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A Discontinuous Finite-Time Consensus Tracking Algorithm for Single Integrator Multi-Agent System with External Disturbance

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Abstract: This paper investigates the consensus tracking with disturbance rejection (CTDR) problem for single integrator multi-agent system (MAS). The communication topology is required to contain a directed spanning tree, where the leader is the root. Based on a discontinuous algorithm, finite-time CTDR is realized in a one-step control process, unlike those existing results derived based on classical two-step control technique, namely sliding mode control. A finite-time stability analysis for CTDR is developed by means of non-smooth Lyapunov analysis. Finally, a numerical example is provided to validate the theoretical analysis.

Key Words: Discontinuous control, finite-time, consensus tracking, non-smooth Lyapunov analysis, directed graph.

1 Introduction

Coordinated control for multi-agent system (MAS) has been paid widespread attention in the past several decades, owing to both its broad applications in practical engineering fields (such as sensor network localization [1–3], formation control [4–6], and so on) and its advantages compared with conventional centralized control algorithm (such as lower communication costs, stronger robustness, and easier implementation).

As the basic issue of coordinated control, consensus problem means designing distributed consensus algorithms for each agent to reach a common state. Consensus tracking is a special case of consensus and has been extensively considered in numerous works [7–9], where there exists a leader which is followed by other agents and plays the role of the group reference state in the MAS. As a result, the final common state of agents can be chosen by selecting the control input to the leader, and therefore it is worthy of investigating consensus tracking for practical consideration. It is noteworthy that when there exists more than one leader, the final common state is not the value of any specific leader but a convex hull spanned by the leaders, namely the so-called containment control [10, 11].

Convergence rate is a key performance measure of consensus algorithm, however, most early works concerning consensus tracking merely guarantee asymptotic consensus over an infinite-time horizon, e.g., [12], and thus may not be applied to practical applications. Actually, it is more appealing to realize consensus after a transient time with finite length in practice. In response to this issue, The notion of finite-time semi-stability is developed in [13], and exploited to realize consensus for single integrator MAS within finite-time. Until now, many excellent results concerning finite-time consensus have been derived [14, 15]. However, these results are derived based on relatively ideal systems without any disturbance. It is imperative to investigate consensus with disturbance rejection since disturbance always exists in MAS and brings serious effects on system performance.

Nowadays a lot of results concerning consensus tracking with disturbance rejection (CTDR) are derived based on sliding mode control. More specifically, the CTDR problem for single integrator MAS is considered in [16] and sliding mode control technique is exploited to realize disturbance rejection. In the theory of sliding mode control, the so-called sliding mode surface needs to be designed and the consensus algorithm is consisting of a discontinuous term and a nominal term. Hence, the objectives of consensus and disturbance rejection are addressed separately in a two-step control process. In the first step, the discontinuous term drives the states with external disturbance onto the sliding mode surface. Then the nominal term plays the role on the sliding mode surface, and the second step can be regarded as a control process to realize consensus for an ideal MAS without any disturbance.

Motivated by the aforementioned discussion, this paper investigates the CTDR problem for single integrator MAS. A consensus tracking algorithm is proposed based on discontinuous control to realize CTDR within finite-time. Noticeably, a finite-time stability analysis for CTDR is developed as a one-step control process by exploiting non-smooth Lyapunov analysis, which is a most significant characteristic in comparison to those existing results derived based on two-step control technique, namely sliding mode control.

The outline of this paper is as follows. Section **2** gives the problem statement, together with the necessary preliminaries. In section **3**, a consensus tracking algorithm is proposed based on discontinuous control and CTDR for single integrator MAS is realized within finite-time, where a one-step finite-time stability analysis is developed based on non-smooth Lyapunov analysis. Section **4** gives simulation results to illustrate the theoretical analysis. Finally section **5** draws conclusions.

Notation: $\mathbb{R}$ and $\mathbb{R}_+$ represent the set of real and positive real number, respectively. $\mathbf{1}$ and $\mathbf{0}$ denote n-dimensional all-one vector and null vector, respectively. Let $\boldsymbol{x}^\top$ and $\mathcal{A}^\top$ be the transpose of vector $\boldsymbol{x}$ and matrix $\mathcal{A}$, respectively. For any vector $\boldsymbol{x} = [x_1, \ldots, x_n]^\top \in \mathbb{R}^n$, $\mathrm{sign}(\boldsymbol{x}) = [\mathrm{sign}(x_1), \ldots, \mathrm{sign}(x_n)]^\top$. $\boldsymbol{x}$ is a positive vector if all of its entries are positive. For any matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$,

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$\lambda_{\min}(\mathcal{A})$ denotes the minimum eigenvalue of $\mathcal{A}$. $\mathcal{A}$ is a positive matrix if $a_{ij} > 0$. Let $\|\mathcal{A}\|_1$ and $\|\mathcal{A}\|_\infty$ be the 1-norm and ∞-norm of $\mathcal{A}$, respectively.

2 Problem Statement and Preliminaries

2.1 Problem Statement

Consider a MAS, which is composed of one static leader and n followers. The dynamics of the leader and followers are depicted as follows

$$\begin{aligned} &\dot{x}_0(t) = 0, \\ &\dot{x}_i(t) = u_i(t) + \nu_i(t) \quad i = 1, \ldots, n, \end{aligned} \tag{1}$$

where $x_0(t) \in \mathbb{R}$ represents the state of the leader. $x_i(t) \in \mathbb{R}$, $u_i(t) \in \mathbb{R}$, and $\nu_i(t) \in \mathbb{R}$ denote the state, the control input, and the external disturbance of the i^{th} follower, respectively. Let $\boldsymbol{x} = [x_1, \ldots, x_n]^\top \in \mathbb{R}^n$, $\boldsymbol{u} = [u_1, \ldots, u_n]^\top \in \mathbb{R}^n$, and $\boldsymbol{\nu} = [\nu_1, \ldots, \nu_n]^\top \in \mathbb{R}^n$. The dependence of the vectors on time t are omitted, and the vectors below are the same.

A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is used to describe the communication topology among the followers, where $\mathcal{V} = \{1, \ldots, n\}$ is the set of the followers, and $\mathcal{E} \subseteq (\mathcal{V} \times \mathcal{V})$ is the set of directed edges. An directed edge $(i, j) \in \mathcal{E}, i \neq j$ means that the i^{th} follower receives information from the j^{th} follower. The weight of the edge (i, j) is denoted by $a_{ij} > 0$ if $(i, j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. Assume that $a_{ii} = 0$, in other words, there is no self-loop in the directed graph $\mathcal{G}$. The Laplacian $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ of $\mathcal{G}$ is defined as

$$l_{ij} = \begin{cases} -a_{ij} & \text{if} \quad (i,j) \in \mathcal{E}, \\ \displaystyle\sum_{j=1, j\neq i}^{n} a_{ij} & \text{if} \quad i = j, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

A directed path from the j^{th} follower to the i^{th} follower is an alternating sequence of directed edges. Furthermore, a diagonal matrix $\mathcal{A} = \operatorname{diag}(a_{10}, \ldots, a_{n0})$ is used to indicate the connection relationship between the leader and followers. If the i^{th} follower receives information from the leader, then $a_{i0} > 0$, otherwise $a_{i0} = 0$.

The following assumptions are formulated to address the CTDR problem.

Assumption 1 The external disturbance ν_i satisfies

$$|\nu_i| \leq \varepsilon \quad i = 1, \ldots, n, \tag{3}$$

where $\varepsilon \in \mathbb{R}_+$ is a known constant.

Assumption 2 There exists a directed spanning tree, where the leader is the root, in other words, there has directed paths from the leader to all the followers.

Lemma 1 [17] Let $\mathcal{L}_{\mathcal{A}} = \mathcal{L} + \mathcal{A}$. If **Assumption 2** holds, then there exists a positive diagonal matrix Ξ such that $\hat{\mathcal{L}}_{\mathcal{A}} = \Xi\mathcal{L}_{\mathcal{A}} + \mathcal{L}_{\mathcal{A}}^\top\Xi$ is positive definite. One specific form of Ξ is given by $\Xi = \operatorname{diag}(\xi)$, where $\xi = [\xi_1, \ldots, \xi_n]^\top = \left(\mathcal{L}_{\mathcal{A}}^\top\right)^{-1}\mathbf{1}$.

2.2 Preliminaries

Consider a nonlinear system

$$\dot{\boldsymbol{x}}(t) = f(t, \boldsymbol{x}), \tag{4}$$

where $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is measurable and locally essentially bounded. Assume that the origin is an unique equilibrium point of (4). Since the case that the righthand side of (4) is discontinuous is not excluded, we intend the Filippov solutions of (4) throughout this paper.

Remark 1 The classical notion of solutions, namely continuously differentiable solutions, cannot be guaranteed to exist when considering discontinuous dynamical systems. Therefore, the notion of Filippov solutions is used to handle such a discontinuity, and the hypotheses on $f(t, \boldsymbol{x})$ in (4) establish mild conditions under which the existence of Filippov solutions of (4) can be guaranteed.

Definition 1 [18] $f : [x, y] \to \mathbb{R}$ is said to be an absolutely continuous function if for any $\epsilon \in \mathbb{R}_+$, there exists $\delta \in \mathbb{R}_+$ such that, for any finite collection of disjoint open intervals $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ on $[x, y]$, when

$$\sum_{i=1}^{n} (y_i - x_i) \leq \delta, \tag{5}$$

the following relation holds

$$\sum_{i=1}^{n} |f(y_i) - f(x_i)| \leq \epsilon. \tag{6}$$

Definition 2 [19] An absolutely continuous function $\boldsymbol{x} : [t_0, t_1] \to \mathbb{R}^n$ is called a Filippov solution of (4) on $[t_0, t_1]$ if $\dot{\boldsymbol{x}} \in \mathbb{K}[f](t, \boldsymbol{x})$ holds for almost all $t \in [t_0, t_1]$, where $\mathbb{K}[f](t, \boldsymbol{x})$ is the so-called Filippov set-valued map of $f(t, \boldsymbol{x})$, and is defined as

$$\mathbb{K}[f](t, \boldsymbol{x}) \triangleq \bigcap_{r>0} \bigcap_{\mu(N)=0} \overline{\operatorname{co}} \left\{ f(t, B(\boldsymbol{x}, r) - N) \right\}, \tag{7}$$

where $\overline{\operatorname{co}}$ means convex closure, μ denotes Lebesgue measure, $B(\boldsymbol{x}, r)$ is an open ball with radius r centered at $\boldsymbol{x}$, and $N \subset \mathbb{R}^n$ is a set with zero Lebesgue measure which can be arbitrarily selected to simplify the calculation.

Definition 3 [20] Let $V : \mathbb{R}^n \to \mathbb{R}$ be locally Lipschitz continuous, then the generalized gradient of V at $\boldsymbol{x}$ is defined as

$$\partial V(\boldsymbol{x}) \triangleq \overline{\operatorname{co}}\{\lim_{i\to\infty} \nabla V(\boldsymbol{x}_i) | \boldsymbol{x}_i \to \boldsymbol{x}, \boldsymbol{x}_i \notin \Omega_V \cup N\}, \tag{8}$$

where $\overline{\operatorname{co}}$ represents convex closure, $\nabla V(\boldsymbol{x})$ means the gradient of $V(\boldsymbol{x})$, Ω_V denotes the set where $\nabla V(\boldsymbol{x})$ does not exist, and $N \subset \mathbb{R}^n$ is a set with zero Lebesgue measure which can be arbitrarily selected to simplify the calculation.

Lemma 2 [21] Let $\boldsymbol{x} : [t_0, t_1] \to \mathbb{R}^n$ be a Filippov solution of (4) and $V : \mathbb{R}^n \to \mathbb{R}$ be locally Lipschitz continuous and regular (see [20] for the definition), then the generalized time derivative of V exists for almost all $t \in [t_0, t_1]$, and is defined as

$$\dot{V}(\boldsymbol{x}) \in p^\top \dot{\boldsymbol{x}}, \tag{9}$$

where $p \in \partial V(\boldsymbol{x})$.

Lemma 3 [22] If 1) $V : \mathbb{R}^n \to \mathbb{R}_+ \bigcup \{0\}$ and $V(\boldsymbol{x}) = 0$ iff $\boldsymbol{x} = 0$, and 2) $\boldsymbol{x} : [t_0, t_1] \to \mathbb{R}^n$ is a Filippov solution of (4) with

$$\dot{V}(\boldsymbol{x}) \le -\rho < 0 \tag{10}$$

holds for almost all $t \in [t_0, t_1]$, then $\boldsymbol{x}$ converges to $\mathbf{0}$ within finite-time, i.e., there exists a positive constant $T_0 \in \mathbb{R}_+$ such that $\boldsymbol{x} = 0, \forall t \ge T_0$.

Problem. The purpose is to devise a distributed consensus tracking algorithm to realize consensus tracking and reject the effects of external disturbance for (1) within finite-time, i.e., there exists a positive constant $T_0 \in \mathbb{R}_+$ such that

$$\|\boldsymbol{x} - x_0 \cdot \mathbf{1}\|_1 = 0 \quad \forall t \ge T_0. \tag{11}$$

3 Main Results

Consider a consensus tracking algorithm based on discontinuous control

$$u_i = -k \cdot \operatorname{sign}\left(\sum_{j=1}^{n} a_{ij}\left(x_i - x_j\right) + a_{i0}\left(x_i - x_0\right)\right), \tag{12}$$

where $k > 0$ is the gain parameter to be determined, and $\operatorname{sign}(\cdot)$ is the signum function.

Define the consensus tracking error vector

$$\boldsymbol{\delta} = \boldsymbol{x} - x_0 \cdot \mathbf{1}. \tag{13}$$

Therefore, one can rewrite the control vector $\boldsymbol{u}$ compactly

$$\boldsymbol{u} = -k \cdot \operatorname{sign}\left(\mathcal{L}_{\mathcal{A}}\boldsymbol{\delta}\right), \tag{14}$$

where $\mathcal{L}_{\mathcal{A}}$ is introduced in **Lemma 1**.

Furthermore, the closed-loop consensus tracking error dynamics of the considered MAS (1) can be derived

$$\dot{\boldsymbol{\delta}} = -k \cdot \operatorname{sign}\left(\mathcal{L}_{\mathcal{A}}\boldsymbol{\delta}\right) + \boldsymbol{\nu}. \tag{15}$$

The finite-time stability analysis for the consensus tracking error $\boldsymbol{\delta}$ is presented in **Theorem 1**.

Remark 2 It has been observed in the published literature that discontinuous control is essential to reject the effects of external disturbance. Most existing results concerning finite-time consensus with disturbance rejection are derived by means of sliding mode control, which is a two-step control technique, and therefore the consensus algorithm therein is proposed based on both discontinuous and nonlinear control (for instance, [23]). By contrast, the consensus tracking algorithm herein is only based on discontinuous control, meanwhile, it still ensures that CTDR can be realized in a one-step control process.

Remark 3 The consensus tracking algorithm (12) is proposed based on signum function, and therefore the consensus tracking error dynamics (15) becomes a discontinuous dynamical system. In [18], it is illustrated that it might be necessary to consider non-smooth Lyapunov functions when dealing with discontinuous dynamical systems, and the generalized gradient notion is available when a non-smooth Lyapunov function fails to be differentiable.

Theorem 1 Consider the MAS (1) and suppose that **Assumption 1 – 2** hold. If the gain parameter k in the proposed discontinuous algorithm (12) satisfies

$$k \ge 2 \cdot \frac{\varepsilon \left\|\Xi\mathcal{L}_{\mathcal{A}}\right\|_\infty + \rho}{\lambda_{\min}(\hat{\mathcal{L}}_{\mathcal{A}})}, \tag{16}$$

where $\rho \in \mathbb{R}_+$ is a positive constant to be chosen, then CTDR for (1) can be realized within finite-time, and the upper bound on transient time T_0 can be estimated as

$$T_0 \le \frac{\left\|\Xi\mathcal{L}_{\mathcal{A}}\boldsymbol{\delta}(0)\right\|_1}{\rho}. \tag{17}$$

Proof. Let $\boldsymbol{z} = \mathcal{L}_{\mathcal{A}}\boldsymbol{\delta}$, and consider a non-smooth Lyapunov function as

$$V(\boldsymbol{z}) = \|\Xi\boldsymbol{z}\|_1 = \sum_{i=1}^{n} \xi_i |z_i|. \tag{18}$$

$V(\boldsymbol{z})$ is convex, and therefore regular. Notice that its corresponding time derivative $\dot{V}(\boldsymbol{z})$ does not exist when there exists at least one $z_i = 0$. Therefore, we intend the Filippov solutions of $\dot{V}(\boldsymbol{z})$ since $\dot{V}(\boldsymbol{z})$ becomes discontinuous. Define two set $Z_0 = \{z_i | z_i = 0\}$ and $Z_{\neq} = \{z_i | z_i \neq 0\}$. Consider the generalized time derivative of $V(\boldsymbol{z})$ by taking into account of **Lemma 2** with relation (9), which yields that

$$\dot{V}(\boldsymbol{z}) \in \sum_{z_i \in Z_0} \operatorname{SIGN}(z_i)\dot{z}_i + \sum_{z_i \in Z_{\neq}} \operatorname{sign}(z_i)\dot{z}_i, \tag{19}$$

where $\operatorname{SIGN}(z_i)$ is the associated Filippov set-valued map of $\operatorname{sign}(z_i)$

$$\operatorname{SIGN}(z_i) = \begin{cases} 1 & z_i > 0, \\ [-1, 1] & z_i = 0, \\ -1 & z_i < 0. \end{cases} \tag{20}$$

In the Filippov sense, if $z_i = 0$ only holds at those isolated time instants of Lebegue measure zero, then $\dot{z}_i$ can be neglected at these time instants. If $z_i = 0$ holds along a time interval with positive Lebegue measure, then it follows that $\dot{z}_i$ exists in such a time interval and takes zero value. Therefore, one can revise the term $\operatorname{SIGN}(z_i)$ to $\operatorname{sign}(z_i)$ and yield that

$$\begin{aligned} \dot{V}(\boldsymbol{z}) = & \sum_{z_i \in Z_0} \operatorname{sign}(z_i)\dot{z}_i + \sum_{z_i \in Z_{\neq}} \operatorname{sign}(z_i)\dot{z}_i \\ = & [\operatorname{sign}(\boldsymbol{z})]^\top \Xi \dot{\boldsymbol{z}} \\ = & [\operatorname{sign}(\boldsymbol{z})]^\top \Xi \mathcal{L}_{\mathcal{A}} \left(-k \cdot \operatorname{sign}(\boldsymbol{z}) + \boldsymbol{\nu}\right) \\ \le & -\frac{k}{2}\lambda_{\min}(\hat{\mathcal{L}}_{\mathcal{A}}) \sum_{i=1}^{n} |\operatorname{sign}(z_i)| \\ & + \left| \sum_{i=1}^{n} \operatorname{sign}(z_i)\xi_i \mathcal{L}_{\mathcal{A}}^i \boldsymbol{\nu} \right|, \end{aligned} \tag{21}$$

where $\mathcal{L}_{\mathcal{A}}^i$ denotes the i^{th} row of $\mathcal{L}_{\mathcal{A}}$. Note that the term with index $z_i \in Z_0$ in (21) takes zero value, however, such term is kept for the sake of analysis. The estimation for the upper bound of the second term can be calculated as

$$\begin{aligned}&\left|\sum_{i=1}^{n}\operatorname{sign}(z_i)\xi_i\mathcal{L}_{\mathcal{A}}^{i}\boldsymbol{\nu}\right|\\ \leq&\sum_{i=1}^{n}\left|\operatorname{sign}(z_i)\xi_i\mathcal{L}_{\mathcal{A}}^{i}\boldsymbol{\nu}\right|\\ \leq&\max_i\left\{\xi_i\mathcal{L}_{\mathcal{A}}^{i}\boldsymbol{\nu}\right\}\sum_{i=1}^{n}\left|\operatorname{sign}(z_i)\right|,\end{aligned}\tag{22}$$

where the maximum term can be evaluated as

$$\begin{aligned}&\max_i\left\{\xi_i\mathcal{L}_{\mathcal{A}}^{i}\boldsymbol{\nu}\right\}\\ =&\max_i\left\{\xi_i\left(\sum_{j=1}^{n}a_{ij}(\nu_i-\nu_j)+a_{i0}\nu_i\right)\right\}\\ \leq&\max_i\left\{\xi_i\left(2l_{ii}+a_{i0}\right)\varepsilon\right\}\\ =&\varepsilon\|\Xi\mathcal{L}_{\mathcal{A}}\|_\infty.\end{aligned}\tag{23}$$

By substituting (22) – (23) into (21), one has that

$$\begin{aligned}\dot{V}(\boldsymbol{z})\leq&-\frac{k}{2}\lambda_{\min}(\hat{\mathcal{L}}_{\mathcal{A}})\sum_{i=1}^{n}\left|\operatorname{sign}(z_i)\right|\\ &+\varepsilon\left\|\Xi\mathcal{L}_{\mathcal{A}}\right\|_\infty\sum_{i=1}^{n}\left|\operatorname{sign}(z_i)\right|.\end{aligned}\tag{24}$$

If finite-time CTDR for (1) has not been realized, then there exists at least one $z_i \neq 0$ such that $|\operatorname{sign}(z_i)| = 1$, which follows that

$$\sum_{i=1}^{n}\left|\operatorname{sign}(z_i)\right|\geq 1.\tag{25}$$

Therefore, it holds that

$$\dot{V}(\boldsymbol{z})\leq-\frac{k}{2}\lambda_{\min}(\hat{\mathcal{L}}_{\mathcal{A}})+\varepsilon\left\|\Xi\mathcal{L}_{\mathcal{A}}\right\|_\infty.\tag{26}$$

By considering the condition of the gain parameter (16) into the righthand side of (26) and taking into account of **Lemma 3** with relation (10), it finally yields that

$$\dot{V}(\boldsymbol{z})\leq-\rho<0\tag{27}$$

holds for almost all $t\in[t_0,t_1]$ as long as finite-time CTDR for (1) has not been realized. Therefore, there exists a positive constant $T_0\in\mathbb{R}_+$ such that all solutions of $\boldsymbol{z}$ converge to $\mathbf{0}$ for all $t\geq T_0$. The upper bound on transient time T_0 can be easily verified as

$$T_0\leq\frac{V\left(\boldsymbol{z}(0)\right)}{\rho}=\frac{\|\Xi\mathcal{L}_{\mathcal{A}}\boldsymbol{\delta}(0)\|_1}{\rho}.\tag{28}$$

Since $\boldsymbol{z}=\mathcal{L}_{\mathcal{A}}\boldsymbol{\delta}$, one has that $\boldsymbol{\delta}=\boldsymbol{x}-x_0\cdot\mathbf{1}=\mathbf{0}$ for all $t\geq T_0$, i.e., finite-time CTDR for (1) can be realized in a one-step control process.

4 Simulations

A numerical example is carried out in this section to verify the theoretical results and show the effectiveness of the proposed discontinuous consensus tracking algorithm.

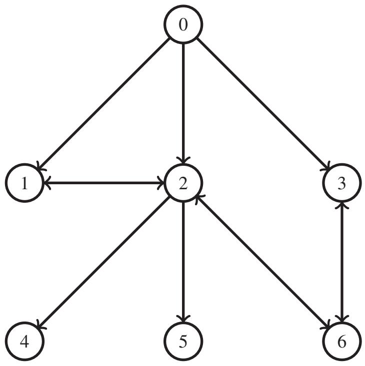

Fig. 1: Directed communication topology.

The considered MAS consists of one leader and 6 followers indexed by 0 and 1–6, respectively. Fig. 1 shows the corresponding communication topology. The corresponding Laplacian matrix is

$$\mathcal{L}=\begin{bmatrix}1&-1&0&0&0&0\\-1&2&0&0&0&-1\\0&0&1&0&0&-1\\0&-1&0&1&0&0\\0&-1&0&0&1&0\\0&-1&-1&0&0&2\end{bmatrix},$$

and the connection matrix is $\mathcal{A}=\operatorname{diag}(1,1,1,0,0,0)$. The initial condition of the leader and followers are chosen as $x_0=0$ and $\boldsymbol{x_0}=[-18,10,12,8,-10,4]^\top$, respectively. The external disturbance of each follower is considered to be $\nu_i(t)=0.1\sin(x_i)$, and the upper bound expressed in (3) is $\varepsilon=0.1$. Furthermore, it is calculated that $\Xi=\operatorname{diag}(1.7,2.5,1.8,1,1,2.6)$, $\|\Xi\mathcal{L}_{\mathcal{A}}\|_\infty=12.3$, and $\lambda_{\min}(\hat{\mathcal{L}}_{\mathcal{A}})=1.6$.

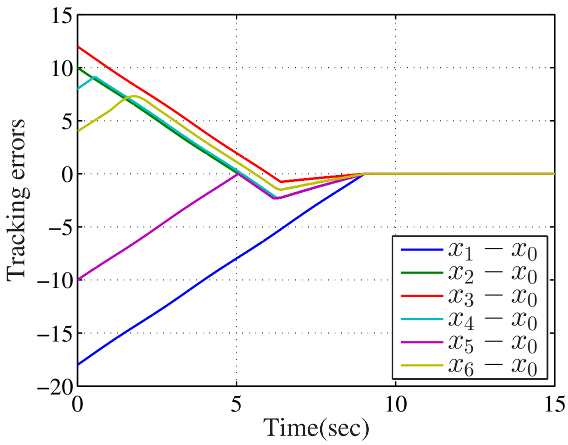

Fig. 2: The evolution of the consensus tracking errors with gain parameter $k=2$.

The arbitrary positive constant given in (16) is selected as $\rho=0.1$, then the minimum control gain to ensure finite-time convergence and robustness according to relation (16) is $\left(2\varepsilon\left\|\Xi\mathcal{L}_{\mathcal{A}}\right\|_\infty+2\rho\right)\Big/\left(\lambda_{\min}(\hat{\mathcal{L}}_{\mathcal{A}})\right)=1.7$, therefore the gain parameter in (12) is adopted as $k=2$ in the first simulation. The evolution of the consensus tracking errors with gain parameter $k = 2$ is shown in Fig. 2. The transient time to realize CTDR is less than 10s.

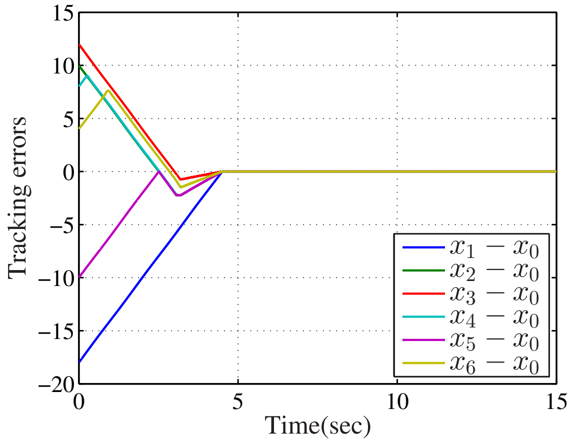

Fig. 3: The evolution of the consensus tracking errors with gain parameter $k = 4$.

In the second simulation, the gain parameter is chosen as $k = 4$, which is twice of that used in the first simulation, and it is roughly shown in Fig. 3 that doubling the control gain reduces the transient time by half.

5 Conclusions

The CTDR problem for single integrator MAS was investigated. The communication topology among the leader and followers was required to contain a directed spanning tree, where the leader is the root. Based on the proposed discontinuous algorithm, finite-time CTDR was realized in a one-step control process, in comparison to those existing works derived based on two-step control technique. A finite-time stability analysis was developed by means of non-smooth Lyapunov analysis, rather than sliding mode control. Finally, a numerical example was carried out to verify theoretical analysis. Further consideration includes extensions to higher-order MAS and agents with more complex dynamics.

References

[1] M. R. Gholami, L. Tetruashvili, E. G. Ström, and Y. Censor. Cooperative wireless sensor network positioning via implicit convex feasibility. *IEEE Transactions on Signal Processing*, 61(23): 5830–5840, 2013.

[2] G. Chai, C. Lin, Z. Lin, and M. Fu. Single landmark based collaborative multi-agent localization with time-varying range measurements and information sharing. *Systems & Control Letters*, 87: 56–63, 2016.

[3] C. Lin, Z. Lin, R. Zheng, G. Yan, and G. Mao. Distributed source localization of multi-agent systems with bearing angle measurements. *IEEE Transactions on Automatic Control*, 61(4): 1105–1110, 2016.

[4] Y. Cao, W. Yu, W. Ren, and G. Chen. An overview of recent progress in the study of distributed multi-agent coordination. *IEEE Transactions on Industrial Informatics*, 9(1): 427–438, 2013.

[5] M. Cao, C. Yu, and B. D. O. Anderson. Formation control using range-only measurements. *Automatica*, 47(4): 776–781, 2011.

[6] Z. Han, K. Guo, L. Xie, and Z. Lin. Integrated relative localization and leader–follower formation control. *IEEE Transactions on Automatic Control*, 64(1): 20–34, 2019.

[7] H. Wang, W. Yu, G. Wen, and G. Chen. Fixed-time consensus of nonlinear multi-agent systems with general directed topologies. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 66(9): 1587–1591, 2018.

[8] Z. Zuo, B. Tian, M. Defoort, and Z. Ding. Fixed-time consensus tracking for multiagent systems with high-order integrator dynamics. *IEEE Transactions on Automatic Control*, 63(2): 563–570, 2018.

[9] J. Ni, L. Liu, C. Liu, and J. Liu. Fixed-time leader-following consensus for second-order multiagent systems with input delay. *IEEE Transactions on Industrial Electronics*, 64(11): 8635–8646, 2017.

[10] M. Ji, G. F. Trecate, M. Egerstedt, and A. Buffa. Containment control in mobile networks. *IEEE Transactions on Automatic Control*, 53(8): 1972–1975, 2008.

[11] Y. Cao, D. Stuart, W. Ren, and Z. Meng. Distributed containment control for multiple autonomous vehicles with double-integrator dynamics: algorithms and experiments. *IEEE Transactions on Control Systems Technology*, 19(4): 929–938, 2011.

[12] Z. Li, X. Liu, W. Ren, and L. Xie. Distributed tracking control for linear multiagent systems with a leader of bounded unknown input. *IEEE Transactions on Automatic Control*, 58(2): 518–523, 2013.

[13] Q. Hui, W. M. Haddad, and S. P. Bhat. Finite-time semistability and consensus for nonlinear dynamical networks. *IEEE Transactions on Automatic Control*, 53(8): 1887–1900, 2008.

[14] W. Ren. On consensus algorithms for double-integrator dynamics. *IEEE Transactions on Automatic Control*, 53(6): 1503–1509, 2008.

[15] H. Du, G. Wen, G. Chen, J. Cao, and F. E. Alsaadi. A distributed finite-time consensus algorithm for higher-order leaderless and leader-following multiagent systems. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 47(7): 1625–1634, 2017.

[16] H. Wang, W. Yu, G. Wen, and G. Chen. Fixed-time consensus tracking of multi-agent systems under a directed communication topology. In *2016 12th IEEE International Conference on Control and Automation (ICCA)*, pages 186–191. IEEE, 2016.

[17] Z. Li, G. Wen, Z. Duan, and W. Ren. Designing fully distributed consensus protocols for linear multi-agent systems with directed graphs. *IEEE Transactions on Automatic Control*, 60(4): 1152–1157, 2015.

[18] J. Cortés. Discontinuous dynamical systems. *IEEE Control Systems Magazine*, 28(3): 36–73, 2008.

[19] A.F. Filippov. *Differential Equations with Discontinuous Righthand Sides: Control Systems*. Springer Science & Business Media, 1988.

[20] F. H. Clarke. *Optimization and nonsmooth analysis*. Wiley, 1983.

[21] D. Shevitz and B. E. Paden. Lyapunov stability theory of nonsmooth systems. *IEEE Transactions on Automatic Control*, 39(9): 1910–1914, 1994.

[22] B. E. Paden and S. S. Sastry. A calculus for computing filippov's differential inclusion with application to the variable structure control of robot manipulators. *IEEE Transactions on Circuits and Systems*, 34(1): 73–82, 1987.

[23] Y. Liu and S. Yu. Finite-time consensus of multi-agent systems with disturbances under directed communication topology. In *2016 35th Chinese Control Conference (CCC)*, pages 752–756, 2016.