

## Tutorial Week 5: Rational Decision Making

### Guidelines

- You can discuss the content of the questions with your classmates.
- However, everyone should work on and be ready to present ALL the solutions.
- Your attendance is marked in the tutorial and participation noted to award class participation marks.

### Problem 1: Allais Paradox

The Allais paradox (Allais, 1953) is a well-known problem potentially suggesting that humans are “predictably irrational” (Ariely, 2009)<sup>1</sup>. People are given a choice between lotteries A and B and then between C and D, which have the following prizes:

A: 80% chance of \$4000    C: 20% chance of \$4000  
B: 100% chance of \$3000    D: 25% chance of \$3000

Most people consistently prefer B over A (i.e., taking the sure payoff), and C over D (taking the higher EMV).

- a) Show that the normative analysis (i.e., describing how a rational agent should act) disagrees. [Hint: Set  $U(\$x) = x$ ; show that the preferences between A, B and C, D are opposites, hence a contradiction.]

#### Solution:

Inequality from preferring B to A:

$$0.8U(\$4000) < U(\$3000)$$

Inequality from preferring C to D:

$$0.2U(\$4000) > 0.25U(\$3000)$$

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<sup>1</sup>For a possible explanation of such a paradox, refer to page 620 of AIMA textbook.

Now multiply by 4:

$$0.8U(\$4000) > U(\$3000)$$

There is a contradiction between the 2 preferences.

In that case, then  $B \succ A$  implies that  $U(\$3000) > 0.8U(\$4000)$ , whereas  $C \succ D$  implies exactly the reverse. In other words, there is no utility function that is consistent with these choices.

- b) Prove that the judgments  $B \succ A$  and  $C \succ D$  in the above Allais paradox violate the axiom of substitutability. [**Hint:** You may wish to consider using the axiom of decomposability.]

**Solution:**

We know  $A \prec B$  and  $C \succ D$ . By the axiom of decomposability,

$$C \sim [0.25, A; 0.75; \$0] \text{ and } D \sim [0.25, B; 0.75, \$0]. \quad (1)$$

Using the above and  $C \succ D$ , we have  $[0.25, A; 0.75; \$0] \succ [0.25, B; 0.75, \$0]$ . This implies that  $EU(A) > EU(B) \implies A \succ B$ .

From  $A \prec B$  and substitutability, we would have  $[0.25, A; 0.75, \$0] \prec [0.25, B; 0.75, \$0]$ .

This is a contradiction, hence substitutability is violated.

## Problem 2: Preference Modelling

Alex is given the choice between two games:

- **Game 1:** a fair coin is flipped and if it comes up heads, Alex receives \$100. If the coin comes up tails, Alex receives nothing.
- **Game 2:** a fair coin is flipped twice. Each time the coin comes up heads, Alex receives \$50, and Alex receives nothing for each coin flip that comes up tails.

Alex prefers Game 2 to Game 1. Argue that Alex would prefer to receive \$50 compared to being allowed to participate in Game 1.

**Solution:**

Since Alex prefers Game 2 to Game 1:

$$\begin{aligned} 0.5U(\$0) + 0.5U(\$100) &< 0.25U(\$0) + 0.5U(\$50) + 0.25U(\$100) \\ \implies 0.25U(\$0) + 0.25U(\$100) &< 0.5U(\$50) \\ \implies 0.5U(\$0) + 0.5U(\$100) &< U(\$50) \end{aligned}$$

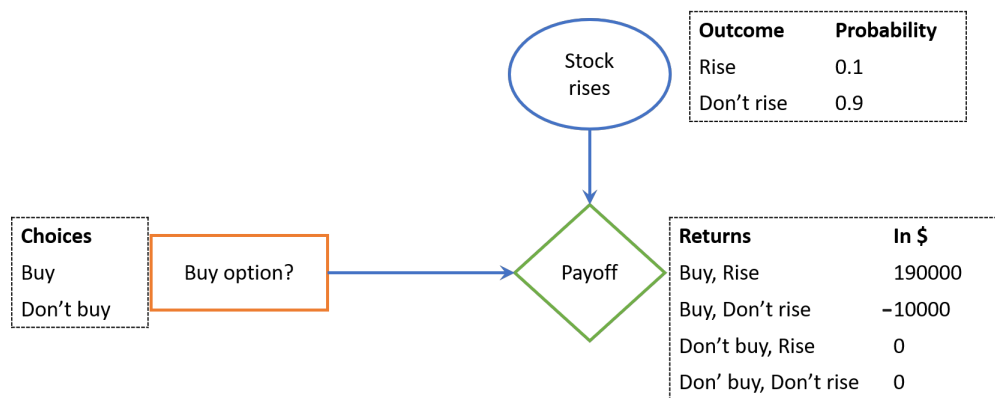
The left hand side is the expected utility of participating in Game 1 while the right hand side is the expected utility of receiving \$50. Since the LHS is less than the RHS, Alex would prefer \$50 over participating in game 1.

### Problem 3: Basic Risky Decision

Richie Bean is trying to strike it big in the stock market during the economic downturn. He is considering buying some options to a very risky stock on a diamond mine in Africa. There is only a 10% chance that the stock price will rise if he exercises his options, but the payoff is \$200,000. It costs \$10,000 to buy and exercise the options. The alternative is not to buy at all, in which case Mr. Bean's profit is zero.

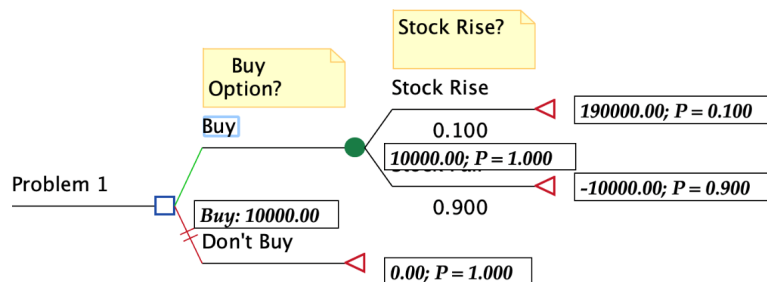
- a. Draw an influence diagram to represent Mr. Bean's problem. Clearly indicate all the options/outcomes and numbers. Should he buy the options? Use the solution approaches mentioned in the lecture to substantiate your answer.

**Solution:**



- b. Draw a decision tree to represent Mr. Bean's problem. Clearly indicate all the options/outcomes and numbers. Should he buy the options? Show all the details in your decision tree.

**Solution:**



EMV for buy \$10000, don't buy \$0.