Elementary mathematics for machine learning

Notes of class

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0.1 Jacobian Matrix

linear maps or linear transformations:

a alternative coordinate system is related with jacobian.

Rememeber that if you alter the base coordinate vectors

note

jacobian determinant: how much areas are scale near to one point.

here is important the following: we defined that is easy find a analytical transformation only indicating how many times

$\alpha_j \hat{j} + \alpha_i \hat{i}$ this allow us determined the new vector in linear transform, and the linear transform must be have the following properties:

$$\begin{gathered} T(k\vec{v}) = k \\ T(\vec{v} + \vec{u}) = T(\vec{v}) + T(\vec{u}) \end{gathered} \tag{1}$$

WHat is base? what is a subspace?

0.2 vector space

0.3 Propierties of vector spaces

Conmutativity $\vec{u} + \vec{v} = \vec{v} + \vec{u}$, null vector, identity element and inverse ($\vec{v} + -\vec{v}) = 0$

0.4 Properties of scalars

multiple a vector $\vec{u}$ of $n - components$ by a scalar k where $k \in R$ then we get $k\vec{u} = (ku_1, ku_2, ..., ku_n)$.

1 ideas or insights

adding two scaled vectors. the unitary vectors $\vec{i}$ and $\vec{j}$ the unitary vectors are the basis vectors of the coordinate system.

2 linear combination

it is called linear combination, to the following

$$k_1\vec{i} + k_2\vec{j} \tag{2}$$

note that $k_1, k_2 \in R$ also take in account that if we fix a vector and vary the another we can get a straight line.

if you take in mind that $\vec{i}, \vec{j}$ it is measure of length then we can get the point (x, y) as the sum of the vectors by the scalars x, y.

2.1 Independence basis and dimension

2.2 Positive define matrices and co-variance matrix

think in the following problem, in OLS you must

think in the following problem in a quadratic form we must sure that the exist a minimun point;

$$ax^2 + by^2 + cxy \tag{3}$$

is this function positive ? note that the squares are also be positive however cxy could be negative the question is that $ax^2 + by^2 - cxy > 0$ always.

we said that a positive definite (or semi-definite) matrix $\mathbf{A}$ if

$$\mathbf{x}^T \mathbf{A} \mathbf{x} \geq 0, \forall \mathbf{x} \in R^n \tag{4}$$

if the inequality is strictly then is definite otherwise is semi.

3 span

is the set of linear combinatios of two vectors i

4 plane

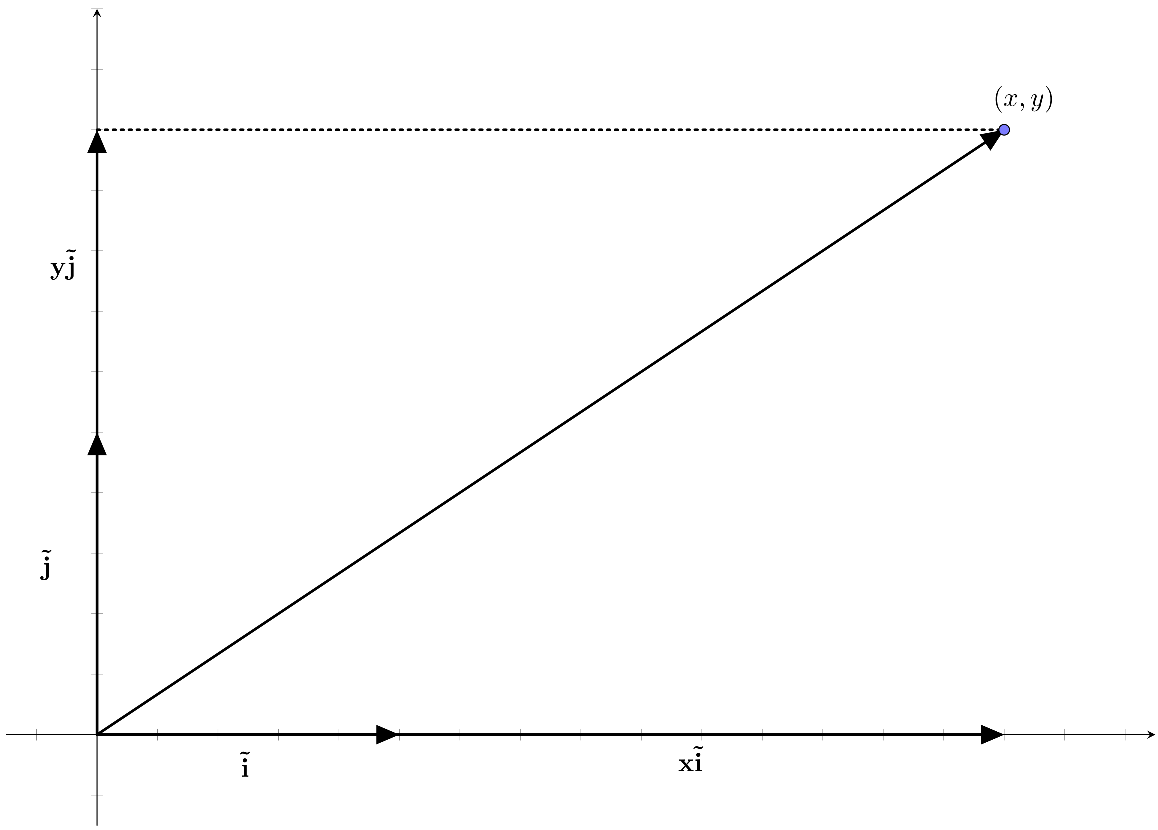
two vectors span a plane. three vectors span a R^3.

5 Basis for a subspace

independent vectors that span the subspace.

6 Dimension

number of basis vectors for the subspace.



Complex numbers

a complex number is composed of a real part and imaginary, $z = r + i$, the imaginary part solve $i^2 = -1$.

7 Eigen values and eigen vectors

take in mind the following operations:

- translation
- inversion
- projections
- stretch
- rotation

$$\mathbf{A}\vec{x} = \lambda\vec{x} \tag{5}$$

Note that the map of A over $\vec{x}$

7.1 Translation

Only consist in pass from the coordinates (x, y) to $(x + \delta x, y)$

$$\begin{pmatrix} x + \delta x \\ y \end{pmatrix} = \begin{pmatrix} w & v \\ z & j \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

therefore;

$$\begin{pmatrix} x + \delta x \\ y \end{pmatrix} = \begin{pmatrix} 1 + \delta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Note that the translation is over the $x - axe$ over the $y - axe$ is only (x, y) to $(x, y + \delta y)$.

7.2 Rotation Matrix

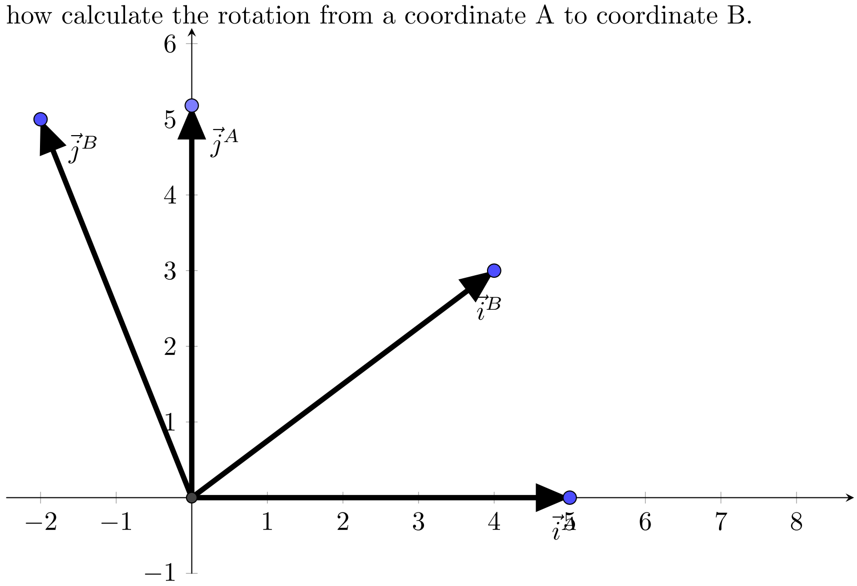
Rotation matrices, rotation is based on the idea of projections.

remember that a vector in a coordinate system (A) could be written as $\vec{x}^A$ also remember that in a analytic way.

in 2D for instance $\vec{x} = a\vec{i} + b\vec{j}$ where $\vec{i}, \vec{j}$ are unitary vectors, in 3D is equal

$$\vec{x}^A = \begin{pmatrix} k_1\vec{i}^A \\ k_2\vec{j}^A \\ k_3\vec{z}^A \end{pmatrix} \tag{6}$$

how calculate the rotation from a coordinate A to coordinate B.



Note that for this case, we pass from coordinates A to coordinates B, we can denote this in R^A_B.

Note that we have project $\vec{i}^B$ over the vectors $\vec{i}^A$, and $\vec{j}^A$.

Note here that the coordinates (x, y) now will be $(\hat{x}, \hat{y})$ think in any vector generated by the translation$\vec{u} = k_1\vec{j}^B + k_2\vec{i}^B$, the question is $\vec{u} = k_1\vec{j}^A + k_2\vec{i}^A$ what is the vector in terms of the A coordinate system.

Assume that the vectors are rotate in θ degrees [1].

thus we can translate the projections over the system A, as follow:

$$\vec{i}^B = proyection(\vec{i}^B)_x + proyection(\vec{j}^B)_y \tag{7}$$

Notice taht Now:

$$cos(\theta) = \frac{proyection(\vec{i}^B)_x}{\vec{i}^B} = \cos(\theta) \tag{8}$$

given that the vector $\vec{i}^B$ it is unitary.

now

$$proyection(\vec{i}^B)_y = \sin(\theta) \tag{9}$$

thus we can get:

$$\vec{i}^B = \cos(\theta) + \sin(\theta). \tag{10}$$

the same is applied to the $\vec{j}^B$ getting:

$$\vec{j}^B = \tag{11}$$

7.3 Getting eigen values

think in mind the following, the span is the entire line that lay over the value.

things to think, why $\vec{i}$ and $\vec{j}$ are the respectively columns and when you multiply, you multiply in inner product.

rotations and projections how think in this operations and how are related?

[1]Remember that $\sin(\theta) = \frac{opposite}{Hypothenuse}$ and $\cos(\theta) = \frac{adjacent}{Hypothenuse}$.

8 Linear approximation

If we have $y = f(x)$ the linear approximation is constructed with tangent line, with in $z = f(x, y)$ then we can think in a tangent plane.

8.1 Quadratic approximation

Is a better approximation (another curve) that a tangent plane.

9 Hessian Matrix

Second derivates of a function: