CS4278/CS5478 Intelligent Robots: Algorithms and Systems

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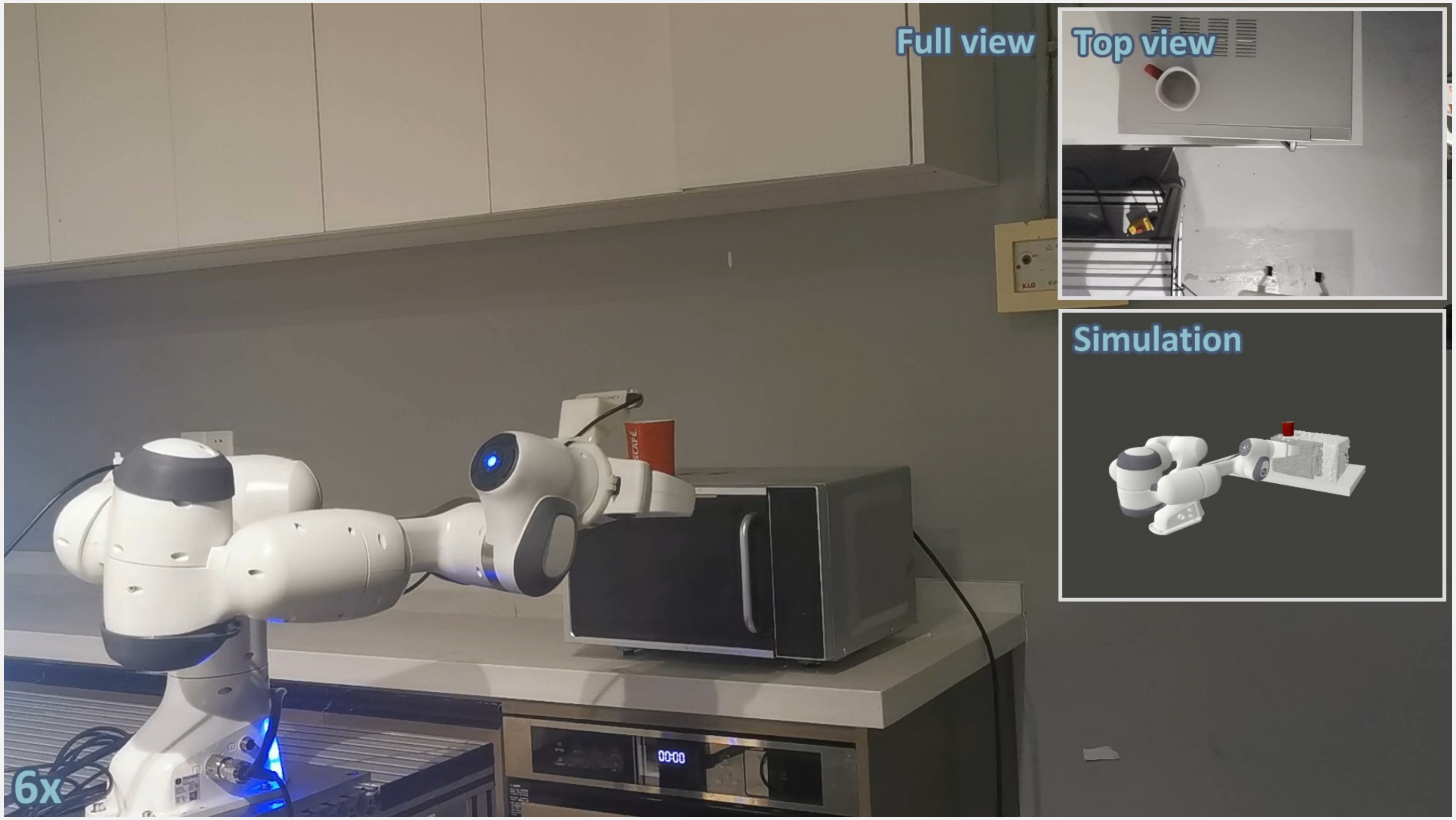
NUS

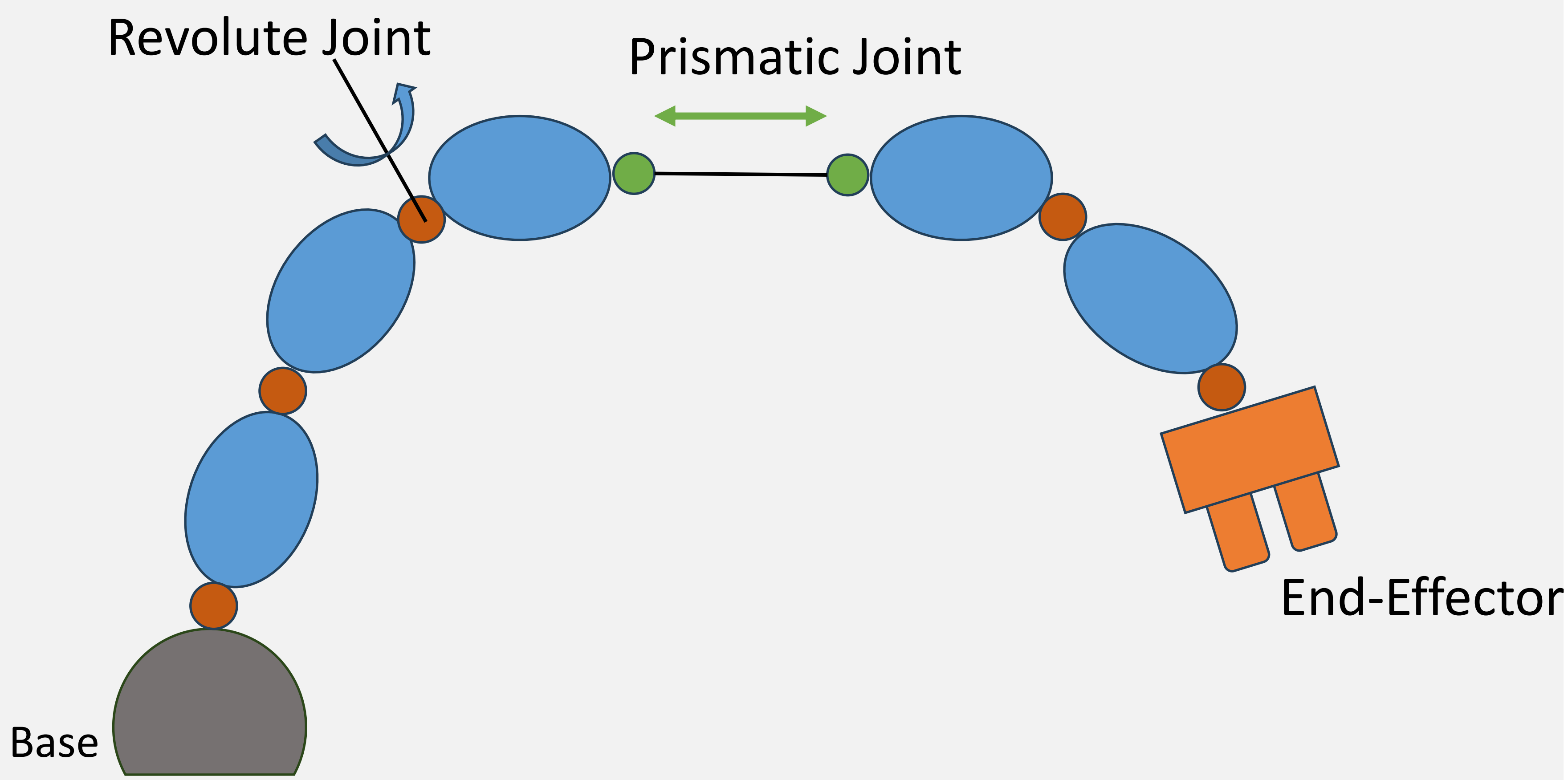
Today's Plan

- Mathematical Models
  - Spatial Descriptions
  - Robot Kinematics
    - Forward Kinematics
    - Instantaneous Kinematics

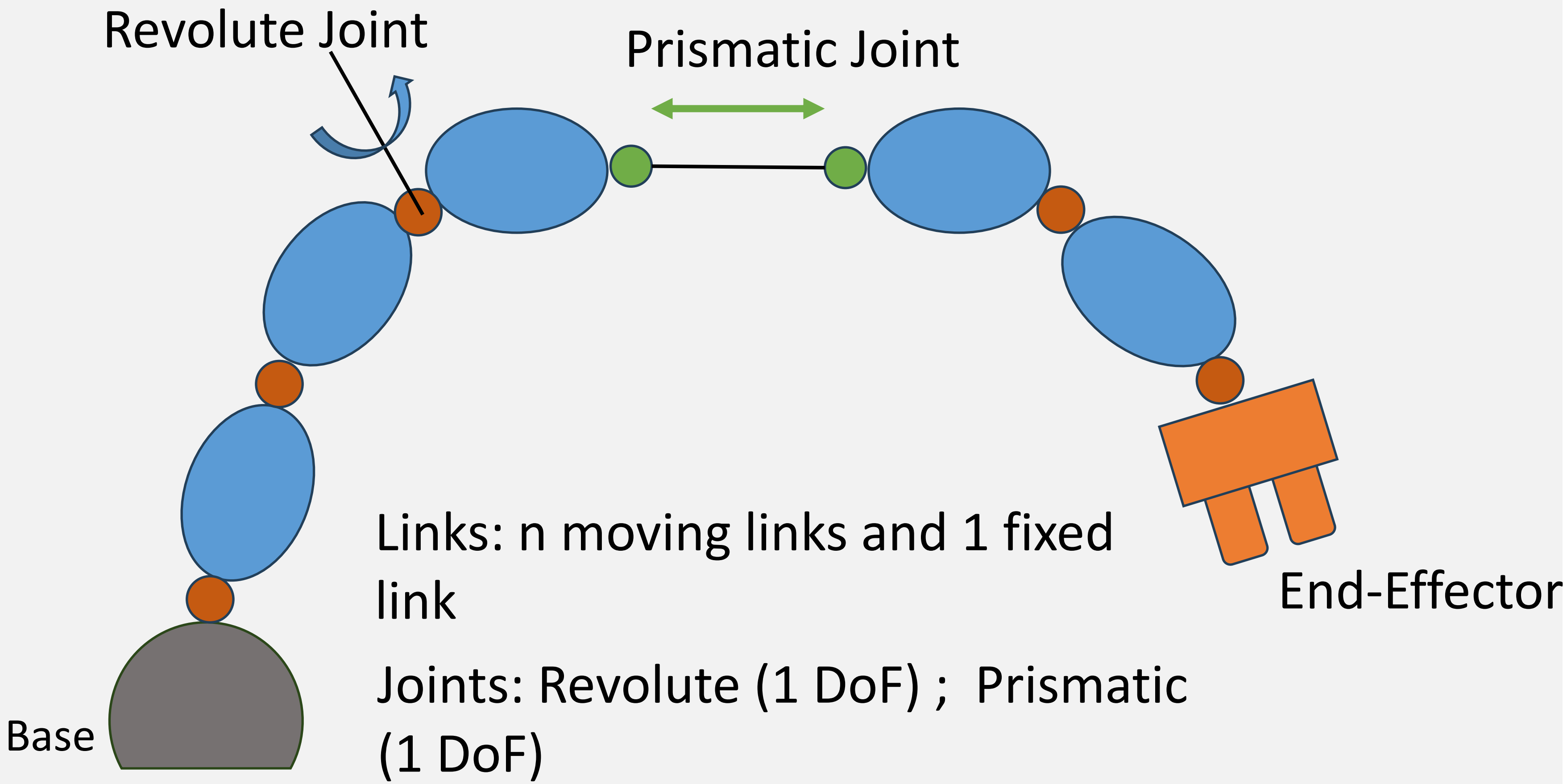
Manipulator



Manipulator



Manipulator

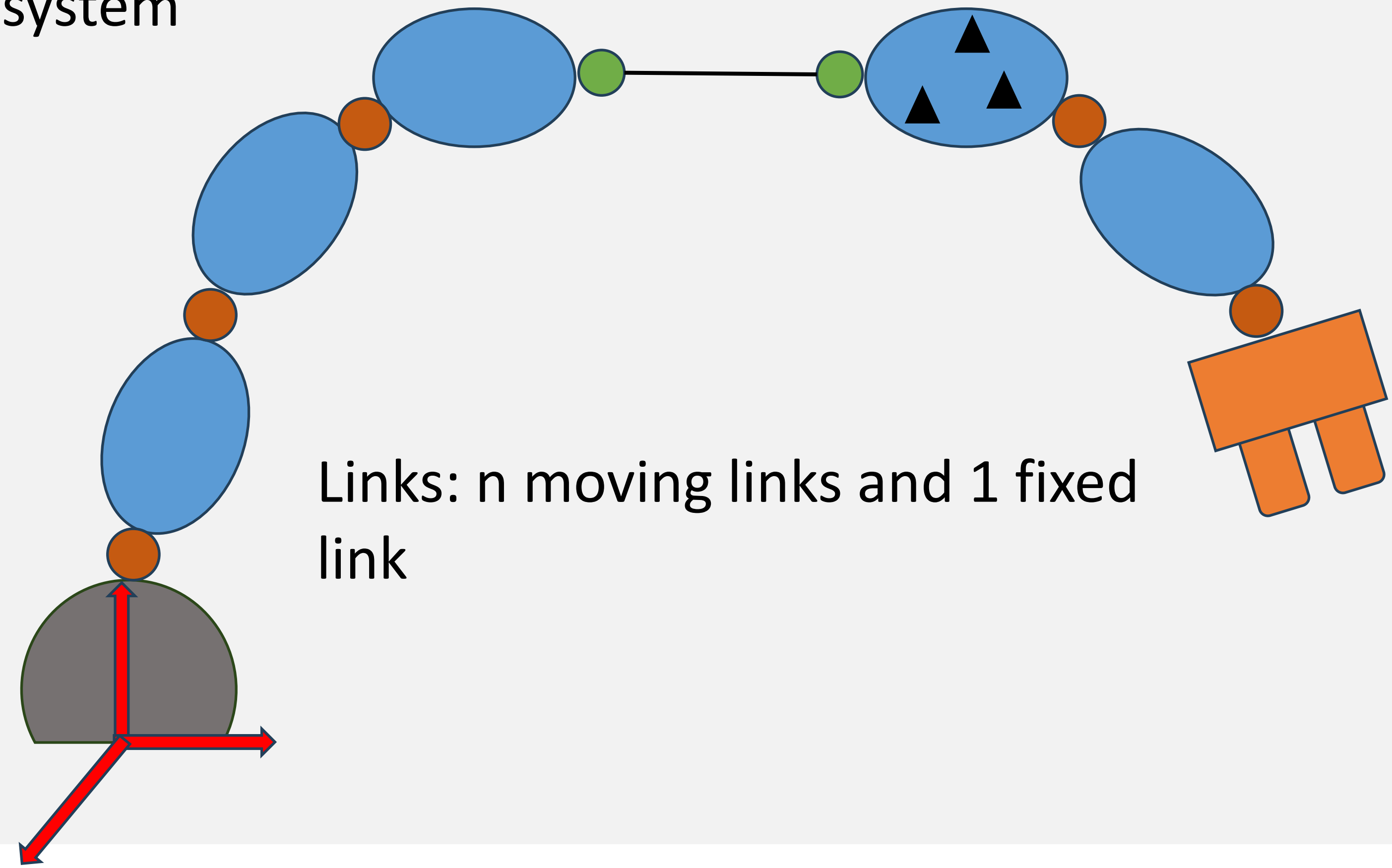


Links: n moving links and 1 fixed link

Joints: Revolute (1 DoF) ; Prismatic (1 DoF)

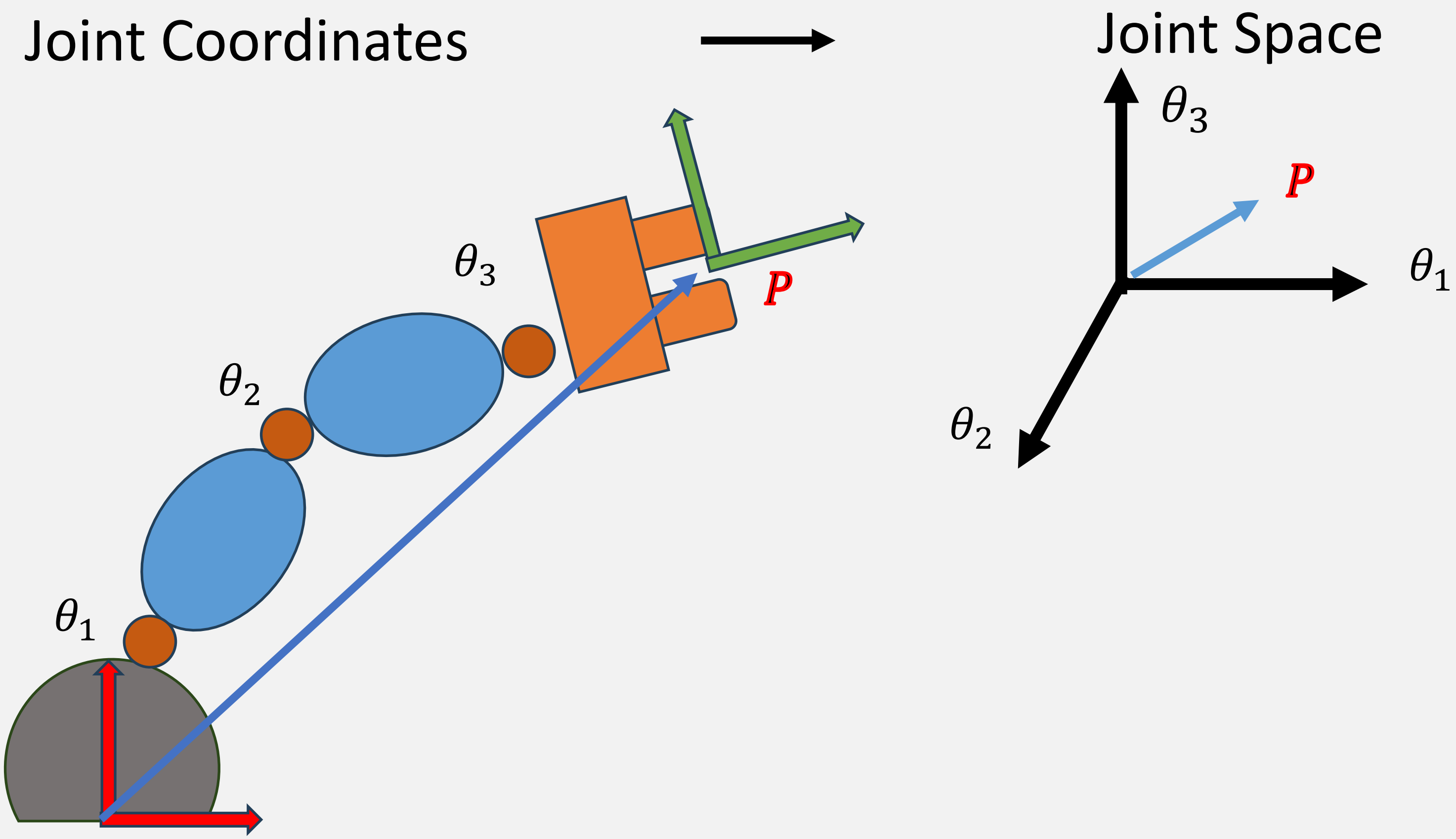
Configuration Parameters

A set of position parameters that describes the full configuration of the system



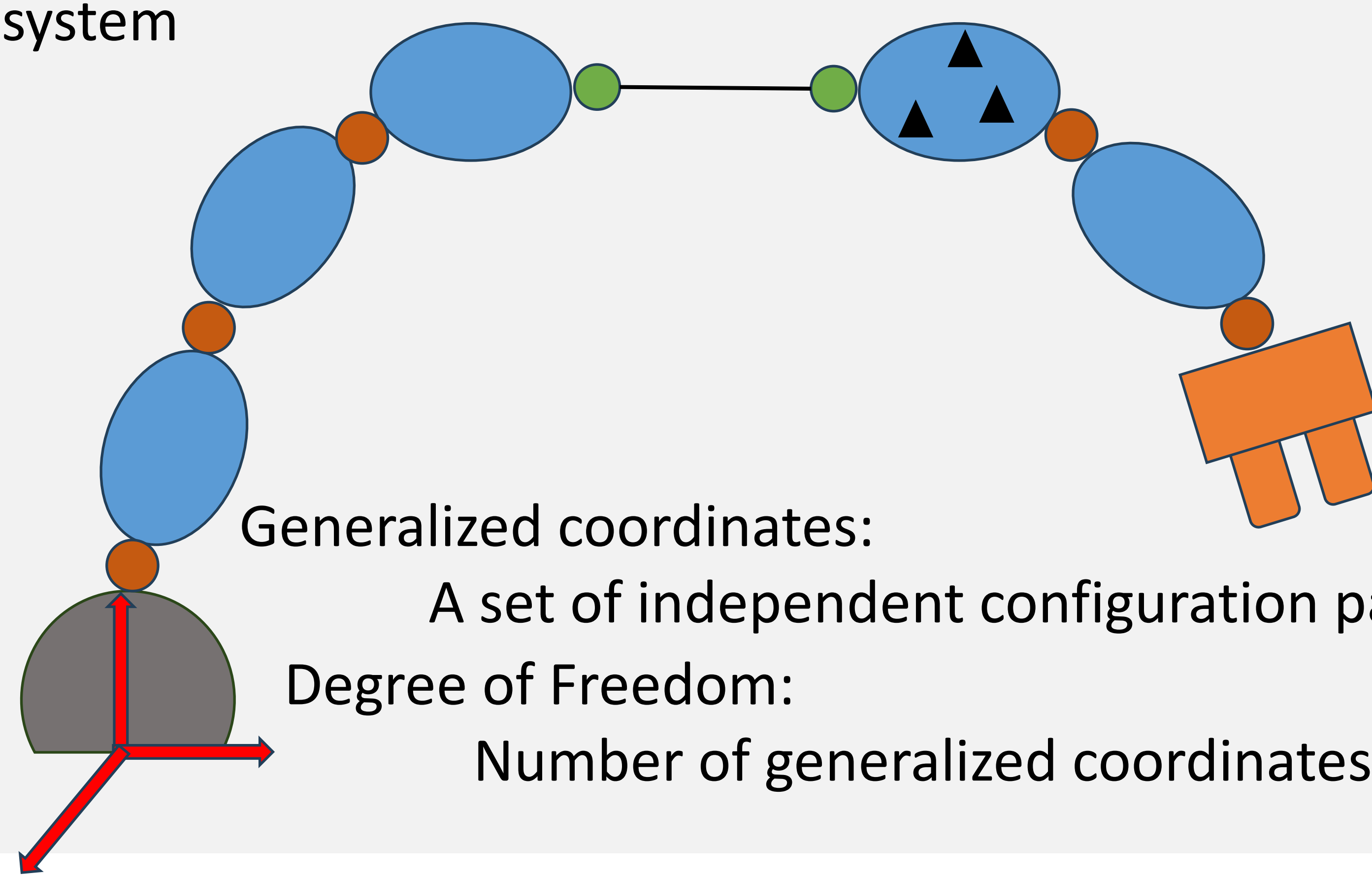
Links: n moving links and 1 fixed link

Configuration Parameters



Configuration Parameters

A set of position parameters that describes the full configuration of the system



Generalized coordinates:

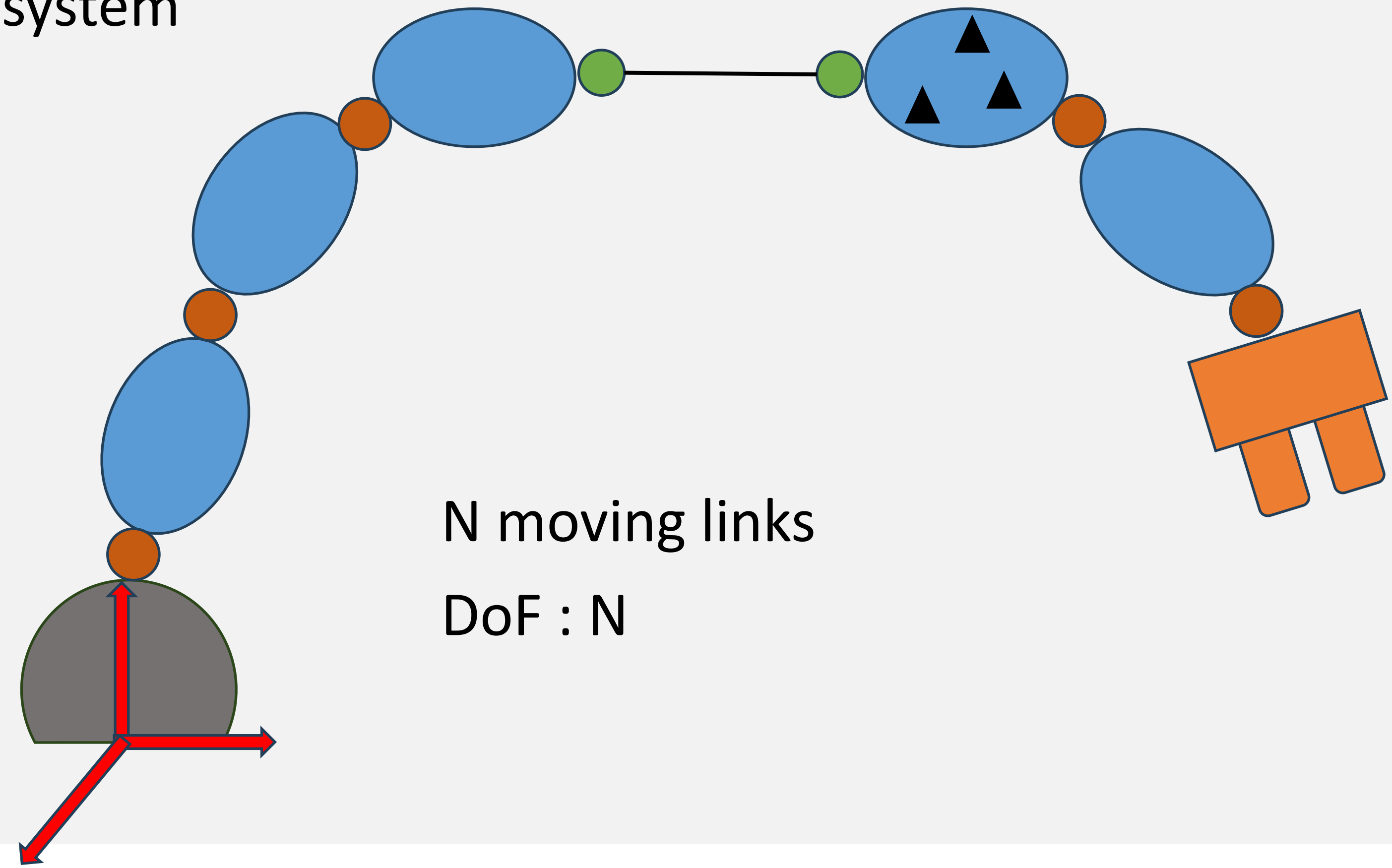
A set of independent configuration parameters

Degree of Freedom:

Number of generalized coordinates

Configuration Parameters

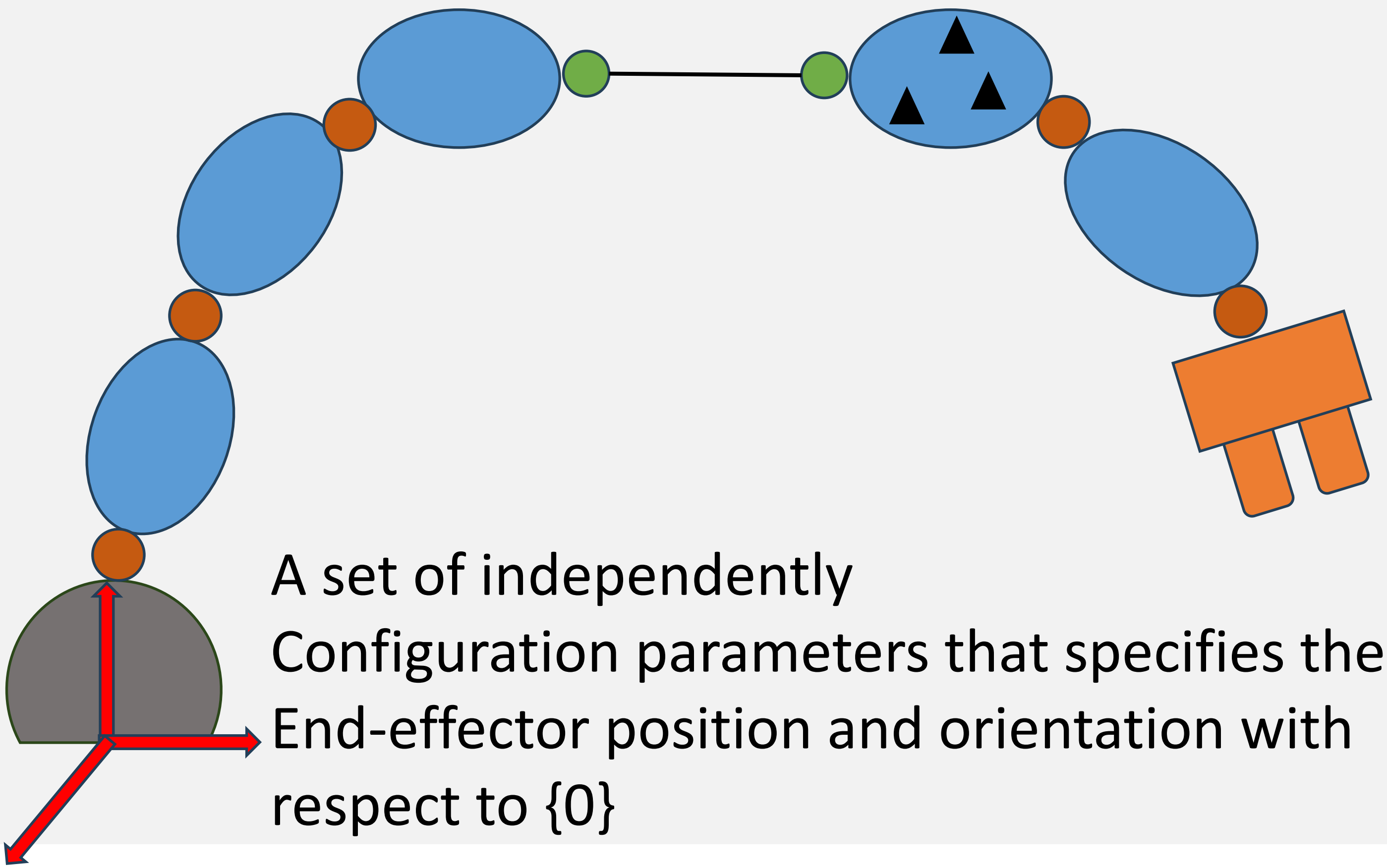
A set of position parameters that describes the full configuration of the system



N moving links

DoF : N

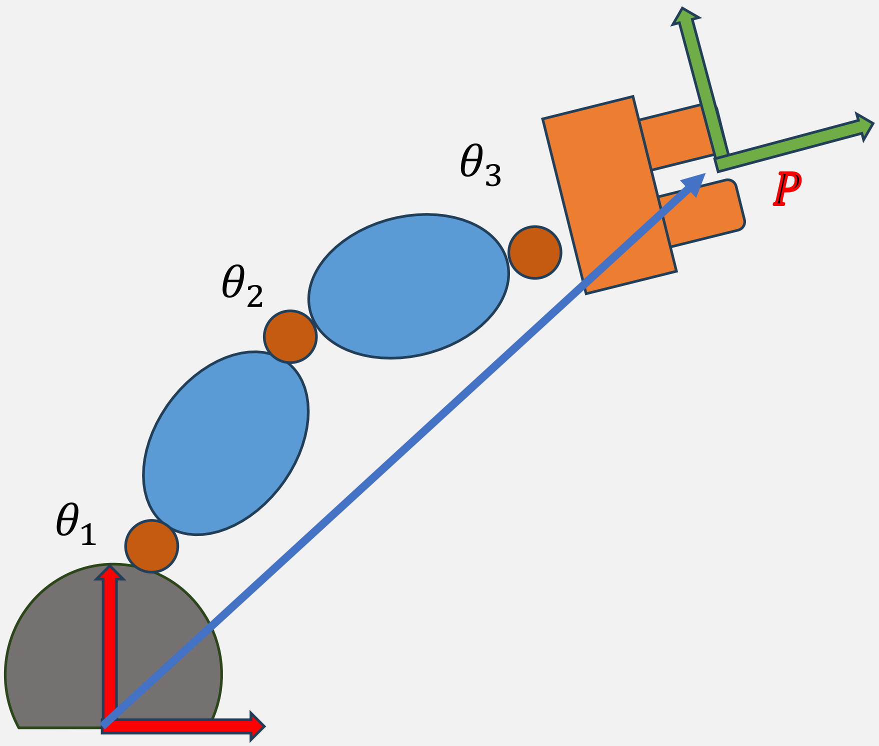
Operational Coordinates

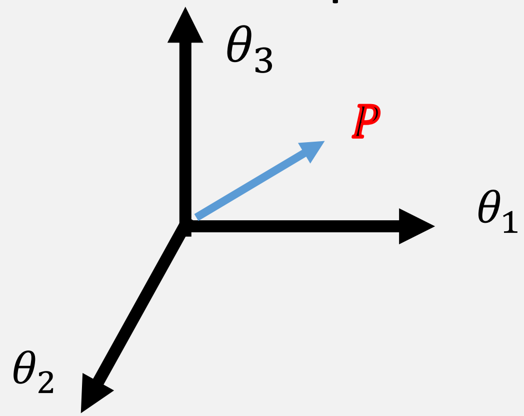


A set of independently
Configuration parameters that specifies the
End-effector position and orientation with
respect to {0}

Configuration Parameters

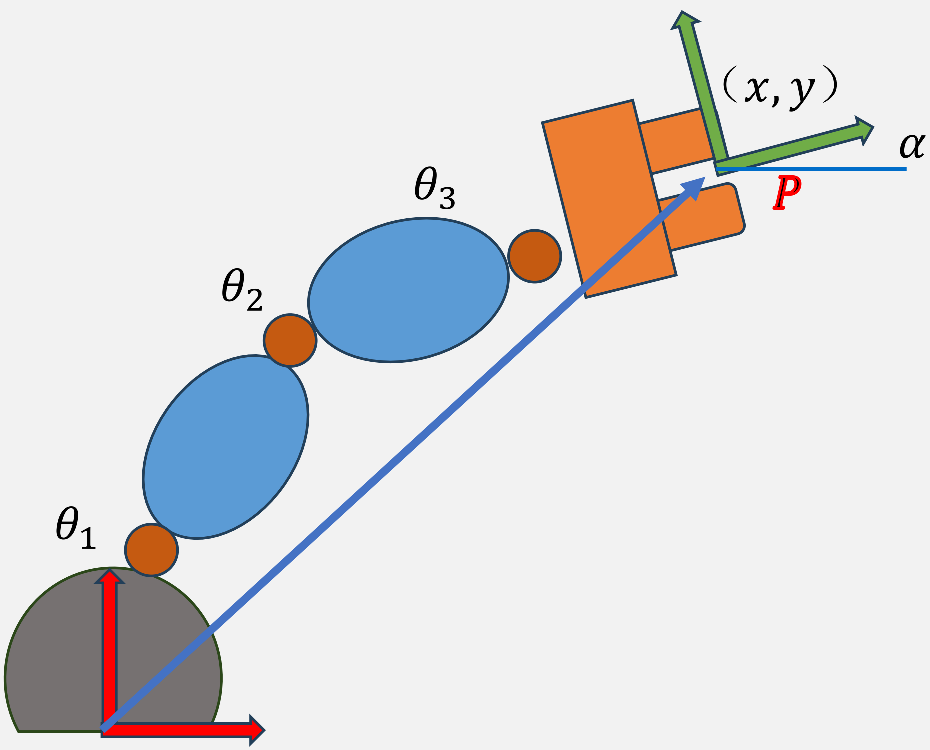
Joint Coordinates → Joint Space

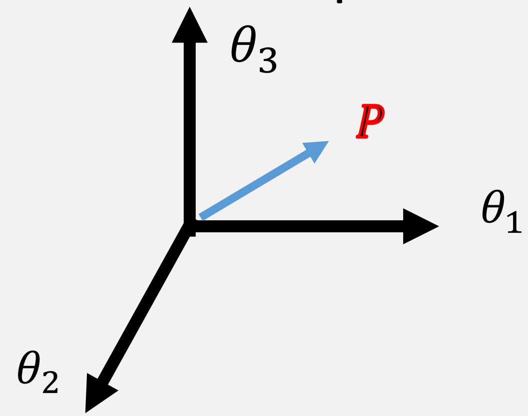


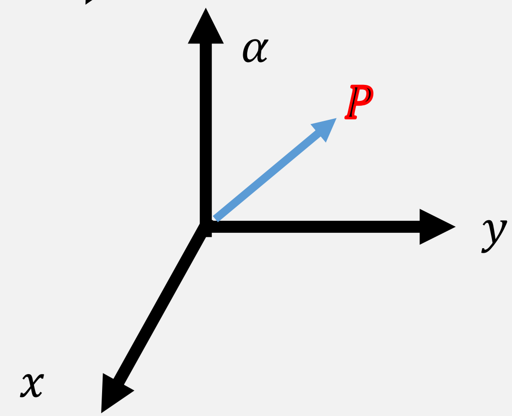


Coordinates

Joint Coordinates → Joint Space

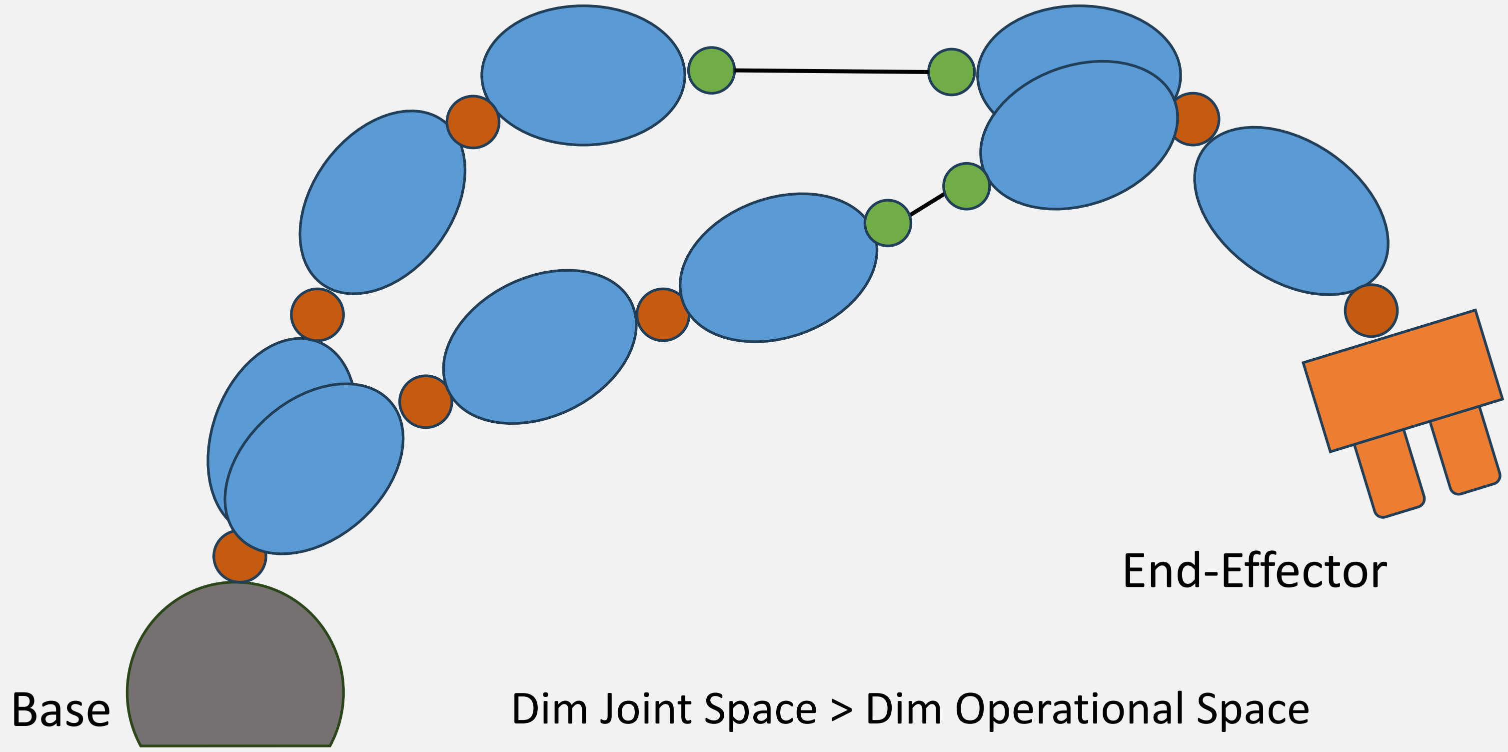






Operational Coordinates → Operational Space

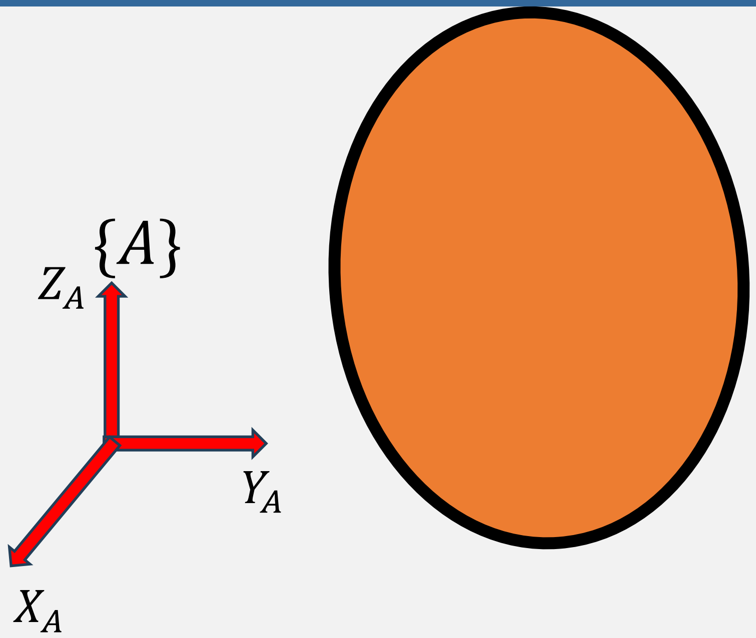
Redundancy



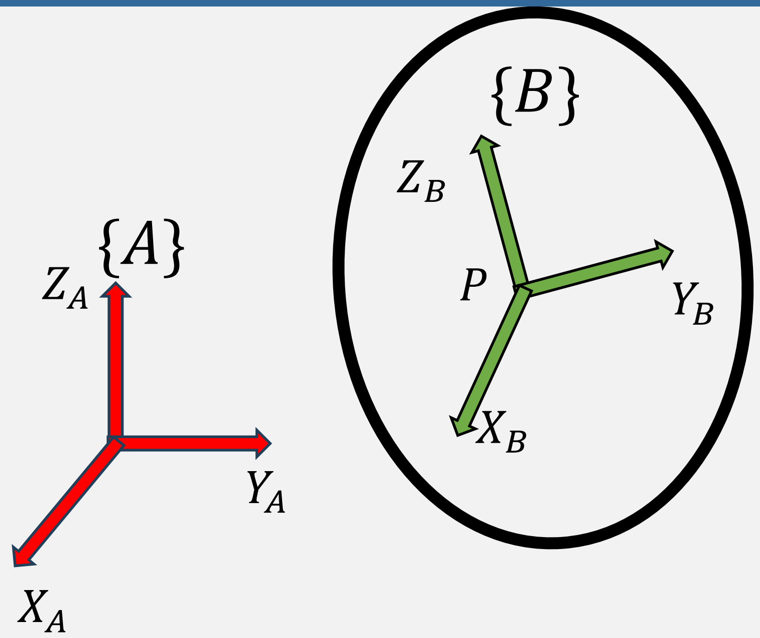
Dim Joint Space > Dim Operational Space

- Manipulator
- Joint Coordinate
- Operational Coordinate

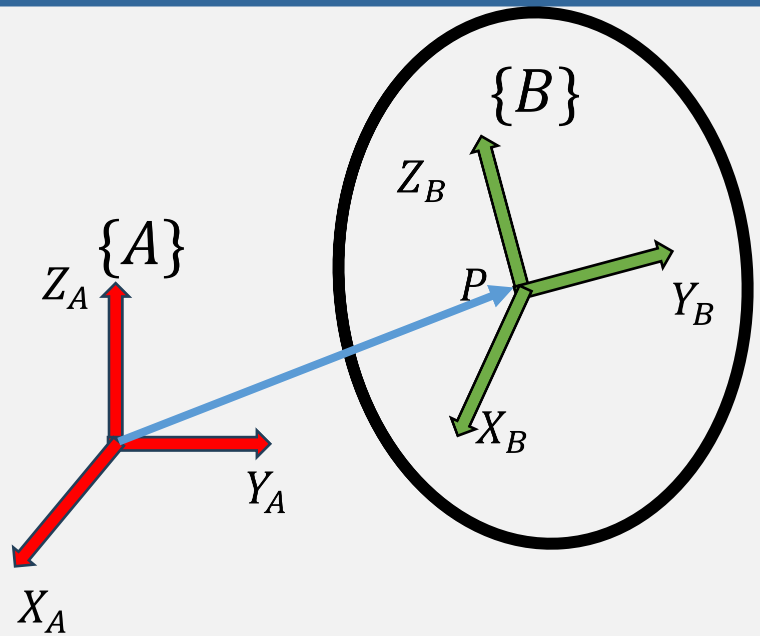
Rigid Body Configuration



Coordinate Frame



Rigid Body Configuration

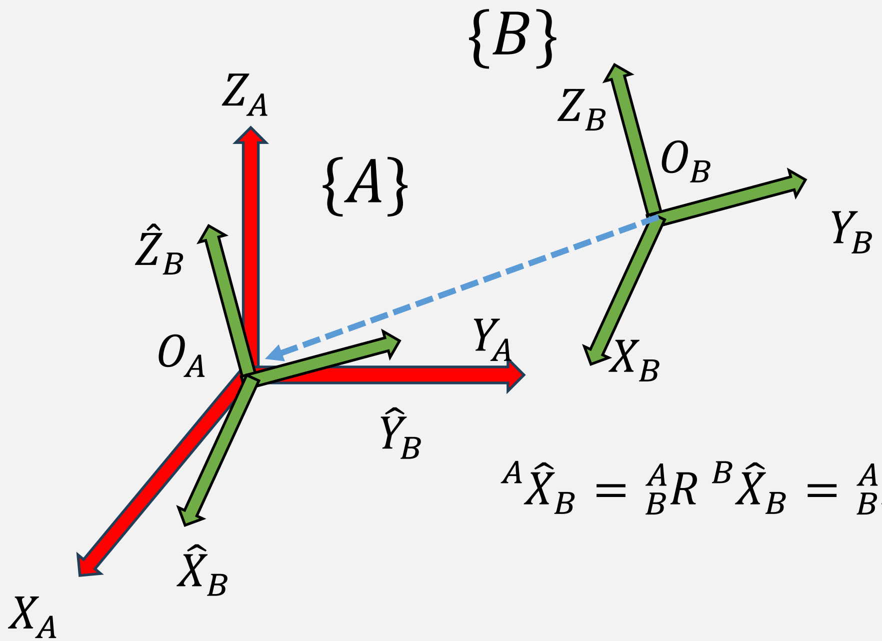


Describe the frame {B} with respect to {A}

Position: ${}^{A}P$

Orientation: $\{ {}^{A}X_B, {}^{A}Y_B, {}^{A}Z_B, \}$

Rotation Matrix



$$
{}^{A}_{B}R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}
$$

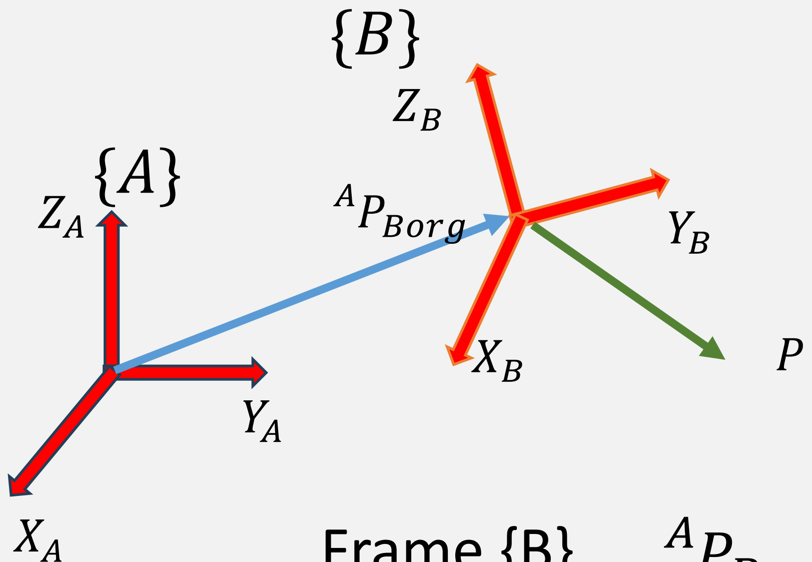
$$
{}^{A}\hat{X}_B = {}^{A}_{B}R \, {}^{B}\hat{X}_B = {}^{A}_{B}R \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
$$

$$
{}^{A}\hat{Y}_B = {}^{A}_{B}R \, {}^{B}\hat{Y}_B = {}^{A}_{B}R \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}
$$

$$
{}^{A}\hat{Z}_B = {}^{A}_{B}R \, {}^{B}\hat{Z}_B = {}^{A}_{B}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
$$

$$
{}^{A}_{B}R = \begin{bmatrix} {}^{A}\hat{X}_B & {}^{A}\hat{Y}_B & {}^{A}\hat{Z}_B \end{bmatrix}
$$

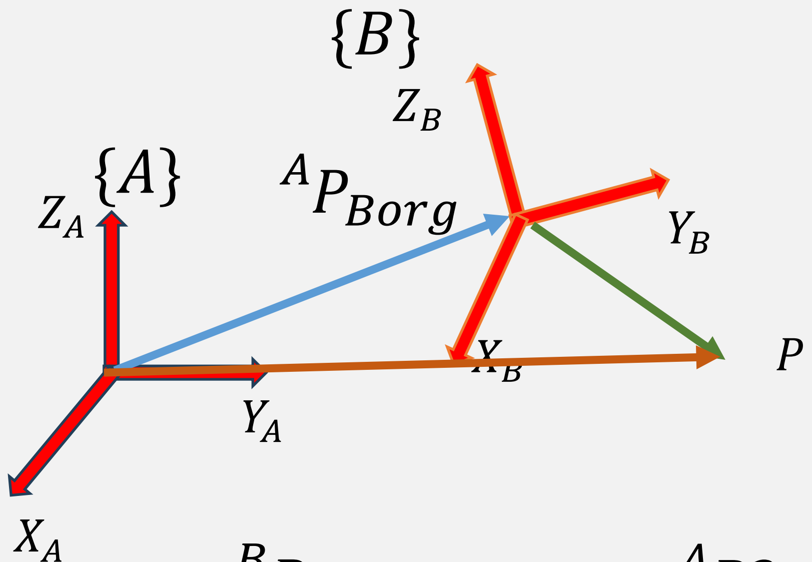
Description of a Frame



Frame {B}, $^{A}P_{Borg}$ $^{A}\hat{X}_B$ $^{A}\hat{Y}_B$ $^{A}\hat{Z}_B$

{B}={$^{A}_{B}R$, $^{A}P_{Borg}$}

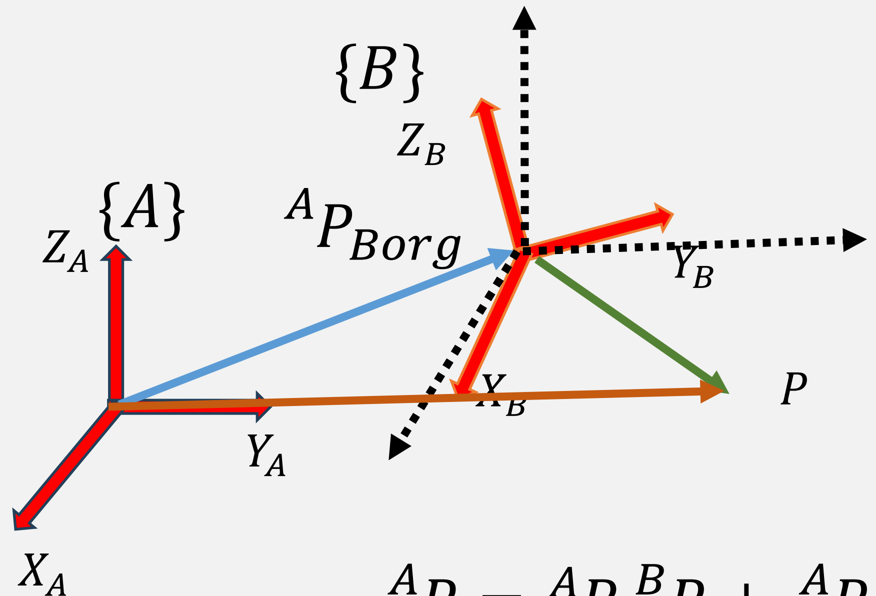
General Transform



${}^{B}P$ ${}^{A}P?$

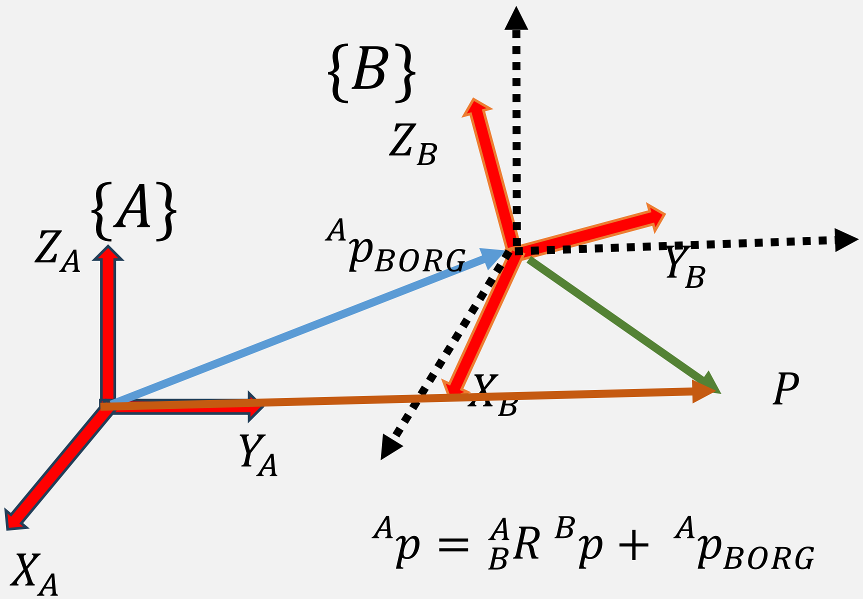
$\{B\} = \{{}^{A}_{B}R,\ {}^{A}P_{Borg}\}$

General Transform



$$ {}^{A}P = {}^{A}_{B}R\, {}^{B}P + {}^{A}P_{Borg} $$

Homogeneous Transform

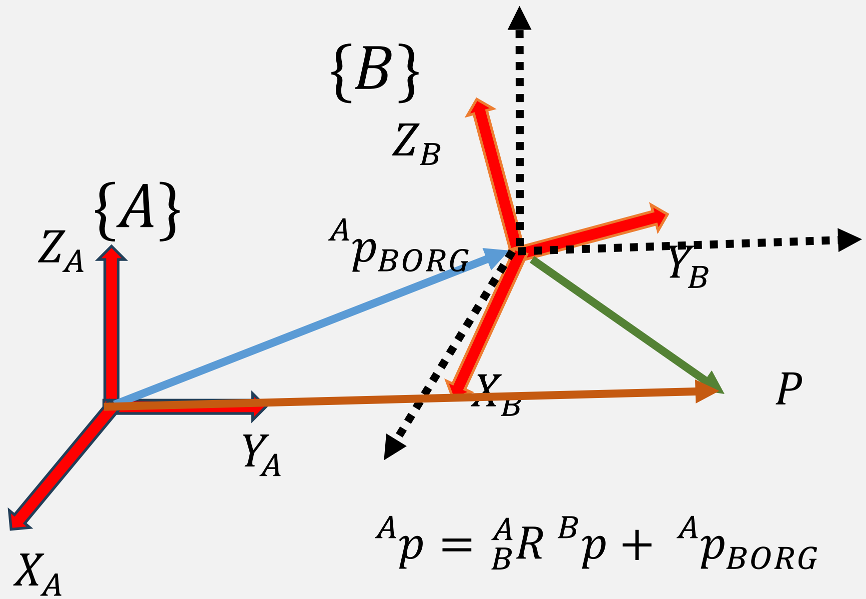


$$ {}^{A}p = {}^{A}_{B}R\ {}^{B}p + {}^{A}p_{BORG} $$

$$ \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & {}^{A}P_{BORG} \\ 0\ 0\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} $$

$$ {}^{A}\tilde{P}_{(4\times 1)} = {}^{A}_{B}T_{(4\times 4)}\ {}^{B}\tilde{P}_{(4\times 1)} $$

Homogeneous Transform



$$ {}^{A}_{B}T_{(4\times4)} = \begin{bmatrix} {}^{A}_{B}R & {}^{A}P_{BORG} \\ 0\ 0\ 0 & 1 \end{bmatrix} $$

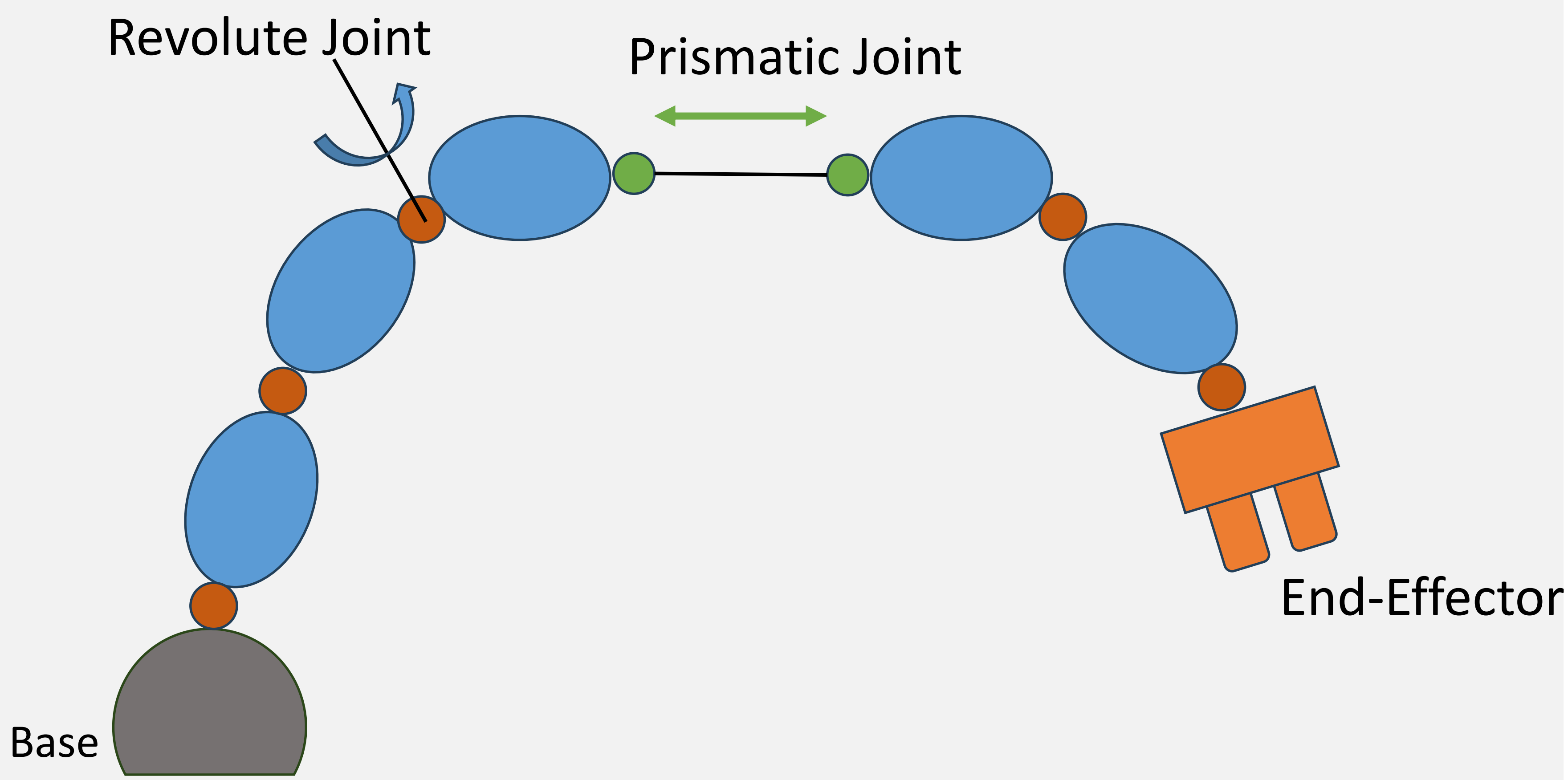
$$ {}^{A}p = {}^{A}_{B}R\ {}^{B}p + {}^{A}p_{BORG} $$

$$ \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & {}^{A}P_{BORG} \\ 0\ 0\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} $$

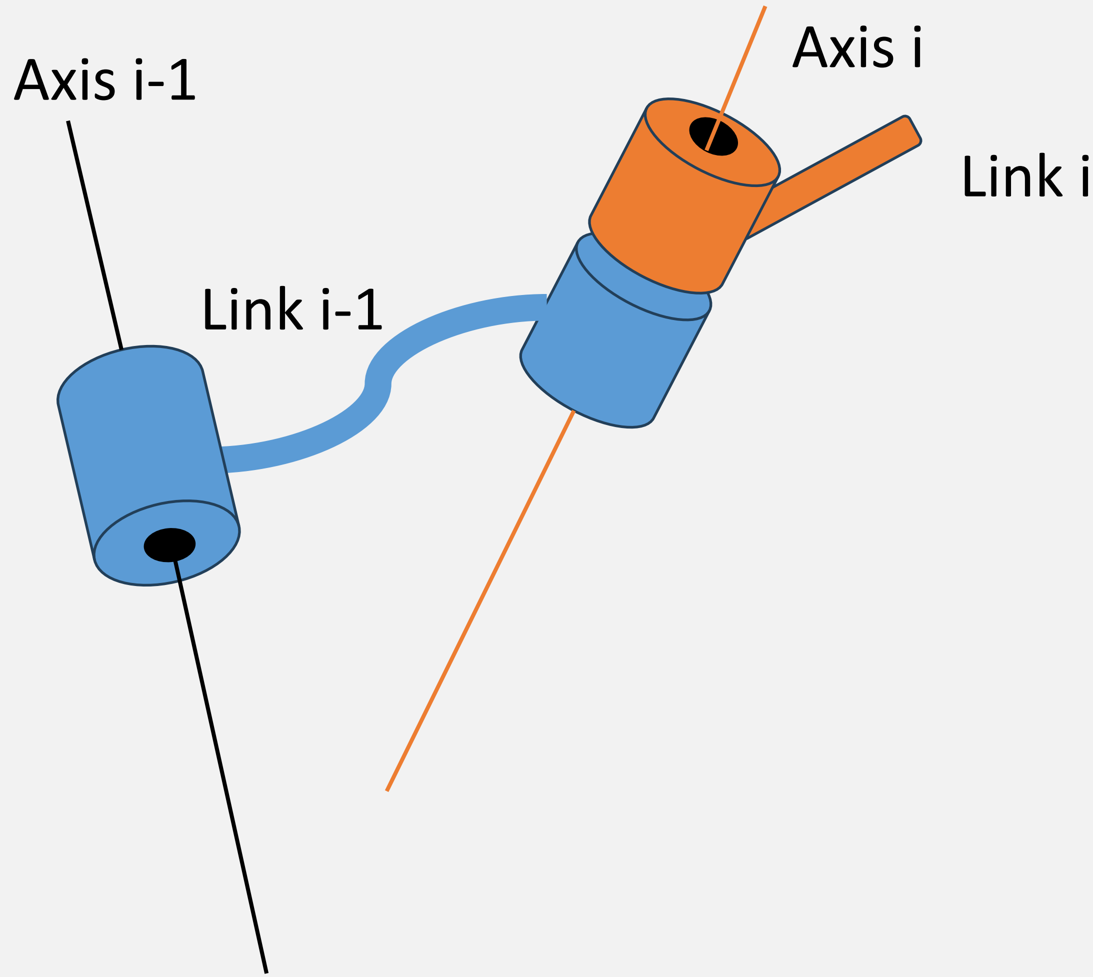
$$ {}^{A}\tilde{P}_{(4\times1)} = {}^{A}_{B}T_{(4\times4)}\ {}^{B}\tilde{P}_{(4\times1)} $$

- Frame
- Rotation Matrix
- Homogeneous Transform

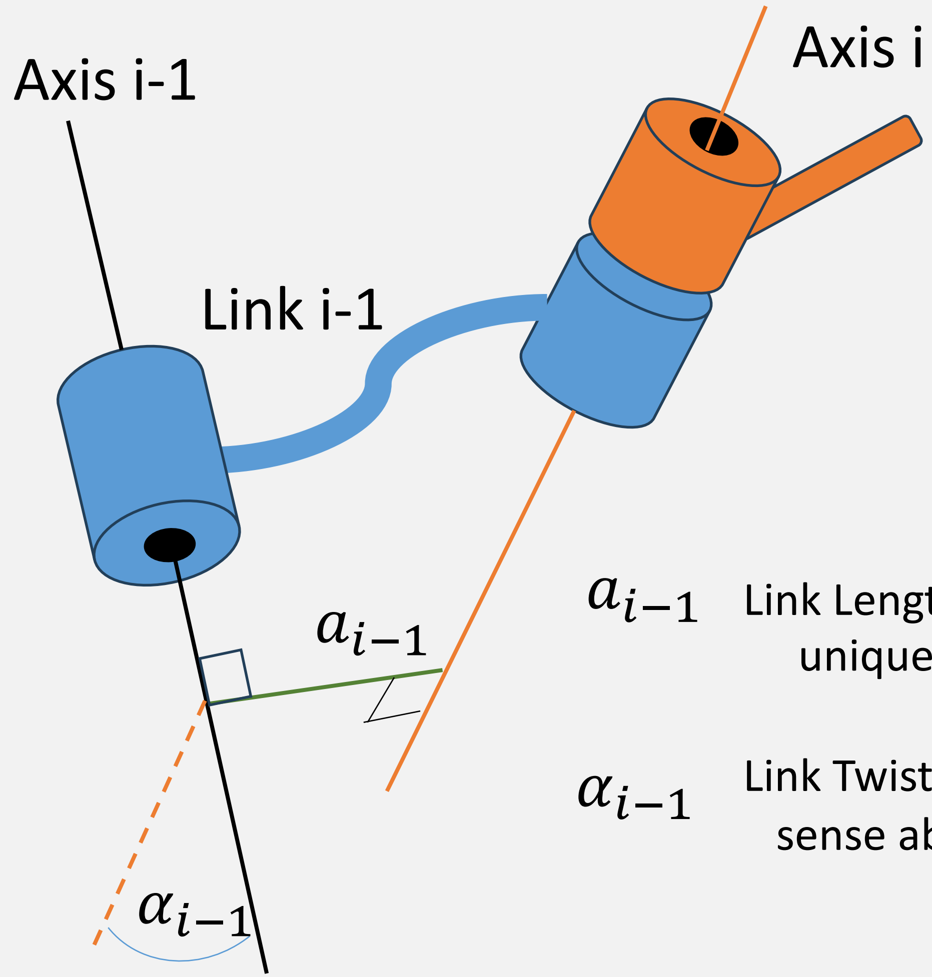
Manipulator



Link Connections



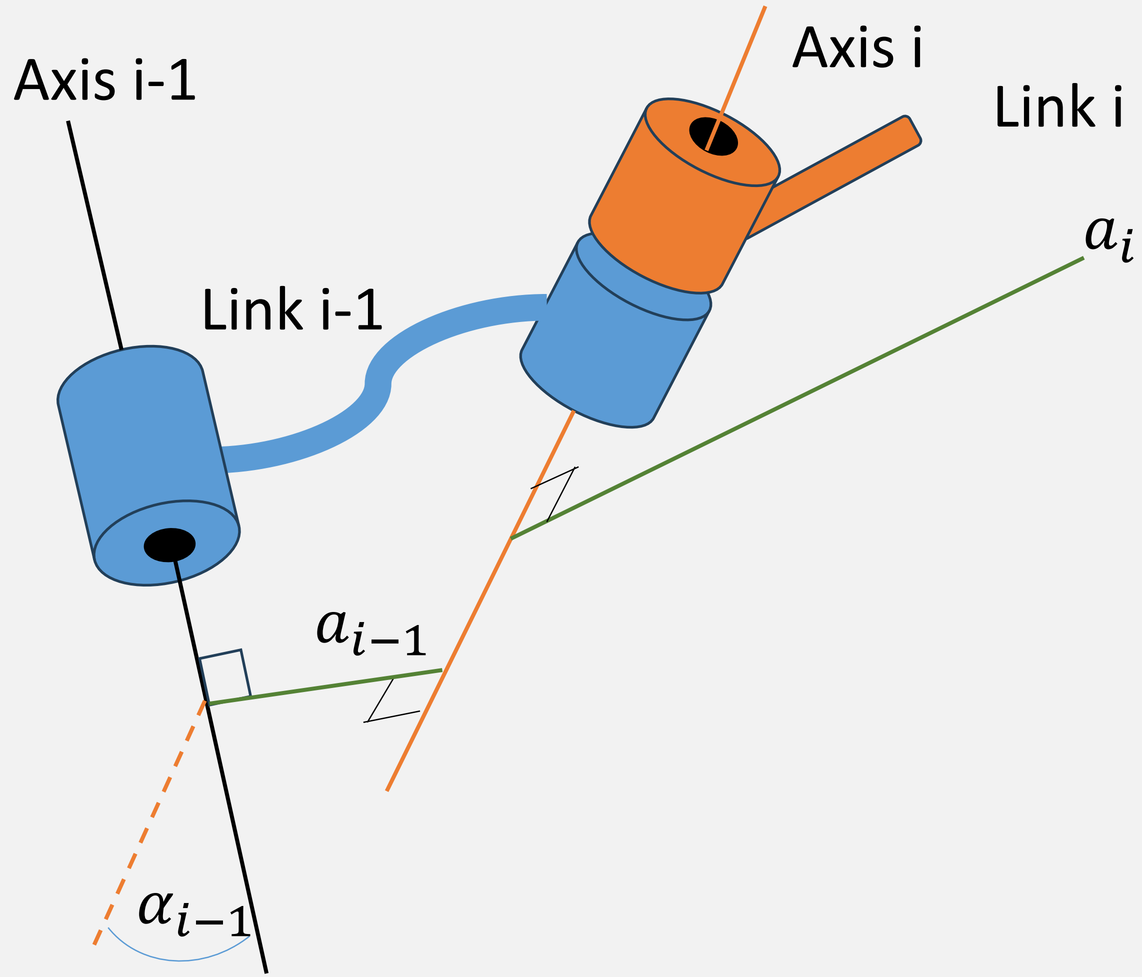
Link Connections



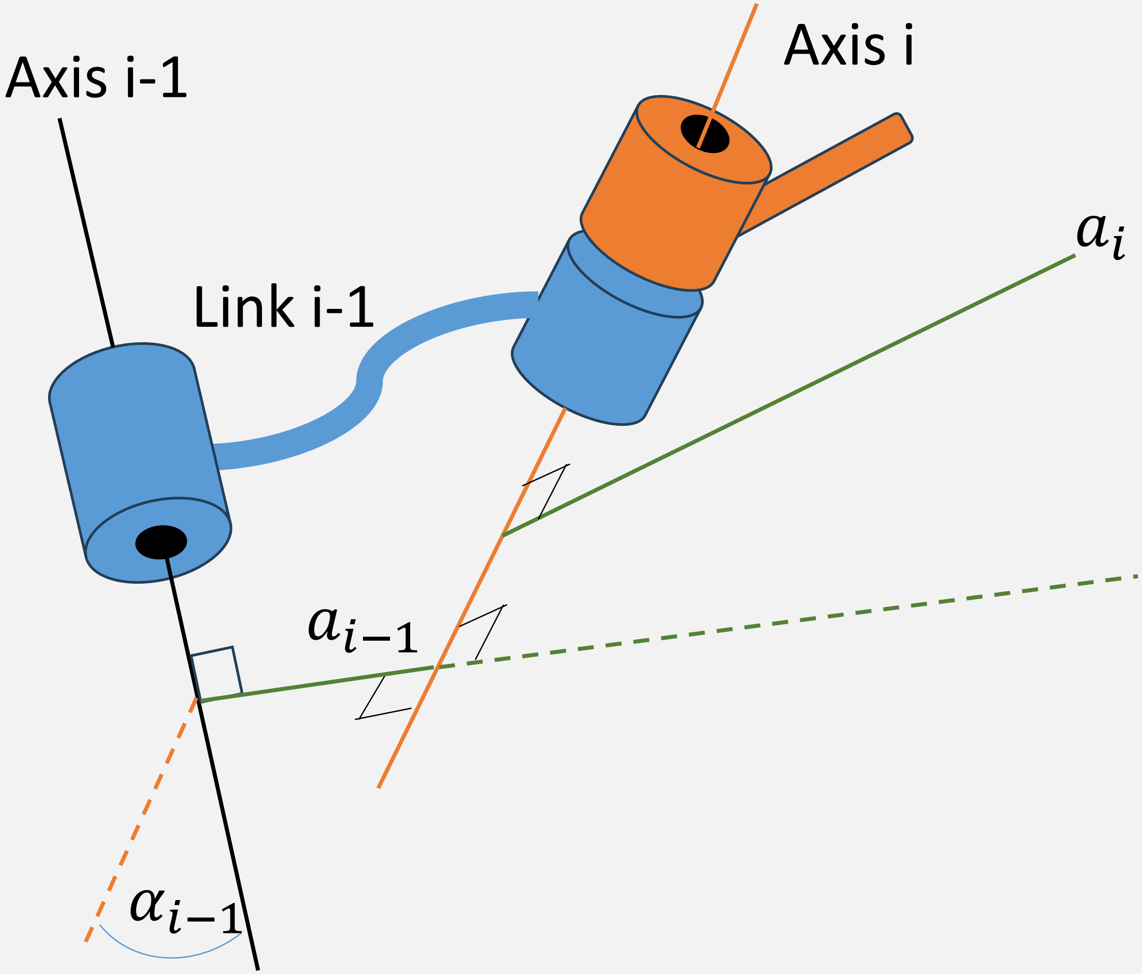
a_{i-1} Link Length: mutual perpendicular
unique except for the parallel axis

α_{i-1} Link Twist: measured in the right-hand sense about a_{i-1}

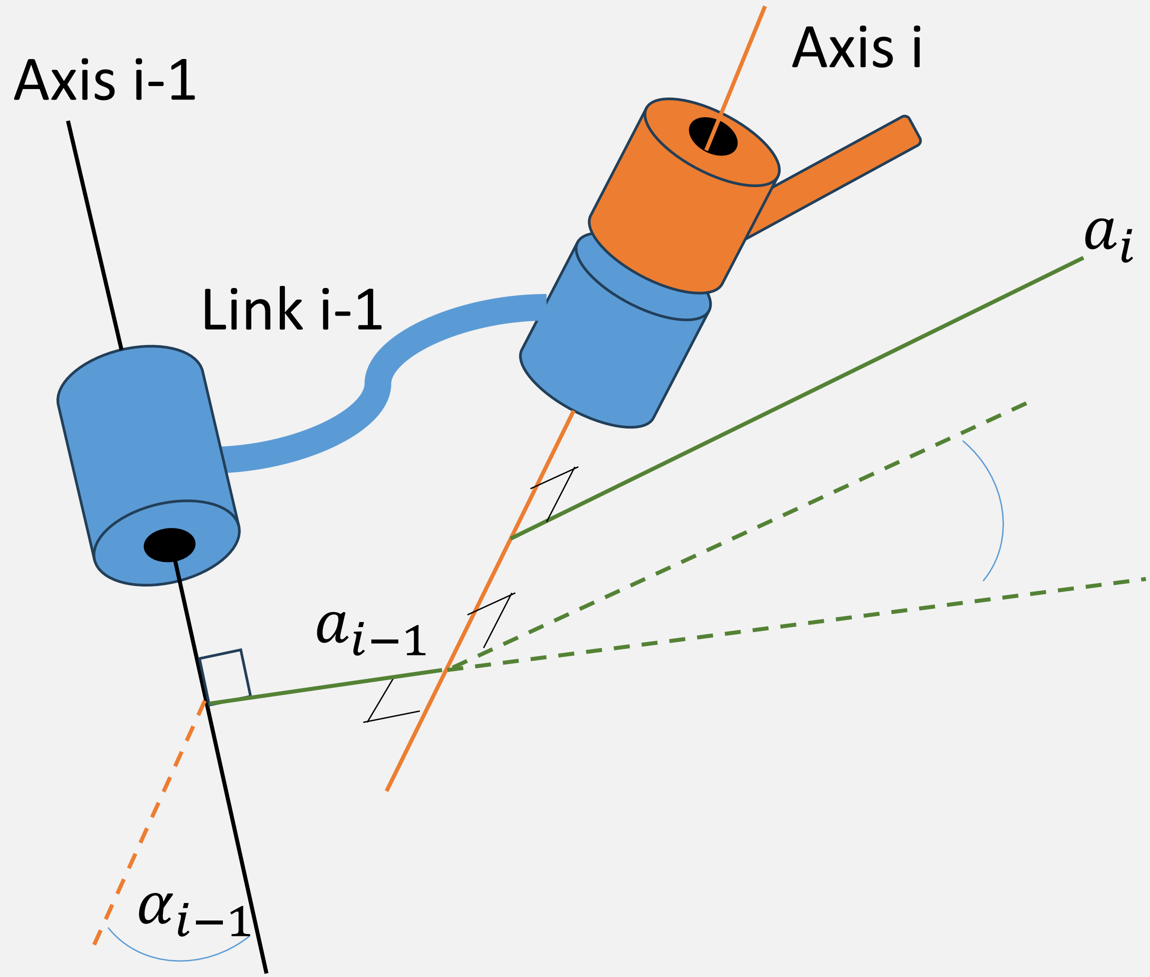
Link Connections



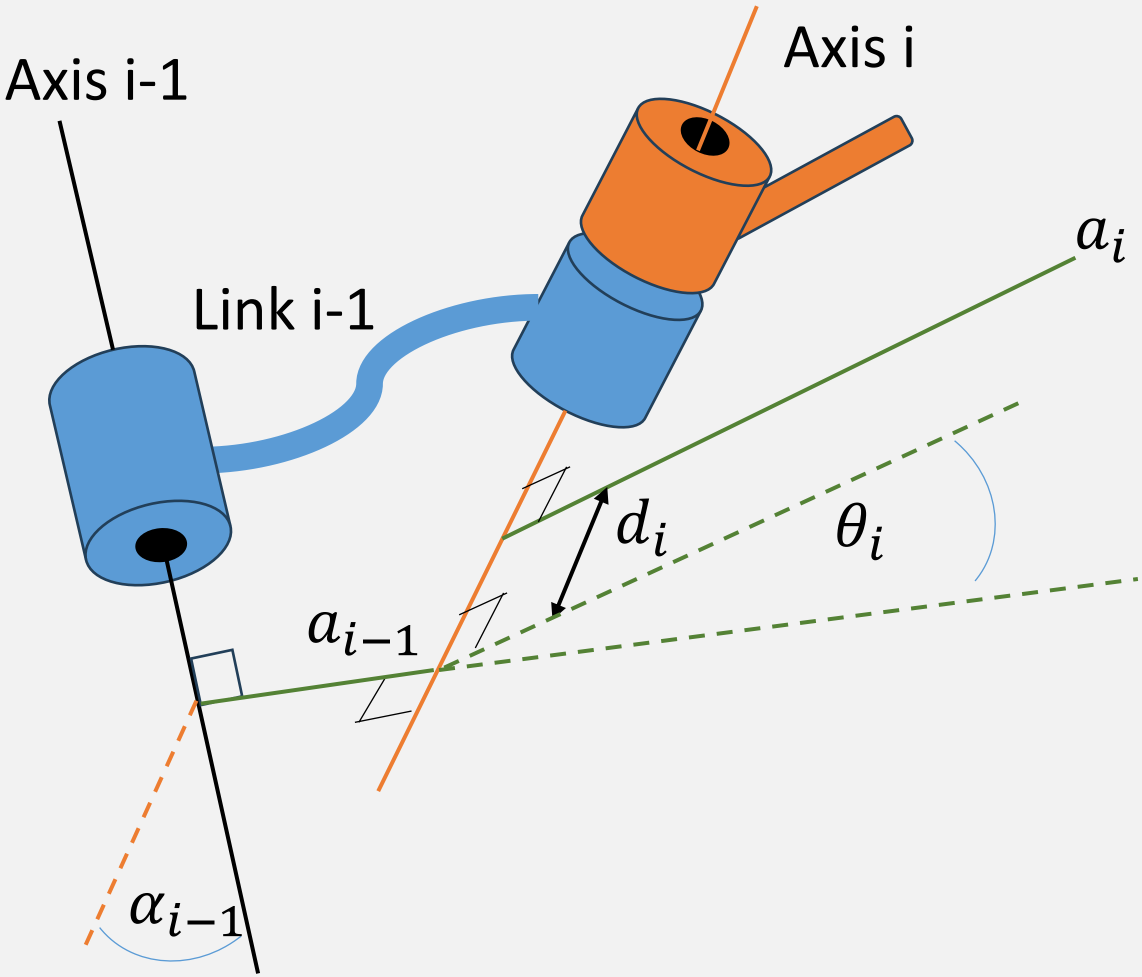
Link Connections



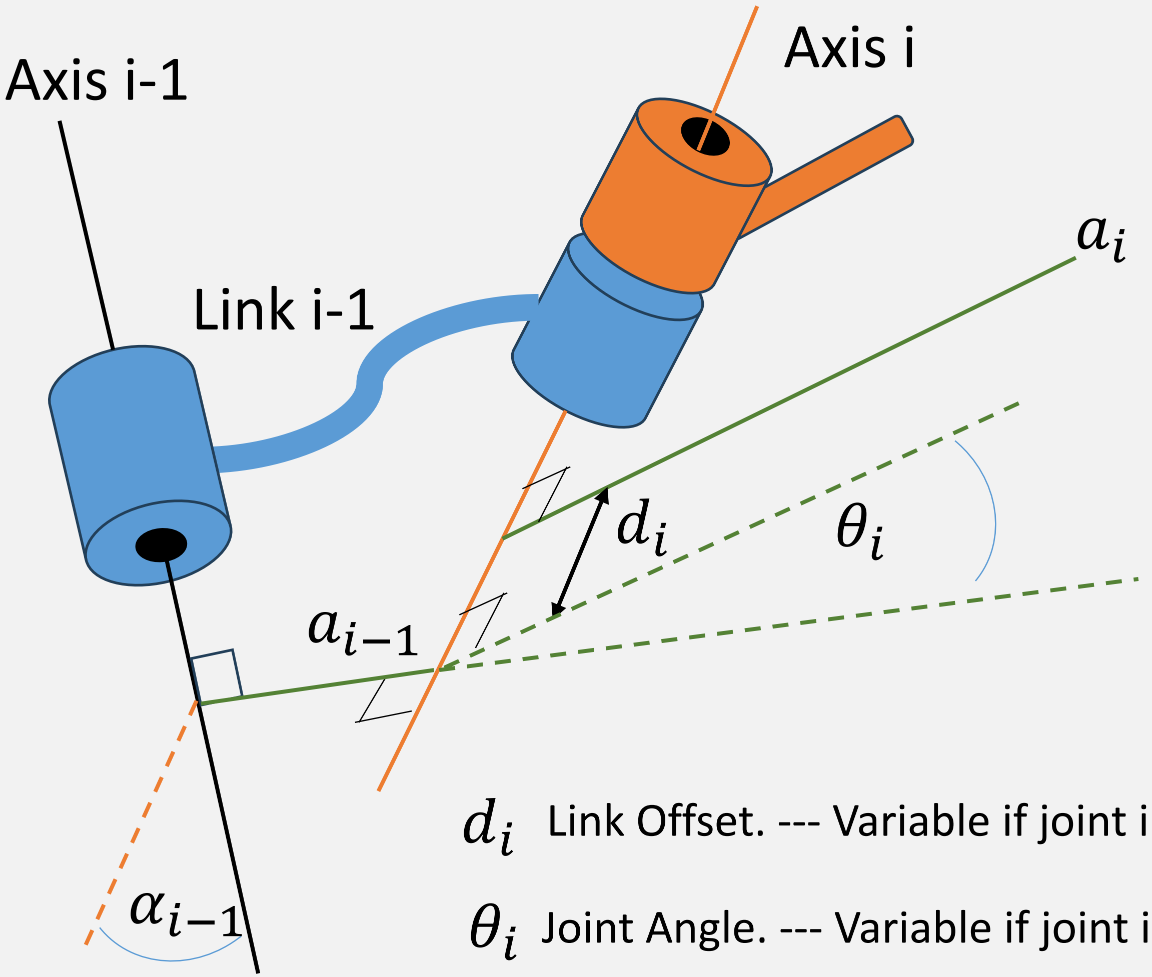
Link Connections



Link Connections



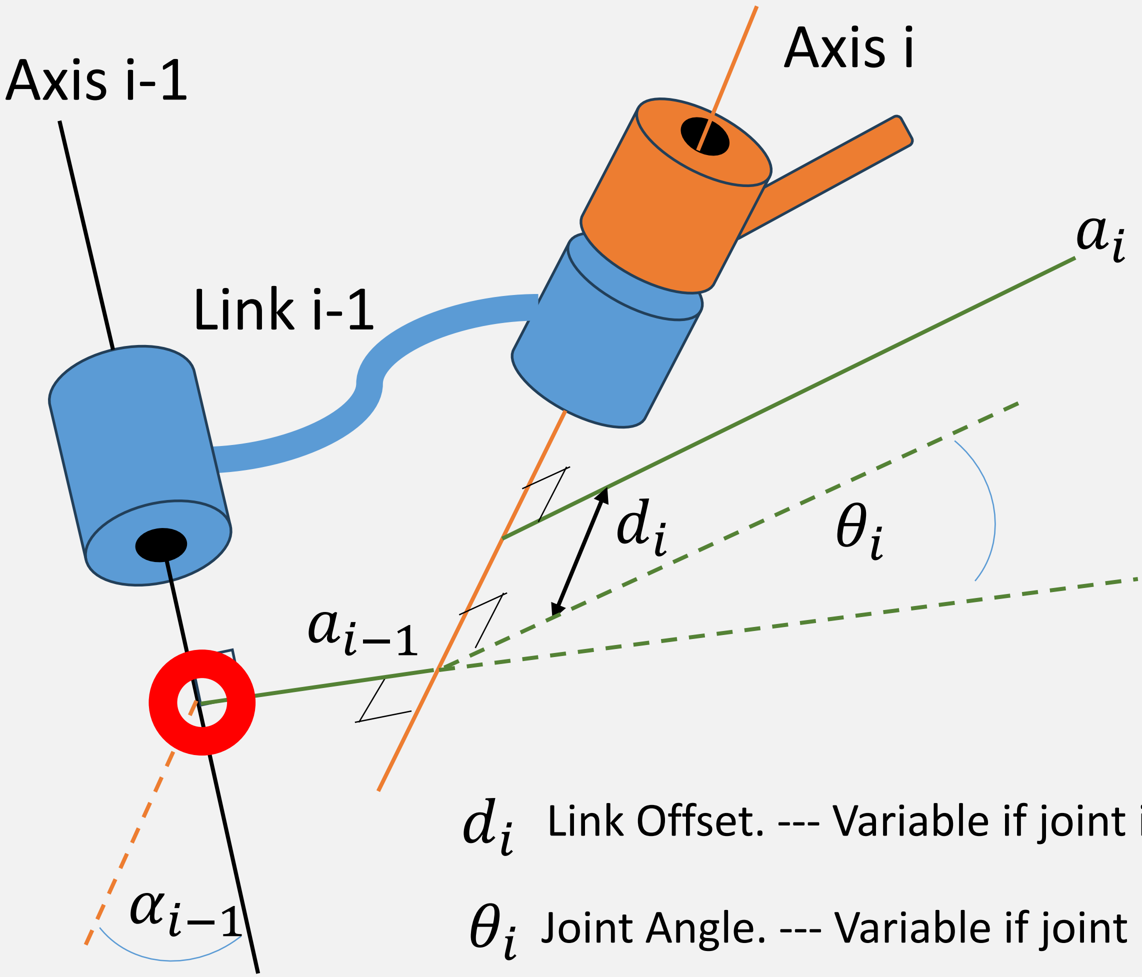
Link Connections



d_i Link Offset. --- Variable if joint i is prismatic

θ_i Joint Angle. --- Variable if joint i is revolute

Frame Attachment



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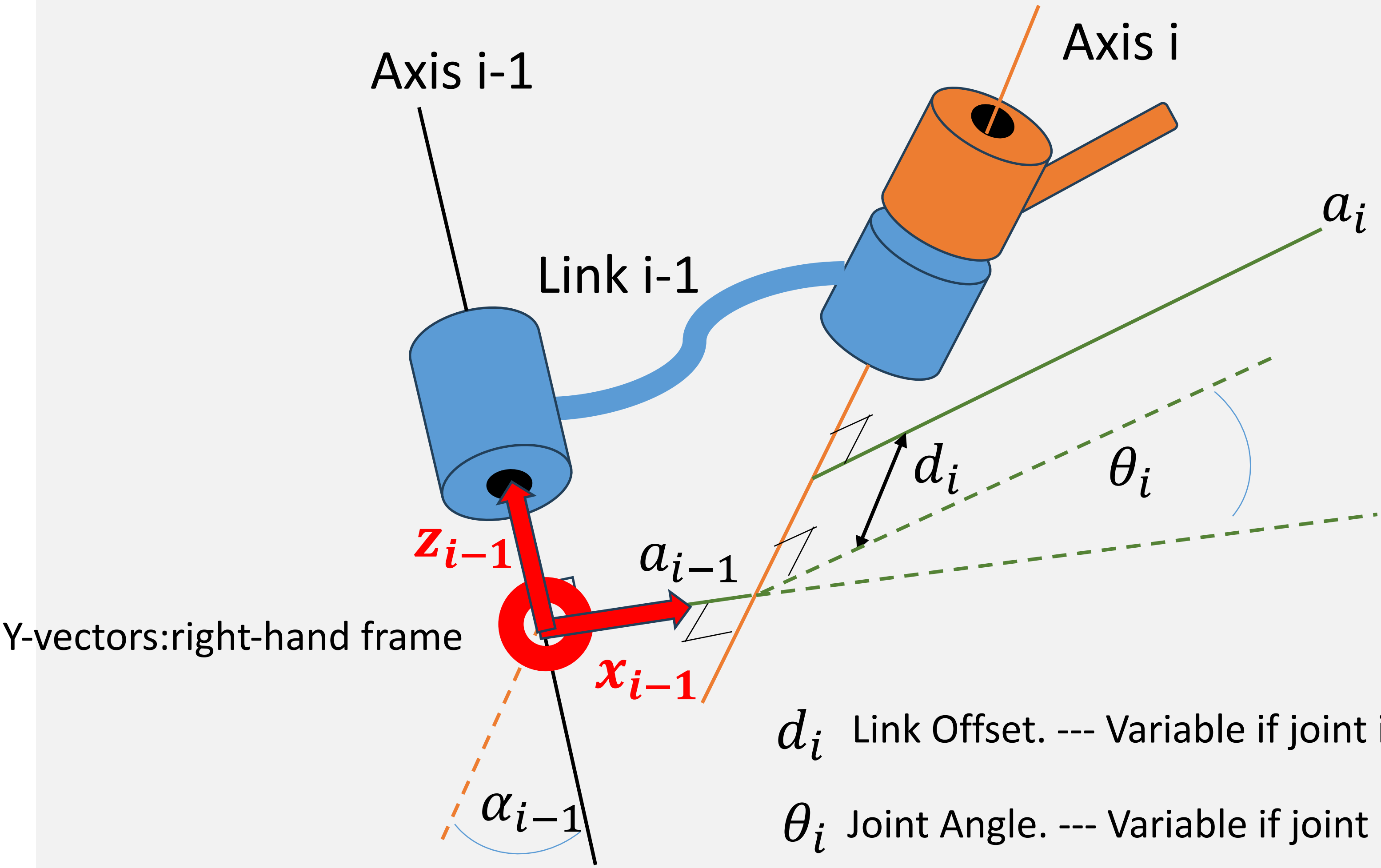

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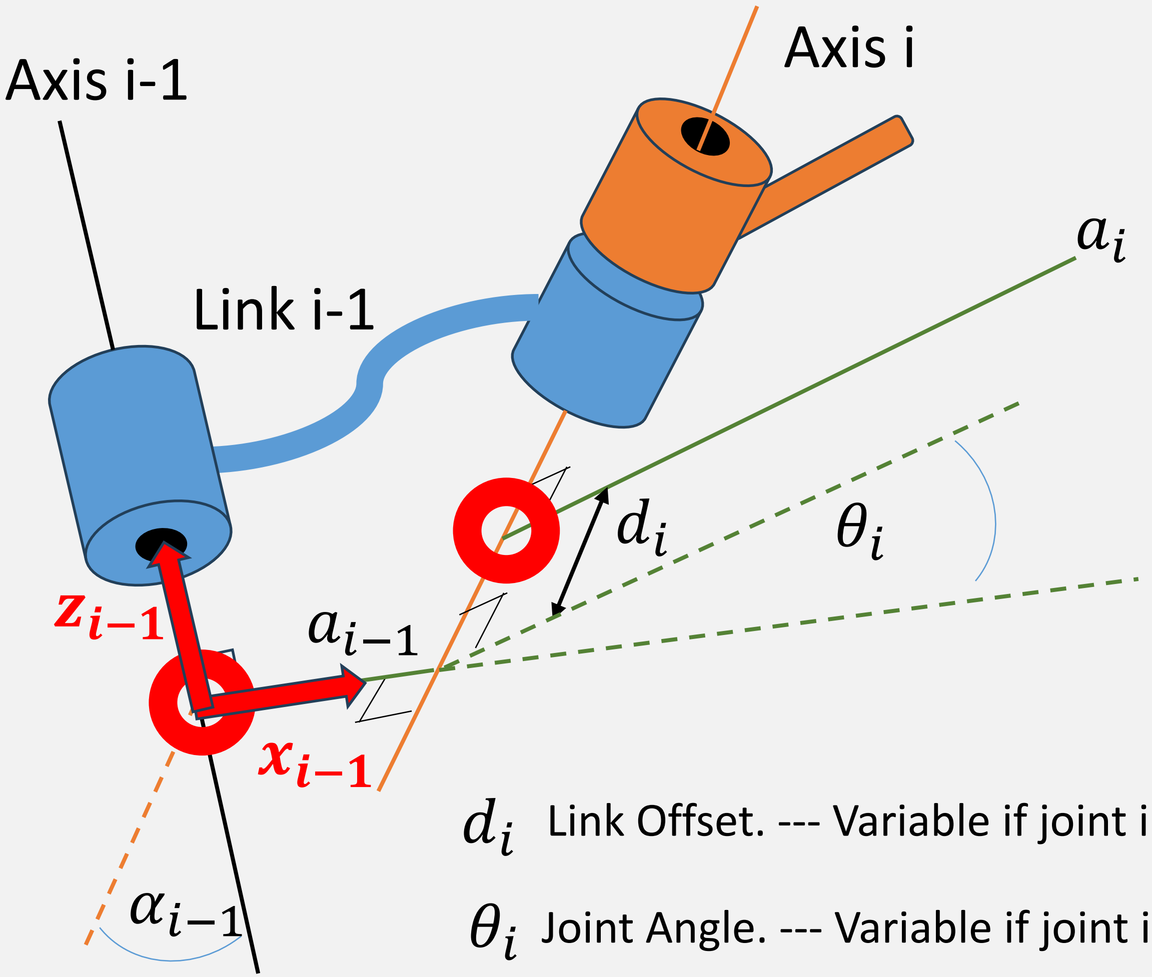

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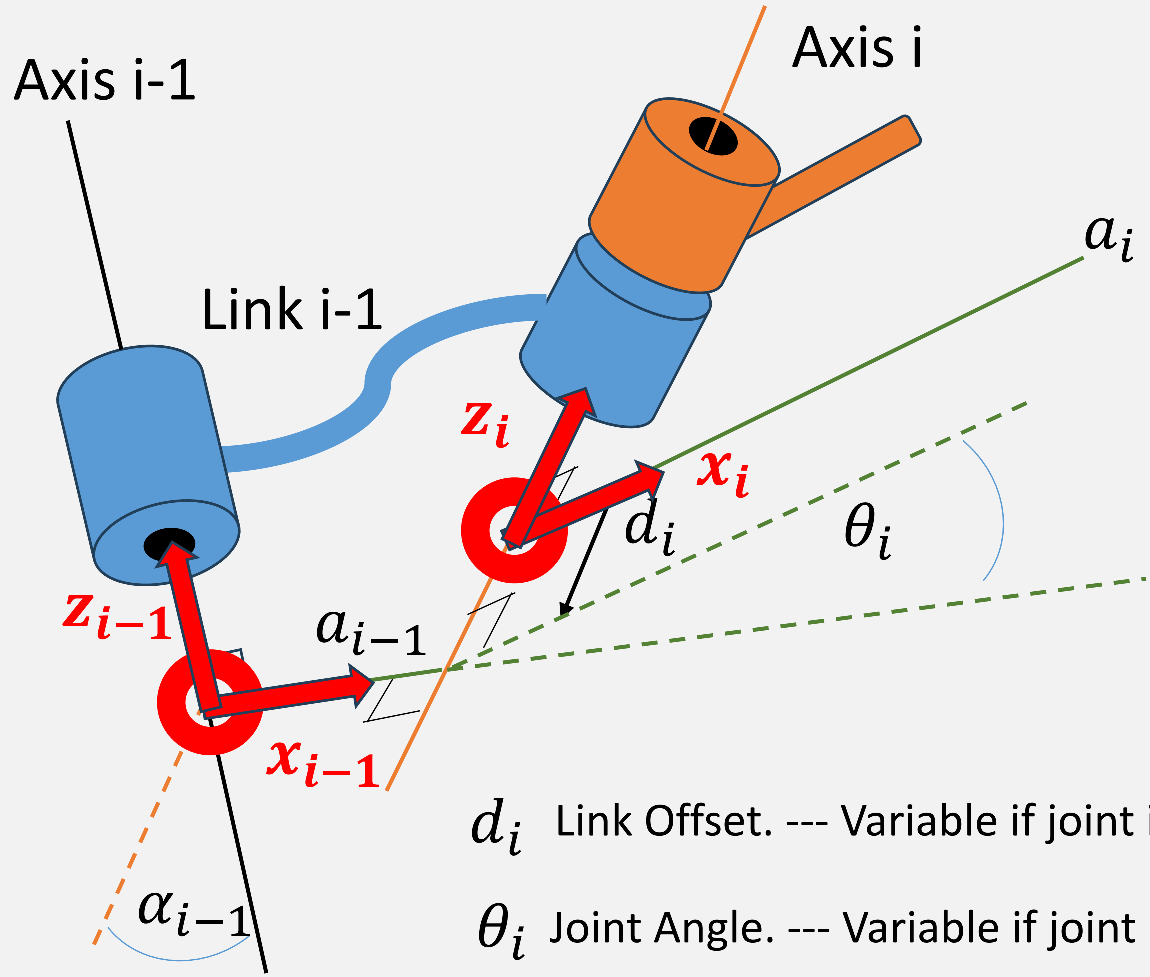

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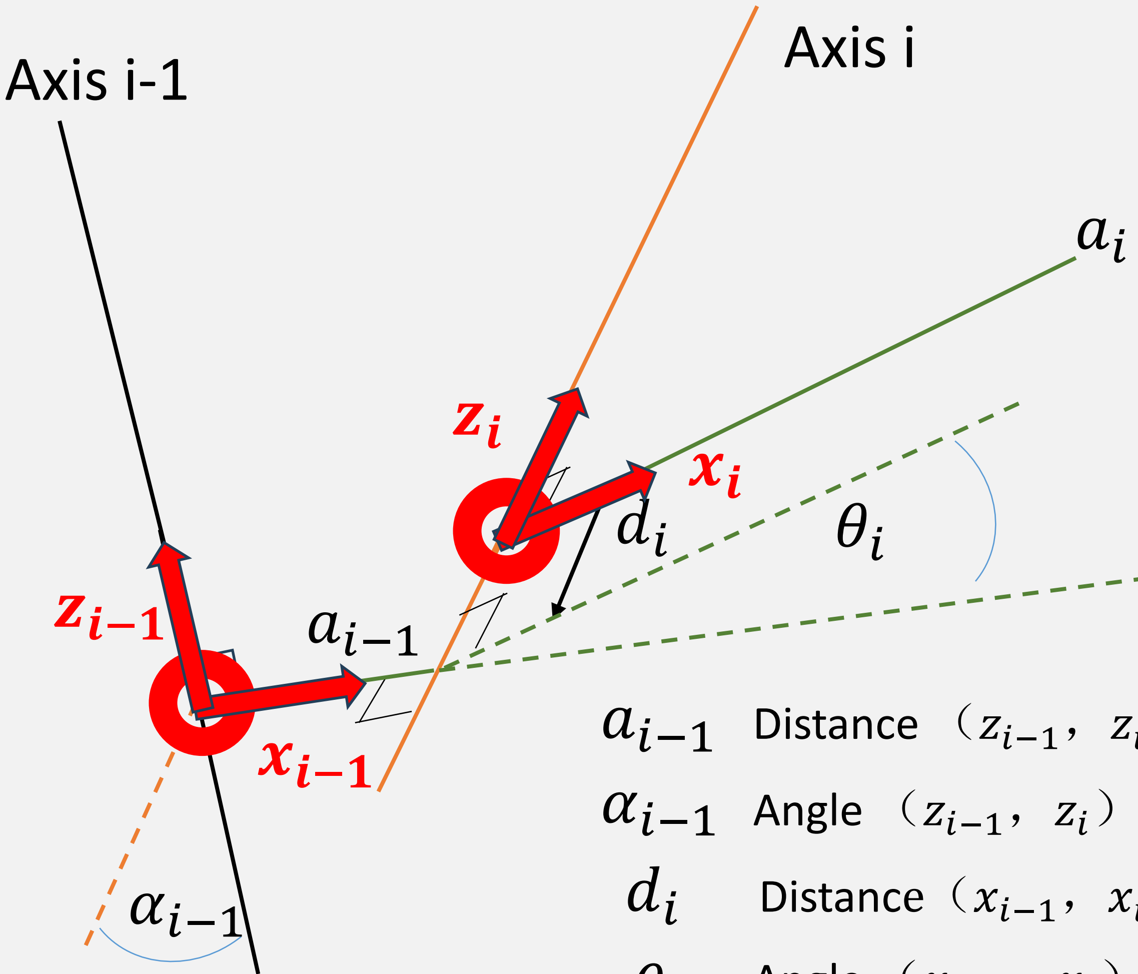


a_{i-1}	Distance （z_{i-1}， z_i） along x_{i-1}
α_{i-1}	Angle （z_{i-1}， z_i） about x_{i-1}
d_i	Distance （x_{i-1}， x_i） about z_i
θ_i	Angle （x_{i-1}， x_i） about z_i

Denavit-Hartenberg Parameters

4 D-H parameters $(a_i, \alpha_i, d_i, \theta_i)$

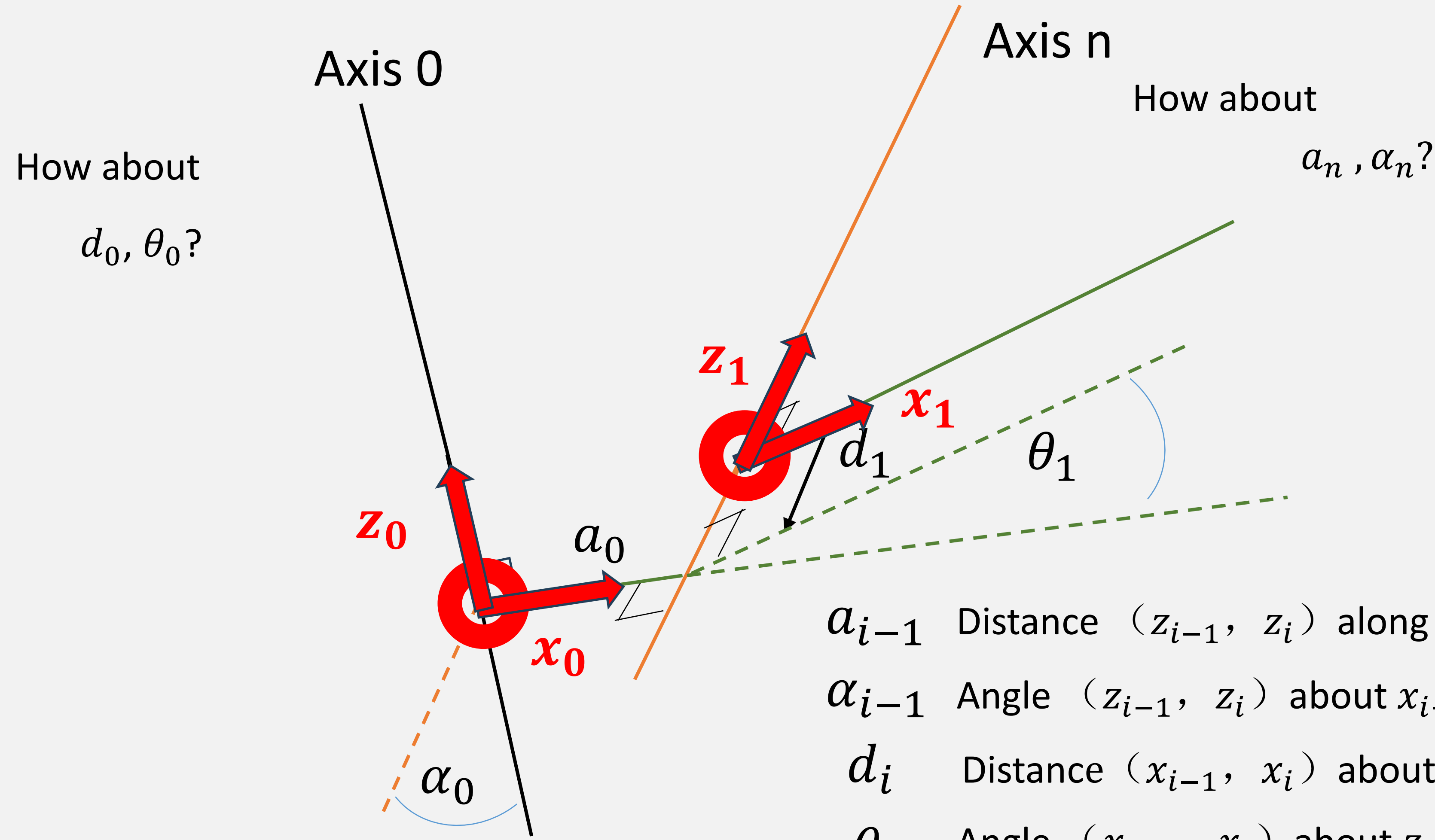
3 fixed link parameters

1 joint variable : $\begin{cases} \theta_i & \text{revolute} \\ d_i & \text{prismatic} \end{cases}$

(a_i, α_i) describes the Link i

(d_i, θ_i) describes the Link's connection

First Frame and Last Frame



a_{i-1} Distance （z_{i-1}， z_i） along x_{i-1}

α_{i-1} Angle （z_{i-1}， z_i） about x_{i-1}

d_i Distance（x_{i-1}， x_i） about z_i

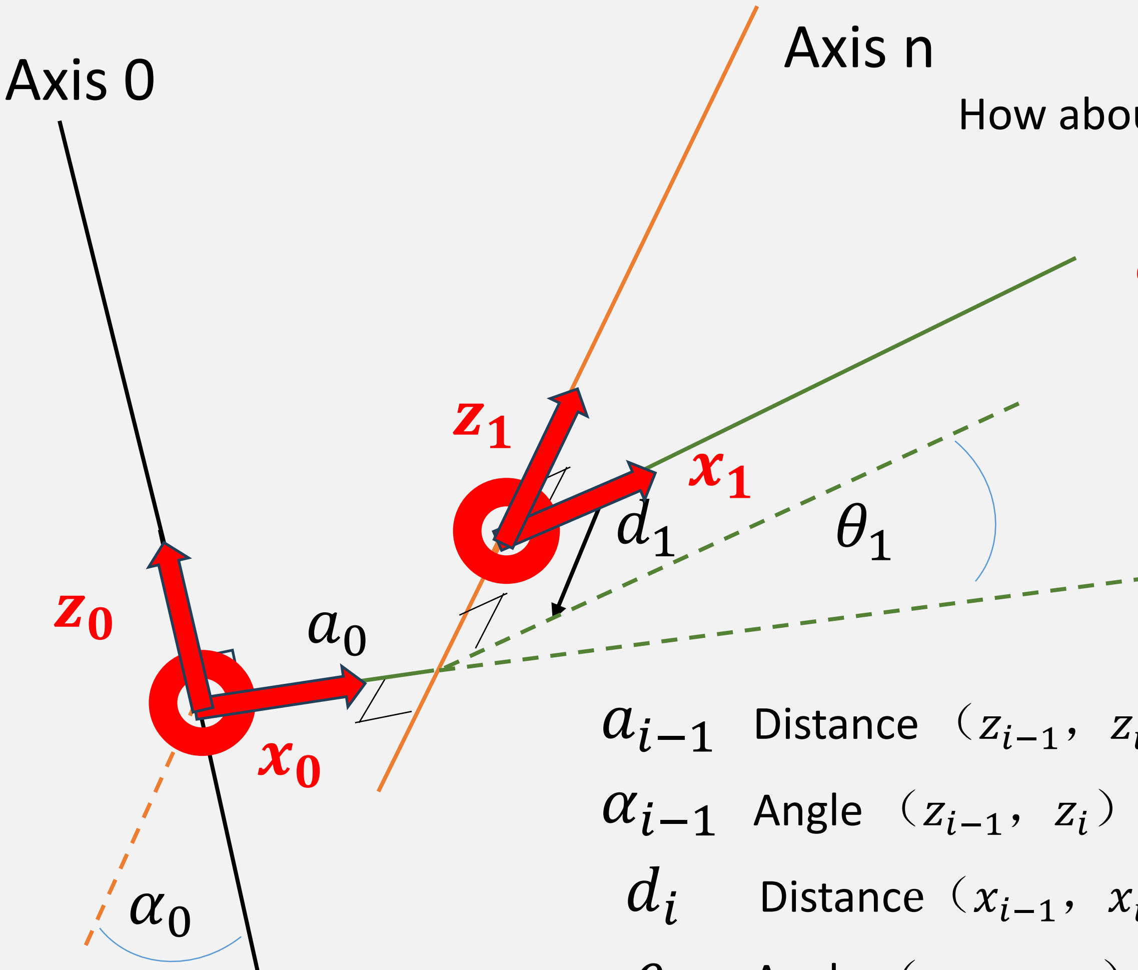
θ_i Angle （x_{i-1}， x_i） about z_i

First Frame and Last Frame

How about

d_0, θ_0?

$d_0 = 0, \theta_0 = 0$



How about

a_n , α_n?

$a_n = 0$, $\alpha_n = 0$

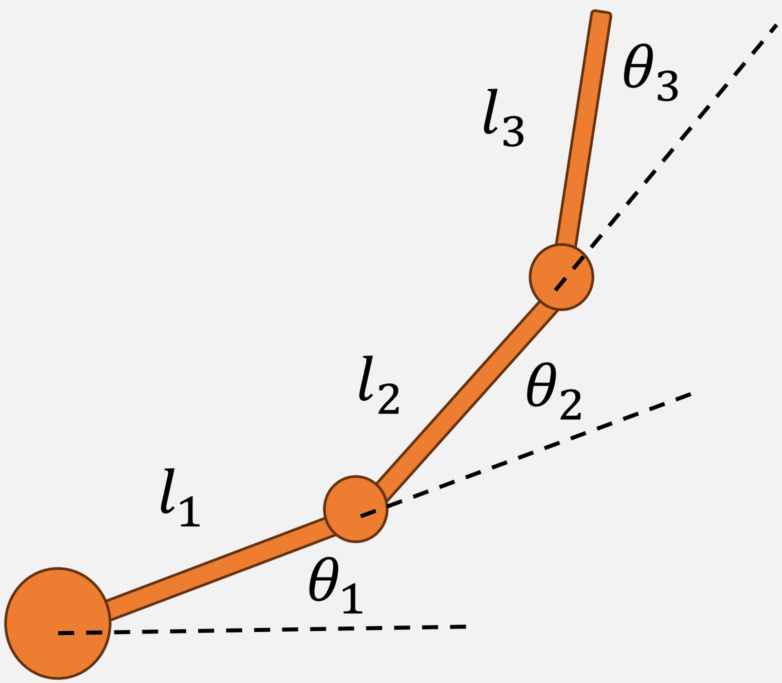
a_{i-1} Distance （z_{i-1}， z_i） along x_{i-1}

α_{i-1} Angle （z_{i-1}， z_i） about x_{i-1}

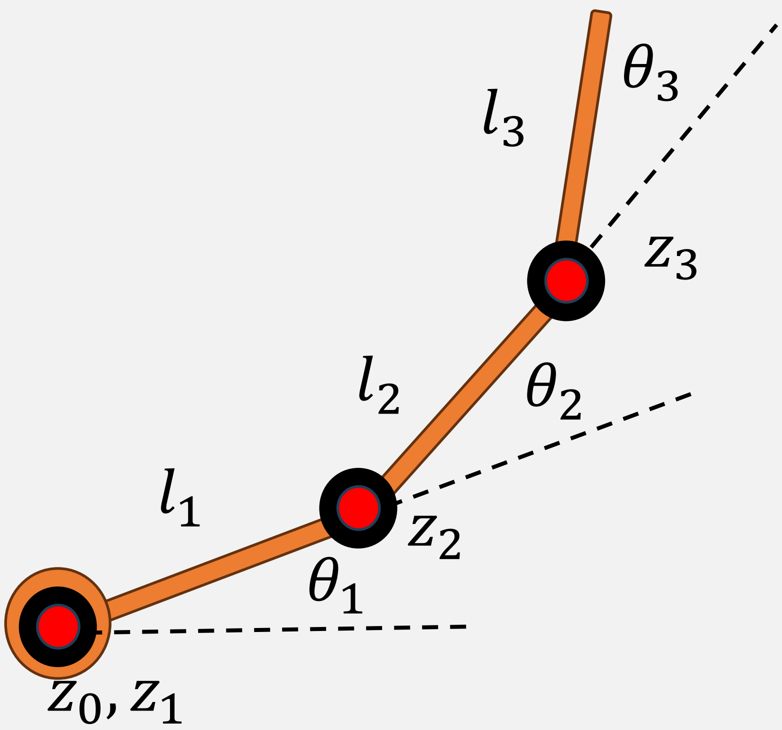
d_i Distance（x_{i-1}， x_i） about z_i

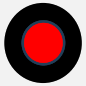
θ_i Angle （x_{i-1}， x_i） about z_i

Example



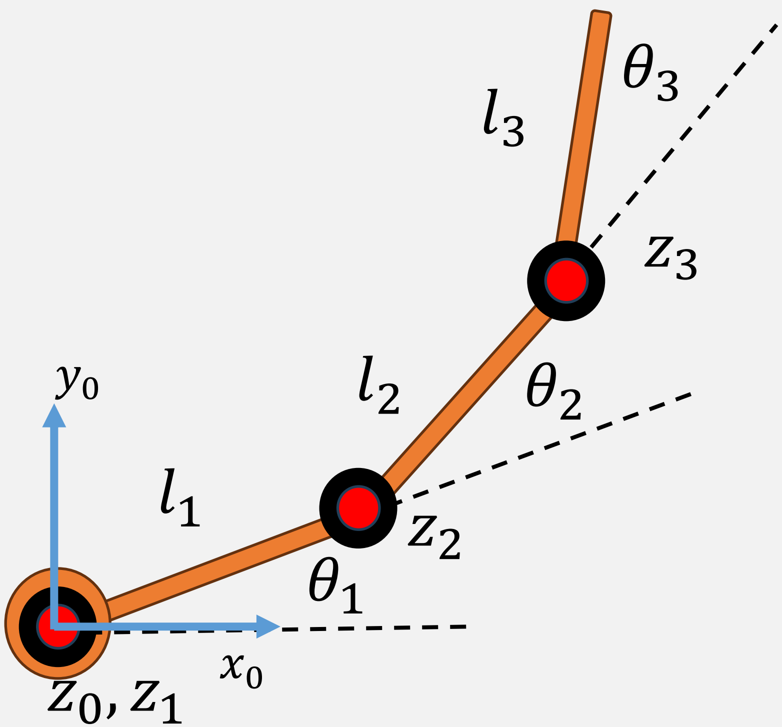
Example

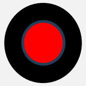




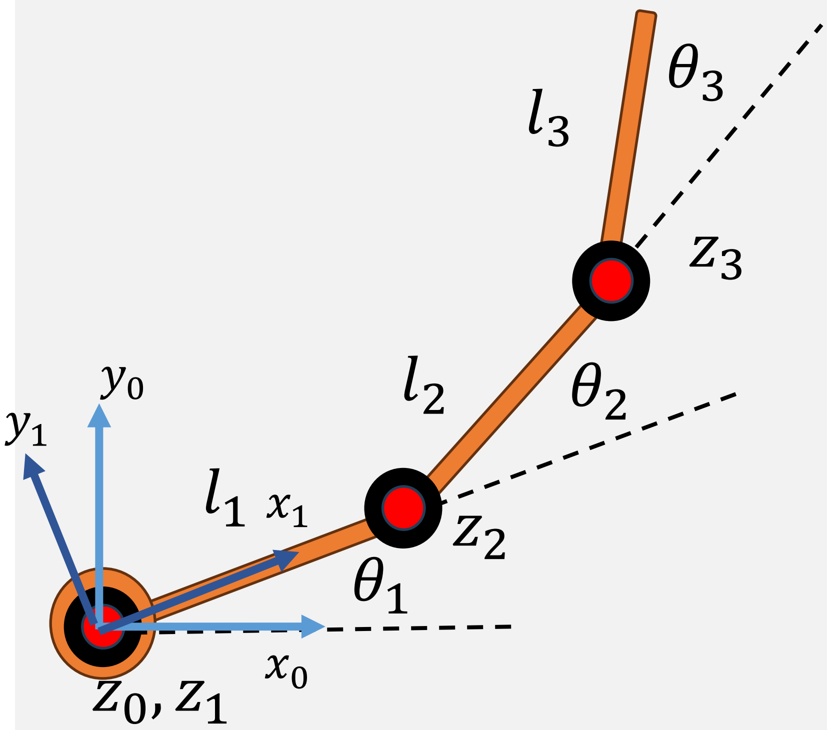
Indicates direction
coming outside of the slides

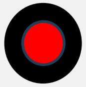
Example



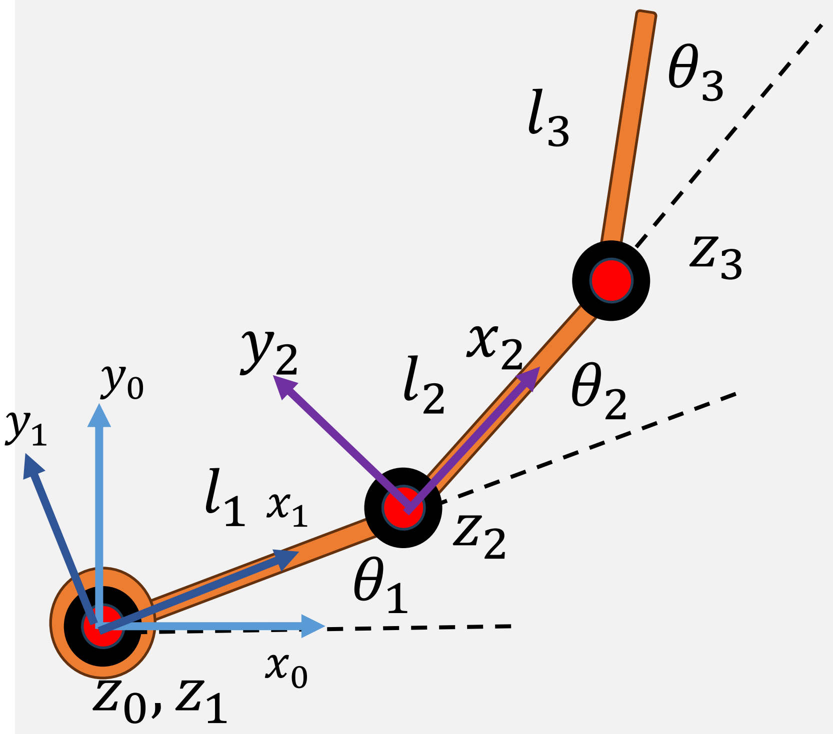

Indicates direction coming outside of the slides

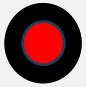
Example




Indicates direction coming outside of the slides

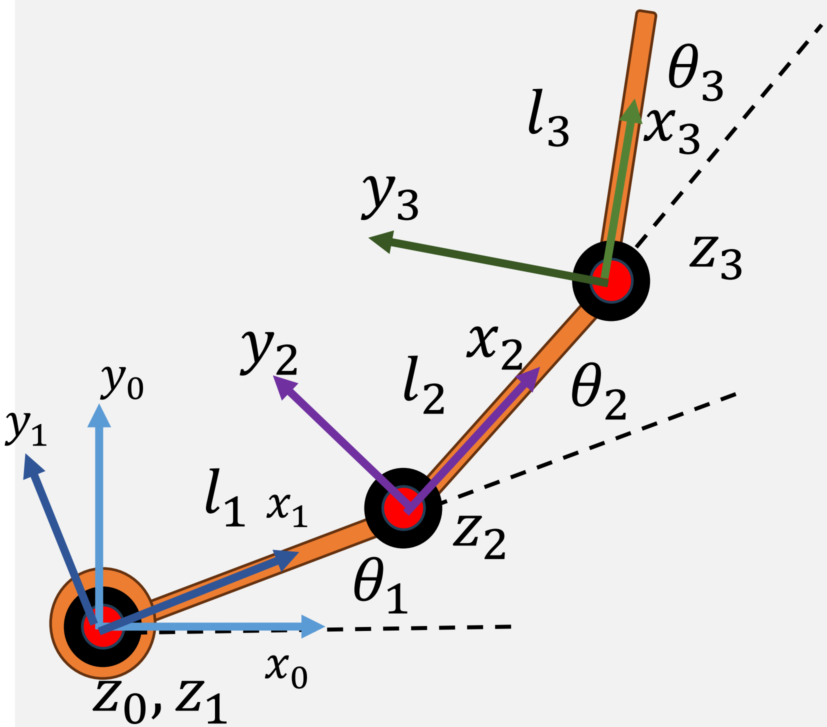
Example



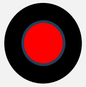


Indicates direction coming outside of the slides

Example

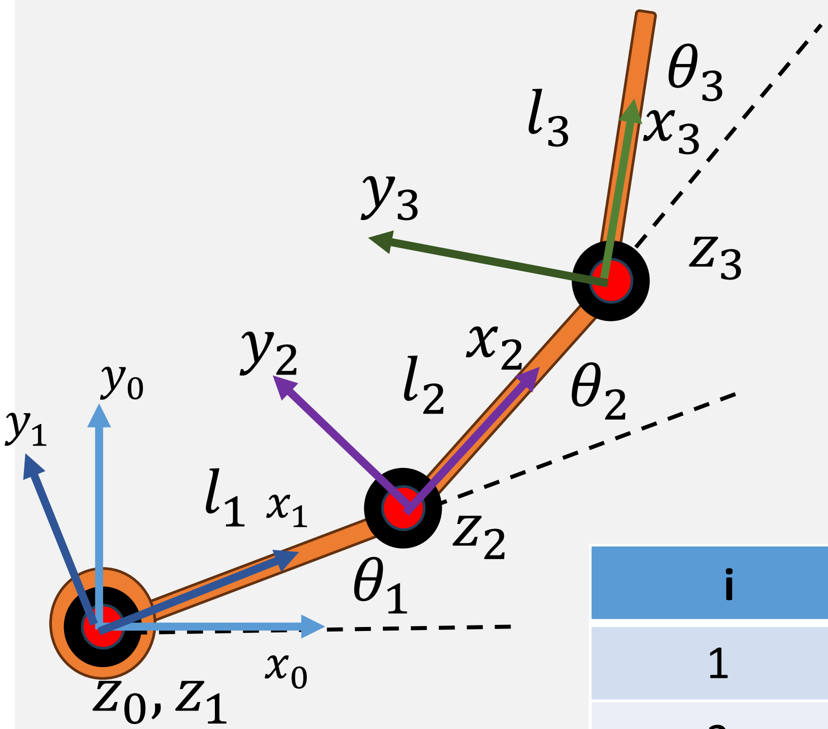


θ_3
l_3
x_3
y_3
z_3
y_2
x_2
l_2
θ_2
y_0
y_1
l_1
x_1
z_2
θ_1
x_0
z_0, z_1



Indicates direction
coming outside of the slides

Example



a_{i-1} Distance（z_{i-1}，z_i）along x_{i-1}

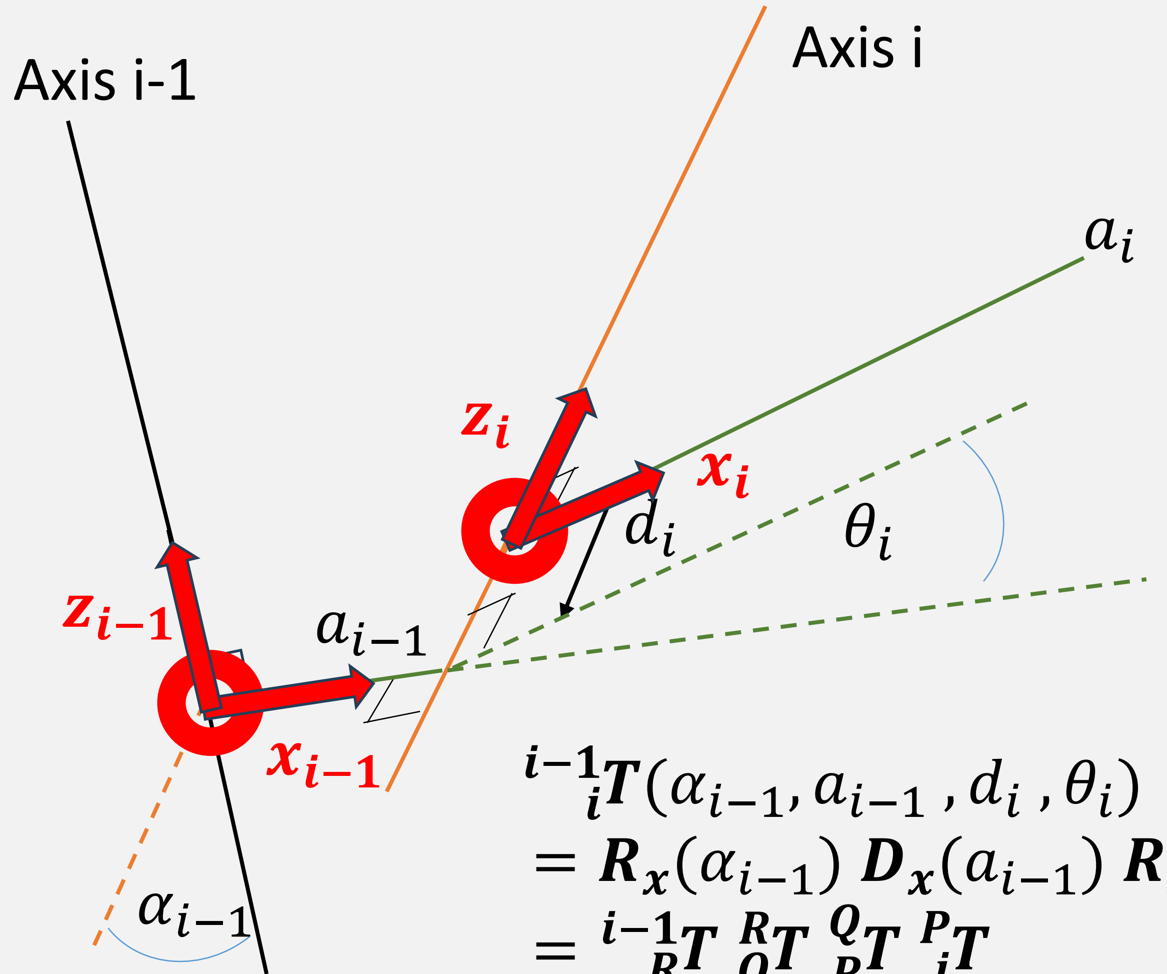
α_{i-1} Angle（z_{i-1}，z_i）about x_{i-1}

d_i Distance（x_{i-1}，x_i）about z_i

θ_i Angle（x_{i-1}，x_i）about z_i

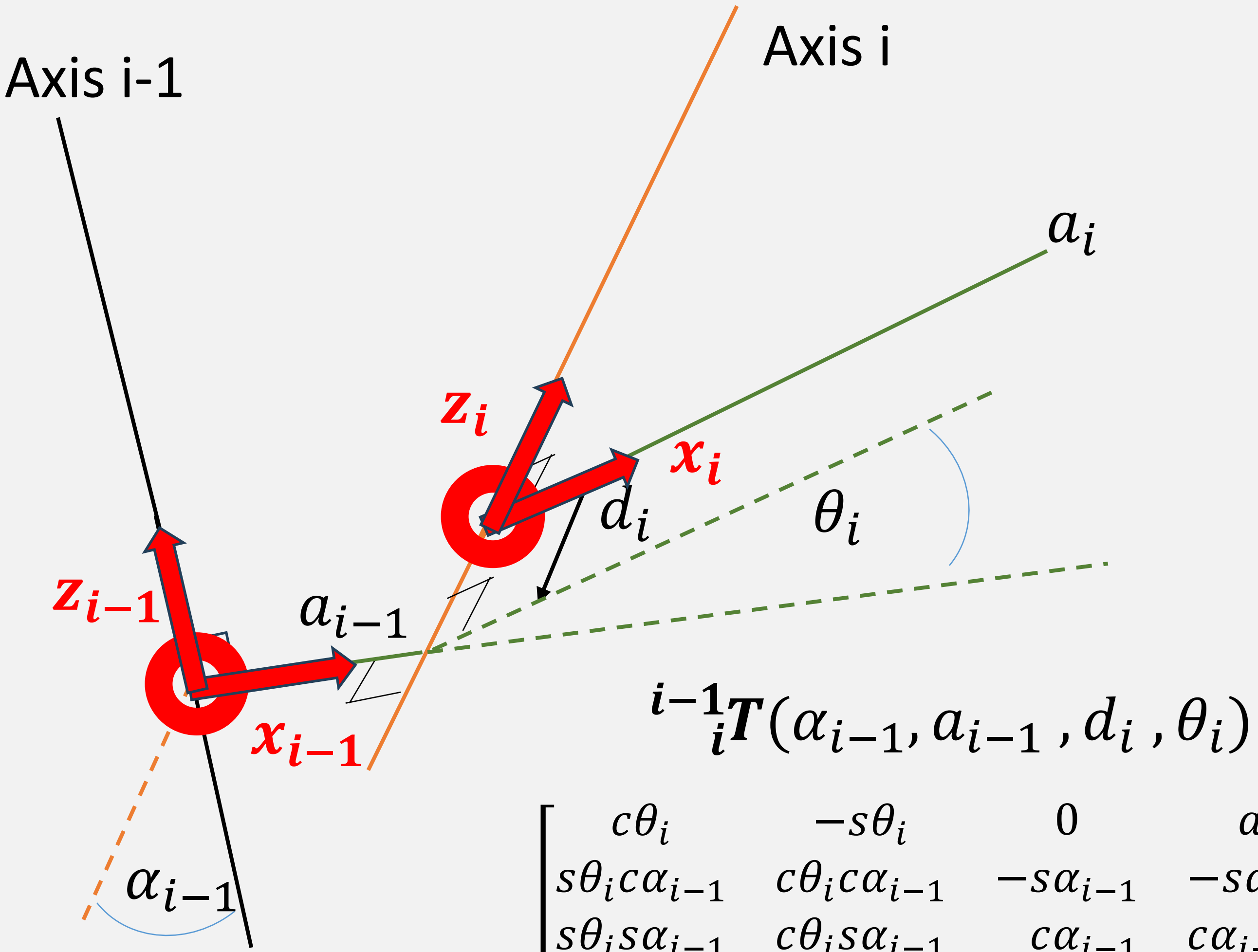
i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	l_1	0	0	θ_2
3	l_2	0	0	θ_3

Frame Attachment



$$
\begin{aligned}
{}^{i-1}_{\;\;i}\boldsymbol{T}(\alpha_{i-1}, a_{i-1}, d_i, \theta_i) \\
&= \boldsymbol{R_x}(\alpha_{i-1})\, \boldsymbol{D_x}(a_{i-1})\, \boldsymbol{R_z}(\theta_i) \boldsymbol{D_z}(d_i) \\
&= {}^{i-1}_{\;\;R}\boldsymbol{T}\; {}^{R}_{Q}\boldsymbol{T}\; {}^{Q}_{P}\boldsymbol{T}\; {}^{P}_{i}\boldsymbol{T}
\end{aligned}
$$

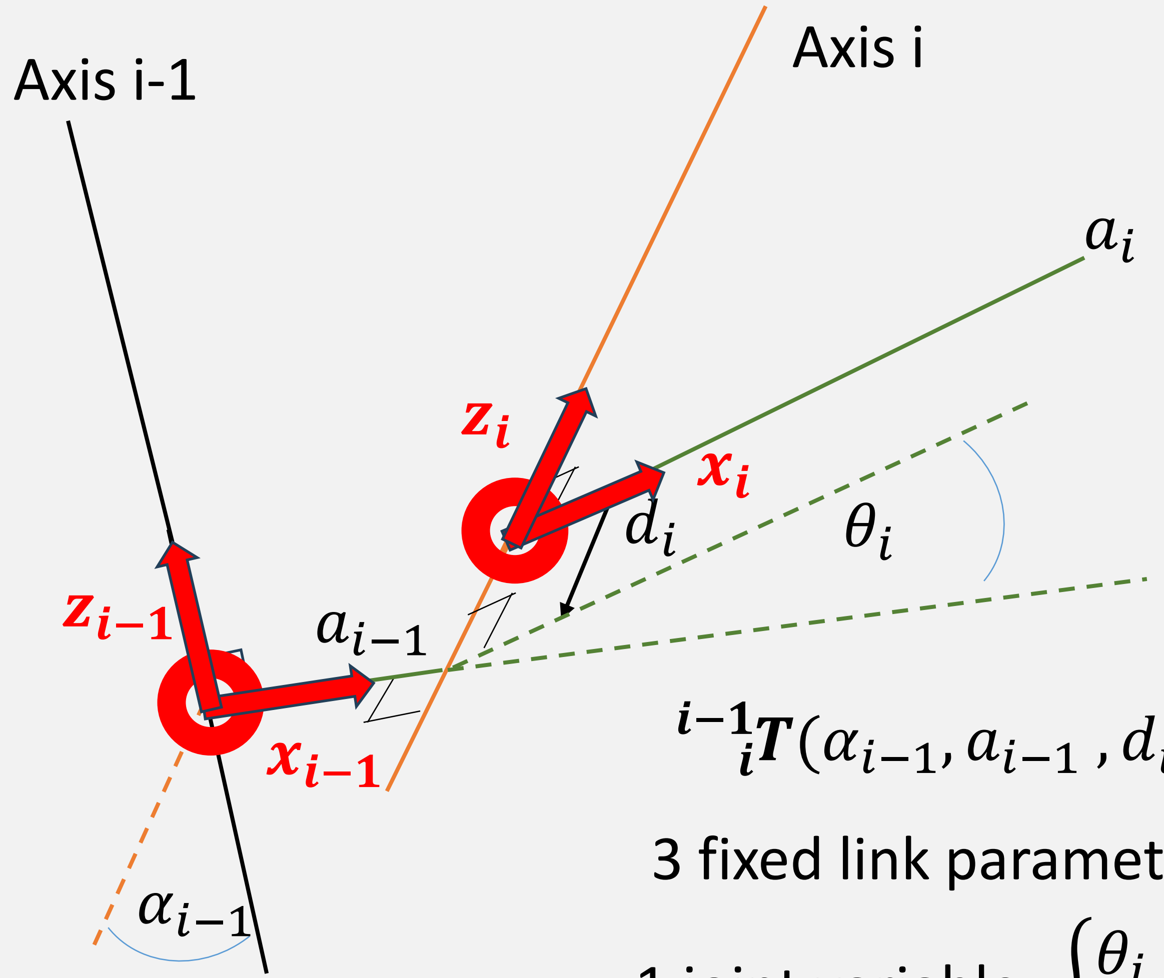
Frame Attachment



$$ {}^{i-1}_{\ \ i}\boldsymbol{T}(\alpha_{i-1}, a_{i-1}, d_i, \theta_i) $$

$$\begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Frame Attachment

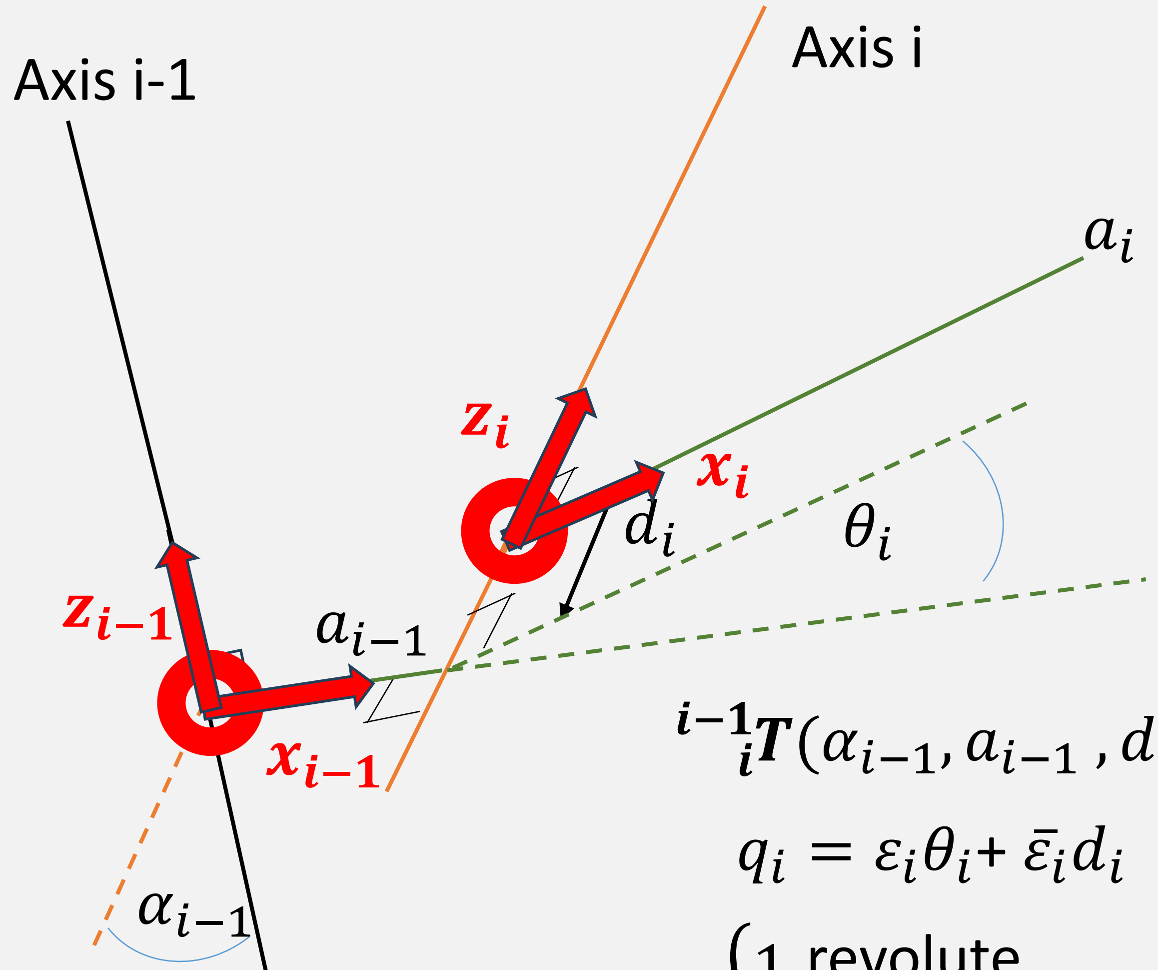


$${}^{i-1}_{\ \ i}\boldsymbol{T}(\alpha_{i-1}, a_{i-1}, d_i, \theta_i)$$

3 fixed link parameters

1 joint variable : $\begin{cases} \theta_i & \text{revolute} \\ d_i & \text{prismatic} \end{cases}$

Frame Attachment

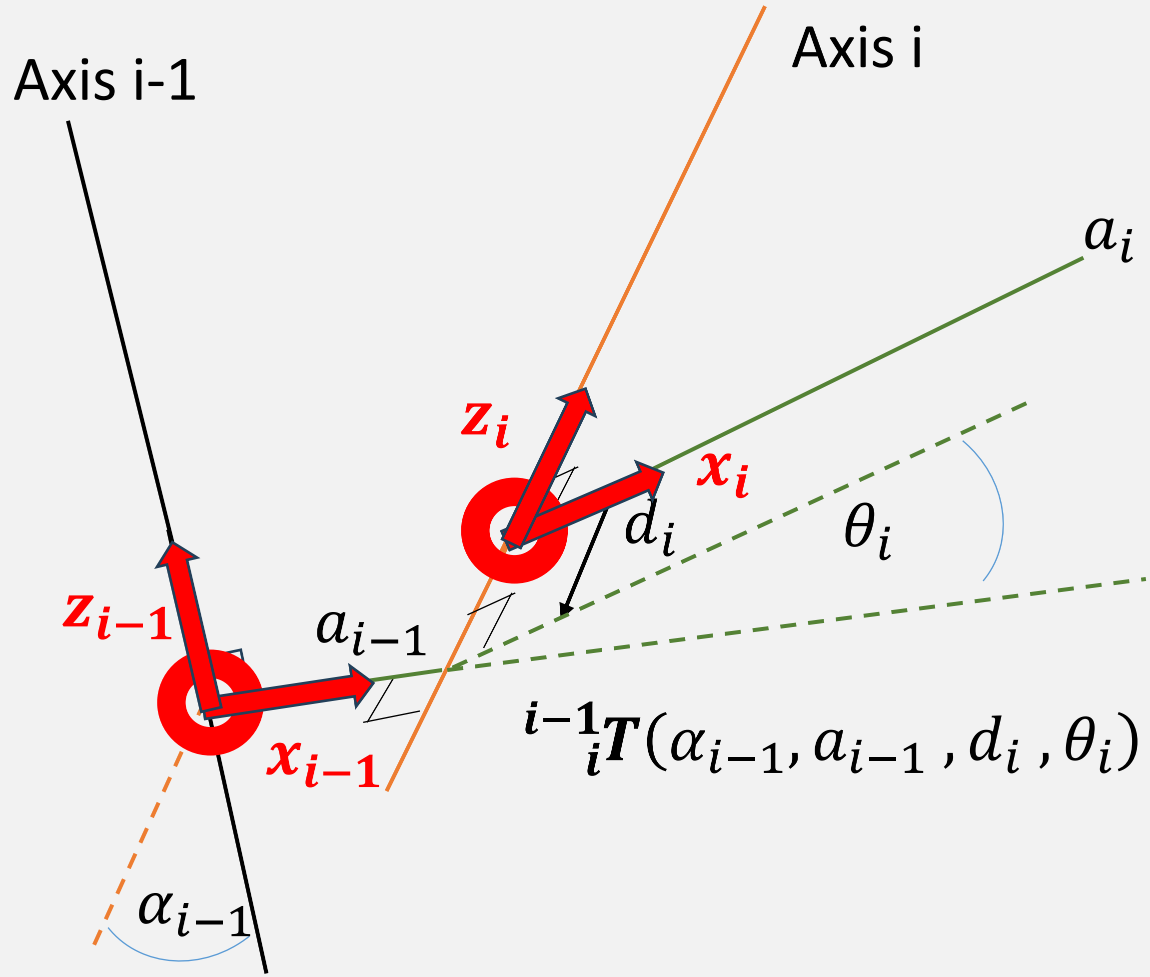


$$ {}^{i-1}_{\ \ i}\boldsymbol{T}(\alpha_{i-1}, a_{i-1}, d_i, \theta_i) $$

$$ q_i = \varepsilon_i \theta_i + \bar{\varepsilon}_i d_i $$

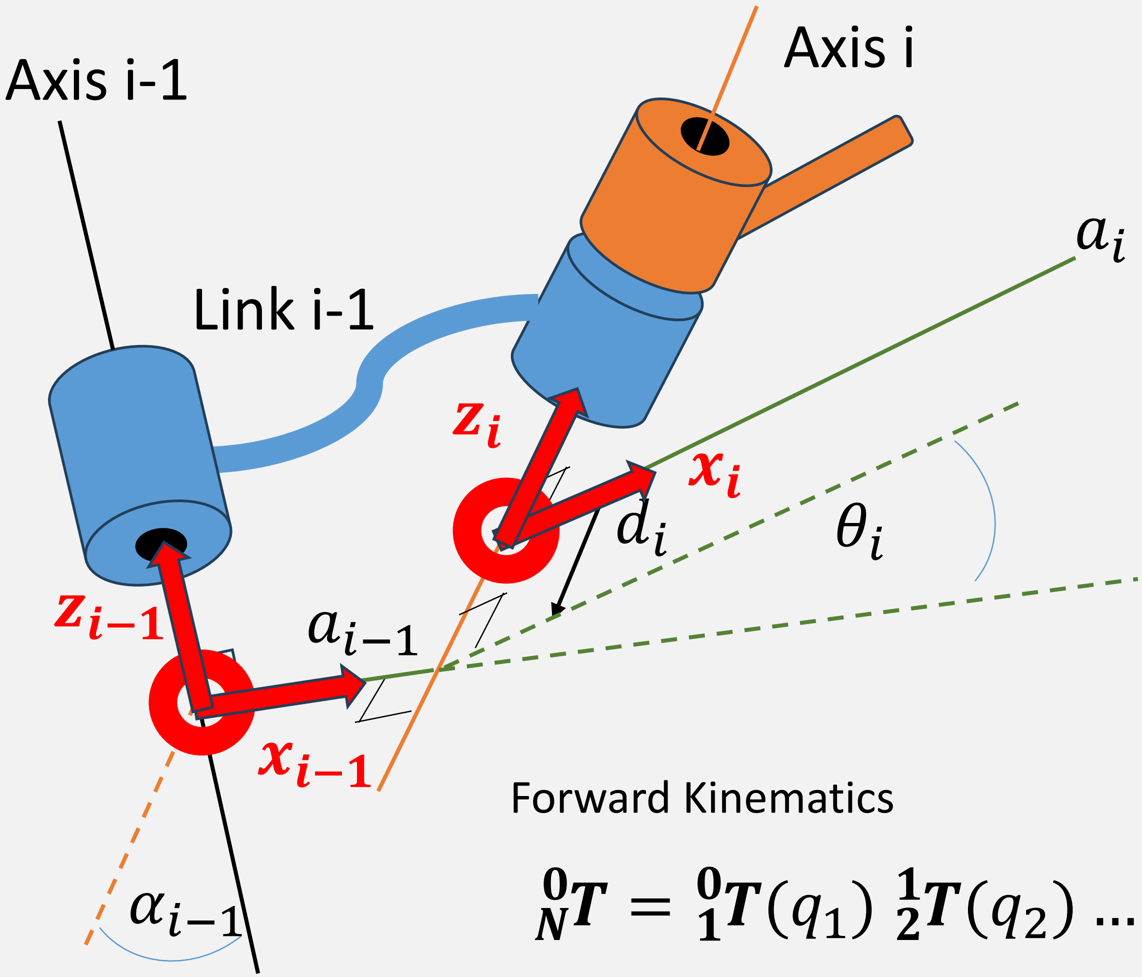
$$ \varepsilon_i : \begin{cases} 1 \text{ revolute} \\ 0 \text{ prismatic} \end{cases} \qquad \bar{\varepsilon}_i = 1 - \varepsilon_i $$

Frame Attachment



$${}^{i-1}_{\ \ i}\boldsymbol{T}(\alpha_{i-1}, a_{i-1}, d_i, \theta_i) = {}^{i-1}_{\ \ i}\boldsymbol{T}(q_i)$$

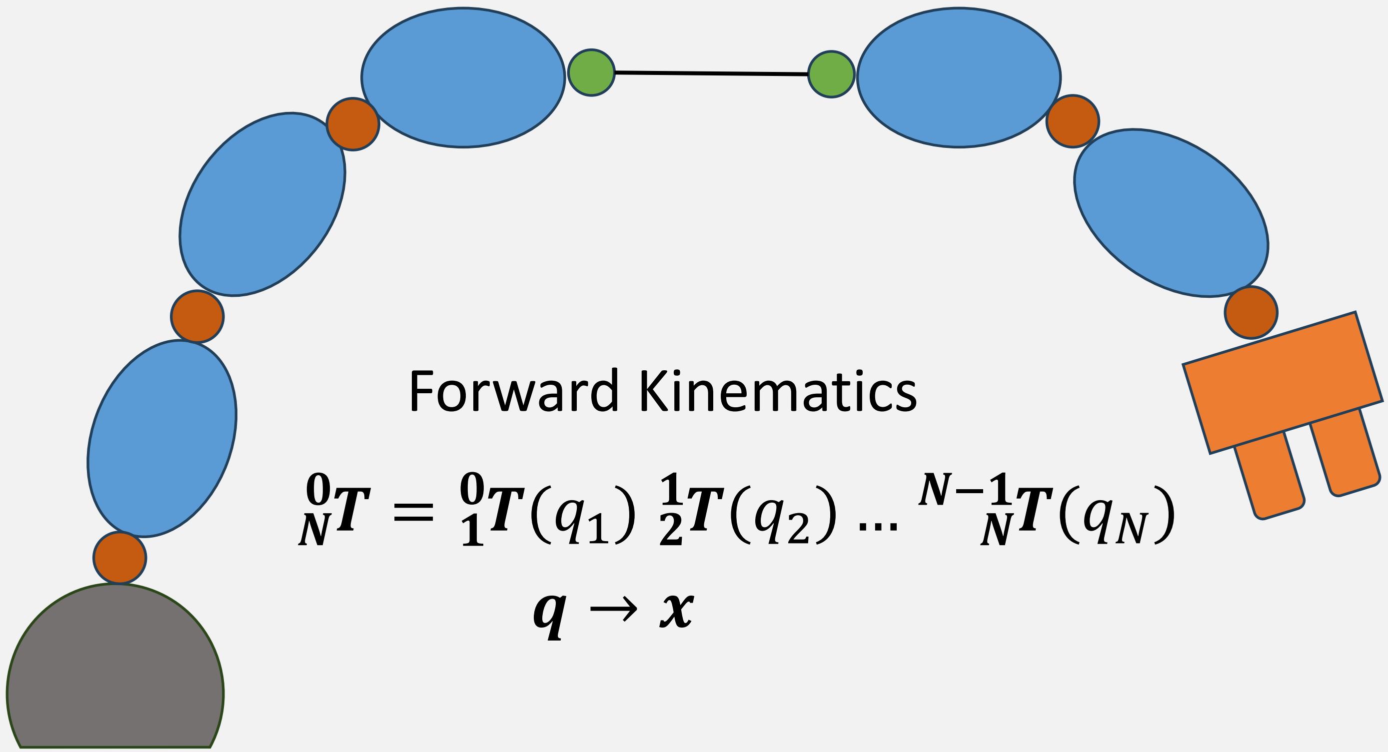
Frame Attachment



Forward Kinematics

$$ {}^{0}_{N}\boldsymbol{T} = {}^{0}_{1}\boldsymbol{T}(q_1)\ {}^{1}_{2}\boldsymbol{T}(q_2) \dots {}^{N-1}_{N}\boldsymbol{T}(q_N) $$

Forward Kinematics

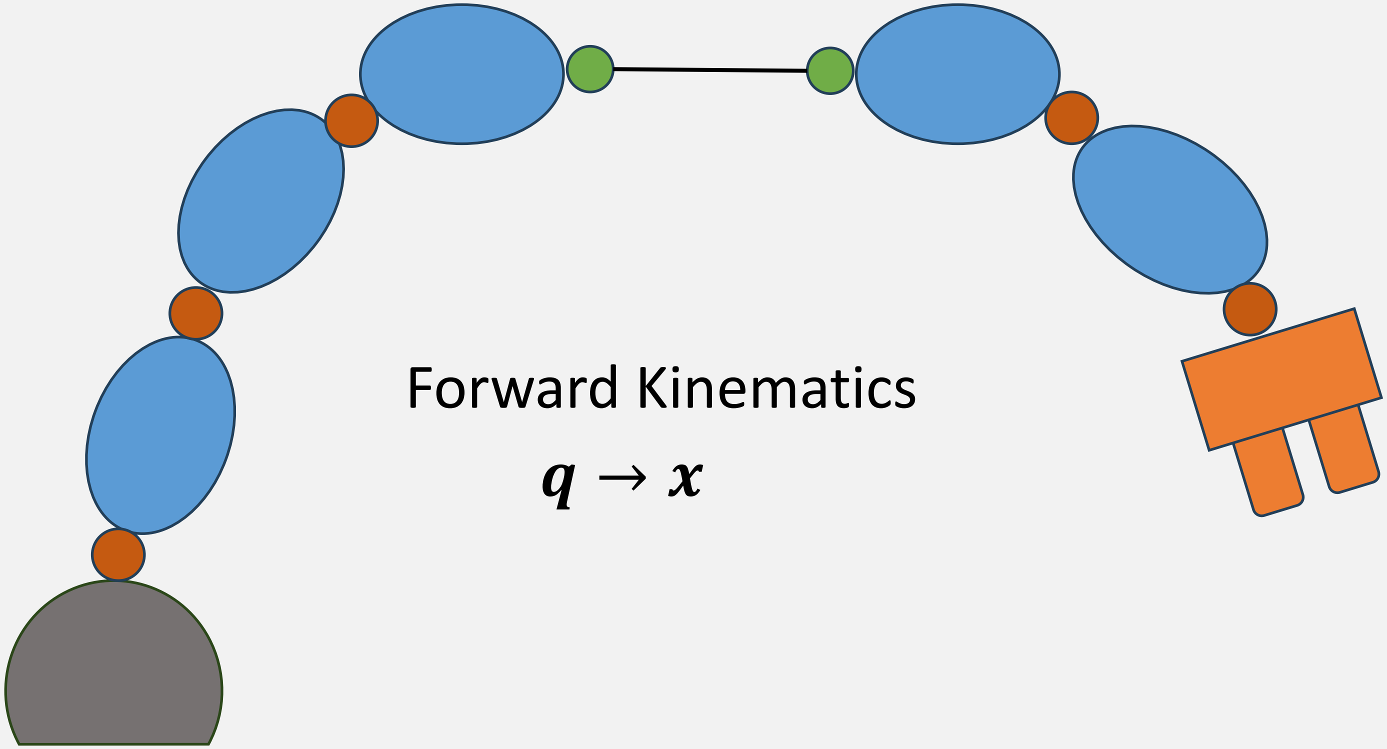


Forward Kinematics

$$ {}^{0}_{N}\boldsymbol{T} = {}^{0}_{1}\boldsymbol{T}(q_1)\ {}^{1}_{2}\boldsymbol{T}(q_2) \ldots {}^{N-1}_{N}\boldsymbol{T}(q_N) $$

$$ \boldsymbol{q} \rightarrow \boldsymbol{x} $$

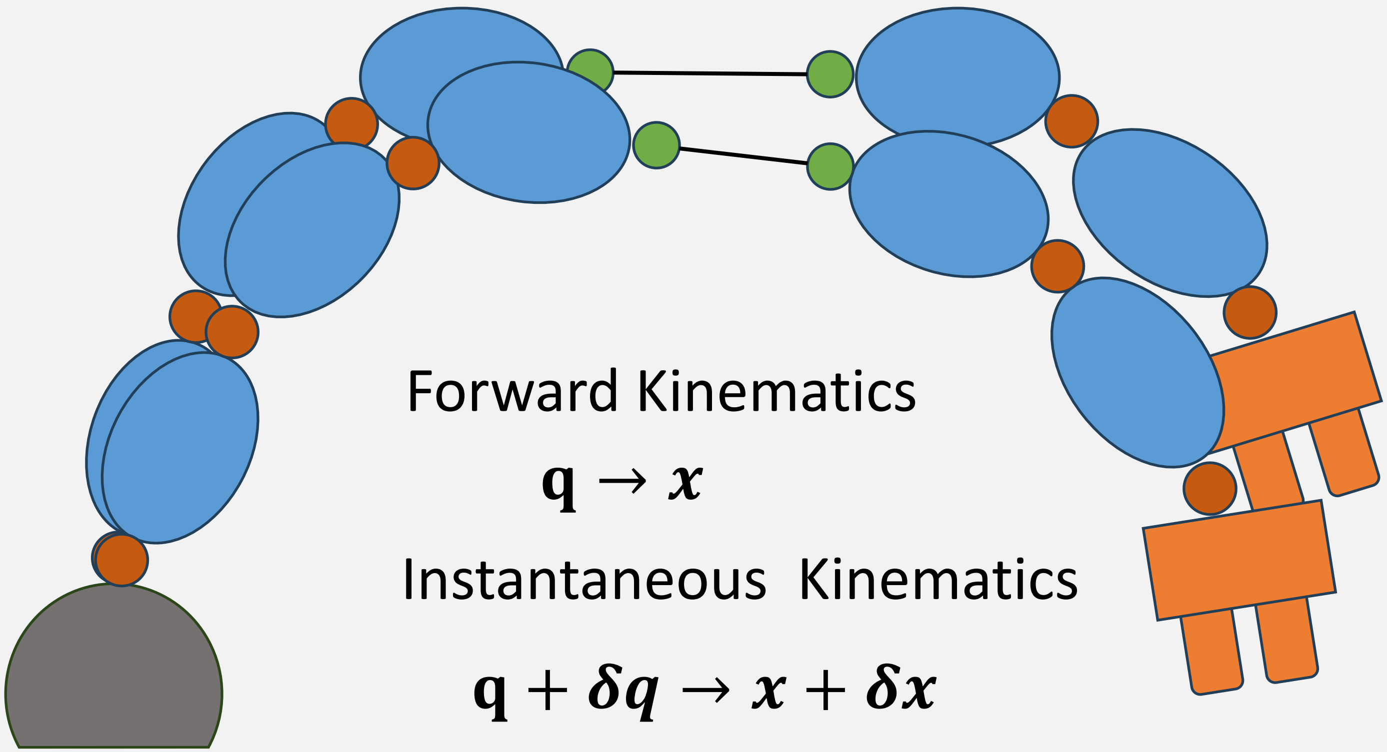
Forward Kinematics



Forward Kinematics

$$\boldsymbol{q} \rightarrow \boldsymbol{x}$$

Instantaneous Kinematics



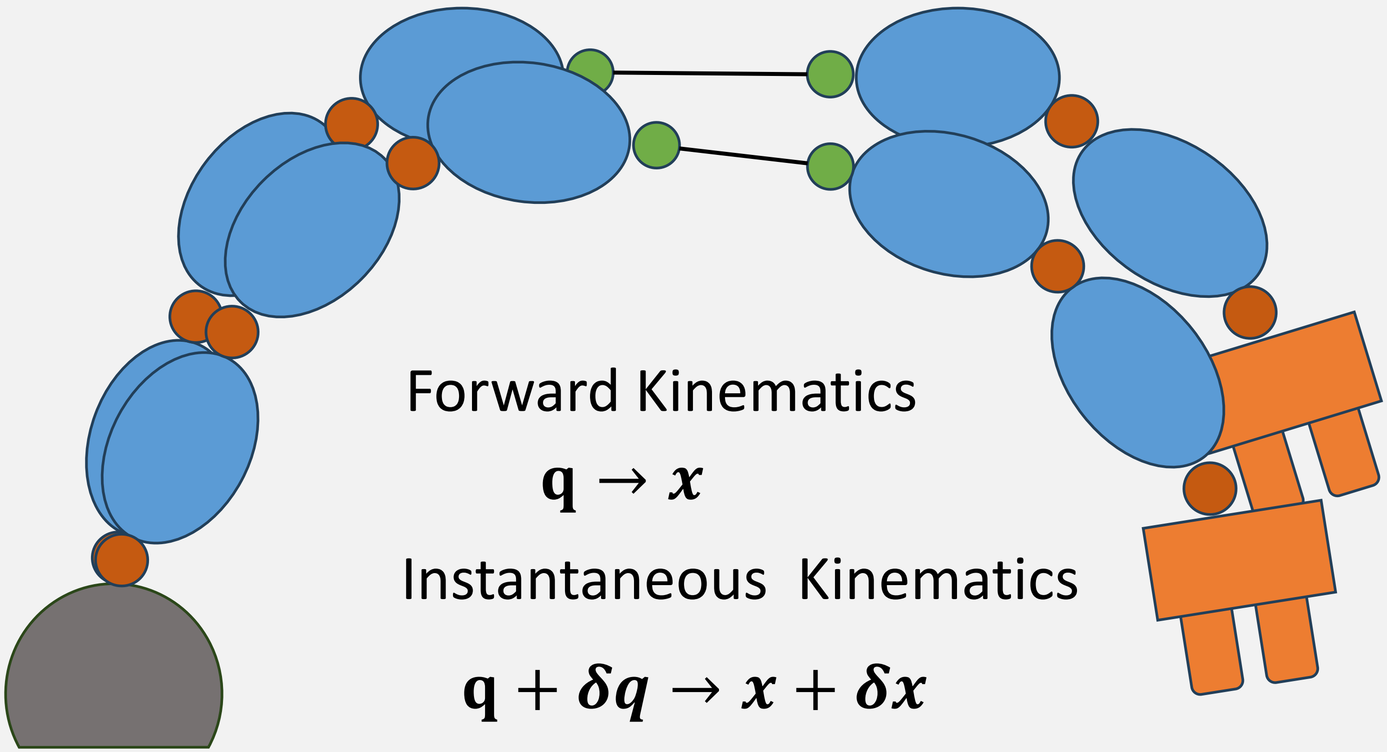
Forward Kinematics

$$\mathbf{q} \rightarrow \boldsymbol{x}$$

Instantaneous Kinematics

$$\mathbf{q} + \boldsymbol{\delta q} \rightarrow \boldsymbol{x} + \boldsymbol{\delta x}$$

Instantaneous Kinematics



Forward Kinematics

$$\mathbf{q} \rightarrow \boldsymbol{x}$$

Instantaneous Kinematics

$$\mathbf{q} + \boldsymbol{\delta q} \rightarrow \boldsymbol{x} + \boldsymbol{\delta x}$$

Relationship: $\boldsymbol{\delta q} \leftrightarrow \boldsymbol{\delta x}$

$\dot{\boldsymbol{q}} \leftrightarrow \dot{\boldsymbol{x}}$ Linear Velocity

Angular Velocity

Joint Coordinates

Coordinate i: $\begin{cases} \theta_i & \text{revolute} \\ d_i & \text{prismatic} \end{cases}$

Joint coordinate: $q_i = \varepsilon_i \theta_i + \bar{\varepsilon}_i d_i$

$$\varepsilon_i : \begin{cases} 1 & \text{revolute} \\ 0 & \text{prismatic} \end{cases}$$

$$\approx = 1 - \varepsilon_i$$

Joint Coordinate Vector: $q = (q_1 q_2 q_3 \cdots q_n)^T$

Jacobian: Direct Differentiation

$$x = f(q) \qquad \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} f_1(q) \\ f_2(q) \\ \vdots \\ f_m(q) \end{bmatrix}$$

$$\delta x_1 = \frac{\delta f_1}{\delta q_1}\delta q_1 + \frac{\delta f_1}{\delta q_2}\delta q_2 + \cdots + \frac{\delta f_1}{\delta q_n}\delta q_n$$

$$\vdots$$

$$\delta x_m = \frac{\delta f_m}{\delta q_1}\delta q_1 + \frac{\delta f_m}{\delta q_2}\delta q_2 + \cdots + \frac{\delta f_m}{\delta q_n}\delta q_n$$

$$\delta x = \begin{bmatrix} \frac{\delta f_1}{\delta q_1} & \cdots & \frac{\delta f_1}{\delta q_n} \\ \vdots & \ddots & \vdots \\ \frac{\delta f_m}{\delta q_1} & \cdots & \frac{\delta f_m}{\delta q_n} \end{bmatrix} \delta q$$

$$\delta x_{(m\times 1)} = J(q)_{(m\times n)}\, \delta q_{(n\times 1)}$$

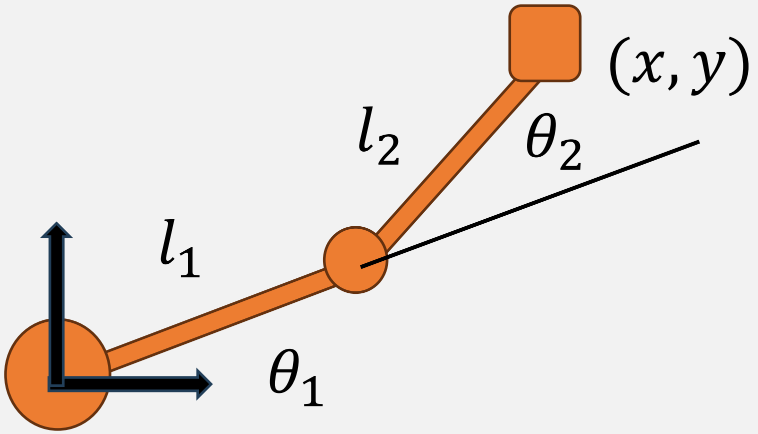
Jacobian

$$\delta x_{(m\times 1)} = J(q)_{(m\times n)}\ \delta q_{(n\times 1)}$$

$$\dot{x}_{(m\times 1)} = J(q)_{(m\times n)}\ \dot{q}_{(n\times 1)}$$

$$J(q)_{(ij)} = \frac{\delta f_i(q)}{\delta q_j}$$

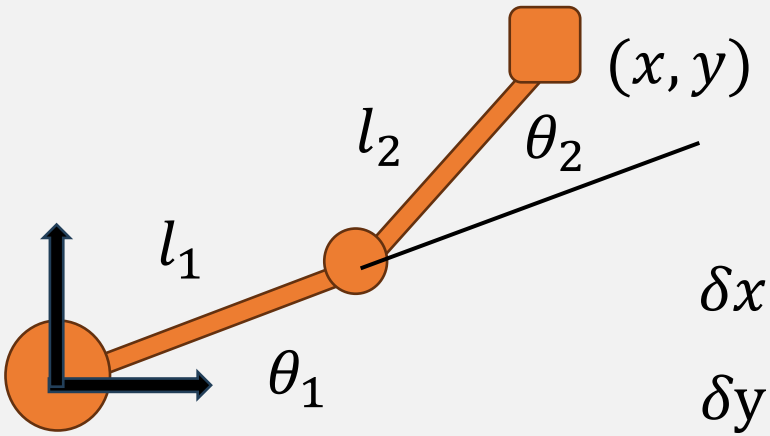
Example



$$x = l_1 c_1 + l_1 c_{12}$$

$$y = l_1 s_1 + l_2\, s_{12}$$

Example



$$x = l_1 c_1 + l_1 c_{12}$$

$$\mathrm{y} = l_1 s_1 + l_2\, s_{12}$$

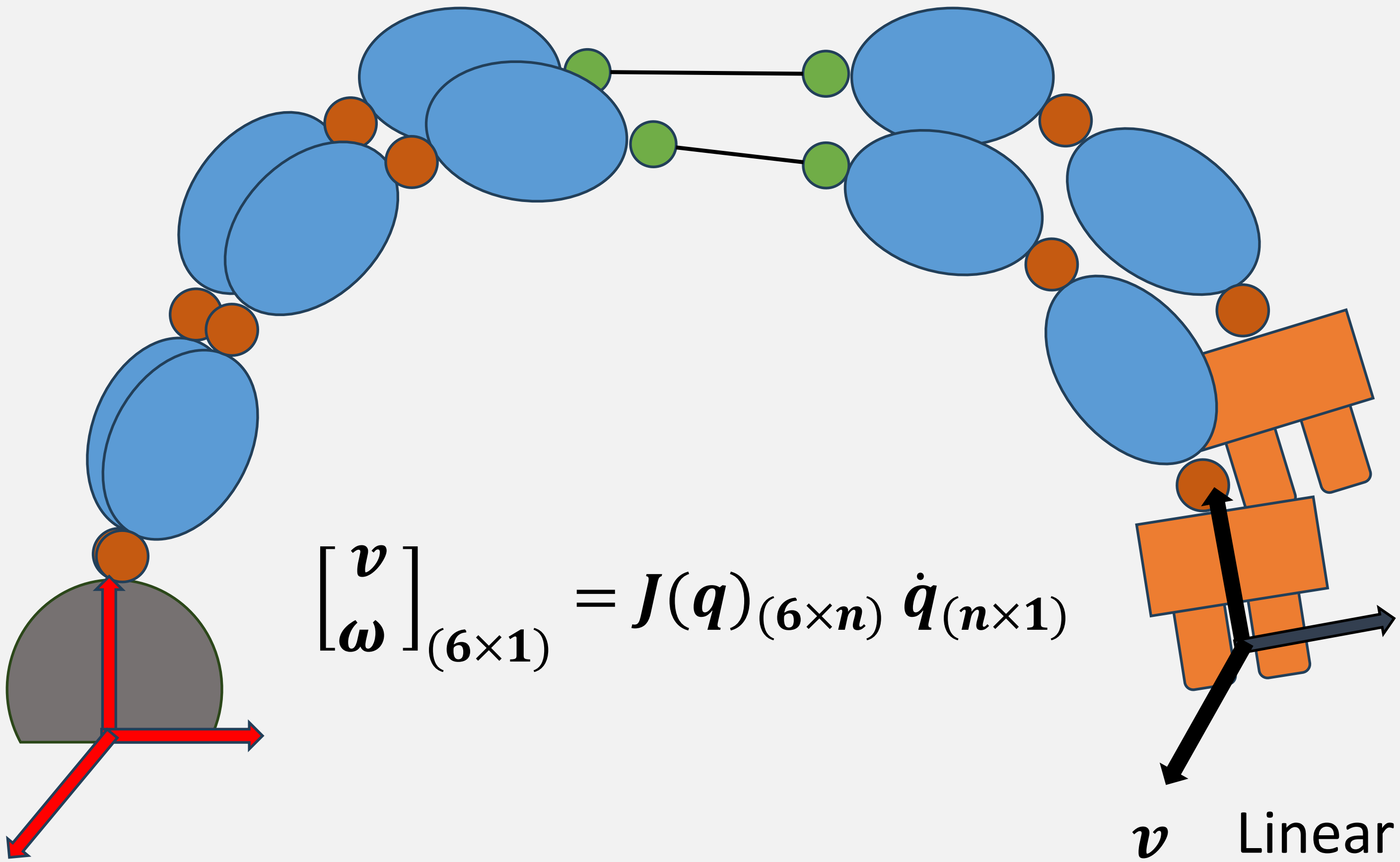
$$\delta x = -(l_1 s_1 + l_2 s_{12})\delta\theta_1 - l_2 s_{12}\delta\theta_2$$

$$\delta \mathrm{y} = (l_1 c_1 + l_2\, c_{12})\, \delta\theta_1 + l_2 c_{12}\delta\theta_2$$

$$\begin{bmatrix} \delta x \\ \delta \mathrm{y} \end{bmatrix} = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} \\ (l_1 c_1 + l_2\, c_{12}) & l_2 c_{12} \end{bmatrix} \begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{\delta x}{\delta\theta_1} & \frac{\delta x}{\delta\theta_2} \\ \frac{\delta y}{\delta\theta_1} & \frac{\delta y}{\delta\theta_2} \end{bmatrix} = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} \\ (l_1 c_1 + l_2\, c_{12}) & l_2 c_{12} \end{bmatrix}$$

Jacobian

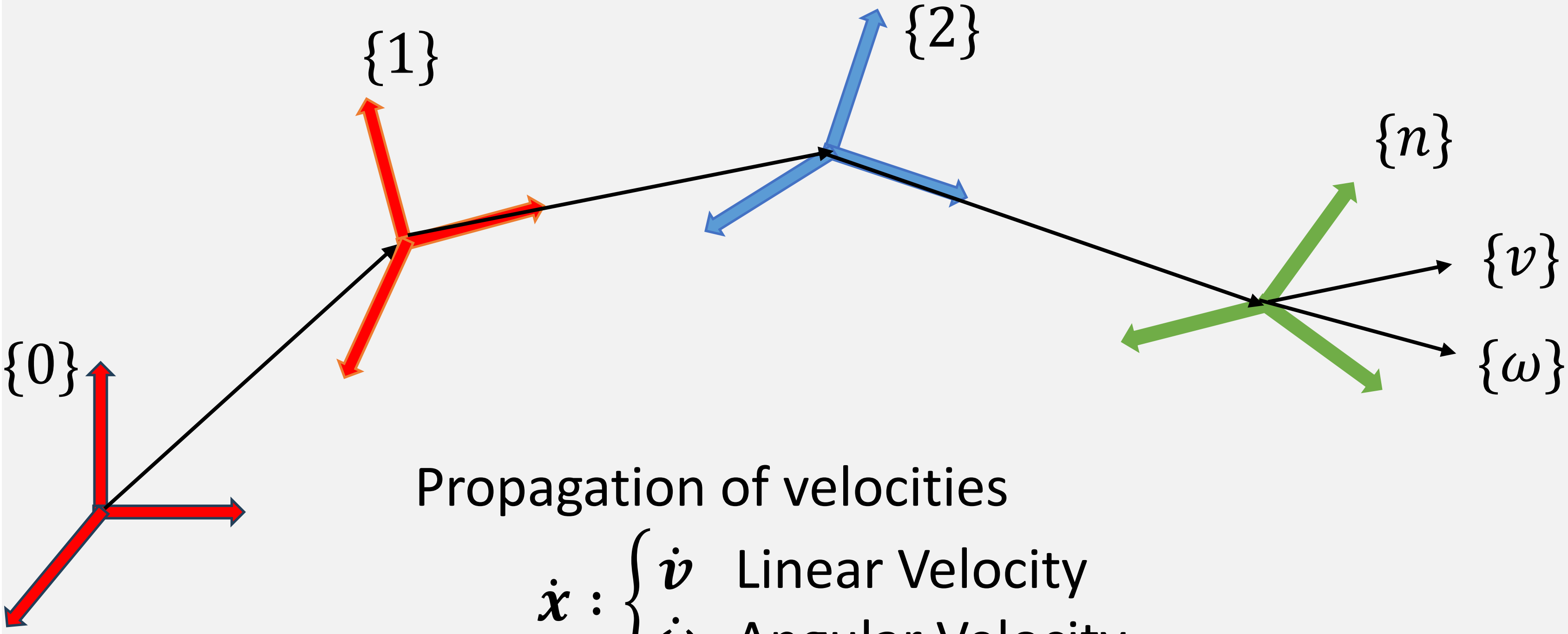


$$\begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix}_{(6\times1)} = \boldsymbol{J}(\boldsymbol{q})_{(6\times n)}\,\dot{\boldsymbol{q}}_{(n\times1)}$$

$\boldsymbol{v}$ Linear Velocity

$\boldsymbol{\omega}$ Angular Velocity

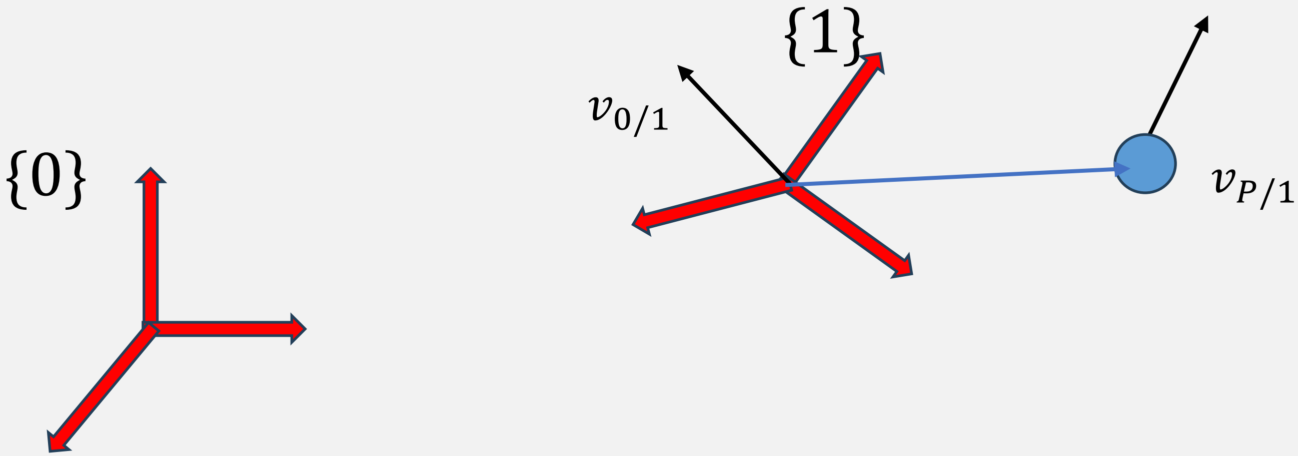
Spatial Mechanisms



Propagation of velocities

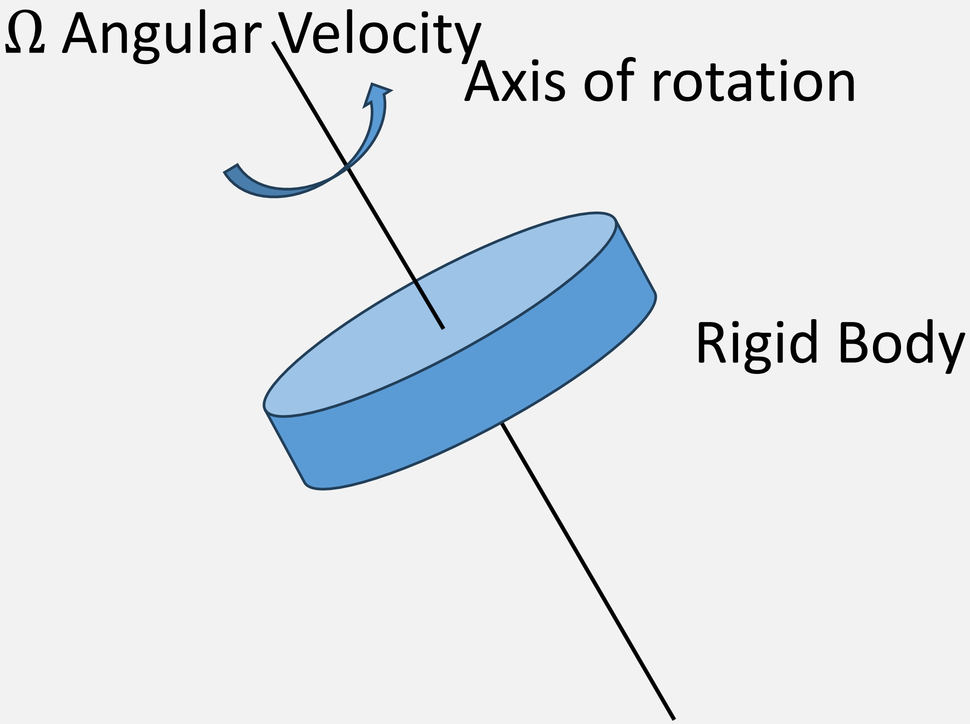
$$\dot{\boldsymbol{x}} : \begin{cases} \dot{\boldsymbol{v}} & \text{Linear Velocity} \\ \dot{\boldsymbol{\omega}} & \text{Angular Velocity} \end{cases}$$

Pure Translation

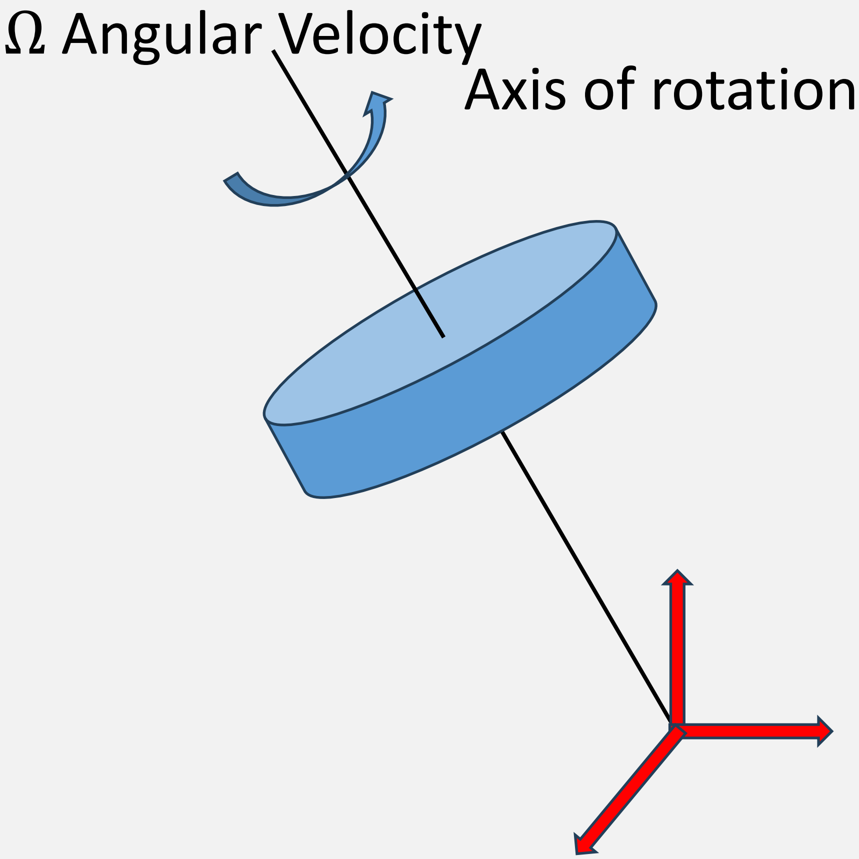


$$v_{P/0} = v_{0/1} + v_{P/1}$$

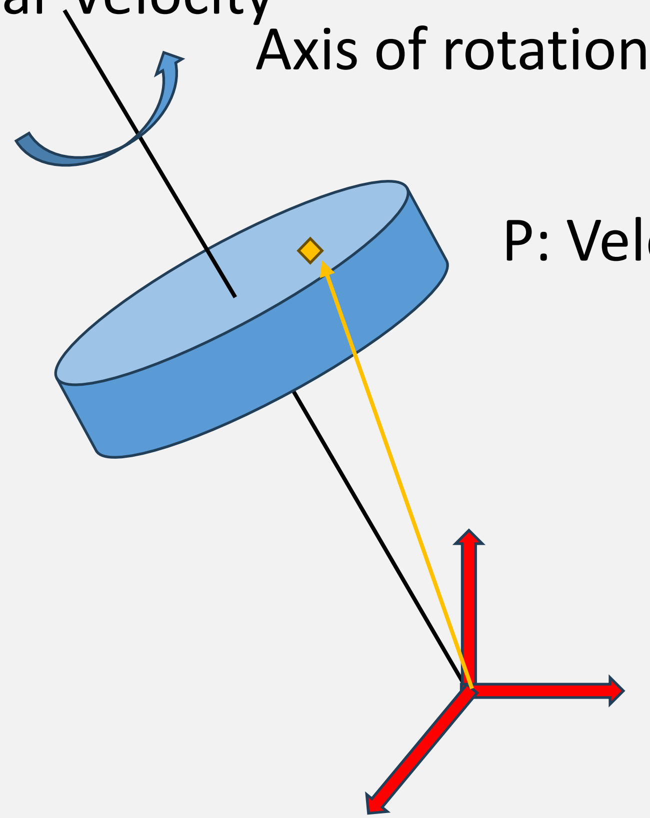
Rotational Motion



Rotational Motion



Rotational Motion



$$v_P = \Omega \times \mathrm{P}$$

Cross Product Operator

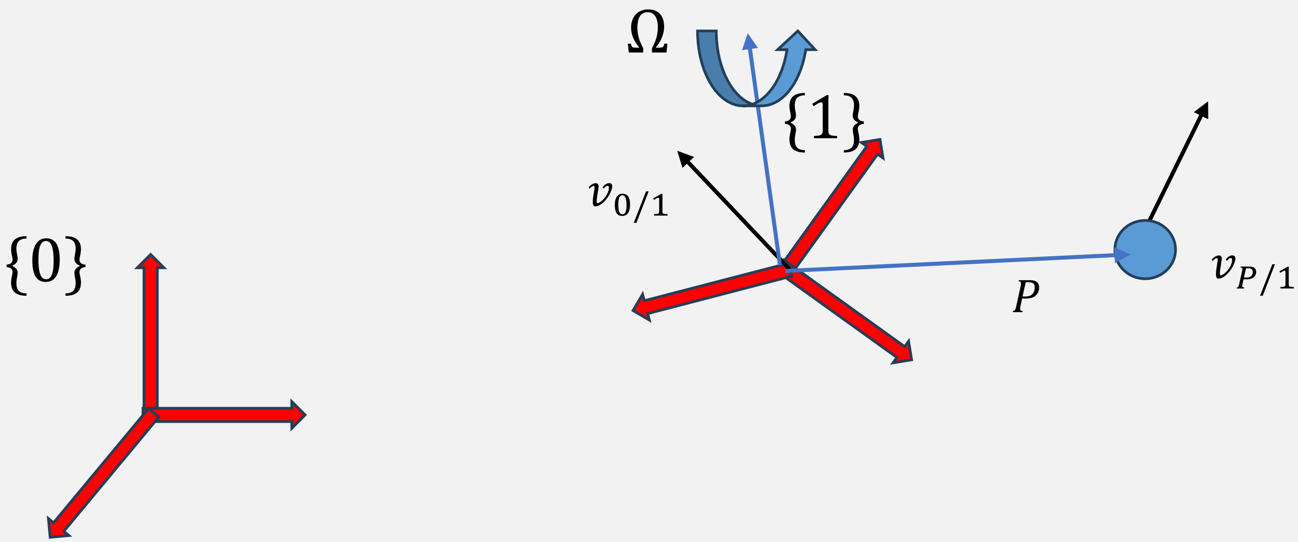
$$v_P = \Omega \times \mathrm{P} \qquad v_P = \widehat{\Omega}\mathrm{P}$$

$$\Omega = \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} \quad P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

$$\Omega\times \rightarrow \widehat{\Omega} \begin{bmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{bmatrix}$$

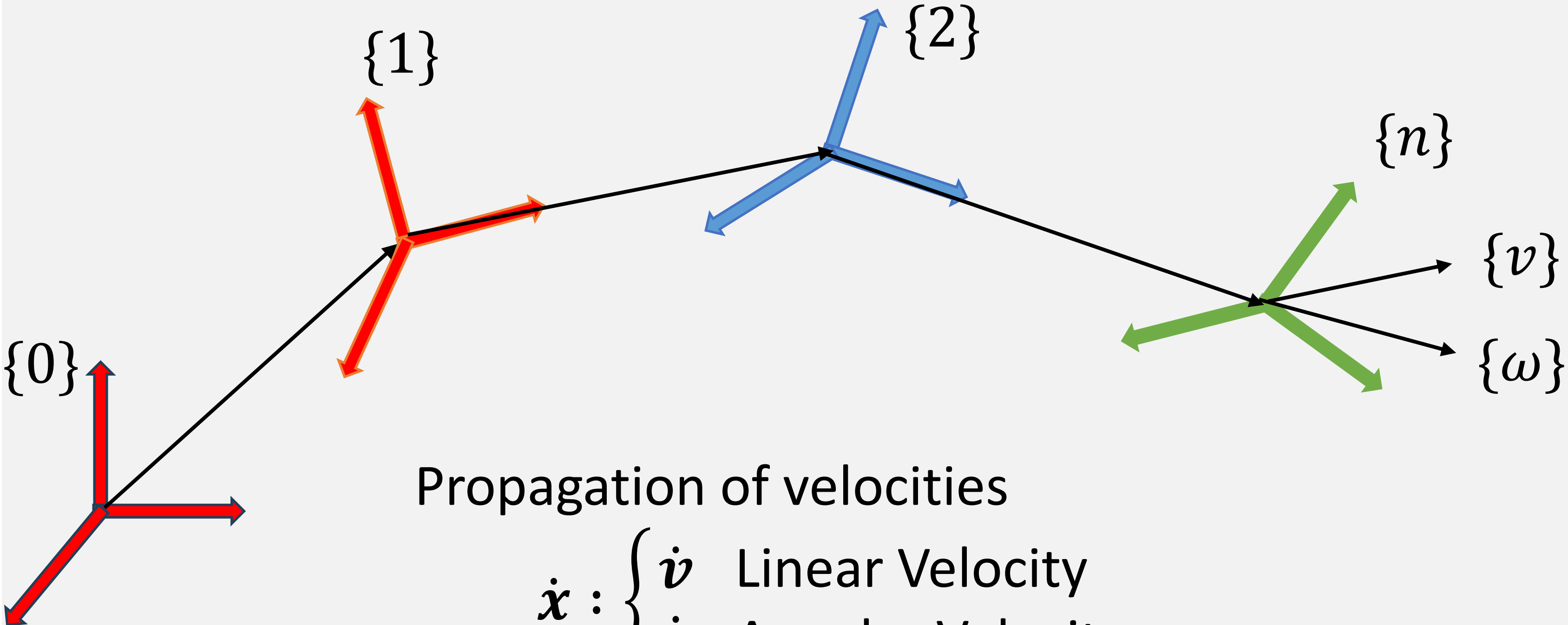
$$\Omega = \begin{bmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

Linear and Angular Motion



$$v_{P/0} = v_{0/1} + v_{P/1} + \Omega \times P$$

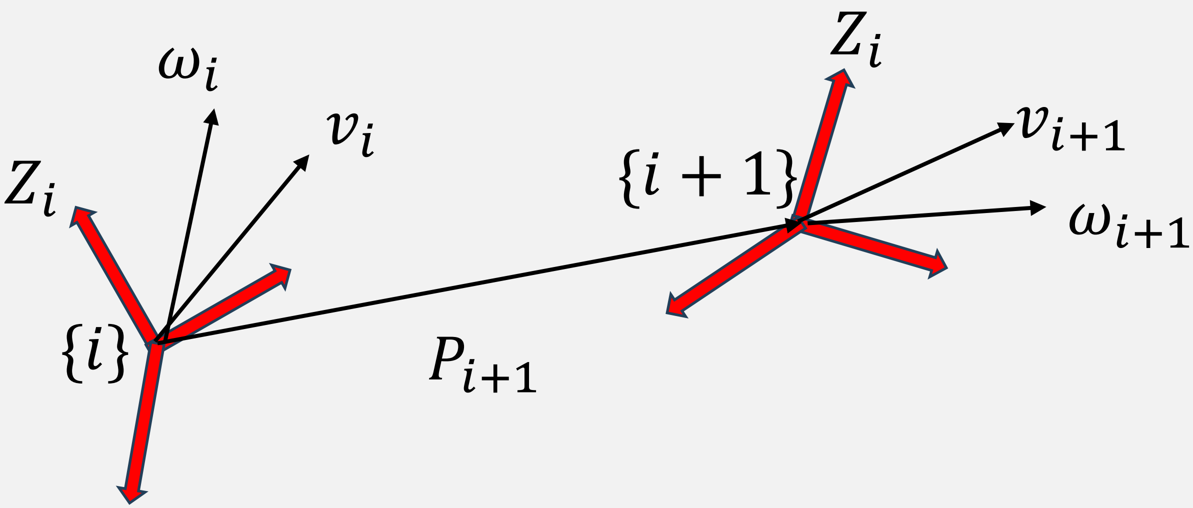
Spatial Mechanisms



Propagation of velocities

$$\dot{\boldsymbol{x}} : \begin{cases} \dot{\boldsymbol{v}} & \text{Linear Velocity} \\ \dot{\boldsymbol{\omega}} & \text{Angular Velocity} \end{cases}$$

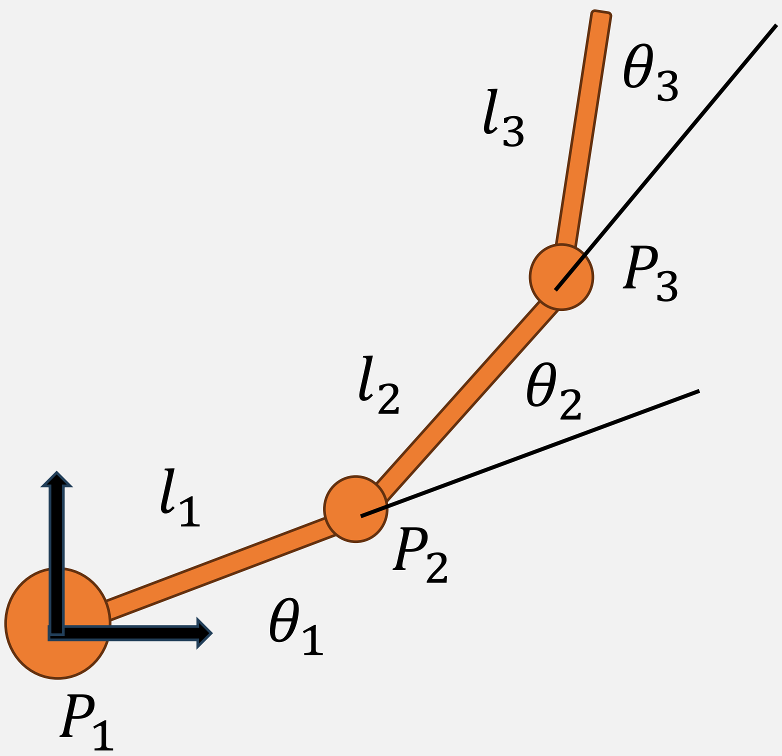
Velocity propagation



Linear $v_{i+1} = v_i + \omega_i \times P_{i+1} + (\dot{d}_{i+1}\, Z_{i+1}\ if\ prismatic)$

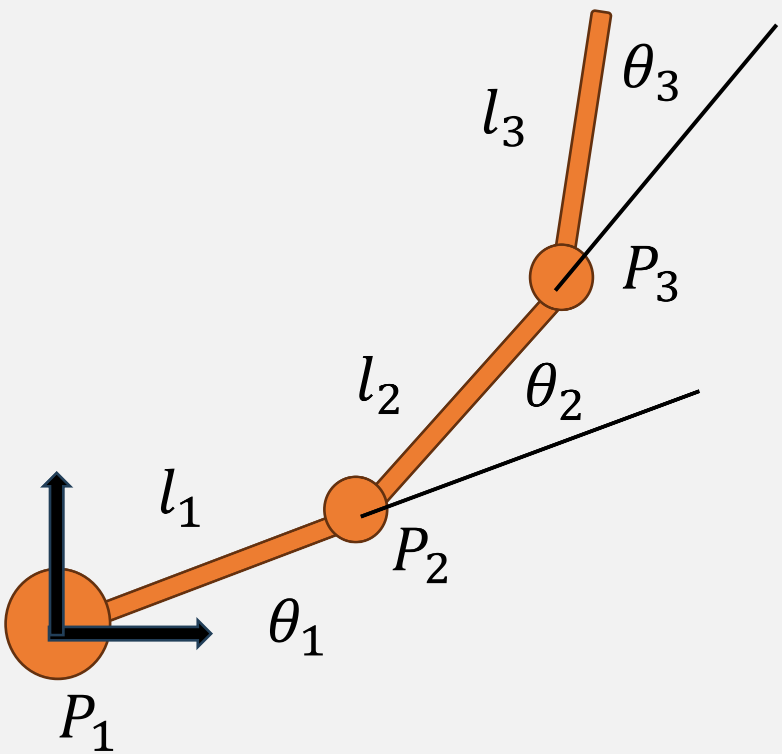
Angular $\omega_{i+1} = \omega_i + (\dot{\theta}_{i+1}\, Z_{i+1}\ if\ revolute)$

Example



$$v_{i+1} = v_i + \omega_i \times P_{i+1}$$

Example



$$v_{i+1} = v_i + \omega_i \times P_{i+1}$$

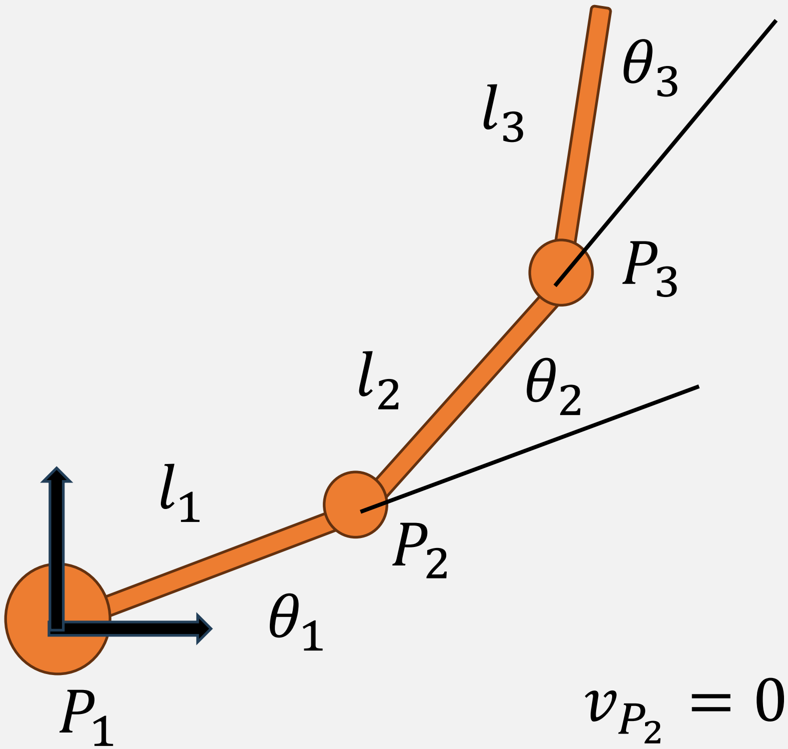
$$v_{P_1} = 0$$

$$v_{P_2} = v_{P_1} + {}^0\omega_1 \times P_2$$

$$v_{P_3} = v_{P_2} + {}^0\omega_2 \times P_3$$

$${}^0\omega_2 = \dot{\theta}_1 Z_1 + \dot{\theta}_2 Z_2$$

Example



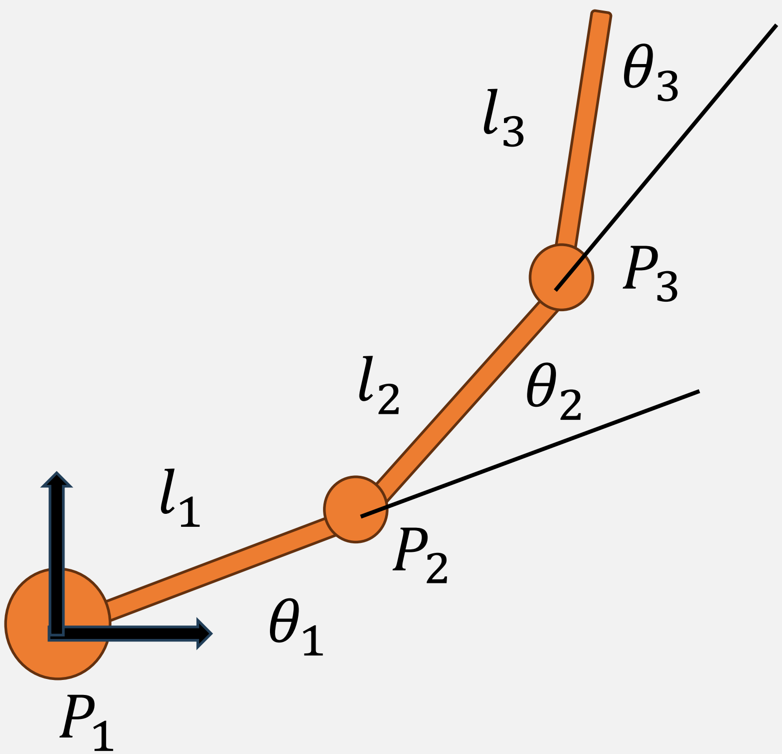
$$v_{i+1} = v_i + \omega_i \times P_{i+1}$$

$$v_{P_1} = 0 \qquad \omega_1 = \dot{\theta}_1 Z_1$$

$$v_{P_2} = v_{P_1} + \omega_1 \times P_2$$

$$v_{P_2} = 0 + \begin{bmatrix} 0 & -\dot{\theta}_1 & 0 \\ \dot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_1 s_1 \\ l_1 c_1 \\ 0 \end{bmatrix} \dot{\theta}_1$$

Example



$$v_{i+1} = v_i + \omega_i \times P_{i+1}$$

$$v_{P_1} = 0 \qquad \omega_1 = \dot{\theta}_1 Z_1$$

$$v_{P_2} = v_{P_1} + {}^{0}\omega_1 \times P_2$$

$$v_{P_3} = v_{P_2} + {}^{0}\omega_2 \times P_3$$

$${}^{0}\omega_2 = (\dot{\theta}_1 + \dot{\theta}_2)$$

Example

$$v_{P_3} = v_{P_2} + \omega_2 \times P_3$$

$$v_{P_3} = \begin{bmatrix} -l_1 s_1 \\ l_1 c_1 \\ 0 \end{bmatrix} \dot{\theta}_1 + \begin{bmatrix} 0 & -(\dot{\theta}_1 + \dot{\theta}_2) & 0 \\ (\dot{\theta}_1 + \dot{\theta}_2) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_2 c_{12} \\ l_2 s_{12} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_1 s_1 \\ l_1 c_1 \\ 0 \end{bmatrix} \dot{\theta}_1 + \begin{bmatrix} -l_2 s_{12} \\ l_2 c_1 \\ 0 \end{bmatrix} \left(\dot{\theta}_1 + \dot{\theta}_2\right)$$

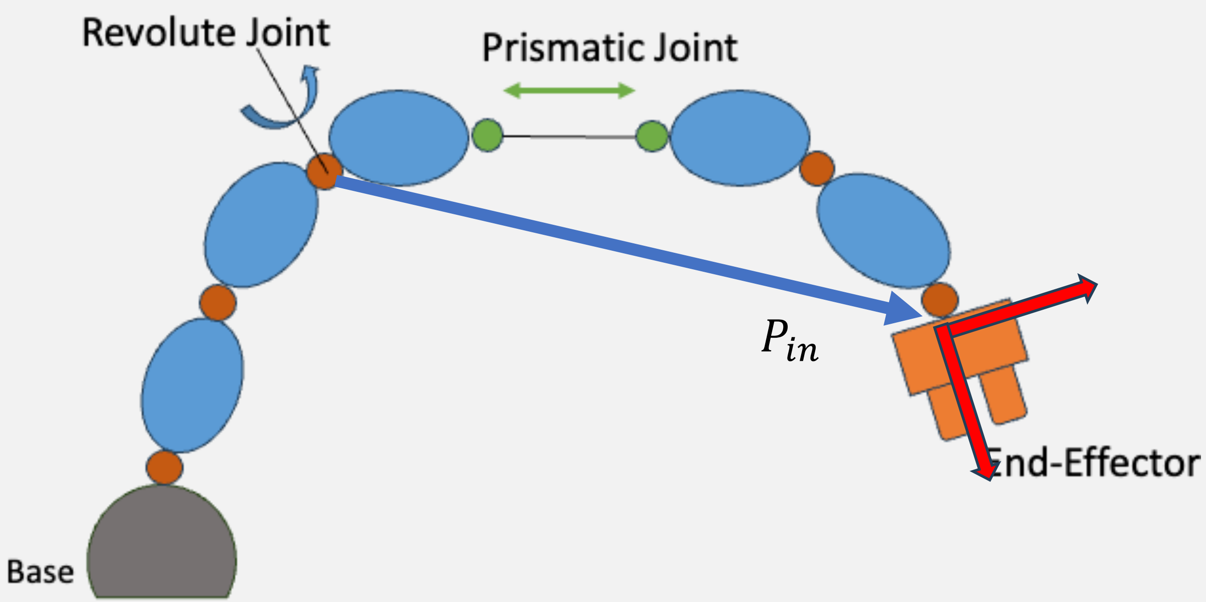
$$= \begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} & 0 \\ (l_1 c_1 + l_2 c_{12}) & l_2 c_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = J_v \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

Example

$$^0\omega_3 = (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)$$

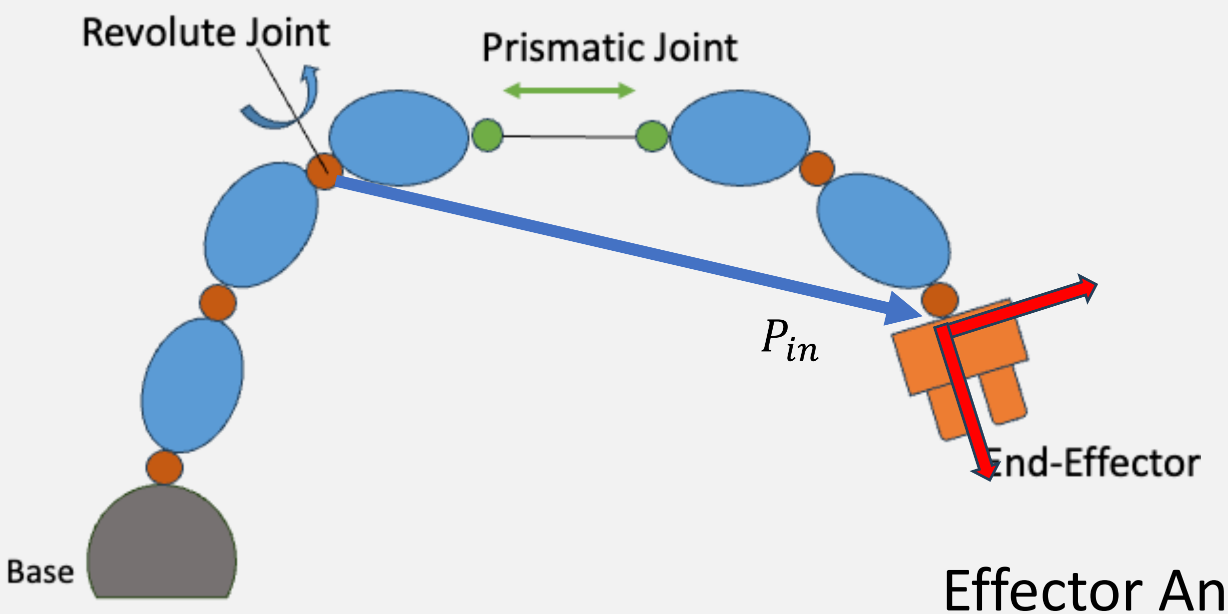
$$^0\omega_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = J_\omega \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

Jacobian (explicit form)



Effector	Prismatic	Revolute
Angular Vel	None	Ω_j
Linear Vel	V_i	$\Omega_j \times P_{in}$

Jacobian (explicit form)



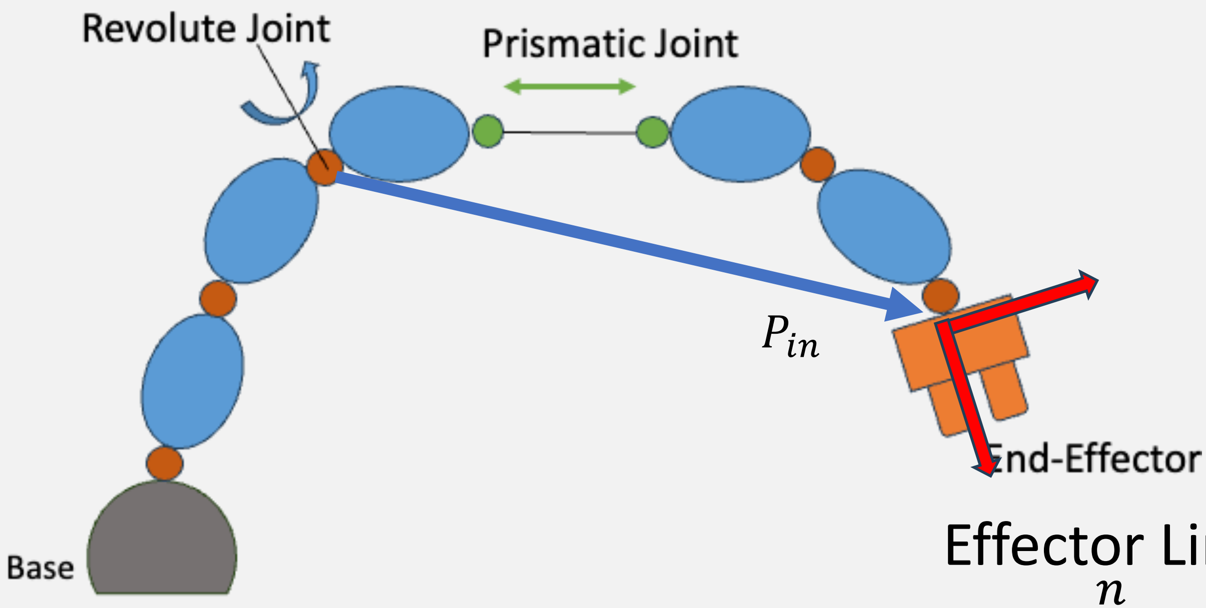
$$\Omega_i = Z_i \dot{q}_i$$

Effector	Prismatic	Revolute
Angular Vel	None	Ω_i
Linear Vel	V_i	$\Omega_i \times P_{in}$

Effector Angular Velocity

$$\omega = \sum_{i=1}^{n} \bar{\varepsilon}_i \Omega_i = \sum_{i=1}^{n} (\bar{\varepsilon}_i Z_i) \dot{q}_i$$

Jacobian (explicit form)



$$\Omega_i = Z_i \dot{q}_i$$

$$V_i = Z_i \dot{q}_i$$

Effector	Prismatic	Revolute
Angular Vel	None	Ω_i
Linear Vel	V_i	$\Omega_i \times P_{in}$

Effector Linear Velocity

$$\mathrm{v} = \sum_{i=1}^{n} \varepsilon_i V_i + \bar{\varepsilon}_i (\Omega_i \times P_{in})$$

$$= \sum_{i=1}^{n} [\varepsilon_i Z_i + \bar{\varepsilon}_i (Z_i \times P_{in})] \dot{q}_i$$

Jacobian (explicit form)

$$\omega = \sum_{i=1}^{n} (\bar{\varepsilon}_i Z_i)\dot{q}_i$$

$$= [\bar{\varepsilon}_1 Z_1 \quad \bar{\varepsilon}_2 Z_2 \quad \dots \quad \bar{\varepsilon}_n Z_n] \begin{bmatrix} \dot{q}_i \\ \dot{q}_i \\ \vdots \\ \dot{q}_i \end{bmatrix} \qquad \omega = J_\omega \dot{q}$$

$$v = \sum_{i=1}^{n} [\varepsilon_i Z_i + \bar{\varepsilon}_i (Z_i \times P_{in})]\dot{q}_i \qquad \mathrm{v} = J_v \dot{q}$$

$$= [\varepsilon_1 Z_1 + \bar{\varepsilon}_1 (Z_1 \times P_{1n}) \quad \varepsilon_2 Z_2 + \bar{\varepsilon}_2 (Z_2 \times P_{2n}) \quad \dots \quad \varepsilon_n Z_n] \begin{bmatrix} \dot{q}_i \\ \dot{q}_i \\ \vdots \\ \dot{q}_i \end{bmatrix}$$