Binomial, normal distribution and sampling distribution

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Random variable

PMF and CDF

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- Probability Mass Function $PMF = P(X = x)$
- Cumulative Distribution Function $CDF = P(X < x)$

Normal

Binomial

Assume that you have 5 trials, you need the probability of get exactly three sucess.

5 trials and 3 success...

Success and failures	**Probability**
$E_1E_2E_3F_4F_5$	$p^3(1-p)^2$
$E_1E_2E_4F_3F_5$	$p^3(1-p)^2$
$E_1E_2E_5F_3F_4$	$p^3(1-p)^2$
$E_1E_3E_4F_2F_5$	$p^3(1-p)^2$
$E_1E_3E_5F_2F_4$	$p^3(1-p)^2$
$E_1E_4E_5F_2F_3$	$p^3(1-p)^2$
$E_2E_3E_4F_1F_5$	$p^3(1-p)^2$
$E_2E_3E_5F_1F_4$	$p^3(1-p)^2$
$E_2E_4E_5F_1F_3$	$p^3(1-p)^2$
$E_3E_4E_5F_1F_2$	$p^3(1-p)^2$

Binomial distribution

We must said that $X \sim B(n, p)$

PMF

$$P(X = x) = \binom{n}{x} P^x (1 - P)^{n-x} \tag{1}$$

CDF

$$P(X \leq x) = \sum_{i=0}^{x} \binom{n}{x} P^x (1 - P)^{n-x} \tag{2}$$

Python

Normal distribution

Abraham moivre uses the formaul to get binomial probabilities.

Normal distribution

$$f(X) = \frac{1}{\sqrt{\pi\sigma}} \exp \frac{a}{b} \tag{3}$$

CLT

Poisson Distribution

According to the former binomial distribution $X \sim b(p, n)$ the two parameter are the shape a form of the distribution. the poisson distribution is the case when the variable follow a binomial distribution with a $n \to \infty$

Frame Title

In the limit case, the occurrence of a only event is only guaranteed in the measure that the space is very small, for instance if the ocurrence of the events is simultaneous, you should not consider a Poisson distribution. the FD we can dervied of a binomial distribution in the following way $E(x) = np = \lambda$, thus:

$$\frac{n!}{(n-k)!k!}(\frac{\lambda}{n})^k(1-\frac{\lambda}{n})^{n-k}$$

$$\frac{(n-k+1)!}{n^k k!}(1-\frac{\lambda}{n})^n(1-\frac{\lambda}{n})^{-k}$$

$e = \lim_{x\to\infty}(1+\frac{1}{n})^n$we must use $t=\frac{n}{k}$, and thus $\frac{n+k}{n} = 1+\frac{k}{n}$

$$\lim_{n\to\infty} = \frac{e^{-k}\lambda^k}{k!}$$

thus a ramdon variable follow a poisson distrbution with a paramter λ $X \sim p(\lambda)$ and its FD is rewritten as:

$$p(X=x) = \frac{e^{-x}\lambda^x}{x!}$$

CLT

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- Population - Parameters
- Sample - Statistics

for instance the mean μ and sample mean $\bar{x}$. in some books σ^2 and S^2 for population and sample variance respectively.

Distribution of sample statistics

Each sample have different values, then statistics are random variables, but what distribution follow?.

Distribution mean

sample

- if $X \sim N(\mu, \sigma)$ then $\bar{x} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$
- by CLT if n is large then X is approximately normal with $N(\mu, \frac{\sigma}{\sqrt{n}})$

How big is?

n?

30 is a practical value