Estimating user-defined nonlinear regression models in Stata and in Mata

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1. Introduction

- Consider nonlinear cross-section regression of y_i on \mathbf{x}_i .
- Example is $y_i | \mathbf{x}_i \sim \text{Poisson}$ with mean $\mu_i = \exp(\mathbf{x}_i' \boldsymbol{\beta})$.
- This talk demonstrates various ways to code up the estimator,
 - using Stata command ml
 - and Mata command optimize

Outline

- Introduction
- Built-in command poisson
- Command ml method lf
- Checking program by simulation
- Command ml methods d0, d1, d2
- Newton-Raphson algorithm in Mata
- Mata command optimize
- NL2SLS example

2. Built-in command poisson

- Data from 2002 U.S. Medical Expenditure Panel Survey (MEPS).
 Data due to Deb, Munkin and Trivedi (2006)
- Aged 25-64 years working in private sector but not self-employed and not receiving public insurance (Medicare and Medicaid)
- Model docvis annual number of doctor visits.

- . use mus10data.dta, clear
- . quietly keep if year02==1
- . describe docvis private chronic female income

| variable name | display format | value label | variable label |
|--|---|----------------|---|
| docvis private chronic female income | %8.0g %8.0g %8.0g %8.0g %9.0g | | number of doctor visits = 1 if private insurance = 1 if a chronic condition = 1 if female Income in \$ / 1000 |

. summarize docvis private chronic female income

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|--|--------------------------------------|--|--|----------------------------------|-------------------------------|
| docvis private chronic female income | 4412 4412 4412 4412 4412 | 3.957389 .7853581 .3263826 .4718948 34.34018 | 7.947601 .4106202 .4689423 .4992661 29.03987 | 0 0 0 0 0 -49.999 | 134 1 1 1 280.777 |

Built-in command poisson

. poisson docvis private chronic female income, vce(robust)

```
Iteration 0: log pseudolikelihood = -18504.413
Iteration 1: log pseudolikelihood = -18503.549
Iteration 2: log pseudolikelihood = -18503.549
```

Poisson regression Number of obs = 4412wald chi2(4) = 594.72Prob > chi2 = 0.0000

| docvis | Coef. | Robust Std. Err. | z | P> z | [95% Conf. | Interval] |
|---------|----------|---------------------|-------|--------|------------|-----------|
| private | .7986652 | .1090014 | 7.33 | 0.000 | .5850263 | 1.012304 |
| chronic | 1.091865 | .0559951 | 19.50 | 0.000 | .9821167 | 1.201614 |
| female | .4925481 | .0585365 | 8.41 | 0.000 | .3778187 | .6072774 |
| income | .003557 | .0010825 | 3.29 | 0.001 | .0014354 | .0056787 |
| _cons | 2297262 | .1108732 | -2.07 | 0.038 | 4470338 | 0124186 |

Note: Nonrobust standard errors are (erroneously) much smaller.

Marginal effects for nonlinear model: $\partial E[y|\mathbf{x}]/\partial x_j = \beta_i \times \exp(\mathbf{x}'\boldsymbol{\beta})$.

. mfx

Marginal effects after poisson y = predicted number of events (predict) = 3.0296804

| variable | dy/dx | Std. Err. | Z | P> z | [95% | C.I.] | Х |
|----------|----------|-----------|-------|--------|---------|---------|---------|
| private* | 4.200068 | .20441 | 9.68 | 0.000 | 1.57755 | 2.37881 | .785358 |
| chronic* | | .27941 | 15.03 | 0.000 | 3.65243 | 4.7477 | .326383 |
| female* | | .17758 | 8.61 | 0.000 | 1.18036 | 1.87645 | .471895 |
| income | | .00331 | 3.25 | 0.001 | .00428 | .017274 | 34.3402 |

- (*) dy/dx is for discrete change of dummy variable from 0 to 1 $\,$
- . margeff

Average marginal effects on E(docvis) after poisson

| docvis | Coef. | Std. Err. | z | P> z | [95% Conf. | Interval] |
|---------|----------|-----------|-------|--------|------------|-----------|
| private | 2.404721 | .2438573 | 9.86 | 0.000 | 1.926769 | 2.882672 |
| chronic | 4.599174 | .2886176 | 15.94 | 0.000 | 4.033494 | 5.164854 |
| female | 1.900212 | .2156694 | 8.81 | 0.000 | 1.477508 | 2.322917 |
| income | .0140765 | .004346 | 3.24 | 0.001 | .0055585 | .0225945 |

3. Command ml method If

• First write a program we call lfpois. This constructs the log-likelihood

$$\textstyle \sum_{i=1}^N \ln f(y_i|\mathbf{x}_i, \boldsymbol{\beta}) = \sum_{i=1}^N \{-\exp(\mathbf{x}_i'\boldsymbol{\beta}) + y_i\mathbf{x}_i'\boldsymbol{\beta} - \ln y_i!\}.$$

- Then give commands
 - ml model lf lfpois (docvis = private chronic female income), vce(robust)
 - ml check
 - ml search
 - ml maximize
- The ml check and ml search are optional.

- y is stored in global macro ML_y1.It is referred to as \$ML_y1
- 2 x is combined with β as the index $\mathbf{x}_i'\beta$ It is referred to as the program argument theta1
- \bullet In $f(y|\mathbf{x}, \boldsymbol{\beta})$ is referred to as the program argument 1f

Arguments, temporary variables and local variables are local macros, referenced in single quotes.

Compute the estimator

. ml maximize

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```
initial:
               log pseudolikelihood = -23017.072
               log pseudolikelihood = -23017.072
rescale:
               log pseudolikelihood = -23017.072
Iteration 0:
               log pseudolikelihood = -19777.405
Iteration 1:
Iteration 2:
               log pseudolikelihood = -18513.54
Iteration 3:
               log pseudolikelihood = -18503.556
               log pseudolikelihood = -18503.549
Iteration 4:
Iteration 5:
               log pseudolikelihood = -18503.549
```

Log pseudolikelihood = -18503.549

Number of obs 4412 wald chi2(4) 594.72 Prob > chi2 0.0000

| docvis | Coef. | Robust Std. Err. | z | P> z | [95% Conf. | Interval] |
|---|--|--|--|---|---|--|
| private chronic female income _cons | .7986654 1.091865 .4925481 .003557 2297263 | .1090015 .0559951 .0585365 .0010825 .1108733 | 7.33 19.50 8.41 3.29 -2.07 | 0.000 0.000 0.000 0.001 0.038 | .5850265 .9821167 .3778187 .0014354 4470339 | 1.012304 1.201614 .6072775 .0056787 |

• Command ml is not restricted to likelihood functions. e.g. For OLS maximize $-\sum_{i=1}^{N}(y_i - \mathbf{x}_i'\boldsymbol{\beta})^2$. quietly replace 'lnf' = -('y'-exp('theta1'))^2 But must then use robust standard errors.

• Command ml can handle models with more than one index. e.g. For negative binomial have two indexes $\mathbf{x}_i'\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$. args lnf theta1 a and ml model lf lfnb (docvis = private chronic female income) ()

Number of numerical derivatives = number of indexes.
 Fast if few indexes.

4. Check program by simulation

Generate sample of size N from

$$y_i \sim \text{Poisson}[\exp(\alpha + \beta x_i)]$$

 $x_i \sim \text{N}[0, 0.5^2]$
 $\alpha = 2; \beta = 1.$

- To check consistency
 - Set *N* = 100,000
 - Does $\widehat{\alpha}=1$? Does $\widehat{\beta}=1$?

- ullet To check computation of the standard errors $s_{\widehat{lpha}}$ and $s_{\widehat{eta}}$.
 - Set N = 500.
 - Draw 2,000 samples of size N and obtain 2,000 estimates using command simulate or command postfile
 - Does $\sqrt{\frac{1}{1999}\sum_{s=1}^{2000}(\widehat{\beta}^{(s)}-\overline{\widehat{\beta}})^2}=\frac{1}{2000}\sum_{s=1}^{2000}s_{\widehat{\beta}}^{(s)}$?
 - i.e. Over the simulations does the st. deviation of $\widehat{\beta}=$ the average st. error of $\widehat{\beta}$?

5. Command ml methods d0, d1, d2

- More general.
- Computes the log-density for each observation.
 This then needs to be summed using mlsum
- Enters parameters β directly, rather than via index $\mathbf{x}'\beta$.
- ullet Method d0 needs to compute q numerical derivatives if q parameters.
- Can provide first derivatives (method d1) and second derivatives (method d2).
 - This speeds up computation.

- For method d0 extra arguments is todo
- ullet mleval converts $oldsymbol{eta}$ to $\mathbf{x}'oldsymbol{eta}$
- mlsum converts $\mathbf{x}_i'\boldsymbol{\beta}$ to $\sum_{i=1}^N \mathbf{x}_i'\boldsymbol{\beta}$.

```
* Method d0: Program d0opois to be called by command ml method d0
program define d0pois
1. version 10.0
2. args todo b lnf // todo is not used, b=b, lnf=lnL
3. tempvar theta1 // theta1=x'b given in eq(1)
4. mleval `theta1' = `b', eq(1)
5. local y $ML_y1 // Define y so program more readable
6. mlsum `lnf' = -exp(`theta1') + `y'* `theta1' - lnfactorial(`y')
7. end
```

- . ml model d0 d0pois (docvis = private chronic female income)
- . ml maximize

```
initial:
               log likelihood = -33899.609
alternative:
               log likelihood = -28031.767
rescale:
               log likelihood = -24020.669
Iteration 0:
               log likelihood = -24020.669
               loa likelihood = -18845.464
Iteration 1:
               log likelihood = -18510.257
Iteration 2:
Iteration 3:
               log likelihood = -18503.552
               log likelihood = -18503.549
Iteration 4:
Iteration 5:
               log likelihood = -18503.549
```

Log likelihood = -18503.549

Number of obs = 4412 Wald chi2(4) = 8052.34 Prob > chi2 = 0.0000

| docvis | Coef. | Std. Err. | z | P> z | [95% Conf. | Interval] |
|---------|----------|-----------|-------|--------|------------|-----------|
| private | .7986653 | .027719 | 28.81 | 0.000 | .7443371 | .8529936 |
| chronic | 1.091865 | .0157985 | 69.11 | 0.000 | 1.060901 | 1.12283 |
| female | .4925481 | .0160073 | 30.77 | 0.000 | .4611744 | .5239218 |
| income | .003557 | .0002412 | 14.75 | 0.000 | .0030844 | .0040297 |
| _cons | 2297263 | .0287022 | -8.00 | 0.000 | 2859815 | 173471 |

- Preceding gives nonrobust standard errors.
- To get robust standard errors need to use method d1 or d2.

```
* Method d2: Program d2pois to be called by command ml method d2
program define d2pois
      version 10.0
      args todo b lnf g negH
                                        // Add g and negH to the arguments list
 tempvar theta1
                                        // theta1 = x'b where x given in eg(1)
      mleval `theta1' = `b', eq(1)
     local y $ML_y1
      local y ML_y1 // Define y so program more readable mlsum `lnf' = -exp(`theta1') + `y'* theta1' - lnfactorial(`y')
      if (`todo'==0 | `lnf'>=.) exit // d1 extra code from here
 8.
      tempname d1
      mlvecsum \inf' d1' = y' - \exp(\theta1')
9.
      matrix `q' = (`d1')
10.
      if ('todo'==0 | 'lnf'>=.) exit // d2 extra code from here
11.
12.
     tempname d11
      mlmatsum `lnf' `d11' = exp(`theta1')
13.
14.
      matrix `negH' = `d11'
15. end
```

6. Newton-Raphson algorithm using Mata

- ullet Iterative algorithms are rules to compute $\widehat{ heta}_{s+1}$ given $\widehat{ heta}_s$.
- Gradient methods use a rule of the form

$$\widehat{m{ heta}}_{s+1} = \widehat{m{ heta}}_s + \mathbf{A}_s \mathbf{g}_s$$

where \mathbf{g}_s is the gradient of the objective function evaluated at $\widehat{m{ heta}}_s$.

• Newton-Raphson (NR) method approximates the objective function at $\widehat{\theta}_s$ by a quadratic function. It chooses $\widehat{\theta}_{s+1}$ to maximize this approximation.

Then

$$\widehat{m{ heta}}_{s+1} = - \mathbf{H}_s^{-1} \mathbf{g}_s$$

where \mathbf{H}_s is the Hessian evaluated at $\widehat{\boldsymbol{\theta}}_s$.



Poisson objective function, gradient and Hessian are:

$$\begin{array}{ll} Q(\pmb{\beta}) &= \sum_{i=1}^N \{-\exp(\mathbf{x}_i'\pmb{\beta}) + y_i\mathbf{x}_i'\pmb{\beta} - \ln y_i!\} \\ \mathbf{g}(\pmb{\beta}) &= \sum_{i=1}^N (y_i - \exp(\mathbf{x}_i'\pmb{\beta}))\mathbf{x}_i \\ \mathbf{H}(\pmb{\beta}) &= \sum_{i=1}^N - \exp(\mathbf{x}_i'\pmb{\beta})\mathbf{x}_i\mathbf{x}_i'. \end{array}$$

So NR is

$$\begin{split} \widehat{\pmb{\beta}}_{s+1} &= \widehat{\pmb{\beta}}_s - \mathbf{H}(\widehat{\pmb{\beta}}_s)^{-1} \times \mathbf{g}(\widehat{\pmb{\beta}}_s) \\ &= \widehat{\pmb{\beta}}_s + \left[\sum_{i=1}^N \exp(\mathbf{x}_i' \widehat{\pmb{\beta}}_s) \mathbf{x}_i \mathbf{x}_i' \right]^{-1} \times \sum_{i=1}^N (y_i - \exp(\mathbf{x}_i' \widehat{\pmb{\beta}}_s)) \mathbf{x}_i. \end{split}$$

Core Mata code is

- Define y and x
 generate cons = 1
 local y docvis
 local xlist private chronic female income cons
- Read these in to Mata using st_view

```
: st_view(y=., ., "'y'")
: st_view(X=., ., tokens("'xlist'"))
```

- Do the analysis and compute b and V
- Pass these back to Stata using st_matrix st_matrix("b",b') st_matrix("V",vb)
- Post results using command ereturn

Do the NR iterations to compute $\widehat{\beta}$.

```
* Complete Mata code for Poisson MLE NR iterations
mata
                                              — mata (type end to exit)
  st_view(y=., ., "`y'")
                                     // read in stata data to y and X
  st_view(X=., ., tokens("`xlist'"))
  b = J(cols(X), 1, 0)
                                     // compute starting values
  n = rows(x)
  iter = 1
                                     // initialize number of iterations
                                     // initialize stopping criterion
  cha = 1
  do {
     mu = exp(x*b)
     grad = X'(y-mu)
                                     // kx1 gradient vector
     hes = makesymmetric((x:*mu)'x) // negative of the kxk hessian matrix
     hold = h
     b = bold + cholinv(hes)*(grad)
     cha = (bold-b)'(bold-b)/(b'b)
     iter = iter + 1
  } while (cha > 1e-16)
                                     // end of iteration loops
```

Compute the variance-covariance matrix of β .

```
mu = exp(x*b)
   hes = (x:*mu)'x
   vgrad = ((X:*(y-mu))'(X:*(y-mu)))
   vb = cholinv(hes)*vgrad*cholinv(hes)*n/(n-cols(X))
                                      // num iterations
   iter
 13
   cha
                                      // stopping criterion
 1.11465e-24
   st_matrix("b",b')
                                      // pass results from Mata to Stata
                                      // pass results from Mata to Stata
   st_matrix("V",vb)
: end
```

User-defined models in Stata

Present results nicely formatted.

- . * Present results, nicely formatted using Stata command ereturn . matrix colnames b = `xlist'
- . matrix colnames V = `xlist'
- . matrix rownames V = `xlist'
- . ereturn post b V
- . ereturn display

| | Coef. | Std. Err. | z | P> z | [95% Conf. | Interval] |
|---------|----------|-----------|-------|--------|------------|-----------|
| private | .7986654 | .1090509 | 7.32 | 0.000 | .5849295 | 1.012401 |
| chronic | 1.091865 | .0560205 | 19.49 | 0.000 | .9820669 | 1.201663 |
| female | .4925481 | .058563 | 8.41 | 0.000 | .3777666 | .6073295 |
| income | .003557 | .001083 | 3.28 | 0.001 | .0014344 | .0056796 |
| cons | 2297263 | .1109236 | -2.07 | 0.038 | 4471325 | 0123202 |

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7. Mata command optimize

- Mata command optimize uses same optimizer as command ml, but different syntax.
- Minimal syntax is
 void evaluator(todo, p, v, g, H)
 where
 p is parameter vector
 v defines objective function, and
 if todo = 0 then gradient g and Hessian H are optional.
- Type v evaluator provides formula for 1 × N vector v, where e'v = f(p).
 Suited to m-estimators (MLE, LS, just-identified NLIV).
- Type d evaluator provides formula for scalar v where $v = f(\mathbf{p})$. Suited to over-identified generalized method of moments (GMM).

Declare the function poissonmle and st_view data

Initialize command optimize and optimize using v2 evaluator.

```
optimize init evaluator(S. &poissonmle())
    optimize init evaluatortype(S. "v2")
    optimize_init_argument(S, 1, y)
    optimize_init_argument(S, 2, X)
    optimize_init_params(S, J(1,cols(X),0))
    b = optimize(S)
Iteration 0: f(p) = -33899.609
Iteration 1:
             f(p) = -19668.697
             f(p) = -18585.609
Iteration 2:
Iteration 3:
             f(p) = -18503.779
Iteration 4:
             f(p) = -18503.549
Iteration 5:
              f(p) = -18503.549
```

S = optimize init()

Compute variance covariance matrix and list results.

```
Vbrob = optimize_result_V_robust(S)
:
    serob = (sqrt(diagonal(vbrob)))'
    b \ serob
                   1
                                   2
                                                   3
                                                                                   5
        .7986653788
                        1.091865108
                                        .4925480693
                                                        .0035570127
                                                                       -.2297263376
 1
        .1090014507
                        .0559951312
                                         .0585364746
                                                        .0010824894
                                                                         .1108732568
: end
```

Note: Can st_matrix back to Stata and ereturn display results.

8. NL2SLS example

- Poisson MLE inconsistent if $E[y \exp(\mathbf{x}'\boldsymbol{\beta})|\mathbf{x}] \neq 0$, due to endogenous regressors.
- Assume there are instruments z such that

$$\mathsf{E}[\mathbf{z}_i(y_i - \exp(\mathbf{x}'\boldsymbol{\beta}))] = 0.$$

• Define the $r \times 1$ vector

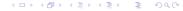
$$\mathbf{h}(\boldsymbol{\beta}) = \left[\sum_{i} \mathbf{z}_{i}(y_{i} - \exp(\mathbf{x}_{i}'\boldsymbol{\beta}))\right].$$

• In just-identified case: # instruments = # regressors (r = K) use the nonlinear instrumental variabels (NLIV) estimator that solves

$$\mathbf{h}(\widehat{\pmb{\beta}}) = \mathbf{0}.$$

• In over-identified case (r > K) the GMM estimator minimizes

$$Q(\boldsymbol{\beta}) = \mathbf{h}(\boldsymbol{\beta})' \mathbf{W} \mathbf{h}(\boldsymbol{\beta}).$$



GMM estimator minimizes

$$Q(\boldsymbol{\beta}) = \mathbf{h}(\boldsymbol{\beta})' \mathbf{W} \mathbf{h}(\boldsymbol{\beta}).$$

• The $K \times 1$ gradient vector is

$$\mathbf{g}(oldsymbol{eta}) = \partial Q(oldsymbol{eta})/\partial oldsymbol{eta} = \mathbf{G}(oldsymbol{eta})' \mathbf{Wh}(oldsymbol{eta}).$$

• The $K \times K$ expected Hessian is

$$\mathsf{H}(\pmb{\beta}) = \partial^2 Q(\pmb{\beta})/\partial \pmb{\beta} \partial \pmb{\beta}' = \mathsf{G}(\pmb{\beta})' \mathsf{WG}(\pmb{\beta})'.$$

Where

$$\mathbf{G}(\boldsymbol{\beta}) = -\sum_{i} \exp(\mathbf{x}_{i}'\boldsymbol{\beta})\mathbf{z}_{i}\mathbf{x}_{i}'$$

$$\mathbf{h}(\boldsymbol{\beta}) = \sum_{i} \mathbf{z}_{i}(y_{i} - \exp(\mathbf{x}_{i}'\boldsymbol{\beta}))$$

$$\mathbf{W} = (\mathbf{Z}'\mathbf{Z})^{-1} = \left(\sum_{i} \mathbf{z}_{i}\mathbf{z}_{i}'\right)^{-1}$$

Declare the function pgmm and st_view data

```
. mata
```

```
mata (type end to exit) -
void pgmm(todo, b, y, X, Z, Qb, g, H)
  xb = x*b'
 mu = exp(xb)
  h = Z'(y-mu)
  W = cholinv(cross(Z.Z))
  Qb = h'w*h
  if (todo == 0) return
  G = -(mu:*Z)'X
  a = (\hat{G}'W*h)'
  if (todo == 1) return
  H = G'W*G
  _makesymmetric(H)
st_view(y=., ., "`y'")
st_view(X=., ., tokens("`xlist'"))
st_view(Z=., ., tokens("`zlist'"))
```

Initialize command optimize and optimize using d2 evaluator.

```
S = optimize init()
    optimize_init_which(S, "min")
    optimize_init_evaluator(S, &pgmm())
    optimize_init_evaluatortype(S, "d2")
    optimize_init_argument(S, 1, y)
    optimize_init_argument(S, 2, X)
    optimize_init_argument(S, 3, Z)
    optimize_init_params(S, J(1,cols(X),0))
    optimize init technique(S."nr")
    b = optimize(S)
Iteration 0:
              f(p) =
                      71995.212
Iteration 1:
              f(p) =
                      9259.0408
Iteration 2:
              f(p) = 1186.8103
Iteration 3:
              f(p) = 3.4395408
Iteration 4:
              f(p) = .00006905
Iteration 5:
              f(p) = 5.672e-14
              f(p) =
                      1.953e-27
Iteration 6:
```

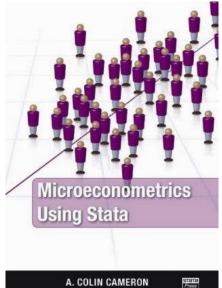
Compute variance covariance matrix (manually) and list results.

```
Compute robust estimate of VCE and se's
 xb = x*b'
 mu = exp(xb)
 h = Z'(y-mu)
 W = cholinv(cross(Z.Z))
 G = -(mu:*Z)'X
 Shat = ((y-mu):*Z)'((y-mu):*Z)*rows(X)/(rows(X)-cols(X))
 Vb = luinv(G'W*G)*G'W*Shat*W*G*luinv(G'W*G)
 seb = (sqrt(diagonal(Vb)))'
 b \ seb
                                2
                                                3
                                                                               5
                1
      1.340291853
                     1.072907529
                                      .477817773
                                                     .0027832801
                                                                   -.6832461817
2
      1.559899278
                     .0763116698
                                     .0690784466
                                                     .0021932119
                                                                    1.350370916
```

User-defined models in Stata

: end

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