A comprehensive guide
to mastering the abacus.

ADVANCED ABACUS

THEORY AND PRACTICE

TAKASHI KOJIMA

Takashi Kojima

ADVANCED ABACUS

Japanese Theory and Practice

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FOREWORD

Mr. Kojima's second book on the abacus gives important information on the further practical use of the abacus and on the principles of its use in business. I believe that his complete explanation of operational methods and their theoretical basis will be of especial help to those foreign students who have no guide or instructor except books.

Aside from its immense utility in business and everyday calculation, the abacus is a far more effective instrument for teaching arithmetic in blind schools than is braille. Moreover, if introduced into ordinary schools, it will prove an excellent time-saver in arithmetic instruction. Half of the problems in arithmetic textbooks are calculation problems and the other half can be reduced to calculation problems by some mathematical reasoning. Consequently, those arithmetic hours allotted for the teaching of abacus operation, by improving the mental arithmetic of students, will enable them to calculate much faster than with pencil and paper, thus creating additional time for a more advanced study of arithmetic.

As Chairman of the die Committee of the International Association of Abacus Operators of the Japan Chamber of Commerce and Industry, I have been most pleased to assist Mr. Kojima by making available the findings of recent technical and theoretical studies and by revising his manuscript in the light of all the latest information.

Yoemon Yamazaki
Professor of Economics, Nihon University
Vice-President, All-Japan Federation of Abacus Operators
Chairman, Committee of the Int. Assn. of Abacus Operators

AUTHOR'S FOREWORD

This book has been written as a sequel to my earlier work *The Japanese Abacus: Its Use and Theory.* In this volume I shall both expand the explanation of some of the basic abacus operations given in the first volume and discuss new operations and new ways of doing the basic operations.

In the first chapter I shall discuss the Oriental history of the abacus in greater detail than was done in my first book. In the second chapter I shall deal with the new problem of negative numbers or negative answers resulting from the subtraction of a larger number from a smaller one. The third and fourth chapters will respectively consider other methods of multiplication and division than the ones explained in my first book. Chapter five is an expanded discussion of decimals in multiplication and division. Chapter six is a practical discussion of how to handle calculations involving more than one unit of measurement (such as inches and feet or pounds and ounces). Chapter seven introduces a method of extracting square roots on the abacus. Finally, chapter eight provides more exercises for the reader to practice on. Through it he will be able to measure his abacus ability by taking actual examinations given to Japanese applicants for proficiency grades eight to one.

I would like to include here some noteworthy statistics concerning the recent license examinations. Of the successful examinees for the third-grade license, about 70% were between thirteen and eighteen years old, 5% were nineteen and over, and 25% were twelve and under (including 0.3% who were nine and under). Of the successful examinees for the second-grade license, about 91% were between thirteen and eighteen, 2% were nineteen and over, and 7% were between ten and twelve. Of the successful examinees for the first-grade license, about 87% were between thirteen and eighteen, 12% were nineteen and over, and 1% were under twelve. These figures do not completely tell the story because among those who pass the first-grade examination every year are some who have already passed it, but either want the practice or a higher score. This explains why the percentage of persons nineteen and over who passed the first-grade examination is larger than the percentage of the same group who passed the second-grade examination.

An interesting conclusion can be drawn from these statistics—it is rather difficult for persons over nineteen and under twelve to pass the first-and second-grade examinations.

The ratio of boys to girls who passed these examinations is also worthy of mention. Of the successful examinees for the third-and second-grade licenses, about 60% were girls and 40% were boys, while the reverse was true of the first-grade examination.

In Japan the abacus is definitely a practical skill. It has found its way into the curriculum of all Japanese grade schools as a fundamental part of arithmetic. Many senior commercial high schools require all students to pass at least the third-grade examination. There have also been many abacus schools established to meet the needs of those preparing to go into business. And, as I hope I am demonstrating in these two books, there is a good reason why the abacus can be found in practically every Japanese household.

I am greatly indebted to several authorities who kindly furnished me with valuable information and suggestions. My most grateful acknowledgments are due to Professor Yoemon Yamazaki of Nihon University. He is the Vice-Chairman of the Abacus Research Institute, and Advisor to the Central Committee of the Federation of Abacus Workers (hereafter referred to simply as the Abacus Committee). These organizations, being under the sponsorship of the Japan Chamber of Commerce and Industry, are far and away the largest and most important of all abacus organizations in Japan.

I also extend my most sincere gratitude to Professor Miyokichi Ban, of the above-mentioned Abacus Committee, who was kind enough not only to furnish this book with a great many exercises especially prepared and arranged for the sake of the foreign student but also to read the book in proof and give me many valuable suggestions.

I also must express my sincere thanks to Mr. Shinji Ishikawa, President of the Japan Association of Abacus Calculation, who spared himself no trouble in reading the manuscript and the proof and furnishing much invaluable up-to-date information.

Grateful acknowledgments are also due to Mr. Hisao Suzuki for his information on the history of the abacus and to Mr. Zenji Arai for valuable suggestions on the uses of the abacus.

I also wish to express my sincere thanks to Mr. Yataro Nagata, Chief of the Abacus Operators' License Examinations Section in the Japan Chamber of Commerce and Industry, for information on the national examinations for abacus operators' licenses and for permission to reprint in this book the problems

presented in the 1959 National License examinations.

Last, but not least, I must thank Mr. William R. Whitney and the editorial staff of the Charles E. Tuttle Company for their valuable suggestions and improvements in both the manuscript and the proof stages.

TAKASHI KOJIMA

I. FURTHER REMARKS ON ABACUS HISTORY

The ancient Chinese books on mathematics which have been preserved furnish hardly any information on the abacus. Accordingly, nothing definite is known about its origin. The only reliable account of the origin of the Oriental abacus is in a book entitled *Mathematical Treatises by the Ancients* compiled by Hsu Yo toward the close of the Later Han dynasty (A.D. 25-220) at the beginning of the third century and annotated by Chen Luan in the sixth century. This book gives some information about various reckoning devices of those days and was one of the *Ten Books on Mathematics* (Suan-hwei-shi-chu) which were included among the textbooks to be read for government service examinations in China and Japan for many centuries.

Chen Luan in his note gives the following description of the calculating device: "The abacus is divided into three sections. In the uppermost and lowest section, idle counters are kept. In the middle section designating the places of numbers, calculation is performed. Each column in the middle section may have five counters, one uppermost five-unit counter and four differently colored one-unit counters."

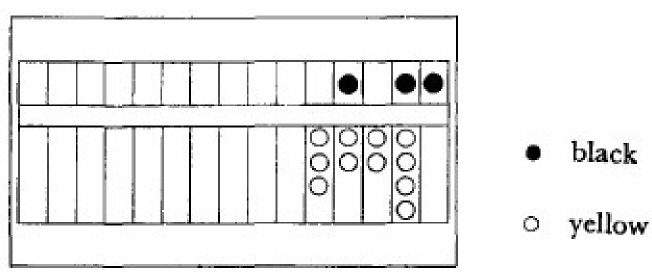


FIG. 1

The above figure represents the abacus as pictured in accordance with the foregoing description. The board represents the number 37,295.

The extent to which the counting board was used may be told by Hsu Yo's poetical description of the board. The verse, which is highly figurative and difficult to decipher, may read: "It controls the four seasons, and coordinates the three orders, heaven, earth, and man." This means that it was used in astronomical or calendar calculations, in geodetic surveys, and in calculations concerning human affairs.

The reader will notice a close similarity between this original Oriental abacus and the Roman grooved abacus, except for the difference that counters were laid down in the former while they were moved along the grooves in the latter. Because of this and other evidence, many leading Japanese historians of mathematics and the abacus have advanced the theory that the above-mentioned prototype of the abacus was the result of the introduction into the East of the Roman grooved abacus.

The following corroborative pieces of evidence in favor of this theory are cited in the latest works by Prof. Yoemon Yamazaki and Prof. Hisao Suzuki of Nihon University.

(1) The original Chinese abacus has a striking resemblance in construction to the Roman grooved abacus, as is evident in the foregoing quotation from Hsu Yo's book, e.g., four one-unit counters and one five-unit counter in each column.

(2) The method of operation of the ancient Chinese abacus was remarkably similar to the ancient Roman method.

In ancient China, multiplication and division were performed by the repetition of addition and subtraction: MULTIPLICATION: Procedure A: 23x5= (23x2)+(23x2)+23=115 (Ans.) Procedure B: 23x5=23+23+23+23+23=115 (Ans.) DIVISION: Procedure A: 115÷23=115-23-46-46=0 (Ans. 5) Procedure B: 115÷23=115-23-23-23-23-23=0 (Ans. 5) In the case of multiplication, each time 23 or 46 was added, 1 or 2 was added to the factor on the left of the board. In the case of division, each time 23 or 46 was subtracted, 1 or 2 was added to the quotient on the left of the board. It is obvious that anyone could easily learn and perform these simple primitive operations.

(3) Traces of reckoning by 5's may be found in the Chinese pictorial representation of reckoning-block calculation as in the Roman numerals, as:

six: VI (5+ 1) seven: VII (5+2)
eight: VIII (5+3) four: IV (5-1)

(4) Trade was carried on between China and Rome. Chinese historical documents written in the Han dynasty (206 B.C.-A.D. 220) furnish descriptions of two land routes, called silk roads, connecting the two great empires.

Inasmuch as even in olden days valuable products or devices made in one country were transmitted to others with astonishing rapidity, the above facts may well substantiate this theory.

Among the dozen other reckoning devices mentioned in this book are the reckoning boards pictured below. These boards are presumed to date back to the days of the Chou dynasty, which ended in 249 B.C.

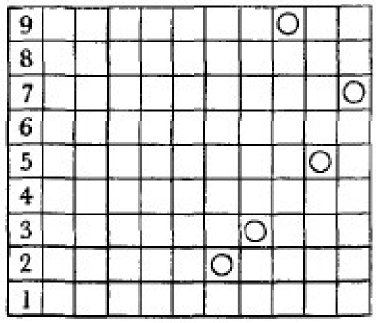


FIG. 2

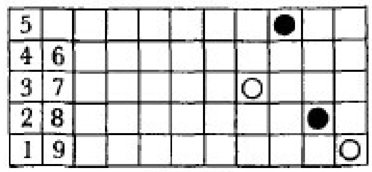


FIG. 3

(The number on the board is 23,957.)

(When yellow counters were used, the squares in each column represented 1, 2, 3, and 4 respectively. When blue ones were used, they represented 5, 6, 7, 8, and 9 respectively. The black balls in the figure stand for blue counters. The number on the board represents 3,581.) These and other reckoning devices are believed to have gone out of use as the previously mentioned abacus developed and gained popularity.

Now two questions present themselves. One is why this ancient abacus developed in the East to become such an efficient calculating machine. The other is why this development did not also take place in the West.

The reasons lay perhaps in the systems of calculation and the numerical nomenclature which were used in the East and West. They differ significantly. In ancient China and Japan numbers were named, written, and set on a calculating board from left to right, from the highest denomination to the lowest. Thus the introduction of the abacus to China provided the Chinese with an ideal tool in terms of their method of naming and using numbers. This compatibility and normal inventiveness caused the primitive abacus to be developed into its modern form during the long development of the Chinese civilization.

The chief calculating devices which are known to have been used in China from before 1,000 B.G. to the days when the abacus came into wide use are reckoning blocks called *ch'eou* in China and *sangi* in Japan and slender bamboo sticks called *chancku* in China and *zeichiku* in Japan. The former device continued to be used in the East for calculation until not many years ago, and the latter device, which was more awkward, was largely replaced by the former for calculating purposes and is presently used only by fortunetellers for purposes of divination.

Until the introduction of Western mathematics, mathematicians in China and Japan utilized reckoning-block calculation, which had not only been developed to the point of performing basic arithmetic operations but was also used to solve quadratic, cubic, and even simultaneous equations. It is presumed that they did not think it worth while to concern themselves with the other reckoning devices, including the abacus, which was, in their eyes, an inferior calculator barely capable of performing multiplication and division by means of the primitive cumulative method of addition and subtraction. Probably another reason which alienated mathematicians from these reckoning devices was that these instruments gave only the result of calculation, and were incapable of showing either the process of calculation or the original problem.

In ancient times China was primarily a nomadic and agricultural country, and business in those days had little need of instruments of rapid calculation. Anyway a millennium after the Han dynasty there was no record of the abacus. During the dozen centuries beginning with its first mention in the Han dynasty until its development, this primitive calculator remained in the background.

However, with the gradual rise of commerce and industry, the need for rapid calculation grew. The modern, highly efficient abacus, which probably appeared late in the Sung dynasty (906-1279), came into common use in the fourteenth century. The great rise and prosperity of free commerce and industry during the Ming dynasty (1368-1636) are presumed to have promoted the use and development of the abacus. A number of books on mathematics brought out in those days give descriptions of the modern Chinese abacus and give accounts of the modern methods of abacus operation, including those of multiplication and division.

Bamboo, indigenous in the East, has furnished an abundant source of ideal material for an efficient and inexpensive abacus. Since the Ming period, on account of its remarkable efficiency, low price, and handiness, the abacus has been the favorite instrument of calculation in the East.

The Chinese abacus of the Ming period had two five-unit counters and five one-unit ones on each rod. The primitive abacus was changed into the present Chinese form to suit the convenience of figuring up the Chinese weights not based on the decimal system. The weights were also important for conversion of currency. Another cogent reason why the Chinese abacus has two five-unit counters on each rod is that a rod with two five-unit counters is more convenient to abacus operation by means of the Chinese method of multiplication and also by means of the older method of division which uses a special division table.

In Europe, the line abacus or counting board appeared first in France about the beginning of the thirteenth century and rapidly became popular. From the fourteenth to the seventeenth century the practice of this manual arithmetic was universal in business and in households, as well as in the departments of government. Its immense popularity may well be illustrated by the following pleasantly expressed stanza attributed to Brebeuf as it is quoted in Francis Pierrepont Barnard's *Casting Counter and the Counting-Board.*

> Les courtisans sont des jetons; Leur valeur depend de leur place; Dans la faveur, des millions; Et des zeros dans la disgrace.

The same book also quotes the phrase, "Faux comme un jeton," which arose from the practice of gilding or plating jettons and passing them as money, or creating a deceptive impression.

The number 2,376 would be expressed by jettons on the line abacus or counting board as in Fig. 4.

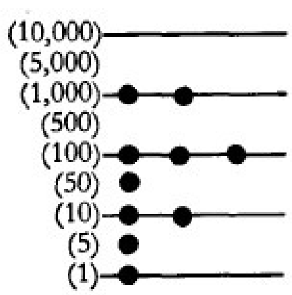


FIG. 4

(Each line upwards is ten times the value of that below it. Each space is five times as much as the line next below it. In addition, the process began at the units, and in subtraction at the higher digits.)

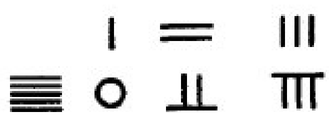
However, in Europe the line abacus failed to develop into the efficient rod abacus, and gradually gave way to the cipher system of greater efficiency, until it was given the *coup de grace* by the French Revolution, which enforced the nation-wide ciphering system. One of the major causes for this result is presumed to be found in the fact that before the introduction of Arabic numerals European countries used diverse systems of numerical notation—duodecimal, binary, sexagesimal, *etc.* The division of daytime into twelve hours and that of one hour into sixty minutes, *etc.* may be mentioned as vestiges of these numerical systems. The rod abacus can never be worked with efficiency on these numerical scales. Another remote cause may be traced back to the way in which the Arabs, who introduced the cipher system into Europe, named their numbers. The Semites, including the Arabs, named their numbers beginning at the units, although they wrote from right to left. Thus for instance, in Arabic, one hundred and twenty-five was called five and twenty and one hundred, the result appearing as 125, as Prof. Cargill G. Knott says in his treatise on the abacus. This is believed to be the primary reason why the Arabs, who achieved remarkable development in mathematics in the medieval ages, made use of their Arabic numerals without recourse to the less efficient line abacus or other calculating devices. Nor could the rod abacus have been used with efficiency by a race which used such a system of naming their numbers. The early Indians, who are credited with the invention of the cipher, spoke like the Arabs although they wrote from left to right. The Chinese named their numbers beginning with the largest denomination, although they wrote from top to bottom, proceeding from right to left.

Now the second question before us is: What causes prevented the adoption of a cipher system in China and Japan? The Chinese and the Japanese write in vertical columns from top to bottom, while the cipher system is worked from left to right. However, this is not considered the primary cause, for in the remote past coeval with the origination of Chinese characters, the Chinese carried out their calculation by means of reckoning blocks working left to right. However, this reckoning-block calculation was cumbersome and was no more fit for rapid operation than the Western line abacus.

A couple of examples of the arrangement of reckoning blocks are given below. The numbers 123 and 5,078 are represented:

123

5,078



In the units, hundreds, and other odd places, the numbers up *to* five are each represented by the corresponding number of vertical strokes, and the numbers from six to nine are each represented by the addition of the requisite number of strokes below a five-unit horizontal line. In the tens, thousands, and other even places, the numbers up to five are each represented by the corresponding number of horizontal strokes, and the numbers from six to nine are each represented by the addition of the requisite number of strokes above a five-unit horizontal line.

The Chinese numerical notation, which was probably the pictorial representation of reckoning-block calculation, was of far less practical use in calculation than reckoning blocks. Accordingly, mathematical calculation was generally performed with reckoning blocks and later also with the abacus. In remote antiquity, probably more than 2,000 years back, reckoning blocks were arranged differently for calculation. In those days the numbers in units and hundreds places were represented by horizontal blocks instead of vertical ones, and numbers in tens and thousands places were represented by vertical blocks. Thus the Chinese numerals, 一 (1), 二 (2), 三 (3), and 百 (100), comprised of horizontal strokes, are pictographs, representing horizontally arranged blocks (一二三), and the numerals, 十 (10), 廿 (20), 卅 (30), and 千 (1,000), comprised of vertical strokes, are pictographic imitations of blocks arranged vertically.

What caused the change in the arrangement of reckoning blocks is a knotty problem, to which no satisfactory solution has been offered. However, some scholars conjecture that because of the great importance of divination in early China the arrangement of reckoning blocks might probably have been influenced by the method of arranging reckoning sticks for divining purposes.

Probably the major cause which prevented the replacement of the cumbersome numerical notation by the cipher system was the development of the abacus, which could meet everyday public needs in business and household calculation. Reckoning-block calculation, which was applied to the primitive abacus, made a remarkable development, and by the Ming dynasty the abacus had become a far more efficient computer than the cipher system.

In the days of feudal government, learning was mostly of classics and was the exclusive heritage of certain officials and limited circles of scholars. Mathematics was studied only by the few who were initiated into this mystery of learning, and many of them formed exclusive esoteric sects of hereditary transmission to preserve their patrimonial positions or living. Under these social conditions it was none of their concern to teach or popularize their secrets of mathematics. The enlightened scholars who were favored with exceptional opportunities to study Western science and mathematics may have been aware of the superiority of Arabic numerals to the cumbersome Oriental numerical notation. But these intellectuals must have been too few and far between and their outcry to initiate the reform too feeble to arouse public attention.

Among the other important causes may be mentioned the want of free international trade and communication, the virtual isolation of Eastern countries from the West, and the consequent lack of understanding of international situations and national prejudice against foreign culture, and among the rest, the conservatism of human nature. The Chinese officialdom was so prominently conservative that it would firmly have resisted any attempts at such reforms or improvements in the hoary customs or time-honored classics of national veneration, many of which had been included among textbooks for government service examinations during the long Chinese historical period extending over twenty centuries.

In Japan it was not until several years after the 1868 political revolution, which overthrew the shogunate (government by the supreme feudal ruler), that the progressive modern government, awakened to the progress of the world, enacted the compulsory education law, including in the curriculum the cipher system, without which the effective teaching of modern mathematics to the public is impossible.

Now the Japanese word for abacus, *soroban*, is probably the Japanese rendering of the Chinese *suan-pan, (soo-pan* in the southern dialect or *sur-pan* in Manchuria). The soroban in Japan did not come into common use until the seventeenth century. However, the historical fact that beginning with the seventh century, there were at times as many as 2,000 Japanese students studying at the then Chinese capital in Chang-an, now called Si-an, furnishes us with reliable evidence that the abacus was introduced into Japan at a far earlier date, although the oldest documentary evidence of the Japanese abacus does not date further back than the sixteenth century.

In any case, once this convenient instrument of calculation gained popularity in Japan, it was studied extensively and intensively by many mathematicians including Seki Kowa (1640-1709), who discovered a native calculus independent of the Newtonian theory. As a result, the form and methods of operating the abacus have undergone one improvement after another. For a long time in Japan two kinds of abacus were used concurrently until the 1868 political revolution: the Chinese-style one with two five-unit counters and five one-unit counters and the older Japanese-style one with one five-unit counter and five one-unit counters. After the time of the revolution, the Chinese-style abacus went completely out of use. Finally since around 1940, the older-style Japanese abacus has largely been replaced by the present more advanced and efficient one with one five-unit counter and four one-unit counters.

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II. NEGATIVE ANSWERS FROM SUBTRACTION

Subtraction of larger numbers from smaller ones is performed by means of complementary numbers.

Fig. 5

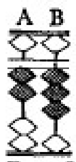


Fig. 6

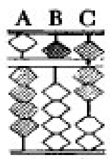


Fig. 7

For this purpose it is necessary to be able to read complementary numbers on the board at a glance.

For example, the complementary digit of 6 with respect to 10 is 4. However, the board shows 3 in the position of the complementary digits (Fig. 5).

The complementary numbers of 23 and 457 are *11* and 543 respectively. However, on the board they appear as 76 and 542 (Figs. 6 and 7). Therefore, on the board, the true complementary number can be obtained by adding one to the complementary digit on the last rod.

The following problem involving complements can be solved without making any mental or written calculations.

PROBLEM: A customer made a purchase of 7 dollars 68 cents and gave a clerk a ten-dollar bill. How much change should the customer receive?

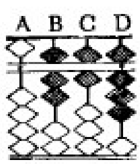


Fig. 8

If the clerk simply sets 768 on the abacus, the answer, 2 dollars 32 cents, will naturally appear on the board of its own accord in the form of the complementary number (Fig. 8).

Here are some examples of abacus calculation by means of complementary digits.

EXAMPLE 1: 2-9=-7

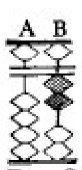


Fig. 9

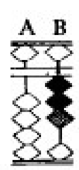


Fig. 10

```
   2
+1 0 ............Number borrowed
 1 2
 − 9 ............Number subtracted
   3
−1 0 ............Number returned
 − 7 ............Answer (complementary
                        number)
```

STEP 1: Set 2 on B (Fig. 9).

STEP 2: Since you cannot subtract 9 from 2, you must borrow 10 from the tens rod A, and subtract 9 from 12. This gives you 3 on B. However, since you borrowed 10 previously, you must return it. In other words, as 10 was added to the 3 on rod B, you must subtract 10 from it. You can do this very simply by setting minus 10 on rod B in your mind. Then the difference between the 3 and the minus 10 on rod B, *i.e.* minus 7, will mechanically appear in the form of the complementary number. This is the answer (Fig. 10). Note that the advantage of the abacus operation is to work out and change the difference between 2 and minus 9, *i.e.* minus 7, into that between 3 and minus 10 and to show the difference in the clearer and more obvious form.

NOTES: (a) In subtracting 9 from 2, do not make the mental calculation of subtracting 9 from 12 but add, to the 2 on B, the complementary number 1, with respect to 10, with the idea that you are subtracting 9 from 10. Nor should you take the trouble of setting 1 on rod A except for practice, (b) In calculation by complementary numbers, it is important for you to remember that the counters or beads which have been moved next to the beam always indicate positive numbers, while their complements represent negative numbers.

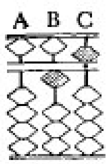


FIG. 11

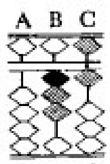


FIG. 12

EXAMPLE 2: 15-84=-69

STEP 1: Set 15on BC (Fig. II).

STEP 2: Borrow 100 from the hundreds rod A, and subtract 84 from 115.

This gives you 31 on BC, and the answer 69 mechanically appears on the board in the form of the complementary number of 31 for 100 (Fig. 12).

NOTE: The above process can be analyzed as follows:

Step 1:	1 5	
Step 2:	+1 0 0	............Number borrowed
	1 1 5	
	− 8 4	............Number subtracted
	3 1	
	−1 0 0	............Number returned
	− 6 9	............Answer (complementary number)

EXAMPLE 3: 29-76+94=47

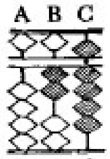


FIG. 13

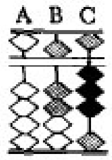


FIG. 14

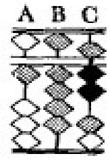


FIG. 15

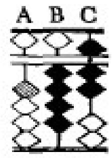


FIG. 16

STEP 1: Set 29 on BC (Fig. 13).

STEP 2: Borrow 100 from A, and subtract 76 from 129. This gives you 53 on BC (Fig. 14).

STEP 3: Add 94 to the 53 on BC. This gives you 147 on ABC (Fig. 15).

STEP 4: As you borrowed 100 previously, you must return it. So remove the 1 on A. The result is 47 on BC (Fig. 16).

NOTE: In step 3, the result 147 which you got is larger than the 100 which you borrowed. This shows that the result is positive. Accordingly, the result is 47 on BC.

	2 9	
+	(1 0 0)	............Number borrowed
	1 2 9	
−	7 6	............Number subtracted
	(5 3)	............The real result is minus 47.
+	9 4	............Number added
	1 4 7	
−	(1 0 0)	............Number returned
	4 7	............Answer

EXAMPLE 4: $628-$936+$864=$556

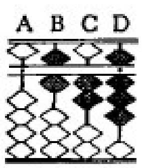


FIG. 17

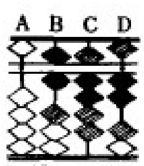


FIG. 18

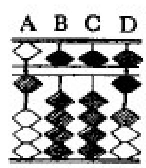


FIG. 19

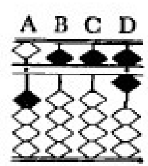


FIG. 20

STEP 1: Set 628 on BCD (Fig. 17).

STEP 2: Borrow 1,000 from rod A, and subtract 936 from 1,628. This gives you 692 on BCD (Fig. 18).

STEP 3: Add 864 to the 692 on BCD. This gives you 1,556 on ABCD (Fig. 19).

STEP 4: Remove the 1 on A which you borrowed in step 2. The answer is $556 (Fig. 20).

Step 1:		6 2 8	
Step 2:	+	(1 0 0 0)	............Number borrowed
		1 6 2 8	
	−	9 3 6	............Number subtracted
		6 9 2	............The real result is minus 308.
Step 3:	+	8 6 4	............Number added
		1 5 5 6	
Step 4:	−	(1 0 0 0)	............Number returned
		5 5 6	............Answer

EXAMPLE 5: 60-84-96=-120

In making successive subtractions, if it is not sufficient to borrow 100 from the first rod to the left, borrow 1,000 from the second rod to the left at the start of your calculations. We shall call this Method A.

The alternative is to borrow 100 from the first rod to the left in making the first subtraction and later borrow 900 when you need another 100. We shall call this Method B.

Although Method A is more efficient in that you only have to borrow once, you may not be able to anticipate your later need for more. In case you haven't borrowed enough at the beginning you have to resort to Method B.

METHOD A:

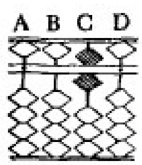


FIG. 21

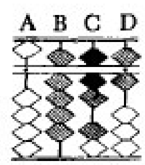


FIG. 22

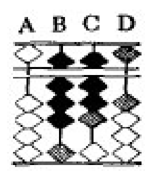


FIG. 23

STEP 1: Set 60 on CD (Fig. 21).

STEP 2: As it will not be sufficient to borrow 100, borrow 1,000 from rod A, and subtract 84 from 1,060. This gives you 976 on BCD. In borrowing 1,000, set 9 on B and, leaving the 9 intact there, shift 100 to B and C and then subtract 84 from 160. In this way you will automatically get the answer, minus 24, on CD in the form of the complementary number of 976 with respect to 1,000 (Fig. 22).

STEP 3: Next subtract 96 from the 976 on BCD. This leaves 880 on BCD. The answer is minus 120, which appears in the form of the complementary number on the board (Fig. 23).

NOTE: This minus 120 is the difference between the 1,000 which you borrowed and the 880 which you have on the board.

```
Step 1:          6 0
Step 2:  +  (1 0 0 0)...........Number borrowed
             1 0 6 0
         -       8 4 ...........First number subtracted
               9 7 6 ...........The real result is minus 24.
Step 3:  -       9 6 ...........Second number subtracted
               8 8 0
         -  (1 0 0 0)...........Number returned
         -     1 2 0 ...........Answer
```

METHOD B:

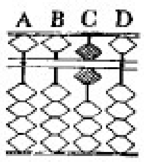


FIG. 24

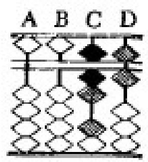


FIG. 25

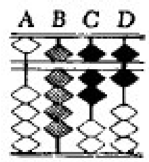


FIG. 26

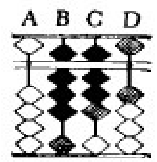


FIG. 27

STEP 1: Set 60 on CD (Fig. 24).

STEP 2: Borrow 100 from rod B, and subtract 84 from the 60 on CD, treating the 60 as 160. This gives you 76 on CD. This number is really minus 24 (Fig. 25).

STEP 3: You cannot subtract 96 from minus 24. So set 9 on B (Fig. 26).

NOTE: Setting 9 on B now produces the same result as having borrowed 1,000 from rod A in the beginning since you borrowed 100 previously from rod B.

STEP 4: Now subtract 96 from the 976 on BCD. This leaves 880 on BCD. The answer is minus 120, which appears in the form of the complementary number (Fig. 27).

```
Step 1:          6 0
Step 2:  +     (1 0 0)...........Number borrowed
               1 6 0
         -       8 4 ...........First number subtracted
                 7 6 ...........The real result is minus 24.
Step 3:  +     (9 0 0)...........Number borrowed again
               9 7 6
Step 4:  -       9 6 ...........Second number subtracted
               8 8 0
         -  (1 0 0 0)...........Number returned
         -     1 2 0 ...........Answer (complementary
                                  number)
```

NOTE: When you have to borrow again, after borrowing 100, never borrow 100 but 900. If you borrow 100 twice, you cannot get the complementary number for 200 on the board, which slows down your operation.

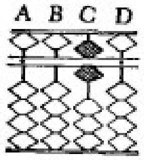


FIG. 28

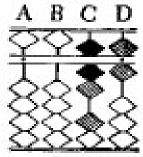


FIG. 29

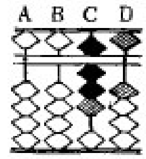


FIG. 30

You will find this in the following steps, in which the above Example 5 is worked by this wrong method.

STEP 1: Set 60 on CD (Fig. 28).

STEP 2: Borrow 100 from rod B, and subtract 84 from the 60 on CD, treating the 60 as 160. This gives you 76 on CD (Fig. 29).

STEP 3: Borrow 100 again from B and subtract 96 from 176, and you get 80 on CD (Fig. 30).

Since you borrowed 200, now you must return 200. But as the board does not show the complementary number of 80 for 200, you must work out the answer by subtracting 80 from 200. Thus this method is rather awkward.

EXAMPLE 6: 55-67-4,297=-4,309

STEP 1: Set 55 on DE (Fig. 31).

STEP 2: Borrow 100 from C, and subtract 67 from 55, treating the 55 as 155. This gives you 88 on DE. The result is minus 12 (Fig. 32).

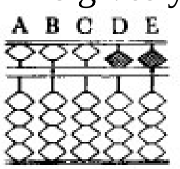


FIG. 31

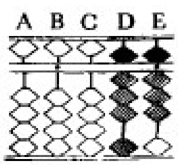


FIG. 32

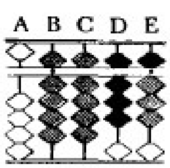


FIG. 33

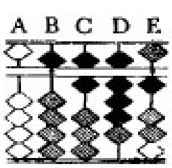


FIG. 34

STEP 3: As you cannot subtract 4,297 from the 88 on DE, you must borrow 10,000 from rod A. So set 9 on both C and B. You must note that setting 9 on both C and B produces the same result as borrowing 10,000 from A, for the reason that you have previously borrowed 100 from C (Fig. 33).

STEP 4: Now subtract 4,297 from the 9,988 on BCDE. This gives you 5,691 on BCDE. The answer is the complementary number of 5,691 with respect to minus 10,000, *i.e.* minus 4,309, which appears on the board (Fig. 34).

Step 1:		5 5	
Step 2:	+	(1 0 0)	...........Number borrowed
		1 5 5	
	−	6 7	...........First number subtracted
		8 8	...........The real result is minus 12.
Step 3:	+	(9 9 0 0)	...........Number borrowed again
		9 9 8 8	
Step 4:	−	4 2 9 7	...........Second number subtracted
		5 6 9 1	
	−	(1 0 0 0 0)	...........Number returned
		−4 3 0 9	...........Answer (complementary number)

Exercises

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
¥167	¥283	¥309	¥436	¥512	¥608	¥274	¥713	¥82	¥903
280	406	472	159	296	152	460	264	39	75
349	137	563	618	947	591	958	380	65	829
265	459	128	703	613	235	215	521	721	6,473
420	261	796	841	308	764	309	849	9,604	146
593	825	204	397	247	318	132	706	76	2,704
178	570	485	204	105	973	341	693	148	92
834	968	601	972	430	896	604	942	85	98,650
672	309	815	120	867	409	897	874	629	3,518
905	147	937	584	698	240	786	150	4,053	637
¥-159	¥-361	¥1,546	¥ 0	¥-1	¥-80	¥-1,944	¥-2,036	¥-14,682	¥105,063

III. OTHER METHODS OF MULTIPLICATION

There are several methods of multiplication that can be used on the abacus. A comparative study of these methods may be interesting. I hope that it will lead the reader to the conclusion that the standard method of multiplication as described in my first book is the best. In addition to discussing other basic methods of multiplication I shall introduce some methods of simplified multiplication.

1. The Variant of the Standard Multiplication

There is a variant method of the standard multiplication, which was popular in Japan around 1930, but which was largely replaced in less than ten years by the standard method of multiplication. The variant is still favored by quite a few experts including entrants in abacus contests, because it is a little faster than the standard method. Here is an example.

EXAMPLE 1: 56x49=2,744

STEP 1: Set the multiplicand 56 on EF and the multiplier 49 on AB (Fig. 35).

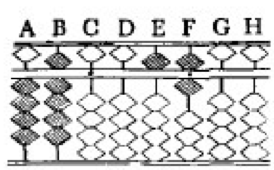


FIG. 35

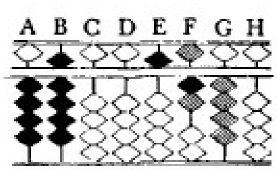


FIG. 36

STEP 2: Multiplying the 4 on A by the 6 on F, set the product 24 on FG after clearing F of the 6. This gives you 24 on FG (Fig. 36).

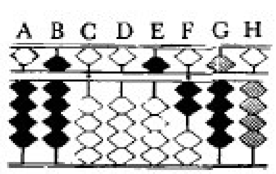


FIG. 37

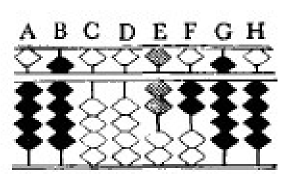


FIG. 38

STEP 3: Multiplying the 9 on B by the same 6 which you remember was on F, set the product 54 on GH. Since you had 24 on FG, you get a total product of 294 on FGH (Fig. 37).

STEP 4: Multiplying the 4 on A by the 5 on E, set the product 20 on EF after clearing E of the 5. This gives you a total of 2,294 on EFGH (Fig. 38).

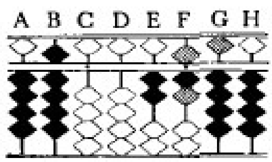
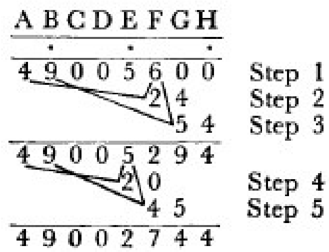


FIG. 39



STEP 5: Multiplying the 9 on B by the same 5 which you remember was on E, set the product 45 on FG. Since you had 2,294 on EFGH, you get, on EFGH, a total of 2,744, which is the answer (Fig. 39).

Advantages and Disadvantages

ADVANTAGE:

This method is a little faster than the standard method, because the distance between the multiplier and the product has been reduced by one rod.

DISADVANTAGES:

(1) The product of this multiplication does not form in the position of the dividend of the standard division, as is the case with the standard multiplication and division.

It should be noted that this method produces its product in the position of the dividend of the older method of division. In other words, it forms the counterpart of the older method of division, the performance of which requires the use of its own special division table. With the decay of the older method of division, this multiplication method has also given way to the standard method of multiplication.

(2) The beginner may find this method a little less easy to follow than the standard one, as he has to remember each digit in the multiplicand after it is removed.

2. Multiplication Starting with the Final Digits of the Multiplier and the Multiplicand

The order of multiplication in this method is identical with that of written multiplication. Accordingly, the student of the abacus will find it interesting and useful to compare this method with the standard method. This arithmetic method is the oldest multiplication method here introduced. It was used extensively in Japan until around 1930, when it was largely replaced by the variant of the standard multiplication method later developed, and which, in turn, was replaced by the standard method itself.

This method may be broken down into two variants. One of them, which formed the counterpart of the older method of division, used to be popular in Japan. We shall call this Variant B. But today, when the older method of division has fallen out of favor, the variant of this method which I shall call Variant A makes a counterpart of the standard method of division and is free from the incidental details of operation that complicate Variant B.

Variant A

EXAMPLE 1: 78x89=6,942

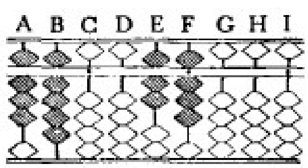


FIG. 40

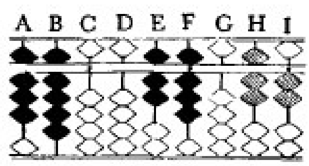


FIG. 41

STEP 1: Set 78 on EF and 89 on AB (Fig. 40).

STEP 2: Multiplying the 9 on B by the 8 on F, set the product 72 on HI (Fig. 41).

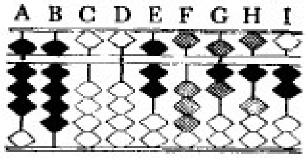


FIG. 42

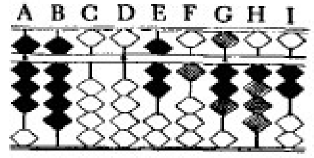


FIG. 43

STEP 3: Multiplying the 8 on A by the same 8 on F, set the product 64 on GH, and clear F of the 8. This gives you a total product of 712 on GHI (Fig. 42).

STEP 4: Multiplying the 9 on B by the 7 on E, add the product 63 to the 71 on GH. This makes a total product of 1,342 on FGHI (Fig. 43).

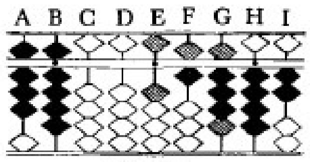
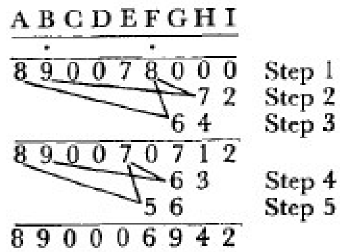


Fig. 44



STEP 5: Multiplying the 8 on A by the same 7 on E, add the product 56 to the 13 on FG, and clear E of its 7. This gives you, on FGHI, a total product of 6,942, which is the answer (Fig. 44).

NOTE: The procedure of this Variant A forms the unit rod of the product on the rod to the right of the unit rod of the multiplicand by as many rods plus one as there are digits in the multiplier.

Variant B

The disadvantages of Variant B may be illustrated best by the same example, since this method can become complicated when both multiplier and multiplicand have large digits. But the complications offer no serious obstacles to experienced operators. Hence, until recently, Variant B has been preferred to Variant A primarily because it is faster and also because it is the counterpart of the older method of division.

EXAMPLE 2: 78x89=6,942

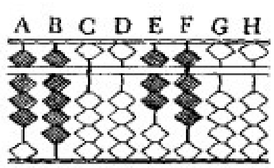


Fig. 45

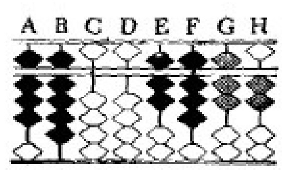


Fig. 46

STEP 1: Set 78 on EF and 89 on AB (Fig. 45).

STEP 2: Multiplying the 9 on B by the 8 on F, set the product 72 on GH (Fig. 46).

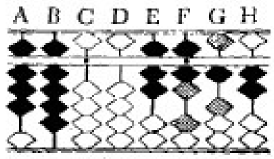


FIG. 47

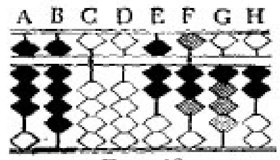


FIG. 48

NOTE: The reader can see that Variant B sets the product one rod closer to the multiplicand and thus is slightly speedier than Variant B.

STEP 3: Multiplying the 8 on A by the 8 on F, set the product 64 on FG after clearing F of its 8. This gives you a total product of 712 on FGH (Fig. 47).

STEP 4: Multiplying the 9 on B by the 7 on E, set the product 63 on FG. This makes a total of 7,342 on EFGH.

In this procedure do not add 1 to the 7 on E, but remember to add it in the next step (Fig. 48).

STEP 5: Multiplying the 8 on A by the 7 on E, set the product 56 on EF after removing the 7 on E.

In this step do not forget the 1 which must be added to E. This leaves you with a total of 6,942 on EFGH, which is the answer (Fig. 49).

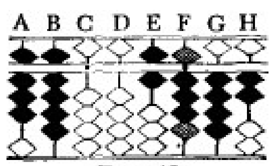
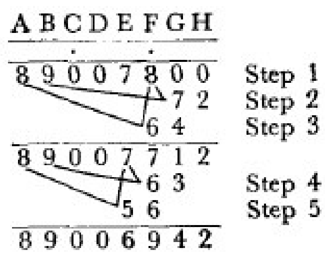


FIG. 49



NOTE: The 1 to be added in step 4 must be remembered till step 5. This kind of situation occurs especially when both the multiplier and the multiplicand are large numbers.

EXAMPLE 3: 78x456=35,568

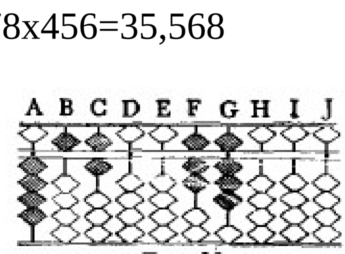


FIG. 50

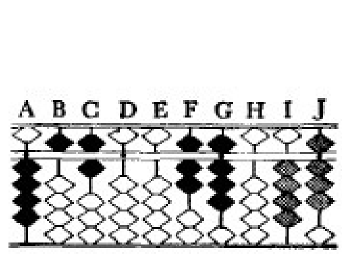


FIG. 51

STEP 1: Set the multiplicand 78 on FG and the multiplier 456 on ABC (Fig.

50).

STEP 2: Multiplying the 6 on C by the 8 on G set the product 48 on IJ (Fig. 51).

STEP 3: Multiplying the 5 on B by the same 8 on G, set the product 40 on HI. This makes a total of 448 on HIJ (Fig. 52).

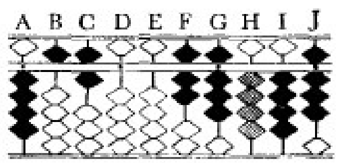


Fig. 52

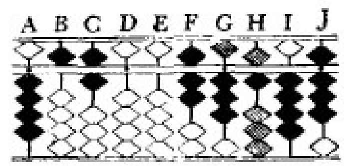


Fig. 53

STEP 4: Multiplying the 4 on A by the same 8 on G, set the product 32 on GH, and clear G of its 8. This makes a total of 3,648 on GHIJ (Fig. 53).

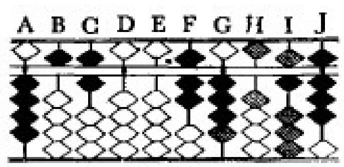


Fig. 54

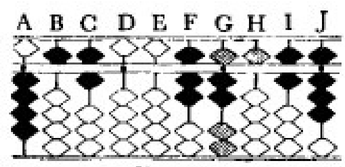


Fig. 55

STEP 5: Multiplying the 6 on C by the 7 on F, set the product 42 on HI. This makes a total of 4,068 on GHIJ (Fig. 54).

STEP 6: Multiplying the 5 on B by the same 7 on F, set the product 35 on GH. This makes a total of 7,568 on GHIJ (Fig. 55).

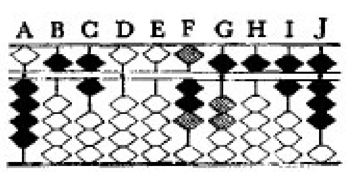


Fig. 56

A	B	C	D	E	F	G	H	I	J	
4	5	6	0	0	7	8	0	0	0	Step 1
								4	8	Step 2
							4	0		Step 3
						3	2			Step 4
4	5	6	0	0	7	3	6	4	8	
							4	2		Step 5
						3	5			Step 6
					2	8				Step 7
4	5	6	0	0	3	5	5	6	8	

STEP 7: Multiplying the 4 on A by the same 7 on F, set the product 28 on FG, and clear F of the 7. This gives you, on FGHIJ, a total of 35,568, which is the answer (Fig. 56).

Advantages and Disadvantages

ADVANTAGES:

(1) Variant A of this method forms the counterpart of the standard method of division, that is, it forms its product in the position of the dividend of the latter method.

(2) Since the order of multiplication is identical with that of written multiplication, the beginner may find it easier to learn.

DISADVANTAGES:

This method is not commonly used for the following reasons.

(1) The operator has to take the trouble of counting the digits in the multiplier. As was previously explained, Variant A requires that the unit rod of the first product be separated from the unit rod of the multiplicand by as many rods plus one as there are digits in the multiplier, while Variant B requires that the unit rod of the first product be separated from the last rod of the multiplicand by as many rods as there are digits in the multiplier. The greater the distance, the greater the inconvenience.

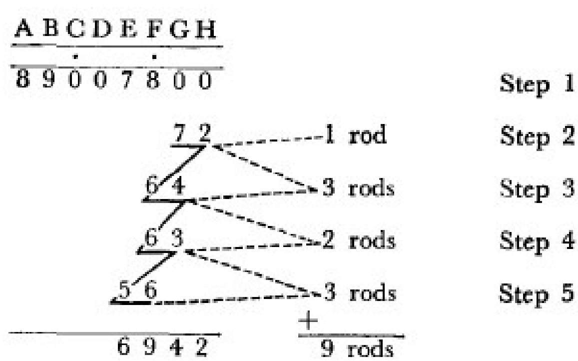
(2) Right-to-left operation makes this method a little slower than the other methods.

(3) Variant B of this method is often complicated by the kind of inconvenience which arose in Example 2 of this section.

Why Is Right-to-Left Operation Slower than Left-to-Right Operation?

The reasons for the slowness of right-to-left operation of this method may be seen through the analytical comparison of this method with the standard one.

The procedure of multiplication illustrated in Example 2 sets the products as given below.

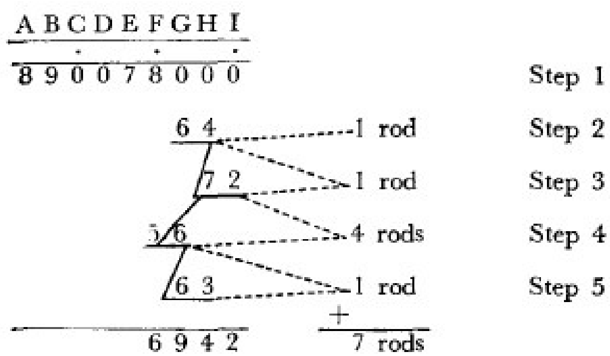


The above shows that in this operation the finger has to travel a total of 9 rods. In step 2, the finger travels 1 rod from G to H. In step 3, it travels 3 rods from H back to F and forward to G. In step 4, it travels 2 rods from G back to F and forward again to G. In step 5, it travels 3 rods from, G back to E and forward to F.

If the same problem, 78x89, is performed by the standard method of multiplication, the finger has to travel no more than 7 rods, as indicated in the figure below.

The figure shows that, in step 2, the finger travels for 1 rod from G to H. In step 3, it travels for 1 rod from H to I. In step 4, it travels for 4 rods from I back to F and forward to G. And in step 5, it travels for 1 rod from G toH.

Furthermore, the right-to-left operation of this method may be said to run counter to the efficient left-to-right operation of the standard method.



3. Multiplication Beginning with the Highest Digits of the Multiplier and Multiplicand

It is also possible to do multiplication beginning with the highest digits of the multiplier and the multiplicand. For this method it is necessary to separate the multiplier and the multiplicand by as many rods plus two as there are digits in the multiplier. Otherwise the product will extend into the multiplicand, and operation will become impossible.

EXAMPLE 1: 43x72=3,096

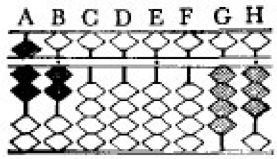


FIG. 57

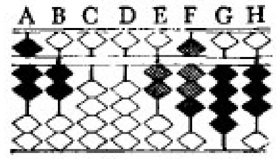


FIG. 58

STEP 1: Set the multiplicand 43 on GH and the multiplier 72 on AB, with four vacant rods between them (Fig. 57).

STEP 2: Multiplying the 7 on A by the 4 on G, set the product 28 on EF (Fig. 58).

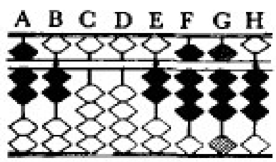


FIG. 59

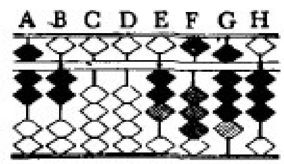


FIG. 60

STEP 3: Multiplying the 2 on B by the same 4 on G, set the product 8 on G after clearing G of the 4, that is, the highest digit of the multiplier. This makes a total product of 288 on EFG (Fig. 59).

STEP 4: Multiplying the 7 on A by the 3 on H, set the product 21 on FG. This makes a total product of 309 on EFG (Fig. 60).

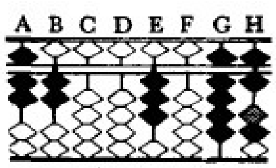
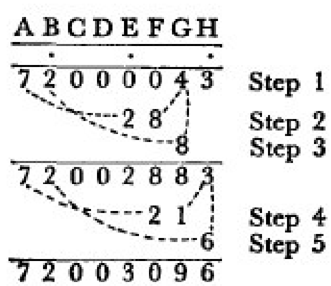


FIG. 61



STEP 5: Multiplying the 2 on B by the same 3 on H, set the product 6 on H after clearing H of the 3, that is, the second digit of the multiplier. This gives you, on EFGH, a total product of 3,096, which is the answer (Fig. 61).

Advantages and Disadvantages

ADVANTAGES:

(1) When the multiplier is a whole number, the unit digit of the product

forms on the unit rod of the multiplicand, so the necessity of searching for the unit rod of the product is eliminated. However, this rule does not hold when the multiplier ends in one or more zeros. The product moves to the right of the multiplicand by as many rods as there are zeros at the end of the multiplier.

(2) As the operation starts by multiplying the first digits of the multiplier and multiplicand, it is convenient for approximations.

DISADVANTAGES:

This method is not used very much for the following reasons.

(1) It necessitates counting the digits of the multiplier. The multiplier must be separated from the multiplicand by as many rods plus two as there are digits in the multiplier. When the multiplier is long, this method becomes rather awkward.

(2) It is inconvenient for the calculation of compound numbers not based on the decimal system. Try using this method on a problem involving hours, minutes, and seconds, for example.

(3) It has no corresponding method of division. In other words, there is no method of division where the quotient has the position of the multiplicand in this method of multiplication.

4. The Chinese Method of Multiplication

The following example introduces the method of multiplication widely practiced in China.

EXAMPLE 1: 345x67=23,115

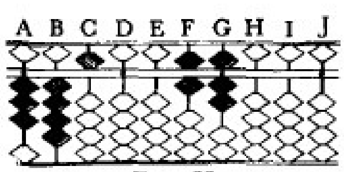


FIG. 62

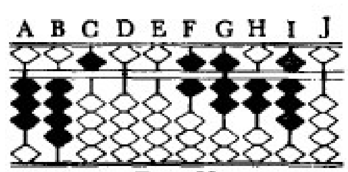


FIG. 63

STEP 1: Set the multiplicand 67 on FG and the multiplier 345 on ABC (Fig. 62).

STEP 2: Multiplying the 4 on B by the 7 on G, set the product 28 on HI (Fig. 63).

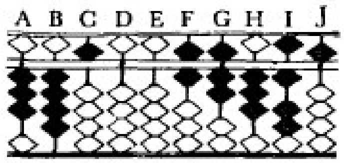


FIG. 64

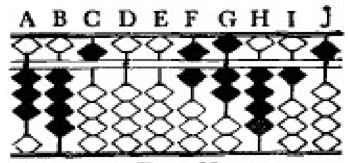


FIG. 65

STEP 3: Multiplying the 5 on C by the same 7 on G, set the product 35 on IJ. This gives you a total of 315 on HIJ (Fig. 64).

STEP 4: Multiplying the 3 on A by the same 7 on G, set the product 21 on GH after clearing G of its 7. This gives you a total product of 2,415 on GHIJ (Fig. 65).

STEP 5: Now multiplying the 4 on B by the 6 on F, set the product 24 on GH. This gives you a total of 4,815 on GHIJ (Fig. 66).

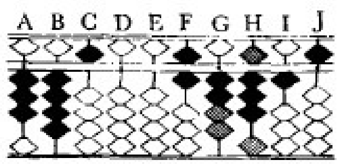


FIG. 66

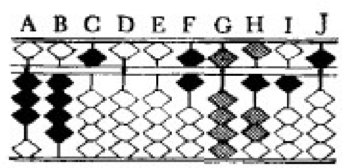


FIG. 67

STEP 6: Multiplying the 5 on C by the same 6 on F, set the product 30 on HI. This gives you a total of 5,115 on GHIJ (Fig. 67).

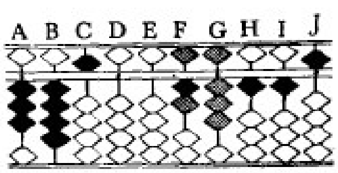


FIG. 68

A	B	C	D	E	F	G	H	I	J	
3	4	5	0	0	6	7	0	0	0	Step 1
							2	8		Step 2
								3	5	Step 3
						2	1			Step 4
3	4	5	0	0	6	2	4	1	5	
						2	4			Step 5
							3	0		Step 6
					1	8				Step 7
3	4	5	0	0	2	3	1	1	5	

STEP 7: Finally, multiplying the 3 on A by the same 6 on F, set the product 18 on FG, after clearing F of its 6. This gives you, on FGHIJ, a total product of 23,115, which is the answer (Fig. 68).

The reader will see that this method of multiplication forms its product in the same position as does the variant of the standard method of multiplication introduced at the beginning of this chapter.

NOTES: (a) When this method of multiplication is followed, it often facilitates calculation for the rods to have two five-unit counters when both the multiplier and multiplicand have large digits. This is one of the reasons why the Chinese abacus has two five-unit counters on each rod. (b) Also notice that in China the multiplicand is set on the left and the multiplier is set on the right.

Advantages and Disadvantages

ADVANTAGES:

In this example, suppose you had used the variant of the standard method of multiplication. Then, in step 2, rod G would have been cleared of the 7 before the product was set on GH. However, in this method, the 7 on G remains on the board until step 4, when all of the operations involving that 7 have been completed.

DISADVANTAGES:

In this example, the additions of the products proceed from left to right till step 3. But in step 4, the hand must shift back to the left to add the product. Since the operation proceeds in this way, it is somewhat slower than the standard method and its variant.

5. The Elimination of the Final Digit of a Multiplier Ending in One

Multiplication can be facilitated by leaving out of calculation the final digit one of the multiplier. The Japanese technical term for this kind of multiplication may be translated as the final-digit-one elimination multiplication.

This multiplication starts its calculation with the highest digits of the multiplier and the multiplicand. The multiplier and the multiplicand must be separated by as many rods plus one or two as there are digits in the multiplier; otherwise the product will extend into the multiplier.

EXAMPLE: 74x31=2,294

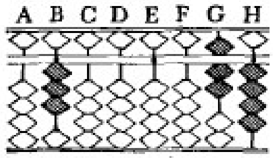


FIG. 69

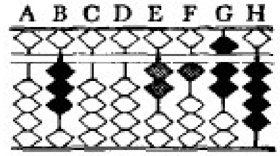


FIG. 70

STEP 1: Set the multiplicand 74 on GH and the 3 of the multiplier 31 on B (Fig. 69).

NOTES: (a) The trouble of setting the final digit one of the multiplier is spared, because it is not used in this operation, (b) A beginner using this method is liable to make errors.

STEP 2: Multiplying the 3 on B by the 7 on G, set the product 21 on EF, and you get 2,174 on EFGH (Fig. 70).

NOTES: (a) In this method of multiplication, the multiplicand 74 is supposed to have been multiplied by the 1 of the multiplier 31, and is left intact on the board, and the products made by multiplying the 3 of 31, *i.e.* 30, by the two digits of the multiplicand each are added to the multiplicand 74. (b) When the multiplier is a whole number, the unit digit of the product forms on the unit rod of the multiplicand, (c) In step 2, the 3 on B is really 30 and the 7 on G is really 70. Accordingly, their product 21 is set on EF, because it is really 2,100.

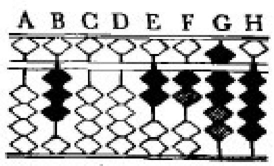


FIG. 71

A	B	C	D	E	F	G	H	
0	3	0	0	0	0	7	4	Step 1
				+2	1			Step 2
0	3	0	0	2	1	7	4	
					+1	2		Step 3
0	3	0	0	2	2	9	4	

STEP 3: Multiplying the same 3 on B by the 4 on H, add the product 12 to the 17 on FG. This gives you, on EFGH, a total of 2,294, which is the answer (Fig. 71).

NOTE: In this step, the 3 on B is really 30, but the 4 on H is a real 4. Accordingly, their product 12 is set on FG, because it is really 120.

This multiplication can be performed by subtracting 1 from a multiplier whose last digit is 2 or a larger digit, although this procedure does not improve calculations. An example will be given below.

EXAMPLE 2: 83x42=3,486

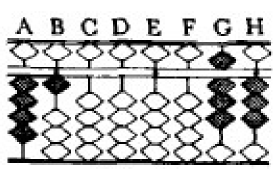


FIG. 72

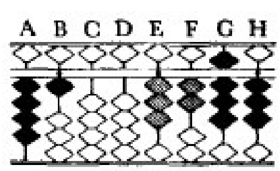


FIG. 73

STEP 1: Set the multiplicand 83 on GH and the multiplier 41 on AB (Fig. 72).

NOTE: In this multiplication, the multiplicand is left intact on the board, and the operation performed can be expressed as 83+[83x(42-1)].

STEP 2: Multiplying the 4 on A by the 8 on G, set the product 32 on EF. This gives you 3,283 on EFGH (Fig. 73).

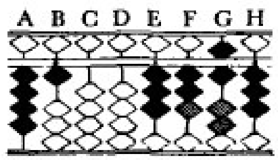


FIG. 74

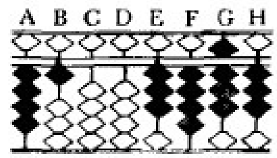


FIG. 75

STEP 3: Next, multiplying the 1 on B by the same 8 on G, set the product 8 on G. This gives you 3,363 on EFGH (Fig. 74).

STEP 4: Now, multiplying the 4 on B by the 3 on H, set the product 12 on FG. This gives you a total of 3,483 on EFGH (Fig. 75).

STEP 5: Finally, multiplying the 1 on B by the same 3 on H, set the product 3 on H. And you get, on EFGH, a total of 3,486, which is the answer (Fig. 76).

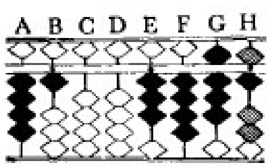
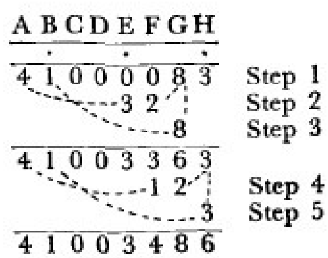


FIG. 76



Advantages and Disadvantages

ADVANTAGES:

This method simplifies calculation by reducing the number of the digits in the multiplier by one.

DISADVANTAGES:

(1) This method necessitates counting the digits in the multiplier so as to separate the multiplier from the multiplicand by as many rods plus one or two as there are digits in the multiplier.

(2) The value of this method is limited to the case in which the final digit of the multiplier is one.

6. The Elimination of the Initial Digit of a Multiplier Beginning with One

Multiplication can also be simplified by leaving out the initial digit one of the multiplier. The Japanese technical term for this multiplication may be translated as the initial-digit-one elimination multiplication.

EXAMPLE 1: 23x12=276

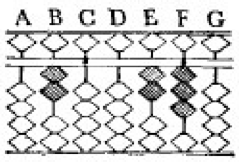


FIG. 77

STEP 1: Set the multiplicand 23 on EF and the 2 of the multiplier 12 on B, the third rod to the left of rod E (Fig. 77).

NOTES: (a)There is no objection to setting the whole multiplier 12 on AB. However, experts who use this method do not customarily set the first digit 1 because it is not used in calculation, and they can remember that the 2 here is the 12 of the multiplier. However, this method of multiplication is apt to give the beginner trouble, (b) In this multiplication, 23 is supposed to have been multiplied by the 10 of the multiplier 12 and is regarded as 230, with G as its unit rod. The product is made by multiplying the 2 of 12 by each digit of 23 and adding the result to the 230 on EFG.

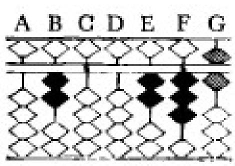


FIG. 78

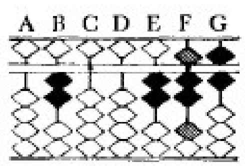


FIG. 79

STEP 2: Multiplying the 2 on B by the 3 on F, set the product 6 on G. This makes a total of 236 on EFG (Fig. 78).

NOTE: Since both the 2 on B and the 3 on F are in the unit place, the product 6 must be set on G, the unit rod of the product.

STEP 3: Next multiplying the same 2 on B by the 2 on E, add the product 4 to the 3 on F. This gives you, on EFG, a total of 276, which is the answer (Fig. 79).

A	B	C	D	E	F	G	
0	2	0	0	2	3	0	Step 1
						+6	Step 2
0	2	0	0	2	3	6	
					+4		Step 3
0	2	0	0	2	7	6	

NOTES: (a) In this step, since the 2 on B is in the unit place and the product 4 must be set on F, the tens rod of the product (b)Operation must start with the last digit of the multiplicand. Otherwise, the product will extend into the multiplicand and render calculation impossible, (c) This method simplifies calculation especially when the second digit of the multiplier is small, i.e., when the multiplier is a number such as 11, 107, 1,008, *etc.*

EXAMPLE 2: 75x103=7,725

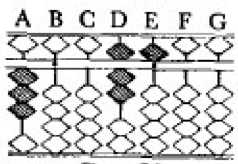


FIG. 80

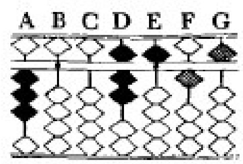


FIG. 81

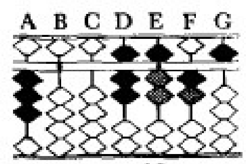


FIG. 82

STEP 1: Set the multiplicand 75 on DE and the 3 of the multiplier 103 on A (Fig. 80).

NOTE: In this problem, the multiplicand 75 is supposed to have been multiplied by the 100 of the multiplier 103. Accordingly, the 75 on DE is regarded as the product 7,500 on DEFG, with G as the unit rod, F as the tens rod, and E as the hundreds rod.

STEP 2: Multiplying the 3 on A by the 5 on E, set the product 15 on FG. This makes a total of 7,515 on DEFG (Fig. 81).

STEP 3: Next multiplying the 3 on A by the 7 on D, add the product 21 to the 51 on EF. This gives you, on EFGH, a total of 7,725, which is the answer (Fig. 82).

A	B	C	D	E	F	G	
·			·				
3	0	0	7	5	0	0	Step 1
				+	1	5	Step 2
3	0	0	7	5	1	5	
			+	2	1		Step 3
3	0	0	7	7	2	5	

NOTE: Since the 7 on D is in the tens place, and the 3 on A in the unit place, the product 21 is really 210 and must be set on EF.

EXAMPLE 3: 78x13=1,024

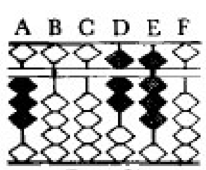


FIG. 83

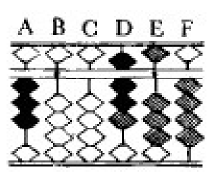


FIG. 84

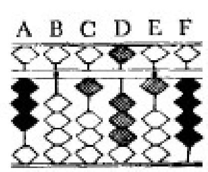


FIG. 85

A	B	C	D	E	F	
3	0	0	7	8	0	Step 1
				2	4	Step 2
3	0	0	7	0	4	
			(1)			
			2	1		Step 3
3	0	1	0	1	4	

This example is to show that when the second digit in the multiplier is not a small one, this method becomes complicated.

STEP 1: Set the multiplicand 78 on DE and the 3 of the multiplier 13 on A (Fig. 83).

NOTE: AS the multiplier is a two-digit number, F becomes the unit rod of the product.

STEP 2: Multiplying the 3 on A by the 8 on E, set the product 24 on EF. This makes a total of 804 on DEF (Fig. 84).

NOTE: However, you must remember till the next step that the digit which was originally on D was not 8 but 7.

STEP 3: Multiplying the 3 on A by the 7 which you remember was on D, set the product 21 on DE. This gives you, on CDEF, a total of 1,014, which is the answer (Fig. 85).

EXAMPLE 4: An article was bought for $250 and sold at a gain of 6.8%. Find the selling price.

Since the selling price is determined by adding the cost and the profit, it can be found by the following multiplication:

$$\$250x(1+0.068)=\$250x1.068=\$267$$

There are two methods for working this multiplication. One is the standard method of multiplication, and the other the method of eliminating the initial digit one of the multiplier. The latter operation is shown below.

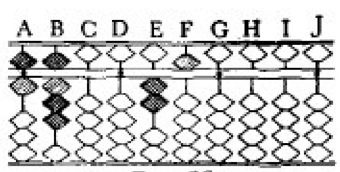


FIG. 86

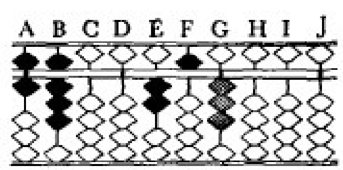


FIG. 87

STEP 1: Set the dividend 250 on EFG, with G as the unit rod, and set the 68 of the divisor 1.068 on AB (Fig. 86).

NOTE: When the multiplier is a mixed decimal, it is generally advisable to set its unit figure on the unit rod. But experts usually do not bother to, since the digit 1 is not used in calculation. As long as you remember that 68 stands for 1.068, it matters little to set the unit digit of the multiplier on the unit rod.

STEP 2: Since the multiplier is a four-digit number, suppose that 250 has been multiplied by 1,000, producing 250,000 on EFGHIJ, with G as the unit rod (Fig. 87).

Now multiplying the 6 on A by the 5 on F, set the product 30 on GH (Fig. 87).

NOTE: As both the 6 on A and the 5 on F are in the tens place, the product 30 must be set on GH.

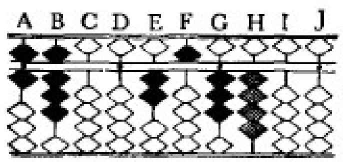


FIG. 88

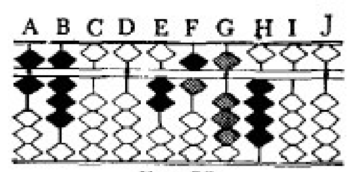


FIG. 89

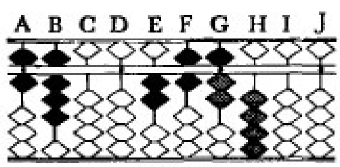


FIG. 90

A	B	C	D	E	F	G	H	I	J	
6	8	0	0	2	5	0	0	0	0	Step 1
						3	0			Step 2
							4	0	0	Step 3
6	8	0	0	2	5	3	4	0	0	
					1	2				Step 4
						1	6			Step 5
6	8	0	0	2	6	7	0	0	0	

STEP 3: Next multiplying the 8 on B by the same 5 on F, set the product 40 on HI. This gives you 34 on GH (Fig. 88).

STEP 4: Next multiplying the 6 on A by the 2 on E, add the product 12 to the 53 on FG. This gives you 654 on FGH (Fig. 89).

STEP 5: Finally multiplying the 8 on B by the same 2 on E, add the product 16 to the 54 on GH. This gives you, on EFG, a total of 267, which is the answer (Fig. 90).

NOTE: It should be noted that when the multiplier is a mixed decimal whose one whole figure is in the unit place, the unit rod of the multiplicand becomes that of the product. The reason for this is that the product may be regarded as having been multiplied by one. Accordingly, in this multiplication, it is quite easy to locate the unit rod of the product.

Rules for Finding the Unit Rod of the Product

RULE 1: When the multiplier is a mixed decimal whose whole figure one is in the unit place—i.e., 1—the unit rod of the quotient forms on that of the multiplicand.

RULE 2: Each time the value of this multiplier is raised by one place, the unit rod of the product shifts by one rod to the right of that of the multiplicand, and each time the value of this multiplier is reduced by one place, the unit rod of the product shifts by one rod to the left of that of the multiplicand.

Rule (1) means that when the multiplier is 1.05 or 1.023, the unit rod of the product forms on that of the multiplicand.

Rule (2) means that when the multiplier is 10.5 or 10.23, the unit rod of the product forms on the first rod to the right of that of the multiplicand, and that when the multiplier is 0.105 or 0.1023, the unit rod of the product forms on the first rod to the left of that of the multiplicand.

Advantages and Disadvantages

ADVANTAGES:

(1) This method of multiplication simplifies calculation especially when the second digit of the multiplier is zero, as in 103, 109, 1.082, *etc.*

When the multiplier is a number whose second digit is small, such as 11.114, 125, 1.078, etc., calculation is often simplified. However, this situation sometimes requires a figure or figures which have to be carried over to the rod next on the left.

(2) As shown in Example 4, when the multiplier is a mixed decimal whose whole figure one is in the unit place, the multiplicand itself is used as the product. In other words, the unit digit of the product forms on the unit rod of the multiplicand. This enables the operator to locate easily the unit rod of the product and to simplify calculations.

Because of these two advantages, this method of multiplication is extensively used in the calculation of percentages in business.

DISADVANTAGES:

When the multiplier is a number whose second digit is large, like 17, 189, etc., this method becomes awkward. It may involve remembering so many digits that calculation becomes extremely difficult. This is especially true when the multiplicand also is a number with many large digits.

7. Multiplication by Complementary Numbers

Multiplication can often be simplified by using the complement of a multiplier. For instance, take the problem 26x98. The 26 is multiplied by 100, becoming 2,600. From this 26x2, 52 is subtracted. In this way the answer 2,548 is obtained. The 2 is, of course, the complement of 98 with respect to 100. This method of computation is better than the ordinary method of multiplication when the multiplier is a number a little smaller than 100 or 1,000, etc., such as 97, 996, *etc*.

EXAMPLE 1: 26x98=2,548

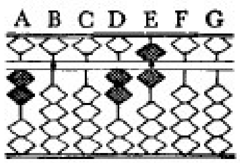


FIG. 91

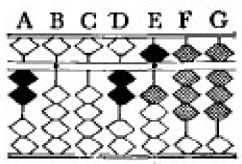


FIG. 92

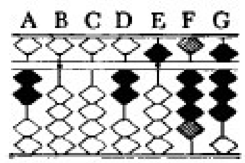


FIG. 93

STEP 1: Set the multiplicand 26 on DE, and on A, set 2, the complement of 98 with respect to 100 (Fig. 91). When the multiplier is a two-figure number, the multiplicand is regarded as having been multiplied by 100. So in this problem the unit rod of the product shifts to G.

A	B	C	D	E	F	G	
·			·				
2	0	0	2	6	0	0	Step 1
					−1	2	Step 2
2	0	0	2	5	8	8	
					−4		Step 3
2	0	0	2	5	4	8	

STEP 2: Multiplying the 2 on A by the 6 on E, subtract the product 12, which is to be set on FG, from the 6 on E. This leaves 2,588 on DEFG (Fig. 92).

STEP 3: Multiplying the 2 on A by the 2 on D, subtract the product 4 from the 8 on F. This leaves 2,548 on DEFG (Fig. 93), the answer.

EXAMPLE 2: A manufacturer wishes to purchase a piece of machinery priced at \$7,250 on which there is a discount of 4% for cash. Find the price he pays in cash.

The answer can be found by the following simplified multiplication.

$$
\begin{aligned}
&7{,}250 \times 0.96 \\
&= 7{,}250 \times (1 - 0.04) \\
&= 7{,}250 - (7{,}250 \times 0.04) \\
&= 6{,}960
\end{aligned}
$$

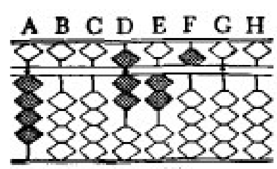


FIG. 94

STEP 1: Set the multiplicand 7,250 on DEFG, and on A, set the 4 of 0.04, which is the complement of 0.96 with respect to 1 (Fig. 94).

NOTE: On the board you have this problem: 7,250—(7,250x0.04). In this problem, the actual multiplier is 0.96, i.e., a decimal with its first significant digit in the tenths place. So you may consider that the answer is obtained by keeping the multiplicand as it is and subtracting from it 7,250x0.04. You should note that when the multiplier is a decimal with its first significant digit in the tenths place, the unit rod of the product remains that of the multiplicand.

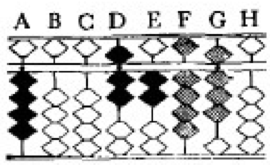


FIG. 95

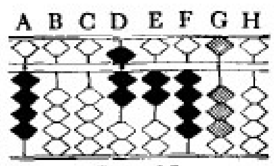


FIG. 96

STEP 2: Multiplying the 4 on A by the 5 on F, subtract the product 20, which is to be set on GH, from the 5 on F. This leaves 7,248 on DEFG (Fig. 95).

NOTE: AS the 5 on F is in the tens place of the dividend, the product 20 must be set on GH.

STEP 3: Multiplying the 4 on A by the 2 on E, subtract the product 8 from the 8 on G. This leaves 724 on DEF (Fig. 96).

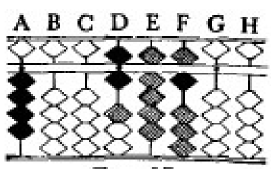


FIG. 97

A	B	C	D	E	F	G	H	
4	0	0	7	2	5	0	0	Step 1
						−2	0	Step 2
4	0	0	7	2	4	8	0	
						−8		Step 3
4	0	0	7	2	4	0	0	
				−2	8			Step 4
4	0	0	6	9	6	0	0	

STEP 4: Multiplying the 4 on A by the 7 on D, subtract the product 28 from the 724 on DEF. This leaves 6,960 on DEFG (Fig. 97). The answer is $6,960.

Rules for Finding the Unit Rod of the Product

RULE 1: When the multiplier is a decimal whose first significant digit is in the tenths place, the unit rod of the multiplicand remains that of the product.

RULE 2: Each time the value of this multiplier is raised by one digit, the unit rod of the product shifts by one rod to the right of that of the multiplicand, and each time the value of this multiplier is reduced by one digit, the unit rod of the product shifts by one rod to the left.

Rule (1) means that when the multiplier is 0.95 or 0.934, the unit rod of the product forms on that of the multiplicand.

Rule (2) means that when the multiplier is 9.5 or 9.34, the unit rod of the product forms on the first rod to the right of that of the multiplicand, and that when the multiplier is 0.095 or 0.0934, the unit rod of the product forms on the first rod to the left of that of the multiplicand.

Advantages and Disadvantages

ADVANTAGES:

(1) The reader can easily see in Example 2 that this method is easier than actually multiplying 7,250x0.96.

(2) As stated in Rule 1 for finding the unit rod of the product, when the multiplier is a mixed decimal whose first significant digit is in the tenths place, the unit rod of the multiplicand remains that of the product. This saves the operator the trouble of finding the unit rod of the product. This is a real advantage.

For these reasons this method of multiplication is extensively employed in the calculation of percentages in business.

DISADVANTAGES:

This method has no particular disadvantages. However, it should be noted that its utility is limited to problems in which the multipliers are numbers a little smaller than 100, 1,000, *etc.* or numbers whose first digit is 9, such as 98, 987, 0.96, *etc.*

Exercises

(1) 24x88=2,112

(2) 37x98=3,626

(3) 851x99=84,249

(4) 169x987=166,803

(5) 408x996=406,368

(6) 3,600x0.92=3,312

IV. OTHER METHODS OF DIVISION

There is an older method of division which is as fast as the standard method of division that I explained in my first book. But this method is no faster and it requires the memorization of a complicated division table. Although this method is still used by the older generation, it is generally becoming obsolete. Since this method is so complicated and since it is hardly used today, I will not spend time explaining it. Instead I shall present some methods of division which are popular for business calculations.

Historically speaking, however, the "standard method of division" is older than what I have called here the older method of division. In ancient China the "standard method of division" was used by means of reckoning blocks. However, the "older method of division," because of its convenience in the calculation of weights not based on the decimal system, became more popular than the "standard method of division." These weights were extremely important in Chinese life because they were used in the conversion of currency. When the abacus was introduced to Japan, it was the "older method of division" which accompanied it. The currency in Japan also was not based on the decimal system, so that although the "standard method of division" was strongly advocated by some mathematicians, it never won real public support. It was not until the new decimal-based currency of the Meiji era replaced the old Japanese currency that the "standard method of division" actually became standard.

1. The Elimination of the Initial Digit of a Divisor Beginning with One

There is a method which simplifies division by leaving out of calculation the initial digit, one, of the divisor. The Japanese technical term for this division may be translated as the initial-digit-one elimination division.

This division is particularly useful when the divisor is a number a little larger than 100, 1,000 etc., such as 103, 1,014, *etc*. It is a counterpart of the multiplication method in which the initial digit one of the multiplier is left out of calculation.

Let us take a very simple example. If you divide 306 by 102, you get the quotient 3. In this case, since the first digit of the divisor 102 is 1, the first digit 3 of the dividend 306 may be used as the quotient. This principle is applied to this method of division on the abacus. On the abacus board, in each step of the division, the first digit of the dividend is used as the trial quotient, and thus the necessity of setting the quotient and that of removing the first digit of the dividend is eliminated, and calculation is considerably simplified and accelerated. Example 1 shows how this division, "306÷102," is actually done on the board.

EXAMPLE 1: 306÷102=3

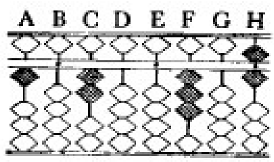


Fig. 98

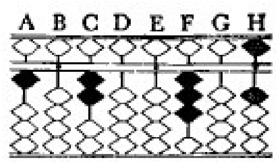


Fig. 99

STEP 1: Set the dividend 306 on FGH with H as the unit rod, and set the divisor 102 on ABC (Fig. 98).

A	B	C	D	E	F	G	H	
1	0	2	0	0	3	0	6	Step 1
							−6	Step 2
1	0	2	0	0	3	0	0	

STEP 2: Suppose that you have divided the 306 on FGH by the 100 of the 102 on ABC and have got the 3 on F as the quotient figure. Multiply the 2 on C by the 3 on F, and subtract the product 6 from the 6 on H. This clears the board of the dividend, and leaves the quotient 3, the answer, on F (Fig. 99).

NOTE: On the board, 306 is divided by 100. But since the actual divisor is 102, the product obtained by the multiplication of 2 (i.e., the difference between 100 and 102) and 3 (i.e., the quotient) must be subtracted from 306. On the board, the procedures of setting the first quotient figure 3 and of subtracting the first digit 3 of the dividend are saved or eliminated.

EXAMPLE 2: 2,575÷103=25

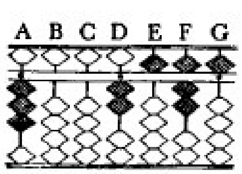


Fig. 100

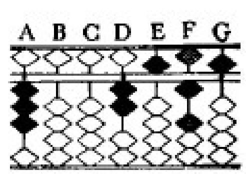


Fig. 101

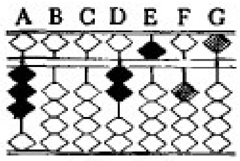


FIG. 102

A	B	C	D	E	F	G	
·			·			·	
3	0	0	2	5	7	5	Step 1
					−6		Step 2
3	9	9	2	5	1	5	
					−1	5	Step 3
3	0	0	2	5	0	0	

STEP 1: Set the dividend 2,575 on DEFG and the 3 of 103 on A (Fig. 100).

NOTE: There is no objection to setting the whole divisor 103 on the board. However, experts do not, because this digit is not actually used in calculation, and so long as they remember that the 3 on A stands for 103 it is not important to set the whole divisor on the board.

STEP 2: As the divisor 103 is a three-digit number, suppose that you have divided the 257 of 2,575 by the 100 of 103 and have tried the 2 on D as the quotient figure.

Multiplying the 3 on A by the 2 on D, subtract the product 6 from the 7 on F. This leaves, on EFG, 515 as the remainder of the dividend (Fig. 101).

NOTES: (a) As the second figure of the divisor 103 is zero, you must set the product 6 on F, skipping over E. (b) In this step, as the 2 on D is used as the first quotient figure, the procedures of setting the first quotient figure 2 and of subtracting the first digit 2 of the dividend 257 are eliminated.

STEP 3: Next, suppose that you have divided the 515 on EFG by 100 and have got the 5 on E as the second quotient figure.

Multiplying the 3 on A by the 5 on E, subtract the product 15 from the 15 on FG. This clears the board of the remaining dividend, and leaves, on DEF, 25 as the quotient (Fig. 102).

As the dividend was practically divided by 100, the unit rod of the quotient moves to the second rod to the left of that of the dividend. Therefore, E becomes the unit rod of the quotient. The answer is 25.

NOTE: In this step, as the 5 on E is used as the second quotient figure, the procedures of setting the second quotient figure and of subtracting the first digit 5 of the dividend 515 are eliminated.

EXAMPLE 3: A merchant's sales increased 0.8% in the second month. The sales for the second month amounted to $3,780. How much were the sales the first month?

This problem can be worked out by the following division:

$3,780÷1.008-$3,750

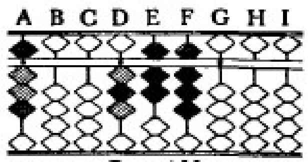


FIG. 103

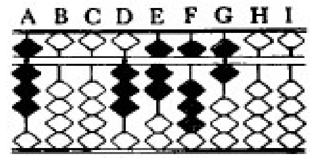


FIG. 104

STEP 1: Set the dividend 3,780 on DEFG and the 8 of the divisor 1.008 on A (Fig. 103).

NOTE: You must remember that 8 stands for 1.008.

STEP 2: As the divisor is a four-digit number, suppose that you have divided the 3,780 on DEFG by 1000 and have got the 3 on D as the first quotient figure.

Multiplying the 8 on A by the 3 on D, subtract the product 24 from the 80 on FG. This leaves, on EFG, 756 as the remainder of the dividend (Fig. 104).

NOTE: AS the second and third figures of 1.008 are zero, you must set the product 24 on FG and not on EF.

STEP 3: Suppose that you have divided the 7,560 on EFGH by 1,000 and have got the 7 on E as the second quotient figure.

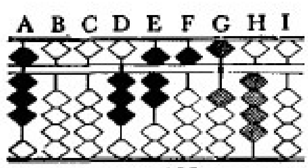


FIG. 105

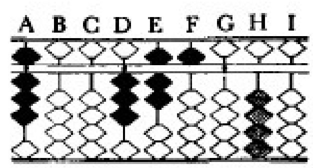


FIG. 106

A	B	C	D	E	F	G	H	I	
·			·			·			
8	0	0	3	7	8	0	0	0	Step 1
					−2	4			Step 2
8	0	0	3	7	5	6	0	0	
						−5	6		Step 3
8	0	0	3	7	5	0	4	0	
							−4	0	Step 4
8	0	0	3	7	5	0	0	0	

Multiplying the 8 on A by the 7 on E, subtract the product 56 from the 560 on FGH. This leaves, on FGH, 504 as the remainder of the dividend (Fig. 105).

NOTE: Again, be sure to set the product 56 on GH.

STEP 4: Suppose that you have divided the 5,040 on FGHI by 1,000, with the 5 on F as the third quotient figure.

Multiplying the 8 on A by the 5 on F, subtract the product 40 from the 40 on HI. This clears the board of the dividend, and leaves 375 as the quotient on DEF (Fig. 106).

As the divisor 1.008 is a mixed decimal, whose whole figure one is in the unit place, the unit rod of the quotient is identical with that of the dividend. The answer is $3,750.

When the divisor is a number whose second digit is not zero, such as 125, 137, etc., it often happens that a figure a little smaller than the first digit of the dividend must be tried as a quotient figure. In such cases this method is less efficient.

Here is an example showing how the division should be worked in such a case.

EXAMPLE 4: A man withdrew his savings from an account after they had earned 12% interest. If the amount he withdrew was $5,376, what was his original investment?

This problem can be worked by the following division:

$$5{,}376 \div 1.12 = 4{,}800$$

STEP 1: Set the dividend 5,376 on EFGH and the last two digits 12 of the divisor 1.12 on AB (Fig. 107).

NOTE: When a divisor is a mixed decimal, it is advisable for the beginner to set its unit digit on a unit rod.

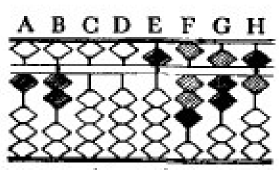


FIG. 107

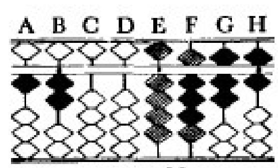


FIG. 108

STEP 2: As the divisor is a three-digit number, suppose that you have divided the 537 on EFG by 100, with the 5 on E as the first quotient figure, and found that 5 is too large. It is because if you multiply the 1 on A by the 5 on E, the product 5 which is to be set on F is larger than the 3 on F. So try 4 as the first trial quotient figure in your mind. Then 1 is left on E as the first figure of the remaining dividend. Multiply the 1 on A by this 4 which is mentally on E, and you will see that you can subtract the product 4 from the 13 which remains mentally on EF. Now subtract the 4 from the 13 by setting 4 on E and adding 6 to the 3 on F. This gives you 49 on EF, with the 4 on E as the first quotient figure.

Of course, in this particular problem if the operator had looked at the problem carefully, he would have tried 4 the first time. An experienced operator can see at a glance that 4 is the proper first number of the quotient.

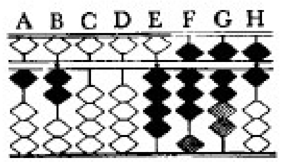


Fig. 109

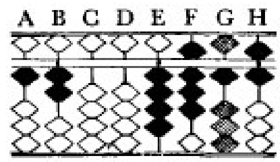


Fig. 110

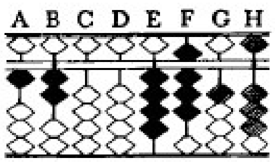
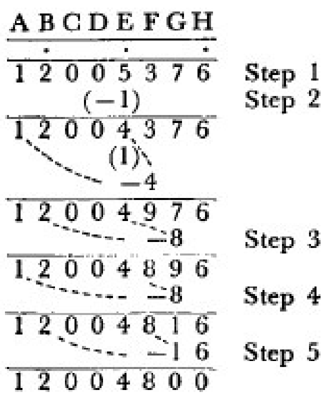


Fig. 111



STEP 3: Now multiply the 2 on B by the 4 on E, and subtract the product 8 from the 97 on FG. This leaves, on FGH, 896 as the remaining dividend (Fig. 109).

STEP 4: Next suppose that, in dividing the 896 on FGH by 100, you have tried the 8 on F as the second quotient figure.

Multiplying the 1 on A by the 8 on F, subtract the product 8 from the 9 on G. This leaves, on GH, 16 as the remaining subtrahend (Fig. 110).

STEP 5: Finally multiplying the 2 on B by the 8 on F, subtract the product 16 from the 16 on GH. This clears the board of the subtrahend and leaves 48 as the quotient on EF (Fig. 111). The answer is $4,800.

NOTE: As the divisor 1.12 is a mixed decimal whose whole figure one is in the unit place, the unit rod of the dividend remains that of the quotient.

Rules for Finding the Unit Rod of the Quotient

RULE 1: When the divisor is a mixed decimal whose whole digit one is in the unit place, the unit rod of the quotient forms on that of the dividend.

RULE 2: Each time the value of this divisor is raised by one digit, the unit rod of the quotient shifts one rod to the left of that of the dividend, and each time the value of this divisor is reduced by one digit, the unit rod of the quotient shifts one rod to the right of that of the dividend.

For instance, when the divisor is 1.05 or 1.023, the unit rod of the quotient forms on that of the dividend. When the divisor is 10.5 or 10.23, the unit rod of the quotient forms on the first rod to the left of that of the dividend. When the divisor is 0.105 or 0.1023, the unit rod of the quotient forms on the first rod to the right of that of the dividend.

This rule may be restated as follows:

RULE 2: The unit rod of the quotient forms on the rod to the left of that of the dividend by as many rods minus one as there are whole digits in the divisor.

The unit rod of the quotient forms on the rod immediately to the right of that of the dividend by as many rods plus one as there are zeros before the first significant decimal figure.

Advantages and Disadvantages

The advantages and disadvantages of this method of division are nearly identical with those of the method of multiplication which leaves the initial digit one of the multiplier out of calculation (see chapter 3, section 6).

ADVANTAGES:

(1) This method of division simplifies calculation when the second digit of the divisor is zero, as in 105, 1.08, *etc.*

When the divisor is a number whose second digit is small, such as 13 or 1.12, etc., often this method simplifies calculation, although the sort of difficulties illustrated in Example 4 may arise.

(2) As shown in Examples 3 and 4, when the divisor is a mixed decimal whose whole digit one is in the units place, the unit rod of the quotient is identical with that of the dividend. This is a great advantage to the abacus operator, as he has no trouble in finding the unit rod of the quotient.

For these reasons this method of division is employed extensively in the calculation of percentages in business, that is, for finding the cost of goods from the selling price and the percent of profit.

DISADVANTAGES:

(1) When the divisor is a number whose second digit is large, such as 19, 183, etc., this method is awkward.

(2) When the divisor is a number whose second and successive digits are large, this method usually is much too complicated.

2. Division by Complementary Numbers

When the divisor is a number a little smaller than 100, 1,000, etc., such as 98, 997, etc., division can be accelerated by using the complement of the divisor with respect to 100, 1,000, *etc*. Let us take two simple examples. 98 goes into 196 twice. So you can divide 196 by 100 with 2 as the quotient if you add, to 196, 4 which is obtained by multiplying 2, the difference between 100 and 98, and 2, the quotient. The 4 is the same number you have subtracted in excess. In the same way you can divide 294 by 100 with 3 as the quotient if you add, to 294, 6 which is obtained by multiplying the 2 and 3, the quotient. The following two rules will help you find the quotient of this division easily, especially if the first digit of the divisor is 9.

RULE 1: The quotient can generally be found by adding 1 to the first digit of the dividend when the quotient is a one-digit number, as in the above examples, and when the final digit of the quotient is to be obtained.

RULE 2: In case the quotient is a more-than-one-digit number, the first digit of each dividend is generally used as the trial quotient figure, except in finding the final digit of the quotient.

EXAMPLE 1: 882÷98=9

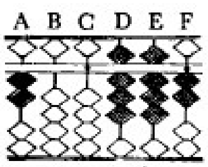


FIG. 112

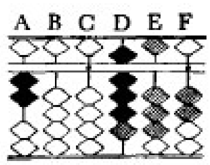


FIG. 113

A	B	C	D	E	F	
2	0	0	8	8	2	Step 1
			(9)			
				+1	8	Step 2
2	0	0	9	0	0	

STEP 1: Set the dividend 882 on DEF with F as the unit rod and set, on A, 2, which is the complementary number of 98 with respect to 100. It is important to note that although 100 is the divisor, only the complementary number of the real divisor with respect to 100 is set on the board.

STEP 2: Since the quotient is a one-digit number, you can apply the above-mentioned Rule 1 to this problem. Add 1 to the first digit 8 of the dividend 882 and you get 9. If 9 is the correct quotient, the following two equations should be found equal.

Equation A: 882+(2x9)=900
Equation B: 100x9=900

Since the above two equations are equal, 9 is the correct quotient. In equation A, 882 is the dividend, 2 is the complementary number, and 9 is the quotient. In equation B, 100 is the divisor.

Now, multiplying the 2 on A by the trial quotient figure 9, add the product

18 to the 82 on EF. This gives you 900 on DEF. Since the divisor 98 is a two-whole-digit number, the unit rod of the quotient is formed on D. The answer is 9 on D (Fig. 113).

EXAMPLE 2: 8,160÷96=85

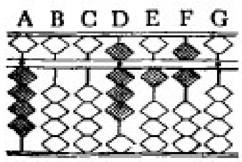


FIG. 114

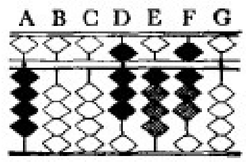


FIG. 115

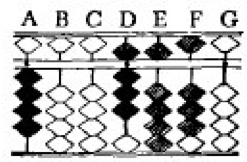


FIG. 116

A	B	C	D	E	F	G	
4	0	0	8	1	6	0	Step 1
				+3	2		Step 2
4	0	0	8	4	8	0	
				(5)			
					+2	0	Step 3
4	0	0	8	5	0	0	

STEP 1: Set the dividend 8,160 on DEFG, and set on A, 4, which is the complement of 96 with respect to 100 (Fig. 114). Since the dividend is a two-digit number, 100 is used as the divisor.

STEP 2: Divide the 816 on DEF by 100, with the 8 on D as the first trial quotient figure. In this step, apply Rule 2. Now, multiplying the 4 on A by the 8 on D, add the product 32 to the 16 on EF. This gives you 480 on EFG (Fig. 115).

STEP 3: In this final step, apply Rule 1 and try 5, which is larger than the 4 on E by 1, as the next trial quotient figure. Multiplying the 4 on A by this 5, add the product 20 to the 80 on FG, and you get 500 on FGH. This number is equal to the 500 obtained by multiplying the divisor 100 by the trial quotient 5. As the divisor is a three-whole-digit number, the unit rod of the quotient is formed on the second rod to the left of that of the dividend. The answer is 85 on DE (Fig. 116).

EXAMPLE 3: 31,428÷0.97=32,400

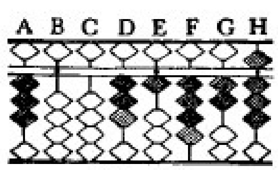


FIG. 117

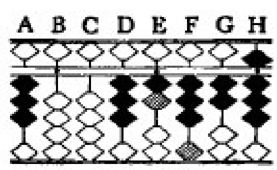


FIG. 118

STEP 1: Set the dividend 31,428 on DEFGH, and set on A, 3, the complement of 97 with respect to 100 (Fig. 117).

STEP 2: Divide the 314 on DEF by 100, with the 3 on D as the first quotient figure.

Multiplying the 3 on A by the 3 on D, add the product 9 to the 4 on F. This leaves, on EFGH, 2,328 as the remainder of the dividend (Fig. 118).

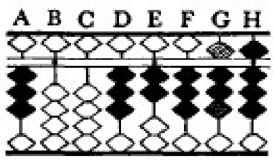


FIG. 119

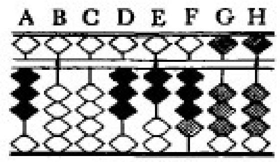


FIG. 120

STEP 3: Now divide the 232 on EFG by 100, with the 2 on E as the second quotient figure.

Multiplying the 3 on A by the 2 on E, add the product 6 to the 2 on G. This leaves 388 on FGH as the remainder of the dividend (Fig. 119).

A	B	C	D	E	F	G	H	
3	0	0	3	1	4	2	8	Step 1
					+9			Step 2
3	0	0	3	2	3	2	8	
						+6		Step 3
3	0	0	3	2	3	8	8	
					(4)			
						+1	2	Step 4
3	0	0	3	2	4	0	0	

STEP 4: In this final step, apply Rule 1 and try 4, which is larger than the 3 on F by 1, as the final trial quotient figure. Add, to the 388 on FGH, the product 12 obtained by multiplying the 3 on A by this 4, and you get 400 on F. This product is the same as that obtained by multiplying the divisor 100 by this same 4. This shows that 4 is the correct third quotient figure. In this division, the divisor is a decimal fraction whose first significant digit is in the tenths' place. Therefore the unit rod of the quotient is identical with that of the dividend. The answer is 32,400 (Fig. 120).

Rules for Finding the Unit Rod of the Quotient

RULE 1: When the divisor is a decimal fraction whose first significant digit is in the tenths place, the unit rod of the dividend remains that of the quotient.

RULE 2: Each time the value of this divisor is raised by one place, the unit rod of the quotient shifts by one rod to the left of that of the dividend, and each time the value of this divisor is reduced by one place, the unit rod of the quotient shifts by one rod to the right of that of the dividend.

For instance, when the divisor is 0.95, the unit rod of the quotient forms on that of the dividend. When the divisor is 9.5, the unit rod of the quotient forms on the first rod to the left of that of the dividend. When the divisor is 0.095, the unit rod of the quotient forms on the first rod to the right of that of the dividend.

Rule 2 may be restated as follows:

RULE 2: The unit rod of the quotient forms on the rod immediately to the left of that of the dividend by as many rods as there are whole digits in the divisor.

The unit rod of the quotient forms on the rod immediately to the right of that of the dividend by as many rods as there are zeros before the first significant decimal figure.

Advantages and Disadvantages

The advantages and disadvantages of this method of division are nearly identical with those of the method of multiplication by means of complementary numbers.

ADVANTAGES:

(1) This method of division simplifies calculations when the divisor is a number a little smaller than 100, 1,000, 1, or 0.01, etc., or especially numbers whose first significant digit is 9, such as 98, 997, 0.96, or 0.0094, *etc.*

(2) As described in Rule 1 for finding the unit rod of the quotient, when the divisor is a decimal fraction whose first significant digit is in the tenths place, the unit rod of the quotient forms on that of the dividend. This is a great advantage to the abacus operator, as he has no trouble in finding the unit rod of the quotient.

Because of these advantages, this method is extensively used in business for the calculation of percentages. For example, it is useful in finding the regular price from the discount price and the discount rate.

DISADVANTAGES:

Its utility is limited to problems in which the divisor is a number just a little smaller than 100, 1,000, *etc.*

When the divisor is a number whose first significant digit is smaller than 9—such as 75, 0.82, etc., Rules 1 and 2 given for finding trial quotient figures do not often apply. In such cases, this method of division is awkward, and the standard method of division is more efficient.

Exercises

(1) 2,116÷92=23

(2) 4,048÷88=46

(4) 382,305÷993=385

(5) 705,042÷897=786

(3) $13,832 \div 988 = 14$

(6) $1,645 \div 0.94 = 1,750$

V. MORE ABOUT DECIMALS

In my first book I included a chapter on decimals. I tried to show in that discussion the Japanese method of handling decimal fractions of various sorts. However, an astute reader may have seen two interesting things about my previous presentation. One thing was simply an oversight on my part, namely that I gave very few clues as to how such a multiplication problem as 0.2x0.4=0.08 would be done on the abacus, *i.e.* a problem having decimals both in the multiplicand and the multiplier. Also I neglected such division problems as 1.2÷3=0.4, i.e., a problem having decimals in the quotient.

The other thing that a Western reader in particular might have found strange is that I presented the following three problems as if they were different and each required a different kind of treatment: 31.36÷0.32=98, 3.136÷0.032=98, and 0.3136÷0.0032=98. The reason this might seem strange is that when an Occidental sets up any of these three problems on paper for long division, he immediately reduces them all to the same problem, namely 3,136÷32=98. These are treated as distinct problems in Japan because of the abacus tradition of avoiding any mental effort and following the line of least resistance. The Western reader may not be satisfied with this solution and may want to work out his own compromise with the Japanese method. I think, however, that after a beginner has practiced this method enough so that it becomes mechanical, he will consider it better than any system which requires a non-mechanical, mental effort—better than the Western method of recognizing the essential simplicity of some problems.

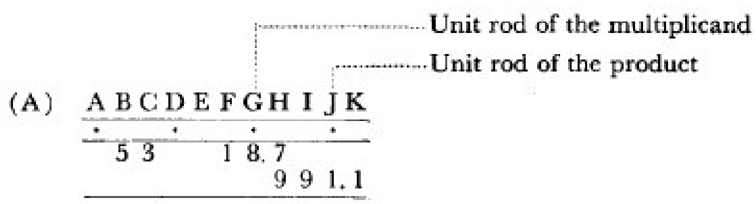
Now let me turn to my original problem. In my first book, I explained how to do the following problems:

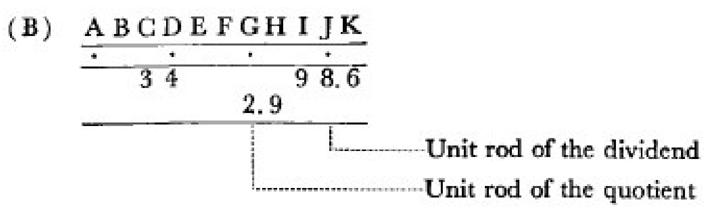
MULTIPLICATION	DIVISION
4x2=8	8÷2=4
25x15=375	375÷15=25
405x123=49,815	49,815÷123=405
34x1.2=40.8	40.8÷1.2=34
32x0.4=12.8	12.8÷0.4=32
98x0.32=31.36	31.36÷0.32=98

32x0.04=1.28	1.28÷0.04=32
98x0.032=3.136	3.136÷0.032=98
32x0.004=0.128	0.128÷0.004=32
98x0.0032=0.3136	0.3136÷0.0032=98

The following five examples should explain how to do any other normal multiplication and division problems.

EXAMPLE 1: (A) 18.7x53=991.1
(B) 98.6÷34=2.9

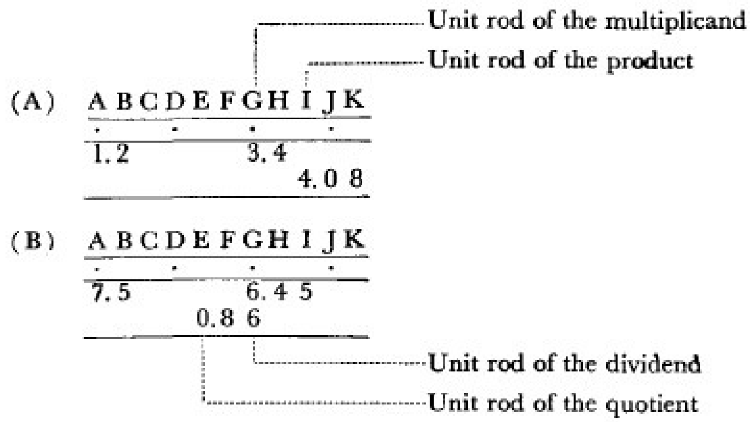




(A) shows that when the multiplier is a two-digit whole number, the unit digit of the product is formed on the third rod to the right of that of the multiplicand, *i.e.* on rod J.

(B) shows that when the divisor is a two-digit whole number, the unit digit of the quotient is formed on the third rod to the left of that of the dividend, *i.e.* on rod G.

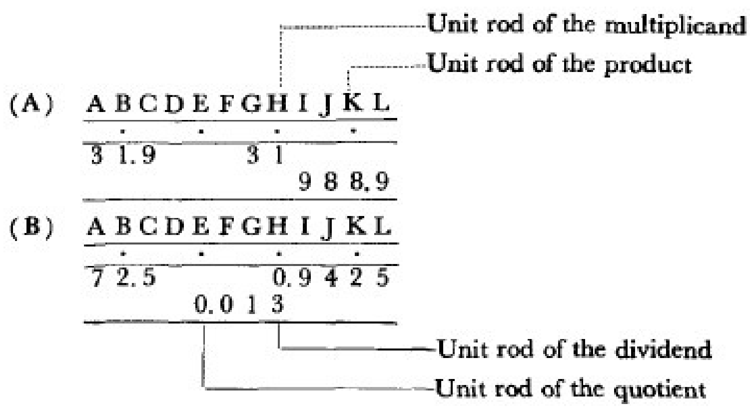
EXAMPLE 2: (A) 3.4x1.2=4.08
(B) 6.45÷7.5=0.86



(A) shows that when the multiplier is a mixed number with only one whole digit in it, the unit rod of the product is formed on the second rod to the right of that of the multiplicand, *i.e.* on rod I.

(B) shows that when the divisor is a mixed number with one whole digit, the unit rod of the quotient forms on the second rod to the left of that of the dividend, E.

EXAMPLE 3: (A) 31x31.9=988.9
(B) 0.9429÷72.5=0.013

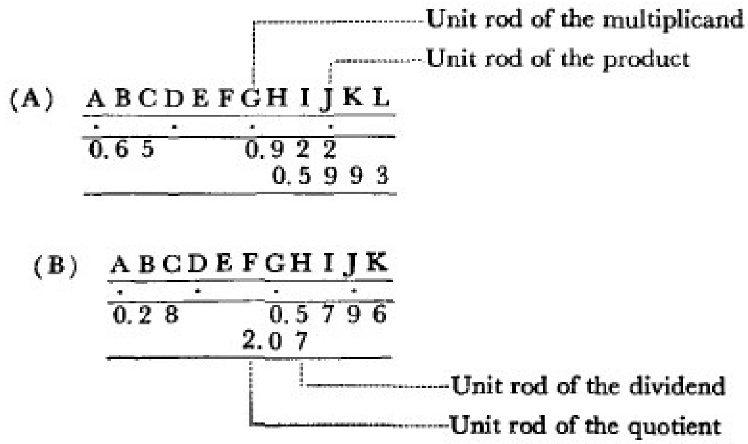


(A) shows that when the multiplier is a mixed number with two whole digits, the unit rod of the product is formed on the third rod to the right of that of the multiplicand, *i.e.* rodK.

(B) shows that when the divisor is a mixed number with two whole digits, the unit rod of the quotient is formed on the third rod to the left of that of the dividend, *i.e.* rod E.

EXAMPLE 4: (A) 0.922x0.65=0.5993

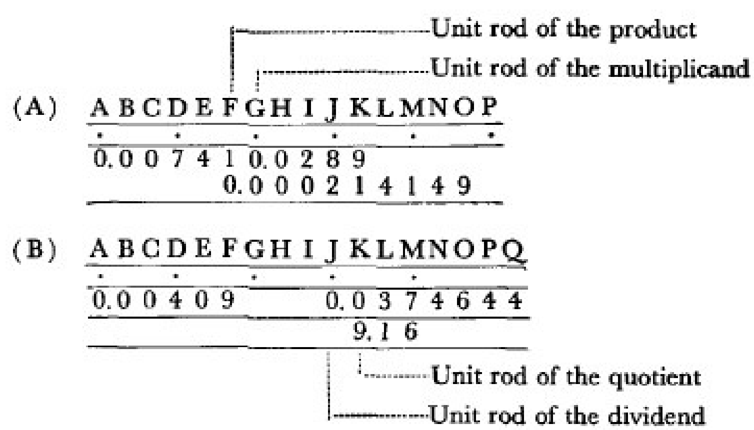
(B) 0.5796÷0.25=2.07



(A) shows that when the divisor is a decimal fraction with its first significant digit in the tenths place, the unit digit of the product is formed on the first rod to the right of that of the multiplicand, *i.e.* on rod H.

(B) shows that when the divisor is a decimal fraction with its first significant digit in the tenths place, the unit digit of the quotient is formed on the first rod *to* the left of that of the dividend, *i.e.* on rod F.

EXAMPLE 5: (A) 0.0289x0.00741=0.000214149
(B) 0.0374644÷0.00409=9.16



(A) shows that when the multiplier is a decimal fraction with its first significant digit in the thousandths place, the unit rod of the product is formed on the first rod to the left of that of the multiplicand, *i.e.* on rod F.

(B) shows that when the divisor is a decimal fraction with its first significant digit in the thousandths place, the unit digit of the quotient is formed on the first rod to the right of that of the dividend, i.e., on rod K.

VI. CALCULATIONS INVOLVING MORE THAN ONE UNIT OF MEASUREMENT

1. Setting Compound Numbers on the Board

In setting a compound number, a measurement expressed in more than one unit, on the board, the part with the highest denomination is first set on a unit rod, and all the other parts are set so that they are separated by at least one vacant rod. Below are examples of how a compound number should be set on the board.

EXAMPLE 1:
£15 14 *s.* 9 *d.*

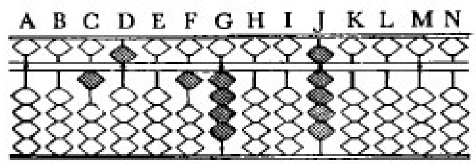


FIG. 121

EXAMPLE 2:
12 hr. 5 min. 20 sec.

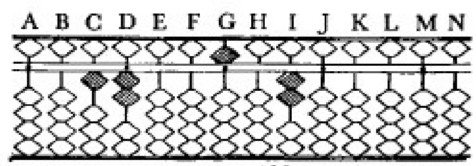


FIG. 122

In Example 1, 14 *s*. is separated from £15 by one vacant rod, while 9 *d*. is separated from 14 *s*. by two vacant rods (Fig. 121), because during calculations the number of pence may require the use of two digits and thus two rods. (There are 12 pence in a shilling and 20 shillings in a pound.) In Example 2, 5 min. is separated from 12 hr. by two vacant rods, because during calculations we may have as many as 59 minutes and thereby need two rods for calculations and one for separation. The 20 seconds is separated from 5 min. by one vacant rod, because no additional rods will be necessary (Fig. 122).

It should be noted that the above arrangements of the compound numbers are the most convenient not only for addition and subtraction but also for multiplication and division. For further details, see Example 3.

EXAMPLE 3:
7 yd. 1 ft. 9 in.

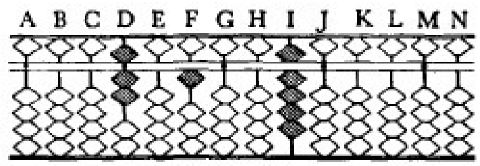


FIG. 123

In Example 3, 1 ft. is separated from 7 yd. by one vacant rod, while 9 in. is separated from 1 ft. by two vacant rods, because during calculations we may have as many as 11 inches and thereby two rods (Fig. 123).

When only addition and subtraction are to be performed, each unit of measurement is often given a separate unit rod, e.g., 1 ft. would be set on G and 9 in. on J. But when multiplication and division are also to be performed, it is more convenient to set the units as in the above examples.

2. Converting Compound Numbers

EXAMPLE 1: Find the value of £37 10 *s.* in dollars if one pound equals $4.03.

This problem can be simplified as follows:

10 *s.* = £0.5 (NOTE: 10 *s.* ÷ 20 *s.* = 0.5)
£37 10 *s.* = £37.5
£37.5 × 4.03 = $151.125

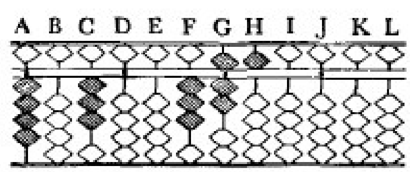


FIG. 124

STEP 1: Set the multiplicand 37.5 on FGH, with G as the unit rod. Set the multiplier 4.03 on ABC, leaving two rods vacant (Fig. 124).

NOTE: It is not really necessary to set the unit figure of the multiplier on a unit rod.

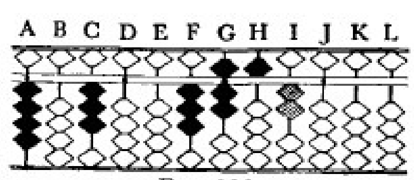


FIG. 125

STEP 2: Multiplying 4 on A by 5 on H, set product 20 on IJ (Fig. 125).

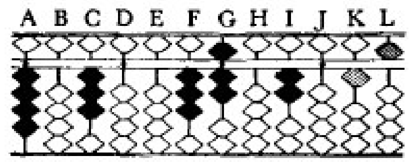


FIG. 126

STEP 3: Multiplying the 3 on C by the same 5 on H, set the product 15 on KL, and clear H of the 5. This makes a total of 2,015 on IJKL (Fig. 126).

NOTE: AS the second figure of the multiplier is zero, set the product 15 on KL, skipping JK.

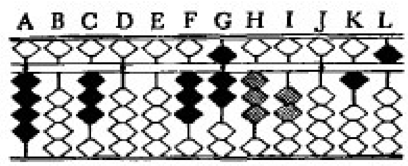


FIG. 127

STEP 4: Next, multiplying the 4 on A by the 7 on G, set the product 28 on HI. This makes a total of 30,015 on HIJKL (Fig. 127).

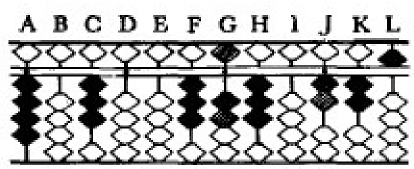


FIG. 128

STEP 5: Multiplying the 3 on C by the same 7 on G, set the product 21 on JK, and clear G of the 7. This makes a total of 30,225 on HIJKL (Fig. 128).

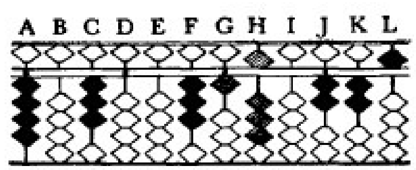


FIG. 129

STEP 6: Next, multiplying the 4 on A by the 3 on F, set the product 12 on GH. This makes a total of 150,225 on GHIJKL (Fig. 129).

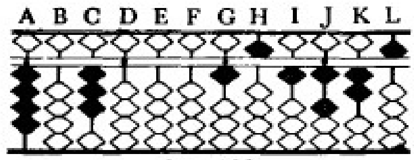


Fig. 130

STEP 7: Finally multiplying the 3 on C by the same 3 on F, set the product 9 on J, and clear F of the 3. This gives you a total of 151,125 on GHIJK (Fig. 130).

As the multiplier is a mixed decimal whose whole figure is in the unit place, the unit rod of the product forms on I, the second rod to the right of G, the unit rod of the multiplicand. The answer is $151,125.

A	B	C	D	E	F	G	H	I	J	K	L	
·			·			·			·			
4	0	3	0	0	3	7	5	0	0	0	0	Step 1
								+2	0			Step 2
										+1	5	Step 3
4	0	3	0	0	3	7	0	2	0	1	5	
							+2	8				Step 4
									+2	1		Step 5
4	0	3	0	0	3	0	3	0	2	2	5	
						+1	2					Step 6
									+9			Step 7
4	0	3	0	0	0	1	5	1	1	2	5	

EXAMPLE 2: Find the value of the following compound number in terms of pence (12 pence=1 shilling; 20 shillings=1 pound).

£12 16*s*. 8*d*.

Written calculations:

£	s.	d.
12	16	8
× 20	+ 240	+3072
240	256	3080
	× 12	
	512	
	256	
	3072	Ans.: 3,080 *d*.

On the abacus, the operation is performed much in the same way as in written calculation. First £12 is multiplied by 20 and the resulting 240 *s.* is added to 16 *s.* Next the sum 256 *s.* is multiplied by 12 and the product 3,072 *d.* is added to 8 *d.* The answer is 3,080 *d.* However, on the abacus, both the multiplication and addition are performed simultaneously, and calculations are much faster.

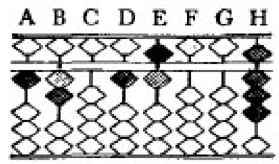


FIG. 131

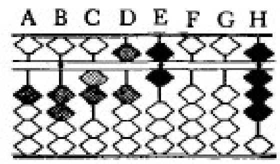


FIG. 132

STEP 1: Set 12 on AB, with B as the unit rod, 16 on DE, with E as the unit rod, and 8 on H, with H as the unit rod (Fig. 131).

STEP 2: First multiply the 2 on B by 20, add the product 40 to the 16 on DE, and clear B of the 2. This makes a total of 56 on DE.

Next multiplying the 1 on A by 20, set the product 20 on CD, and clear A of the 1. This gives you a total of 256 on CDE (Fig. 132).

NOTE: In this procedure multiply the 2 on B by 20 first and the 1 on A by 20 next. If you multiply the digit on A first, the product may extend to rod B. This will happen if the digit on A is large. If, for example, the number on A were 8, setting the product 16 on BC would be rather confusing.

STEP 3: Now the 256 on CDE must be multiplied by 12 and added to the 8 on H.

First multiply the 6 on E by 10, and set the product 60 on GH. This makes a total of 68 on GH.

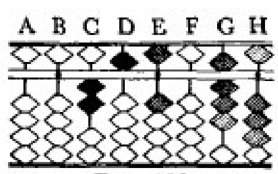


FIG. 133

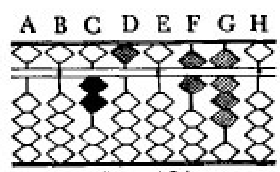


FIG. 134

Next multiplying the same 6 on E by 2, add the product 12 to the 68 on GH, and clear E of the 6. This gives you a total of 80 on GH (Fig. 133).

NOTE: Be sure to dispose of the 6 of 256 first, next the 5, and finally the 2.

STEP 4: Multiplying the 5 on D by 10, set the product 50 on FG. This makes a total of 580 on FGH.

Next, multiplying the same 5 on D by 2, add the product 10 to the 58 on FG, and clear D of the 5. This gives you a total of 680 on FGH (Fig. 134).

STEP 5: Multiplying the 2 remaining on C by 10, set the product 20 on EF. This makes a total of 2,680 on EFGH.

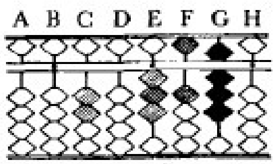


FIG. 135

A	B	C	D	E	F	G	H	
	·			·			·	
1	2	0	1	6	0	0	8	Step 1
			+4	0				Step 2
		+2	0					
0	0	2	5	6	0	0	8	
						+6	0	Step 3
						+1	2	
0	0	2	5	0	0	8	0	
					+5	0		Step 4
					+1	0		
0	0	2	0	0	6	8	0	
				+2	0			Step 5
					+4			
0	0	0	0	3	0	8	0	

Finally, multiplying the same 2 on C by 2, add the product 4 to the 6 on F, and clear C of the 2. This gives you a total of 3,080 on EFGH (Fig. 135). The answer is 3,080 *d.*

NOTE: Experts usually save the trouble of setting such simple multipliers as 20 and 12 on the board.

EXAMPLE 3: Find the value of 6,190 *d.* in terms of pounds and shillings.

Written calculation:

```
     515            25
12)6190        20)515
    60             40
    19            115
    12            100
    70             15
    60
    10
```

Ans.: £25 15 *s.* 10 *d.*

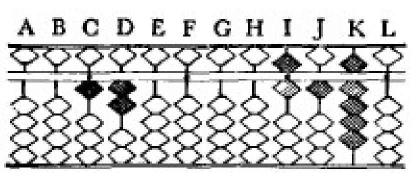


FIG. 136

On the abacus, the operation is performed in the same way as in written calculation. First 6,190 is divided by 12, next the quotient 515 is divided by 20.

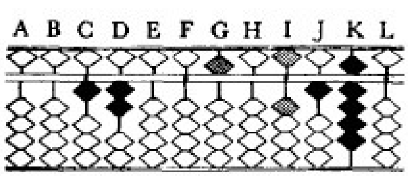


FIG. 137

STEP 1: Set 6,190 on IJKL, with L as the unit rod, and set 12 on CD. See that there are four vacant rods between the two numbers (Fig. 136).

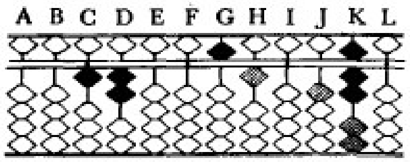


Fig. 138

STEP 2: Comparing the divisor 12 and the 61 in 6,190, set the quotient figure 5 on G. Now multiply the 1 on C by this 5, and subtract the product 5 from the 6 on I. This leaves 1,190 on IJKL.

Next multiply the 2 on D by the same 5 on G, and subtract the product 10 from the 11 on IJ. This leaves 190 on JKL (Fig. 137).

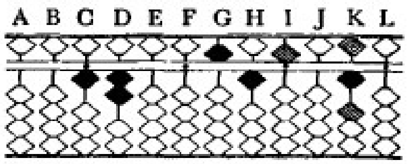


Fig. 139

STEP 3: Comparing the divisor 12 and the 19 on JK, set the second quotient figure 1 on H. Now multiply the 1 on C by the 1 on H, and subtract the product 1 from the 1 on J. This leaves 90 on KL. Next multiply the 2 on D by the same 1 on H, and subtract the product 2 from the 9 on K. This leaves 70 on KL (Fig. 138).

STEP 4: Comparing the divisor 12 and the 70 on KL, set the third quotient figure 5 on I. Now multiply the 1 on C by this 5 on I, and subtract the product 5 from the 7 on K. This leaves 20 on KL. Next multiply the 2 on D by the same 5 on I, and subtract the product 10 from the 20 on KL. This leaves 10 on KL (Fig. 139).

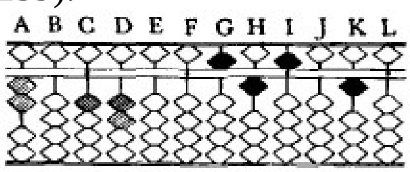


Fig. 140

STEP 5: Now leaving the 10 *d.* on KL as it is, you must divide the 515 *s.* on GHI by 20 to convert it into pounds.

Clear CD of its 12, and set the divisor 20 on AB, leaving four vacant rods between it and the 515 on GHI (Fig. 140).

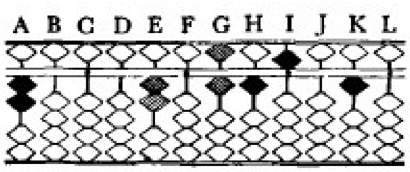


Fig. 141

STEP 6: Now comparing the 2 on A and the 5 on G, set the quotient 2 on E. Multiplying the 2 on A by this 2 on E, subtract the product 4 from the 5 on G. This leaves 115 on GHI (Fig. 141).

STEP 7: Next compare the 2 on A and the 11 on GH, and set the quotient figure 5 on F. Multiplying the 2 on A by this 5 on F, subtract the product 10 from the 11 on GH. This leaves 15 *s.* on HI. The answer is £25 15 *s.* 10 *d.* (Fig.

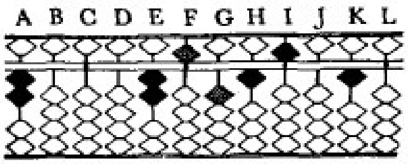


FIG. 142

142).

NOTE: Why are the pounds, shillings, and pence figures set as they are in Examples 2 and 3?

The close arrangement of these figures is the most efficient arrangement for performing the four arithmetic operations. Any closer arrangement would cause confusion.

This arrangement affords another special advantage in that the standard methods of multiplication and division can be used most efficiently.

A	B	C	D	E	F	G	H	I	J	K	L	
		·			·			·			·	
0	0	1	2	0	0	0	0	6	1	9	0	Step 1
						5						Step 2
							−5					
							−1	0				
0	0	1	2	0	0	5	0	0	1	9	0	
							1					Step 3
								−1				
									−2			
0	0	1	2	0	0	5	1	0	0	7	0	
								5				Step 4
									−5			
									−1	0		
0	0	1	2	0	0	5	1	5	0	1	0	
2	0	0	0	0	0	5	1	5	0	1	0	Step 5
				2								Step 6
					−4							
2	0	0	0	2	0	1	1	5	0	1	0	
					5							Step 7
					−1	0						
2	0	0	0	2	5	0	1	5	0	1	0	

Examine step 2 in Example 2, where pounds are converted to shillings and you can see that the standard method of multiplication forms the unit digit of the product on the unit rod of the shillings figure on the board.

Examine steps 2, 3, and 4 in Example 3, where the pence are converted to shillings, and you can see that the standard method of division forms the unit digit of the quotient on the unit rod of the shillings figure on the board. Any other arrangement of the pounds, shillings, and pence figures would require the use of methods of multiplication and division other than Step 7 the standard methods and would make calculations less efficient.

3. Adding and Subtracting Compound Numbers

EXAMPLE 1:

21 yd.	1 ft.	8 in.	
15 yd.	2 ft.	7 in.	
+29 yd.	2 ft.	11 in.	Ans.: 67 yd. 1 ft. 2 in.
67	1	2	

There are two ways of adding measures. One is to add each compound number from higher to lower denominations, and the other is to add up column by column from lower to higher denominations as in written calculation. The former method is generally used when both addition and subtraction are to be performed, and it must always be followed when successive compound numbers are dictated to the operator. The latter is considered to be a little faster if it can be used.

METHOD A: Adding from higher to lower denominations:

yd.		ft.		in.	
21		1		8	Step 1
+15		2		7	
36		3		15	Step 2
+29		2		11	
65		5		26	Step 3
+ 2		+ 2			
67	Step 7	7	Step 5	2	
				12)26	Step 4
		2		24	
		3)7	Step 6	2	
		6			
		1			

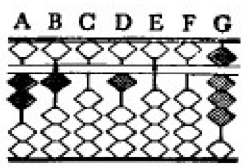


FIG. 143

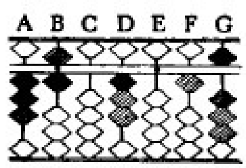


FIG. 144

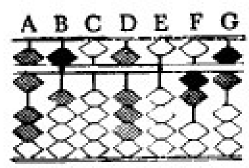


FIG. 145

STEP 1: Set 21 yd. on AB, with B as the unit rod. Set 1 ft. on D and 8 in. on G (Fig. 143).

Refer to Fig. 123 at the beginning of this chapter.

STEP 2: Add 15 to the 21 on AB. This makes a total of 36 on AB.

Now add 2 to the 1 on D. This makes a total of 3 on D.

Next add 7 to the 8 on G. This makes a total of 15 on FG (Fig. 144).

STEP 3: Add 29 to the 36 on AB. This makes a total of 65 on AB.

Next add 2 to the 3 on D. This makes a total of 5 onD.

Finally add 11 to the 15 on FG. This makes a total of 26 on FG (Fig. 145).

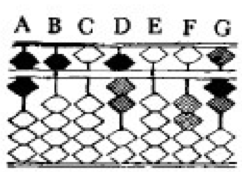


FIG. 146

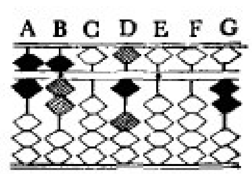


FIG. 147

STEP 4: Now divide the 26 on FG by 12, and add the quotient 2 to the 5 on D, leaving the remainder 2 on G. This makes a total of 7 on D (Fig. 146).

NOTE: In dividing the 26 on FG, experienced operators generally do not set the divisor 12 on the board.

STEP 5: Finally divide the 7 on D by 3, and add the quotient 2 to the 65 on AB, leaving the remainder 1 on D. This makes a total of 67 on AB. The answer is 67 yd. 1 ft. 2 in. (Fig. 147).

METHOD B: Adding from lower to higher denominations:

yd.	ft.	in.	
2	2		
21	1	8	
15	2	7	
+29	2	11	
67 Step 5	7 Step 3	26 Step 1	Ans.: 67 yd. 1 ft. 2 in.
	2	2	
	3) 7 Step 4	12) 26 Step 2	
	6	24	
	1	2	

STEP 1: Set 8 on unit rod G. Add 7 and 11 to the 8 on G, and you get a total of 26 on FG (Fig. 148).

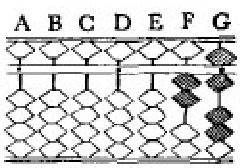


FIG. 148

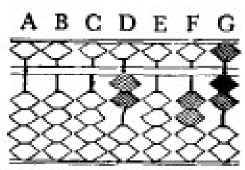


FIG. 149

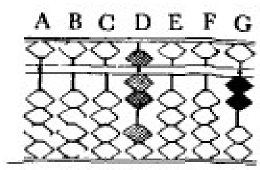


FIG. 150

STEP 2: Divide the 26 on FG by 12, and set the quotient 2 on D, which is the proper rod on which to set the quotient. This gives you 2 on D and leaves 2 on G (Fig. 149).

STEP 3: Add 1, 2, and 2 successively to the 2 on D. This makes a total of 7 on D (Fig. 150).

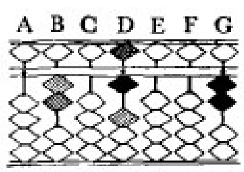


FIG. 151

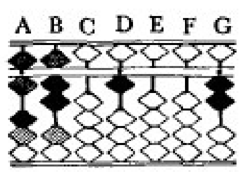


FIG. 152

STEP 4: Divide the 7 on D by 3, and set the quotient 2 on B, which is the proper rod on which to set the quotient. This gives you 2 on B and leaves 1 on D (Fig. 151).

STEP 5: Finally add 21, 15, and 29 successively to the 2 on B. This makes a total of 67 on AB. The answer is 67 yd. 1 ft. 2 in. on AB, D, and G respectively (Fig. 152).

NOTE: Why are the yards, feet, and inches figures set as they are in this example?

The great advantage of this arrangement of these figures is that the standard methods of multiplication and division can be used. Notice the use of the standard method of division in steps 4 and 5 of Method A, and in steps 2 and 4 of Method B in this example. For further details see Examples 2 and 3, section 2 of this chapter.

EXAMPLE 2: Subtract 12 days 17 hours 48 minutes from 25 days 12 hours 43 minutes.

da.	hr.	min.	
25	12	43	
−12	−17	−48	
12	18	55	Ans.: 12 da. 18 hr. 55 min.

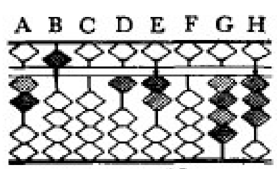


FIG. 153

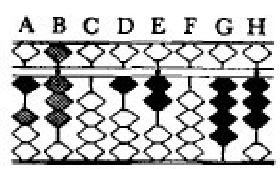


FIG. 154

STEP 1: Set 25 on AB, with B as the unit rod, set 12 on DE, with E as the unit rod, and set 43 on GH, with H as the unit rod (Fig. 153).

STEP 2: Subtract 12 from the 25 on AB. This leaves 13 on AB (Fig. 154).

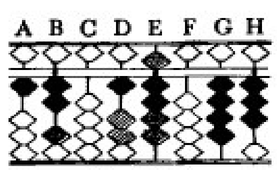


FIG. 155

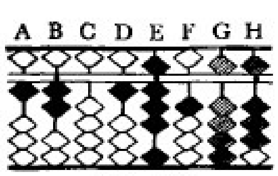


FIG. 156

	A	B	C	D	E	F	G	H	
	2	5	0	1	2	0	4	3	Step 1
−	1	2							Step 2
	1	3	0	1	2	0	4	3	
		−1							Step 3
				+2	4				
				−1	7				
	1	2	0	1	9	0	4	3	
					−1				Step 4
							+6	0	
							−4	8	
	1	2	0	1	8	0	5	5	

STEP 3: Since you cannot subtract 17 from the 12 on DE, borrow 1 from B, and subtracting 17 from 24, add 7 to the 12 on DE. This makes 19 on DE (Fig.

155).

NOTE: In borrowing from B, you may add 24 to the 12 on DE, and subtract 17 from the sum 36. This gives you the same result 19.

STEP 4: Since you cannot subtract 48 from the 43 on GH, borrow 1 from the 9 on E, and subtracting 48 from 60, add the result 12 to the 43 on GH. This makes 55 on GH. Now you have 12 on AB, 18 on DE, and 55 on GH. The answer is 12 days 18 hours 55 minutes (Fig. 156).

EXAMPLE 3:

£	s.	d.	
95	7	3	
+12	8	5	
− 7	19	10	
+26	2	7	Ans.: £125 18 *s.* 5 *d.*
125	18	5	

In the above example, the second and fourth compound numbers are to be added, and the third to be subtracted.

When both additions and subtractions are to be performed, calculations generally move from higher to lower denominations on each row. But they can also move down each column—completing each column before moving to the higher denomination.

METHOD A: Calculating from higher to lower denominations: STEP 1: Set £95 on BC, with C as the unit rod, set 7 *s.* on unit rod F, and set 3 *d.* on unit rod I (Fig. 157).

STEP 2: Add 12 to the 95 on BC, producing 107 on ABC.

Add 8 to the 7 on F, yielding 15 on EF.

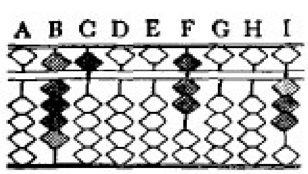


FIG. 157

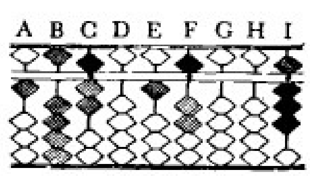


FIG. 158

Add 5 to the 3 on I, yielding 8 on I.

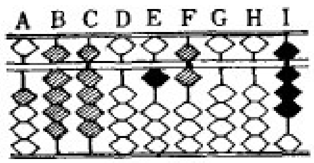


FIG. 159

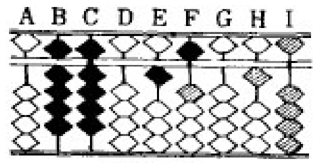


FIG. 160

STEP 3: Subtract 7 from the 107 on ABC, leaving 100 on ABC.

As you cannot subtract 19 from the 15 on EF, borrow 1 from rod C. Subtracting 19 from 20, add 1 to the 15 on EF. This gives you 16 on EF and 99 on BC (Fig. 159).

STEP 4: As you cannot subtract 10 from the 8 on I, borrow 1 from the 16 on EF, and subtracting 10 from 12, add the result 2 to the 8 on I. This gives you 10 on HI and 15 on EF (Fig. 160).

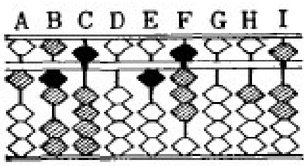


FIG. 161

STEP 5: Now add 26 to the 99 on BC, and you get 125 on ABC.

Add 2 to the 15 on EF, leaving 17 on EF.

Add 7 to the 10 on HI, yielding 17 on HI.

Finally subtract 12 from this 17 and carry 1 to the 17 on EF. This gives you 5 on I and 18 on EF. The answer is £125 18*s*. 5*d*. (Fig. 161).

METHOD B: Calculating from lower to higher denominations: An alternative is to calculate column by column from lower to higher denominations.

First do the pence column on HI by means of complementary numbers, yielding 5 *d.* on I.

Next do the shillings column on EF by means of complementary numbers, and you get minus 2 *s.* on F. So borrow £1 from £95 to rectify this minus quantity, and you get 18*s*. on EF and £94 left on BC.

In subtracting 19 *s.* from 15 *s.*, you may borrow *£1* instead of calculating by means of complementary numbers.

Finally setting £94 on BC, calculate the pound column, which leaves £125 on ABC.

This gives you the same result, £125, 18 *s.*, and 5 *d.* on ABC, EF, and I respectively.

4. Multiplying Compound Numbers

EXAMPLE 1: Multiply the following compound number by 18.

£4 2*s*. 6*d*.

On the abacus the computation is made mostly in the same way as in the following written calculation.

£ 4		2 *s.*		6 *d.*	Step 1
×18	Step 6	×18	Step 4	×18	Step 2
72		36		12) 108 (9	Step 3
+ 2		+ 9	Step 4	108	
74	Step 6	20) 45 (2	Step 5	0	
		40			
		5		Ans.: £74 5 *s.*	

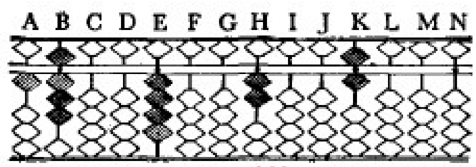


FIG. 162

STEP 1: Set 4 on unit rod E, and 2 and 6 on H and K respectively. Set the multiplier 18 on AB (Fig. 162).

NOTE: Experts, however, often do not actually set the multiplier.

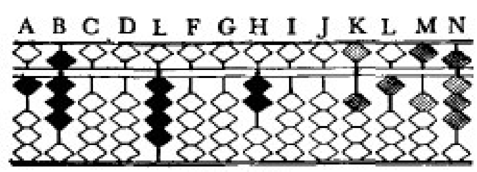


FIG. 163

STEP 2: First multiply the 6 on K by 18, and you get the product 108 on LMN. Now clear K of its 6 (Fig. 163).

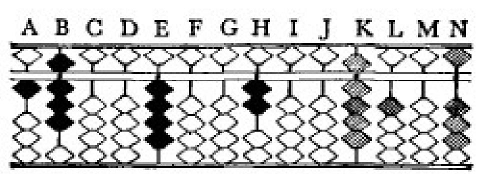


FIG. 164

STEP 3: Now divide the 108 on LMN by 12 and you get the quotient 9 on K, having cleared LMN of the 108. Here you should remember the divisor 12 without setting it on the board (Fig. 164).

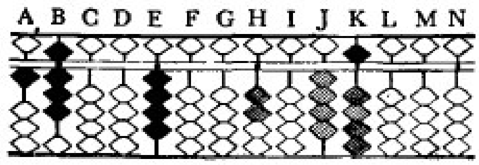


FIG. 165

STEP 4: Next you must multiply the 2 on H by 18, and add the product 36 to the 9 on K. In this procedure, first multiply the 1 on A by the 2 on H, then set the product 2 on J. This gives you 29 on JK. Next multiply the 8 on B by the same 2 on H; add the product 16 to the 29 on JK, and clear H of its 2. This gives you a total of 45 on JK (Fig. 165).

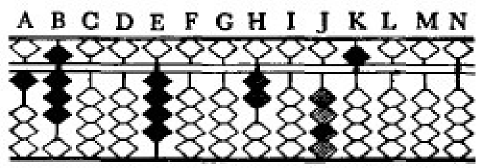


FIG. 166

STEP 5: Now divide the 45 on JK by 20, and you get the quotient 2 on H, with 5 left over on K (Fig. 166).

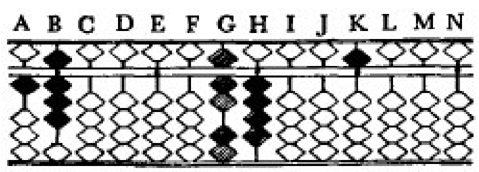


FIG. 167

STEP 6: Finally you must multiply the 4 on E by 18, and add the product 72 to the 2 on H. In this process, first multiply the 1 on A by the 4 on E, and set the product 4 on G. This gives you 42 on GH. Next multiply the 8 on B by the same 4 on E, add the product 32 to the 42 on GH, and clear E of its 4. This gives you a total of 74 on GH. The answer is £74 5*s.* (Fig. 167).

NOTE: An alternative method for working this problem is to convert the numerical figures of this problem to pence, and then to multiply the result by 18. At the end of the problem the product is reconverted to pounds, shillings, and pence. This method is often used when the multiplier is a decimal fraction.

5. Dividing Compound Numbers

EXAMPLE 1: Divide the following compound number by 14.

£243 2*s*. 8*d*.

On the abacus the division is performed much the same way as in the following written calculation:

£243 Step 1	2 *s*. Step 1	8 *d*. Step 1
	$+\ 5\times 20$ Step 3	$+\ 4\times 12$ Step 5
	102	56
$14\overline{)243}$ quotient 17 Step 2	$14\overline{)102}$ quotient 7 Step 4	$14\overline{)56}$ quotient 4 Step 6
14	98	56
103	4	0
98		
5		

Ans.: £17 7*s*. 4*d*.

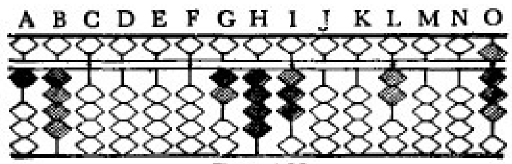


FIG. 168

STEP 1: Set 243 on GHI, with I as the unit rod, and set 2 on unit rod L and 8 on unit rod O. The divisor 14 may be set either on AB or on BC, though preferably on the former with four vacant rods between B and G (Fig. 168).

STEP 2: Divide the 243 on GHI by 14, and you get the quotient 17 on EF and a remainder of 5 on I (Fig. 169).

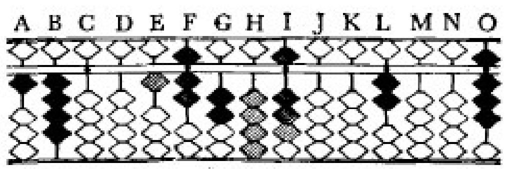


FIG. 169

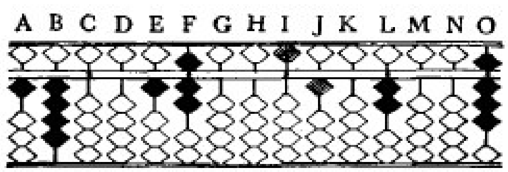


FIG. 170

STEP 3: Multiply the 5 on I by 20, and add the product 100 to the 2 on L, clearing the 5 on I, and you get 102 on JKL (Fig. 170).

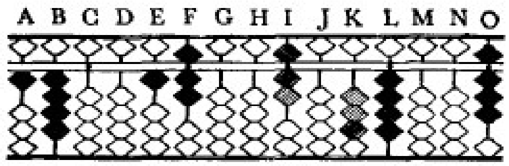


FIG. 171

STEP 4: Divide the 102 on JKL by 14, leaving a quotient of 7 on I and a remainder of 4 on L (Fig. 171).

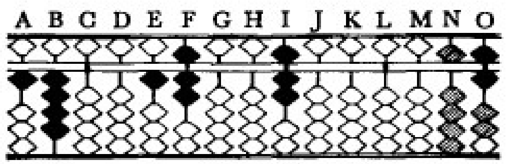


FIG. 172

STEP 5: Multiply the 4 on L by 12, and add the product 48 to the 8 on O, clearing the 4 on L, and you get 56 on NO (Fig. 172).

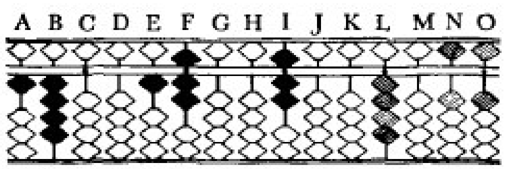


FIG. 173

STEP 6: Divide the 56 on NO by 14, and you get the quotient 4 on L with no remainder. The answer is £17 7*s*. 4*d*. (Fig. 173).

NOTES: (a) An alternative method for this problem is to convert everything to pence and then to divide the result by 14. This result is converted to pounds, shillings, and pence. When the figure of the highest denomination is smaller than the divisor, division cannot be performed unless it is converted into that of the lower denomination and makes a figure larger than the divisor, (b) On the board, multiplication is performed simultaneously with addition, and division simultaneously with subtraction. This means that to multiply numbers is to add numbers and that to divide numbers is to subtract numbers. Accordingly, multiplication and division are all the faster on the board than on paper.

VII. EXTRACTING SQUARE ROOTS

There are several methods of finding both the square and cube roots of numbers on the abacus. They are all adaptations of algebraic methods.

Since the methods of finding cube roots are complicated, and are beyond the scope of the interest and need of ordinary abacus operators, this book only concerns itself with three typical methods of extracting square roots.

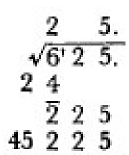
The first is the exact algebraic method. The second is considered the best by some authoritative experts. The third is the most representative traditional method and is the most widely accepted by modern abacus operators.

The method of finding the two-figure square root of a number is founded upon the method of finding the square of an ordinary binomial (or two-membered) expression, (a+b). The number is analyzed into (a^2+2ab+b^2) as in algebra, and the square root is obtained.

Let us take the number 625 as an example of finding a two-figure square root.

In all the three methods introduced here, the reader will see that the first step consists in subtracting the square of a or 20 (=a^2 or 400), the next of subtracting 2ab or 2x20x5 (=200), and the third of subtracting the square of b or 5 (=b^2 or 25).

As a refresher, the standard written method of extracting square roots is as follows:



625 is divided into pairs of digits beginning from the decimal point and moving right and left (thus 18,967.103 is divided 1'89'67.10'30). Then the largest perfect square which matches the first set of numbers is selected—4 is the largest perfects square smaller than or equal to 6.2—and the square root of 4, 2, is written both above the 6 and to the side. The product, 4, is written below the 6. The 4 is subtracted from 6 and the next pair of numbers is brought down—making 225. The number which is above the square root sign, 2, is doubled and brought down to the side—here the 4 of the 45 at the side. Then a number is selected, 5, such that when placed with 4 to make a two-digit number, 45, and that number is multiplied by the original

number selected, 45x5, the result is the largest number less than or equal to the "quotient," which at this point is 225.

The following analysis will also help the reader to find the square root of 625 on the abacus. Note that in the series of algebraic equations given here, *a* is in substance 20 and *b* is 5.

$$
\begin{aligned}
&(a+b)^2 \\
&=a^2+2ab+b^2 \\
&=20^2+(2\text{x}20\text{x}5)+5^2 \\
&=400+200+25 \\
&=625
\end{aligned}
$$

Both the abacus method and this written method are based on algebraic equations. In a simple problem (one involving a two-digit answer) both methods of extracting a square root can be described by the binomial equation $(a+b)^2-a^2-2ab-b^2=0$. In a slightly more complicated problem (one involving a three-digit answer) both can be described by the equation $(a+b+c)^2-a^2-2ab-b^2-2ac-2bc-c^2=0$. To understand this compare the examples below, which show the standard written method (left) and the abacus method (right).

```
      2   5.     (a+b)                  2   5.     (a+b)
    √6'2  5.     (a+b)²               √6'2  5.     (a+b)²
 2   4           (−a²)            2    4           (−a²)
     2 2 5                             2 2
45   2 2 5       (−2ab−b²)        4    2 0         (−2ab)
                                         2 5
                                  5      2 5       (−b²)
```

In a trinomial problem this is what happens. At left is the written method and at right the abacus method.

```
                                          3   4   6.     (a+b+c)
                                      √1 1'9 7'1  6.     (a+b+c)²
                                     3   9               (−a²)
                                         2 9
      3   4   6.    (a+b+c)          6   2 4             (−2ab)
  √1 1'9 7'1  6.    (a+b+c)²               5 7
3     9             (−a²)            4     1 6           (−b²)
      2 9 7                                4 1
64    2 5 6         (−2ab−b²)        6     3 6           (−2ac)
        4 1 1 6                              5 1
686     4 1 1 6     (−2ac−2bc−c²)    8       4 8         (−2bc)
                                               3 6
                                     6         3 6       (−c²)
```

This means that the numbers handled in the Japanese abacus method as opposed to the Western written method are smaller and simpler. On the other hand, the number of manipulations required is larger. Nevertheless, once an operator becomes accustomed to using this Japanese method he can obviously do a problem faster than he would have been able to on paper.

EXAMPLE 1: Find the square root of 625.

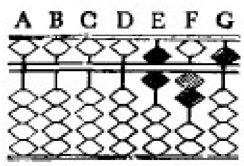


FIG. 174

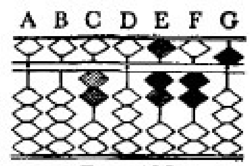


FIG. 175

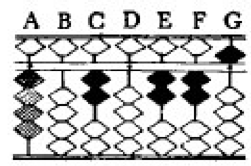


FIG. 176

METHOD A: STEP 1: Set 625 on EFG, with G as the unit rod (Fig. 174). Mark off the digits in sets of twos. The largest perfect square less than 6 is 4.

STEP 2: Set, on C, 2, the root of the 4. Square this 2, and subtract the product 4 from the 6 on E. This leaves 225 on EFG (Fig. 175).

NOTE: The 2 on C corresponds to the a of the expression (a^2+2ab+b^2).

STEP 3: Double the 2 on C, and set the product 4 on A (Fig. 176).

NOTE: Now the 4 on A corresponds with the *2a* of the algebraic expression.

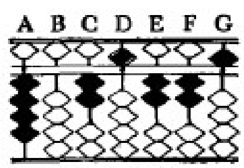


FIG. 177

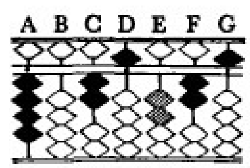


FIG. 178

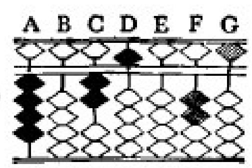


FIG. 179

STEP 4: The 4 on A divides into the 22 on EF five times. Set the second trial quotient figure 5 on D (Fig. 177).

NOTE: The 5 on D corresponds with the *b* of the algebraic expression.

STEP 5: Multiplying the 4 on A by the 5 on D, subtract the product 20 from the 22 on EF. This leaves 25 on FG (Fig. 178).

NOTE*: (4x5) corresponds with the 2ab of the expression.*

STEP 6: Square the 5 on D, and subtract the product 25 from the 25 on FG.

This clears the board of 625, and shows that the 5 on D is the correct second quotient figure. The answer is 25 on CD (Fig. 179).

NOTE: 5^2 or 25 corresponds with the b^2 of the expression. Also note that this method corresponds more strictly to the written algebraic method than the other methods introduced below.

METHOD B: $2\sqrt{625}$.

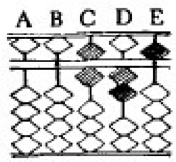


FIG. 180

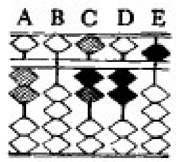


FIG. 181

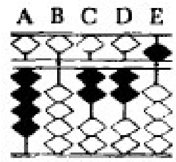


FIG. 182

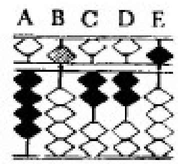


FIG. 183

STEP 1: Set 625 on CDE, with E as the unit rod (Fig. 180).

Mark off the digits in sets of twos. The largest square less than 6 is 4.

STEP 2: Set, on A, 2, the root of the 4. Square this 2, and subtract the product 4 from the 6 on C. This leaves 225 on CDE (Fig. 181).

STEP 3: Double the 2 on A, making 4 on A (Fig. 182).

STEP 4: The 4 on A goes into the 22 on CD five times. Set the second trial quotient figure 5 on B (Fig. 183).

NOTE: The 5 on B corresponds with the *b* of the expression.

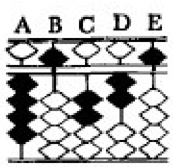


FIG. 184

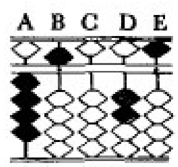


FIG. 185

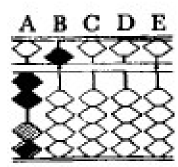


FIG. 186

STEP 5: Multiply the 4 on A by the 5 on B, and subtract the product 20 from the 22 on CD. This leaves 25 on DE (Fig. 184).

STEP 6: Square the 5 on B, and subtract the product 25 from the 25 on DE, and the board is cleared of the dividend (Fig. 185).

STEP 7: Finally halve the 4 on A into its original digit 2. Our square root is 25 on AB (Fig. 186).

METHOD C: $2\sqrt{625}$

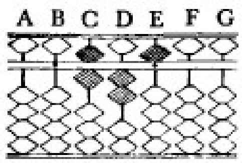


FIG. 187

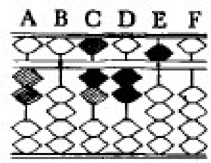


FIG. 188

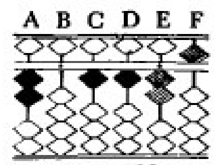


FIG. 189

STEP 1: Set 625 on CDE, with E as the unit rod (Fig. 187).

Mark off the digits in sets of twos. The largest square less than 6 is 4.

STEP 2: Set, on A, 2, the root of the 4. Square this 2, and subtract the product 4 from the 6 on C. This leaves 225 on CDE (Fig. 188).

NOTE: The 225 on CDE corresponds to the $(2ab+b^2)$ of the algebraic expression.

STEP 3: Multiply the 225 on CDE by 0.5, and set the product 112.5 on CDEF (Fig. 189).

NOTES: (a) If you find it hard to do this division mentally, you may first set the product 112.5 on FGHI and later set it on CDEF. (b) The 112.5 on CDEF corresponds to $(ab+\frac{b^2}{2})$ of the algebraic expression.

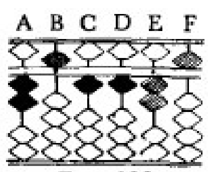


FIG. 190

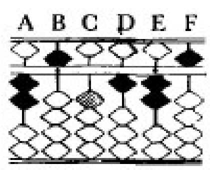


FIG. 191

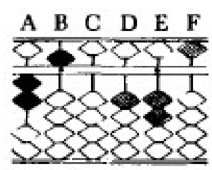


FIG. 192

STEP 4: The 2 on A divides into the 11 on CD five times. Set the second trial quotient figure 5 on B (Fig. 190).

NOTE: The 5 on B corresponds with the *b* of the expression.

STEP 5: Multiply the 2 on A by the 5 on B, and subtract the product 10 from the 11 on CD. This leaves 12.5 on DEF (Fig. 191). *b* 2

NOTE: The 12.5 on DEF corresponds to $\frac{b^2}{2}$ of the expression.

STEP 6: Square the 5 on B mentally, and you get 25. Next either divide this 25 by 2 or multiply it by 0.5, and you get 12.5. If you find it hard to do this calculation mentally you can do it on the board.

Now subtract this 12.5 from the 12.5 on DEF. This clears the board of the 625, and leaves the answer 25 on AB (Fig. 192).

EXAMPLE 2: Find the square root of 4,489 (Method B).

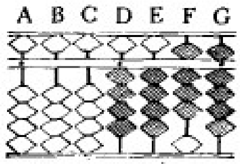


Fig. 193

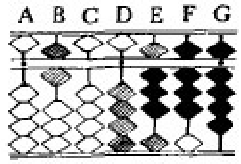


Fig. 194

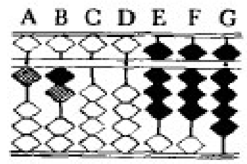


Fig. 195

STEP 1: Set 4,489 on DEFG, with G as the unit rod (Fig. 193). Mark off the digits in sets of twos. In the first pair 44, the highest square root is 6.

STEP 2: Set, on B, 6, which is the root of the 44 on DE. Square this 6, and subtract the product 36 from the 44 on DE. This leaves 889 on EFG (Fig. 194).

STEP 3: Double the 6 on B into 12 on AB (Fig. 195).

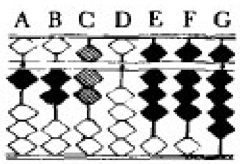


Fig. 196

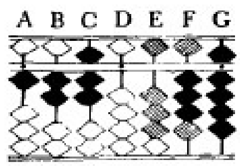


Fig. 197

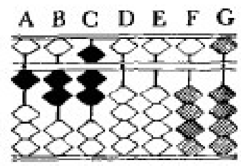


Fig. 198

STEP 4: The 12 on AB divides into the 88 on EF seven times. Set the second trial divisor figure 7 on C (Fig-196).

NOTE: The 7 on C corresponds with the *b* of the expression.

STEP 5: Now multiplying the 12 on AB by the 7 on C, subtract the product 84 from the 88 on EF. This leaves 49 on FG (Fig. 197).

NOTE: The 12 on AB multiplied by 7 corresponds with $2ab$ or (120x7).

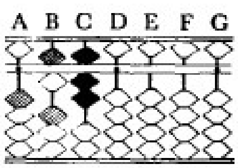


Fig. 199

STEP 6: Next square the 7 on C, and subtract the product 49 from the 49 on FG. This clears the board of the remainder of the number or the b^2 of the expression (Fig. 198).

STEP 7: Finally halve the 12 on AB into its original 6 on B.

Our square root is 67 on BC (Fig. 199).

EXAMPLE 3: Find the square root of 119,716.

The method of finding the three-digit square root of a number is founded upon the algebraic method of finding the square of an ordinary trinomial (or three-membered) expression, $(a+b+c)$. A number is analyzed as in algebra, into $(a^2+2ab+b^2+2ac+2bc+c^2)$ and the square root is found.

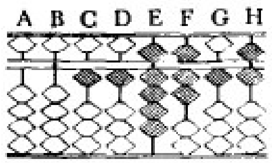


FIG. 200

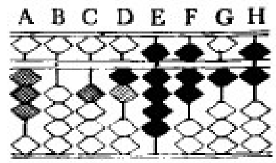


FIG. 201

STEP 1: Set 119,716 on CDEFGH, and mark off the digits in sets of twos.

In the first pair 11, the highest square root is 3 (Fig. 200).

STEP 2: Set, on A, 3, the first trial figure in the root. Square the 3, and subtract the product 9 from the 11 on CD. This leaves 29,716 on DEFGH (Fig. 201).

NOTE: 3 corresponds to the *a* of the algebraic expression ($a^2+2ab+b^2+2ac+2bc+c^2$).

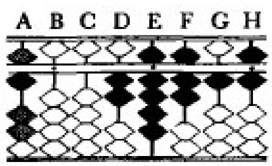


FIG. 202

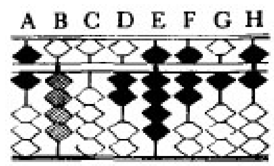


FIG. 203

STEP 3: Double the 3 on A making 6 on A (Fig. 202).

STEP 4: Figuring how many times the 6 on A goes into the 29 on DE, we find that 4 is correct. So we set 4 on B as the second trial figure in the root (Fig. 203).

NOTE: The 4 on B corresponds to the b of the algebraic expression.

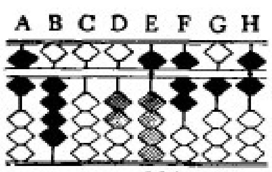


FIG. 204

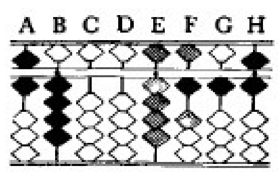


FIG. 205

STEP 5: Multiply the 6 on A by the 4 on B, and subtract the product 24 from the 29 on DE. This leaves 5,716 on EFGH (Fig. 204).

NOTE: (4x6) or 24 corresponds to the lab of the algebraic expression.

STEP 6: Now square the 4 on B, and subtract the product 16 from the 57 on EF.

This leaves 4,116 on EFGH (Fig. 205).

NOTE: The square of 4 corresponds to the b^2 of the algebraic expression.

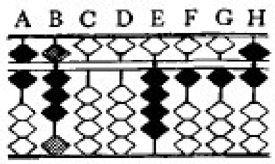


Fig. 206

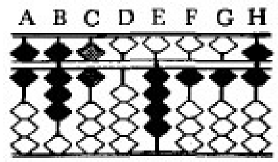


Fig. 207

STEP 7: Double the 4 on B, making 8 (Fig. 206).

NOTE: When the second trial divisor figure is 5 or larger, move the first divisor figure to the first rod to the left, and set double the second trial quotient figure on AB.

STEP 8: Figuring how many times the 6 on A goes into the 41 on EF, we find that 6 is the proper figure. Set 6 on C as the third trial figure in the root (Fig. 207).

NOTE: The 6 on C corresponds to *c* of the expression.

STEP 9: Multiply the 6 on A by the 6 on C, and subtract the product 36 from the 41 on EF. This leaves 516 on FGH (Fig. 208).

NOTE: 6x6 corresponds to the 2*ac* of the algebraic expression.

STEP 10: Multiply the 8 on B by the same 6 on C, and subtract the product 48 from the 51 on FG. This leaves you 36 on GH (Fig. 209).

NOTE: (8 on Bx6 on C) corresponds to the 2*bc* of the algebraic expression.

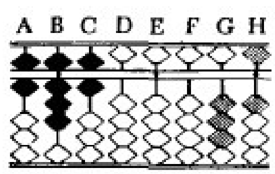


Fig. 210

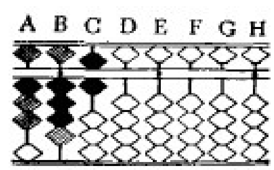


Fig. 211

STEP 11: Square the 6 on C, and subtract the product 36 from the 36 on GH. This clears the board of the remainder of the number, which corresponds to the c^2 the algebraic expression (Fig. 210).

STEP 12: Now halve the 6 on A and the 8 on B into the original 3 and 4 respectively. Our square root is 346 on ABC (Fig. 211).

NOTE: The 68 which was on AB corresponded to the 2*ab* of the algebraic expression.

VIII. MORE EXERCISES

The exercises for the first, second, and third grades were actually used in recent national abacus license examinations and are here reprinted through the courtesy of the Japan Chamber of Commerce and Industry.

Note: Exercises begin on the following page.

1. Eighth-Grade Operator GROUP A

(1 set per minute, or entire group with 70% accuracy in 10 minutes)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
¥62	¥35	¥71	¥20	¥86	¥14	¥90	¥57	¥82	¥49
15	70	45	38	92	20	26	69	54	80
40	29	36	92	50	97	-83	32	-73	21
71	86	50	13	29	65	59	70	-38	68
84	43	-92	56	-74	28	42	31	10	32
26	58	-27	90	51	73	-15	46	29	14
97	10	83	84	63	41	-60	18	76	95
80	64	19	47	-30	56	47	94	-61	70
39	17	-60	75	-48	30	31	25	40	53
53	92	84	61	17	89	78	80	95	76
¥567	¥504	¥209	¥576	¥236	¥513	¥215	¥522	¥214	¥558

	GROUP B ***(70% accuracy, 10 minutes)***	GROUP C ***(70% accuracy, 10 minutes)***
(1)	45x8=360	72÷2=36
(2)	73x4=292	135÷5=27
(3)	52x2=104	246÷3=82
(4)	38x7=266	126÷9=14
(5)	94x3=282	413÷7=59
(6)	15x6=90	300÷4=75

(7)	86x9=774	744÷8=93
(8)	67x5=335	288÷6=48
(9)	21x8=168	183÷3=61
(10)	79x6=474	435÷5=87
(11)	134x3=402	8,829÷9=981
(12)	509x2=1,018	420÷4=105
(13)	876x8=7,008	2,634÷6=439
(14)	623x4=2,492	752÷2=376
(15)	258x9=2,322	4,336÷8=542
(16)	480x6=2,880	4,050÷5=810
(17)	942x5=4,710	1,906÷2=953
(18)	715x7=5,005	2,124÷3=708
(19)	301x4=1,204	1,068÷4=267
(20)	697x3=2,091	4,368÷7=624

2. Seventh-Grade Operator *Assortment 1*

GROUP A

(1 set per minute, or entire group with 70% accuracy in 10 minutes)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
¥45	¥63	¥87	¥31	¥56	¥94	¥26	¥79	¥70	¥12
78	95	60	19	27	70	98	16	45	60
30	59	31	40	68	19	-20	84	61	85
59	-80	48	95	90	51	-57	40	13	-32
61	-43	24	-87	31	35	41	62	90	-78
96	78	12	-28	47	60	93	97	24	45
20	51	39	30	82	89	-65	23	57	90
83	-27	50	72	43	46	10	95	38	76

52	82	67	16	10	18	74	60	47	-53
17	90	29	-57	24	42	58	35	96	10
93	-64	15	20	93	37	-89	21	80	34
40	-37	98	65	51	68	-13	74	12	-89
62	10	54	-43	80	27	30	83	35	-71
81	46	70	-64	69	50	76	18	26	94
74	12	36	98	75	23	42	50	89	67
¥891	¥335	¥720	¥207	¥846	¥729	¥304	¥837	¥783	¥250

	GROUP B ***(70% accuracy, 10 minutes)***	GROUP C ***(70% accuracy, 10 minutes)***
(1)	7,832x6=46,992	402÷2=201
(2)	4,096x7=28,672	6,776÷7=968
(3)	6,348x8=50,784	3,282÷6=547
(4)	1,573x4=6,292	2,430÷3=810
(5)	8,105x9=72,945	5,536÷8=692
(6)	2,619x3=7,857	1,380÷4=345
(7)	5,980x6=35,880	3,180÷6=530
(8)	3,467x5=17,335	7,101÷9=789
(9)	9,024x9=81,216	3,408÷8=426
(10)	2,751x2=5,502	865÷5=173
(11)	3,647x6=21,882	3,896÷4=974
(12)	9,502x4=38,008	966÷7=138
(13)	6,138x3=18,414	1,184÷2=592
(14)	8,420x9=75,780	4,830÷6=805
(15)	4,975x2=9,950	807÷3=269
(16)	1,769x8=14,152	2,250÷5=450
(17)	7,081x4=28,324	2,468÷4=617

(18)	2,153x2=4,306	682÷2=341
(19)	5,304x5=26,520	5,648÷8=706
(20)	8,296x7=58,072	7,407÷9=823

Assortment 2

GROUP A

(70% accuracy, 10 minutes)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
¥61	¥47	¥21	¥86	¥34	¥53	¥95	¥78	¥16	¥41
72	16	90	47	68	61	59	40	83	68
30	28	64	91	-50	42	10	61	-24	90
84	60	-39	20	-37	90	-46	35	-70	42
12	59	-70	35	64	57	-89	84	25	37
89	32	94	58	15	32	71	27	93	56
93	75	18	24	40	18	20	95	41	27
10	39	56	10	-83	25	64	50	-67	10
56	40	-82	73	29	70	-23	32	80	96
34	18	25	69	71	64	87	19	35	53
59	83	43	26	97	86	30	94	78	39
67	71	80	49	-21	39	15	87	-46	28
75	50	-67	18	-60	80	-78	23	-59	70
20	94	-71	30	58	74	-62	16	90	14
48	26	53	75	92	91	34	60	12	85
¥810	¥738	¥215	¥711	¥317	¥882	¥187	¥801	¥287	¥756

	GROUP B ***(70% accuracy, 10 minutes)***	GROUP C ***(70% accuracy, 10 minutes)***
(1)	2,604x3=7,812	4,0774÷9=453
(2)	5,219x7=36,533	2,6674÷7=381
(3)	8,045x5=40,225	1,6124÷2=806
(4)	1,573x6=9,438	2,810÷5=562
(5)	3,987x2=7,974	8824÷3=294
(6)	6,820x4=27,280	5,096÷7=728
(7)	9,751x9=87,759	6804÷4=170
(8)	7,406x8=59,248	9574÷3=319
(9)	4,132x3=12,396	7,2404÷8=905
(10)	9,368x7=65,576	3,8824÷6=647
(11)	6,972x6=41,832	5,4814÷7=783
(12)	3,285x3=9,855	2,100÷6=350
(13)	5,049x2=10,098	6,246÷9=694
(14)	1,856x4=7,424	1,195÷5=239
(15)	8,391x8=67,128	3,488÷4=872
(16)	4,703x9=42,327	4354÷3=145
(17)	2,168x4=8,672	1,0544÷2=527
(18)	9,027x3=27,081	900÷5=180
(19)	7,430x7=52,010	7,744÷8=968
(20)	6,514x5=32,570	832÷2=416

3. Sixth-Grade Operator *Assortment 1*

GROUP A

(1 set per minute, or entire group with 70% accuracy in 10 minutes)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
¥35	¥64	¥9,017	¥781	¥29	¥83	¥459	¥6,082	¥54	¥207
409	5,013	928	350	67	962	104	94	592	18
852	937	241	19	8,024	38	-76	350	9,364	173
14	790	-58	28	71	5,093	-395	71	-108	14
3,267	28	-703	974	-438	126	7,201	259	-46	3,059
198	576	342	6,831	-92	40	68	37	15	86
41	102	96	402	684	57	890	25	687	542
620	58	430	56	305	814	24	68	30	69
93	4,681	73	98	586	71	16	1,704	879	250
8,065	25	16	34	90	2,708	-73	219	13	67
47	872	65	2,097	-1,632	345	-248	73	205	938
971	34	-809	516	-741	96	-3,091	854	-36	8,401
53	609	-7,521	45	-53	607	67	380	-4,071	97
702	91	-86	27	910	54	582	69	-728	45
86	43	54	603	57	219	35	146	92	623
¥15,453	¥13,923	¥2,085	¥12,861	¥7,867	¥11,313	¥5,563	¥10,431	¥6,942	¥14,589

	GROUP B ***(70% accuracy, 10 minutes)***	GROUP C ***(70% accuracy, 10 minutes)***
(1)	46x754=34,684	1,288÷56=23
(2)	703x13=9,139	1,568÷98=16
(3)	28x972=27,216	1,554÷21=74
(4)	159x69=10,971	1,505÷35=43
(5)	82x407=33,374	4,030÷62=65
(6)	514x18=9,252	4,324÷47=92
(7)	37x230=8,510	2,573÷83=31
(8)	950x51=48,450	1,102÷19=58

(9)	64x805=51,520	1,406÷74=19
(10)	821x36=29,556	8,004÷92=87
(11)	798x84=67,032	516÷43=12
(12)	45x289=13,005	918÷27=34
(13)	172x52=8,944	5,472÷72=76
(14)	630x65=40,950	1,302÷14=93
(15)	84x906=76,104	3,936÷96=41
(16)	209x47=9,823	4,930÷58=85
(17)	51x743=37,893	5,963÷89=67
(18)	306x19=5,814	899÷31=29
(19)	97x308=29,876	3,640÷65=56
(20)	63x261=16,443	2,592÷54=48

Assortment 2

GROUP A

(70% accuracy in 10 minutes)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
¥586	¥41	¥873	¥268	¥184	¥725	¥92	¥308	¥657	¥468
93	705	345	981	510	86	743	96	14	274
610	539	62	705	61	97	10	473	280	59
139	285	204	-53	647	830	451	89	906	412
304	129	583	-178	79	15	-809	21	45	160
78	-450	691	94	803	957	-65	140	-12	358
42	-683	78	360	52	401	586	897	-138	907
175	-42	306	412	923	648	31	205	-879	13
762	801	452	-29	490	312	708	34	923	76
407	93	18	-506	735	60	-342	516	498	802
863	217	790	-635	96	502	-74	925	60	35

51	-964	29	840	318	243	-623	650	327	791
920	-78	145	27	574	89	290	874	-705	623
98	360	67	491	26	173	968	731	-46	580
245	76	910	73	208	694	157	62	531	49
¥5,373	¥1,029	¥5,553	¥2,850	¥5,706	¥5,832	¥2,123	¥6,021	¥2,461	¥5,607

	GROUP B ***(70% accuracy, 10 minutes)***	GROUP C ***(70% accuracy, 10 minutes)***
(1)	403x73=29,419	888÷74=12
(2)	26x208=5,408	782÷23=34
(3)	589x74=43,586	3,953÷59=67
(4)	81x861=69,741	476÷17=28
(5)	170x49=8,330	7,990÷85=94
(6)	68x617=41,956	2,576÷46=56
(7)	945x14=13,230	6,643÷91=73
(8)	307x32=9,824	1,558÷38=41
(9)	69x506=34,914	5,518÷62=89
(10)	72x985=70,920	945÷27=35
(11)	34x194=6,596	4,335÷85=51
(12)	512x83=42,496	3,570÷42=85
(13)	49x508=24,892	882÷14=63
(14)	301x27=8,127	2,652÷68=39
(15)	758x91=68,978	7,068÷93=76
(16)	93x609=56,637	5,586÷57=98
(17)	206x35=7,210	783÷29=27
(18)	87x402=34,974	994÷71=14
(19)	65x253=16,445	1,512÷36=42
(20)	124x76=9,424	684÷19=36

4. Fifth-Grade Operator ***Assortment 1***

GROUP A

(1 set per minute, or entire group with 70% accuracy in 10 minutes)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
¥576	¥8,294	¥603	¥3,918	¥702	¥423	¥186	¥307	¥9,684	
2,089	435	481	706	9,318	357	534	1,273	509	7
961	3,108	894	327	529	4,108	129	918	763	
846	912	5,021	986	437	689	9,705	134	-2,157	
417	593	372	8,031	-654	1,594	671	8,059	-406	
6,752	237	9,140	795	-231	380	830	462	-813	5
408	1,689	975	641	-3,048	516	-3,284	391	6,045	
7,813	860	256	4,823	197	7,634	-157	4,907	526	
359	7,046	-517	590	6,851	290	5,021	746	270	9
9,210	572	-8,306	254	960	825	789	598	3,481	
783	904	421	1,085	-893	2,067	-493	7,610	934	
5,034	745	6,359	143	-2,786	942	-7,260	852	-317	6
192	8,061	-738	562	402	8,759	-948	625	-8,059	
624	753	-2,069	476	1,975	301	6,402	583	298	
305	621	-847	7,209	5,064	176	365	2,064	172	4
¥36,369	¥34,830	¥12,045	¥30,546	¥18,823	¥29,061	¥12,490	¥29,529	¥10,930	¥37

GROUP B

(70% accuracy, 10 minutes)

GROUP C

(70% accuracy, 10 minutes)

(1)	601x138=82,938	9,180÷108=85
(2)	247x375=92,625	8,232÷42=196
(3)	5,630x87=489,810	50,934÷653=78
(4)	415x746=309,590	49,282÷82=601
(5)	879x213=187,227	15,141÷309=49
(6)	358x601=215,158	28,917÷51=567
(7)	926x950=879,700	66,822÷74=903
(8)	104x594=61,776	26,236÷937=28
(9)	72x8,062=580,464	7,550÷25=302
(10)	983x429=421,707	30,784÷416=74
(11)	¥459x946=¥434,214	¥14,196÷39=¥364
(12)	¥703x217=¥152,551	¥8,333÷641=¥13
(13)	¥198x683=¥135,234	¥5,508÷204=¥27
(14)	¥942x825=¥777,150	¥70,590÷78=¥905
(15)	¥61x3,901=¥237,961	¥45,568÷512=¥89
(16)	¥205x472=¥96,760	¥51,085÷85=¥601
(17)	¥862x730=¥629,260	¥9,280÷16=¥580
(18)	¥3,807x69=¥262,683	¥65,016÷903=¥72
(19)	¥174x504=¥87,696	¥22,372÷47=¥476
(20)	¥536x158=¥84,688	¥31,850÷325=¥98

Assortment 2

GROUP A

(70% accuracy in 10 minutes)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(
¥1,063	¥274	¥319	¥4,657	¥530	¥6,841	¥805	¥956	¥728	¥4,
542	8,029	753	208	3,714	265	154	7,209	156	
951	367	8,406	431	862	1,079	7,061	567	4,809	

674	3,510	837	-1,629	495	682	142	498	237	1,
8,502	781	264	-370	7,023	236	869	3,015	416	
349	-608	1,472	593	971	8,903	237	947	6,038	
175	-2,153	950	8,042	839	579	9,740	572	345	2,
928	-467	3,029	416	9,506	713	-392	1,086	-293	
6,730	892	145	359	648	2,054	-128	621	-9,507	
481	9,046	581	6,283	187	928	-5,093	835	184	8,
7,209	431	936	107	2,368	435	817	2,690	451	
856	584	7,620	-925	240	197	6,284	723	7,069	
293	-5,670	814	-768	571	4,780	356	134	-273	5,
4,018	-239	6,095	-5,890	124	305	-3,479	408	-690	
367	915	728	174	6,059	146	-605	8,341	-5,182	
¥33,138	¥15,692	¥32,949	¥11,688	¥34,137	¥28,143	¥16,768	¥28,602	¥4,488	¥26,

	GROUP B ***(70% accuracy, 10 minutes)***	GROUP C ***(70% accuracy, 10 minutes)***
(1)	723x815=589,245	51,874÷74=701
(2)	38x4,079=155,002	45,301÷509=89
(3)	901x687=618,987	14,118÷26=543
(4)	467x942=439,914	42,151÷691=61
(5)	235x351=82,485	7,585÷37=205
(6)	6,702x14=93,828	8,676÷82=18
(7)	149x530=78,970	29,056÷908=32
(8)	584x603=352,152	3,302÷13=254
(9)	816x296=241,536	37,060÷85=436
(10)	950x728=691,600	68,482÷706=97
(11)	¥257x498=¥127,986	¥11,856÷624=¥19
(12)	¥183x532=¥97,356	¥40,356÷57=¥708

(13)	¥49x6,705=¥328,545	¥52,136÷931=¥56
(14)	¥802x103=¥82,606	¥33,615÷405=¥83
(15)	¥716x789=¥564,924	¥59,512÷86=¥692
(16)	¥364x217=¥78,988	¥2,856÷14=¥204
(17)	¥9,408x31=¥261,648	¥26,274÷302=¥87
(18)	¥521x856=¥445,976	¥8,428÷28=¥301
(19)	¥390x264=¥102,960	¥77,104÷79=¥976
(20)	¥675x940=¥634,500	¥6,885÷153=¥45

Assortment 3

GROUP A

(70% accuracy in 10 minutes)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(
¥741	¥2,908	¥456	¥1,063	¥601	¥397	¥8,509	¥516	¥907	¥
6,102	847	321	759	2,738	206	971	4,089	132	
819	758	9,604	3,941	573	421	128	934	8,021	
4,057	3,416	259	528	697	1,035	613	5,129	346	6,
938	673	512	709	528	647	2,760	761	238	
692	-105	7,364	247	7,089	531	597	830	-176	4,
8,720	-369	947	5,482	316	785	381	6,208	-2,759	
143	-5,280	396	620	-8,172	4,190	435	792	-681	7,
9,567	721	4,018	135	-450	928	-7,043	831	3,460	9,
285	954	130	8,062	849	6,273	-816	627	594	
306	-1,470	685	594	9,403	806	564	7,283	-815	
928	-536	8,207	317	-961	5,469	9,308	345	-5,092	
5,034	182	793	486	-245	714	-726	590	4,978	5,
476	6,093	278	901	-3,016	852	-4,952	1,054	703	
315	942	1,850	6,873	254	3,098	-240	476	564	

¥39,123 ¥9,734 ¥35,820 ¥30,717 ¥10,204 ¥26,352 ¥10,489 ¥30,465 ¥10,420 ¥37,

	GROUP B ***(70% accuracy, 10 minutes)***	GROUP C ***(70% accuracy, 10 minutes)***
(1)	415x948=393,420	55,647÷81=687
(2)	952x852=811,104	56,870÷605=94
(3)	381x176=67,056	7,599÷149=51
(4)	64x6,307=403,648	36,868÷52=709
(5)	890x219=194,910	2,808÷216=13
(6)	167x563=94,021	67,200÷75=896
(7)	523x490=256,270	8,289÷307=27
(8)	7,048x75=528,600	41,495÷43=965
(9)	279x284=79,236	28,952÷94=308
(10)	306x301=92,106	5,376÷128=42
(11)	¥675x369=¥249,075	¥18,666÷306=¥61
(12)	¥1,948x54=¥105,192	¥7,605÷15=¥507
(13)	¥403x890=¥358,670	¥30,848÷64=¥482
(14)	¥21x4,701=¥98,721	¥7,884÷219=¥36
(15)	¥906x548=¥496,488	¥72,071÷743=¥97
(16)	¥587x132=¥77,484	¥82,156÷92=¥893
(17)	¥862x607=¥523,234	¥34,983÷507=¥69
(18)	¥359x286=¥102,674	¥57,024÷81=¥704
(19)	¥730x913=¥666,490	¥8,136÷452=¥18
(20)	¥124x725=¥89,900	¥7,790÷38=¥205

5. Fourth-Grade Operator *Assortment 1*

GROUP A

(1 set per minute, or entire group with 70% accuracy in 10 minutes)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
¥629	¥7,084	¥13,472	¥8,563	¥698	¥40,615	¥293	¥5,471	¥
3,167	653	4,081	795	5,907	123	6,748	186	
82,036	3,471	958	6,041	654	7,850	913	42,305	
578	49,680	65,412	359	3,761	584	1,095	-627	
9,247	-8,163	7,504	7,261	80,342	1,609	78,932	-6,741	
15,402	-794	963	50,178	531	90,476	689	20,567	
139	162	80,721	98,432	4,270	6,398	17,504	492	
8,576	27,905	3,852	-7,510	65,492	213	9,827	91,058	
30,951	8,741	139	-42,087	789	27,064	451	-9,683	
745	91,036	26,598	-623	10,928	947	82,076	-539	
2,810	56,327	9,605	39,401	5,163	54,801	1,463	-31,870	
71,684	209	437	896	73,095	1,326	60,852	5,324	
493	-852	70,216	-8,304	817	37,892	3,174	908	
4,508	-30,915	8,647	-712	1,204	935	52,630	3,216	
60,392	-2,548	390	26,945	42,386	8,752	405	80,947	
¥291,357	¥201,996	¥292,995	¥179,635	¥296,037	¥279,585	¥317,052	¥201,014	¥3

	GROUP B ***(70% accuracy, 10 minutes)***	GROUP C ***(70% accuracy, 10 minutes)***
(1)	4,321x607=2,622,847	130,977÷567=231
(2)	9,108x938=8,543,304	440,700÷975=452

(3)	7,685x452=3,473,620	336,226÷341=986
(4)	2,934x120=352,080	81,624÷456=179
(5)	5,017x543=2,724,231	622,800÷720=865
(6)	8,652x789=6,826,428	67,402÷134=503
(7)	3,476x204=709,104	502,854÷802=627
(8)	1,809x671=1,213,839	90,746÷289=314
(9)	6,523x316=2,061,268	294,240÷613=480
(10)	7,940x895=7,106,300	724,584÷908=798
(11)	¥2,463x271=¥667,473	¥821,215÷851=¥965
(12)	¥6,289x613=¥3,855,157	¥99,144÷408=¥243
(13)	¥3,045x492=¥1,498,140	¥493,425÷675=¥731
(14)	¥8,251x507=¥4,183,257	¥77,089÷127=¥607
(15)	¥7,836x324=¥2,538,864	¥31,968÷296=¥108
(16)	¥5,970x749=¥4,471,530	¥248,992÷502=¥496
(17)	¥9,012x830=¥7,479,960	¥84,940÷310=¥274
(18)	¥4,758x186=¥884,988	¥567,207÷963=¥589
(19)	¥8,617x905=¥7,798,385	¥136,090÷439=¥310
(20)	¥1,394x658=¥917,252	¥667,968÷784=¥852

Assortment 2

GROUP A

(70% accuracy in 10 minutes)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
¥4,068	¥371	¥93,416	¥8,506	¥74,289	¥1,903	¥6,081	¥25,937
975	30,142	6,904	75,243	315	246	943	3,216
43,106	5,489	21,586	759	1,082	36,152	3,257	64,108
1,279	901	7,843	60,138	471	580	87,109	967
50,813	6,492	397	-7,865	16,208	29,365	548	-7,091

497	72,054	6,052	-971	4,539	5,078	20,816	-459	4
7,160	-523	87,931	1,084	293	8,914	9,457	620	
9,582	-64,370	728	32,416	90,614	57,681	872	30,415	
251	-7,865	240	-9,302	5,367	409	5,139	732	1
86,304	218	5,072	-615	86,970	1,853	726	-54,803	
629	45,806	53,419	-48,127	9,156	60,729	43,065	-1,278	
20,374	-9,762	805	793	304	498	2,193	-564	
8,735	-689	70,693	6,540	762	7,031	76,340	8,925	
281	10,935	2,581	924	8,045	247	629	90,183	
95,346	8,317	164	20,398	57,823	46,372	15,084	7,846	7
¥329,400	¥97,516	¥357,831	¥139,921	¥356,238	¥257,058	¥272,259	¥168,754	¥3

	GROUP B ***(70% accuracy, 10 minutes)***	GROUP C ***(70% accuracy, 10 minutes)***
(1)	9,634x426=4,104,084	346,800÷850=408
(2)	2,761x593=1,637,273	64,390÷274=235
(3)	4,508x180=811,440	419,152÷536=782
(4)	7,985x867=6,922,995	89,027÷701=127
(5)	3,120x305=951,600	80,850÷165=490
(6)	6,842x751=5,138,342	250,158÷482=519
(7)	1,076x918=987,768	130,472÷347=376
(8)	8,453x472=3,989,816	586,112÷608=964
(9)	5,917x609=3,603,453	787,319÷923=853
(10)	3,209x234=750,906	95,559÷159=601
(11)	¥6,013x801=¥4,816,413	¥368,193÷623=¥591
(12)	¥3,497x487=¥1,703,039	¥26,520÷195=¥136
(13)	¥9,084x712=¥6,467,808	¥216,832÷308=¥704
(14)	¥4,526x206=¥932,356	¥603,880÷974=¥620

(15)	¥8,761x658=¥5,764,738	¥140,322÷546=¥257
(16)	¥2,905x923=¥2,681,315	¥666,462÷831=¥802
(17)	¥5,430x165=¥895,950	¥88,665÷257=¥345
(18)	¥1,258x594=¥747,252	¥402,047÷409=¥983
(19)	¥3,872x370=¥1,432,640	¥326,876÷782=¥418
(20)	¥7,619x439=¥3,344,741	¥469,090÷610=¥769

Assortment 3

GROUP A

(70% accuracy in 10 minutes)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
¥2,819	¥314	¥80,593	¥4,295	¥63,017	¥7,694	¥579	¥18,024	
15,038	2,639	921	50,973	4,361	853	4,106	9,132	
946	708	1,650	391	25,703	4,081	95,248	389	
71,405	4,512	789	-8,406	892	725	134	65,708	
571	98,056	6,432	-829	7,210	28,139	3,079	-8,421	
2,795	67,321	958	37,146	982	4,506	18,620	-968	
346	-485	29,076	1,065	80,124	13,472	493	90,614	
49,630	-7,249	513	94,752	6,873	956	7,851	2,837	
7,512	-30,967	71,408	218	58,907	2,780	89,315	-756	
86,057	8,293	4,817	-15,630	764	91,352	407	-36,571	
983	51,420	43,160	-728	9,658	6,041	1,986	-4,720	
6,108	-137	5,249	-6,203	435	467	50,732	945	
729	-6,905	763	871	42,059	36,908	276	51,092	
20,463	584	67,024	60,534	913	219	6,023	7,653	
8,234	10,876	2,385	9,487	1,546	70,583	42,568	304	
¥275,636	¥198,980	¥315,738	¥227,936	¥303,444	¥268,776	¥321,417	¥195,262	¥2

	GROUP B ***(70% accuracy, 10 minutes)***	GROUP C ***(70% accuracy, 10 minutes)***
(1)	1,975x419=827,525	93,063÷463=201
(2)	4,702x165=775,830	653,650÷850=769
(3)	6,819x382=2,604,858	45,310÷197=230
(4)	8,431x674=5,682,494	141,360÷304=465
(5)	3,587x503=1,804,261	546,336÷672=813
(6)	1,064x926=985,264	894,132÷918=974
(7)	5,293x708=3,747,444	102,910÷205=502
(8)	7,648x853=6,523,744	72,954÷86=189
(9)	9,320x497=4,632,040	337,608÷521=648
(10)	2,056x201=413,256	267,393÷749=357
(11)	¥3,674xl59=¥584,166	¥213,852÷502=¥426
(12)	¥7,831x840=¥6,578,040	¥74,889÷471=¥159
(13)	¥9,502x765=¥7,269,030	¥756,286÷943=¥802
(14)	¥1,425x238=¥339,150	¥199,080÷316=¥630
(15)	¥2,368x406=¥961,408	¥694,848÷704=¥987
(16)	¥5,093x687=¥3,498,891	¥97,552÷268=¥364
(17)	¥8,956x924=¥8,275,344	¥498,261÷921=¥541
(18)	¥2,709x173=¥468,657	¥84,525÷805=¥105
(19)	¥6,147x302=¥1,856,394	¥524,286÷657=¥798
(20)	¥4,081x591=¥2,411,871	¥37,674÷138=¥273

The National Examination for the
Third-Grade Abacus License

Held under the auspices of
the Japan Chamber of Commerce and Industry

GROUP A

(Entire group of ten sets with 70% accuracy in ten minutes)

(1)	(2)	(3)	(4)	(5)
¥37,258	¥20,139	¥954	¥638	¥7,524
826	360,728	780,241	301,492	1,630
432,501	46,170	2,516	59,710	849,075
96,317	8,397	37,690	834	26,914
4,620	27,816	-8,215	6,185	-604,132
81,453	819,605	-496,507	720,943	-98,206
708,319	3,547	-21,365	62,397	715
5,492	64,052	619	958	3,854
975	573	84,035	36,270	179,038
67,154	945,610	7,283	917,846	43,927
849,560	8,324	209,714	513	-879
2,738	21,496	-95,480	6,905	-50,186
683	952	-347	80,673	-2,937
58,027	102,873	57,981	654,812	564
907,614	38,109	430,692	28,041	624,105
¥3,236,418	¥2,449,206	¥989,495	¥2,885,103	¥1,010,740

(6)	(7)	(8)	(9)	(10)
¥406,725	¥6,208	¥872	¥5,930	¥18,357
157	37,461	46,237	20,718	9,041
54,860	801,537	695,018	463,802	378,260
3,746	972	79,561	549	56,834
760,912	-42,796	6,409	-87,056	672
538	-194,083	84,357	-109,867	29,138
28,301	-8,615	360,172	97,385	710,953
804,139	23,590	483	8,476	5,280
6,852	314	51,290	714	429

19,026	260,475	3,625	689,502	69,704
497	7,132	976,108	-54,923	804,519
96,813	-56,084	741	-361	235
570,289	-392	29,083	-1,274	1,627
31,794	904,875	4,539	246,013	48,706
2,435	69.581	508,412	93,251	963,415
¥2,787,084	¥1,810,175	¥2,846,907	¥1,372,859	¥3,097,170

GROUP B

(70% accuracy, 10 minutes. Calculate Problems 1-10 to the nearest thousandth; 11-20 to the nearest yen.)

(1)	2,931x819=2,400,489
(2)	6,052x235=1,422,220
(3)	84.67x90.7=7,679.569
(4)	9,234x572=5,281,848
(5)	70,496x86=6,062,656
(6)	1,507x0.461=694.727
(7)	218x1,059=230,862
(8)	0.3875x6.48=2.511
(9)	5,689x3.24=18,432.36
(10)	0.4103x0.073=0.030
(11)	¥6,379x108=¥688,932
(12)	¥2,014x0.429=¥864
(13)	¥1,493x963=¥1,437,759
(14)	¥9,584x0.875=¥8,386
(15)	¥765x504.8=¥386,172
(16)	¥2,301x637=¥l,465,737

(17)	¥40,678x91=¥3,701,698
(18)	¥5,896x20.5=¥120,868
(19)	¥8,125x0.736=¥5,980
(20)	¥3,207x214=¥686,298

GROUP C

(70% accuracy, 10 minutes. Calculate Problems 1-10 to the nearest thousandth; 11-20 to the nearest yen.)

(1)	172,894÷631=274
(2)	0.20592÷3.96=0.052
(3)	361,444÷829=436
(4)	76.296÷408=0.187
(5)	50.735÷0.073=695
(6)	186,296÷584=319
(7)	212.795÷26.5=8.03
(8)	0.47693÷0.917=0.520
(9)	639,184÷7,024=91
(10)	11,526÷15=768.4
(11)	¥324,563÷463=¥701
(12)	¥185÷0.625=¥296
(13)	¥646,919÷7,109=¥91
(14)	¥455÷0.547=¥832
(15)	¥341,105÷85=¥4,013
(16)	¥66÷0.176=¥375
(17)	¥78,196÷90.4=¥865
(18)	¥282,048÷312=¥904
(19)	¥1,820÷2.08=¥875

(20) ¥602,196÷938=¥642

The National Examination for the Second-Grade Abacus License

Held under the auspices of
the Japan Chamber of Commerce and Industry

GROUP A

(Entire group of ten sets with 80% accuracy in ten minutes)

(1)	(2)	(3)	(4)	(5)
¥1,289,307	¥483,971	¥3,029,176	¥62,501,478	¥974,362
675,243	81,602,459	15,849	729,360	4,265,103
90,428,165	5,871,304	436,250	3,968,251	58,130,279
17,436	-90,562	20,754,318	91,087	86,341
7,360,294	-264,918	8,109,567	-46,083,529	679,854
6,548	-6,738,095	3,291	-857,901	1,502,463
874,310	10,947,283	97,846	5,340,782	7,318
48,039,152	306,947	521,738	19,648	80,264,597
2,157,896	54,732	70,983,462	654,270	392,086
93,405	7,086,529	1,240,573	-2,593	18,759
526,781	-32,169,840	612,984	-9,145,806	2,840,915
3,185,069	-7,615	54,601	-36,714	76,051,432
43,972	9,520,736	9,072,345	20,794,136	419,687
54,702,631	64,187	865,913	1,283,097	23,590
951,820	815,023	65,378,024	476,835	3,705,241
¥210,352,029	¥77,482,141	¥181,175,937	¥39,732,401	¥229,352,027

(6)	(7)	(8)	(9)	(10)

¥537,641	¥312,650	¥8,029,137	¥86,374	¥80,625,749
7,241,930	1,287,094	79,630,821	10,342,569	79,853
35,082,469	64,238	948,675	493,720	5,986,072
697,508	40,175,982	52,149	6,205,917	395,846
-4,803,621	3,674	-2,407,536	8,253	10,793
-28,159	5,214,803	-163,904	274,068	9,753,261
-90,154,682	793,165	60,245,817	31,679	148,507
763,240	83,620,941	74,396	7,052,841	7.3,062,918
17,563	19,576	5,720,681	54,189,302	2,814,730
2,058,374	6,048,239	819,570	924,516	2,684
-386,019	852,497	-6,438	8,760,193	490,172
-9,785	41,056	-31,057,294	17,245	36,405
10,964,827	9,536,718	-98,763	536,980	41,205,396
8,435,719	27,418,305	4,502,698	3,875,421	6,897,013
70,296	907,523	381,052	90,641,835	541,628
¥-29,512,699	¥176,296,461	¥126,671,061	¥183,440,913	¥222,051,027

GROUP B

(80% accuracy, 10 minutes. Calculate Problems 1-10 to the nearest thousandth; 11-20 to the nearest yen.)

(1)	60,937x2,154=131,258,298
(2)	42,618x7,309=311,494,962
(3)	804,752x962=774,171,424
(4)	18,509x0.5817=10,766.6853
(5)	93.125x6.048=563.22
(6)	76,084x8,539=649,681,276
(7)	0.20436x470.6=96.1718

(8)	8,741x39,281=343,355,221
(9)	31,597x1.475=46,605.575
(10)	0.52963x0.0623=0.0330
(11)	¥47,512x8,063=¥383,089,256
(12)	¥90,386x0.7421=¥67,075
(13)	¥2,975x508.96=¥1,514,156
(14)	¥76,128x0.4375=¥33,306
(15)	¥38,604x2,869=¥110,754,876
(16)	¥65,093x0.0942=¥6,132
(17)	¥17,249x3,517=¥60,664,733
(18)	¥504,831x658=¥332,178,798
(19)	¥49,625x1.024=¥50.816
(20)	¥81,307x7,391=¥600,940,037

GROUP C

(80% accuracy, 10 minutes. Calculate Problems 1-10 to the nearest thousandth; 11-20 to the nearest yen.)

(1)	8,485,074÷2,754=3,081
(2)	44,080,524÷5,436=8,109
(3)	0.2059086÷0.7068=0.2913
(4)	52,622,371÷61,403=857
(5)	0.6793975÷9.371=0.0725
(6)	2,984,785÷482=6,192.5
(7)	1,709.6483÷3,592=0.4760
(8)	5.74902÷0.0615=93.48
(9)	12,426,698÷1,987=6,254
(10)	610.42124÷82.09=7.436

(11)	¥11,393,544÷3,918=¥2,908
(12)	¥641÷0.0784=¥8,176
(13)	¥12,945,709÷287=¥45,107
(14)	¥2,187÷0.4306=¥5,079
(15)	¥52,899÷5.496=¥9,625
(16)	¥6,593,836÷1,637=¥4,028
(17)	¥1,131÷0.8l25=¥l,392
(18)	¥59,415,129÷92,403=¥643
(19)	¥360,804÷70.25=¥5,136
(20)	¥51,693,213÷6,591=¥7,843

GROUP D

MENTAL CALCULATION

(70% accuracy, 2 minutes)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
¥58	¥130	¥61	¥987	¥83	¥704	¥32	¥501	¥23	¥436
716	97	389	19	206	16	284	48	741	28
65	681	24	870	45	860	91	17	308	510
901	458	85	21	731	29	105	930	49	67
45	76	150	36	52	487	53	376	85	940
234	802	93	603	417	72	314	29	132	79
82	37	704	546	180	918	790	862	64	12
320	26	417	89	94	63	85	75	250	386
79	940	52	405	39	590	67	409	96	807
143	59	236	72	625	35	426	68	571	95
¥2,643	¥3,306	¥2,211	¥3,648	¥2,472	¥3,774	¥2,247	¥3,315	¥2,319	¥3,360

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GROUP A

(Entire group of ten sets with 80% accuracy in ten minutes)

(1)	(2)	(3)	(4)	
¥52,089,461	¥432,186,075	¥9,135,842	¥4,170,326,958	¥247,013,5
7,153,820	8,145,903,267	304,827,196	9,852,036	5,928,3
3,045,861,297	9,247,183	96,705	12,493,870	36,740,2
179,625,408	-62,098,541	2,036,754,981	-736,981,425	27,1
43,162	-762,309	1,408,273	-572,908	1,029,846,7
81,904,753	-5,280,931,764	83,927,654	1,630,857	3,507,4
9,286,041	3,679,452	702,149,836	6,058,249,731	574,268,1
7,306,154,289	24,805,691	5,368,071	93,175,648	61,930,5
8,479,503	901,326,785	6,520,743,189	567,903,412	459,8
297,580,364	4,270,516	49,386,527	-2,647,103	8,702,164,5
165,937	71,058,934	610,945	-3,901,728,564	6,382,0
64,327,810	-358,712,640	9,017,254,368	-28,157,496	418,079,6
4,710,698,325	-6,450,839	168,032,579	845,069,273	92,543,7
5,732,649	1,097,534,628	2,571,460	24,019	1,605,9
923,517,086	83,917	50,492,813	4,615,380	7,350,291,8
¥16,692,619,905	¥4,981,140,355	¥18,952,760,439	¥7,093,253,688	¥18,530,789,8

(6)	(7)	(8)	(9)	(
¥2,709,658,341	¥7,421,630	¥921,376,048	¥6,194,873	¥314,726,9
5,182,069	65,049	6,347,208,195	804,725,361	2,645,7
890,246,753	4,086,739,215	-70,153,264	16,249	59,832,0
-731,482	615,908,743	-694,503	30,954,187	8,031,276,5

-46,029,378	27,851,409	162,980,375	1,054,678,932	694,7
-9,017,486,523	9,372,586	3,015,467,928	7,240,156	3,501,6
3,592,710	7,251,094,863	7,935,681	201,536,498	968,135,4
109,264,538	36,281,597	84,029,135	871,250	25,840,3
72,503,694	548,617,320	3,748,256	76,109,382	6,401,279,8
65,871	3,946,075	-456,102,987	5,192,783,604	7,453,1
-1,874,305	8,905,173,246	-8,561,740	8,459,736	46,087,9
-380,516,924	639,182	-2,609,845,317	420,368,915	172,943,0
5,024,387,196	4,280,951	27,439	69,547,023	5,318,9
61,740,935	92,314,708	7,890,251	9,043,182,567	69,2
8,961,247	170,523,864	93,014,862	5,073,829	7,890,421,3
¥-561,035,258	¥21,760,230,438	¥7,498,320,359	¥16,921,742,562	¥23,930,227,0

GROUP B

(80% accuracy, 10 minutes. Calculate Problems 1-10 to the nearest thousandth; 11-20 to the nearest yen.)

(1) 432,159x68,194=29,470,650,846
(2) 751,083x95,036=71,379,923,988
(3) 628,407x23,871=15,000,703,497
(4) 0.97815x80.6253=78.86364
(5) 360,798x0.17409=62,811.32382
(6) 584,162x75,382=44,035,299,884
(7) 23.6054x40.617=958.78053
(8) 8,079,231x3,925=31,710,981,675
(9) 127,496x5.4178=690,747.8288
(10) 0.603945x0.02649=0.01600
(11) ¥576,183x42,817=¥24,670,427,511

(12) ¥862,054x0.06759=¥58,266

(13) ¥3,915,478x5,603=¥21,938,423,234

(14) ¥740,625x0.84512=¥625,917

(15) ¥184,397x739.24=¥136,313,638

(16) ¥902,871x0.28196=¥254,574

(17) ¥451,063x92,031=¥41,511,778,953

(18) ¥273,509x54,768=¥14,979,540,912

(19) ¥68,192x360.875=¥24,608,788

(20) ¥390,246x13,904=¥5,425,980,384

GROUP C

(80% accuracy, 10 minutes. Calculate Problems 1-10 to the nearest thousandth; 11-20 to the nearest yen.)

(1) 9,373,655,025÷48,237=194,325

(2) 59,530,739,332÷70,652=842,591

(3) 0.1271344371÷2.14863=0.05917

(4) 52,274,789,792÷56,104=931,748

(5) 40.87907525÷0.13807=296.075

(6) 330,085.21078÷69,018=4.78260

(7) 7,534,346,391÷35,769=210,639

(8) 0.4164695782÷0.07391=5.63482

(9) 80,233,503,322÷9,254=8,670,143

(10) 624.88295605÷84.925=7.35806

(11) ¥3,931,366,928÷15,896=¥247,318

(12) ¥635,615÷0.98013=¥648,501

(13) ¥44,731,890÷63.125=¥708,624

(14) ¥7,643,415,038÷37,142=¥205,789

(15) ¥254,071,068÷296.71=¥856,294
(16) ¥46,113÷0.06459=¥713,934
(17) ¥52,348,910,242÷58,067=¥901,526
(18) ¥37,108,174,071÷4,057=¥9,146,703
(19) ¥174,669÷0.83424=¥209,375
(20) ¥39,176,554,843÷729,803=¥53,681

GROUP D

Mental Calculation

(80% accuracy, 2 minutes)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
¥321	¥5,680	¥759	¥6,014	¥495	¥8,720	¥147	¥3,098	¥254	¥9
2,038	173	9,370	397	8,150	479	6,250	917	8,190	
716	3,527	164	4,680	-243	6,093	863	1,340	-436	2
1,542	8,601	-527	251	-3,961	158	-235	654	-5,021	
653	912	2,843	7,406	518	7,826	4,012	2,801	617	
9,704	485	682	5,937	2,074	613	9,581	579	4,203	7
285	7,034	4,105	873	1,632	2,760	729	465	3,568	
5,410	298	-3,524	695	-925	594	-1,936	7,602	-937	4
869	6,940	-913	1,208	784	3,408	-348	8,937	784	6
4,937	796	8,061	982	6,307	951	5,704	286	1,592	
¥26,475	¥34,446	¥21,020	¥28,443	¥14,831	¥31,602	¥24,767	¥26,679	¥12,814	¥32

Mathematics 900795

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