frac{w}{2} \\ \frac{(2(w-\alpha)c + (2\alpha - w)d)}{w} \le x, & if\ \alpha > \frac{w}{2} \end{cases} \quad (A.10)$$

Corollary A.1. (Kundu et al., 2019) *Given a trapezoidal fuzzy variable* $\xi = (a, b, c, d, w), a < b \le c < d, 0 < w \le 1$ *and a predefined* *credibility level* $0 < \alpha \leq 1$, *we have*:

$$Cr\{\xi \geq x\} \geq \alpha \Rightarrow \begin{cases} \dfrac{((w-2\alpha)d + 2\alpha c)}{w} \geq x \quad , \text{ if } \alpha \leq \dfrac{w}{2} \\ \dfrac{(2(w-\alpha)b + (2\alpha - w)a)}{w} \geq x, \text{ if } \alpha > \dfrac{w}{2} \end{cases} \tag{A.11}$$

Appendix. B Type-2 fuzzy set

Definition B.1. (Mendel and John, 2002) A *Type-2 fuzzy set* (T2-FS) can be regarded as a special T1-FS of which the MF is also a *fuzzy set*. A T2-FS $\tilde{A}$ can be defined as follows:

Ordered Pairs:

$$\tilde{A} = \{((x,\varphi), \mu_{\tilde{A}}(x,\varphi)) : \forall x \in U, \forall \varphi \in J_x \subset [0,1]\} \tag{B.1}$$

Zadeh:

$$\tilde{A} = \begin{cases} \sum_{x \in U} \sum_{\varphi \in J_x} \mu_{\tilde{A}}(x,\varphi)/(x,\varphi), U \text{ is finite set} \\ \iint_{x \in U, \varphi \in J_x} \mu_{\tilde{A}}(x,\varphi)/(x,\varphi) \quad , U \text{ is infinite set} \end{cases} \tag{B.2}$$

vector:

$$\tilde{A} = \{\mu_{\tilde{A}}(x,\varphi) : \forall x \in U, \forall \varphi \in J_x \subset [0,1]\} \tag{B.3}$$

where J_x is the *primary membership* of $x \in U$; $\mu_{\tilde{A}}(x,\varphi) \in [0,1]$ is the *MF* of T2-FS.

Definition B.2. (MJ.RI and L. F, 2006) For a $x \in U$, its secondary membership function $\tilde{\mu}_{\tilde{A}}(x)$ is given as follows:

$$\tilde{\mu}_{\tilde{A}}(x) = \int_{\varphi \in J_x} \mu_{\tilde{A}}(x,\varphi)/\varphi \tag{B.4}$$

where J_x is the domain of $\tilde{\mu}_{\tilde{A}}(x)$.

Definition B.3. (Kundu et al., 2019) If $\forall (x,\varphi), \exists\ \mu_{\tilde{A}}(x,\varphi) = 1$, then $\tilde{A}$ is an *interval type-2 fuzzy set* (IT2-FS). An IT2-FS can be descripted as the *uncertain footprint* (UF) as follows:

$$\mathrm{UF}(\tilde{A}) = \bigcup_{x \in U} J_x \tag{B.5}$$

where the UF can be regarded as a region bounded by two boundary MFs (denoted as $\overline{\mu_{\tilde{A}}}(x)$, $\underline{\mu_{\tilde{A}}}(x)$). Usually, an IT2-FS $\tilde{A}$ is written as $\tilde{A} = \begin{pmatrix} \tilde{A}^U \\ \tilde{A}^L \end{pmatrix}$ where $\tilde{A}^U$ and $\tilde{A}^L$ are T1-FSs defined on $\overline{\mu_{\tilde{A}}}(x)$ and $\underline{\mu_{\tilde{A}}}(x)$ respectively.

Definition B.4. (Liu and Liu, 2010) Suppose that $(\Theta, \mathscr{P}, \widetilde{Pos})$ is the *fuzzy possibility space*. A function $\tilde{\xi} : (\Theta, \mathscr{P}, \widetilde{Pos}) \rightarrow \Re$ is called a *Type-2 fuzzy variable* (T2-FV), which is defined as a map from Θ to $\Re$. $\forall\ t \in \Re$, $\{\gamma \in \Theta | \tilde{\xi}(\gamma) \leq t\} \in \mathscr{P}$.

Definition B.5. (Liu and Liu, 2010) Let $\tilde{\xi}$ be a T2-FV. Its *secondary possibility distribution function* is a map $\Re \rightarrow [0,1]$, and $\tilde{\mu}_{\tilde{\xi}}(x) = \widetilde{Pos}\{\gamma \in \Theta | \tilde{\xi}(\gamma) = x\}, x \in \Re$. Its *type-2 possibility distribution function* is a map $\Re \times J_x \rightarrow [0,1]$, and $\mu_{\tilde{\xi}}(x,\varphi) = Pos\{\tilde{\mu}_{\tilde{\xi}}(x) = \varphi\}, (x,\varphi) \in \Re \times J_x$.

Definition B.6. If $\forall (x,\ \varphi) \in \Re \times J_x, \exists \mu_{\tilde{\xi}}(x,\varphi) = 1$, then $\tilde{\xi}$ is an *interval type-2 fuzzy variable* (IT2-FV).

Example B.7. A trapezoidal IT2-FV $\tilde{\xi}$ determined by $\begin{pmatrix} a^U, b^U, c^U, d^U, w^U \\ a^L, b^L, c^L, d^L, w^L \end{pmatrix}, 0 < w^U, w^L \leq 1$ is shown in Fig. B. 1(a). When $b^U = c^U \&\& b^L = c^L$, $\tilde{\xi}$ is a triangular IT2-FV (Fig. B. 1(b)).

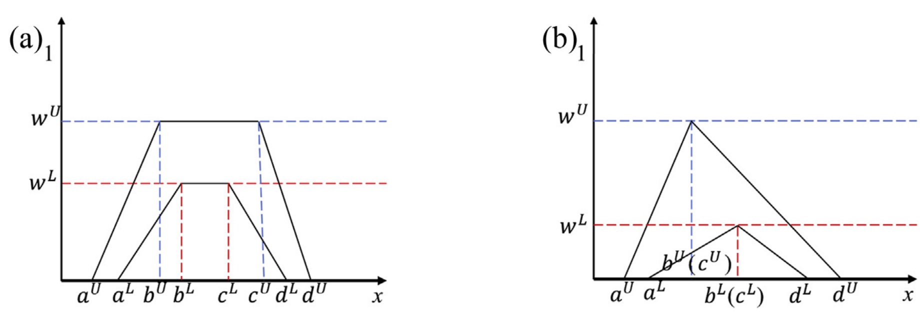

Fig. B 1. . A trapezoidal IT2-FA $\tilde{\xi}$

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