Proba2 IIND-2027

Inference - application in counting german tanks

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Problem

Counting panzers in world war 2; assume a population with a total of B tanks, you take a label random with the goal of estimate the parameter B.

Discrete Uniform distribution

we said that $X \sim U(A, B)$

pmf

$pmf = \frac{1}{B-A+1} \quad x \in \{A, A+1, ..., B\}$

cdf

$P(X \leq x) = \sum_{i=A}^{x} \frac{1}{B-A+1} = \frac{x-A+1}{B-A+1}$

likelihood function

we have the following.

...

$$L(A, B \mid X) = \prod f(x_i \mid A, B)) = \left(\frac{1}{B - A + 1}\right)^n \tag{1}$$

now we need

...

$$\min(A - B + 1) \quad \text{s.t. } A \leq x_i \leq B, \ \forall i \tag{2}$$

...

$$\begin{aligned} \hat{A} &= A_{MLE} = \min(x_1, ..., x_n) \\ \hat{B} &= B_{MLE} = \max(x_1, ..., x_n) \end{aligned} \tag{3}$$

Maximum distribution

...

$$Y = \max\{x_1, ..., x_n\} \tag{4}$$

...

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in a vector X of random variables **iid** we are defining the distribution of $Y = max\{x_1, ..., x_n\}$ the approach is get the cumulative distribution function (CDF) and after derived:

$$CDF(y) = F_Y(y) = P(Y \leq y) = P(\max\{x_1, ..., x_n\} \leq y) \tag{5}$$

using the property that are **iid** then:

$$F_Y(y) = P(x_1 \leq y)...P(x_n \leq y) \tag{6}$$

The probability of each **iid** x_i random variable is lesser than y is $F_X(y)$ therefore

$$F_Y(y) = (F_X(y))^n \tag{7}$$

...

$$dist(y) = f_Y(y) = n\left(F_X(y)\right)^{n-1} f_X(y) \quad continue - case$$

Uniform example

using the cdf we ge the pdf

$$dist(max(U)) = P(max_Y = x) = \left(\frac{x - A + 1}{B - A + 1}\right)^n - \left(\frac{x - A}{B - A + 1}\right)^n \quad (8)$$

Maximum as estimator

we are interested in the expected value of the estimator!

$$E(max(x_1, ...x_n)) \tag{9}$$

$$E(x) = \sum x \left(\left(\frac{x - A + 1}{B - A + 1} \right)^n - \left(\frac{x - A}{B - A + 1} \right)^n \right) \tag{10}$$

ir our problem tanks begin in 1 until B.

Tank problem

using the continus approximation of the pmf and with A = 1

$$E(x) = \sum x \frac{n}{B} \left(\frac{x}{B}\right)^{n-1} \tag{11}$$

$$E(x) = \frac{n}{B^n} \sum x^n \tag{12}$$

now we can study $S_n = \sum_{m=1}^{B} x^n$ as $\int_1^B x^n dx = \frac{B^{n+1}}{n+1}$

...

$$E() = \frac{n}{B^n} \frac{B^{n+1}}{n+1} = \frac{n}{n+1} B = B - \frac{B}{n+1} \tag{13}$$

this shows that the estimator underestimated B.

now the extepected value of estimator $\hat{B} = max(x_1, ..., x_n)$ understimated therefore we need adjust

...

$$\begin{aligned} E[c\hat{B}] &= B \\ cE[\hat{B}] &= B \\ c\frac{n}{n+1}B &= B \end{aligned} \tag{14}$$

finally our estimator is

...

$$\hat{B} = \frac{n+1}{n} max(x_1, ..., x_n) \tag{15}$$

lab

Counting Tanks!

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