Decision Analysis

CS4246/CS5446

AI Planning and Decision Making

Recap: Types of Decision Theory

- Normative decision theory
  - Describes how ideal, rational agents should behave
- Descriptive decision theory
  - Describes how actual agents (humans) really behave
- Prescriptive decision theory
  - Prescribes guidelines for agents to behave rationally

Topics

- The decision analysis framework
  - Formulating decision models
    - Decision networks: Influence diagrams (15.5)
    - Decision trees
  - Analyzing decision networks
    - Sensitivity analysis and robust decision models (15.6.6)
- Information value theory (15.6)
  - Expected value of perfect information (15.6.1-15.6.3)
  - Implementing an information gathering agent (15.6.4)

Value of Information

Information and Decision Making

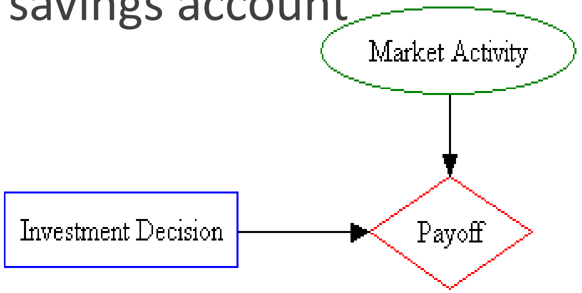
- Information is gathered to reduce uncertainty in decision making
  - Consult experts
  - Conduct surveys
  - Perform mathematical or statistical analyses
  - Do research
  - Read books, journals, newspapers
  - Learn from past data
- Relevant questions:
  - How to evaluate or measure the “value” or usefulness of the information?
  - What does it mean for a knowledge source to provide perfect information?
  - Shall we invest effort or pay $X for additional information to help problem solving?

Value of Information

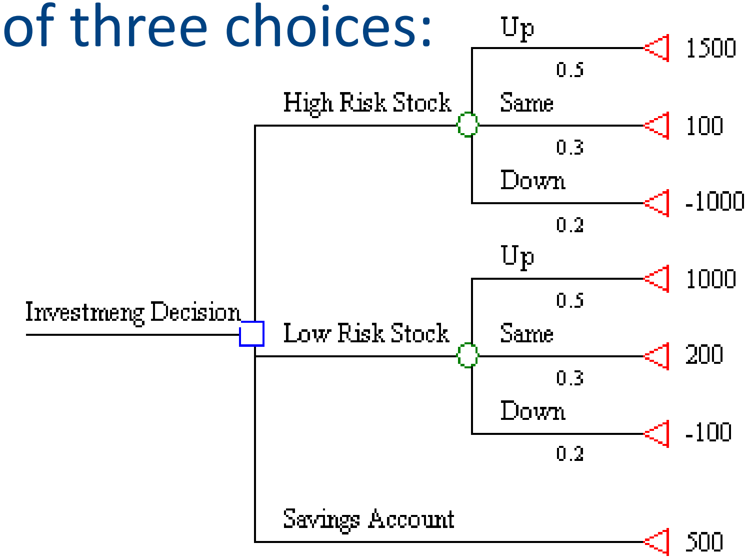
- Costly information gathered to reduce uncertainty
  - How to place value on information in a problem?
  - How to decide whether or not to gather more information?
- Main ideas:
  - Information has value to the extent that it is likely to cause a change of plan and to the extent that the new plan is significantly better than the old one
  - Use conditional probabilities and Bayes' Theorem to model the expected value of information

Example: Stock Investment

- An investor may invest in one of three choices:
  - A high-risk stock
  - A low-risk stock
  - A savings account



Influence diagram



Decision tree

Expected Value of Information

- Expected value of information (EVI)
  - Indicates if information is worth gathering
  - Lower bound: zero expected value
  - Upper bound: expected value of perfect information
- Information has:
  - no value or zero expected value if the same choice will be made before and after obtaining information
  - positive expected value if it leads to a different choice
  - maximum expected value if information is perfect
- EVI is defined in terms of the decision context
  - Different people in different situations may place different values on the same information

Stock Investment Example (cont.)

Should investor consult an expert?

- If the expert always provides perfect information:
  - P(Exp says "Up" | Market Up) = 1
  - P(Exp says "Down" | Market Up) = 0
  - P(Exp says "Up" | Market Down) = 0
- To show there is no uncertainty after hearing the expert, apply Bayes' theorem:

P(Market Up | Exp says "Up")

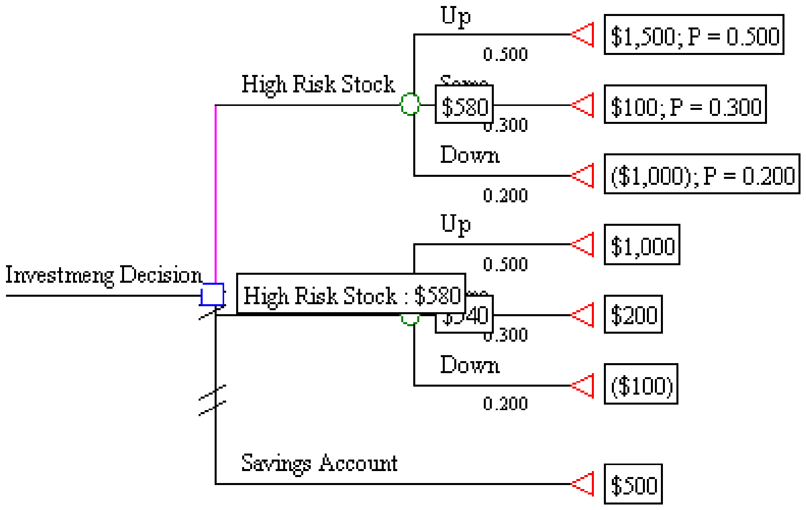
$$= \frac{P(\text{Exp says "Up"} \mid \text{Market Up})\, P(\text{Market Up})}{[P(\text{Exp says "Up"} \mid \text{Market Up})P(\text{Market Up}) + P(\text{Exp says "Up"} \mid \text{Market Down})P(\text{Market Down})]}$$

$= 1$

- Note:
  - The posterior probability is equal to 1 regardless of the prior probability
  - How do real problems differ from the above situation?

Stock Investment Example (cont.)

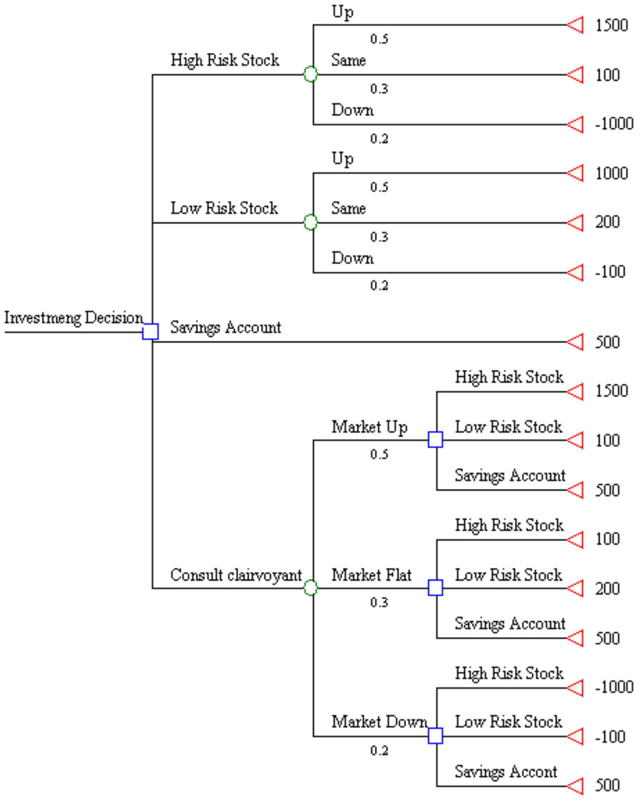
- The optimal choice is the high-risk stock with EMV $580
- Assumption: optimistic about market (Up, 0.5 prob)
- How much would the investor be willing to pay for information about the market activity?



Expected Value of Perfect Information

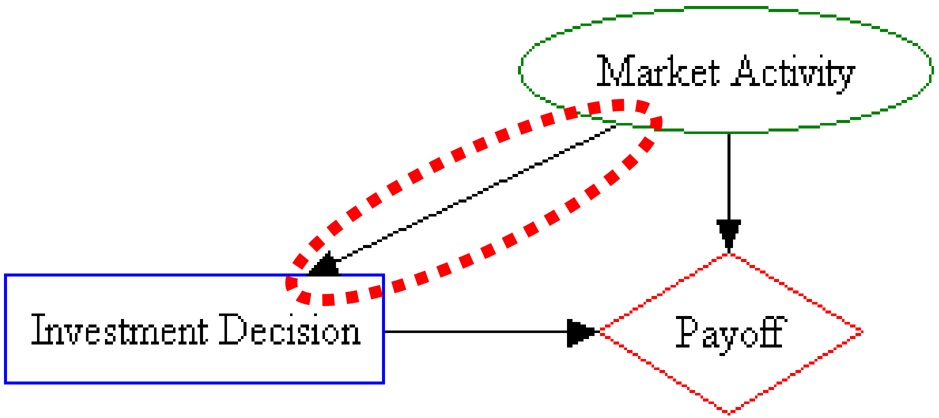
- Max amount that the decision maker is willing to pay for perfect information
- To find expected value of perfect information (EVPI):
  - Modify the decision model to indicate perfect information
  - Solve the model and find the EMV
  - EVPI = EMV (with perfect information) - EMV(original)

Stock Investment Example (w/ PI)



Modifying Influence Diagram for EVPI

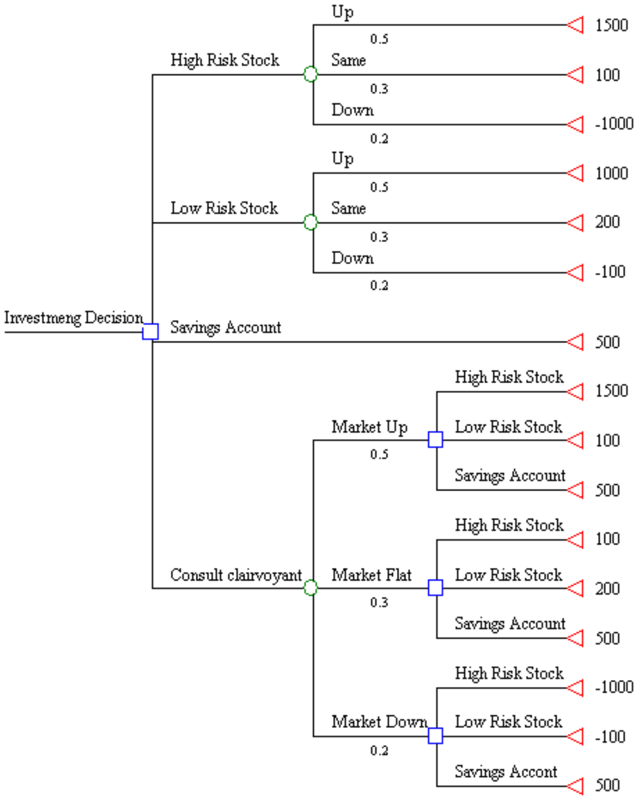
- Impose order on the decision and uncertain event nodes
- The uncertainty nodes for which perfect information is available comes before the decision node



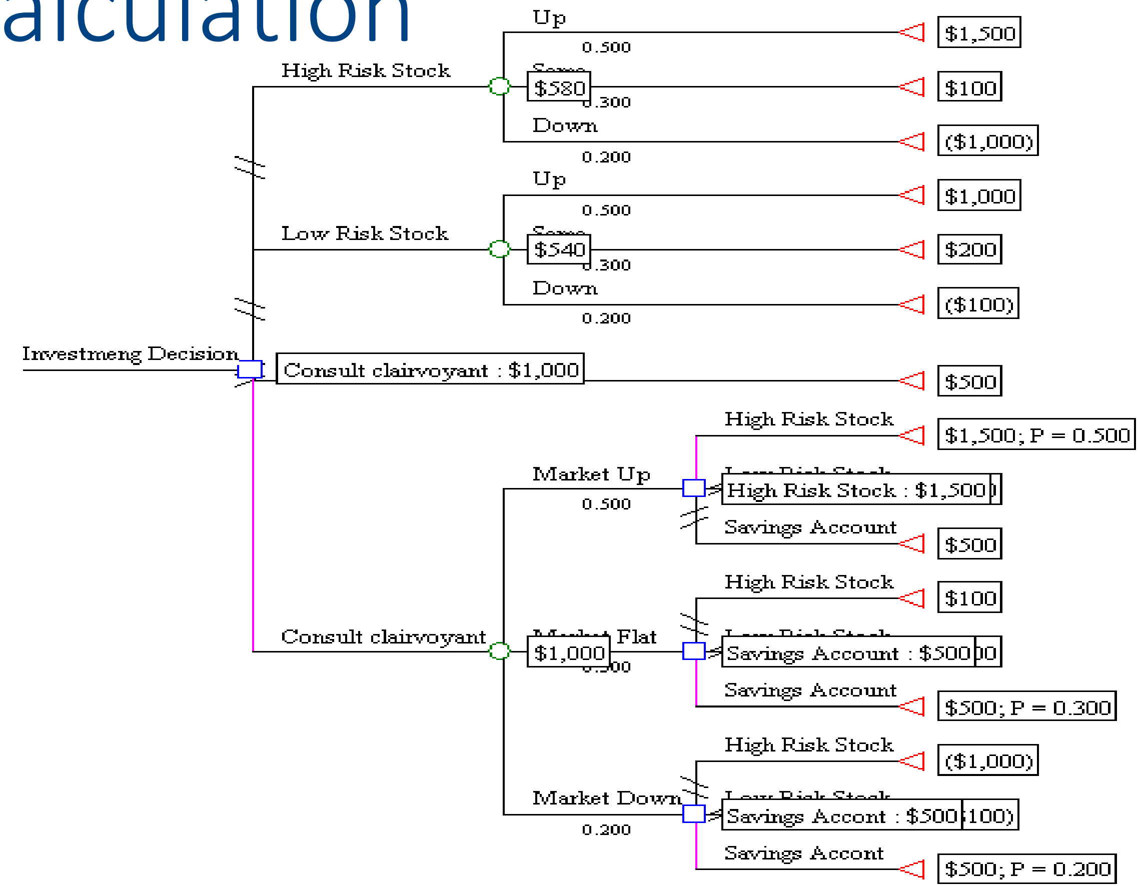
Influence diagram with perfect information

Modifying Decision Tree for EVPI

- Reorder the decision and the uncertain event nodes
- The uncertainty nodes for which perfect information is available comes before the decision node



EVPI Calculation



Expected Value of Perfect Information

- Max amount that the decision maker is willing to pay for perfect information
- To find expected value of perfect information (EVPI):
  - Modify the decision model to indicate perfect information
  - Solve the model and find the EMV ($1000)
  - EVPI = EMV (with perfect information) - EMV(original)

    = $1000 - $580 = $420

(Expected) Value of Perfect Information

Assume exact evidence about variable E_j; compute value of perfect information (VPI)

- Given expected utility with current best action α:

$$EU(\alpha) = \max_a \sum_{s'} P(Result(a) = s')U(s')$$

- Value of the best new action after $E_j = e_j$ is obtained

$$EU\left(\alpha_{e_j} \middle| e_j\right) = \max_a \sum_{s'} P\left(Result(a) = s' \middle| e_j\right)U(s')$$

- Variable E_j can take multiple values e_j, so on averaging:

$$VPI(E_j) = \sum_{e_j} P(E_j = e_j)EU\left(\alpha_{e_j} \middle| E_j = e_j\right) - EU(\alpha)$$

Properties of Value of information

- Expected value of information is always non-negative

$$\forall j \;\; VPI(E_j) \geq 0$$

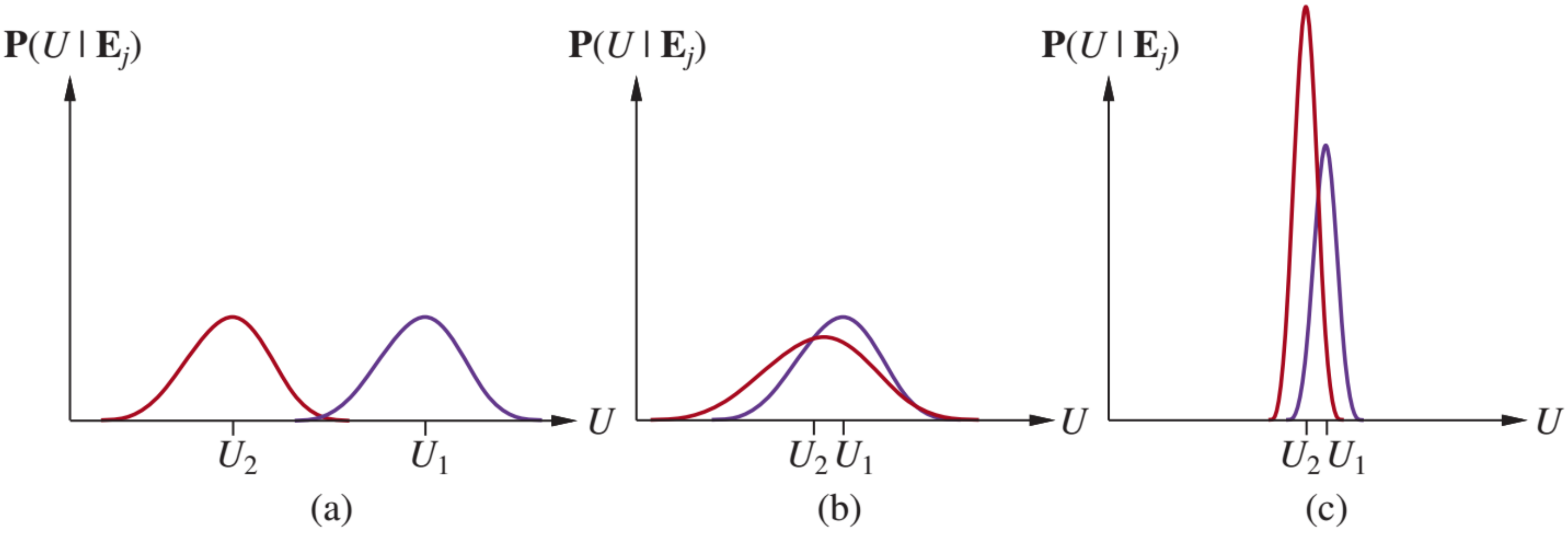
- VPI is not additive

$$VPI(E_j, E_k) \neq VPI(E_j) + VPI(E_k)$$

- VPI is order independent

$$VPI(E_j, E_k) = VPI(E_j) + VPI(E_k|E_j) = VPI(E_k) + VPI(E_j|E_k) \quad = VPI(E_k, E_j)$$

When to Gather More Information?



Source: RN Figure 15.8

Information Gathering, Decision-Theoretic Agent

- Agent should gather information before taking actions, if possible
  - Cost associated with getting the information, so how to choose which variable to get more information about?

function INFORMATION-GATHERING-AGENT(*percept*) **returns** an *action*
  persistent: D, a decision network

  integrate *percept* into D
  $j \leftarrow$ the value that maximizes $\boxed{VPI(E_j)\ /\ C(E_j)}$
  if $VPI(E_j) > C(E_j)$
    then return $Request(E_j)$
  else return the best action from D

Unit gain per unit cost
OR
Information value

Source: RN Figure 15.9

Summary

- Decision analysis
  - A prescriptive framework for guiding systematic, rational decision making
  - Involve formulation of explainable decision models and solutions
  - Extensive applications in practice
  - Theoretical foundations and methodological bases for AI decision systems – decision-theoretic AI
- Challenges and opportunities for AI
  - Human-machine collaboration in decision making
  - Uncertainty modeling with expert judgment and observational data
  - Preference modeling in complex, changing and uncertain conditions
  - Responsible AI – ethical, governance, and regulatory conditions

Homework

- Readings:
  - RN: 15.6.1-15.6.4 (Value of information)
  - *Optional:*
    - Howard, R.A., Decision Analysis: Practice and Promise. Management Science, 1988. 34(6): p. 679-695. [Accessible through NUS Library e-Resources]
- Reviews:
  - RN: 13.2-13.5; 14.2-14.4 (Conditional probability and Bayesian networks)
  - Charniak, E., Bayesian networks without tears: making Bayesian networks more accessible to the probabilistically unsophisticated. AI Mag., 1991. 12(4): p. 50–63.