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Introduction to fuzzy logic

Notes of class

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1 Fuzzy logic

linguistic variable of the following elements:

- X variable name
- $T(x)$ set of terms
- ϖ universe of discourse
- G syntax rules
- M semantic rules

degree of belonging $x \in A = \mu_A(x) = 1$, otherwise $x \notin A = \mu_A(x) = 0$. now we can said that $\mu(x) \in [0, 1]$ the degree of belonging of x to a any set is a number in the closed interval $[0, 1]$.

we can said that ϖ discrete (countable), a set could be written as $\tilde{A} = \{\frac{\mu_{\tilde{A}}(x_1)}{x_1} + \frac{\mu_{\tilde{A}}(x_2)}{x_2} + ... + \frac{\mu_{\tilde{A}}(x_n)}{x_n}\}$ or in a compact way in $\tilde{A} = \{\sum_i^n \frac{\mu_{\tilde{A}}(x_i)}{x_i}\}$. in continuous ϖ (uncountable) we can write $\tilde{A} = \{\int \frac{\mu_{\tilde{A}}(x)}{x}\}$.

Until now the major point is the breaking the law of "tercero excluido".

1.1 Support

The support of a fuzzy set $S()$, those elements that have a degree of belonging greater than zero $\mu_{\tilde{A}}(x) > 0$.

$$S(A) = \{x | \mu_A(x) > 0\} \tag{1}$$

remember that a set is considered regarding a ϖ.

1.1.1 Singleton

The support is defined only for one value of the all domain for example x_0 and $\mu_A(x_0) = 1$.

1.2 α cut-off

suppose that A is fuzzy set of ϖ then by $\alpha \in [0, 1]$ A_α is:

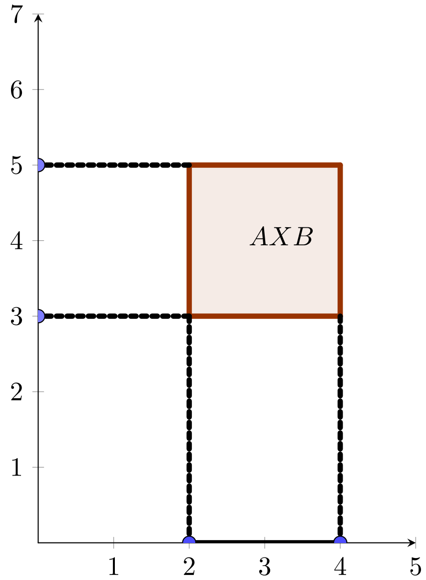
$$A_\alpha = \begin{cases} x \in \varpi | \mu_A(x) \geq \alpha, & \text{if } \alpha \geq 0 \\ S(A), & \text{if } \alpha = 0 \end{cases} \tag{2}$$

2 Cartesian product

by two sets A and B the Cartesian product is defined as all possible pairs formed with the sets.

$$AXB : \{(a,b)|a \in A \text{ and } b \in B\} \tag{3}$$

for instance $A = \{a_1, a_2\}$ and $B = \{b_1, b_2, b_3\}$ therefore $AXB = \{(a_1, b_1), (a_1, b_2), (a_1, b_3), (a_2, b_1), (a_2, b_2), (a_2, b_3)\}$. when you think in two intervals for instance $A = [2, 4]$ and $B = [3, 5]$ we can interpreted as region in $\mathbb{R}^2$.



3 Orthogonal extension

we have two universes of discourses ϖ_1 and ϖ_2 we can think in the cartesian product of them $\varpi_1 X \varpi_2$.

ϖ_1/ϖ_2	$\tilde{A}$	a_1	a_2	a_3
b_1	0.3	(b_1, a_1)	(b_1, a_2)	(b_1, a_3)
b_2		(b_2, a_1)	(b_2, a_2)	(b_2, a_3)
b_3	0.9	(b_3, a_1)	(b_3, a_2)	(b_3, a_3)

now the extension is:

ϖ_1/ϖ_2	$\tilde{A}$	a_1	a_2	a_3
b_1	0.3	0.3	0.3	0.3
b_2	0	0	0	0
b_3	0.9	0.9	0.9	0.9

we can establish that $\mu_{\vec{\tilde{A}}}(x, y) = \mu_{\tilde{A}}(x)$

4 Orthogonal projection

This problem is inverse to the problem defined previously.

ϖ_1/ϖ_2	a_1	a_2	a_3
b_1	0.8	0.9	1
b_2	0	0	0.8
b_3	0.4	0.5	0.7

this is only take the major value for each element in the different combinations, for instance for $b_1 = 1$ and for $a_1 = 0.8$. We can establish that $\mu_{\overleftarrow{A}}(x) = \max \mu_A(x, y)$.

ua	ub	U
0	0	0
0	1	1
1	0	1
1	1	1

4.1 basic operations

the Union operation with membership functions.

in summary we can uses $\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x))$.

make the table for intersection and complement.

4.2 T-norm and S-norm

remember the unitary cubic

4.3 fuzzy relations

We have $A \subset \varpi_1$ and $B \subset \varpi_2$ a R is a subset of pairs, the fuzzy relation $\tilde{R} = \frac{\mu_{a_1b_1}}{(a_1b_1)} + ...+$.

in mathematical terms we can said that a fuzzy relation between two fuzzy sets is a subset of the Cartesian product among the universe of discourses. $\tilde{R} = \tilde{A}X\tilde{B}$ is a subset of the cartesian product of the universe of discourses, and the degree of belonging is defined as $\mu_{\tilde{R}}(x, y) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y))$. Remember that we can uses any t-norm.

4.3.1 Compound relations

Suppose $\tilde{A}\tilde{R_1}\tilde{B}$ and $\tilde{B}\tilde{R_2}\tilde{C}$ how we can establish a relation between $\tilde{A}$ and $\tilde{C}$.

$$\tilde{R_1} = \begin{bmatrix} 0.3 & 0.9 & 0.1 \\ 0.1 & 0 & 0.4 \\ 0.8 & 0.6 & 0.7 \end{bmatrix}, \tilde{R_2} = \begin{bmatrix} 0.9 & 0.5 & 0.1 \\ 0.3 & 0.7 & 0.3 \\ 0.4 & 0.9 & 0.5 \end{bmatrix}$$

Think in the composition of two functions $f \circ g(x) = f(g(x))$ therefore $\tilde{R_1} \circ \tilde{R_2}$ is defined in the a similar way with a multiplication matrix take the $i-th$ row of $\tilde{R_1}$ and compare with the $i-th$ column of $\tilde{R_2}$ instead of apply the multiplication uses the min operator for instance the 1 row of $\tilde{R_1}$ is $[\tilde{r_{111}}, \tilde{r_{112}}, \tilde{r_{113}}...\tilde{r_{11n}}]$ and the 1 column of $\tilde{R_2}$ is $[\tilde{r_{211}}, \tilde{r_{221}}, \tilde{r_{231}}...\tilde{r_{2m1}}]^T$. the first element of the composition matrix will be $\max[min(\tilde{r_{111}}, \tilde{r_{211}}), min(\tilde{r_{112}}, \tilde{r_{221}})...min(\tilde{r_{11n}}, \tilde{r_{2m1}})]$. applied the steps over all rows and columns respectively.

$$\tilde{R_1} \circ \tilde{R_2} = \begin{bmatrix} 0.4 & 0.7 & 0.3 \\ 0.4 & 0.4 & 0.5 \\ 0.8 & 0.7 & 0.5 \end{bmatrix}$$

5 Linguistic variables

6 fuzzy inference

7 Fuzzy C-means