16th International Science Youth Forum

Collaborative Science Activity

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1 Pentagon Drawing

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Your Task

As a group, please construct a pentagon with only a compass and a straight edge. Mathematical reasoning and proof will greatly enhance your answer.

Bonus: Why can't a heptagon be constructed? Good reasoning will add points to your answer :)

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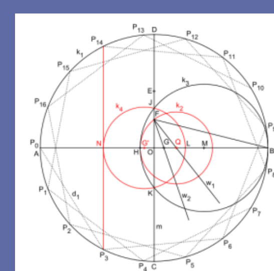
Introduction

In 1796, Gauss, 19, successfully constructed a 17 sided regular pentagon using a compass and a straight edge.

Subsequently, the Gauss-Wantzel Theorem was discovered, which stated that some n-gons could be drawn with a compass and a straight edge, but others could not be drawn.



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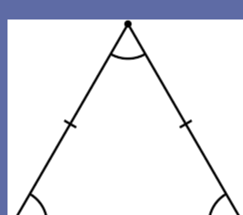
Background

To start off, we can draw an equilateral triangle using a compass and a straight edge.

Note: A straight edge means a ruler, but you cannot use the markings on it to measure length.



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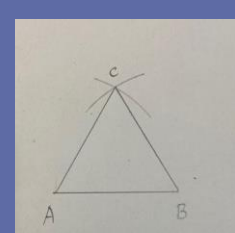
Background

To do this, we can draw a line and let its endpoints be AB.

From here, we aim to construct a point C where AC = AB = BC. Draw arcs from both points A and B, with distance AB from each point. The intersection point is point C, and it gives

AC = AB = BC.



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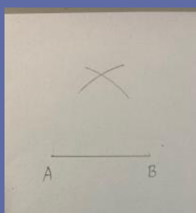
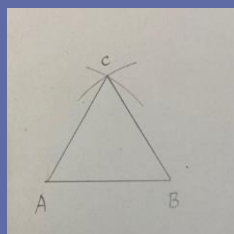
Construction





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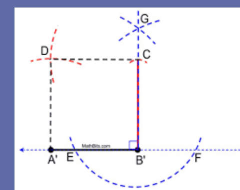
Background

To construct a square, we draw a base first.

Next, the idea behind this is that if we are able to construct the 3rd point, then constructing the last point can be done with the solution for the triangle.

How to construct the 3rd point?



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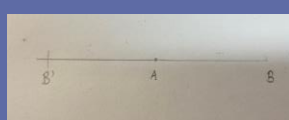
Background

We can use the idea of constructing perpendicular bisectors on one point.

First construct line AB of arbitrary length as one side of the square. Then extend AB beyond A to some point B', where B'A = AB.



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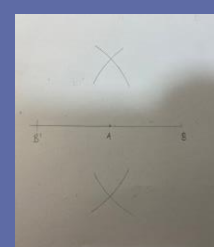
Background

From here, extend the compass beyond the length AB and draw arcs from both points B and B'. The intersection points should lie on the perpendicular bisector of BB', and from here we can connect the 2 points to form the bisector.



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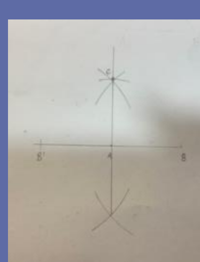
Background

To get the third point, extend the compass to have length AB, then draw an arc from A and the intersection between the arc and the new line is the third point.

Then construct the last point using the solution for triangle



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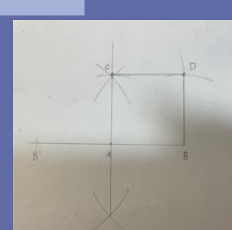
Construction



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2 Falling Ruler

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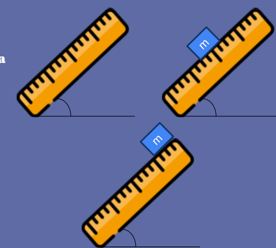
Introduction

Consider 3 three different scenarios: a normal ruler, a ruler with some mass m rigidly attached to its centre, and another ruler with the same mass rigidly attached to its edge. Let's say that all three rulers are placed at an angle of θ from a frictionless, horizontal ground. Which ruler will topple to the ground the fastest, and why? You will be required to utilise scientific reasoning to justify your answers.



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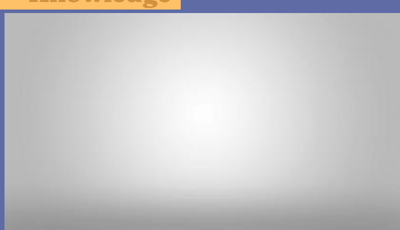
Background Knowledge



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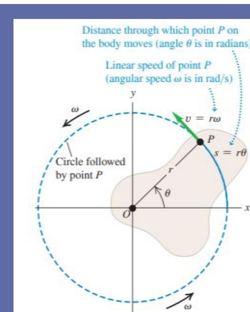
Background Knowledge

- S is the arclength of a circle, defined as $S = r\theta$
- For our case, $\frac{ds}{dt} = r\frac{d\theta}{dt}$
- Which can be written as: $v = r\omega$
- And in extension, $a_t = \frac{dv}{dt} = r\frac{d^2\theta}{dt^2} = r\ddot{\theta}$
- Angular acceleration, $\alpha = \ddot{\theta}$, the rate of change of angular velocity, hence
- $a_t = r\alpha$



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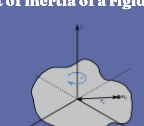
Background Knowledge

What is the moment of inertia?
It is defined as the quantity expressed by the body resisting angular acceleration.
If a body is rotating about a fixed axis, the moment of inertia of a rigid body can be calculated by:

Moment of inertia of a body for a given rotation axis $\rightarrow I = m_1r_1^2 + m_2r_2^2 + \cdots = \sum_i m_ir_i^2$

Masses of the particles that make up the body

Perpendicular distances of the particles from rotation axis



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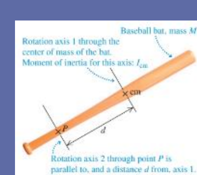
Background Knowledge

Parallel-Axis Theorem:
A body doesn't just have one moment of inertia, but infinitely many, due to the fact that it can rotate about infinitely many axes.
This theorem illustrates the relationship between the moment of inertia of a body about an axis through its center of mass and the moment of axis about any other axis parallel to the original axis.



Parallel-axis theorem:
Moment of inertia of a body for a rotation axis through point P $\rightarrow I_P = I_{cm} + Md^2$

Mass of body

Distance between two parallel axes

Moment of inertia of body for a parallel axis through center of mass

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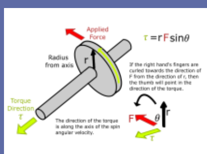
Background Knowledge: Torque

It is a measure of a force that can cause an object to rotate about an axis.
Torque can be expressed in two ways:



Rotational analog of Newton's second law for a rigid body:

$$\sum \tau_z = I\alpha_z$$

Net torque on a rigid body about z-axis; Moment of inertia of rigid body about z-axis; Angular acceleration of rigid body about z-axis

where $\alpha = ra_{tan}$

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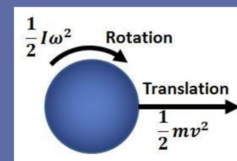
Background Knowledge

Rotational Kinetic Energy
This is simply the kinetic energy of a rotating rigid body.

Rotational kinetic energy of a rigid body rotating around an axis

$$K = \tfrac{1}{2}I\omega^2$$

Moment of inertia of body for given rotation axis; Angular speed of body



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Background Knowledge

Translational	Rotational
Velocity, $v = \frac{dx}{dt}$ **where x is displacement**	**Angular Velocity,** $\omega = \frac{d\theta}{dt}$ **where** θ **is angular displacement**
Acceleration, $a = \frac{dv}{dt}$	**Angular Acceleration,** $\alpha = \frac{d\omega}{dt}$
Mass, m	**Moment of Inertia,** I
Force, $F = ma$	**Moment of Inertia,** $\tau = I\omega$
Kinetic Energy, $K = \frac{1}{2}mv^2$	**Kinetic Energy,** $K = \frac{1}{2}I\omega^2$
Work done, W = Fs	**Work done,** $W = \tau\theta$
Power, $P = Fv$	**Power,** $P = \tau\omega$

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