Intelligent Robots (CS4278/CS5478) Homework 1

(Autumn 2023) Due: **11:59pm September 10**

Name ____________________________

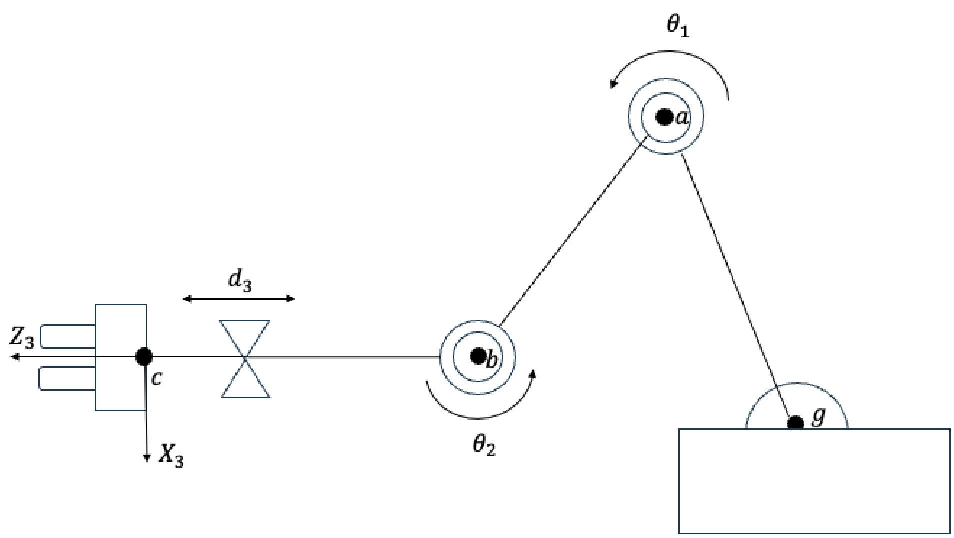
1. Given two frames {B} and {C} that are initially coincident with each other. First, we rotate {C} about $\hat{Z}_C$ by θ_1 degrees. Then, we rotate the resulting frame {C} about the new $\hat{Y}_C$ by θ_2.

   (a) Determine the 3×3 rotation matrix, ${}^B_C R$, that will change the description of a vector P in frame {C}, ${}^C\mathbf{P}$, to frame {B}, ${}^B\mathbf{P}$.

   (b) What is the value of ${}^B_C R$, if $\theta_1 = 45°$, $\theta_2 = 60°$?

(c) We then define a new frame A which translates from the frame B along the vector of ${}^{B}\mathbf{q} = [q_1, q_2, q_3]^T$. Write down the homogeneous transformation ${}^{A}_{C}T$ from frame C to frame A.

2. Consider the following manipulator with two revolute joint and one prismatic joint.



(a) Draw the frames of this manipulator. Define l_1 to the length connecting points g and a, and l_2 to be the length connecting points a and b. Note that frame 3 has been done for you, and your solution needs to be consistent with the given frame 3.

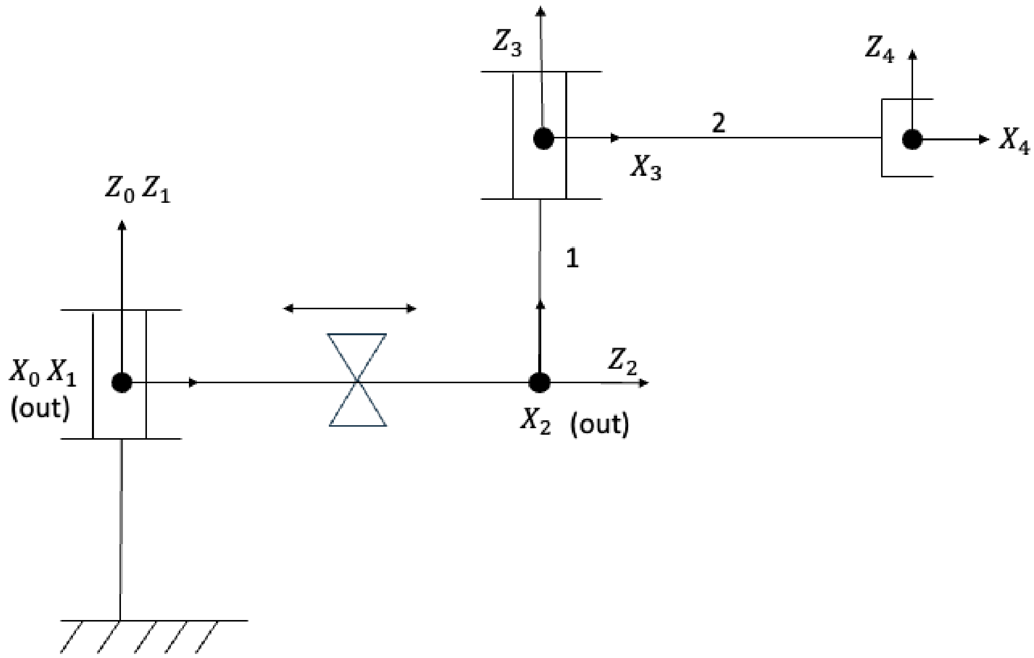
Hint: Frame 0 is not located at point g

(b) Find the Denavit-Hartenberg parameters for this manipulator and fill in the entries of the following table

i	a_{i-1}	α_{i-1}	d_i	θ_i
1				
2				
3				

(c) Given $\theta_1 = 225°$, $\theta_2 = 45°$, $l_1 = 0.5$, $l_2 = 0.4$, and $d_3 = 0.25$, find the matrix ${}^0_3T$ at the configuration from part (a). You may write down the answer as a product of matrices.

3. Let us consider the RPR manipulator with 3 links represented in the schematic below. The schematic is drawn in the configuration $\theta_1 = 0, \theta_3 = 90°$



Luckily, you do not need to compute the forward kinematics, because they are given to you here (note that $c_{13} = \cos(\theta_1 + \theta_3)$):

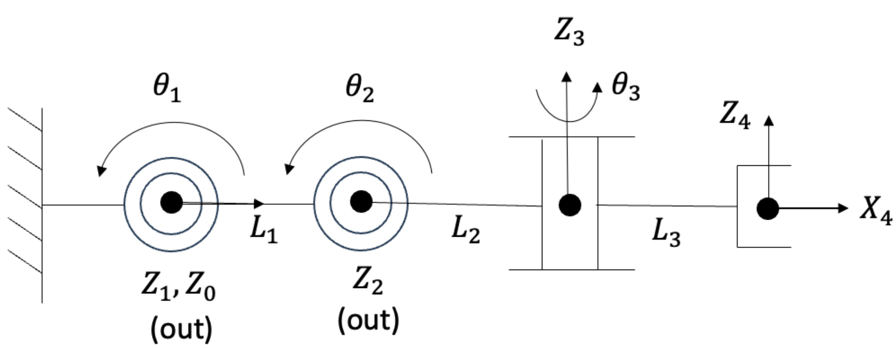
$$
{}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^0_{,2}T = \begin{bmatrix} c_1 & 0 & -s_1 & -d_2 s_1 \\ s_1 & 0 & c_1 & d_2 c_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^0_{,3}T = \begin{bmatrix} c_{13} & -s_{13} & 0 & -d_2 s_1 \\ s_{13} & c_{13} & 0 & d_2 c_1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_ET = \begin{bmatrix} c_{13} & -s_{13} & 0 & 2c_{13} - d_2 s_1 \\ s_{13} & c_{13} & 0 & 2s_{13} + d_2 c_1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

(a) Find the linear velocity and angular velocity of the end-effector in frame $\{0\}$ as a function of the joint variables. Obtain the linear velocity by differentiation.

(c) If the robot is stationary (i.e. $\dot{\mathbf{q}} = \mathbf{0}$ and $\ddot{\mathbf{q}} = \mathbf{0}$), and we apply a force (measured in frame $\{0\}$) of ${}^0F = [F_x\ F_y\ F_z]^T$ on the end-effector, what are the resulting joint torques?

4. You are presented with the RRR manipulator below. L_1, L_2, and L_3 are strictly positive.



(a) Find the Denavit-Hartenberg parameters for this manipulator. Assign the frames such that all your a_i are positive.

i	a_{i-1}	α_{i-1}	d_i	θ_i
1				
2				
3				
4				

(b) The position of the end-effector is:

$$
{}^{0}P_{4} = \begin{bmatrix} L_1c_1 + L_2c_{12} + L_3c_{12}c_3 \\ L_1s_1 + L_2s_{12} + L_3s_{12}c_3 \\ -L_3s_3 \end{bmatrix},
$$

where $c_{12} = cos(\theta_1 + \theta_2)$.

Derive the linear Jacobian ${}^{0}J_{v}$.

(c) Find the singular configurations of this manipulator. For each singularity, draw the robot configuration and clearly state how the movement is restricted (in terms of frame axes).

Hint: The linear Jacobian in frame $\{2\}$ is given to you here:

$$
{}^2J_v = \begin{bmatrix} -L_1 s_2 & 0 & -L_3 s_3 \\ L_1 c_2 + L_2 + L_3 c_3 & L_2 + L_3 c_3 & 0 \\ 0 & 0 & -L_3 c_3 \end{bmatrix}
$$

5. Let us consider the manipulator RPRP shown below, find the linear jacobian ${}^0J_v$ and the angular jacobian ${}^0J_\omega$ for the end effector point (origin of frame $\{4\}$), expressed in frame $\{0\}$.