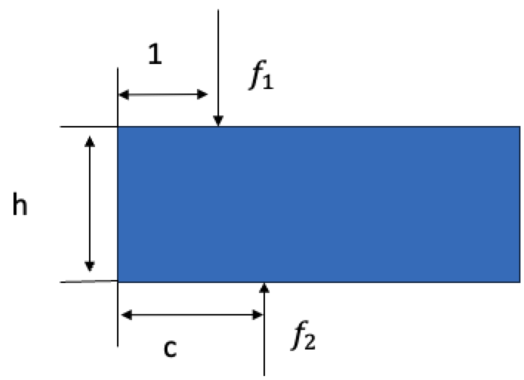
Intelligent Robots (CS4278/CS5478) Homework 2

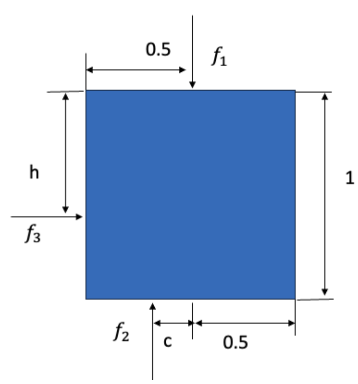
(Autumn 2023) Due: **11:59pm October 15**

Name ____________________________

1. Figure 1 shows a rectangle in 2D space with two contact points. The rectangle has a height of h. f_1 and f_2 are two point contacts with friction coefficient μ. Derive the range of μ such that the grasp yields force closure and write it as function of h and c.

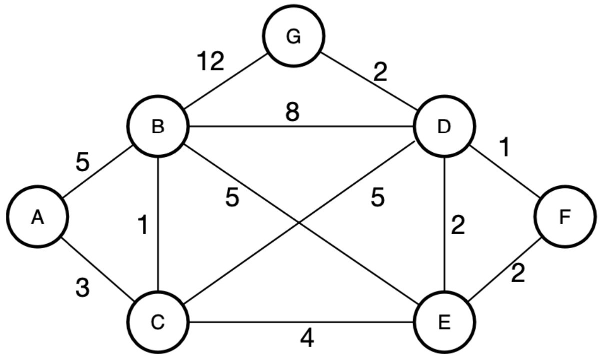


2. Figure 2 shows a square that is restrained by three contact points. f_1 is a point contact with friction coefficient μ, while f_2 and f_3 are frictionless point contacts. If $c = 0.25$ and $h = 0.5$, find one value of μ such that the grasp yields force closure.



3. Find the shortest path to node A from every other node in the weighted graph below.

   (a) Apply the backward dynamic programming algorithm. Let $V^*(s)$ be the shortest-path length from node s to A and $V_i(s)$ be the estimated shortest-path length in the i'th iteration of the



dynamic programming algorithm. Show the values for $V_0(s)$, $V_1(s)$, and $V_2(s)$ as well as $V^*(s)$ for all nodes in the graph.

	A	B	C	D	E	F	G
V_0							
V_1							
V_2							
V^*							

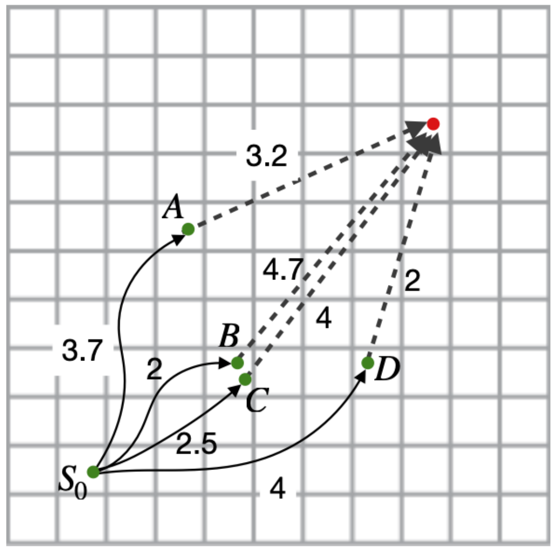
(b) Apply the Dijkstra's algorithm. The shortest paths form a tree. Draw the shortest-path tree.

4. Give the dimension of the configuration space for the following systems. Briefly justify your answer.
   (a) An articulated robot in a 2D plane with a fixed base and two revolute joints.
   (b) Two mobile robots freely translate and rotate in the plane.
   (c) An aerial manipulator consisting of two manipulators attached to an unmanned aerial vehicle (UAV). Each manipulator has 6 revolute joints.

5. This problem explores the configuration space of lines and that of line segments.
   (a) Consider an infinite line l that translates and rotates freely in 3-D space. Give two different parameterizations of the configuration space C for l: one that makes use of angles and one that makes no use of angles.
   (b) What is the dimension of C?
   (c) Consider a straight-line segment s that translates and rotates freely in 3-D space. What is the dimension of the configuration space for s? Can you use the two parameterizations in part (a) for s? What modifications would be needed if any?

6. Suppose that the configurations space $\mathcal{C}$ is the unit space $[0,1]\times[0,1]$. The multi-query PRM algorithm, first samples n collision-free configurations and then tries to connect these milestones by calling **LINK**.
   (a) If the algorithm calls **LINK** for every pair of roadmap nodes, give an asymptotic upper bound on the number of calls to **LINK**.
   (b) Suppose that the algorithms calls **LINK** only if the Euclidean distance between two milestones is smaller than a threshold t. Give an asymptotic bound on the number of calls to LINK when $t = O(\sqrt{1/n})$. You may assume that the milestones are distributed roughly uniformly in C.

7. Consider the hybrid A* algorithm described in the class and apply it to plan the motion of an autonomous robot car. Suppose that the A* search starts at the node S, shown in the figure below. By applying four candidate actions, it reaches new nodes: A, B, C, and D. The cost-to-come and the heuristic estimate of the cost-to-go for all the new nodes are shown in the figure.



   (a) What is the dimension of the grid used in the hybrid A* search?

   (b) What does the priority queue contain at the stage of the hybrid A* search illustrated in the figure? For each item, in the priority queue, specify the node and its associated f -value.

8. A key step in applying the A^* algorithm in practice is to design a good heuristic function. Suppose that we want to apply A^* to a shortest-path problem and have two heuristic functions $h_1(x)$ and $h_2(x)$, both of which are admissible.

   (a) Show that $h(x) = \max\{h_1(x), h_2(x)\}$ is also admissible.

   (b) Of the three heuristic functions, $h_1(x)$, $h_2(x)$, and $h(x)$, which one would you use? Why? Give a proof if you can.