Introduction to Linear Algebra

Insights for IA

Iván Andrés Trujilllo Abella

Facultad de Ingenieria
Pontificia Universidad Javeriana

trujilloiv@javeriana.edu.co

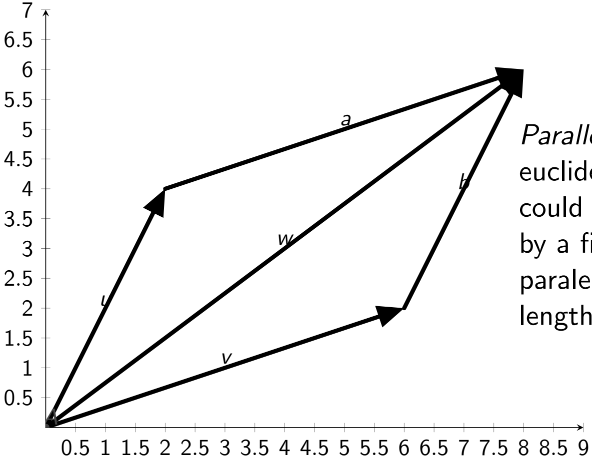
Inner product

the vectors $\vec{x}, \vec{y} \in R^n$ the . or inner product is defined as follow:
$\vec{x} = (x_1, x_2, \ldots, x_n)$ and $\vec{y} = y_1, y_2, \ldots, y_n$.

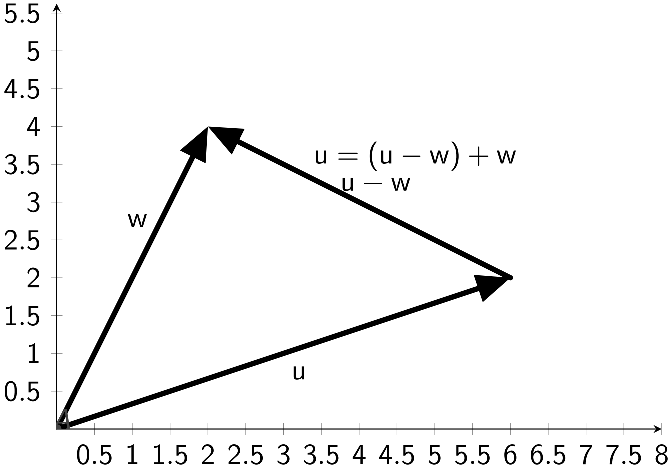
$$\vec{x}.\vec{y} = \sum_{i=1}^{n} = x_i y_i \tag{1}$$

the sum of each pair of components in each vector is the inner product.

Paralelogram



Parallelogram in euclidean geometry could be represented by a figure with paralels lines of equal length.


5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8
u = (u − w) + w
u − w
w
u

Unit vector

the length of a vector or norm $\|x\|$ it also important and will be defined using the pitagoras theorem, as illustrate the following graph.
we can said that a vector $\bar{v}$ is unitary of $\|\bar{v}\| = 1.$ think that a vector y is parallel to a x if for instance exist λ such that $y = \lambda x$.
to normalize a vector you only need divide by a scalar in this case its longitude thus:

$$\bar{v_u} = \frac{\bar{v}}{\|v\|} \tag{2}$$

Cauchy - swartch inequality

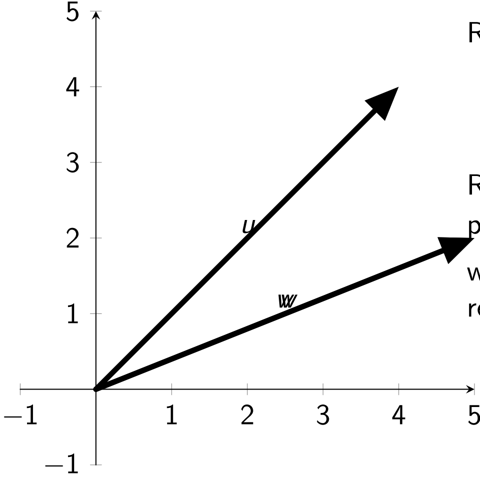
$$|\bar{x}.\bar{y}| \leq \|\bar{x}\|\|\bar{y}\| \tag{3}$$

given the angle and inner product $\bar{x}\bar{y} = \|\vec{x}\|\|\vec{y}\|cos\theta$ take into account that $|\cos\theta| \leq 1, \forall\theta \in \mathbb{R}$

$$|\vec{x}.\vec{y}| = |\|\vec{x}\|\|\|\|\vec{y}\|\|| \cos\theta| \tag{4}$$

Projection

there is a k scalar such that the vectors $\vec{x}, \vec{y}$ be orthogonal.



Remember that;

$$\cos\theta = \frac{\bar{u}\bar{w}}{\|\vec{u}\|\|\vec{w}\|} \tag{5}$$

Remember that the project make a parallel line therefore $\cos\theta = \frac{\|v_p\|}{\|\vec{u}\|}$ where $\vec{W}$ it is the hypohenuse of rectangle.

$$\|\vec{u}\|\cos\theta = \|v_p\| \tag{6}$$

we calculate the scalar v_p now we are interested in the vector, therefore:

$$\frac{\vec{w}}{\|W\|} V_p = \vec{V_p} \tag{7}$$

Notice that $\vec{v_p}$ refer to the vector while in other case $\vec{v_p}$ refer to the magnitude.

$$\vec{v_p} = \frac{\vec{u}\vec{w}}{\|\vec{w}\|}\vec{w} \tag{8}$$