Matrices

Matrix: A matrix is a rectangular arrangement of numbers, symbols, or expressions organized in rows and columns. For example, consider a 2x3 matrix A:

```
A = | 1  2  3 |
    | 4  5  6 |
```

Determinant: The determinant is a scalar value that can be computed from a square matrix. It is often denoted as "det(A)" for a square matrix A. For example, for a 2x2 matrix B:

```
B = | 2  3 |
    | 4  5 |
```

The determinant of B is: det(B) = (2*5) - (3*4) = 10 - 12 = -2.

Order of a Matrix:
The order of a matrix is represented as "m x n," where "m" is the number of rows, and "n" is the number of columns. For example, if you have a matrix R with 2 rows and 3 columns:

```
R = | 1  2  3 |
    | 4  5  6 |
```

The order of matrix R is 2 x 3.

Transpose: The transpose of a matrix switches its rows with columns. For a matrix H:

```
H = | 1  2 |
    | 3  4 |
```

The transpose of H, denoted as H^T, is:

```
H^T = | 1  3 |
      | 2  4 |
```

Trace:
The trace of a matrix is the sum of the elements on its main diagonal, which is the diagonal that runs from the top-left to the bottom-right of a square matrix. The trace is often denoted as "Tr(A)" for a matrix A.

For example, if you have a 3x3 matrix B:

```
B = | 1  2  3 |
    | 4  5  6 |
    | 7  8  9 |
```

The trace of matrix B would be:

Tr(B) = 1 + 5 + 9 = 15

Types of Matrices:

Row Matrix: A row matrix has only one row and multiple columns. For example, a 1x3 row matrix C:

$$C = \begin{vmatrix} 1 & 2 & 3 \end{vmatrix}$$

Column Matrix: A column matrix has only one column and multiple rows. For example, a 3x1 column matrix D:

$$D = \begin{vmatrix} 4 \\ 5 \\ 6 \end{vmatrix}$$

Square Matrix: A square matrix has the same number of rows and columns. For instance, a 2x2 square matrix E:

$$E = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$$

Types of Square Matrices:

Diagonal Matrix: A diagonal matrix has non-zero elements only on its main diagonal (top-left to bottom-right), and all other elements are zero. Example of a 3x3 diagonal matrix F:

$$F = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{vmatrix}$$

Scalar Matrix: A scalar matrix is a special case of a diagonal matrix where all diagonal elements are equal. Example of a 2x2 scalar matrix G:

$$G = \begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix}$$

Identity Matrix: The identity matrix is a special diagonal matrix where all diagonal elements are 1, and all other elements are 0. The symbol for the identity matrix is often "I," and it is commonly denoted as "I" or "I_n" for an n x n matrix. Example of a 3x3 identity matrix:

$$I = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Operation on matrix

Matrix Addition:

Matrices can be added together if they have the same dimensions (i.e., the same number of rows and columns). The addition is performed element-wise.

Example: If you have two matrices A and B:

```
A = | 1  2 |
    | 3  4 |

B = | 5  6 |
    | 7  8 |
```

The sum of A and B (A + B) is:

```
A + B = | 1+5  2+6 |
        | 3+7  4+8 |
      = | 6  8 |
        | 10 12 |
```

Matrix Subtraction:
Similar to addition, matrices of the same dimensions can be subtracted element-wise.

Example: If you have matrices C and D:

```
C = | 5  8 |
    | 2  1 |

D = | 1  4 |
    | 3  7 |
```

The result of C - D is:

```
C - D = | 5-1  8-4 |
        | 2-3  1-7 |
      = | 4  4 |
        | -1 -6 |
```

Scalar Multiplication:
You can multiply a matrix by a scalar (a single number) by multiplying every element of the matrix by that scalar.

Example: If you have a matrix E and a scalar value k (let's say k = 2):

```
E = | 3  6 |
    | 1  4 |
```

The result of k * E is:

```
2 * E = | 2*3  2*6 |
        | 2*1  2*4 |
      = | 6  12 |
        | 2  8 |
```

Matrix Multiplication:
Matrix multiplication is a bit more complex. Two matrices can be multiplied if the number of columns in the first matrix matches the number of rows in the second matrix.

Example: If you have matrices F and G:

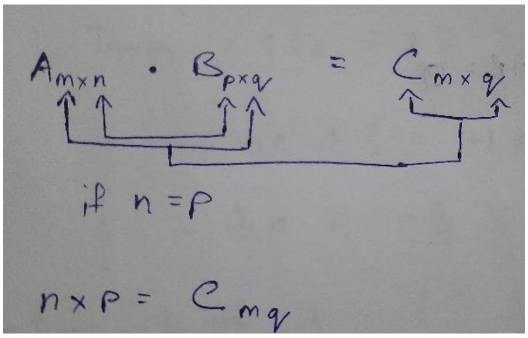
F = | 1 2 |
      | 3 4 |

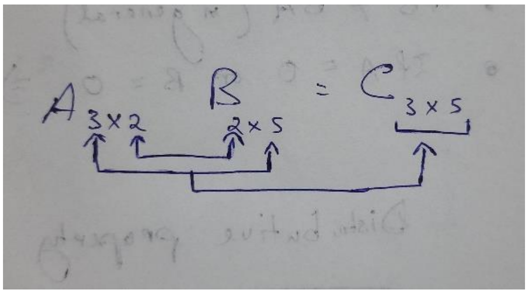
G = | 5 6 |
      | 7 8 |

The product of F and G (F * G) is calculated as follows:

F * G = | (1*5 + 2*7) (1*6 + 2*8) |
            | (3*5 + 4*7) (3*6 + 4*8) |
         = | 19 22 |
            | 43 50 |

A symmetric matrix is a square matrix.





Matrix multiplication examples:

METHOD -1

Find $\begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}$

$$= \begin{array}{c|c} & (3,2) \qquad\qquad (5,1) \\ \hline \begin{array}{c} (2,4) \\[2ex] (1,5) \end{array} & \begin{bmatrix} 6+8 & 10+4 \\[2ex] 3+10 & 5+5 \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} 14 & 14 \\ 13 & 10 \end{bmatrix}_{2\times 2}$$

METHOD -2

Find $\begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$$

$a_{11} = 6 + 8 = 14$

$a_{12} = 10 + 4 = 14$

$a_{21} = 3 + 10 = 13$

$a_{22} = 5 + 5 = 10$

PROBLEM 1:

If $\begin{bmatrix}1&1\\0&1\end{bmatrix}\cdot\begin{bmatrix}1&2\\0&1\end{bmatrix}\cdot\begin{bmatrix}1&3\\0&1\end{bmatrix}\cdots\cdots\begin{bmatrix}1&n-1\\0&1\end{bmatrix}=\begin{bmatrix}1&78\\0&1\end{bmatrix}$ Then, find n.

$$\begin{bmatrix}1 & 1+2+3+\cdots\cdots+(n-1)\\0 & 1\end{bmatrix}=\begin{bmatrix}1&78\\0&1\end{bmatrix}$$

$$\frac{(n-1)n}{2}=78$$

$$\Rightarrow (n-1)\,n = 2\times 2\times 39$$

$$\Rightarrow 2\times 2\times 3\times 13$$

$$\Rightarrow 12\times 13$$

PROBLEM 2:

Let $A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$ and $B=\begin{bmatrix}\alpha\\\beta\end{bmatrix}\neq\begin{bmatrix}0\\0\end{bmatrix}$ such that $AB=B$ and

$A+D=2021$, ~~that~~ then the value of $ad-bc$ is equal to –

$$a=0,\ d=2021$$

$$A=\begin{bmatrix}0&b\\c&2021\end{bmatrix}\cdot\begin{bmatrix}\alpha\\\beta\end{bmatrix}$$

$$A=\begin{bmatrix}0&b\\c&2021\end{bmatrix}\qquad B=\begin{bmatrix}\alpha\\\beta\end{bmatrix}$$

$$\begin{bmatrix}0&b\\c&2021\end{bmatrix}\cdot\begin{bmatrix}\alpha\\\beta\end{bmatrix}=\begin{bmatrix}b\beta\\c\alpha+2021\beta\end{bmatrix}=\begin{bmatrix}\alpha\\\beta\end{bmatrix}$$

$$b\beta=\alpha \qquad c\alpha+2021\beta=\beta$$

$$\frac{\alpha}{\beta}=b \qquad c\alpha=-2020\beta$$

$$\frac{\alpha}{\beta}=\frac{-2020}{c}$$

$$b=\frac{-2020}{c} \qquad \therefore -bc=2020$$

Determinants

Determinants are always square.

Representation:

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$= \det|A|$

Determinant value of 1x1 & 2x2

$|A| = |\,-2\,|_{1x1} = -2$ -2 is the value

$$|B| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}_{2x2}$$

= Here will be cross multiplication between (axd) - (bxc) = ad - bc

Here are some examples:

1. The value of $\begin{vmatrix} a+1 & a-2 \\ a+2 & a-1 \end{vmatrix}$

$= (a+1)(a-1) - (a+2)(a-2)$
$= (a^2-1) - (a^2-4)$
$= -1 + 4$
$= 3$

2. The value of $\begin{vmatrix} 1+\cos\theta & \sin\theta \\ \sin\theta & 1-\cos\theta \end{vmatrix}$

$= (1 + \cos\theta)(1 - \cos\theta) - \sin^2\theta$
$= (1 - \cos^2\theta) - \sin^2\theta$
$= \sin^2\theta - \sin^2\theta$
$= 0$

Minors :

The number of minors depends on the number of elements or,
no. of elements = no of minors

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad D = \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \qquad D = \begin{vmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} \\ \cancel{a_{21}} & a_{22} & a_{23} \\ \cancel{a_{31}} & a_{32} & a_{33} \end{vmatrix}$$

M_{11} = Delete 1st row & 1st column

$$M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} \qquad D = \begin{vmatrix} a_{11} & \cancel{a_{12}} & a_{13} \\ \cancel{a_{21}} & \cancel{a_{22}} & \cancel{a_{23}} \\ a_{31} & \cancel{a_{32}} & a_{33} \end{vmatrix}$$

M_{22} = Delete 2nd row & 2nd column

Co-factors:

The number of co-factors also depends on the number of elements or,
no. of minors/no. of elements = no of co-factors

$C_{ij} = (-1)_{ij}\ M_{ij}$

If $_{i+j}$ = odd, then add (-)
If $_{i+j}$ = even, then don't add (-)

$C_{11} = (-1)_{i+j}\ M_{11} = M_{11}$
$C_{12} = (-1)_{i+j}\ M_{12} = -M_{12}$
$C_{13} = (-1)_{i+j}\ M_{13} = M_{13}$

Expanding (open) w.r.t R1

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$= a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13}$
= value of determinant

Expanding (open) w.r.t C2

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$= a_{12}c_{12} + a_{22}c_{22} + a_{32}c_{32}$

= value of determinant

Expanding w.r.t R1

$$\begin{vmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{vmatrix}$$

Delete rows and columns of the in which the following numbers 2, -3, 1 are present.

$$= 2\begin{vmatrix} 0 & -1 \\ 4 & 5 \end{vmatrix} + 3\begin{vmatrix} 2 & -1 \\ 1 & 5 \end{vmatrix} + 1\begin{vmatrix} 2 & -3 \\ 2 & 0 \end{vmatrix}$$

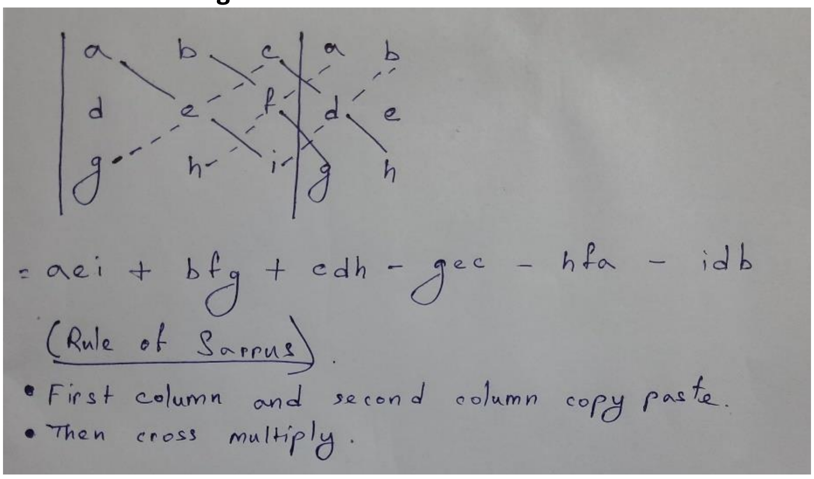
Now cross multiply

= 2(0-(-4) + 3(10-(-1)) + 1(0-(-6))

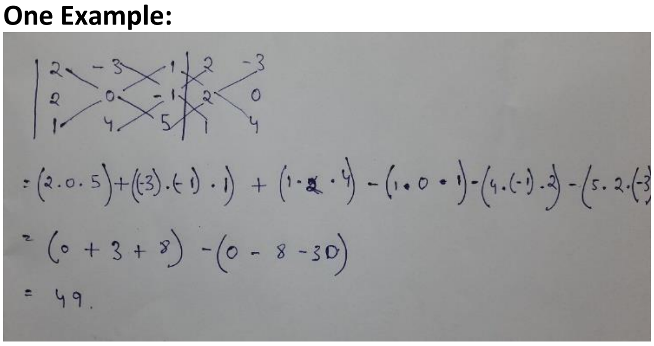
= 8 + 33 + 7

= 48

Shortcut of finding determinant



One Example:



PROBLEM 1:

The value of $\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix}$ is

$$\begin{array}{ccc|cc} -1 & 2 & 3 & 1 & 2 \\ -4 & 3 & 6 & -4 & 3 \\ 2 & -7 & 9 & 2 & -7 \end{array}$$

$= (-27) + (24) + (+84) - (18) + (-42) + (-72)$

$= (-27 + 24 + 84) - (18 + 42 + (-72))$

$= 231.$

<u>**Properties of Determinants:**</u>

1. **Transpose:**

Rows will become column & columns will become rows.

|A| = |a b c|
|d e f|
|g h i|

$|A^T|$ = |a d g|
|b e h|
|c f I|

The value of both |A| and $|A^T|$ will be same.
$|A| = |A^T|$

2. **If any two rows or columns of a determinant be interchanged, the value of determinant is changed in sign only.**

One example:

$A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = (-2).$

$A = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = 6 - 4 = (2).$

3. If row and columns are rotated in cyclic order then value of determinant is unchanged.

One example:

$$\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} = \begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix}$$

Here row1 has became row2, row2 has became row3 and row3 has became row1.
Still the value of the determinant will be same.

4. If a determinant has any two rows or columns identical, then it's value will be 0.

Example:

| a b c |
| a b c |
| x y z |

= 0

5. Scaler multiplication will be multiplied in any one row or column.

If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then $KD = \begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

or, $\begin{vmatrix} a_1 & Kb_1 & c_1 \\ a_2 & Kb_2 & c_2 \\ a_3 & Kb_3 & c_3 \end{vmatrix}$

6. | kA | = k^n|A|, where n is the order of A.

NOTE: The value of a skew symmetric determinant of odd order is zero.

| 0 a -b |
| -a 0 c |
| b -c 0 |$_{3x3}$

- Diagonal = 0
- Odd order = 3 [3x3]

Value will be 0.

If the skew symmetric determinant is of even order then the value will be $\neq 0$.

| 0 1 |
|-1 0 |$_{2x2}$

7. **Adding determinants**

You can add two determinants if two rows and columns of the determinants are same.

Example:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+x & b_1+y & c_1+z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

8. **Splitting determinants**

You can split determinants and the condition is you have to keep two rows and columns same.

Find $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$

$$= \begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{vmatrix}$$

= Take 2 common from 2nd determinant row 2.

= Then answer will be 0.

9. **| AB |= |A| |B|**

$A = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} 1 & 0 \\ 1 & 4 \end{vmatrix}$ Then $AB = \begin{vmatrix} 2 & 4 \\ 3 & 8 \end{vmatrix}$

$|A| = 2 - 1 = 1$ $\quad |B| = 4$ $\quad |AB| = 16 - 12 = 4$

10. If det|A| = 0, then A is known as singular matrix.

$$|A| = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix}$$

$$= 2 - 2 = 0.$$

11. The value of determinant remains same if we apply elementary transformation.

$R_1 \to R_1 + kR_2 + mR_3$ or $C_1 \to C_1 + kC_2 + mC_3$

Here k & m represents constant numbers, it may be positive numbers, negative numbers of it may be 0.

PROBLEM 1

Prove that $\begin{vmatrix} b+c & a+b & a \\ c+a & b+c & b \\ a+b & c+a & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$

$$\Rightarrow \overset{0}{\begin{vmatrix} b+c & a & a \\ c+a & b & b \\ a+b & c & c \end{vmatrix}} + \begin{vmatrix} b+c & b & a \\ c+a & c & b \\ a+b & a & c \end{vmatrix}$$

$$\Rightarrow \overset{0}{\begin{vmatrix} b & a & a \\ c & c & b \\ a & a & c \end{vmatrix}} + \begin{vmatrix} c & b & a \\ a & c & b \\ b & a & c \end{vmatrix}$$

$$\Rightarrow - \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$\Rightarrow -(-(a^3 + b^3 + c^3 - 3abc))$$

$$= a^3 + b^3 + c^3 - 3abc$$

<u>Adjoint of Matrix</u>

Adjoint = (co-factor)^T

First, find the co-factor of a matrix then find the adjoint.

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Now, find the co-factor

$$\text{Co-factor of } A = \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix}$$

Now make adjoint, means simply make the transpose of (co-factor)A.

$$\text{Adj } A = \begin{vmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{31} & c_{32} & c_{33} \end{vmatrix}$$

Example:

Find the adjoint of the matrix

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

First find out minor of A

$$\text{Minor}(A) = \begin{bmatrix} 2 & -2 & 6 \\ 1 & -2 & 5 \\ -2 & 2 & -8 \end{bmatrix} \qquad \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

$$\text{Co-factor}(A) = \begin{bmatrix} 2 & 2 & 6 \\ -1 & -2 & -5 \\ -2 & -2 & -8 \end{bmatrix} \qquad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\text{co-factor transpose}(A) : \begin{bmatrix} 2 & -1 & -2 \\ 2 & -2 & -2 \\ 6 & -5 & -8 \end{bmatrix}$$

Properties of adjoint:

1. **A (adj A) = (adj A) A = $|A|I_n$**

Example:

$$A(\text{adj } A) = (\text{adj } A)A = |A| I_n$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\text{co-factor } (A) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

Now, $A \cdot \text{adj } A$

$$A \cdot \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

$$= \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$$

$$= |A| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= |A| I_n$$

2. **$|\text{adj } A| = |A|_{n-1}$**

3. **Adj(AB) = (adj B) x (adj A)**

Earlier in transpose we have learned that $(AB)^T = B^T \times A^T$

4. **Adj(kA) = k^{n-1} (adj A), (k € R)**

Earlier in determinant we have learned a property $|kA| = k^n |A|$, where if you take k out then you will get k^n but here in adjoint if you take k out then you will get $k^{n-1.}$ Where n is the order of the particular matrix.

5. **Adj (adj A) = $|A|^{n-2}$ A**
6. **$|\text{adj (adj } A)| = |A|^{(n-1)^2}$**

Inverse of a matrix:

A square matrix A said to be invertible (non singular) if there exists a matrix B such that AB = I = BA
B is called the inverse of A and is denoted by A^{-1}. Thus $A^{-1} = B \leftrightarrow AB = I = BA$

Previously in matrix we have learned about additive inverse but now we will learn about multiplicative inverse.

Solving system linear equations:

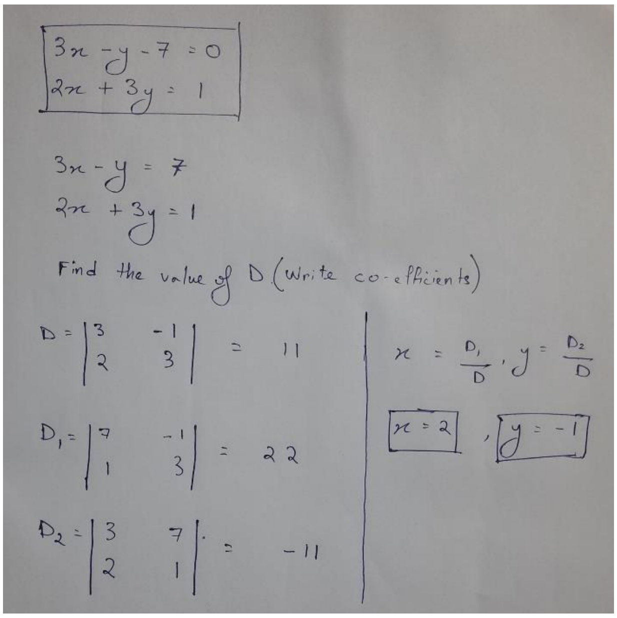
- Determinant method (Cramer's rule)
- Matrix method (Gauss-Jordan method)

Cramer's Rule:

2 equations 2 variables

$a_1x + b_1y = c_1$
$a_2x + b_2y = c_2$

Where $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$; $D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$; $D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$



$\boxed{\begin{array}{l} 3x - y - 7 = 0 \\ 2x + 3y = 1 \end{array}}$

$3x - y = 7$
$2x + 3y = 1$

Find the value of D (Write co-efficients)

$D = \begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix} = 11$

$D_1 = \begin{vmatrix} 7 & -1 \\ 1 & 3 \end{vmatrix} = 22$

$D_2 = \begin{vmatrix} 3 & 7 \\ 2 & 1 \end{vmatrix} = -11$

$x = \frac{D_1}{D}, y = \frac{D_2}{D}$

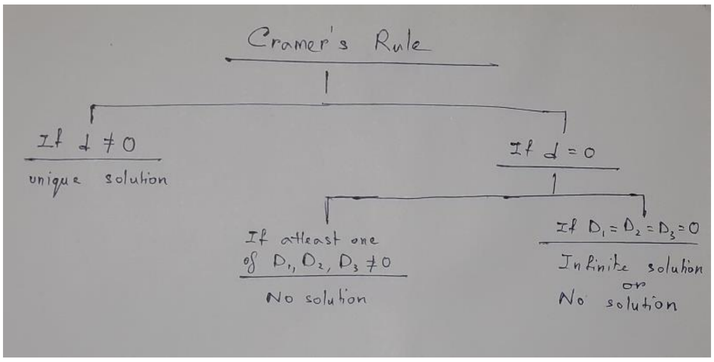
$\boxed{x = 2}$, $\boxed{y = -1}$

Here to find the value of D_1 and D_2.

- First, put the right side value in 1st column of D_1 and 2nd column of D_2.
- Then replace the value of D with the 2nd column of D_1 and 1st column of D_2.

IMPORTANT TERMS

1. **Consistent :** solution exists (unique or infinite solution).
2. **Inconsistent :** solution does not exist (no solution).
3. **Homogeneous equation :** constant terms are 0.
4. **Trivial solution :** all variables = 0 i.e., x = 0, y = 0, z = 0.



Homogeneous Linear Equation

Homogeneous Linear Equations

$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$a_3x + b_3y + c_3z = 0$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \qquad D_1 = \begin{vmatrix} 0 & b_1 & c_1 \\ 0 & b_2 & c_2 \\ 0 & b_3 & c_3 \end{vmatrix} = 0$$

$$D_2 \begin{vmatrix} a_1 & 0 & c_1 \\ a_2 & 0 & c_2 \\ a_3 & 0 & c_3 \end{vmatrix} = 0 \qquad D_3 = \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} = 0$$

$$x = \frac{d_1}{d}, \quad y = \frac{d_2}{d}, \quad z = \frac{d_3}{d}$$

$$D_1 = D_2 = D_3 = 0$$

Matrix method (Gauss Jordan Method):

From the linear equations make :

- 1 matrix of coefficients
- 1 matrix of variables
- 1 matrix of constants

Matrix Method

Gauss Jordan method

$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x + y - z = 1$$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}}_{\text{Co-efficients (A)}} \cdot \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\text{Variables (X)}} = \underbrace{\begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}}_{\text{Constants B}}$$

$AX = B$

~~Now~~ cross multiply

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$