

# ARIMA – Fixed-Income and Credit Derivatives

## Assignment on Rating Momentum

Hermann Elendner

Winter Term 2023

Assume there are 2 “official” non-default ratings  $A$  and  $B$ , and a default rating  $D$ . In the model specification we add a state  $B^*$  which firms enter into when downgraded into the  $B$  rating. This “excited” state is suspected to have a higher default intensity and a lower upgrade intensity than the “normal” state. Firms may, however, normalize after a certain period, i.e. move to state  $B$ .

Consider a sample of 100 firms initially rated  $A$  and 100 firms initially rated  $B$ . Assume we follow the rating histories of these firms for 2 years. The evolution of those 25 firms from this sample which migrated during the observation period is given in the table below. (Note that time is given in months.) The rest of the firms did not migrate during the observation period.

1. By using the cohort estimator, compute the (1-year) transition probability matrices both in the (Markov) case with a richer state space and in the (non-Markov) case with a poorer state space.
2. By using the direct ML estimator, compute the (1-year) generator matrices both in the (Markov) case with a richer state space and in the (non-Markov) case with a poorer state space.
3. For both state spaces, explain the difference in the 1-year default probabilities between the discrete-time and the continuous-time estimator.
4. What happens to the 1-year default probability from the “official” states  $A$  and  $B$  when there is an additional state? Do the results correspond to the interpretation of  $B^*$  as an “excited” state?

Firm	Initial state	Time in state	New state	Time in state	New state	Time in state
#1	A	1	B*	3	B	20
#2	A	2	B*	14	B	8
#3	A	4	B*	8	B	12
#4	A	9	B*	4	D	11
#5	A	11	B*	13	—	—
#6	A	14	B*	6	A	4
#7	A	14	B*	9	D	1
#8	A	19	B*	3	B	2
#9	A	21	B*	3	—	—
#10	A	23	B*	1	—	—
#11	A	24	—	—	—	—
⋮	⋮	⋮				
#100	A	24	—	—	—	—
#101	B	1	A	23	—	—
#102	B	3	A	18	B*	3
#103	B	7	A	17	—	—
#104	B	12	A	9	B*	3
#105	B	15	A	5	B*	4
#106	B	18	A	6	—	—
#107	B	18	A	4	B*	2
#108	B	20	A	4	—	—
#109	B	21	A	3	—	—
#110	B	23	A	1	—	—
#111	B	2	D	22	—	—
#112	B	8	D	16	—	—
#113	B	18	D	6	—	—
#114	B	19	D	5	—	—
#115	B	22	D	2	—	—
#116	B	24	—	—	—	—
⋮	⋮	⋮				
#200	B	24	—	—	—	—

*Good Luck!*