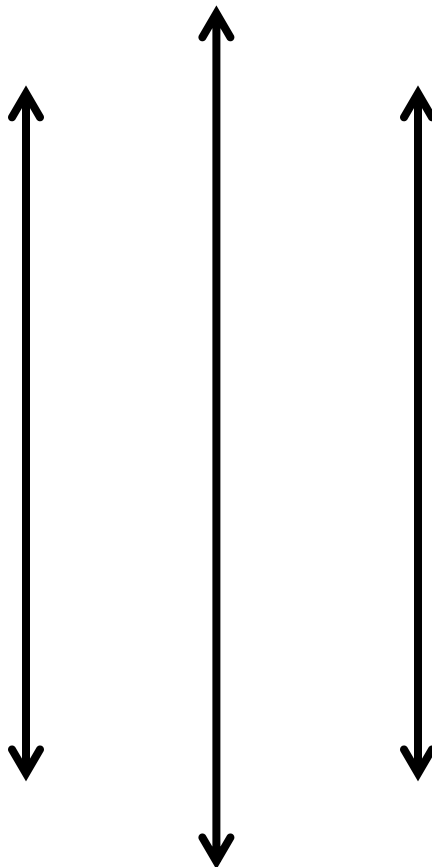




NOTES
ON
ECONOMICS ENGINEERING



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CHAPTER ONE

INTRODUCTION

Economic engineering is a specialized field, incorporating a knowledge of **engineering** and basic **micro-economics**. Its main function is to facilitate decision-making based on the **economic** comparison of different technological alternatives for investment.

Engineering economics deals with the methods that enable one to take economic decisions towards minimizing costs and/or maximizing benefits to business organizations.

According to A.M. Wellington “ Engineering economic is the art of doing well with one dollar which any bungler can do with two after a fashion”

“Economics” is the study of **“how societies and individuals use limited resources to satisfy “unlimited” human wants???”**. Resources are limited because they are scarce. Resources are scarce because there are more possible uses for the resources than there are resources available to be employed in those uses. When scarce resources confront "unlimited" wants, choices must be made. We can restate our definition of economics as follows: Economics is the study of how societies and individuals choose among alternative uses for scarce resources, in the never-ending effort to satisfy "unlimited" wants.

Objectives of economic engineering

1. Generating a high level of employment
2. Stabilising the price levels
3. Economic efficiency
4. Equitable distribution of income
5. Economic growth.

Principals economic engineering

1. **Develop the alternative** :Remember! Quality derives from quantity.
2. **Focus of the differences**: Only differences, don't get biased. Be very neutral and impartial.
3. **Use a consistent viewpoint**: What is your position? Be consistent.
4. **Use a common unit of measure**: Uniformity in measurement, analysis and result. Helps interpret easily.
5. **Consider all relevant criteria**: All good looking results may not, sometimes, be good! Try to underpin your decisions from all quarters.
6. **Make uncertainty explicit** Don't fire blindly.
7. **Revisit your decision**: After all we are mankind and remember and we, yes, we, may commit mistakes.

Decision making

The act of deciding on matters of the economy.

Facts of decision making

- I. The process involves parallel activities, feedback loops and repeated steps
- II. It is an iterative process and repeats as results are refined
- III. Communications skills are extremely important!
- IV. Each decision is one of many to be made
- V. Politics are very important!

Steps of decision making

1. Define/formulate problem (opportunity), boundary
2. Choose objective(s)/set goals
3. Identify alternatives
4. Evaluate consequences
5. Apply criteria for selection
6. Select preferred course of action
7. Specify and implement solution, adjust as required
8. Audit/monitor results, revise as necessary



1.1 Business and accounting terminology

Value

Value is the worth a person ascribes to a good service. The value is inherent in what someone wants to pay for

It and differs from the cost . The cost of production, the price asked and the price paid are connected but they are not the same. The value changes from person to person.

Utility

Utility is a measure of the power of a good or service to satisfy human wants. As in value it is the person who sets the utility and the satisfaction derived is the utility. Value is utility to a person in terms of money.

Economic Life

The period of time (years) that results in the minimum Equivalent Uniform Cost (EUAC) of owning and operating an asset is called economic life .It is sometimes called minimum-cost life or optimum replacement interval. For a new asset, economic life can be computed if capital investment, annual expenses, and year-by-year market values are known or can be estimated .

Ownership Life

Period between the date of acquisition and date of disposal by a specific owner is called ownership life. A given asset may have different categories of use during this period, and accordingly the ownership life.

Physical Life

Period of time between original acquisition and final disposal of an asset over its succession of owners is called physical life.

Useful Life

The time period in years that an asset is kept in productive services either in primary or backup mode is called useful life. It is an estimate of how long an asset is expected to be used in a trade or business to produce income.

Capital

Capital refers to the wealth in the form of money or property that can be used to produce more wealth from investment opportunities. Based on the source from which it is derived in the business, capital is classified as borrowed capital (it is obtained from the lenders) and equity capital (it is owned by individuals who have invested their money or property in a business in the hope of receiving a profit.

Salvage Value(s)

It is receipt at project termination for disposal of the equipment and can also be a salvage cost in some cases.

Market value

The value of an asset when sold in an arms-length free market environment is called market value. But, this can only be accurate if we actually sell it. Therefore, this is usually an estimate, even based on a formal appraisal by experts and found to be interchangeably used with salvage value.

Depreciation

Depreciation involves a procedure where the cost of an asset less its estimated salvage value is distributed over the asset's estimated useful life in a systematic and rational manner.

Inflation

The decrease of money's purchasing power or an increase in the amount of money necessary to obtain the same amount of goods or services before the inflated price was present.

Deflation

It is the opposite of inflation where the currency buys more than before (or prices come down is another version of this) i.e. the money gains its purchasing power



Opportunity Costs

It is the contribution to income that is foregone by not using a limited resource in its best use. In other words, it is defined as the cash flows that could be generated from an asset the firm already owns, provided they are not used for the alternative in question.

Sunk Costs

The cost that has already been incurred by past actions is called sunk costs. A sunk cost is irrelevant to decision making process because they cannot be changed regardless of what decision is made now or in the future.

Marginal Costs

It is the added cost that would result from increasing the rate of output by a single unit.

1.2 Cash flow

A **cash flow** describes a real or virtual movement of money. Cash flows are narrowly interconnected with the concepts of **value**, interest rate and **liquidity**. Cash flow are **economically equivalent** if they have the same economic value when evaluated at the same point in time. From an economic point view equivalent cash flow have equal value and hence can be subjected or traded for one another. For e.g. Rs. 50000 is received today is economically equivalent to Rs. 73,205 received four years from now at an interest rate of 10% per year.

a. Single cash flow:

This type of cash flow involves the financial transaction only once in the cash flow diagram. The cash flow may be in initial point or at the end or somewhere in between them

At the end of 1st year = $P + Pi = P(1+i)$

At the end of 2nd year = $P(1+i) + P(1+i)i = P(1+i)^2$

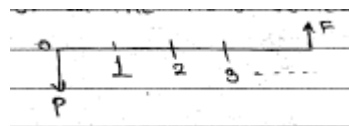
At the end of 3rd year = $P(1+i)^2 + P(1+i)^2i = P(1+i)^3$

.

At the end of Nth year = $P(1+i)^2 + P(1+i)^2i = P(1+i)^N$

$\therefore F = P(1+i)^N$

The bracketed term $P(1+i)^N$ is referred to as a **single payment compound amount factor** & is written as $(F/P, i, N)$ and read as find F when P, i, N is given.



b. Equal payment uniform series

This type of cash flow involves the equal payment at the end of each interest period. Commercial installment, rental payment, insurance payment, plans are example at equal payment series

The future amount, F, of the equal payments, A, at the end of each year at an interest rate of i for a period of N is calculated as follows:

The future sum of this deposit is given by

$$F = F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + \dots + F_N$$

$$F_1 = A(1+i)^{N-1}$$

$$F_2 = A(1+i)^{N-2}$$

$$F_3 = A(1+i)^{N-3}$$

.

.

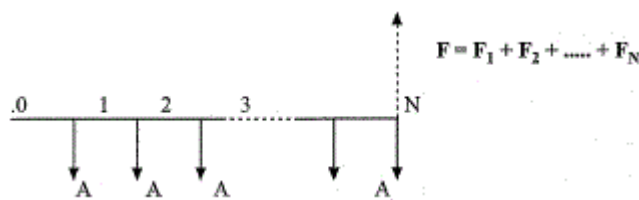
$$F_N = A(1+i)^{N-N} = A$$

Now,

$$F = A(1+i)^{N-1} + A(1+i)^{N-2} + A(1+i)^{N-3} + \dots + A$$

It can be written as

$$F = A + A(i+1) + A(1+i)^2 + \dots + A(1+i)^{N-3} + A(1+i)^{N-2} + A(1+i)^{N-1} \dots (i)$$





Multiplying both side by $(1+i)$ we get

$$F(1+i) = A(1+i) + A(1+i)^2 + A(1+i)^3 + \dots + A(1+i)^{N-2} + A(1+i)^{N-1} + A(1+i)^N \dots \dots (ii)$$

Subtracting equation (i) from (ii)

$$F(1+i) - F = A(1+i)^N - A = A((1+i)^N - 1)$$

$$\therefore F = A \left[\frac{(1+i)^N - 1}{i} \right]$$

$$\therefore F = A \left(F/A, i\%, N \right)$$

In the above equation $(F/A, i\%, N)$ is called **uniform series compound amount factor**

From equation (b) we can write

Present worth factor–find P, given A, i% and N

We know

$$F = A \left[\frac{(1+i)^N - 1}{i} \right] \dots \dots (a)$$

$$F = P(1+i)^N \dots \dots (b)$$

From equation (a) & (b), we get

$$P(1+i)^N = A \left[\frac{(1+i)^N - 1}{i} \right]$$

$$\therefore P = A \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right]$$

$$\therefore P = A \left(P/A, i, N \right)$$

Where $(P/A, i, N)$ is called **uniform series present worth factor**

Sinking fund factor- Find A, given F, i% and N

A fixed sum deposited each interest period in an interest – bearing account for the purpose of replacing fixed assets is called **Sinking Fund**.

We know that

Then,

$$A = F \left[\frac{i}{(1+i)^N - 1} \right]$$

$$\therefore A = F \left(A/F, i\%, N \right)$$

where $(A/F, i\%, N)$ is called **uniform series sinking fund factor** or simply **sinking fund factor**

Capital recovery (Annuity) factor- Find A, given P, i% and N

We know that

$$P = A \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right]$$

which gives

$$A = P \left[\frac{i(1+i)^N}{(1+i)^N - 1} \right]$$

$$A = P \left(A/P, i\%, N \right)$$

Where $(A/P, i, N)$ is called **capital recovery factor**



1.3 Economic systems

Every economy has to perform three vital functions-production, consumption and growth or investment. The purpose of all economic activity IS the satisfaction of human wants through consumption of goods and services and the institutional framework within which a country carries out its economic activities is called economic system. The economic system may be of the following three types..

a. Free Enterprise or Capitalist Economy

In the capitalist economy, the private business enterprises will be the most Significant and has the following characteristics

- Private Property
- Absence of central planning
- Economic freedom
- Profit motive and individual initiative

Merits

- Individual initiative
- Perfect competition
- Dynamic economy

Demerits

- Inequality of incomes
- Inefficient production
- Monopoly and exploitation
- Unemployment

b. Socialist or Controlled Economy

In socialist economy, government enterprises will dominate the scenario and has the following characteristics:

- Socialist economy has a purpose
- Central planning
- Collective ownership
- Equality of incomes
- Existence of pricing process

Merits

- In the absence of private profit, production will be shifted from more profitable goods to more useful goods.
- Many things, the consumption of which is considered essential for health and efficiency, may be supplied free or below cost.
- Socialist economy will prevent cyclical fluctuations in business activity and will bring about smooth working of the economy.

Demerits

- Bureaucratic running of the system.
- It will lead to concentration of both political and economic power in the hands of the government.
- There is no proper basis of cost calculation and in the absence of such a basis, the economy cannot function in an efficient manner or allocate the resources in the best possible way.

c. Mixed Economy

Mixed Economy is the outcome of the compromise between two widely different schools of thought. Capitalism and socialism and the concept admits the possibility of the existence of private enterprise side by side with public enterprise. But private enterprise should reconcile the element of social interest and may not be allowed to figure prominently in every sector of the economy There may even be certain sectors, which may be regarded as of strategic and national importance, to which private enterprise may not be allowed at all.



Some solved numericals problems

Q.1. Suppose you invested Rs 15000 at 7% interest rate compounded annually. How much would you have after 8 years?

Solution:

Given,

P = Rs 15000

i = 7% per year, compounded annually

N = 8 years

F = ?

We know that,

$$F = P(F/P, i\%, N)$$

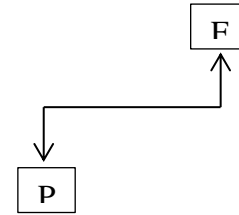
which gives

$$F = 15000(F/P, 7\%, 8)$$

$$F = 15000(1 + 7\%)^8$$

$$F = \text{Rs } 25772.79$$

$$\therefore F = P(1+i)^N$$



Q.2 How long does it take for an investment to triple itself if the interest rate is 9%, compounded annually?

Solution:

Let the investment, P will become 3P after N years.

Then, Investment, P = P

Future sum, F = 3P

We know that,

$$F = P(F/P, i\%, N)$$

$$\text{or, } 3P = P(F/P, 9\%, N)$$

$$\text{or, } \frac{3P}{P} = (1 + 0.09)^N$$

$$\text{or, } 1.09^N = 3$$

Taking log on both sides,

$$N \log 1.09 = \log 3$$

$$N = \frac{\log 3}{\log 1.09} = 12.74$$

$$\therefore N = 12.74 \text{ years}$$

Q.3. For an interest rate of 8% compounded annually, find how much can be loaned now if Rs 25000 will be repaid at the end of 5 years?

Solution:

Given,

P = ?

F = Rs 25000

i = 8% per year

N = 5 years

We know that,

$$P = F(P/F, i\%, N)$$

$$= F(1+i\%)^{-N}$$

$$= 25000(1+8\%)^{-5}$$

$$P = \text{Rs } 17014.57993$$

Q.4. A man wants to have Rs 200000 for the studies of his son after a period of 10 years. How much money does he have to deposit each year in a saving account that earns 8% every year?

Solution:

Given,

F = Rs 200000

N = 10 years

i = 8% per year

N = 10 years

A = ?

We know that,

$$A = F(A/F, i\%, N)$$



$$= F \left[\frac{i}{(1+i)^N - 1} \right] = 200000 \left[\frac{8\%}{(1+8\%)^{10} - 1} \right]$$

A = Rs 13805.897

Q.5. How much money should you deposit now in a saving account earning 10% compounded annually so that you may make eight end of year withdrawals of Rs 2000 each?

Solution:

Given,

$i = 10\%$ per year $N = 8$ years $A = \text{Rs } 2000$

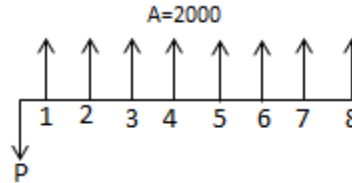
$P = ?$

We know that,

$P = A(P/A, i\%, N)$

$$P = A \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right] = 2000 \left[\frac{(1+10\%)^8 - 1}{i(1+10\%)^8} \right]$$

$\therefore P = \text{Rs } 10669.85$



Q.6. Part of the income that a machine generates is put into a sinking fund to replace the machine when it wears out. If Rs 2000 is deposited annually at 7% interest, how many years must the machine be kept before a new machine costing Rs 30000 can be purchased?

Solution:

Given,

$A = \text{Rs } 2000$ $i = 7\%$ per year $F = \text{Rs } 30000$

$N = ?$

We know that,

$$F = A \left[\frac{(1+i)^N - 1}{i} \right]$$

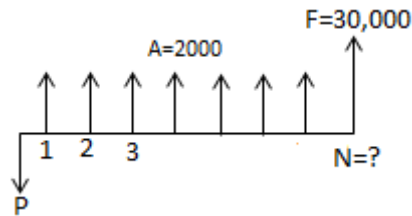
$$30000 = 2000 \left[\frac{(1+7\%)^N - 1}{7\%} \right]$$

$$1.07^N = 2.05$$

Taking log on both sides,

$$N \log(1.07) = \log(2.05)$$

$$\therefore N = \frac{\log(2.05)}{\log(1.07)} = 10.61 \text{ years}$$





CHAPTER TWO

COST CLASSIFICATION AND ANALYSIS

2.1 The elements of cost

Cost is a resource sacrificed or foregone to achieve a specific objective. It is usually measured as the monetary amount that must be paid to acquire goods and services. A cost must not be confused with an expense, that is the part of costs or goods or services that has been used up in the process of generating revenues.

$$\begin{aligned}\text{Total cost} &= \text{Direct cost} + \text{Indirect cost} \\ &= \text{Prime cost} + \text{Overhead cost}\end{aligned}$$

The elements of cost are as follow

- a. Material cost
- b. Labor cost
- c. Expenses

2.2 Classification of cost: overhead cost, prime cost

A. Manufacturing cost

Manufacturing costs include all costs incurred by a manufacturer in operating a factory while converting raw materials into completed products. Manufacturing costs can further be divided into

a. Prime Costs (or Direct Costs)

Prime costs are those costs related to a given cost object and that can be traced into it in an economically feasible way. Prime costs is the aggregate of following three elements.

i) Direct Material Cost

It is the cost associated with those materials and components that can be directly and conveniently traced to a unit of product. Cost of paper in book, wood in furniture, steel in bridge, windscreen-wipers or gearbox in a car etc. are some examples of direct material costs.

ii) Direct Labor Costs

The costs of production labor that can be directly and conveniently traced to a unit of product are called direct labor costs. Costs associated with chef in a restaurant, welders in metal fabrication, carpenters or masons in building works, machine operators in manufacturing operations are some examples of direct labor costs.

iii) Direct Expenses

The production expenses that can be directly and conveniently traced to a unit of product are called direct expenses. Special designs and drawings, hire of special tools and equipment for manufacturing job and their maintenance are some examples of direct expenses.

b. Overhead Costs (or Indirect Costs)

Overhead costs are related to the particular cost object that cannot be traced to specific units of product. Overhead costs is the aggregate of following three elements.

i) Indirect Material Costs

The costs of materials not directly traceable, and those extremely small in monetary value are called indirect material cost. Cost of glue used to bind the book, screws in a furniture factory, dishwasher detergent in a fast-food restaurant etc. are some examples of indirect material costs.

ii) Indirect Labor Costs

The labor costs that are not directly traceable, or those extremely small in monetary value are called indirect labor costs. Costs associated with storekeepers, night security guards, material handlers, supervisors etc are some example of indirect labor costs.

iii) Indirect Expenses

The expenses that are not directly traceable, or those extremely small in monetary value are called indirect expenses. Factory rent, depreciation of plant and equipment, insurance of the factory, factory heating and lighting etc. are some examples of indirect expenses.

B. Non-manufacturing Costs

Non-manufacturing costs include all costs associated with the activities carried out in support of any manufacturing operations.

A company may incur non-manufacturing costs for:

- Selling and Marketing
- Distribution



- Research and Development
- General and Administrative
- Finance

Opportunity cost: The value of benefits sacrificed in selecting a course of action among alternatives. The value of the next best opportunity foregone by deciding to do one thing rather than another. (create value of own)

Sunk cost: Cost already paid, that is not relevant to the decision concerning the future that is being made, capital already invested that for some reason cannot be retrieved

Lifecycle cost: It is applied to alternative with cost estimated over the entire system life span. Cost from very early stage to the finishing final stage e.g. New construction, manufacturing plants, commercial aircraft, new automobile models etc.

Marginal cost: Marginal cost are year by year estimates at the cost to own and operate an asset for that year. Marginal cost of product is the cost of producing an additional unit of the cost

2.3 Cost variance analysis

It is the calculation process by which differences are analyzed. It is also simply known as variance analysis.

Variance analysis provides a framework for business managers to breakdown the overall performance of an organization, so that each individual element of the business can be isolated and analyzed in turn.

Actual cost < standard cost => Favorable

Actual cost > standard cost => Adverse

Variance

Variance is defined as the difference between an actual amount of something and the amount it was supposed to be according to the budget. Variance is used to: assist managers in planning and control, evaluate performance, and suggest changes in strategies. So, Variance analysis is to measure performance, correct inefficiencies, and deal with the accountability function.

Variance Fall into 2 Categories

- Adverse variances that arise when the standard allowance is exceeded by actual expenditure. Adverse/ Unfavorable variances are debits and they increase production costs.
- Favorable variances are due to actual expenditure being less than the standard allowance. Favorable variances are credits and they reduce production costs.

It should be remembered that a favourable variance does not necessarily mean good, nor does an unfavorable variance mean bad. Management should analyze all variances to determine the cause

- determine if standard is correct
- consider costs vs. benefit in reviewing standards

A. Direct Material Cost Variances

Direct Material Cost Variance is the difference between the actual expenditure and budgeted expenditure of direct materials used in production process. It is the comparison of what we did actually pay or use compared to what we should have paid or used using the standard cost per unit. Accordingly, It is the aggregate of following:

i) Direct Material Price variance (DMPV)

The proportion of the budget variance that is caused by paying more/less than the standard price for the actual purchase

$$\text{DMPV} = \text{AQ}(\text{AR} - \text{SR})$$

Where,

AQ = Actual quantity consumed or purchased

AR = Actual rate of actual price per unit

SR = Standard rate or standard price per unit

ii) Direct material usage variance (DMUV)



The proportion of the budget variance that is due to using more/less material in production than the standard quantity

$$\text{DMUV} = \text{SR} (\text{AQ} - \text{SQ})$$

Where,

SR = Standard rate . .

AQ = Actual quantity consumed or purchased .

SQ = Standard quantity allowed or budgeted

B. Direct Labour Variances (or Direct wages variances)

Direct Labour budget Variance is the difference between the actual payroll and the standard labour cost of actual production. Direct labour budget variance is the aggregate of the following.

i) Direct Labour Wage Rate Variance (DLWRV)

The proportion of the budget variance that is caused by paying more/less than the standard wage for the actual labour hours used.

$$\text{DLWRV} = \text{AH}(\text{AR} - \text{SR})$$

Where,

AH = Actual hours worked

AR = Actual labour rate

SR = Standard labour rate

ii) Direct labour efficiency variance (DLEV)

The proportion of the budget variance that is due to using more/less labour hours in production than the standard hours.

$$\text{DLEV} = \text{AH} \times \text{SR} - \text{AP} \times \frac{\text{SH}}{\text{unit}} \times \text{SR}$$

Where,

AH = Actual hours worked

AR = Actual labour rate

SR = Standard labour rate

AP = Actual production units

SH = Standard labour hours allocated

C. Variable Overhead Variances

Variable overhead variance is the difference between the actual variable overhead and the standard variable overhead cost of for production. Variable overhead variance is the aggregate of the following: .

i) Variable Overhead Expenditure Variance (VOE_xV)

The proportion of the budget variance. that is due to using more/less labour hours in production than the standard overhead.

$$\text{VOE}_x\text{V} = \text{AH}(\text{AR} - \text{SR})$$

Where,

AH = Actual hours worked .

AR = Actual rate of variable overhead

SR = Standard rate of variable overhead

ii) Variable Overhead Efficiency Variance (VOE_fV)

$$\text{VOE}_f\text{V} = \text{AH} \times \text{SR} - \text{AP} \times \frac{\text{SH}}{\text{unit}} \times \text{SR}$$

Where,

SH = Standard hours worked

AP = Actual production units

D. Fixed Overhead Variances

Fixed overhead variance is the difference between actual fixed overhead incurred and standard cost of fixed overhead absorbed in the actual output. It is the aggregate of the following:

i) Fixed overhead expenditure variance (FOE_xV)

$$\text{FOE}_x\text{V} = \text{Actual fixed overhead} - \text{Budgeted fixed overhead}$$

ii) Fixed overhead capacity variance (FOCV)

$$\text{FOCV} = \text{Budgeted fixed overhead} - \text{AH} \times \text{SR}$$



Where,

AH = Actual hours worked

SR = Standard rate of fixed overhead

iii) Fixed overhead efficiency variance (FOEN)

$$FOEN = AH \times SR - AP \times \frac{SH}{\text{unit}} \times SR$$

Where,

SH = Standard hours worked

AP = Actual production units

2.4 Job and process costing (Costing method)

The process of determination of total cost of production and unit cost of production is known as product costing. There are two basic methods for product costing:

A. Job Costing

It allocates costs to products that are readily identified by individual units or batches, each of which is independently identifiable. When using the job cost system, costs are accumulated for each individual unit produced, or each separate order of products. This method is especially useful when producing something that is unique or custom-made. Job costing would be used by a caterer, a garage, a helicopter manufacturer, a construction company, a textbook publisher etc.

General approach to job costing

The following seven step approach is used to assign actual costs to individual jobs:

- Identify the chosen cost object(s)
- Identify the direct cost of the job
- Select the cost-absorption base(s)
- Identify the indirect costs associated with each cost-absorption base
- Compute the rate per unit of each cost absorption base to allocate indirect costs to jobs
- Compute the indirect costs allocated to the job
- Compute the cost of the job by adding all direct costs assigned to it.

B. Process costing

Process costing is a method that is applied when goods or services are produced from a series of repetitive or continuous processes or operations and costs of processing are charged to the process as a whole before being averaged out over the units produced during the period. It is also known as continuous operation costing.

A process costing system involves the costs of producing similar items being accumulated and allocated to the products by averaging costs over large number of nearly identical products. The average cost per unit is calculated by dividing the total production cost by the number of units produced. Process costing would be used by businesses such as food processors, household product manufacturers; chemical processors and oil refiners.

General approach to process costing

- Collect cost data for the period on production cost report
- Prepare statement of physical flows and equivalent units of output for the period
- Ascertain the total costs to be accounted for this period
- Calculate the cost per equivalent unit
- Apportion cost between finished output and work-in-progress, and.
- Check that all costs accounted for

Comparison between job costing and process costing

Job costing	Process costing
1. Job costing helps management control the efficiency of operations, materials and machines by suitable comparison.	1. Cost obtained merely forms reference for future use and of no use in efficiency
2. It is very expensive,	2. It is simpler and less expensive.
3. Unit costs can be computed periodically at short intervals e.g. daily; weekly etc.	3. Periodical unit cost calculation is not possible.



4. It helps in production planning and estimating costs of similar job to be taken up in future.	4. Managerial control is possible by evaluating the performance of each process.
5. Errors may occur because of increased 'clerical-works	5. Error may occur because of cost averaging.

Some Numerical Examples

1) The following information has been obtained from a company:

	standard	Actual
Production(units)	9000	8000
Direct material	40500kg@Rs20/kg	32000kg@Rs 19/kg
Direct labor	72000hrs@Rs105/hr	60000hrs@rs 100/hr

Calculate (a) total material cost variance, and (b) total wage variance indicating and separating the components.

Solution

A. Total Material Cost Variance

a) Direct Material Price Variance (DMPV)

$$\text{DMPV} = \text{AQ}(\text{AR}-\text{SR})$$

where,

$$\text{AQ} = 32000\text{kg}$$

$$\text{AR} = \text{Rs } 19/\text{kg}$$

$$\text{SR} = \text{Rs } 20/\text{kg}$$

$$\text{Therefore, DMPV} = 32000(19-20)$$

$$= \text{Rs } 32000 \text{ (-ve)}$$

$$\therefore \text{DMPV} = \text{Rs } 32000 \text{ (F)}$$

$$\text{Total material cost variance} = \text{Rs } 32000\text{(F)} + \text{Rs } 170000\text{(F)}$$

$$= \text{Rs } 201000\text{(F)}$$

b) Direct Material Usage Variance (DMUV)

$$\text{DMUV} = \text{SR}(\text{AQ}-\text{SQ})$$

Where,

$$\text{BR} = \text{Rs } 20/\text{kg}$$

$$\text{AQ} = 32000\text{kg}$$

$$\text{SQ} = 40500 \text{ kg}$$

$$\text{DMUV} = 20(3200-40500)$$

$$= \text{Rs } 170000 \text{ (F)}$$

B. Total Wage Variance

(a) Direct Labour Wage Rate Variance (DLWRV)

$$\text{DLWRV} = \text{AH}(\text{AR}-\text{SR})$$

Where,

$$\text{AH} = 60000\text{hrs}$$

$$\text{AR} = \text{Rs } 100/\text{hr}$$

$$\text{SR} = \text{Rs } 105/\text{hr}$$

$$\text{DLWRV} = 60000(100-105)$$

$$\text{DLWRV} = \text{Rs } 300000\text{(F)}$$

(b) Direct Labour Efficiency Variance (DLEV)

$$\text{DLEV} = \text{AH} \times \text{SR} - \text{AP} \times \frac{\text{SH}}{\text{unit}} \times \text{SR}$$

Where,

$$\text{AH} = 60000\text{hrs}$$

$$\text{AR} = \text{Rs } 100/\text{hr}$$

$$\text{SR} = \text{Rs } 105/\text{hr}$$

$$\text{AP} = 8000 \text{ units}$$

$$\text{SH} = 72000\text{hrs}$$

$$\text{DLEV} = 60000 \times 105 - 8000 \times \frac{72000}{9000} \times 105$$

$$= \text{Rs } 420000\text{(F)}$$

$$\text{Hence, Total wage variance} = \text{Rs } 300000\text{(F)} + \text{Rs } 420,000\text{(F)} = \text{Rs } 720000\text{(F)}$$

2. A garment factory in Bauddha manufactures and sells a designer jacket that requires tailoring and many hand operations. Sales are made to the distributors who sell to independent clothing stores and retail chains.

Factory's only costs are manufacturing costs. Find variable overhead variance.

Cost Item/Allocation Base	Actual Result	Flexible-Budget Amount
Output units (Jackets)	10,000	10,000
Machine-hours	4500	4000
Variable Manufacturing overhead costs	Rs 130500	Rs 120000

Solution:



i) Variable Overhead Expenditure Variance

$$VOExV = AH(AR - SR)$$

Where,

AH = Actual hours worked = 4500 hrs.

AR = Actual rate of variable overhead = Rs 130500/4500 hrs = Rs 29/hrs.

SR = Standard rate of variable overhead = Rs 120000/4000 hrs = Rs 30/hrs.

Therefore,

$$VOExV = 4500(29 - 30) = -4500 \text{ Rs, which is negative.}$$

i.e. favourable.

$$\therefore VOExV = \text{Rs } 4500(F)$$

ii) Variable Overhead Efficiency Variance (VOE_fV)

$$VOEN = AH \times SR - AP \times \frac{SH}{\text{unit}} \times SR$$

Where,

AH = 4500 hrs.

SR = Rs 30/hrs.

AP = Actual production = 10000 nos.

Therefore, $VOE_fV = 4500 \times 30 - 10000 \times \frac{4000}{10000} \times 30 = \text{Rs } 15000$, which is positive i.e. variance is adverse.

$$VOE_fV = \text{Rs } 15000 (A)$$

Hence, Variable overhead variance = Rs 4500(F) + Rs 15000(A)

$$= \text{Rs } 10500 (A)$$

3. Based on the following information, calculate (a) total material cost variance, (b) total wage variance, (c) variable overhead variance and (d) fixed overhead variance.

	standard	Actual
Production (units)	10000	7500
Direct material (kg)	50000	56250
Direct material cost (Rs)	1250000	1350000
Direct labor hours	100000	67500
Direct labour cost (Rs)	11500000	7425000
Fixed overhead (Rs)	13800000	7796250
Variable overhead (Rs)	9200000	5197500

Solution

A. Direct material cost variance

a) Direct Material Price Variance (DMPV)

$$DMPV = AQ(AR - SR)$$

Where,

AQ = 56250 kg

AR = Rs 1350000/56250 kg = Rs 24/kg

SR = Rs 1250000/50000 kg = Rs 25/kg

Therefore,

$$DMPV = 56250(24 - 25) = \text{Rs } 56250/kg$$

$$\text{Total material cost variance} = \text{Rs } 56250(F) + \text{Rs } 156250(A) = \text{Rs } 100000(A)$$

b) Direct Material Usage Variance (DMUV)

$$DMUV = SR(AQ - SQ)$$

Where,

SR = Rs 25/kg

AQ = 56250 kg

SQ = 50000 kg

$$DMUV = 25(56250 - 50000) = \text{Rs } 156250(A)$$

B. Total wage variances

**b) Direct Labor Wage Rate Variance (DLWRV)**

$$DLWRV = AH(AR-SR)$$

Where,

$$AH = 67500 \text{ hrs}$$

$$SR = \text{Rs } 11500000/100000 \text{ hr} = \text{Rs } 115/\text{hr}$$

$$AR = \text{Rs } 7425000/67500 \text{ hr} = \text{Rs } 110/\text{hr}$$

$$DLWRV = 67500(115-110)$$

$$= \text{Rs } 337500(\text{F})$$

c) Direct Labor Efficiency Variance (DLEV)

$$DLEV = AH \times SR - AP \times \frac{SH}{\text{unit}} \times SR$$

Where,

$$AH = 67500 \text{ hrs}$$

$$SR = \text{Rs } 115/\text{hr}$$

$$SH = 100000 \text{ hrs}$$

$$AR = \text{Rs } 115/\text{hr}$$

$$AP = 7500 \text{ units}$$

$$DLEV = 67500 \times 115 - 7500 \times \frac{100000}{10000} \times 115$$

$$= \text{Rs } 862500(\text{F})$$

$$\text{Hence, Total wage variance} = \text{Rs } 337500(\text{F}) + \text{Rs } 862500(\text{F}) = \text{Rs } 1200000(\text{F})$$

C. Variable overhead variances**a. Variable Overhead Expenditure Variance (VOE_xV)**

$$VOE_x V = AH(AR-SR)$$

Where,

$$AH = 67500 \text{ hrs}$$

$$AR = \text{Rs } 5197500/67500 \text{ hrs} = \text{Rs } 77/\text{hrs}$$

$$SR = \text{Rs } 9200000/100000 \text{ hrs} = \text{Rs } 92/\text{hrs}$$

$$VOE_x V = AH(AR-SR)$$

$$= 67500(77-92)$$

$$= \text{Rs } 1012500(\text{F})$$

b. Variable Overhead Efficiency Variance (VOE_tV)

$$VOE_t V = AH \times SR - AP \times \frac{SH}{\text{Unit}} \times SR$$

Where,

$$AH = 67500 \text{ hrs}$$

$$SR = \text{Rs } 92/\text{hrs}$$

$$SH = 100000 \text{ hrs}$$

$$AP = 7500$$

$$VOE_t V = 67500 \times 92 - 7500 \times \frac{100000}{10000} \times 92$$

$$= \text{Rs } 690000(\text{F})$$

$$\text{Hence, variable overhead variance} = \text{Rs } 1012500(\text{F}) + \text{Rs } 690000(\text{F}) = \text{Rs } 1702500(\text{F})$$

D. Fixed overhead variances**a) Fixed Overhead Expenditure Variance (FOE_xV)**

$$FOE_x V = \text{Actual fixed overhead} - \text{Budgeted fixed overhead}$$

$$= \text{Rs } 7796250 - \text{Rs } 13800000$$

$$= \text{Rs } 6003750(\text{F})$$

b) Fixed Overhead Capacity Variance (FOCV)

$$FOCV = \text{Budgeted fixed overhead} - AH \times SR$$

Where,

$$AH = 67500 \text{ hrs}$$

$$SR = \text{Rs } 13800000/100000 \text{ hrs} = \text{Rs } 138/\text{hr}$$

$$FOCV = \text{Budgeted fixed overhead} - AH \times SR$$

$$= \text{Rs } 13800000 - 67500 \times 138 = \text{Rs } 4485000(\text{A})$$

c) Fixed Overhead Efficiency Variance (FOE_tV)

$$FOE_t V = AH \times SR - AP \times \frac{SH}{\text{unit}} \times SR$$

Where,

$$AH = 67500 \text{ hrs}$$

$$SH = 100000 \text{ hrs}$$

$$SR = \text{Rs } 138/\text{hr}$$

$$AP = 7500 \text{ units}$$

$$FOE_t V = 67500 \times 138 - 7500 \times \frac{100000}{10000} \times 138 = \text{Rs } 1035000(\text{F})$$

$$\text{Hence, Fixed overhead variance} = \text{Rs } 6003750(\text{F}) + \text{Rs } 4485000(\text{A}) + \text{Rs } 1035000(\text{F}) = \text{Rs } 2553750(\text{F})$$

4. Product X. is produced after three distinct processes the following information obtained from the accounts of a



period:

Items	Total	Process		
		I	II	III
Direct material(Rs)	3000	2000	500	500
Direct wage(Rs)	500	100	300	100
Direct expenses(Rs)	600	300	100	200

Total production overhead is Rs 1000 and is 200% of direct wages. Production was 100kg. No opening or closing stocks. Prepare process cost accounts assuming no process loss.

Solution:

For process I

Direct material:	Rs2000
Direct wage:	Rs 100
Direct expenses:	Rs300
Production overhead:	200% of Rs 100=Rs200
Total I	Rs2600

For process II

Carry over from process I	Rs. 2600
Direct material	Rs.500
Direct wage	Rs.300
Direct expenses	Rs.100
Production overhead	200% of Rs 300=Rs600
Total II	4100

For process III

Carry over from process I	Rs. 4100
Direct material	Rs.500
Direct wage	Rs.100
Direct expenses	Rs.200
Production overhead	200% of Rs 100=Rs200
Total III	5100

Hence, total cost of production = Rs 5100

Unit production cost= Rs 5100/100kg = Rs 51/kg

5. Suppose there are two processes A and B and the cost data for a period is as follows:

	Process A	Process B
--	-----------	-----------



Raw materials(Rs)	80000	10000
Direct labor(Rs)	10000	15000
Overhead (Rs)	15000	5000
Completed	10000	10000
Partially completed	1000 units(50%)	3000 units(25%)

Calculate cost per unit of the product.

Solution:

For process A

Raw materials (Rs) 80000

Direct labor(Rs) 10000

Overhead (Rs) 15000

Sub-total(Rs) 105000

Output 10000units+50% of 1000 units=10500 units

Cost per unit Rs 105000/10500= Rs10/unit

For process B

Raw materials (Rs) 10000

Direct labor(Rs) 15000

Overhead (Rs) 5000

Sub-total(Rs) 30000

Transfer from process A 10000 units @Rs10/unit=Rs100000

Transfer from process B Rs 30000+Rs 100000=Rs130000

Output 10000units+25% of 3000 units=10750 units

Cost per unit Rs 130000/10750= Rs12.093/unit

6.



CHAPTER THREE

INTEREST AND THE TIME VALUE OF MONEY

Interest

Interest rate is a percentage that is periodically applied and added to an amount of money over a specified length of time: It is expressed as % per time period . For example, 9% interest rate means that for every Rupee lent, Rs.0.09 is paid in interest for each time period. If no time period is specified, assumption is per year.

Whenever we go for any investment, we will have to consider the following three factors:

- a) **Liquidity** Once it is invested, it is not so easy to convert it to cash and when needed immediately, we will not be able to spend on another project or on other financial expenses. In other words, it is the reward for not being able to use your money while you are holding the stock or mortgage or promise.
- b) **Risk premium** There is always a certain degree of risk associated with any financial investment. For example, if you lend someone Rs.10000, it is not sure that you may get it back either because of his nature or market scenario. The situation is worse in case when you are making investment on businesses, shares etc. where you might also lose your principal amount. It is common that most of the people fear for investing, knowingly or unknowingly they are conscious about the risks associated with it. In other words, it is the reward for any chance that you would not get your money back or that it will have declined in value while invested.
- c) **Inflation factor** Purchasing power of money goes down at a constant rate annually and we call it inflation. The money we invested should at least earn to cover the loss in its value due to inflation; In other words, it is the compensation for decrease in purchasing power between the time you invest it and the time it is returned to you.

Every investor, because of these factors, looks for some return on their investment and charges a cost of investment known as **Interest Rate**.

3.1 Simple interest, compound interest, interest tables, interest charts

Interest calculating schemes

There are two computational schemes for calculating the interest earned on the investment of financial transactions loaning and repaying:

a) Simple interest :

Simple interest is a type of interest that is applied to the amount borrowed or invested for the entire duration of the loan without taking any other factor into account such as past interest and any other financial consideration.

In this scheme, interest earned or charged is only on the principal amount during each interest period and does not earn additional interest in the remaining periods. Simple interest is calculated as:

$$I = P \cdot N \cdot i$$

Where,

I = Interest earned at the end of the period

i = Interest rate per compounding period

N = number of periods during the term

P = amount of money invested

And the residual payment at the end of the term (amount that must be repaid at the end of the term), F , is calculated as $F = P(1 + iN)$

Simple interest is not much used in current practices and so is in Engineering Economics.

b) Compound Interest

Compound interest is the interest which is computed as a percentage of revised Principal i.e. original Principal plus accumulated interest that has been left in the account, at the end of the previous period. Simple interest is calculated as: $I = P \cdot N \cdot i$

Suppose, you have lent Rs 'P', that earns interest rate of $i\%$ per year. The following table shows how the interest earning process is repeated and how the future amount is calculated under this scheme.

Beginning of period	Amount lent	Interest amount	Amount owned at period End
1	P	Pi	P(1+i)
2	P(1+i)	P(1+i)i	P(1+i) ²



3	$P(1+i)^2$	$P(1+i)^2i$	$P(1+i)^3$
...
N	$P(1+i)^{N-1}$	$P(1+i)^{N-1}i$	$P(1+i)^N$

By using compound interest calculation scheme, the future accumulated value is computed as
 $F = P(1+i)^N$

Difference between simple interest and compound interest

simple interest	Compound interest
<ol style="list-style-type: none"> 1. It is interest charged on the Principal for entire period 2. Principal remains constant 3. Calculation is easy 4. It is calculated as $P.N.i$ 	<ol style="list-style-type: none"> 1. It is interest compounded on both Principal and previously earned interest 2. Principal changes due to effect of compounding 3. Calculation is vast comparing to simple interest 4. It is calculated as $P(1+i)^N - i$

3.2 Nominal and effective interest rates

The **Nominal interest rate (Annualized Percentage Rate or APR)** is the periodic interest rate multiplied by the number of periods per year. For example, a nominal annual interest rate of **12%** based on monthly compounding means a **1%** interest rate per month (compounded).

The Effective Annual Interest Rate (EAR) is the interest rate that would have earned the same amount of interest in one year. Another way to view it is as the annual rate of interest for compounding yearly that would have earned the same money. If we are compounding yearly, then the EAR is equal to APR. The table below shows how the effective interest rate is calculated and the future value of Rs 10000 at an interest rate of 9% compounded quarterly.

I quarter	Base amount + Interest(2.25%)	Rs 10,000 + Rs 225
II quarter	=New base amount+ Interest (2.25%)	= Rs 10,225 + Rs 230.06
III quarter	=New base amount+ Interest (2.25%)	= Rs 10,455.06 + Rs 235.24
IV quarter	=New base amount+ Interest (2.25%)=value after one year	= Rs 10,690.30 + Rs 240.53= Rs 10,930.83

We can compute this value by using a **simple relationship between Effective interest rate and Nominal interest rate, given by**

$$i = \left(1 + \frac{r}{M}\right)^M - 1$$

Where,

i = Effective annual interest rate

r = Nominal interest rate

M = No. of compounding periods per year

3.3 Continuous compounding and continuous compounding formula

Continuous compounding is the procedure of obtaining interest on top of interest in a monthly, quarterly semiannual basis. In this method interest is obtained continuously.

$$A = Pe^{rt}$$

where

P = principal amount

i = interest rate

t = numbers of year

A = amount after time t

3.4 Interest calculations for uniform gradient

There are two types of linear gradient series:

A. Strict gradient series

The gradient series in which there is no cash flow in the first year is called strict gradient series. Depending upon the pattern of the gradient, it may be of the following types:-

if $G > 0$, the series is called increasing gradient series

if $G < 0$ the series is called decreasing gradient series.

**Present worth factor**

$$P = G \left[\frac{((1+i)^N - iN - 1)}{i^2(1+i)^N} \right]$$

$$= G(P/G, i\%, N)$$

Where $(P/G, i\%, N)$ is called **gradient series present worth factor**.

Gradient to uniform series conversion factor - Find A, given G, i% and N

We know,

$$P = A \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right] \dots \dots \dots (i)$$

$$P = G \left[\frac{((1+i)^N - iN - 1)}{i^2(1+i)^N} \right] \dots \dots \dots (ii)$$

From (i) & (ii)

$$A = G \left[\frac{((1+i)^N - iN - 1)}{i(1+i)^N - i} \right]$$

$$= G(A/G, i\%, N)$$

Where $(A/G, i\%, N)$ is called **gradient to uniform series conversion factor**.

Future worth factor- Find F, given G, i% and N

We know that

$$P = \frac{F}{(1+i)^N} \dots \dots \dots (i)$$

$$P = G \left[\frac{((1+i)^N - iN - 1)}{i^2(1+i)^N} \right] \dots \dots \dots (ii)$$

From (i) & (ii)

$$F = G \left[\frac{((1+i)^N - iN - 1)}{i^2} \right]$$

$$= G(F/G, i\%, N)$$

Where $(F/G, i\%, N)$ is called **Future worth factor**.

B. Composite Gradient series

In this series, there is an initial payment in the first interest period and it increases by a gradient G over rest of the interest periods.

In order to compute the equivalent worth of the composite gradient series, we first convert the gradient series into the uniform one and go through the following strategies:

If the series is increasing,

$$A = A_1 + G(A/G, i\%, N)$$

If the series is decreasing,

$$A = A_1 - G(A/G, i\%, N)$$

Some solved numerical problems

Q.N.1. You lend Rs 100 for 3 years at 10% interest compounded annually. How much would you earn interest and get at the end of the three years?

Solution

Given:

$$P = \text{Rs } 100$$

$$N = 3 \text{ years}$$

$$i = 10\% \text{ per year} = 0.1$$

$$F = ?$$

Simple interest method:

$$\text{Total interest earned, } I = P \cdot N \cdot i = 100(3)(0.1) = \text{Rs } 30$$

$$\text{Future accumulated value} = 100 + 30 = \text{Rs } 130$$

Or we may compute it directly using the formula,

$$F = P(1 + iN) = 100(1 + 0.1 \times 3) = 130$$

**Compound interest method**

The calculation is simplified in the tabular form as below,

Start of year	Amount lent	Interest earned	Amount owed at year end
1	100	100(0.1)	110.0
2	110	110.0(0.1)	121
3	121	121(0.1)	133.10

Hence, Total interest earned = Rs 133.10 - Rs 100 = Rs 33.10

Or we may compute it directly using the formula,

$$F = P(1+i)^N$$

$$= 100(1 + 0.1)^3$$

$$= \text{Rs } 133.10$$

Q.N.2. A bank charges 1% per month on car loans. What is the APR? What is the EAR?

Solution:

Annual Percentage Rate, (APR) $r = 1\%$ per month = 12% per year

$$\text{Effective Annual Interest Rate, } i = \left(1 + \frac{r}{M}\right)^M - 1$$

Where, nominal interest rate, $r = 12\%$ per year

no. of compounding periods in a year, $M = 12$

[\because Compounding is monthly and there are 12 months in a year]

$$i = \left(1 + \frac{0.12}{12}\right)^{12} - 1$$

$$\therefore i = 12.6825\%$$

Hence APR = 12% per year

EAR = 12.6825% per year

Q.N.3. What will be the maturity amount of Rs 15000 after 5 years for nominal rate of 9% per year, when compounded (i) yearly, and (ii) quarterly?

Solution:

Given: Nominal rate, $r = 9\%$

When compound is yearly, $M = 1$

$$\text{Effective Annual Interest Rate, } i = \left(1 + \frac{r}{M}\right)^M - 1$$

$$i = \left(1 + \frac{0.09}{1}\right)^1 - 1 = 9\%$$

Maturity amount, $F = 15000(F/P, 9\%, 5)$

$$F = 15000(1 + 9\%)^5 = 23079.359$$

$$\mathbf{F = 23079.359}$$

When compound is yearly, $M = 4$

$$\text{Effective Annual Interest Rate, } i = \left(1 + \frac{r}{M}\right)^M - 1$$

$$i = \left(1 + \frac{0.09}{4}\right)^4 - 1 = 9.308\%$$

Maturity amount, $F = 15000(F/P, 9.308\%, 5)$

$$F = 15000(1 + 9.308\%)^5 = 23407.28$$

$$\mathbf{F = 23407.28}$$

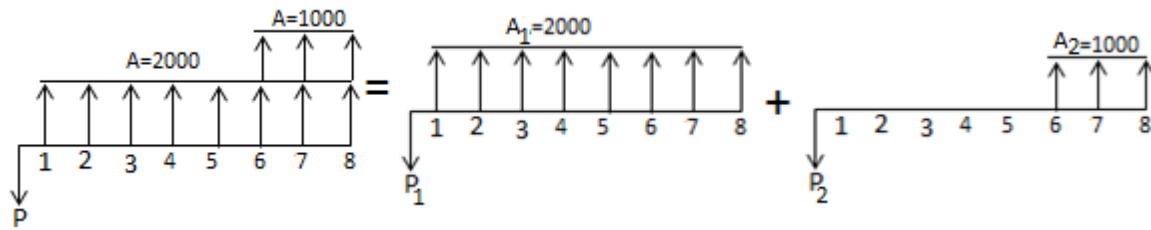
Q.N.4. What amount P, I will get if an amount of Rs 2,000 is deposited annually for 5 years. An amount of Rs 3,000 is annually deposit for the next 3 years at an interest of 10%.

Solution,

Given,

$$i = 10\% = 0.1$$

According to question, 2000 is deposited for 5 year and 3000 is deposited for next three year. The cash flow diagram can be written as follow



Let

$$A_1=2000$$

$$A_2=1000$$

$$N_1=8$$

$$N_2=3$$

Then

$$P=P_1+P_2$$

Now,

$$P_1 = A_1 \left[\frac{(1+i)^{N_1}-1}{i(1+i)^{N_1}} \right] = 2000 \left[\frac{(1+0.1)^8-1}{0.1(1+0.1)^8} \right] = 10669.32$$

$$P_2(F) = A_2 \left[\frac{(1+i)^{N_2}-1}{i(1+i)^{N_2}} \right] = 1000 \left[\frac{(1+0.1)^3-1}{0.1(1+0.1)^3} \right] = 2486.85$$

$$P_2 = F(1+i)^{-n} \text{ where } n=5$$

$$P_2=2486.85(1+0.1)^5=1544.138$$

$$\therefore P=2486.85+1544.138=12213.98$$

$$\therefore P=12213.98$$

Q.N.5. A person is planning for his retired life and has 10 more years of service. He would like to deposit 20% of his salary, which is Rs 10000 at the end of the first year and thereafter he wishes to deposit the same amount (Rs 10000) with an annual increase of Rs 2000 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10 year of the above series.

Solution:

From the question, it is a linear, composite gradient series of increasing *type*. In this series,

$$A_1= \text{Rs } 10000$$

$$N = 10 \text{ years}$$

$$i = 15\% \text{ per year}$$

$$G = \text{Rs } 2000$$

$$F=?$$

Converting gradient series into uniform series, we get,

$$A_2= 2000(A/G, 15\%, 10)$$

$$A_2 = G \left[\frac{((1+i)^N)-iN-1}{i(1+i)^N-i} \right]$$

substituting values

$$A_2 = 2000 \left[\frac{((1+0.15)^{10})-0.15*10-1}{0.15(1+0.15)^{10}-0.15} \right]$$

$$= \text{Rs } 6766.38$$

So, the given gradient series can be replaced by a uniform series having annual cash flow of A, calculated as below:

$$A= A_1 + A_2$$

$$= \text{Rs}(10000 + 6766.38)$$

$$= \text{Rs } 16766.38$$

Now, Future amount, F, is calculated as follows:

$$F = 16766(F/A, 15\%, 10)$$

$$F = 16766 \left[\frac{(1+15\%)^{10}-1}{15\%} \right]$$

$$= \text{Rs } 340420.092$$



Q.N.6. Five annual deposits in the amounts of (Rs 1200, Rs 1000, Rs 800, Rs 600 and Rs 400) are made into a fund that pays interest at a rate of 9%, compounded annually. Determine the amount in the fund immediately after the 5th deposit.

Solution:

This is a decreasing gradient series where,

$$A_1 = \text{Rs } 1200$$

$$G = \text{Rs } 200$$

$$N = 5 \text{ year}$$

$$i = 9\% \text{ per year}$$

$$F = ?$$

This series can be replaced by a uniform series having equal payments A as follows:

$$A = A_1 - 200(A/G, 9\%, 5) = 1200 - 200(A/G, 9\%, 5)$$

$$= 1200 - 200 \left[\frac{((0.09+1)^5) - 0.09 \times 5 - 1}{0.09(0.09+1)^5 - 0.09} \right]$$

$$= \text{Rs } 834.36$$

Now,

$$F = 834.36(F/A, 9\%, 5)$$

$$F = 834.36 \left[\frac{(1+9\%)^5 - 1}{9\%} \right]$$

$$= \text{Rs } 4993.40$$

Q.N.7. What is the equal payment series for 10 years that is equivalent to a payment series of Rs 12000 at the end of the first year, decreasing by Rs 1000 each year over 10 years. Interest rate is 8% compounded annually.

Solution:

The given series is a decreasing gradient series where

$$A_1 = \text{Rs } 12000$$

$$G = \text{Rs } 1000$$

$$N = 10 \text{ years}$$

$$i = 8\% \text{ per year}$$

The given series can be transformed into a uniform series having an equal payments as follows:

$$A = A_1 - 1000(A/G, 8\%, 10)$$

$$= 12000 - 1000(A/G, 8\%, 10)$$

$$= 12000 - 1000 \left[\frac{((0.08+1)^{10}) - 0.08 \times 10 - 1}{0.08(0.08+1)^{10} - 0.08} \right]$$

$$= \text{Rs } 8128.69$$



CHAPTER FOUR

BASIC METHODOLOGIES OF ENGINEERING ECONOMIC STUDIES

The following methodologies are extensively used in, the economic evaluation of any project proposal in hands.

- b) **The payback period method**
- c) **Equivalent worth methods**
 - **The present worth**
 - **The future worth**
 - **The annual worth**
- d) **Rate of return methods**
 - **Internal rate of return method**
 - **External rate of return method**

4.1 Present worth and annual worth methods**Present worth method**

Present Worth method discounts future amounts to the present by using the interest rate over the appropriate study period as

$$PW(i\%) = \sum_{k=0}^N F_k (1 + i)^{-k}$$

Where,

i = effective interest rate, or MARR per compounding period.

k = index for each compounding period

F_k = future cash flows at the end of period K

N = number of compounding periods in study period

While evaluating a project by PW method, the following rule is applicable

If $PW(i\%) > 0$, accept the project

If $PW(i\%) = 0$, remain indifferent

If $PW(i\%) < 0$, reject the project

Annual worth method

Annual Worth is the equivalent worth of a lump-sum amount converted into a series of equal payments at the end of each period and is calculated as

$$AW(i\%) = \underline{R} - \underline{E} - CR(i\%)$$

Where,

\underline{R} = equivalent revenues

\underline{E} = equivalent expenses

$CR(i\%)$ = Capital recovery amount .

While evaluating a project by AW method, following rule is applicable.

If $AW(i\%) > 0$, accept the project

If $AW(i\%) = 0$, remain indifferent

If $AW(i\%) < 0$, reject the project

Advantages of AW

Annual worth method has the following advantages over other equivalent worth methods:

- ❖ Annual Worth is a popular analysis technique that is easily understood, and the results are reported in Rs/time period.
- ❖ AW analysis method is applicable to a variety of engineering economy studies
 - Asset Replacement
 - Breakeven Analysis
 - Make or Buy Decisions
 - Studies dealing with manufacturing Costs
 - Economic Value Added analysis (EVA)

So, AW analysis is recommended over NPW analysis in many key real-world situations for the following reasons:



- i. In Many financial reports, an annual equivalent value is preferred to a present worth value.
- ii. Calculation of unit costs is often required to determine reasonable pricing for sale items.
- iii. Calculation of cost per unit of use is required to reimburse employees for business use of personal cars.
- iv. Make-or-buy decisions usually require the development of unit costs for the various alternatives ..
- v. Minimum cost analysis is easy to do when based on annual equivalent worth.

4.2 Future worth method

Future Worth of a project is the equivalent worth of all cash flows, both inflows and outflows, at the end of the planning horizon at an interest rate that is generally the MARR. The FW is calculated as

$$FW(i\%) = \sum_{k=0}^N F_k (1 + i)^{N-k}$$

Where;

i = effective interest rate, or MARR per compounding period.

k = index for each compounding period

F_k = future cash flows at the end of period k

N = number of compounding periods in study period

While evaluating a project by FW method, the following rule is applicable

If $FW(i\%) > 0$, accept the project

If $FW(i\%) = 0$, remain indifferent

If $FW(i\%) < 0$, reject the project

4.3 Internal rate of return method(IRR)

Internal rate of return of a project is the breakeven interest rate, i' , at which equivalent worth of the project's cash flow is zero. At this particular rate of return, the equivalent worth of revenues generated by the project is enough to bear the equivalent worth of expenses absorbed by the project, without imposing any financial load to the firm.

Based on PW formulation to find out this interest rate, the corresponding equation becomes,

$$PW(i\%) = \sum_{k=0}^N R_k(P/F, i\%, k) - \sum_{k=0}^N E_k(P/F, i\%, k) = 0$$

Where,

R_k = revenues for the K^{th} year .

E_k = expenditures (including investment) for the K^{th} year

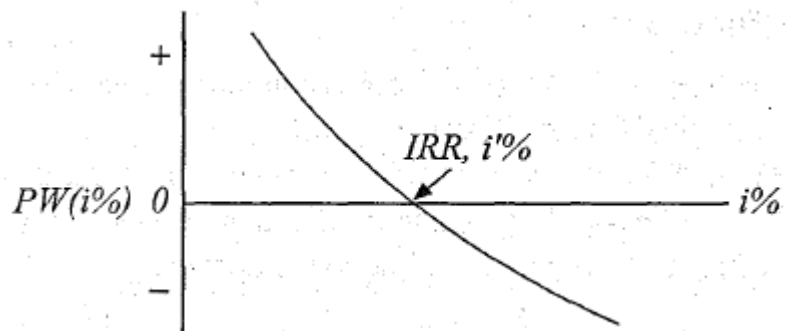
N = project life or study period

Similarly at the IRR,

$FW(i'\%) = 0$

$AW(i'\%) = 0$

If we plot a curve of present worth for different values of $i\%$, the IRR can be computed by spotting where the PW is zero. It is represented graphically as aside:



Methods for calculating the IRR:

The following steps are followed to calculate the IRR of any project cash flow: .

Step-1: Develop an equation for equivalent worth, generally the PW, at an interest rate, whose value is to be found out.

Step-2: Equate the equation developed as above to zero.

Step-3: Solve it to get the value of, which is IRR.

4.4 Drawbacks of the internal method

Internal Rate of Return method is more widely understood, used and possesses a single value of merit. At the same time, it has the following drawbacks:

- a. The IRR method is based on the assumption that recovered funds if not consumed in each time period, are



reinvested at rather than at MARR. This is not always practical.

- b. As we sum amounts from left to right, the cash flow equation changes sign at least once. It'll behaved cash flow may change sign more than once, indicating multiple roots For example,

End of year	Net cash flow
0	-1000000
1	2300000
2	-1320000

For the above cash flow pattern, we find= 10% and 20%, but both of them are incorrect. Thus we may abandon the IRR for practical purposes and use the NPW criterion to make the decision.

- c. Since the IRR measure ignores the scale of the investment, it has flaws in ranking mutually exclusive projects and can be misleading while making selection. For example

End of year	Mutually exclusive project cash flow	
	A1	A2
0	-1000	-5000
1	2000	7000
IRR	100%	40%
PW(10%)	818	1364

Both projects are acceptable at MARR = 10%, but A2 with higher PW is worth more to the stockholders, whereas from IRR point of view, A1 seems better. Since IRR is a relative measure of investment worth, this inconsistency in ranking occurs. Under this confusion, we would select the project with higher PW or go for incremental analysis.

- d. IRR method involves linear interpolation of non-linear function and when solved manually by trial and error method, may not give accurate results and is more time consuming.

4.5 External rate of return method (ERR)

Support a firm's MARR is 15% per year and it has invested in a project yielding an IRR of 30% per year. IRR assumption states that the amount generated or borrowed by this project can be reinvested at IRR again. But in the market, MARR is prevailing and if the firm uses or invests its cash proceeds outside the firm, it will earn around MARR. This complication leads to the computation of another rate of return called **External Rate of Return(ERR)**. ERR method use a different discount rate, called **the External Reinvestment Rate**.

This method uses a different discount rate called external reinvest rate notated as "E%", The following three steps are followed in calculating the ERR.

- a) Discount all expenses to the Present (i.e. time 0) at the external reinvestment rate.

$$\sum_{K=0}^N E_k(P/F, E\%, K)$$

- b) Project all income to the Future (i.e. at the end of the project) at the external reinvestment rate, a%

$$\sum_{K=0}^N R_k(F/p, E\%, N-K)$$

- c) ERR, i*%, is the interest rate that establishes equivalence between these terms. So, solve the values obtained from step 1 and 2 by using equivalence concept.

$$\sum_{K=0}^N E_k \left(\frac{P}{F}, E\%, K \right) \left(\frac{E}{P}, I^*\%, N \right) = \sum_{K=0}^N R_k(F/p, E\%, N-K)$$

Where,

Rk = net revenues in period k



E_k = net expenses in period k

N = project's life

$E\%$ = external reinvestment rate

The external rate of return thus determined has the following advantages over IRR:

- It does not need trial and error process to solve for i^* .
- There is no possibility of multiple rates of return.

4.6 Minimum attractive rate of return method(MARR)

A minimum return the company will accept on the money it invests is called Minimum Attractive Rate of Return (MARR). MARR can also be regarded as the minimum return required to get investors to put up the money. It is usually calculated by financial analysts in the company and provided to those who evaluate projects.

MARR is different for each firm and the firm sees this as their cost of capital i.e. the firm sees MARR as an opportunity cost because investing in one project may mean giving up another. MARR can be developed from existing projects and may be different from time to time within the same firm.

The following factors greatly influence the determination of MARR for a firm:

- The amount of fund available for investment and its source;
- The nature of investment alternatives;
- The amount of risk perceived in the investment; and
- The type of organizations involved.

4.7 The payback (pay-out) period method

It is a project-screening tool which is the first formal method to be used to evaluate investment projects. Payback period method for screening the projects is excessively used in the following circumstances:

- To measure cash recovery speed
- When the firm has cash flow problem;
- When the product is built to last only for a short period of time; and
- When the machine itself is known to have short market life.

Based on the way we compute the payback period, it can be classified into two types:

A. Simple Payback Period

The simplest index of the economic feasibility of investing money in order to save or generate money in the future is the simple payback. The simple payback indicates the amount of time to break even on an investment i.e. it measures a project's liquidity and is computed as follows:

$$SP = \frac{\text{initial investment}}{\text{expected saving or revenue per year}}$$

In other way, the simple payback period for a project having one time investment at time 0 can be computed as follows:

$$\sum_{k=1}^{\theta} (R_k - E_k) - I > 0$$

Where,

I = Capital investment

R_k = Revenues at the end of year k

E_k = Expenses at the end of year k

θ = simple payback period

The use of the Payback Period as a Capital Budgeting decision rule specifies that all *independent* projects with a Payback Period less than a specified number of years should be accepted. When choosing among *mutually exclusive* projects, the project with the quickest payback is preferred.

The conventional payback period method has the following benefits as well as flaws:

Benefits:

- Simplicity
- May eliminate some alternatives, thus by reducing a firm's need to make further analysis efforts.
- Reduces information search by focusing on that time at which the firm expects to recover the initial investment.

Flaws/drawback:



- It ignores time value of money
- It measures liquidity of a project rather than its profitability
- It neglects the cash flow after the pay back period

B. Discounted Payback Period

It is defined as the number of years required to recover the investment from discounted cash flows i.e. considering the Time value of money. Thus, the discounted payback period for a project having one time investment at time 0 can be computed as follows:

$$\sum_{k=1}^{\theta'} (R_k - E_k) \left(\frac{P}{F}, i\%, K \right) - 1 > 0$$

Where,

I == Capital investment

R_k = Revenues at the end of year k

E_k = Expenses at the end of year k

i = Minimum Attractive Rate of Return

θ' = Discounted payback period

It should be remembered that the discounted payback method can overcome one of the shortcomings of simple payback period and it is consideration of time value of money. However, all other short-comings are still found in this technique.

Some solved numericals

Q.N.1. Evaluate the acceptability of the following proposal if maximum allowable simple payback period is 5 years and maximum allowable discounted payback period is 6 years. Assume minimum attractive rate of return= 15% per year,

cash flow estimates	
year	amount (in thousands)
0	-1500
1	200
2	400
3	450
4	450
5	600
6	900
7	1100

Solution

$$i = 15\% = 0.15$$

computation of simple payback period

End of year	Net cash flow	Cumulative cash flow
0	-1500	-1500
1	200	-1300
2	400	-900
3	450	-450
4	450	0
5	600	600
6	900	1500
7	1100	2600

Since cumulative cash flow becomes zero at the end of 4 years, Simple payback period, θ = 4 years

**Computation of discounted payback period**

$$PW_0 = -1500(1+.15)^0 = -1500$$

$$PW_2 = 400(1+.15)^2 = 302.46$$

$$PW_4 = 450(1+.15)^4 = 257.29$$

$$PW_6 = 900(1+.15)^6 = 389.09$$

$$PW_1 = 200(1+.15)^1 = 173.91$$

$$PW_3 = 450(1+.15)^3 = 295.88$$

$$PW_5 = 600(1+.15)^5 = 298.31$$

$$PW_7 = 1100(1+.15)^7 = 413.53$$

End Of Year	Cash Flow	Pw At 15%	Cumulative Cash Flow
0	-1500	-1500	-1500
1	200	173.91	-1326.087
2	400	302.46	-1023.629
3	450	295.88	-727.747
4	450	257.29	-470.458
5	600	298.31	-172.152
6	900	389.09	216.943
7	1100	413.53	630.473

From the above table, cumulative PW at 15% has -ve sign at the end of 5 year and +ve at the end of 6 year. Therefore, the discounted payback period lies in between 5 and 6 years and is calculated as below:

$$\theta' = 5 + \frac{172.152}{389.09} = 5.44$$

Decision

$\theta = 4 \text{ year} < 5 \text{ year}$, the proposal is acceptable

$\theta' = 5.44 \text{ years} < 6 \text{ years}$, the proposal is acceptable

Q.N.2. Compute the IRR for the following project. Also show the unrecovered investment balance in the graphical and tabular form. [P.U. 2005]

End of year	0	1	2	3	4	5	6	7	8
Net cash flow (in million)	-10	1.8	1.8	1.8	1.8	1.8	1.8	1.8	2.8

Solution

$$PW(i\%) = -10 + 1.8(P/A, i\%, 8) + 1(P/F, i\%, 8) = 0$$

$$PW(i'\%) = -10 + 1.8 \left[\frac{(1+i')^8 - 1}{i'(1+i')^8} \right] + 1(1+i')^{-8}$$

By hit and trial

Put $i' = 5\%$

Then,

$$PW(5\%) = -10 + 1.8 \left[\frac{(1+5\%)^8 - 1}{i'(1+5\%)^8} \right] + 1(1+5\%)^{-8}$$

$$PW(5\%) = 2.3105 \text{ million}$$

$$\text{Error} = 0 - 2.3105 = -2.3106$$

Again,

Put $i' = 15\%$

Then,

$$PW(15\%) = -10 + 1.8 \left[\frac{(1+15\%)^8 - 1}{i'(1+15\%)^8} \right] + 1(1+15\%)^{-8}$$

$$PW(15\%) = -1.5959 \text{ million}$$

$$\text{Error} = 0 - (-1.5959) = 1.5959$$

Now use interpolation

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$



$$\frac{y-5}{15-5} = \frac{0-(-2.3106)}{1.5959-(-2.3106)}$$

$$y = 10.91$$

∴ IRR=10.91%

Un-recovered Investment Calculation (In tabular form)

Year	Cash Flow	Un-recovered investment		(1+i')× unrecovered
		Start of year	End of year	
0	-10	-10	-10	-11.091
1	1.8	-11.09	=1.8-11.09 = -9.29	-10.30
2	1.8	-10.30	-8.50	-9.43
3	1.8	-9.43	-7.63	-8.47
4	1.8	-8.47	-6.67	-7.39
5	1.8	-7.39	-5.59	-6.20
6	1.8	-6.20	-4.40	-4.88
7	1.8	-4.88	-3.08	-3.42
8	2.8	-3.42	-0.62	-0.69

Q.N.3. Calculate the IRR for the following investment alternative

End of year	0	1	2	3	4	5
Net cash flow (Rs.)	-100000	-8600	23000	84000	142000	-40000

Solution:

Using PW method:

$$PW(i') = -100000 - 8600(P/F, i', 1) + 23000(P/F, i', 2) + 84000(P/F, i', 3) + 142000(P/F, i', 4) - 40000(P/F, i', 5)$$

$$= 0$$

$$\text{or, } PW(i') = -100000 - 8600(1 + i')^{-1} + 23000(1 + i')^{-2} + 84000(1 + i')^{-3} + 142000(1 + i')^{-4} - 40000(1 + i')^{-5}$$

Put $i' = 15\%$

$$PW(15\%) = -100000 - 8600(1 + 15\%)^{-1} + 23000(1 + 15\%)^{-2} + 84000(1 + 15\%)^{-3} + 142000(1 + 15\%)^{-4} - 40000(1 + 15\%)^{-5}$$

$$PW(15\%) = 26449$$

$$\text{Error} = 0 - 26449 = -26449$$

Put $i' = 25\%$

$$PW(25\%) = -100000 - 8600(1 + 25\%)^{-1} + 23000(1 + 25\%)^{-2} + 84000(1 + 25\%)^{-3} + 142000(1 + 25\%)^{-4} - 40000(1 + 25\%)^{-5}$$

$$PW(25\%) = -4096$$

$$\text{Error} = 0 - (-4096) = 4096$$

Now using interpolation we have

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 15}{25 - 15} = \frac{0 - (-26449)}{4096 - (-26449)}$$



$$y = 23.65\%$$

$$\therefore \text{IRR} = 23.65\%$$

Q.N.4. A company has just brought a new machine at a cost of Rs. 105000 to replace one that had a salvage value of Rs. 20000. The projected annual after-tax savings via improved efficiency, which will exceed the investment cost, are as follows:

End of year	0	1	2	3	4	5	6
cash flow	-105000+20000	15000	25000	35000	45000	45000	35000

Soln

End Of Year	Cash Flow	Cumulative Cash Flow
0	-85000	-85000
1	15000	-70000
2	25000	-45000
3	35000	-10000
4	45000	35000
5	45000	80000
6	35000	115000

$$\text{The simple payback period} = 3 + \frac{10000}{45000} = 3.22 \text{ years.}$$

Q.N.5. Check the feasibility of the following investment by using Present Worth, Future Worth and Annual Worth method. What is the Capital Recovery amount of this project?

First investment (Rs) = 100000

Salvage value (Rs) = 25000

Project period (Yrs.) = 10

Annual revenue (Rs) = 20000

Annual cost (Rs) = 5000

MARR = 10% per year

solution

Solution:

Present Worth method

$$\text{PW}(10\%) = -100000 + (20000 - 5000)(P/A, 10\%, 10) + 25000(P/F, 10\%, 10)$$

$$= -100000 + 15000 \frac{(1+10\%)^{10} - 1}{10\%(1+10\%)^{10}} + 25000(1+10\%)^{-1}$$

$$= \text{Rs } 1807$$

Since $\text{PW}(10\%) > 0$, the project is acceptable.

Future Worth method

$$\text{FW}(10\%) = -100000(F/P, 10\%, 10) + (20000 - 5000)(F/A, 10\%, 10) + 25000$$

$$= -100000(1+10\%)^{10} + 15000 \frac{(1+10\%)^{10} - 1}{10\%} + 25000$$

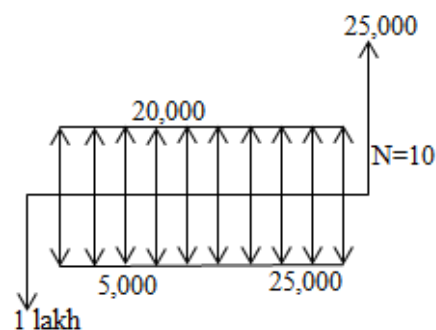
$$= \text{Rs } 4691$$

Since $\text{FW}(10\%) > 0$, the project is acceptable.

Annual Worth method

$$\text{AW}(10\%) = -100000(A/P, 10\%, 10) + (20000 - 5000) + 25000(A/F, 10\%, 10)$$

=





=Rs 298

Since $AW(10\%) > 0$, the project is acceptable.**Capital Recovery amount**

$$CR(10\%) = 100000(A/P, 10\%, 10) - 25000(A/F, 10\%, 10) = \text{Rs } 14706$$

Q.N.6 Calculate the payback period for the following project:**Investment = 10000****Annual Revenue= 5310****Annual expense= 3000****Salvage value = 2000****Solution, given,**

Investment = 10000

Annual Revenue= 5310

Annual expense= 3000

Salvage value = 2000

$$\therefore \text{simple payback period} = \frac{\text{initial payment}}{\text{annual saving}} = \frac{10000}{5310 - 3000} = 4.329 \text{ yrs}$$

Q.N.7. Assuming the minimum attractive .rate of return as 10% per year, ' screen the best one from the following projects. Use payback period method for screening.

Year	Project Cash Flow		
	A	B	C
0	-15000	-4000	-4500
1	300	2000	2000
2	300	1500	2000
3	300	1500	2000
4	300	500	5000
5	300	500	5000
6	300	1500	
7	300		
8	300		

Soln**For project A**

EOY	Cash flow	PW(10%)	Cum Pw
0	-1500	-1500	-1500
1	300	272.7273	-1227.27
2	300	247.9339	-979.339
3	300	225.3944	-753.944
4	300	204.904	-549.04
5	300	186.2764	-362.764
6	300	169.3422	-193.422
7	300	153.9474	-39.4744
8	300	139.9522	100.4779

$$\text{Discounted payback period} = 7 + \frac{39.47}{139.95} = 7.28 \text{ yrs}$$

**For project B**

EOY	Cash flow	PW(10%)	Cum Pw
0	-4000	-4000	-4000
1	2000	1818.18	2181.82
2	1500	1239.67	-942.15
3	1500	1126.97	184.82
4	500	341.51	526.33
5	500	310.46	836.79
6	1500	846.71	1683.50

Discounted payback period $= 2 + \frac{942.15}{1126.97} = 2.83$ yrs

For project C

EOY	Cash flow	PW(10%)	Cum Pw
0	-4500	-4500	-4500
1	2000	1818.18	-2681.82
2	2000	1652.89	-1028.93
3	2000	1502.63	473.70
4	5000	3415.07	3888.77
5	5000	3104.61	6993.38

Discounted payback period $= 2 + \frac{1028.93}{1502.62} = 2.68$ yrs.

∴ the best one is project C

Q.N.8. What is the simple payback period (SPB) for a Rs. 100,000 investment that yields Rs. 30,000 per year (paid at year end) in the first and second year, and Rs 18,000 per year in the third to sixth years?

Solution

EOY	Cash flow	Cum cash flow
0	-100000	-100000
1	30000	-70000
2	30000	-40000
3	18000	-22000
4	18000	-4000
5	18000	14000
6	18000	32000

Simple Pay Back period $= 4 + \frac{4000}{18000} = 4.22$ yrs.



CHAPTER FIVE

COST/BENEFIT ANALYSIS

Cost/Benefit analysis is the decision making tool used to symmetrically develops useful information about the desirable and undesirable effects of public project. It estimates and total up the equivalent money value of benefits and to the community of projects to established whether there are worthwhile. This projects may be dams, irrigation, water supply, highways etc. In other word cost/benefit analysis is defined as the ratio of equivalent work of benefit to the equivalent worth of cost.

5.1 Conventional benefit/cost ratio

It is the ratio of gross benefits to costs and expressed as

$$\frac{\text{benefit}}{\text{investment} + \text{operational and maintenance cost} - \text{salvage}}$$

For present worth formulation

$$\text{BCR} = \frac{\text{PW(B)}}{\text{PW(I)} + \text{PW(O\&M)} - \text{PW(S)}}$$

For Future worth formulation

$$\text{BCR} = \frac{\text{FW(B)}}{\text{FW(I)} + \text{FW(O\&M)} - \text{FW(S)}}$$

For Annual worth formulation

$$\text{BCR} = \frac{\text{AW(B)}}{\text{AW(I)} + \text{AW(O\&M)} - \text{AW(S)}}$$

5.2 Modified benefit/cost ratio

It is the ratio of gross benefits to costs and expressed as

$$\frac{\text{benefit} - \text{operational and maintenance cost}}{\text{investment} - \text{salvage}}$$

For present worth formulation

$$\text{BCRM} = \frac{\text{PW(B)} - \text{PW(O\&M)}}{\text{PW(I)} - \text{PW(S)}}$$

For Future worth formulation

$$\text{BCRM} = \frac{\text{FW(B)} - \text{FW(O\&M)}}{\text{FW(I)} - \text{FW(S)}}$$

For Annual worth formulation

$$\text{BCRM} = \frac{\text{AW(B)} - \text{AW(O\&M)}}{\text{AW(I)} - \text{AW(S)}}$$

NOTE:

If B/C > 1 accept

If B/C = 1 Remain indifferent

If B/C < 1 Reject

5.3 Break-even analysis

It is point at which revenue is equal to cost. At the break-even point no profit is being made nor is any loss incurred.

If there are more than one investment opportunities under consideration depending upon a single and common parameter, the value of this parameter can be determined by equating these EW for which the conclusion is standoff.

For example,

$$\text{EWA} = f_1(x), \text{ and}$$

$$\text{EWA} = f_2(x)$$

Break-even point is the value of x for which,

$$F_1(x) = f_2(x)$$

It is based on two types of cost .i.e.



Fixed cost: these are the cost that are sent regardless of how many item you sell. It remains constant over a wide range of activities as long as the business does not permanently discontinue operations. All start of cost such as rent, insurance and computers etc. are considered fixed cost since you have to make these analysis before you sell your first item

Variable cost: these are recurring cost that you observed with each unit you sell. Variable cost changes with production level, work fee size and other parameters. It includes cost such as direct level, materials, indirect cost, advertisement and warranty

Let

s=selling price per unit

v= variable price per unit

Fc= Fixed price per unit

Q= quantity of production

then,

Total sells(S)=s × Q. (i)

Total cost of firm= variable cost + fixed cost

or, Tc= v × Q + Fc

At break-even point

Total cost = Total sales

or, variable cost + fixed cost= s × Q

or, v × Q + Fc = s × Q

or, Fc= Q(s-v)

∴ $Q = \frac{Fc}{s-v}$ units

SOME SOLVED NUMERICAL PROBLEMS

Q.N.1. Find the both types of B/C ratios by using PW formulation.

Investment	Rs. 100000
Annual revenue	Rs. 40000
Annual cost	Rs. 19000
Salvage Value	Rs. 20000
MARR	10%
Numbers of year	10

Solution:

i=10%

N=10

PW(I) = Rs 100000

$PW(B) = 40000(P/A, 10\%, 10) = A \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right] = 40000 \left[\frac{(1+0.1)^{10} - 1}{0.1(1+0.1)^{10}} \right] = \text{Rs } 245782.6842$

$PW(O\&M) = 19000(P/A, 10\%, 10) = 19000 \left[\frac{(1+0.1)^{10} - 1}{i(1+0.1)^{10}} \right] = \text{Rs } 116746.775$

$PW(S) = 20000(P/F, 10\%, 10) = F(1+i\%)^{-N} = 20000(1+10\%)^{-N} = \text{Rs } 7710.865$

Now,

Conventional B/C ratio:

$$BCR = \frac{PW(B)}{PW(I) + PW(O\&M) - PW(S)} = \frac{245784}{100000 + 116747.4 - 7710}$$

=1.17 > 1, The project is acceptable.

Modified B/C ratio

$$BCRM = \frac{PW(B) - PW(O\&M)}{PW(I) - PW(S)} = \frac{245784 - 116747.4}{100000 - 7710} = 1.39 > 1, \text{ The project is acceptable.}$$



Q.N.2. Calculate the both types of B/C ratio in FW formulation for the project with the following cash flow. Use gradient formula to convert the gradient cash flow into FW. First cost = Rs 200000, Project life= 5 years, Salvage value= Rs 40000, Annual O&M costs = Rs 20000 and interest rate = Rs 17%. The benefit from the project at the end of first year is Rs 5 0000, at the end of the second year is Rs 75000 and so on i.e. increasing by Rs 25000 each year for five years.

Solution: Given

Present cost(P)=200000

Interest rate(i)=17%

Annual O&M(A)=20000

Salvage value(S)=40000

Now,

$$FW(I) = 200000(F/P, 17\%, 5) = 200000(1 + 17\%)^5 = \text{Rs } 438489.61$$

$$FW(S) = \text{Rs } 40000$$

$$FW(O\&M) = 20000(F/A, 17\%, 5) = 20000 \left[\frac{(1+17\%)^5 - 1}{17\%} \right] = \text{Rs } 140288$$

Future worth of Benefit:

Benefit from the project is in increasing gradient series, where G = Rs 25000.

Let us first convert the gradient series into Uniform series.

$$A = 50000 + 25000(A/G, 17\%, 5) = 50000 + 25000 \left[\frac{((17\%+1)^5) - 17\% \cdot 5 - 1}{17\%(17\%+1)^5 - 17\%} \right]$$

$$= \text{Rs } 92232.45$$

$$\text{Now, } FW(B) = 92232.45(F/A, 17\%, 5) = 92232.45 \left[\frac{(1+17\%)^5 - 1}{17\%} \right] = \text{Rs } 646955.32$$

Conventional B/C Ratio

$$BCR = \frac{FW(B)}{FW(I) + FW(O\&M) - FW(S)} = \frac{646955.32}{438489.61 + 140288 - 40000}$$

$$= 1.20 > 1, \text{ The project is acceptable}$$

Modified B/C Ratio

$$BCRM = \frac{FW(B) - FW(O\&M)}{FW(I) - FW(S)} = \frac{646955.32 - 140288}{438489.61 - 40000}$$

$$= 1.27 > 1, \text{ The project is acceptable}$$



CHAPTER SIX

INVESTMENT DECISIONS

6.1 Comparison of alternatives having same useful life

When useful lives of the alternatives being compared are the same and are also equal to the study period, we may take help of the following techniques for comparing mutually exclusive alternatives.

➤ **By using Equivalent worth (PW, FW, AW) methods:**

Step-1: Calculate equivalent worth of each alternative at MARR

Step-2: Prefer project with the greatest equivalent worth

The following example will help understand the steps outlined above.

Example:

- A. Out of the following projects, recommend the best one to be implemented by using Equivalent Worth (PW,FW and AW) methods. Study period is 15 Years each and MARR = 9% per year.

	Project		
	A	B	C
Fist cost (RS)	40000	360000	725000
Net annual revenue (RS)	110000	79000	166000
Salvage value(RS)	34000	28600	53000

Solution

a.PW Method:

Present Worth of A,

$$PW_A(9\%) = -400000 + 110000(P/A, 9\%, 15) + 34000(P/F, 9\%, 15)$$

$$= -400000 + 110000 \left[\frac{(1+9\%)^{15} - 1}{9\%} \right] + 34000(1+9\%)^{-15} = \text{Rs } 496010$$

Present Worth of B,

$$PW_B(9\%) = -360000 + 79000(P/A, 9\%, 15) + 28600(P/F, 9\%, 15)$$

$$= -360000 + 79000 \left[\frac{(1+9\%)^{15} - 1}{9\%} \right] + 28600(1+9\%)^{-15} = \text{Rs. } 284646$$

Present Worth of C,

$$PW_C(9\%) = -725000 + 166000(P/A, 9\%, 15) + 53000(P/F, 9\%, 15)$$

$$= -725000 + 166000 \left[\frac{(1+9\%)^{15} - 1}{9\%} \right] + 53000(1+9\%)^{-15} = \text{Rs. } 627625$$

it is found that $PW_C(9\%) > PW_A(9\%) > PW_B(9\%)$

Hence select project C.

b.FW Method:

Future Worth of A,

$$FW_A(9\%) = -400000(F/P, 9\%, 15) + 110000(F/A, 9\%, 15) + 34000$$

$$= -400000(1+9\%)^{15} + 110000 \left[\frac{(1+9\%)^{15} - 1}{9\%} \right] + 34000 = \text{Rs. } 1806699$$

Future Worth of B,

$$FW_B(9\%) = -360000(F/P, 9\%, 15) + 79000(F/A, 9\%, 15) + 28600$$

$$= -360000(1+9\%)^{15} + 79000 \left[\frac{(1+9\%)^{15} - 1}{9\%} \right] + 28600 = \text{Rs } 1036811$$

Future Worth of C,

$$FW_C(9\%) = -725000(F/P, 9\%, 15) + 166000(F/A, 9\%, 15) + 53000$$

$$= -725000(1+9\%)^{15} + 166000 \left[\frac{(1+9\%)^{15} - 1}{9\%} \right] + 53000 = \text{Rs } 2286096$$

It is found that $FW_C(9\%) > FW_A(9\%) > FW_B(9\%)$.

Hence select project C.

**c. AW Method:****Annual Worth of A,**

$$AW_A(9\%) = -400000(A/P, 9\%, 15) + 110000 + 34000(A/F, 9\%, 15)$$

$$= -400000 \left[\frac{9\%(1+9\%)^{15}}{(1+9\%)^{15}-1} \right] + 110000 + 34000 \left[\frac{9\%}{(1+9\%)^{15}-1} \right] = \text{Rs. } 61519$$

Annual Worth of B,

$$AW_B(9\%) = -360000(A/P, 9\%, 15) + 79000 + 28600(A/F, 9\%, 15)$$

$$= -360000 \left[\frac{9\%(1+9\%)^{15}}{(1+9\%)^{15}-1} \right] + 79000 + 28600 \left[\frac{9\%}{(1+9\%)^{15}-1} \right] = \text{Rs. } 35300$$

Annual Worth of C,

$$AW_C(9\%) = -725000(A/P, 9\%, 15) + 166000 + 53000(A/F, 9\%, 15)$$

$$= -725000 \left[\frac{9\%(1+9\%)^{15}}{(1+9\%)^{15}-1} \right] + 166000 + 53000 \left[\frac{9\%}{(1+9\%)^{15}-1} \right] = \text{Rs. } 77835$$

It is found that $AW_C(9\%) > AW_A(9\%) > AW_B(9\%)$.

Hence select project C.

➤ **By using Rate of Return methods:**

As inconsistent ranking will occur while sorting the projects based on IRR. So, we should not select the alternative having the greatest IRR, rather we go for incremental analysis. Hence, we would go through the following steps:

Step-1: Compute IRR for each alternative;

Step-2: Compare it with the MARR.

If $IRR \geq MARR$, consider it for incremental analysis

What is Incremental Analysis?

Incremental analysis evaluates the difference, or the "increment" between two or more mutually exclusive alternatives. This approach is required to correctly apply for IRR and B/C ratio measures to make selection from mutually exclusive alternatives. The steps in incremental analysis are:

1. Order alternatives in increasing order of First Costs to ensure that the increments have cash flow patterns corresponding to investments
2. Establish a base alternative
 - Alternative having the least capital investment in. established as the base and it should have been pre-qualified i.e. $IRR > MARR$.
3. Use iteration to evaluate differences (incremental cash flows) between alternatives until no more alternatives exist.
4. Calculate the IRR for each incremental cash flow
5. Check for the Justification of incremental cash flows
 - If the increment is justified, select the alternative having greater investment; otherwise select the former one.
 - Simply repeat the procedure until final decision is made.

Example:

- A. An electronics company is trying to determine to which new product they should commit their limited capital resources. The information below shows the estimated net cash flow for each of the two products. If the MARR = 10% per year, make a selection by IRR method.**

End of year	Product 1	Product 2
0	-150000	-520000
1	50000	30000
2	50000	140000
3	50000	220000
4	50000	335000

Solution:

Determination of IRR

$$PW_1(i\%) = -150000 + 50000(P/A, i\%, 4) = 0$$



$$\text{or, } -15000 + 50000 A \left[\frac{(1+i)^4 - 1}{i(1+i)^4} \right] = 0$$

$$i' \% = 12.6\%$$

$$PW_2(i' \%) = -520000 + 30000(P/F, i' \%, 1) + 140000(P/F, i' \%, 2) + 220000(P/F, i' \%, 3) + 335000(P/F, i' \%, 4) = 0$$

$$\text{or, } -520000 + 30000(1+i' \%)^{-1} + 140000(1+i' \%)^{-2} + 220000(1+i' \%)^{-3} + 335000(1+i' \%)^{-4} = 0$$

$$\therefore i' \% = 11.2\%$$

Comparison with MARR:

$IRR_1 = 12.6\% > \text{MARR (i.e. } 10\%)$, Product 1 is acceptable

$IRR_2 = 11.2\% > \text{MARR (i.e. } 10\%)$, Product 2 is acceptable

Both the product are acceptable at $\text{MARR} = 10\%$ per year. Since we are required to select any one of them, we perform incremental analysis.

Incremental analysis

Product 1 has been treated as a base case as it has less investment cost than that of Product 2 and the increment is done with this as a reference.

End of year	incremental cash flow	
	Product 1	Product 1 → Product 2 (i.e. CF of 2 – CF of 1)
0	-150000	-520000 – (-150000) = -370000
1	50000	30000 – 50000 = -20000
2	50000	140000 – 50000 = 90000
3	50000	220000 – 50000 = 170000
4	50000	335000 – 50000 = 285000
incremental IRR	12.60%	10.70%
is incremental justified? (i.e. greater than MARR)	Yes	Yes

$$PW'_2(i' \%) = -370000 - 20000(P/F, i' \%, 1) + 90000(P/F, i' \%, 2) + 170000(P/F, i' \%, 3) + 285000(P/F, i' \%, 4) = 0$$

$$\text{or, } -370000 - 20000(1+i' \%)^{-1} + 90000(1+i' \%)^{-2} + 170000(1+i' \%)^{-3} + 285000(1+i' \%)^{-4} = 0$$

$$\therefore i' \% = 110.70\%$$

Decision: From incremental analysis, it is seen that if we go for selecting Product 2, leaving behind- the once, the increment it still justified or good. Hence, select Product 2.

B. An engineering firm is considering the following mutually exclusive alternatives

EOY	Projects			
	A1	A2	A3	A4
0	-2500	-1200	-3600	-2000
1	1200	400	1700	800
2	1400	800	2000	700
3	1500	1000	1600	850

Which project would you select based on IRR method, assuming that $\text{MARR} = 20\%$ per year?

Solution:

Determination of IRR

$$PW_{A1}(i' \%) = -2500 + 1200(P/F, i' \%, 1) + 1400(P/F, i' \%, 2) + 1500(P/F, i' \%, 3) = 0$$

$$\text{or, } -2500 + 1200(1+i' \%)^{-1} + 1400(1+i' \%)^{-2} + 1500(1+i' \%)^{-3} = 0$$

$$\therefore i' \% = 28.19\%$$

$$PW_{A2}(i' \%) = -1200 + 400(P/F, i' \%, 1) + 800(P/F, i' \%, 2) + 1000(P/F, i' \%, 3) = 0$$

$$\text{or, } -1200 + 400(1+i' \%)^{-1} + 800(1+i' \%)^{-2} + 1000(1+i' \%)^{-3} = 0$$

$$\therefore i' \% = 31.84\%$$

$$PW_{A3}(i' \%) = -3600 + 1700(P/F, i' \%, 1) + 2000(P/F, i' \%, 2) + 1600(P/F, i' \%, 3) = 0$$

$$\text{or, } -3600 + 1700(1+i' \%)^{-1} + 2000(1+i' \%)^{-2} + 1600(1+i' \%)^{-3} = 0$$

$$\therefore i' \% = 22.33\%$$

$$PW_{A4}(i' \%) = -2000 + 800(P/F, i' \%, 1) + 700(P/F, i' \%, 2) + 850(P/F, i' \%, 3) = 0$$

$$\text{or, } -2000 + 800(1+i' \%)^{-1} + 700(1+i' \%)^{-2} + 850(1+i' \%)^{-3} = 0$$



$$\therefore i' \% = 8.43\%$$

Comparison with MARR

$$IRR_{A1} = 28.19 > MARR$$

$$IRR_{A2} = 31.84 > MARR$$

$$IRR_{A3} = 22.33 > MARR$$

$$IRR_{A4} = 8.43 < MARR, \text{ it is not qualified for incremental analysis.}$$

Incremental Analysis

We are now left with three projects - A1, A2 and A3 for incremental analysis.

Out of these alternatives, A2 has the least investment (i.e., Rs. 1200) and we treat it as a base case. Then with increasing investment, we go for analysis as below:

EOY	Projects		
	A ₂	A ₂ →A ₁ (A ₁ *)	A ₁ →A ₃ (A ₃ *)
0	-1200	=-25000-(-12000)=-1300	=-36000-(-25000)=-1100
1	400	=1200-400=800	=1700-1200=500
2	800	=1400-800=600	=2000-1400=600
3	1000	=1500-1000=500	=1600-1500=100
incremental IRR	31.84%	23.87%	5.39%
is incremental justified?	Yes	Yes	No

$$PW_{A1*} = -1300 + 700(P/F, i' \%, 1) + 600(P/F, i' \%, 2) + 500(P/F, i' \%, 3) = 0$$

$$\text{or, } -1300 + 700(1+i' \%)^{-1} + 600(1+i' \%)^{-2} + 500(1+i' \%)^{-3} = 0$$

$$\therefore i' \% = 23.87\%$$

$$PW_{A2*} = -1100 + 500(P/F, i' \%, 1) + 600(P/F, i' \%, 2) + 100(P/F, i' \%, 3) = 0$$

$$\text{or, } -1100 + 500(1+i' \%)^{-1} + 600(1+i' \%)^{-2} + 100(1+i' \%)^{-3} = 0$$

$$\therefore i' \% = 5.39\%$$

Comparison with MARR

$$IRR_{A1*} = 23.87 > MARR \text{ it is justified.}$$

$$IRR_{A2*} = 5.39 < MARR \text{ it is not justified.}$$

Decision

It is seen that

if we select A₁ leaving behind A₂, it is justified.

if we select A₃ leaving behind A₂, it is not justified.

Hence select A₁.

C. From the following two mutually exclusive alternatives, make a selection based on IRR method assuming that MARR = 12% per year

EOY	Alternatives	
	P	Q
0	-12000	-12000
1	4500	5000
2	4200	4000
3	4100	4000
4	4000	3500

Solution:

Determination IRR

$$PW_P(i' \%) = -12000 + 4500(P/F, i' \%, 1) + 4200(P/F, i' \%, 2) + 4100(P/F, i' \%, 3) + 4000(P/F, i' \%, 4) = 0$$

$$\text{or, } -12000 + 4500(1+i' \%)^{-1} + 4200(1+i' \%)^{-2} + 4100(1+i' \%)^{-3} + 4000(1+i' \%)^{-4} = 0$$

$$i' \% = 15.3\%$$

$$PW_Q(i' \%) = -12000 + 5000(P/F, i' \%, 1) + 4000(P/F, i' \%, 2) + 4000(P/F, i' \%, 3) + 3500(P/F, i' \%, 4) = 0$$

$$\text{or, } -12000 + 5000(1+i' \%)^{-1} + 4000(1+i' \%)^{-2} + 4000(1+i' \%)^{-3} + 3500(1+i' \%)^{-4} = 0$$

$$i' \% = 15\%$$

Comparison with MARR



$$IRR_P = 15.3\% > MARR$$

$$IRR_Q = 15.01\% > MARR$$

Both of them are qualified for incremental analysis.

Incremental Analysis

Since initial investment is same for both the alternatives, we cannot establish the base case by considering the investment only. The base alternative is selected in such a way that the first incremental cash flow becomes negative. So choose Alternative Q as a base.

EOY	Alternatives	
	Q	Q→P(P*)
0	-12000	0
1	5000	-500
2	4000	200
3	4000	100
4.00%	3500	500
incremental IRR	15%	23%
is incremental justified?	yes	yes

$$PW_P(i'%) = 0 - 500(P/F, i'%, 1) + 200(P/F, i'%, 2) + 100(P/F, i'%, 3) + 500(P/F, i'%, 4) = 0$$

$$\text{or, } -500(1+i'%)^{-1} + 200(1+i'%)^{-2} + 100(1+i'%)^{-3} + 500(1+i'%)^{-4} = 0$$

$$\therefore i'% = 22.70\% > MARR$$

Decision

Select Alternative P.

6.2 Comparison of alternatives having different useful life

When the alternatives under consideration have different useful lives, we first establish a common length by using either Repeatability assumption or Co-terminated assumption as below:

Repeatability Assumption:

The analysis period over which alternatives are being compared is either indefinitely long or spans a common multiple of the involved assets. The economic consequences that are estimated to happen in an alternative's initial life span will repeat in all the succeeding life spans. This depends on the assumption that assets will be repeated by successors having identical cost characteristics. For example if the alternatives extend for 4 and 6 years, a 12-year period is used for analysis.

Co-terminated assumption:

This assumption uses a finite and identical study period for all feasible alternatives and analysis is made over this period. The planning horizon, thus chosen, could be

- life of shorter lived alternative
- life of longer lived alternative
- less than the shorter lived alternative
- greater than the longer lived alternative
- in between the shortest and longest lived alternatives

Examples

A. Make a selection from the following two mutually exclusive alternatives:

alternatives	A	B
capital investment (Rs)	450000	600000
annual revenue (Rs)	22000	26000
annual expenses (Rs)	7450	11020
useful life(yrs.)	6	8
market value (Rs)	25000	28000
MARR	10% per year	



Make any assumption and use any method.

Solution:

Since two alternatives have unequal useful lives, we have to fix the analysis period first and this can be done by using repeatability or co-terminated assumption.

Study period:

Using repeatability assumption,

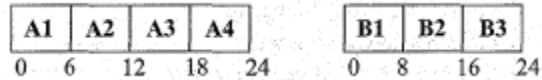
L.C.M. of 6 & 8 = 24 years

Then, study period = 24 years

Comparison

Alternative, A should be repeated $24/6 = 4$ times

Alternative B should be repeated $24/8 = 3$ times



$$\begin{aligned} PWA(10\%) &= -450000[1 + (P/F, 10\%, 6) + (P/F, 10\%, 12) + (P/F, 10\%, 18) + (P/F, 10\%, 24)] + (22000 - 7450)(P/A, 10\%, 24) + 25000[(P/F, 10\%, 6) + (P/F, 10\%, 12) + (P/F, 10\%, 18) + (P/F, 10\%, 24)] \\ &= -450000[1 + (1+10\%)^{-6} + (1+10\%)^{-12} + (1+10\%)^{-18} + (1+10\%)^{-24}] + (22000 - 7450)(P/A, 10\%, 24) + 25000[(1+10\%)^{-6} + (1+10\%)^{-12} + (1+10\%)^{-18} + (1+10\%)^{-24}] \\ &= -768510 \text{ Rs} \end{aligned}$$

$$\begin{aligned} PWB(10\%) &= -600000[1 + (P/F, 10\%, 8) + (P/F, 10\%, 16) + (P/F, 10\%, 24)] + (26000 - 11020)(P/A, 10\%, 24) + 28000[(P/F, 10\%, 8) + (P/F, 10\%, 16) + (P/F, 10\%, 24)] \\ &= -600000[1 + (1+10\%)^{-8} + (1+10\%)^{-16} + (1+10\%)^{-24}] + (26000 - 11020)(1+10\%)^{-24} + 28000[(1+10\%)^{-8} + (1+10\%)^{-16} + (1+10\%)^{-24}] \\ &= -853872 \text{ Rs} \end{aligned}$$

Decision

$PWA(10\%) > PWB(10\%)$, Select alternative A.

B. Solve the above problem by using co-terminated assumption.

Solution:

Study period

While fixing study period using co-terminated assumption, it would be wise to take the value in such a way that study period is either equal to or greater than useful lives of all the alternatives. In other words, study period can be taken as the life of longer lived alternative. In this example, it would be taken as 8 years. When study period happens to be lesser than useful lives (say 3 years in this example), we have to curtail down the cash flow to the end of study period and suitable market value should be assigned to the alternative. It can be done by using *the imputed market value technique* which is discussed latter. Now, we go ahead with 8 years study period.



Equivalent worth:

From the above figures, we see that there is no adjustment required for alternative B. However, in case of alternative A, study period is 2 year greater than its useful life. When study period is greater than useful life, it is assumed that the cash flows accumulated at the end of useful life will be reinvested for the extended periods.

Therefore, we proceed with FW calculation.

$$\begin{aligned} FW_B(10\%) &= -600000(F/P, 10\%, 8) + (26000 - 11020)(F/A, 10\%, 8) + 28000 \\ &= -600000(1+10\%)^{-8} + (26000 - 11020) \frac{(1+10\%)^8 - 1}{10\%} + 28000 \\ &= 1086850 \text{ Rs} \end{aligned}$$

$$\begin{aligned} FW_A(10\%) &= [-450000(F/P, 10\%, 6) + (22000 - 7450)(F/A, 10\%, 6) + 25000](F/P, 10\%, 2) \\ &= [-450000(1+10\%)^{-6} + (22000 - 7450) \frac{(1+10\%)^6 - 1}{10\%} + 25000](1+10\%)^{-2} \\ &= \text{Rs } -798527.7926 \text{ Rs} \end{aligned}$$

Decision

$FWA(10\%) > FWB(10\%)$, select alternative A.

C. Two. mutually exclusive alternatives are shown below:



alternatives	M	N
capital investment (Rs)	500000	700000
annual revenue (Rs)	110000	110000
useful life(yrs.)	3	5
market value (Rs)	40000	40000
MARR	10% per year	

Select the best project by using PW method for a predetermined study period of 5 years.

Solution:

Study period

We are required to analyze the alternatives over 5 years study period.

Comparison



PW calculation

Since the projects are being compared over 5 years, it implies that there is a projection of additional 2 years for Alternative M and 1 year for Alternative N. So, we first calculate FW at the end of 5th year and convert it into PW.

$$FW_M(10\%) = [-500000(F/P, 10\%, 3) + 110000(F/A, 10\%, 3) + 40000](F/P, 10\%, 2)$$

$$= [-500000(1+10\%)^3 + 110000 \frac{(1+10\%)^3 - 1}{10\%} + 40000] (1+10\%)^2$$

$$= -261400 \text{ Rs}$$

$$\text{Therefore, } PWM(10\%) = -261400(P/F, 10\%, 5) = -162303 \text{ Rs}$$

$$FW_N(10\%) = [-700000(F/P, 10\%, 4) + 110000(F/A, 10\%, 4) + 40000](F/P, 10\%, 1)$$

$$= [-700000(F/P, 10\%, 4) + 110000 \frac{(1+10\%)^4 - 1}{10\%} + 40000](F/P, 10\%, 1)$$

$$= \text{Rs } -521796$$

$$\text{Therefore, } PW_N(10\%) = -521796 (P/F, 10\%, 5) = -521796 (1 + 10\%)^{-5} = \text{Rs } -323994.2627$$

Decision

$PWM(10\%) > PW_N(10\%)$, select Alternative M.

D. Solve the above problem by using repeatability assumption and AW method.

Solution:

Study period

LCM of 3 & 4 = 12 years

Annual Worth calculation

It should be remembered that AW over the initial useful life of each alternative is the same as its AW over the length of study period. So, we calculate AW over each useful life rather than 45 years.

$$AW_M(10\%) = -500000(A/P, 10\%, 3) + 110000 + 40000(A/F, 10\%, 3)$$

$$= -500000 \left(\frac{10\%(1+10\%)^3}{(1+10\%)^3 - 1} \right) + 110000 + 40000 \left[\frac{10\%}{(1+10\%)^3 - 1} \right]$$

$$= -78966 \text{ Rs}$$

$$AW_N(10\%) = -700000(A/P, 10\%, 4) + 110000 + 40000(A/F, 10\%, 4)$$

$$= -700000 \left(\frac{10\%(1+10\%)^4}{(1+10\%)^4 - 1} \right) + 110000 + 40000 \left(\frac{10\%}{(1+10\%)^4 - 1} \right)$$

$$= -102230 \text{ Rs}$$

Decision

$AW_M(10\%) > AW_N(10\%)$, Select Alternative M.

6.3 Comparison of alternatives including of excluding the time value of money

6.4 Comparison of alternatives using the capitalized worth method

Public project usually accrue benefits in the long run. For example, Hydropower projects, Road facilities, Irrigation schemes, Bridge structures etc. have substantially longer service lives and the benefits they generate is also



extended over this period. While evaluating or comparing such projects, we introduce a special case of PW in which PW of a cash flow series is computed over an infinite or extremely long period of time (40 years or more) in order to make effective analysis is referred to as Capitalized worth. This criterion for evaluating and comparing the alternative is useful in places where

- The project has a perpetual service life
- Planning horizon is extremely long because of project's service life
- Repeatability assumption is applicable

We know that PW of uniform series is calculated by the relation,

$$PW(i\%) = A(P/A, i\%, N)$$

When $N \rightarrow \infty$, The PW is called as Capitalized worth.

This gives,

$$CW(i\%) = PW \rightarrow \infty(i\%) = A(P/A, i\%, \infty) = A \left[\lim_{N \rightarrow \infty} \frac{(1+i)^N - 1}{i(1+i)^N} \right] = \frac{A}{i}$$

Thus the following steps involve in computing the Capitalized worth of any investment proposal

- Find the AW of the project
- Divide this AW by the MARR or interest rate

Examples

A. A mobile company is taking quotations on the purchase, installation, and operation of microwave towers:

	Quotation A	Quotation B
Equipment cost(Rs.)	6500000	5800000
Installation cost (Rs.)	1500000	2000000
Annual maintenance cost (Rs.)	100000	125000
Annual extra taxes(Rs.)	0	50000
Life(yrs.)	40	35
Salvage value(Rs.)	0	0

Which is the most economical quotation, if the interest rate is considered to be 12% per year?

Solution:.

Since both the quotations have shown that the alternative have longer useful lives, we use capitalized worth method for comparison.

For quotation A,

$$\begin{aligned} AW_A(12\%) &= -6500000(A/P, 12\%, 40) - 1500000(A/P, 12\%, 40) - 100000 \\ &= -6500000 \left(\frac{12\%(1+12\%)^{40}}{(1+12\%)^4 - 1} \right) - 1500000 \left(\frac{12\%(1+12\%)^{40}}{(1+12\%)^4 - 1} \right) - 100000 \\ &= -1070429 \text{ Rs} \end{aligned}$$

For quotation B,

$$\begin{aligned} AW_B(12\%) &= -5800000(A/P, 12\%, 35) - 2000000(A/P, 12\%, 35) - 125000 - 50000 \\ &= -5800000 \left(\frac{12\%(1+12\%)^{35}}{(1+12\%)^{35} - 1} \right) - 2000000 \left(\frac{12\%(1+12\%)^{35}}{(1+12\%)^{35} - 1} \right) - 125000 - 50000 \\ &= -1129069.631 \text{ Rs} \end{aligned}$$

Capitalize worth

$$CW_A(12\%) = \frac{AW_A(12\%)}{i} = -\frac{1070429}{0.12} = -8920241$$

$$CW_B(12\%) = \frac{AW_B(12\%)}{i} = -\frac{1129069.631}{0.12} = -9408913.59$$

We found that $CW_A(12\%) > CW_B(12\%)$ Hence, select quotation A so as to reduce the cost.

B. Assuming infinite project life, recommend one of the following mutually exclusive projects if MARR = 12%

Item	Product A	Product B
First cost(Rs.)	500000	1200000
Salvage value(Rs.)	100000	180000
Annual costs(Rs.)	90000	60000
Useful Life(Yrs.)	20	50

Solution:



In the given example, infinite project life is assumed which guides us to use the Capitalized Worth method for comparing these alternatives.

Annual Worth:

$$AW_A(12\%) = -500000(A/P, 12\%, 20) + 100000(A/F, 12\%, 20) - 90000$$

$$= -500000 \left(\frac{12\%(1+12\%)^{20}}{(1+12\%)^{20}-1} \right) + 100000 \left[\frac{12\%}{(1+12\%)^{20}-1} \right] - 90000$$

$$= -155560 \text{ Rs}$$

$$AW_B(12\%) = -1200000(A/P, 12\%, 50) + 180000(A/F, 12\%, 50) - 60000$$

$$= -1200000 \left(\frac{12\%(1+12\%)^{50}}{(1+12\%)^{50}-1} \right) + 180000 \left[\frac{12\%}{(1+12\%)^{50}-1} \right] - 60000$$

$$= -204425 \text{ Rs}$$

Capitalize worth

$$CW_A(12\%) = \frac{AW_A(12\%)}{i} = -\frac{155560}{0.12} = -1296333$$

$$CW_B(12\%) = \frac{AW_B(12\%)}{i} = -\frac{204425}{0.12} = -1703541.64$$

We found that $CW_A(12\%) > CW_B(12\%)$, Project A recommended.

6.5 Definition of mutually exclusive investment alternatives in terms of combinations of projects

While performing capital budgeting analysis, the firm must bring all the alternatives under consideration into decision activities. The firm should try to make the combination of these alternatives in order to maximize its present worth so long as the budget limitation does not exceed. In this process, the nature of each alternative and the relationship among alternatives are of paramount importance. These are described below: .

1. Independent Projects

A project is said to be independent if its selection is free from the accept-reject decision of any other projects in the group. Accordingly, expected costs and benefits of each project do not depend on whether or not the other one is chosen.

2. Dependent Projects

The projects related to one another in such a way that the acceptance or rejection of one project influences the acceptance of others are called dependent projects. The dependencies among these projects may be of the following types.

- Mutually exclusive projects
- Contingent projects

Mutually exclusive projects

Mutually exclusive projects are those when one project is chosen, all the other ones are excluded. Such cases come into decision making process mostly because of limitation of capital to invest in .

Contingent projects

Two or more projects are said to be contingent if the acceptance of one requires the acceptance of another. For example, the decision to connect internet is dependent upon the purchase of a computer, however, the computer can be purchased without internet connection.

Combination of projects

In many instances, investors face the situation where many investment alternatives are present. Depending upon the availability of capital budgets, the investors may require making the combination of more than one project and choosing the very combination for implementation. Thus, the number combination that could be made depends upon the nature of projects available for analysis.

A. Independent projects

Suppose we have two independent projects A and B. We can make the combinations as follows:

Mutually Exclusive combination	Explanation	Projects	
		A	B
1	Do nothing	0	0
2	Accept A	1	0
3	Accept B	0	1
4	Accept A, Accept B	1	1

**B. Mutually exclusive projects**

Suppose we have three mutually exclusive projects A,B and C We can make the combinations as follows:

Mutually Exclusive combination	Explanation	Projects		
		A	B	C
1	do nothing	0	0	0
2	Accept A	1	0	0
3	Accept B	0	1	0
4	Accept C	0	0	1

C. Contingent projects

Suppose we have three projects A,B and C where the project C is contingent on the acceptance of B and acceptance of B is contingent on acceptance of A. We can make the combinations as follows:

Mutually Exclusive combination	Projects		
	A	B	C
1	0	0	0
2	1	0	0
3	1	1	0
4	1	1	1

Examples:

A. Engineering projects A, B1, B2 and C are being considered with cash flows estimated over ten year as shown in the table below. The capital investment budget limit is Rs. 100000, and the MARR is 12% per year.

a. List all possible alternatives.

b. Develop the net cash flows for all feasible alternatives.

c. Which investment alternative (Combination of projects) should be selected? use the PW method.

	A	B1	B2	C
Capital investment	30000	22000	70000	82000
Annual Revenue	8000	6000	14000	18000
Market Value	3000	2000	5000	7000
B1 & B2	Mutually exclusive			
C	Dependent on acceptance of B2			
A	Dependent on acceptance of B1			

Solution



Present Worth calculation of each project:

	A	B1	B2	C
Capital Investment	30000	22000	70000	82000
Annual revenues	8000	6000	14000	18000
Market value	3000	2000	5000	7000
PW(12%)	16168	12545	10713	21958

Mutually exclusive project combination

Mutually exclusive combination	Project			
	A	B1	B2	C
1	0	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	1	1
5	1	1	0	0

Combined project cash flow:

Items	Mutually exclusive combination				
	1	2	3	4	5
Capital Investment	0	22000	70000	152000	52000
Annual revenues	0	6000	14000	32000	14000
Market value	0	2000	5000	12000	5000
PW(12%)	0	12545	10713	32671	28713

As our capital investment budget is limited to Rs. 100000, the mutually exclusive combination #4 is excluded from the list.

Out of the remaining combinations, the combination.#5 has the highest Present Worth at MARR, $PW(12\%) = \text{Rs } 28713$. The independent investment projects are being considered

B. Three independent investment projects are being considered

Items	Mutually exclusive combination				
	1	2	3	4	5
Capital Investment	0	22000	70000	152000	52000
Annual revenues	0	6000	14000	32000	14000
Market value	0	2000	5000	12000	5000
PW(12%)	0	12545	10713	32671	28713

Assuming a study period of 15 years and $MARR = 10\%$ per year, which project(s) should be chosen if investment funds are limited to Rs. 250000? State any assumptions.

Solution:

The projects has unequal useful lives and they study period is fixed as 15 years. Application of co-terminated assumption best suits here. While using co-terminated assumption, we first compute the FW at the end of each project's useful life and extend it to the end of 15 years shown a below:

$$FW_x(10\%) = [-100000(F/P, 10\%, 10) + 16280(F/A, 10\%)](F/P, 10\%, 5) \\ = \text{Rs. } 146.34$$

$$PW_x(10\%) = 146.31(P/F, 10\%, 15) = \text{Rs. } 35.026$$

$$FWY(10\%) = -150000(F/P, 10\%, 15) + 22020(F/A, 10\%, 15) = \text{Rs. } 73050.45$$

$$PWY(10\%) = 73050.45(P/F, 10\%, 15) = \text{Rs. } 17488.28$$

$$FW_z(10\%) = [-200000(F/P, 10\%, 8) + 40260(F/A, 10\%, 8)]$$



$(F/P, 10\%, 7) = \text{Rs. } 61753$

$PW_z(10\%) = 61753(P/F, 10\%, 15) = \text{Rs. } 14783.67$

Mutually exclusive combination of independent projects

Mutually exclusive Combination	Project		
	X	Y	Z
1	0	0	0
2	1	0	0
3	0	1	0
4	0	0	1
5	1	1	0
6	1	0	1
7	0	1	1
8	1	1	1

Combined project cash flow:

Mutually exclusive Combination	Capital Investment	PW(10%)
1	0	0
2	100000	35.026
3	150000	17488.28
4	200000	14783.67
5	250000	17523
6	300000	14773
7	350000	32272
8	450000	32307

(Note: Since co-terminated assumption is used, there will be no additional investment for the projects. So, capital investment remains the same)

As our capital budget is limited to Rs. 250000, the combinations # 6, 7, 8 are excluded. Out of the remaining combinations, the combination# 5 has the highest PW, i.e. $PW(10\%) = \text{Rs } 17523$. Hence select the combination# 5 (Projects X and Y)

6.6 Comparison of mutually exclusive alternative

Like Internal Rate of Return (IRR), benefit cost analysis provides the relative measure of cash flows i.e. benefits and costs. So, selecting projects with higher BCR does not confirm the best project selection ~s inconsistent ranking occurs in doing so. In order to overcome this technical difficulty, we perform incremental analysis while comparing mutually exclusive alternatives by BCR and proceed as follows:

- Calculate B/C of each alternative and eliminate the ones having B/C ratio less than 1.
- Arrange the remaining alternatives in the increasing order of denominator.
- Compute the. Incremental differences of each alternatives and compute B/C for it
- If B/C thus calculated is still greater than 1, select the second alternative and go on repeating this comparison until final decision is made.



CHAPTER SEVEN

RISK ANALYSIS

Risk analysis

Risk analysis is a technique used to identify and assess factors that may jeopardize the success of project or achieving goal. It is the process of defining and analyzing the danger to individuals, businesses and government agencies posed by potential natural and human-caused adverse events.

7.1 Projects operating under conditions of certainty

Sometimes in decision it is reasonable to assume that only one state is relevant and then that the decision as if the states were certain to occur (our initial analyses were based on this)

7.2 Projects operating under conditions of uncertainty

Decision situation where several states are possible and sufficient information is not available to assign probability value to their occurrence

Sources of Uncertainty

The main source of uncertainty is always rooted in the data items affecting the cash components of the estimate: P, A, F, i, G, N and tax rates, etc. The sources of these uncertainties are:

- Measurement errors
- Unclear specifications
- Volatile and unpredictable future

Since Engineering Economy deals with the future, this uncertainty cannot be avoided. Thus sensitivity analysis must be used because:

- technology and global competition changes rapidly
 - Competitors have become very much "smarter"

7.3 Decision tree

A decision tree is a graphical representation of the logical structure of a decision problem in terms of the sequence of decisions and outcomes chance events. Decision Trees are excellent tools for helping us choose between several courses of action. They provide a highly effective structure within which we can lay out options and investigate the possible outcomes of choosing those options. They also help us form a balanced picture of the risks and rewards associated with each possible course of action. Alternative evaluation may require a series and decision where the outcome from one stage is important to the next stage of decision making. When each alternative is clearly defined and probability estimates can be made to account for risk it is helpful to perform the evaluation using decision tree.

Decision tree is a powerful means of facilitating the analysis of important problem especially those that involve sequential decision and variable outcome overtime

Decision trees are usually used when:

- Many future scenarios exists
- Large number of alternate solutions exist
- Some or many of the alternatives can be suitable for sequential decision making

A decision tree includes

- a. More than one stage of alternatives selection
- b. Selection of an alternative at one stage that leads to another stage
- c. Expect results from a decision at each stage
- d. Probability estimates for each outcomes
- e. Estimates of economic value (cost or revenue) for each outcome
- f. Measure of worth as the selection criterion such as E (PW)

Components of decision tree

- a. The decision tree is constructed left to right and includes each possible decision and outcome
- b. **Decision Node:** A square represents a decision node for making decision by a decision maker. e.g. Produce or outsource TV screens.
- c. **Chance node:** It represent an even whose outcome is uncertain e.g., uncertain demand for TV screens. It is represented by a circle which represent probability node with the possible outcomes and estimated probability of the branches
- d. **Branch:** It is a line connecting nodes from left to right of the diagram, depicting the sequence of possible decisions and chance events.



e.Node: The probability that is estimated must sum to 1 for each set of outcomes (branches) that results from decision.

7.4 Sensitivity analysis

Sensitivity analysis is aimed at establishing the effects of estimated input values on the recommendation for action. It provides us information on how sure are we that the data used was good? Or what is the risk in the decision due to data inaccuracies?

Sensitivity analysis is the study of relative magnitude of change in the measure of merit (e.g., PW, AW, IRR) caused by one or more changes in estimated study parameters and answers questions such as: would a 10% cost reduction still allow this to be a good project?

If the cost overrun exceeds 15% will this project become a loser?

If interest rates change, what be the effect?

Sensitivity analysis must be used because:

- It aids in making better decisions .
- Enables the decision maker to decide which factors it is worth to obtain additional information about, refine estimates
- Focuses managerial attention on the key variables during implementation

Sensitivity Graph

After preparing sensitivity table, it is customary to present the result in graphical form and this graph is called sensitivity graph or spider plot. While constructing spider plot, the following points should be kept in mind:

- a) PW, FW, AW or IRR etc. is plotted on y-axis;
- b) "% error in estimate of parameter value " or "% of base case value" is plotted on x,-axis
- c) The base-case values are used to calculate the PW which is then plotted on the graph. All curves will pass through this point, since 0% change in the base case value is part of each curve
- d) The curves on the graph are determined by calculating the resulting PW when 1 input variable is varied through its uncertainty range (may be linear or nonlinear). This step is repeated with the other variables.

Interpretation of Sensitivity Graph

- a) On a plot, there are 2 directions to measure uncertainty:
 - On the x-axis, the uncertainty in the input variable is measured
 - On the y-axis is measured the impact of that uncertainty on PW
- b) The slopes of the lines show how sensitive the PW is to changes in each of the inputs: the steeper the slope, the more sensitive the PW is to a change in a particular variable
- c) The graph allows us to identify the crucial variables that most affect the final outcome
- d) We should look for the values of the variables that lead to the PW = 0 or lower and then assess how likely is it that these values would occur.
- e) A properly drawn spider plot shows the following:
 - limits of uncertainty for each cash flow element
 - impact of each cash flow element on the PW, EAC or IRR (each can be used as Y-axis)
 - identification of each cash flow element that might change the recommendation

Some Numerical examples

Example

Q.N.1. Perform sensitivity analysis using PW method over a range of $\pm 40\%$ in (a) initial investment, (b) net annual revenue, (c) salvage value, and (d) useful life.

Initial investment (Rs) 200000

Annual revenues (Rs) 50000

Annual expenses (Rs) 5000

Salvage value (Rs) 25000

Useful life 10 yrs.

MARR 12% per year

Draw also the sensitivity graph. [T.U. 2059]

Solution

Prime equation:

$$PW(12\%) = -200000 + (50000 - 5000)(P/A, 12\%, 10) + 25000(P/F, 12\%, 10) = \text{Rs. } 62309$$

For $\pm 40\%$ fluctuation in:



(a) investment

$$PW(12\%) = -200000(1 \pm \frac{40}{100}) + (50000 - 5000)(P/A, 12\%, 10) + 25000(P/F, 12\%, 10)$$

The calculation has been shown in the table below.

(b) Net annual revenue

$$PW(12\%) = -200000(50000 - 5000)(1 \pm \frac{40}{100})(P/A, 12\%, 10) + 25000(P/F, 12\%, 10)$$

The calculation has been shown in the table below.

(c) salvage value

$$PW(12\%) = -200000(50000 - 5000)(P/A, 12\%, 10) + 25000(1 \pm \frac{40}{100})(P/F, 12\%, 10)$$

The calculation has been shown in the table below.

(d) useful life

$$PW(12\%) = -200000 + (50000 - 5000)$$

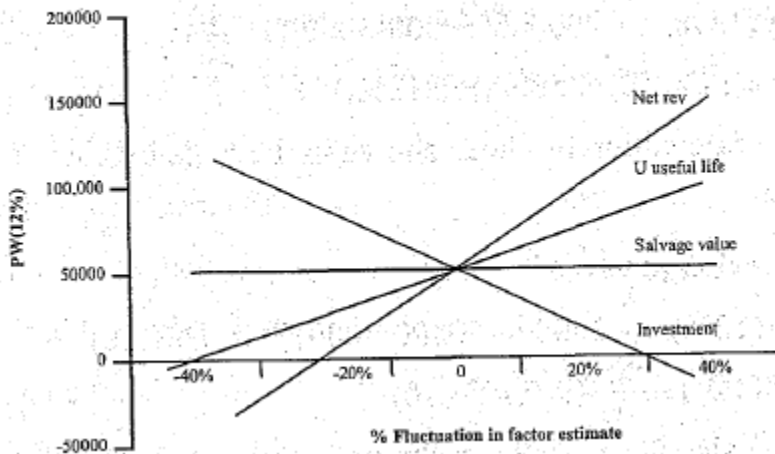
$$[P/A, 12\%, 10(1 \pm \frac{40}{100})] + 250000[P/F, 12\%, 10(1 \pm \frac{40}{100})]$$

The calculation has been shown in the table below.

Calculation Table:

	PW(12%)				
	-40%	-20%	0	20%	40%
Investment	142309	102309	62309	22309	-17691
Net rev	-39394.6	11457.2	62309	113160.8	164012.6
Salvage value	59089	60699	62309	63919	65529
Useful life	-2205	33639.5	62309	85165.5	103384

Sensitivity Graph



From the sensitivity graph, the slope of Net annual Revenue line is found to be the greatest and hence is more sensitive ..

Scenario Analysis

Scenarios analysis technique emphasizes the impact of combinations of uncertainty on the total outcome. It considers the sensitivity of the result e.g. PW, to simultaneous changes in key input variables over the range of likely values, Often a decision maker considers a 'worst case' scenario (e.g. low sales volume~, low price ,high costs) and a "best case scenario. The PW under both scenarios is calculated and compared to the PW of the base-case (or "most likely") scenario.Scenario analysis, however, doesnot provide the probability of these scenarios which would otherwise have been helped us interpret and decide.

Q.N.2. Suppose that for an engineering project, the optimistic, most likely and pessimistic estimates are as shown below:



	Optimistic	Most Likely	Pessimistic
Capital Investment (Rs)	80000	95000	120000
Useful life (yrs)	12	10	6
Market value (Rs)	30000	20000	0
Net annual cash flow (Rs)	35000	30000	20000
MARR	12%	12%	12%

What is the AW for each of the three estimation condition?

Solution:

For optimistic estimate,

$$AW(12\%) = -80000(A/P, 12\%, 12) + 35000 + 30000(A/F, 12\%, 12)$$

$$= \text{Rs. } 23330$$

For most likely estimate,

$$AW(12\%) = -95000(A/P, 12\%, 10) + 30000 + 20000(A/F, 12\%, 10)$$

$$= \text{Rs. } 14325$$

For pessimistic estimate,

$$AW(12\%) = -120000(A/P, 12\%, 6) + 20000$$

$$= \text{Rs. } -9184$$

If the AW for pessimistic and optimistic were positive, we would have made go decision but the result has shown that further computation is necessary.



CHAPTER EIGHT

TAXATION SYSTEM IN NEPAL

8.1 Taxation law in Nepal

Nepal has a long history in the taxation. Land tax has been found to be practiced already in Lichhavi period and excise duty in Rana regime. Interestingly, these taxes are still in existence. Up to the Rana regime, tax system was not institutionalized and method of collection too was not systematic. For example, custom duties used to be collected through contractors, land revenues by Mukhiya, Zimwal or Zamindar in different parts of the country and they were authorized by the then power centers.

Modern taxation system was rationalized by introducing the Income Tax in Nepal by the first Parliamentary Government in 1959. Income Tax Act 1962 was enacted in 1962 replacing business, Profit and Remuneration Tax Act of 1959. The Income Tax Act, 1962 was replaced by Income Tax Act, 1974, which was amended for eight times and existed for a period of 28 years. The Income Tax Act, 1974 and all the income tax related provisions made under other special enactment have been repealed and the existing Income Tax Act, 2058 became effective since Chaitra 19, 2058 (01, April 2002). The Act governs all income tax matters and is applicable throughout the Kingdom of Nepal. It is also applicable to residents residing wherever outside Nepal.

The Inland Revenue Department (IRD) is currently responsible for the enforcement of Tax Laws and administration of the following taxes: Income Tax, Value Added Tax, Excise Duty, Vehicle Tax and certain fee and duties like Entertainment fee (Film Development) fee, Tourism Development Fee, Special fee, etc. Likewise, The Department is also responsible for monitoring the non-tax revenue such as dividends, royalties etc. of the Government.

Tax Structure in Nepal

The tax structure in Nepal is divided into two types

A. Direct Tax

Direct tax is levied on individual person or corporations and tax burden ultimately lie on taxpayer. This tax has further been divided into:

- (i) Personal Tax, and
- (ii) Company or Corporate Tax:

Personal Tax

Profits from a business, salaries and benefits, income from house rent, interest and dividends are taxable income.

Income tax in case of individuals is the most encountered tax system and has the following rates:

The taxable income of a resident individual for an income-year will be taxed at the following rates:

Up to Rs. 85,000- Not taxable

From Rs. 85,000 - up to Rs. 1,55,000 - @ 15%

Above Rs. 1,55,000- @ 25% plus Rs. 11250

- The taxable income of a couple, if they chose to be treated as a couple will be taxed at the following rates;

Up to Rs.100,000- Not taxable

From Rs. 100,000,- upto Rs. 1,75,000-@ 15%

Above Rs. 1,75,000-@ 25% plus Rs. 11250.

Company or Corporate Tax

This tax is exempted on dividends declared by the company. Companies listed in the stock exchange are subject to 33% corporate tax. Expenses wholly and exclusively incurred in the generation of income are deducted from taxable corporate and personal income. 5 % of total gross income of industrial units may be deducted for advertisement, entertainment and other contingencies. Depreciation on all plant and machinery equipment ranges between 15%-25% per annum and industrial and commercial buildings between 5%- 7% per annum. Transport vehicles may be depreciated between 15%-25% per annum and furniture between 10%-15% per annum.

B. Indirect tax

Indirect tax is imposed on goods manufactured in or imported in to Nepal. The rate of tax varies depending on the nature of goods manufactured or imported. There are several types of indirect tax:

**i. Customs Duty:**

A customs duty is imposed on any goods imported. The duty varies from 0% to 110%. Live animals, fish and most primary products are exempt from import duties. Machinery or goods related to basic needs are charged 5% duty. For products hazardous to health such as cigarettes and liquor, duty is 110%. For luxury goods such as cosmetics, air conditioners etc. the duty is 40%. Any goods imported from The South Asia Association for Regional Co-operation (SAARC) which includes Nepal, India, Pakistan, Bangladesh, Bhutan, Sri Lanka and the Maldives will be charged 10% and 5% less on total customs duty respectively. In an effort to help combat air pollution in the Kathmandu Valley, the Government recently approved temporary reduction of duty to 10% and 5% for vehicles and parts respectively.

ii. Sale Tax:

There are two levels for sales tax: 10% and 20%. The tax is 10% on basic needs products like tea, cooking, oil, etc. And 20% on luxury products like cigarettes, TV, VCR, watches, vehicle, liquor, etc.

iii. Excise Duty or Countervailing Tax:

Goods imported are charged a countervailing tax whereas goods manufactured in Nepal are charged an excise duty only. The rate of excise duty or the countervailing tax varies depending on the nature of the goods in question. For example, cigarettes are taxed on the number of sticks and the duty on television is charged as a total percentage of the price.

iv. Value Added Tax (VAT):

VAT is a new tax system for Nepal. VAT was justified in the light of government fiscal imbalances and need for extra revenue mobilization through an efficient tax system. Although the VAT Act was passed in 1995 and the VAT regulation was approved in 1996, the VAT could only be introduced from November 16, 1997. The VAT replaced the then Sales tax, Hotel tax, Contract tax and Entertainment tax.

There is persistent increase in the number of VAT registrants. As the taxpayers are increasing, the amount of revenue collection and the level of tax compliance are improving too. The information regarding VAT has been given in detail under separate topics.

v. Octroi:

This tax is charged by municipalities on the transportation of goods from one municipality to another. The normal charge is 1% on the total value of goods.

8.2 Depreciation rates for buildings, equipment, furniture, etc.**Depreciation**

An asset starts losing value as soon as it is purchased and this gradual decrease in utility of fixed assets with use and time is called "Depreciation". However, it is not the case for art, collectibles, etc. The best example of depreciation is a car, which loses about 20-30% of its value when the customer takes delivery.

Guidelines for computing it is 'also in practice and in doing so, the following depreciation rates are applied:

Depreciation rates for building

Category	Depreciation rate per year	Total years
1	3%	25
2	2%	30
3	1%	70
4	0.75%	100

Where,

Building Category 1 → Walls with raw bricks inside and fine bricks outside in mud mortar and all types of timber houses

Building Category 2 → All fine bricks or stone masonry in mud mortar

Building Category 3 → Fine bricks in lime mortar or cement mortar

Building Category 4 → RCC framed structure

**Depreciation rates for others**

Machineries	10%
Telephone, typewriter, photocopy machine etc.	10%
Furniture	
–Metallic	8%
–Wooden and others	15%
Vehicles	
–Trucks, Bulldozers, tractor etc.	20%
–Bus, minibus, Van, Car, Jeep	15%
–By-cycle, motorbike, Tempo, Rickshaw	15%

8.3 Recaptured depreciation

When the depreciable asset used in business is sold for an amount greater than the estimated book value, we say the depreciation recaptured and the difference is called **recaptured depreciation**.

Recaptured Depreciation = Cost base - Book value

The recaptured depreciation is also treated as a taxable income under the existing tax law in Nepal.

The following relation describes the relationship and difference between Capital gains and Recaptured Depreciation:

Gains = Salvage value - Book Value
 = [Salvage value - Cost base] + [Cost base - Book Value]
 = Capital gains + Recaptured Depreciation

8.4 Taxes on normal gains

The gains from the sale of product or inventories of an enterprise by using circulating capital are known as normal gains. These gains are taxable under the current tax net.

The tax act allows the business form or enterprises to deduct the normal losses while calculating net taxable income.

8.5 Taxes on capital gains

Capital assets for corporation include all depreciable assets and non-depreciable assets. When capital assets are sold for more or less than the purchase price, a capital gain (profit) or capital loss may be realized

Capital gain (loss) = selling price (Salvage value) - Cost base

Where cost base = Purchase cost + Incidental costs

The costs associated with the incidental activities like installation, freight etc are called incidental costs~ Capital gains (losses) must be considered in the calculation of taxable income and capital gains are taxed at the rate of 10% in Nepal.

Suppose you bought a computer at Rs 25000 and sold it for Rs 35000, the capital gain from this transaction is Rs 10000 and will be taxed accordingly.

8.6 Value Added Tax (VAT)

In Nepal, Value Added Tax (VAT) was introduced on 16 Nov. 1997. This tax was levied in place of the manufacturing level, Sales Tax, Hotel Tax, Contract Tax and Entertainment Tax. However, it could not be implemented fully until the FY 1998/99 due to political instability and strong opposition from the business community.

VAT replaces the old Sales Tax, the Contract Tax, the Hotel Tax and the Entertainment Tax. It has been designed to collect the same revenue as the four taxes it replaced. Since the collection of both custom duties and income tax depends to a great extent upon the effectiveness of VAT, it is expected to help enhance revenue collection.

The (VAT) is a broad-based tax as it covers the value added to each commodity by a firm during all stages of production and distribution. It is a modern tax system to improve the collection of taxes, to increase efficiency and to lessen tax evasion. It is also regarded as the backbone of income tax system in Nepal.

The current threshold for VAT registration is Rs. 2 million. Those vendors whose annual turnover is below the threshold can, however, register voluntarily.



There is persistent increase in the number of VAT registrants. IT has crossed the 30,000 mark. At the time of conversion from the then sales tax to the VAT a total of 2045 taxpayers were converted as VAT registrants. As the taxpayers are increasing, the amount of revenue collection and the level of tax compliance is improving too.

How does it Works?

VAT is a tax imposed on the value added to goods and services consumed in Nepal. The tax is based on the Principal that each producer or distributor adds value, in some way, to the materials they have purchased and it is this added value that is taxed at each stage of the production and distribution chain. There is the presumption that VAT is shifted forward completely to the Consumer.

In the VAT system producers, distributors and people providing services impose VAT on the product or the services sold or provided. The difference between the VAT collected on sales and the VAT charged on purchases determines the amount a registrant must remit or the amount that may be claimed as a refund.

In other words, if the tax on sales is more than the tax on purchases the person remits the difference. If the tax on sales is less than the tax on purchases, the person may carry forward this credit for next month.

What is taxed and what is not?

VAT divides all goods and services into two basic categories, taxable and tax-exempt.

Goods and services are either taxed at the standard rate of 10 percent or they are taxed at 0%. Those taxed at the standard rate include all goods and services except those which are specified as taxed at 0% or tax-exempt.

What Is Tax-Exempt?

The purchaser will NOT pay VAT on tax-exempt goods and services and the supplier is not allowed input tax credits on purchases related to the following goods and services provided:

- (a) Goods and services of basic needs which include rice, pulses flour, fresh fish, meat, eggs, fruits, flowers, edible oil, piped water, wood fuel, and kerosene.
- (b) Basic agricultural products are also tax-exempt, for example, paddy, wheat, maize, millet, cereals and vegetables.
- (c) The expense of buying goods and services required to grow basic agricultural products are tax exempt. This. Includes live animals, agricultural inputs including machinery, manure, fertilizer, seeds, and pesticides.
- (d) Social welfare services including medicine, medical services, veterinary services and educational services.
- (e) Goods made for the use of disabled persons.
- (f) Transport services.
- (g) Educational and cultural goods and services such a book and other printed materials, radio and television transmissions, artistic goods, cultural programs, non-professional sporting events and admissions to educational and cultural facilities.
- (h) Personal services are also tax-exempt. These are services provided, for example, by professionals, by actors and other entertainers, people charging for academic and technical research and computer services.
- (i) Exemption from tax is also extended to the purchase and rent of land and buildings
- (j) Financial and insurance services.
- (k) Postage and revenue stamps, bank notes, cheque books.

Are Imported Goods Taxed?

The VAT Act, Schedule I lists imports which are tax-exempt. Some of these include: prescription drugs, basic groceries, medical devices and agricultural products.

Most imports, however, are fully taxable at the time of importation. Thereafter they are treated on the same basic as domestically Produced goods

The VAT on imported goods is collected on the duty paid value of the goods, in other words, on the value of the goods including transportation, insurance, freight and commissions PLUS any duty or other payable on the goods Registrants may claim input tax credit for the VAT paid on imported goods are used in their commercial activities.

Are Exports Taxed?

The VAT is meant to apply only to the consumption of goods and services in Nepal. Supplies made in Nepal that are exported are taxable at 0%

Exporters are allowed to claim input tax credits for VAT paid or payable on purchases of goods and services relating to their commercial activities.

Exports taxed at 0% include exports of both goods and services.



Who must register?

Registration is required for any business:

- (a) With annual taxable turnover of more than Rs. 20,00,000.
- (b) Belonging to an associated group which has aggregate annual taxable turnover exceeding Rs. 20,00,000.

Who is affected by VAT?

In addition to consumers, persons involved in commercial activities are affected by VAT.

A person means an individual, firm, company, association, cooperative, institution, joint business, partnership; trust, government body or religious organization.

There are categories of persons and organizations which are not required to collect VAT nor allowed to claim a refund of the VAT they have paid in producing their goods and services for sale. These would include unregistered small suppliers, that is, persons with annual sales of taxable goods and services of Rs. 20,00,000 or less, who have chosen not to register.

What are the obligations of VAT registrants?

- (a) VAT registrants are required to:
- (b) Submit VAT return and pay tax within 25th day of the following month.
- (c) Provide their customers with a tax invoice
- (d) Maintain Purchase Book, Sales Book, VAT Account.
- (e) Keep their VAT records for a period of 6 years
- (f) Inform the IRO of changes to the business including new address, telephone number or a reorganization of a partnership within 15 days.
- (g) Put their Certificate of Registration in the premises where customers may observe it, and
- (h) Allow tax officers to enter the business to examine the business records and the stock on hand.

Is it necessary to maintain A special accounting system?

Most business will require only minor modifications to their record keeping. In order to complete his VAT return a taxpayer will need to ensure that his books and records will provide:

- (a) The amount of VAT paid on purchases
- (b) The amount of VAT collected on sales
- (c) A method of distinguishing between taxable and exempt sales
- (d) The time the goods and services were supplied, and
- (e) Evidences that goods were exported.

What books and records must be kept?

A taxpayer must keep the following books and records:

- (f) A purchase book,
- (g) A sales book, and
- (h) A VAT account.



CHAPTER NINE

DEMAND ANALYSIS AND SALES FORECASTING

9.1 Demand analysis

Demand analysis is performed to discover the forces that affect products sales and to establish the relationship between these forces and sales. It can be done with the help of following two approaches

A. Econometrics or statistical analysis

It is the collection of statistical techniques available for testing economic theories by empirically measuring relationships among economic variables. This approach quantifies economic reality bridge the gap between abstract theory and real world human activity and preferred when we have sufficient empirical data to analyze. However, the data available or collected for analysis may be of the following types:

a. Time series data

Time series data are collected over several time periods, usually at regular intervals and thus are *dynamic* in nature. Data detailing . the number of building permits issued in Kathmandu metropolitan city in each year sales for a company overtime etc. are some examples of time series data. Before making any analysis for this type of data, the following points should be considered:

1. If data in monetary terms, an inflation adjustment is necessary.
2. Technology may change over time.
3. Population adjustment is necessary.
4. Production function assumes that production takes place where input combination is most efficient.

b. Cross-sectional data

Cross-sectional data are collected at the same or approximately the same point in time and thus are *static* in nature. For example, data detailing the number of building permits issued in Poush 2061 in each of the municipalities of Nepal, sales of 10 companies in the music industry at one point in time etc. Before making any analysis for this type of data, the following points should be considered:

1. No technological changes over time, but all plants in the investigation are assumed to have same technology.
2. Adjustments across different geographical areas must be made. - Wages and price level
3. No guarantee that each plant operates at the most efficient input combination for the period examined.

Time Series vs. Cross sectional data

The following differences can be noted in these two types of data:

- Time series data has a temporal ordering, unlike cross-section data
- Will need to alter some of our assumptions to take into account that we no longer have a random sample of individuals
- Instead, we have one realization of a stochastic (i.e. random) process

B. Non-econometric Techniques or Market research techniques

Non-econometric techniques are suitable when we are introducing a new product in the market for which we do not have sufficient statistical data. Since this approach is qualitative one, we may find it very useful when we intend to test responses to changes in , qualitative parameters such as taste, preference and consumers' expectations. *The following methods can be applied to conduct demand analysis by using market research techniques:*

a. Consumer Surveys

Potential or current customers are asked for their reactions to changes in demand variables (e.g. price, advertisement; income etc.) through carefully prepared questionnaires. The survey may ask the questions like .

- "How many bags of chips would you buy if the price was Rs 12/bag?"
- "How many cases of beer would you buy if the price of beer was Rs 840/case?"

Having collected responses, the firm then compares different individuals' responses and establish a relationship between responses and questions.

Advantages:

- Flexible
- Relatively inexpensive to conduct

Disadvantages

- Many .potential biases (Strategic, Information, Hypothetical, Interviewer etc.)



b. Market Experiments

In this method, Firms carry out experiments in actual market condition by varying one or more of the demand determinants such as prices and/or advertising and comparing consumer behavior over time (*e.g., before and after rebate offer*) or over space (*e.g., compare Pokhara and Biratnagar consumption when prices are varied between two regions*). While conducting market experiment the following potential problems may occur:

- Control of other factors not guaranteed.
- "Playing" with market prices may be risky.
- Expensive

c. Consumer Clinics and Focus Groups

In this method, simulated market setting is established and consumers are given income to spend on a variety of goods: It is a controlled experiment in which the experimenters control income, prices, advertising, packaging, etc. The demand variables such as product price, the price of competing goods can easily be manipulated from the reactions shown by the consumers.

Advantages

- Flexibility

Disadvantages

- Selectivity bias
- Very expensive

9.2 Correlation of price and consumption rate

9.3 Multiple correlation of price and consumption rate

9.4 Market research

9.5 Sales forecasting

Sales forecasting is the process of organizing and analyzing information in a way that makes it possible to estimate what your sales will be. In other words, forecasting is about extrapolating the past into future using qualitative and quantitative methods

Importance of forecasting

Sales forecasting is thought to be important because of its following beauty:

- Production output and input schedules are built around projected demand
- Output: How much and when to provide product?
- Input: Raw materials, capital investments, human resources, and other inputs are purchased based on expected sales.
- Forecasting sales is an important tool to understanding future revenue streams.

Steps in Forecasting

Whenever we are about to make any sales forecast, we go through the following steps:

1. Determine the use of the forecast
2. Select the items to be forecast
3. Determine the time horizon of the forecast
4. Select the forecasting model(s)
5. Gather the data
6. Make the forecast
7. Validate and implement results

Forecasting Approaches

There are two approaches for sales forecasting:

A. Qualitative Methods



This approach is followed when situation is vague & little data exist. Such situation arises when we have to make forecasts of new products and new technology. Qualitative approach for forecasting involves intuition and experience. e.g., forecasting future sales on internet

Qualitative forecasts give subjective results and are more judgmental. It can be done in the following ways:

- a. **Jury of executive opinion:** Pool opinions of high level executives, sometimes augment with statistical models
- b. **Sales force composite:** Estimates from individual salespersons are reviewed for reasonableness, then aggregated
- c. **Delphi method:** Panel of experts, queried iteratively often using email
- d. **Consumer intention Survey:** Ask the customer what their intention is

B. Quantitative Methods

This approach is followed when situation is '*stable*' & *historical data exist*. Such situation arises when we have to make forecasts of existing products and current technology. It involves mathematical techniques e.g., forecasting sales of DVDs and color televisions *or* Qualitative forecasts give objective results and can be done in the following ways:

- (i) **Causal**—consider the effects of outside influences/factors
What causes something to be at a certain level – mathematically predict effect of another variable on what you are trying to predict
 - e.g. reject rate if have certain level of quality bonus
 - Extra demand for ice-cream in hotter than normal weather pattern
 - Impact of interest rates on demand for new houses
- (ii) **Projective**—looks at past patterns and extrapolate/extend this trend into the future. Considers impacts of internal factors within our control
10 rejects, 20 rejects, 30 rejects, 40 rejects?

The sales forecasting is the critical one for most business. Key decision that are derived for a sales forecast includes:

- i) Employment level required
- ii) Promotional mix
- iii) Investment in production capacity

The selection of which type of forecasting to use depends on several factors

- i. The degree of accuracy required
- ii. Availability of data and information
- iii. Time horizon that the self-forecast is intended to cover
- iv. The position of the product in its lifecycle

9.6 Criteria for desirable sales forecasting procedures

9.7 Factors affecting accuracy of forecasting

The following factors affect the accuracy of forecasting:

- I. **Availability and accuracy of historical data:** This factor is of vital importance while forecasting sales of any product as it would be extrapolated from what we have in the past.
- II. **Time available to make analysis:** The more time we have to make analysis, the better the forecasts would be
- III. **Degree of accuracy expected:** Our expectation from forecast influences largely the accuracy of forecasts. Sometimes, we may not want to have higher order of accuracy of forecasts and so will be the results.
- IV. **Length of forecast period:** Because of dynamic nature of market, forecasts made for shorter periods tend to be more accurate.
- V. **Cost of making analysis:** Forecasting will be more accurate when we make extensive analysis and use modern computer software packages. If we can bear more cost, we could obviously have accurate projections.



NUMERICALS

Q.1. The given tables shows the consumption of fish in kg when price in Rs are as shown. Calculate the hypothesized regression equation. What will be the consumption if the price is set at Rs 100/kg?

S.N	Price Rs/Kg	Consumption Kg
1	106	55
2	90	63
3	112	50
4	86	58
5	136	46
6	98	54
7	112	52
8	150	42
9	83	67
10	128	46

Soln

As per the question, we are required to find the regression equation of consumption on price. In other words, price is independent variable while consumption is dependent variable on price. So, we proceed with the calculations as below:

S.N	Price Rs/Kg(x)	Consumption Kg(y)	xy	x^2
1	106	55	5830	11236
2	90	63	5670	8100
3	112	50	5600	12544
4	86	58	4988	7396
5	136	46	6256	18496
6	98	54	5292	9604
7	112	52	5824	12544
8	150	42	6300	22500
9	83	67	5561	6889
10	128	46	5888	16384
	$\sum x=1101$	$\sum y=533$	$\sum xy=57209$	$\sum x^2=125693$

Hypothesized regression equation of Y on X is

$$\hat{Y} = a + b\hat{x}$$

Where,

$$\sum y = na + b \sum x$$

$$553 = 10a + 1101b \dots \dots \dots (1)$$

$$\sum xy = a \sum x + b \sum x^2$$

$$57209 = 1101a + 125693b \dots \dots \dots (2)$$

Solving equation (1) and (2) we get

$$a = 89.63$$

$$b = -0.33$$

Now the equation is

$$\hat{Y} = 89.63 - 0.33\hat{x}$$

When the price is set at Rs 100/kg, i.e. X= 100

$$\hat{Y} = 89.63 - 0.33 \times 100 = 56.63 \text{ kg}$$



Q.N.2. The following is the supply of oranges. What is the price per kg of oranges, when the demand is 500 kg? Use regression method.

S.N	Price Rs/Kg(y)	Demand Kg(X)
1	64	625
2	53	750
3	67	500
4	52	690
5	82	575
6	59	680
7	67	630
8	90	570
9	50	800
10	77	550
11	88	550
12	71	250

Soln

As per the question, we are required to find the price when the demand is given. In other words, demand is independent variable while price is dependent variable. So, we proceed with the calculations as below:

S.N	Price Rs/Kg(y)	Demand Kg(x)	xy	x ²
1	64	625	40000	390625
2	53	750	39750	562500
3	67	500	33500	250000
4	52	690	35880	476100
5	82	575	47150	330625
6	59	680	40120	462400
7	67	630	42210	396900
8	90	570	51300	324900
9	50	800	40000	640000
10	77	550	42350	302500
11	88	550	48400	302500
12	71	250	17750	62500
	$\sum y=820$	$\sum x=7170$	$\sum xy=478410$	$\sum x^2=4501550$

Hypothesized regression equation of Y on X is

$$\hat{Y} = a + b\hat{x}$$

Where,

$$\sum y = na + b \sum x$$

$$820 = 12a + 7170b \dots \dots \dots (1)$$

$$\sum xy = a \sum x + b \sum x^2$$

$$478410 = 7170a + 4501550b \dots \dots \dots (2)$$

Solving equation (1) and (2) we get

$$a = 100$$

$$b = -0.053$$

Now the equation is

$$\hat{Y} = 100 - 0.053\hat{x}$$

When the price is set at Rs 500kg, i.e. X= 500

$$\hat{Y} = 100 - 0.053 \times 500 = 73.5 / \text{kg}$$



Q.N.3 Use the conventional B/C ratio method with AW formulation to select the preferred design from the following mutually exclusive projects.

Factor	Alternative Design		
	1	2	3
Capital investment	1240000	1763000	1475000
Salvage Value	90000	150000	120000
Annual O & M cost	215000	204000	201000
Annual benefits to the User group A	315000	367000	355000
Annual benefits to the other User groups	147800	155000	130500

Assume that MARR = 9% per year and the analysis period of 15 years each.

Solution,

Factor	Alternative Design		
	1	2	3
Capital investment, I	1240000	1763000	1475000
Salvage Value, S	90000	150000	120000
Annual O & M cost, O&M	215000	204000	201000
Annual Benefits, B	$=315000+147800=462800$	$=367000+155000=522000$	$=355000+130500=485500$
AW(I)	153833.0145	218715.8101	182986.8519
AW(S)	3065.299439	5108.83	4087.065
AW(O&M)	215000	204000	201000
AW(B)	462800	522000	48500
convectional B/C	1.27	1.25	1.28