ARIMA – Fixed-Income and Credit Derivatives
Assignment on Asset Volatility in the Merton model

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In the Merton (1974) model, the underlying is the asset-value process V_t which is unobservable to the researcher in practice. (Remember the no-arbitrage argument does rely on it being a tradable asset.) Consequently, neither can its initial value V_0 nor its volatility σ_V be observed. However, those are necessary to employ the model!

Different approaches of estimating them exist. Your task is to implement the Vassalou and Xing (2004) approach, an iterative procedure based on the following insights:

- The Black–Scholes formula links the value of the underlying with the value of the derivative (given all other variables). This equality holds both ways: given observations of the time series of the stock price, the time series the asset values is exactly determined.
- If the asset values are determined based on an incorrect volatility parameter, their volatility will be incorrect, too, yet closer to the true volatility. Therefore, iterating both steps usually converges quickly.

More specifically, you should:

1. Download adjusted closing prices for the three companies (tickers) Tesla, Inc. (`TSLA`), Siemens AG (`SIE.DE`) and OMV AG (`OMV.VI`) for each available day in the ten years 2014 to 2023.

   In `GNU/R` you can use the `quantmod` package:

   ```
   library(quantmod)
   getSymbols("TSLA")
   ```

2. Use the Merton model to calculate implied asset values. Make intelligent assumptions and document them clearly.
3. Calculate the volatility of your backed-out asset-value process. Is it consistent with your assumptions? If not, iterate until it is. What asset values and asset volatilities do you find for each firm at the end of 2023?
4. Compare asset volatilities to stock-price volatilities and comment briefly.
5. For each firm, if you estimate asset volatility separately for the first four years and the last five years of your sample, what do you conclude?
6. Are your asset value processes consistent with the assumption that they follow geometric Brownian motions (GBM)? Do stock prices? Can you judge if either does more so?

References

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance*, 29(2):449–470.

Vassalou, M. and Xing, Y. (2004). Default risk in equity returns. *Journal of Finance*, 59(2):831–868.

Good Luck!