CS5340:
Tutorial 1

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Announcements

- To Bell-Curve or Not To Bell Curve? Survey (Quiz) on Canvas
- Past year Quiz questions uploaded.
- Tutorial Solutions will be uploaded the next day.

1. Two Numbers Game

- Team 1:
  - Pick 2 **different** numbers between 0 and 10 (inclusive).
  - Write each number on a piece of paper each.
  - Turn the papers face down.
- Team 2:
  - Objective is to pick the **larger number**.
  - Pick one of the pieces of paper.
  - Have a peek at the number.
  - **Decide:** do you keep this number or *switch*?
- **Question:** Can Team 2 win more than 50% of the time?

Yes!

Team 2 (Cheating Strategy)

- Team 2 has a **spy** in Team 1
- Team 1 picks two numbers L and H
- Spy tells you a number between L and H
  - If L and H are next to one another, then spy tells you L
- Example:
  - Team 1 picks 3 and 8
  - Spy says 5
  - Team 2 picks randomly.
  - Team 2: Do you switch?

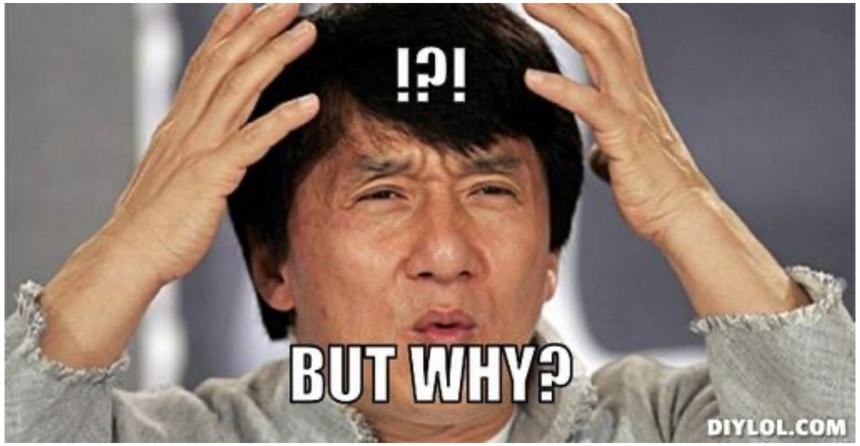
- Team 2 has a 100% of winning!

From Lecture 1:
Academic Honesty

- Please be **academically honest**.
  - *"Give credit where credit is due"*
- Do ***NOT*** cheat
- **Strict** Plagiarism policy:
  - If you cheat, we will report you to the disciplinary board
  - If found guilty, you will get an F (University Policy)

Team 2 Strategy

- Randomly pick $z \in [0, 10)$
- Take a peek at one of the numbers, lets call this x
- If $x \leq z$, switch, else stick with x

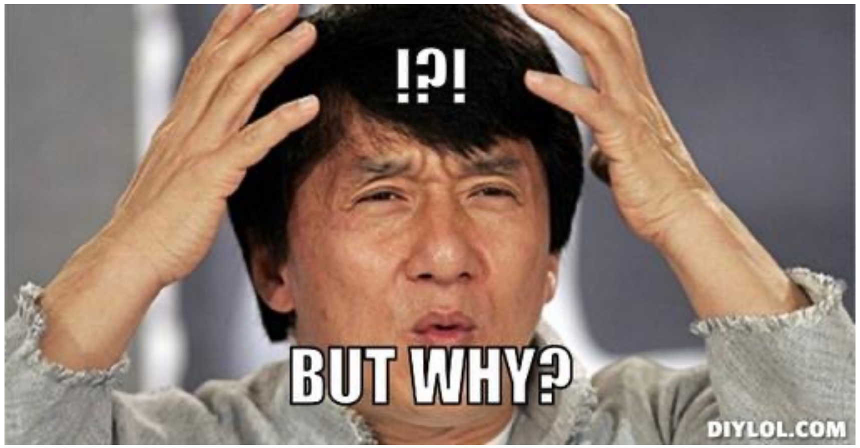


In Groups

- Introduce yourselves.
- Split into Team 1 and Team 2 and play this game.
- Discuss and analyze why the randomized strategy works.
  - Use the techniques learnt in the lecture (tree diagram)
- What is the probability Team 2 wins using this strategy?
- If you are done, move on to the other problems in the tutorial.

Team 2 Strategy

- Randomly pick $z \in [0, 10)$
- Take a peek at one of the numbers, lets call this x
- If $x \leq z$, switch, else stick with x



Strategy Analysis

- Let $L \ < H$ be the numbers chosen by Team 1.
- 3 possible cases:
  - **Case** just-right: $L \leq Z < H$:
    - Team 2 wins always!
    - $p(win|\text{justright}) = 1$ and $p(M) \geq \frac{1}{10}$
  - **Case** too-high: $H \ \leq Z$:
    - Team 2 switch. Only wins if picked L
    - $p(win|\text{toohigh}) = \frac{1}{2}$
  - **Case** too-low: $Z \ < L$:
    - Team 2 stays. Only wins if picked H
    - $p(win|\text{toolow}) = \frac{1}{2}$

$$p(\text{win}) \geq \left(1 \times \frac{1}{10}\right) + \frac{1}{2}\left(1 - \frac{1}{10}\right)$$
$$p(\text{win}) \geq \frac{11}{20}$$

More complete derivation in: MIT Math for CS, Chapter 18.3.3
https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-042j-mathematics-for-computer-science-spring-2015/readings/MIT6_042JS15_Session31.pdf

Strategy Analysis (general case)

- Team 1 chooses 2 numbers $[0, \dots, n]$
- Team 2 picks half-integers:

$$\frac{1}{2}, \frac{3}{2}, \dots, \frac{2n-1}{2}$$

- Analysis is similar to before
- 3 cases:
  - $p(z$ just right): $(H - L)/n$
  - $p(z$ too low): L/n
  - $p(z$ too high): $(n - H)/n$

Source: https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-042j-mathematics-for-computer-science-spring-2015/readings/MIT6_042JS15_Session31.pdf

Strategy Analysis (general case)

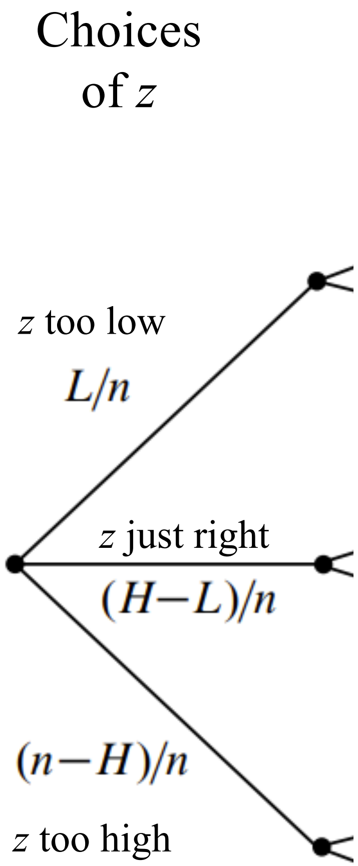


Figure 18.3 The tree diagram for the numbers game.

Strategy Analysis (general case)

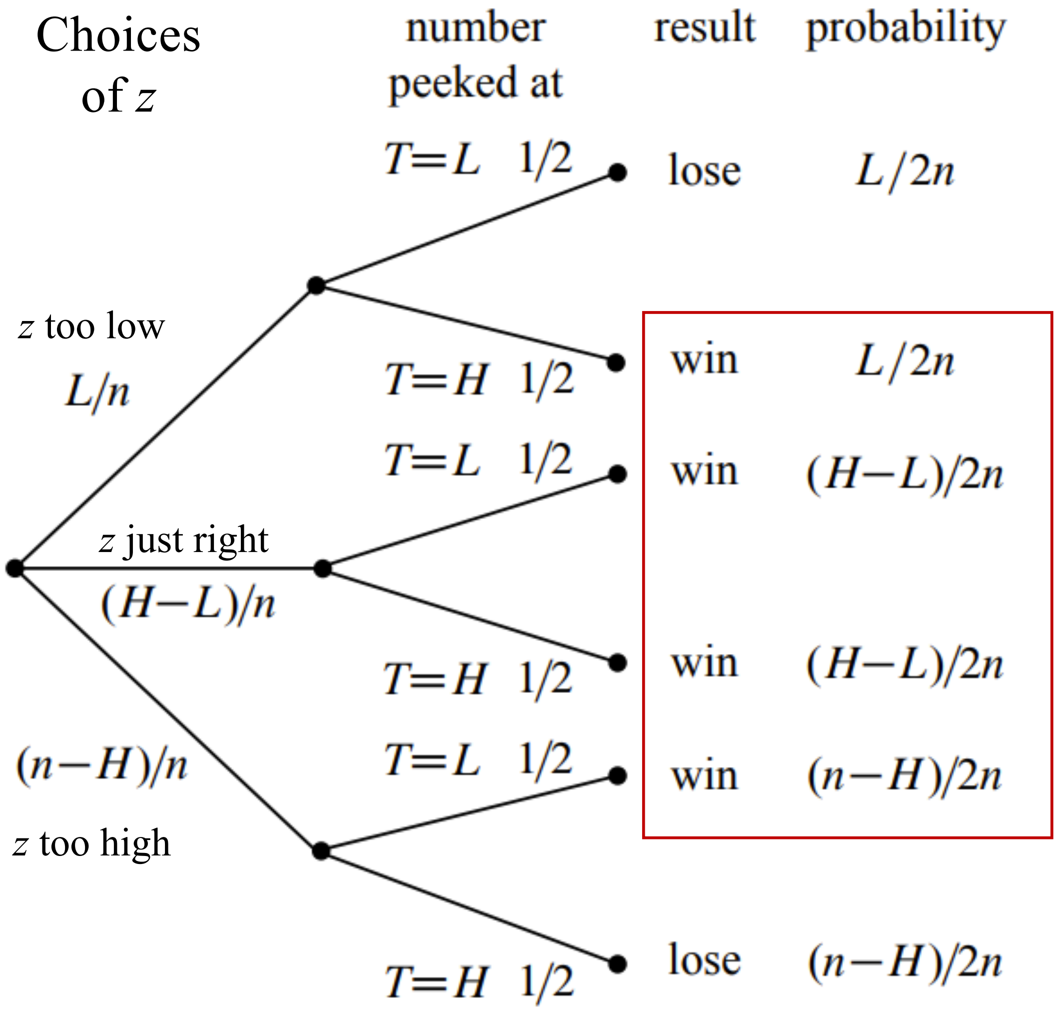


Figure 18.3 The tree diagram for the numbers game.

Probablity of win:

$$
\begin{aligned}
&p(\text{win})\\
&= \frac{L}{2n} + 2\frac{H-L}{2n} + \frac{n-H}{2n}\\
&= \frac{1}{2} + \frac{H-L}{2n}\\
&\geq \frac{1}{2} + \frac{1}{2n}
\end{aligned}
$$

Source: https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-042j-mathematics-for-computer-science-spring-2015/readings/MIT6_042JS15_Session31.pdf

https://www.quantamagazine.org/solution-information-from-randomness-20150722/

More rigorous: https://arxiv.org/pdf/1608.01899.pdf

Questions?

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2. Legal Reasoning

Setup:

- Blood found at a scene.
- Blood type present is type S.
- Type S blood is found in 1% of the population.

Prosecutor: *"there is a 1% chance the defendant would have blood type S if he was innocent. Thus, there is a 99% chance he is guilty!"*

Is the prosecutor correct? Justify your answer.

What is your decision?

- You are the Jury. Do you vote to convict the Defendant?
- Put in your vote onto polleverywhere

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2. Legal Reasoning

The argument is **wrong**. Let:

- A = *"person has blood type S"*
- B = *"person is innocent"*

Prosecutor has quoted $p(A|B)$ (or $1 - p(A|B)$).
However, what is relevant is $p(B|A)$

$$p(\text{innocent}|S) = \frac{p(S|\text{innocent})\, p(\text{innocent})}{p(S)}$$

This is known as the **prosecutor's fallacy**

Questions?

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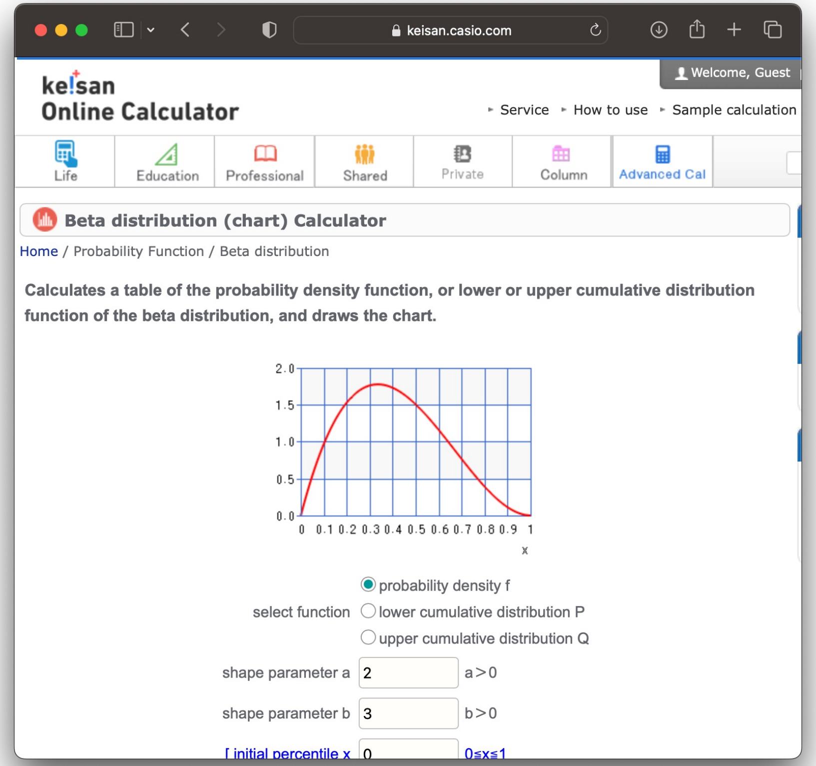
3. Conjugate Analysis

Show that the Beta Distribution is conjugate to the Binomial Distribution.

https://keisan.casio.com/exec/system/1180573226



3. Conjugate Analysis

Show that the Normal distribution is conjugate to the Normal distribution with unknown mean but known variance.

4. Variance of a Sum

Show that

$$V[X+Y] = V[X] + V[Y] + 2\mathrm{Cov}[X,Y]$$

where $\mathrm{Cov}[X,Y] = E[XY] - E[X]E[Y]$
and $V[X] = E[X^2] - E[X]^2$

Questions?

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Please Prepare for Next Week

- Complete Tutorial 1.
- Watch the videos.
- Do Tutorial 2.

Slides

- L2-FittingSimpleModels.pdf

Video Lectures

- L2 - Part 1 (Introduction)
- L2 - Part 2 (MLE)
- L2 - Part 3 (Bayesian Inference)
- L2 - Part 4 (MAP)
- L2 - Part 5 (Prediction)
- L2 - Part 6 (Exponential Family)

Tutorial Sheet (for next week)

- Tut2_23.pdf
- To Bell-Curve or Not To Bell-Curve, that is the question.
  25 Jan | 0 pts

$\epsilon \sim \mathcal{N}(\epsilon \mid \mu, \sigma_*^2)$

$\mathcal{N}(\mu_0, \sigma_0^2)$, $\mu_0 = 0$

$D = \{\epsilon_1, \epsilon_2, \ldots, \epsilon_n\}$

$p(\mu \mid D)$

$p(\mu \mid \hat{\mu}, \hat{\sigma}^2)$

$p(D \mid \mu, \sigma^2)$