Linear Regression Analysis

using python.

Iván Andrés Trujilllo Abella

Facultad de Ingenieria
Pontificia Universidad Javeriana

trujilloiv@javeriana.edu.co
addajaveriana

Ordinary least squares

$$u = \sum (y_i - \hat{y}_i)^2 \tag{1}$$

note that $\hat{y}_i = \beta_0 + \beta_1 x_i$.

$$u^2 = \sum (y_i - \beta_0 - \beta_1 x_i)^2 \tag{2}$$

the first order condition require $\frac{\partial u^2}{\partial \beta_0} = 0$, $\frac{\partial u^2}{\partial \beta_1} = 0$.

Chain rule

to get

$$\frac{\partial u^2}{\partial \beta_0} = \frac{\partial u^2}{\partial u}\frac{\partial u}{\partial \beta_0} \tag{3}$$

$$\frac{\partial u^2}{\partial \beta_1} = \frac{\partial u^2}{\partial u}\frac{\partial u}{\partial \beta_1} \tag{4}$$

Remember that $\frac{d\sum g_i(x)}{dx} = \frac{\sum dg_i(x)}{dx}$. Therefore $\frac{\partial u^2}{\partial u} = 2u$ and $\frac{\partial u}{\partial \beta_0} = -1$, $\frac{\partial u}{\partial \beta_1} = -x_i$.

Partial derivatives

$$\frac{\partial u^2}{\partial \beta_0} = -2 \sum (y_i - \beta_0 - \beta_1 x_i) \tag{5}$$

$$\frac{\partial u^2}{\partial \beta_1} = -2x_i \sum (y_i - \beta_0 - \beta_1 x_i) \tag{6}$$

note that this will be zero if we know the exactly parameters.

Gradient

The gradient is the vector of partial derivates evaluated in a point p

$$\frac{\partial u^2}{\partial \boldsymbol{\beta}} = \nabla(u^2) = \begin{pmatrix} \frac{\partial(u^2)}{\partial \beta_0} \\ \\ \frac{\partial(u^2)}{\partial \beta_1} \end{pmatrix} \tag{7}$$

Gradient descend

in this equation α is the learning rate.

$$\boldsymbol{\beta_i} = \boldsymbol{\beta_{i-1}} - \alpha\nabla(u^2(\boldsymbol{\beta_{i-1}})) \tag{8}$$

python implementation

e

Standardized coefficients

Suppose a $\boldsymbol{X}$ vector (exogenous) and y (endogenous) that are transformed in Z punctuation and for instance in the regression: x_1 have associated β_1. The interpretation is:

Interpreation

The increase of one standard deviation in x_1 is associated with the increase (reduction) of y in β_1 standard deviations.

Pontificia Universidad JAVERIANA Bogotá

Get standardized from OLS

β_1 is a no-standardized coefficient, and β_{1std} is obtained from:

$$\beta_{1std} = \frac{\sigma_x}{\sigma_y}\beta_1 \tag{9}$$

where σ_x and σ_y are estimated standard deviations.

Which have the major relative importance?

Statsmodels

Multivariate parameters estimation

Concepts needed

Inverse Matrix

some

Operations

Matrix differentiation

What is the result of correlation among variables?

Testing in lab.

Quantile regression

3