Hypothesis testing - P value.

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Hypothesis testing

Is a part of statistical that allow us make inference about Θ.

α Significance level

Will be our decicision rule...

Decision rule

According to the statistical value, we are going to accept or not our hypothesis

$$P - value < \alpha \tag{1}$$

Fair coin - Experiement

Experiment - flip coin 100 times

Suppose that you need test that three coin (independent) are fair or not, then you collect data and need test.

Real parameter (P)	**Successes**	**P value**	**Claim**
0.50	20	0.898	Fair!

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0.67	30	0.059	Fair!

Fair coin - Experiement

Experiment - flip coin 100 times

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Real parameter (P)	**Successes**	**P value**	**Claim**
0.50	20	0.898	Fair!
0.67	30	0.059	Fair!
0.72	35	0.001	Biased

Statemets

	True H_0	**False** H_0
Accept H_0	TP	Type 2 error
Reject H_0	Type 1 error	TN

α is type 1 error!!!

(See simulations)

CI and hypothesis testing

Lab

- **(See lab hypothesis)**

P-value

...

$$P(X \mid H_0) \tag{2}$$

It is important note that P value tell us the probability of observed this data given that H_0 is true...

Hypothesis

...

- H_0 Hypothesis null
- H_a Alternative hypothesis

Test statistic

...

$$\text{Test-statistic} = \frac{\hat{\Theta} - \Theta_{H_0}}{SE(\hat{\Theta})} \quad (3)$$

...

Θ_{H_0} is our believe in the fair coin example $\Theta_{H_0} = P = 0.5$

Proportion

By normal approximation

SE

$$SE(\hat{P}) = \sqrt{\frac{P(1-P)}{n}} \tag{4}$$

ESE

$$SE(\hat{P}) = \sqrt{\frac{P_{H_0}(1-P_{H_0})}{n}} \tag{5}$$

Test statistics

Test statistics z_{H_0}

How many estimated standard errors be our estimated parameter from the null parameter.

Questions

Now and accordning to hypothesis you could interested in:

- $P(X > z_{H_0} \mid H_0)$
- $P(X < z_{H_0} \mid H_0)$
- $P(X > z_{H_0} \mid H_0) + P(X < z_{H_0} \mid H_0)$

Finally, $P - value$ is the result of estimate the probabilities over **standard normal distribution**.

Greater

$$
\begin{aligned}
H_0 :& \Theta \\
H_a :& \Theta > c
\end{aligned} \tag{6}
$$

We need compute $P(X > z_{H_0} \mid H_0)$

Lesser

Not equal

$$
\begin{aligned}
H_0 &: \Theta = c \\
H_a &: \Theta \neq c
\end{aligned} \tag{7}
$$

We need compute $P(X > z_{H_0} \mid H_0) + P(X < z_{H_0} \mid H_0)$.

Decision

The we can reject or fail to reject the null hypothesis.

Decision rule

$P - value < \alpha$ **Reject the null hypothesis**

$P - value > \alpha$ **Fail to reject the null hypothesis**

Difference in population proportion

...

Two populations A and B with a feature ϕ we are interested in determined if there are difference in both...

$$\Theta_A - \Theta_B = P_A - P_B \tag{8}$$

There is a significant difference among parameters?

Difference in population proportion

Set the significance level α.

...

$$\begin{aligned} H_0 :& P_1 - P_2 = 0 \\ H_1 :& P_1 - P_2 \neq 0 \end{aligned} \tag{9}$$

Assumptions

Random samples and common proportion at least 10 (no's and yes's) by each group.

$$\hat{P} = \frac{n_A \hat{P}_A + n_B \hat{P}_B}{n_A + n_B} \tag{10}$$

$$\begin{aligned} \hat{P} n_i &\geq 10 \quad i = A, B. \\ (1 - \hat{P}) n_i &\geq 10 \quad i = A, B. \end{aligned} \tag{11}$$

Difference in population proportion

test statistics

...

$$\frac{(\hat{P}_A - \hat{P}_B) - 0}{ESE(\hat{P}_A - \hat{P}_B)} = \frac{\hat{P}_A - \hat{P}_B}{\sqrt{\hat{P}(1 - \hat{P})\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} \tag{12}$$

Laboratory

(Proportion difference)

Mean

SE

$$SE = \frac{\sigma}{\sqrt{n}} \tag{13}$$

ESE

$$ESE = \frac{S}{\sqrt{n}} \tag{14}$$

Test statistic

$$\frac{\bar{x} - \mu_{H_0}}{ESE(\bar{x})} \sim t(df - 1) \tag{15}$$

where df is the degree of freedoms of the t-distribution.

P-value

...

Consider that we are testing the probability of get a equal or more extreme value of test statistics if the null hypothesis is true.