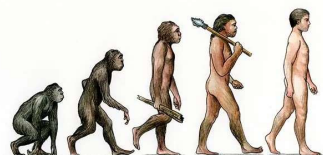
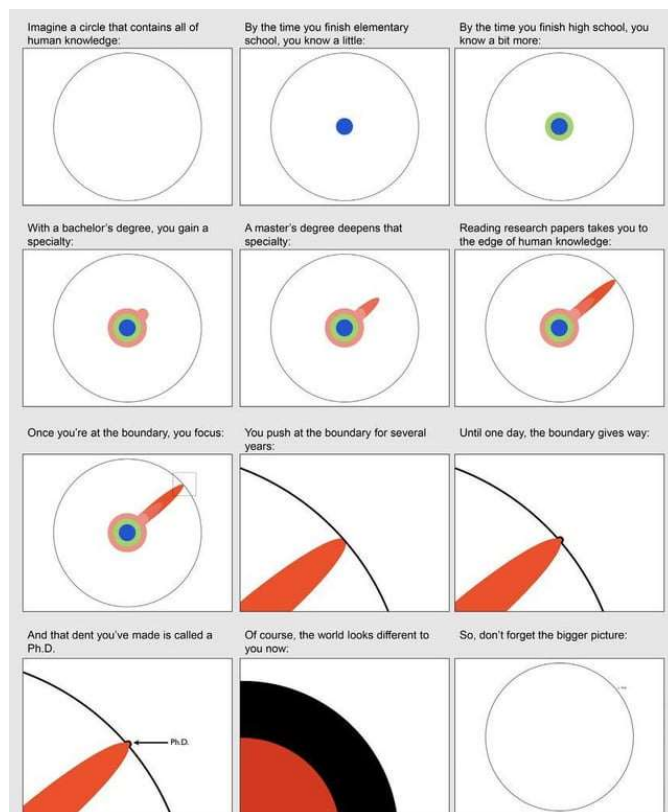


# Real Analysis in Stat 6410 (Iowa State)

Wednesday, June 26, 2024 3:06 PM



More than just a catchy phrase, "ontogeny recapitulates phylogeny" is the foundation of recapitulation theory. Recapitulation theory posits that the development of individual organisms (ontogeny) follows (recapitulates) the same phases of the evolution of larger ancestral groups of related organisms (phylogeny). Feb 10, 2017

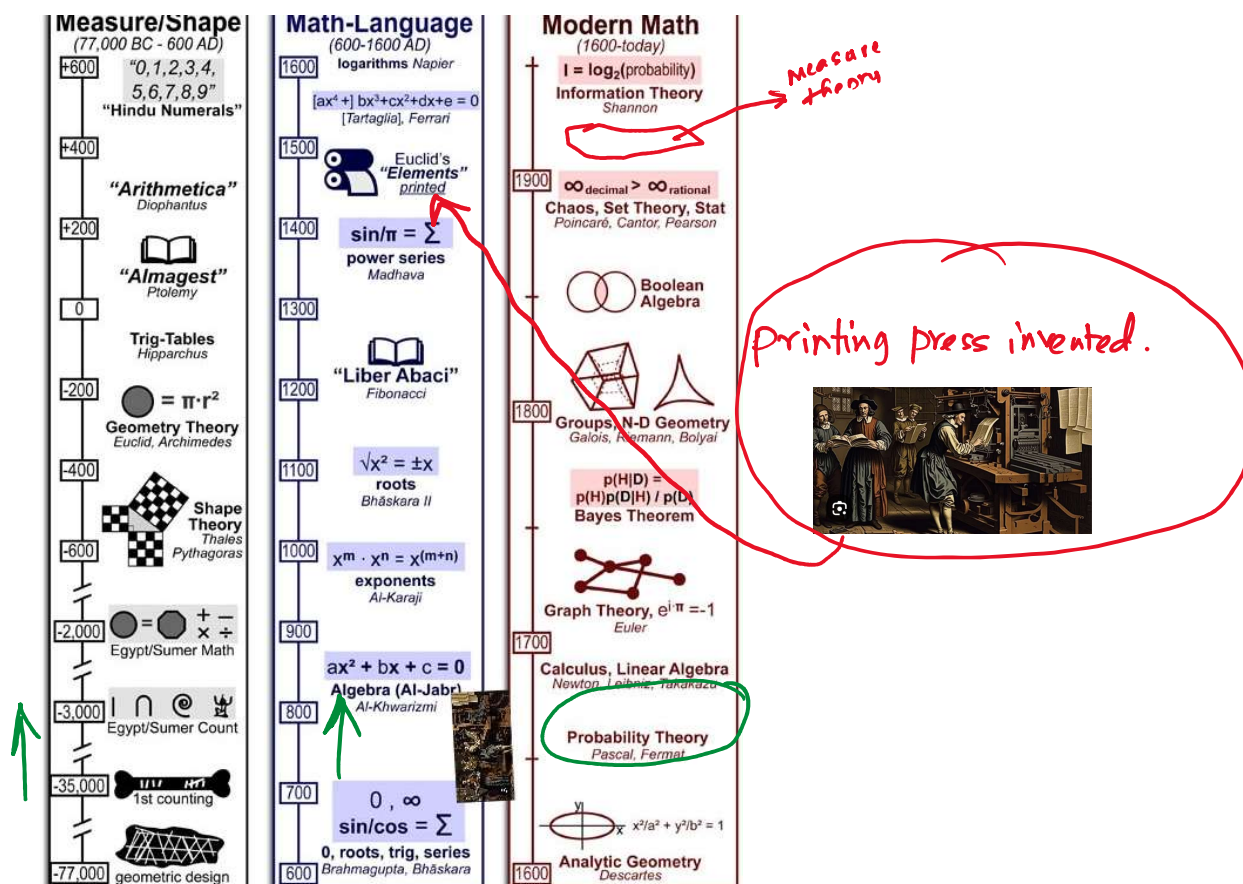
NYBG New York Botanical Garden  
<https://www.nybg.org/blogs/science-talk/2017/02>

A Catchy Phrase, But is It True? - New York Botanical Garden

About featured snippets Feedback

<b>Measure/Shape</b> (77,000 BC - 600 AD) +600 "0,1,2,3,4,5,6,7,8,9" "Hindu Numerals"	<b>Math-Language</b> (600-1600 AD) logarithms Napier $[ax^4 + ] bx^3 + cx^2 + dx + e = 0$	<b>Modern Math</b> (1600-today) $I = \log_2(\text{probability})$ Information Theory Shannon
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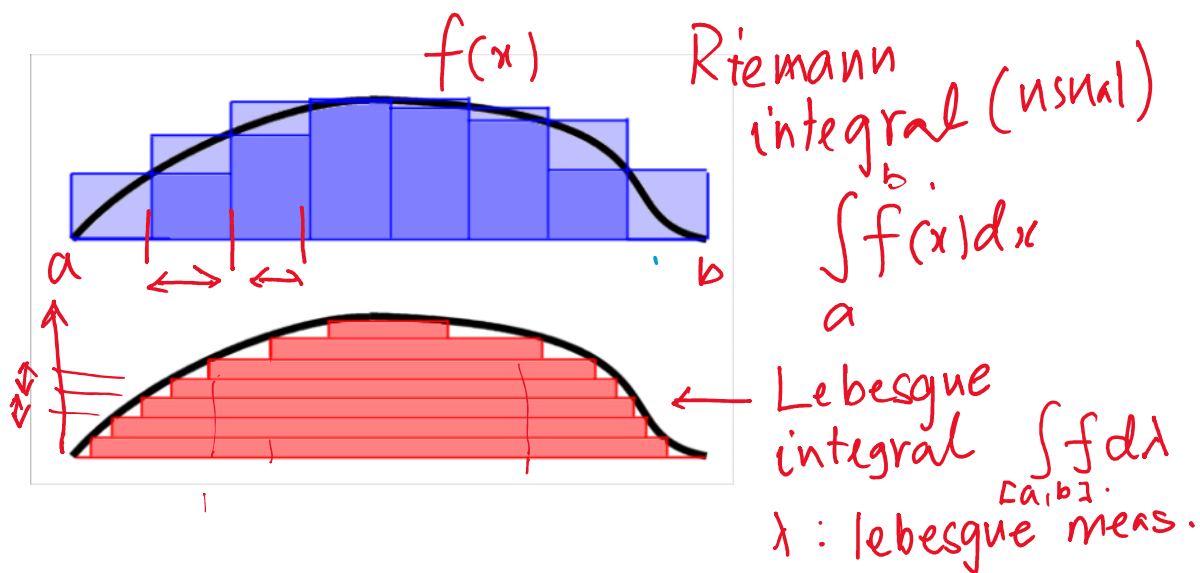
Measure theory



Stat 6410 : measure theory

- need basics of analysis/calculus.
- extends those ideas to stable math foundation for prob. theory.

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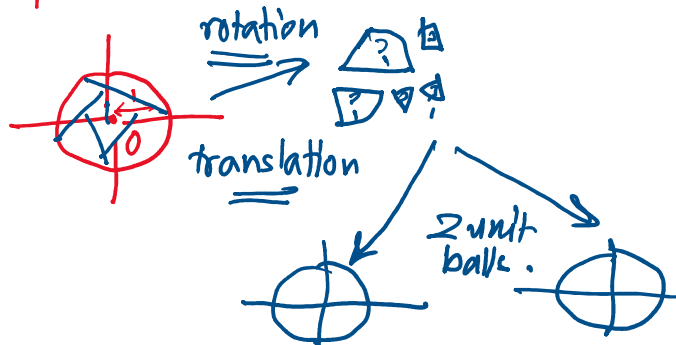
## Intro to measure theory

(everything here will be discussed in detail later)

- \* Shortcomings of Riemann integration / calculus.
- \* Connecting loose ends of prob. theory!
- \* Math paradoxes that lead to unreasonable results

\* Math paradoxes that lead to unreasonable results

eg:



Banach-Tarski paradox

(Reasons for Measure theory)

$$(\Omega, \mathcal{F}, \mathbb{P}) \quad \overset{X_i}{\rightarrow} \quad (\mathbb{R}, \mathcal{B}(\mathbb{R}))$$

**Theorem 5.5.9 (Strong Law of Large Numbers)** Let  $X_1, X_2, \dots$  be iid random variables with  $EX_i = \mu$  and  $\text{Var } X_i = \sigma^2 < \infty$ , and define  $\bar{X}_n = (1/n) \sum_{i=1}^n X_i$ . Then, for every  $\epsilon > 0$ ,

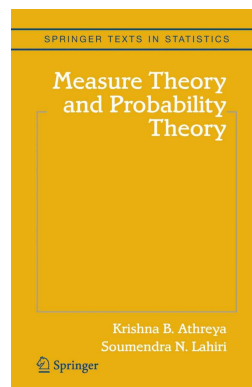
$$\mathbb{P} \left( \lim_{n \rightarrow \infty} |\bar{X}_n - \mu| < \epsilon \right) = 1; \quad \Leftrightarrow \quad \mathbb{P} \left( \lim_{n \rightarrow \infty} |\bar{X}_n - \mu| = 0 \right) = 1$$

or  $\bar{X}_n - \mu \rightarrow 0 \text{ a.e. } (\mathbb{P})$

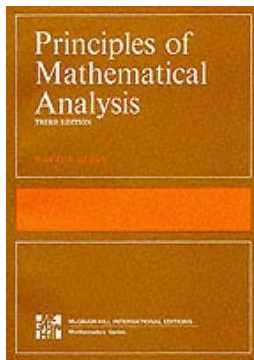
that is,  $\bar{X}_n$  converges almost surely to  $\mu$ .

Usual text for 641

<https://www.amazon.com/Measure-Theory-Probability-Springer-Statistics/dp/038732903X>



Classic reference for analysis (advanced calculus):



Rudin's textbook for real analysis.