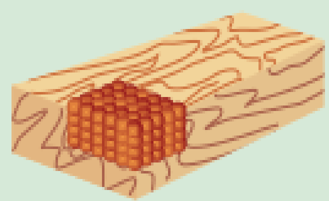
PHYSICS

6. Mechanical Properties of Solids

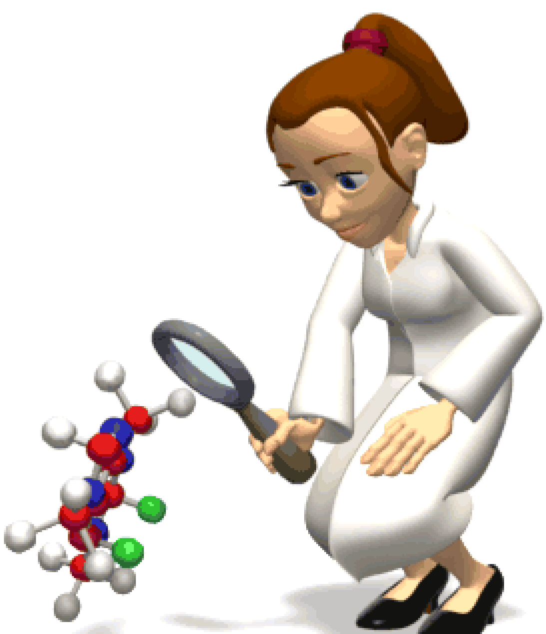
Solids, Liquids and Gases

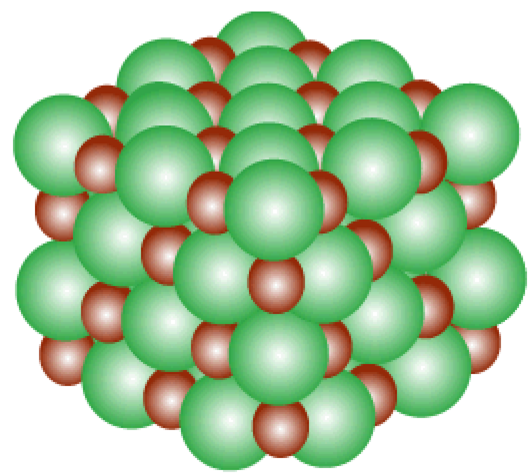
Prof. Ashwini Gonsalves

	Properties	Solids	Liquids	Gases
1	**Mass**	Definite	Definite	Definite
2	**Shape**	Definite	Acquires the shape of the container	Acquires the shape of the container
3	**size**	Definite	Definite	Indefinite
4	**Compressibility**	Not possible	Almost Negligible	Highly Compressible
5	**Fluidity**	Not possible	Can flow	Can flow
6	**Rigidity**	Highly rigid	Less rigid	Not rigid
7	**Diffusion**	Slow	Fast	Very fast
8	**Space between particles**	Most closely packed 	Less closely packed 	Least closely packed 
9	**Interparticle force**	strongest	Slightly weaker than in solids	Negligible

6.1 Introduction

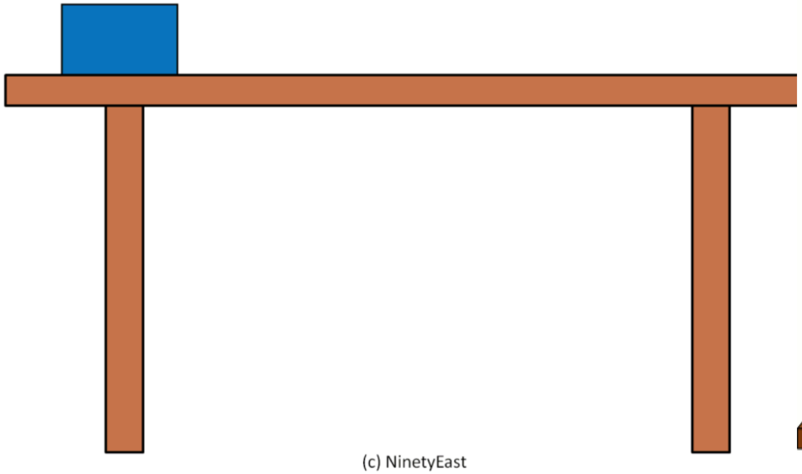
Have u ever wondered why solids have definite shape and size?

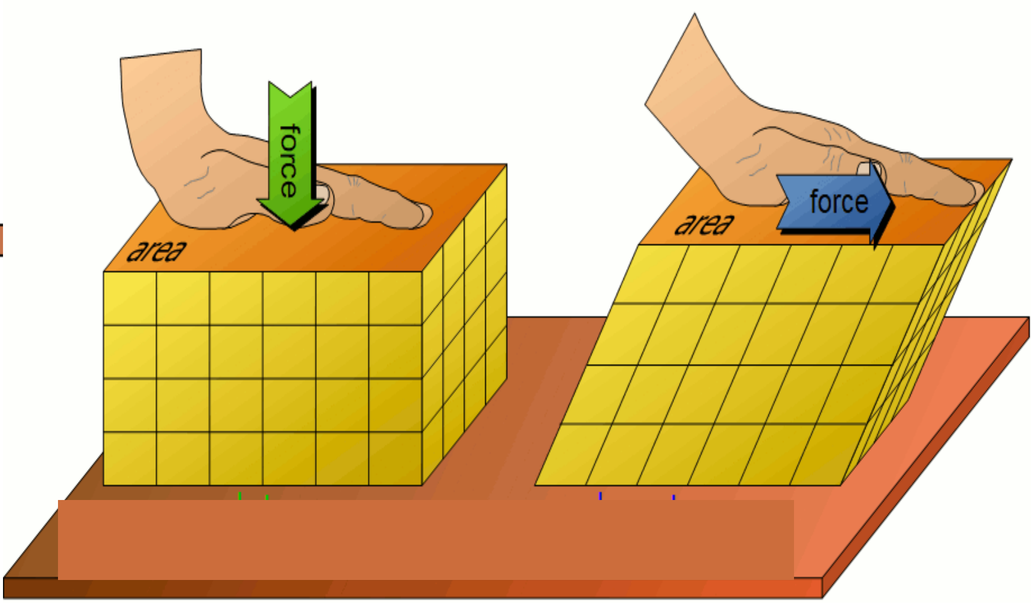




Solid state

What will happen when force is applied to a solid object?

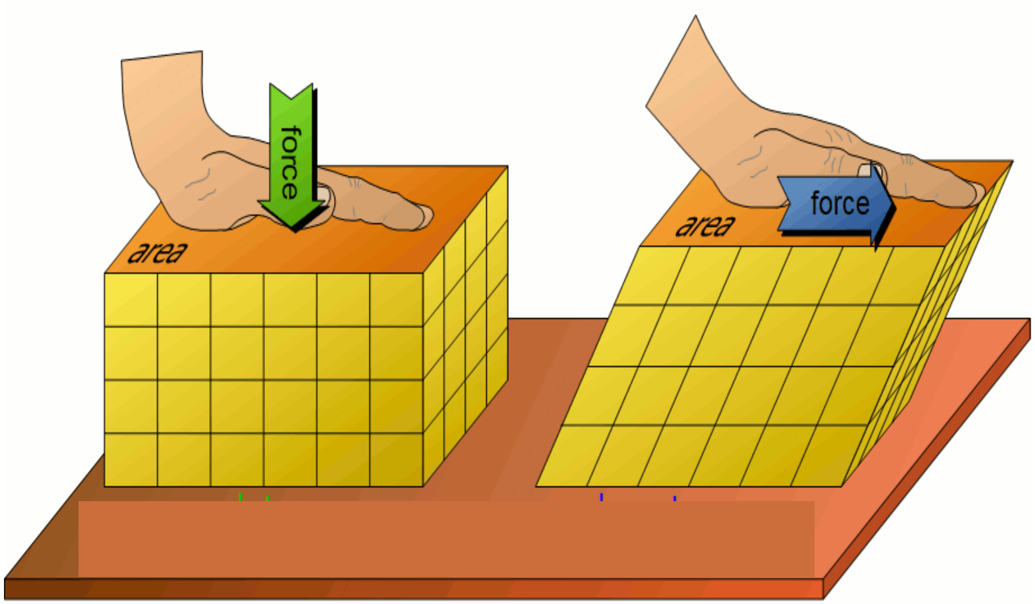


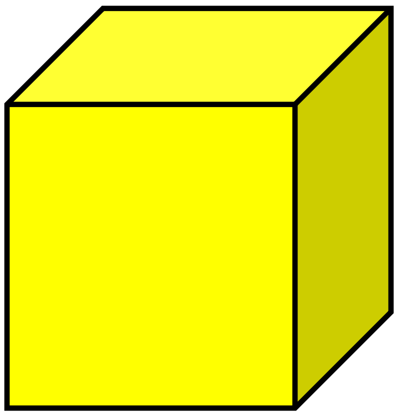


CASE 1: Object is free to move

CASE 2: Object is not free to move

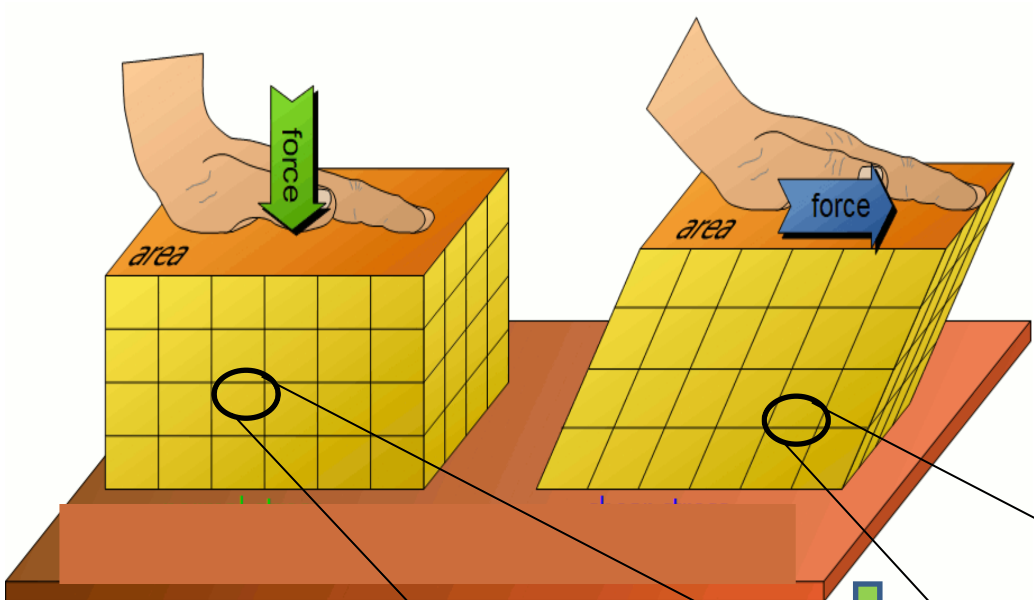
What will happen when force is applied to a solid when it is no free to move?



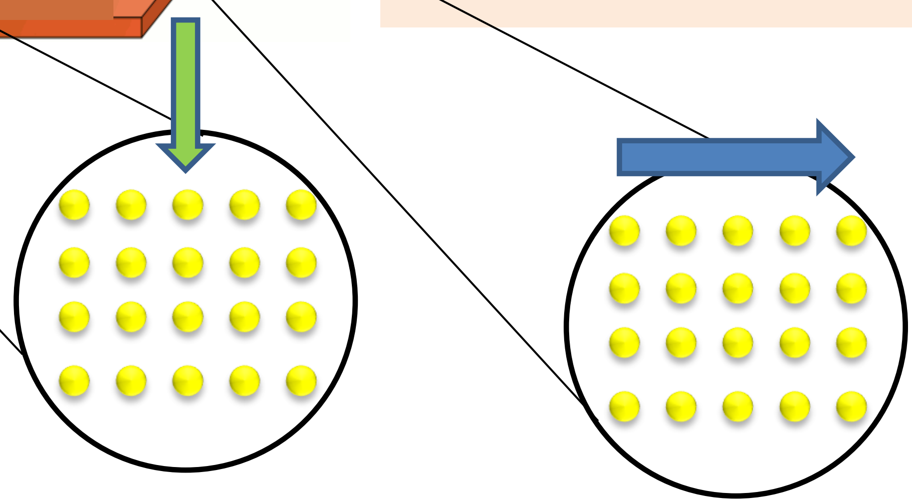


The change in **shape or size or both of a body** due to an external force is called **DEFORMATION.**

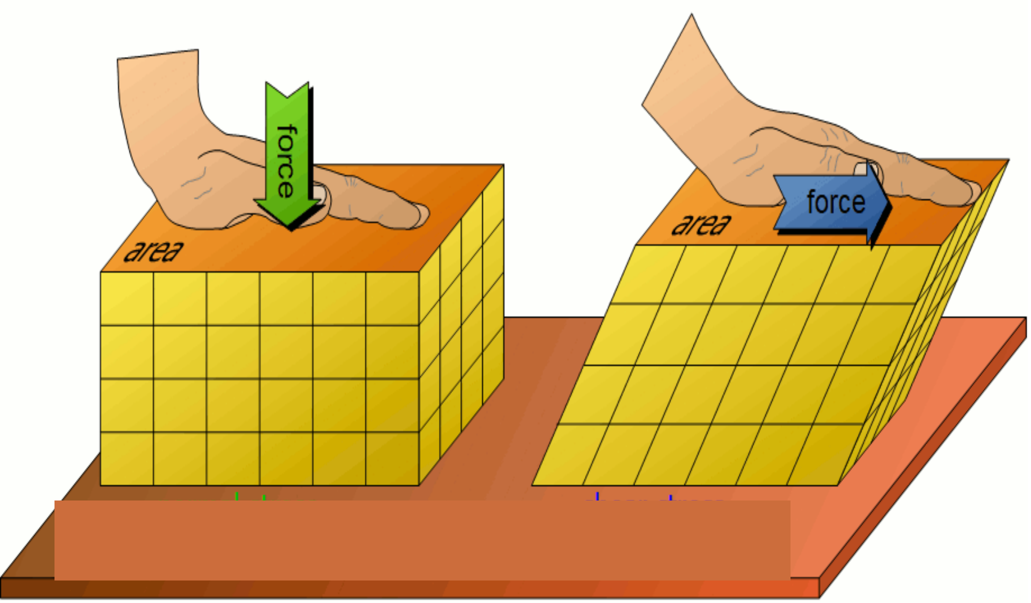
What will happen when force is applied to a solid when it is no free to move?

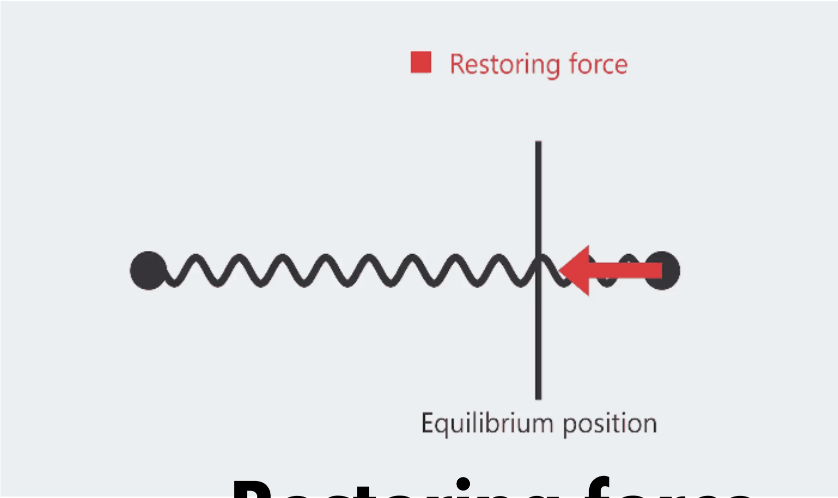


When a force is applied to a solid (which does not free to move), the **size or shape or both change** due to changes in relative positions of molecules. Such a force is called **DEFORMING FORCE**.

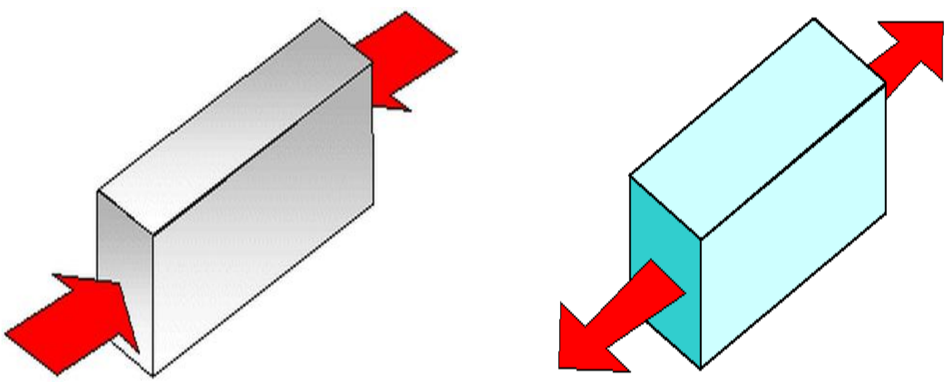


After removing this deforming force what will happen to an object?





Restoring force



If deforming force is applied to rubber, clay or dough ,then what happens?

Observation 1

Observation 2

Observation 3

- These observations indicate that rubber and clay are different in nature.

The property that decides this nature is called **Elasticity/plasticity**

Observation 4

6.2 Elastic Behaviour of Solids

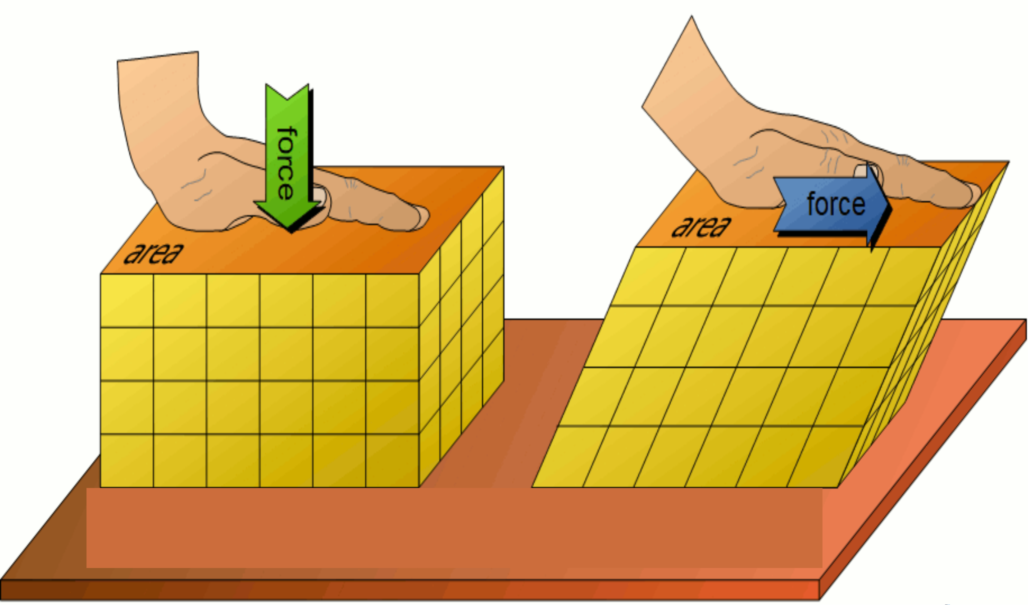
Elasticity- If a body regains its original shape and size after removal of the deforming force, it is called as **elastic body** and the property is called **elasticity**.

If a body regains its original shape and size completely and instantaneously upon the removal of deforming force, then it is called as **perfectly elastic**.

Plasticity -If a body does not regain its original shape and size and retains its altered shape or size upon removal of the deforming force, it is called **plastic body** and the property is called **Plasticity**.

6.2 Stress and Strain

Elastic properties of a body are described in terms of stress and strain.



Stress

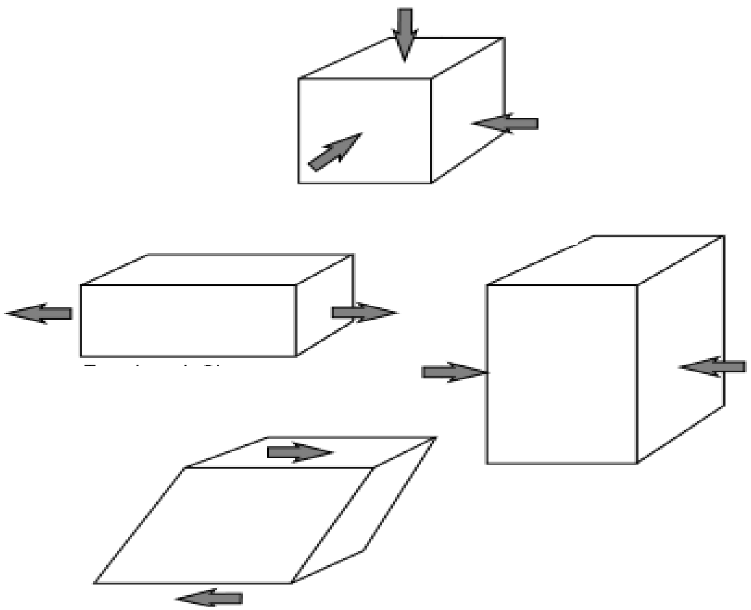
The deforming force per unit area of the body is called as stress.

$$\text{stress} = \frac{\text{deforming force}}{\text{area}} = \frac{|\vec{F}|}{A} \quad \text{--- (6.1)}$$

Where F is external applied deforming force.
SI unit of stress is N $/m^2$ or pascal (Pa).
The dimensions of a stress are
[$L^{-1}M^{1}T^{-2}$].

6.2 Stress and Strain

Elastic properties of a body are described in terms of stress and strain.



- Strain is a measure of the deformation of a body

Strain is defined as ratio of change in dimensions of the body to its original dimensions

$$\text{Strain} = \frac{\text{change in dimensions}}{\text{original dimensions}} \quad \text{--- (6.2)}$$

- It is a dimensionless quantity and does not have units.

Types of Stress

- There are three types of stress:

A) **Tensile Stress** (compressive stress)-

B) **Volume Stress**

C) **Shearing Stress**

Types of Strain

- There are 03 types of strain:

1. Tensile strain.
2. Volume strain.
3. Shear strain.

A) Tensile stress or compressive stress & Tensile strain

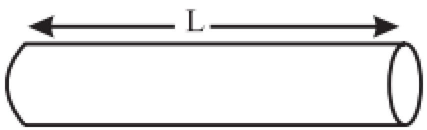


Fig. 6.1 (a): Tensile stress.

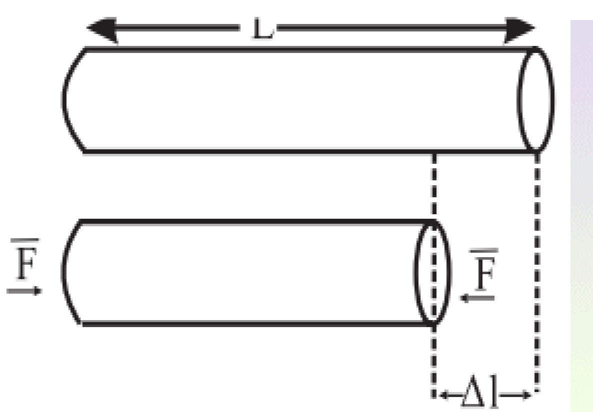


Fig. 6.1 (b): Compressive stress.

Suppose a force $\vec{F}$ is applied along the length of a wire, or perpendicular to its cross section A. This produces an elongation in the wire and the length of the wire increases accordingly, as shown in Fig. 6.1 (a).

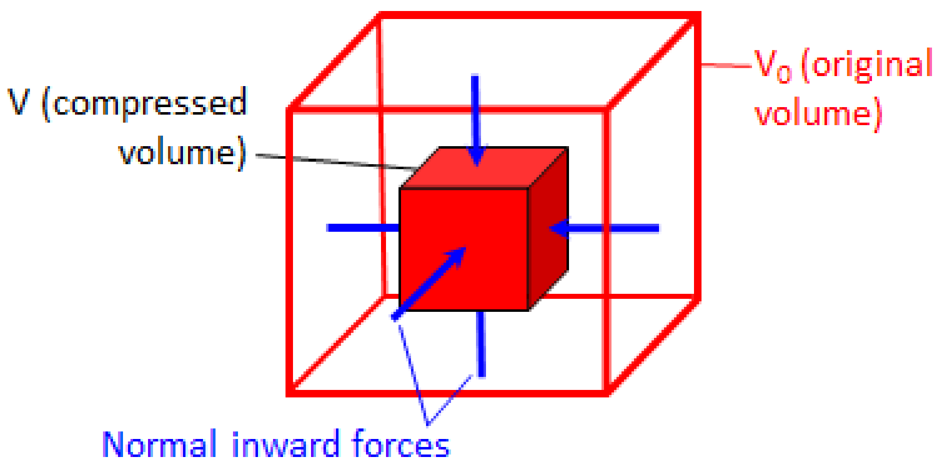
$$\text{Tensile stress} = \frac{|\vec{F}|}{A} \qquad \text{--- (6.3)}$$

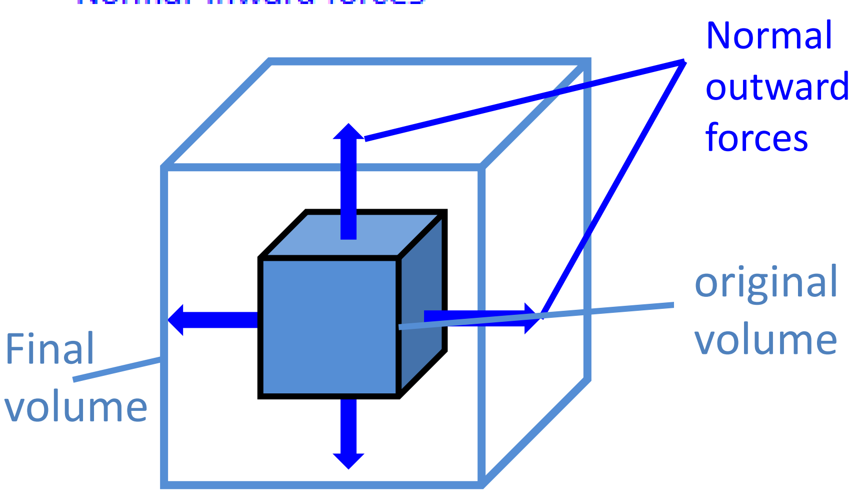
$$\text{Compressive stress} \; \frac{|\vec{F}|}{A} \qquad \text{--- (6.4)}$$

If L is the original length and Δl is the change in length due to the deforming force, then

$$\text{Tensile strain} = \frac{\Delta l}{L} \qquad \text{--- (6.5)}$$

B) Volume stress or Hydraulic stress & Volume strain





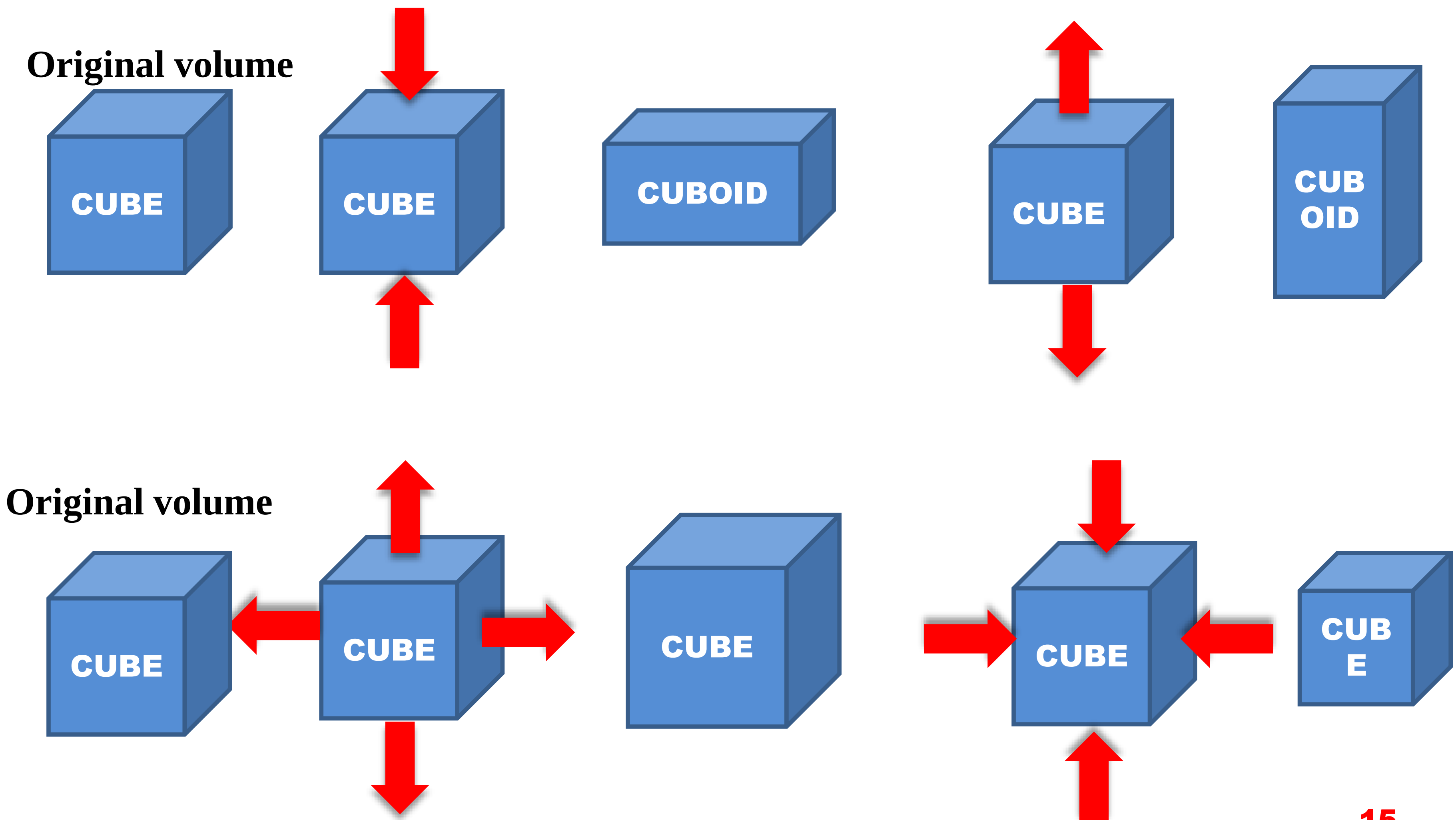
Let $\vec{F}$ be a force acting perpendicular to the entire surface of the body. It acts normally and uniformly all over the surface area A of the body. Such a stress which produces change in size but no change in shape is called volume stress.

$$\text{Volume stress} = \frac{|\vec{F}|}{A} \qquad \text{--- (6.6)}$$

A deforming force acting perpendicular to the entire surface of a body produces a volume strain. Let V be the original volume and ΔV be the change in volume due to deforming force, then

$$\text{Volume stain} = \frac{\Delta V}{V} \qquad \text{--- (6.7)}$$

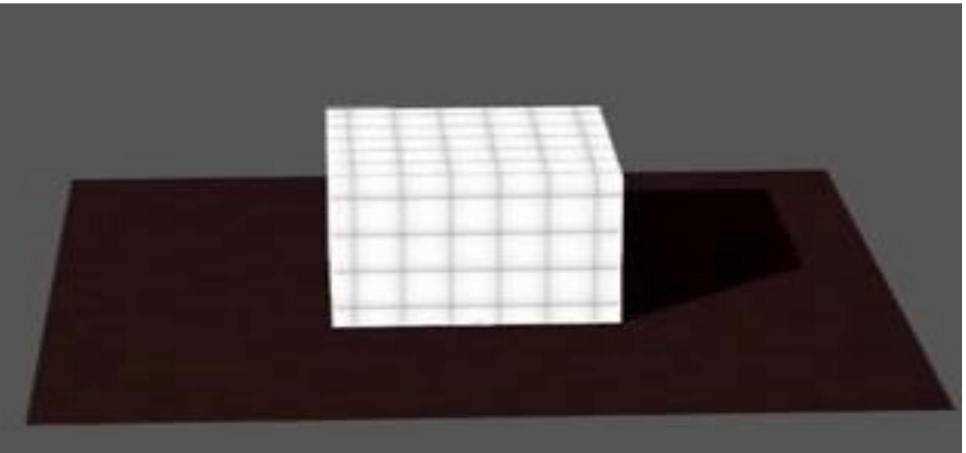
B) Volume stress or Hydraulic stress & Volume strain



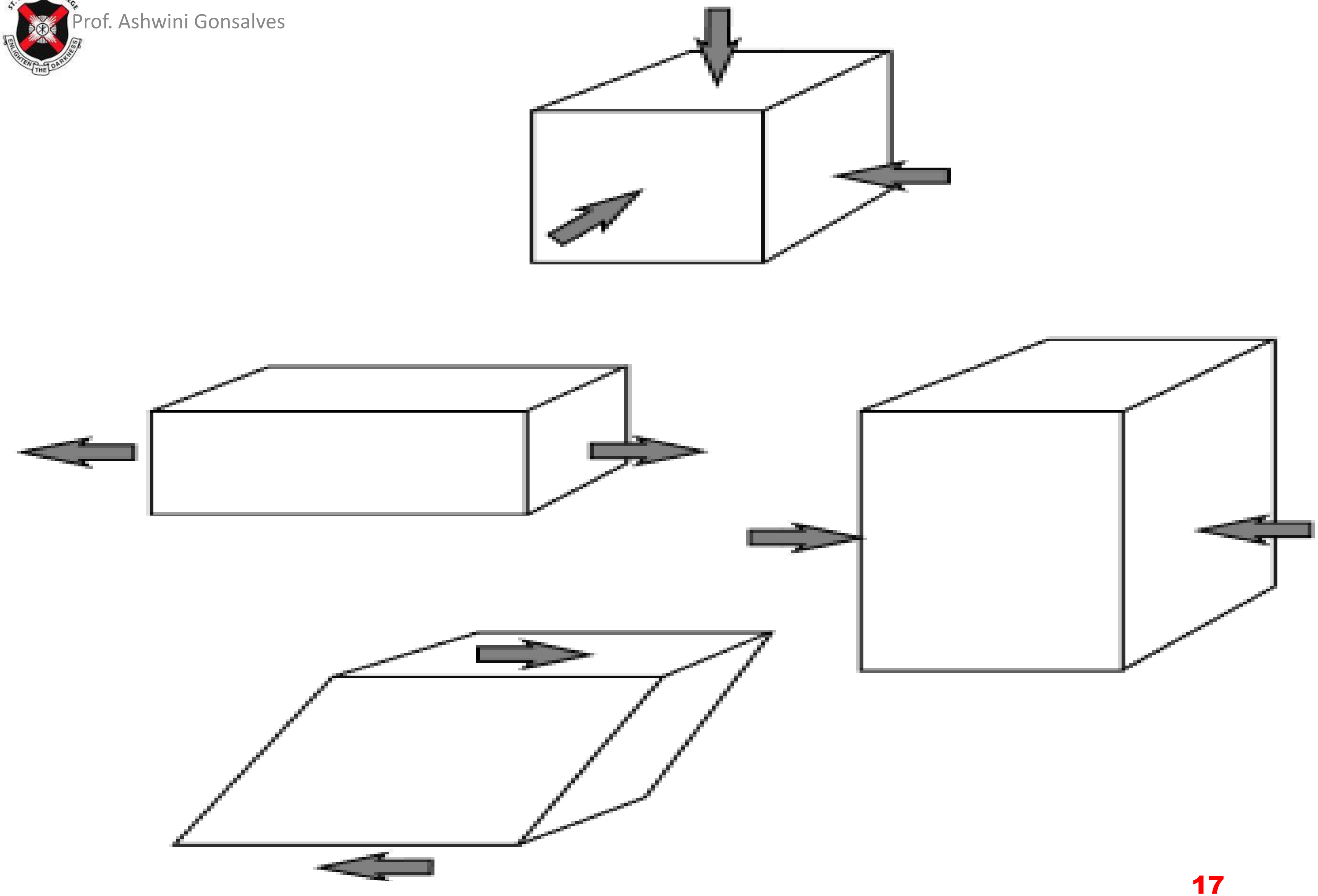
C) Shearing stress & Shearing strain

$$\text{Shearing strain } \frac{\Delta l}{l} = \tan\theta = \theta \quad \text{--- (6.9)}$$

when the relative displacement Δl is very small.



$$\text{Shearing stress} = \frac{\text{Tangential force}}{\text{Area}} \text{--- (6.8)}$$



6.4 Hook's Law

Within elastic limit, stress is directly proportional to strain.

$$\frac{\text{Stress}}{\text{Strain}} = \text{constant}$$

The constant is called the **modulus of elasticity**

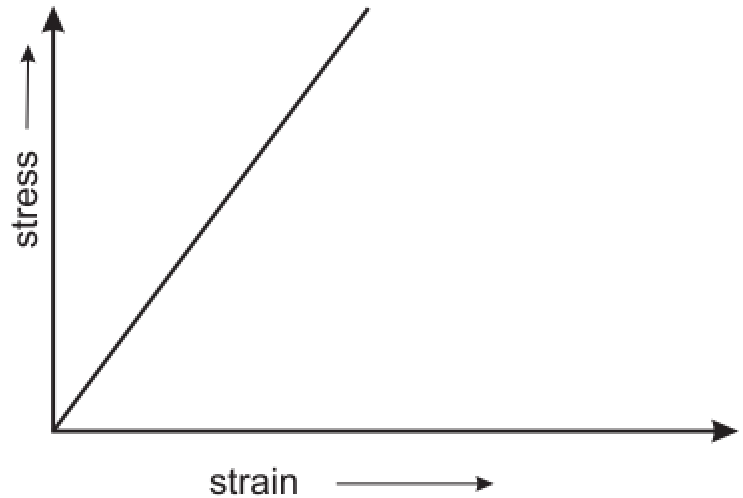


Fig 6.4: Stress versus strain graph within elastic limit for an elastic body.

The maximum value of stress up to which stress is directly proportional to strain is called the **elastic limit**.

6.5 Elastic modulus

- Modulus of elasticity of a material is the ratio of stress to the corresponding strain.
- Its SI unit is N/m^2
- Its dimensions are $[L^{-1} M^1 T^{-2}]$

Three types of modulus of elasticity

B) Bulk Modulus

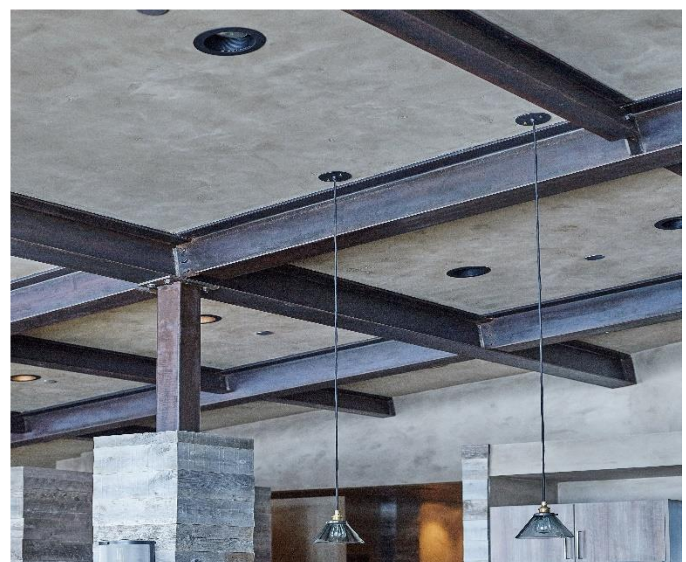
A) Young's modulus

C) Modulus of rigidity

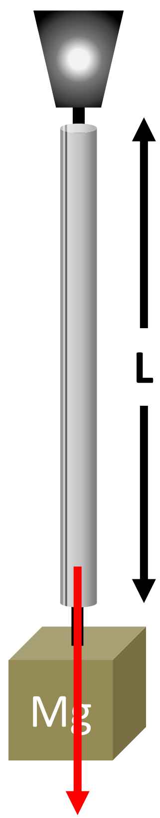
6.5.1 Young's Modulus (Y)

It is modulus of elasticity related to **change in length** of an object like a metal wire, rod, beam etc, due to the applied deforming force.

Beam



6.5.1 Young's Modulus (Y)



Deforming Force acting on wire

$$Longitudinal\ stress = \frac{Applied\ force}{Area}$$

$$= \frac{F}{A}$$

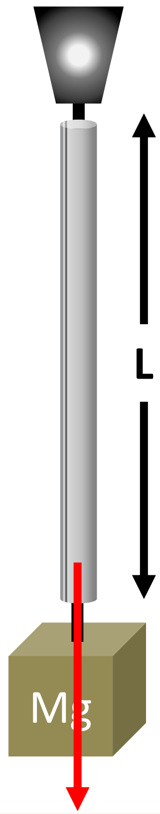
$$= \frac{Mg}{\pi r^2} \quad \text{--- (6.10)}$$

It produces a change in length of the wire. If (L+l) is the new length of wire, then l is the extension or elongation in wire.

$$Longitudinal\ strain = \frac{change\ in\ length}{original\ length}$$

$$= \frac{l}{L} \quad \text{-- (6.11)}$$

6.5.1 Young's Modulus (Y)



Young's modulus is the ratio of longitudinal stress to longitudinal strain.

$$Young's\,modulus = \frac{longitudinal\ stress}{longitudinal\ strain} \quad \text{-- (6.12)}$$

$$Y = \frac{\dfrac{Mg}{\pi r^2}}{\dfrac{l}{L}}$$

$$Y = \frac{MgL}{\pi r^2 l} \quad \text{---(6.13)}$$

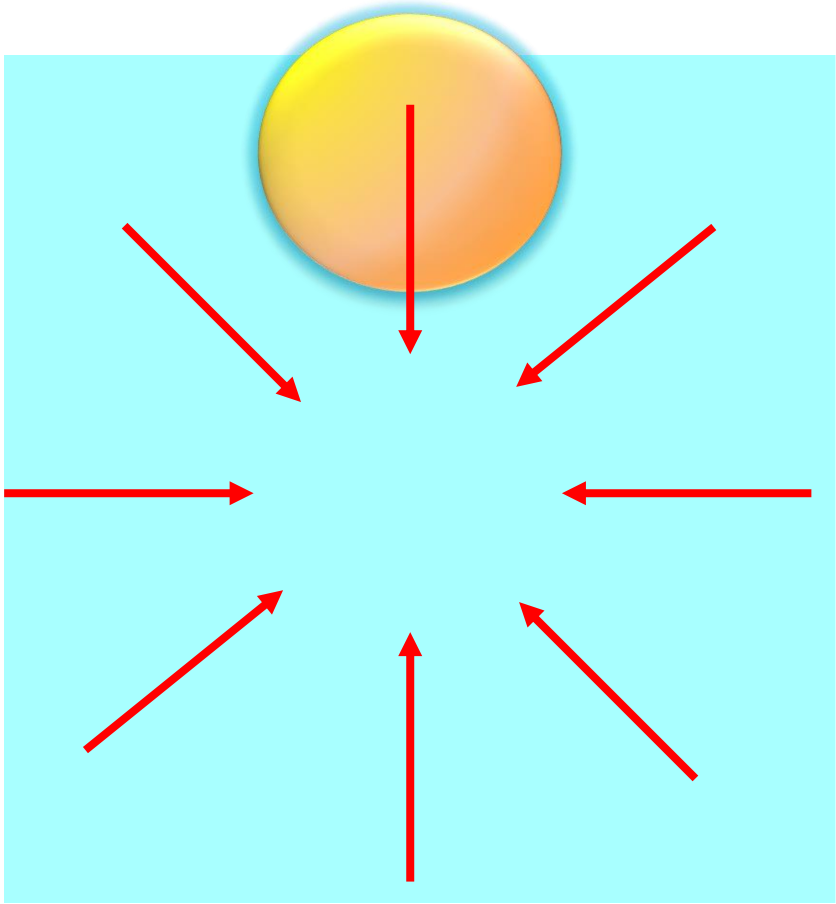
SI unit of Young's modulus is N/m^2. Its dimensions are $[\,L^{-1}\,M^{1}\,T^{-2}\,]$.

Defo
actin

Young's modulus indicates the resistance of an elastic solid to elongation or compression

6.5.2 Bulk Modulus (K)

It is the modulus of elasticity related to **change in volume** of an object due to applied deforming force



Stress $= \frac{F}{A}$ → **Compressive Stress** $= \frac{F}{A}$

Change is Pressure = dP = $\frac{F}{A}$

Let the change in pressure **dP** and let the change in volume be **dV.**

If the original volume of the sphere is V, then volume strain is defined as

$$\text{Volume strain} = \frac{\text{change in volume}}{\text{original volume}}$$

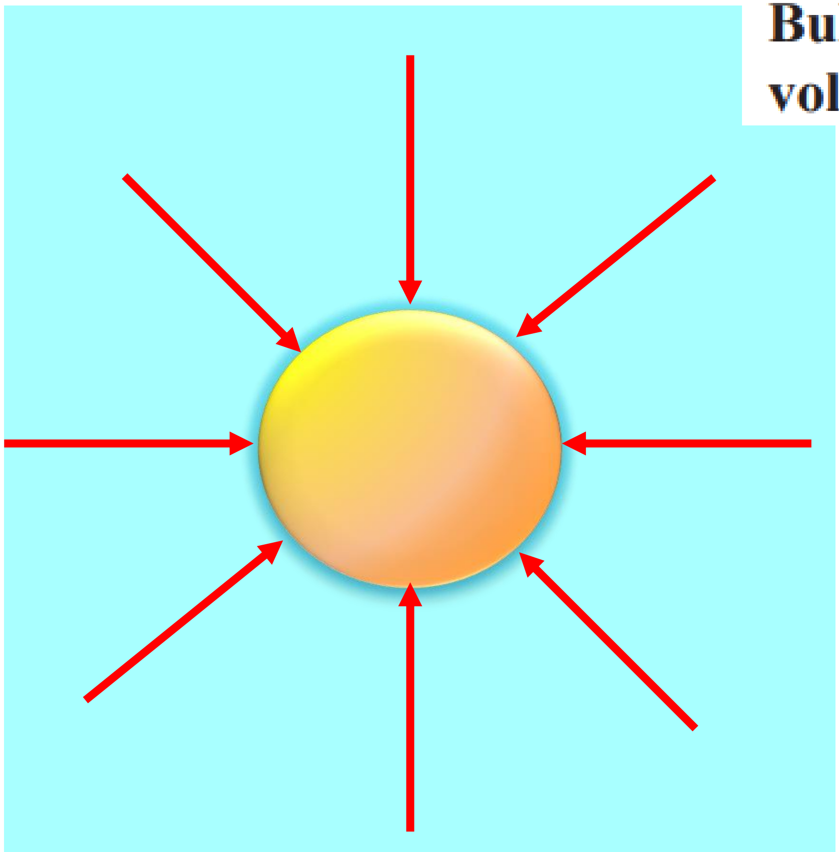
$$= -\frac{dV}{V} \qquad \text{--- (6.14)}$$

The magnitude of the volume strain $= \frac{dV}{V}$

6.5.2 Bulk Modulus (K)

It is the modulus of elasticity related to **change in volume** of an object due to applied deforming force

Change is Pressure= Volume Stress = dP & volume strain $=\dfrac{dV}{V}$



Bulk modulus is defined as the ratio of volume stress to volume strain.

$$\text{Bulk modulus} = \frac{\text{volume stress}}{\text{volume strain}} = K$$

$$K = \frac{dP}{\left(\frac{dV}{V}\right)} = V\frac{dP}{dV} \quad \text{--- (6.17)}$$

SI unit of bulk modulus is N/m^2. Dimensions of K are $[L^{-1} M^1 T^{-2}]$.

Bulk modulus measures the resistance offered by gases, liquids or solids while an attempt is made to change their volume.

6.5.3 Modulus of rigidity (η)

The modulus of elasticity related to **change in shape** of an object is called rigidity modulus

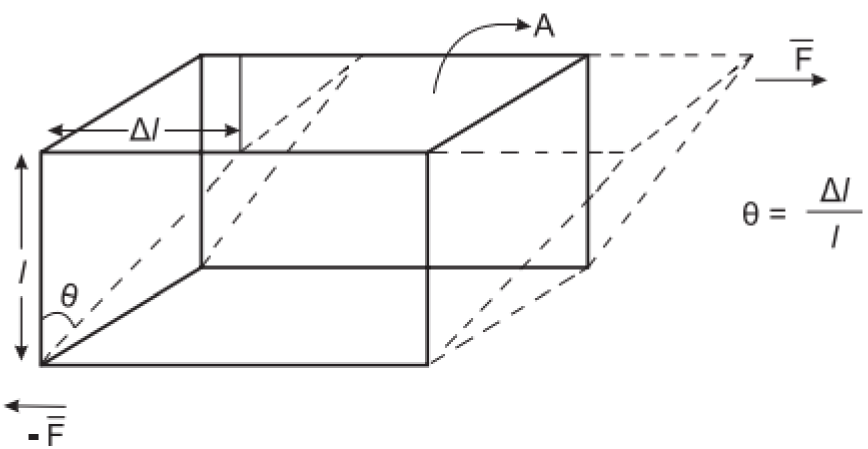


Fig. 6.5: Modulus of rigidity, tangential force F and shear strin θ.

Shear stress $= \dfrac{F}{A}$ **Shear strain** $= \dfrac{\Delta l}{l}$

Shear modulus or modulus of rigidity: It is defined as the ratio of shear stress to shear strain within elastic limits.

$$\eta = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta} \quad \text{--- (6.17)}$$

Rigidity modulus indicates the resistance offered by a solid to change in its shape.

6.5.4 Poisson's ratio

Suppose a wire is fixed at one end and a force is applied at its free end so that the wire gets stretched.

Length of the wire increases and at the same time, its diameter decreases

i.e., the wire becomes longer and thinner as shown in Fig. 6.6 (a).

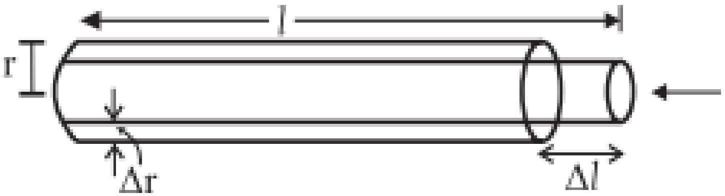


Fig. 6.6 (a): When a wire is stretched its length increases and its diameter decreases.

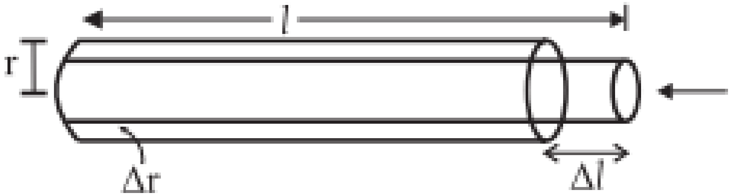


Fig. 6.6 (b): When a wire is compressed its length increases and its diameter increases.

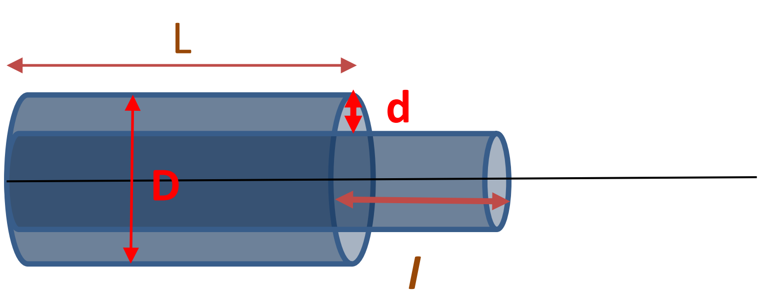
6.5.4 Poisson's ratio

The ratio of change in dimensions to original dimensions in the direction of the applied force is called **linear strain**

The ratio of change in dimensions to original dimensions in a direction perpendicular to the applied force is called **lateral strain.**

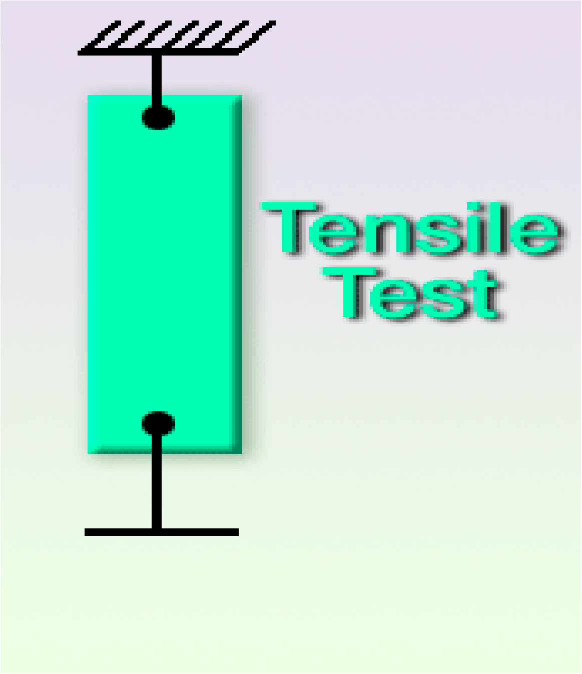
Within elastic limit, the ratio of lateral strain to the linear strain is called the **Poisson's ratio.**

If *L* is the original length of wire, *l* is increase/decrease in length of wire, D is the original diameter and d is corresponding change in diameter of wire then, Poisson's ratio is given by



$$\sigma = \frac{Lateral\ strain}{Linear\ strain} = \frac{d/D}{l/L} = \frac{d.L}{D.l} \qquad \text{--- (6.18)}$$

6.6 Stress-Strain Curve



- Suppose a metal wire/ strip is suspended vertically from a rigid support and stretched by applying load to its lower end.
- The load is gradually increased in small steps until the wire breaks.
- The elongation produced in the wire is measured during each step.
- Stress and strain is noted for each load and a graph is drawn by taking tensile strain along x-axis and tensile stress along y-axis.

6.6 Stress-Strain Curve

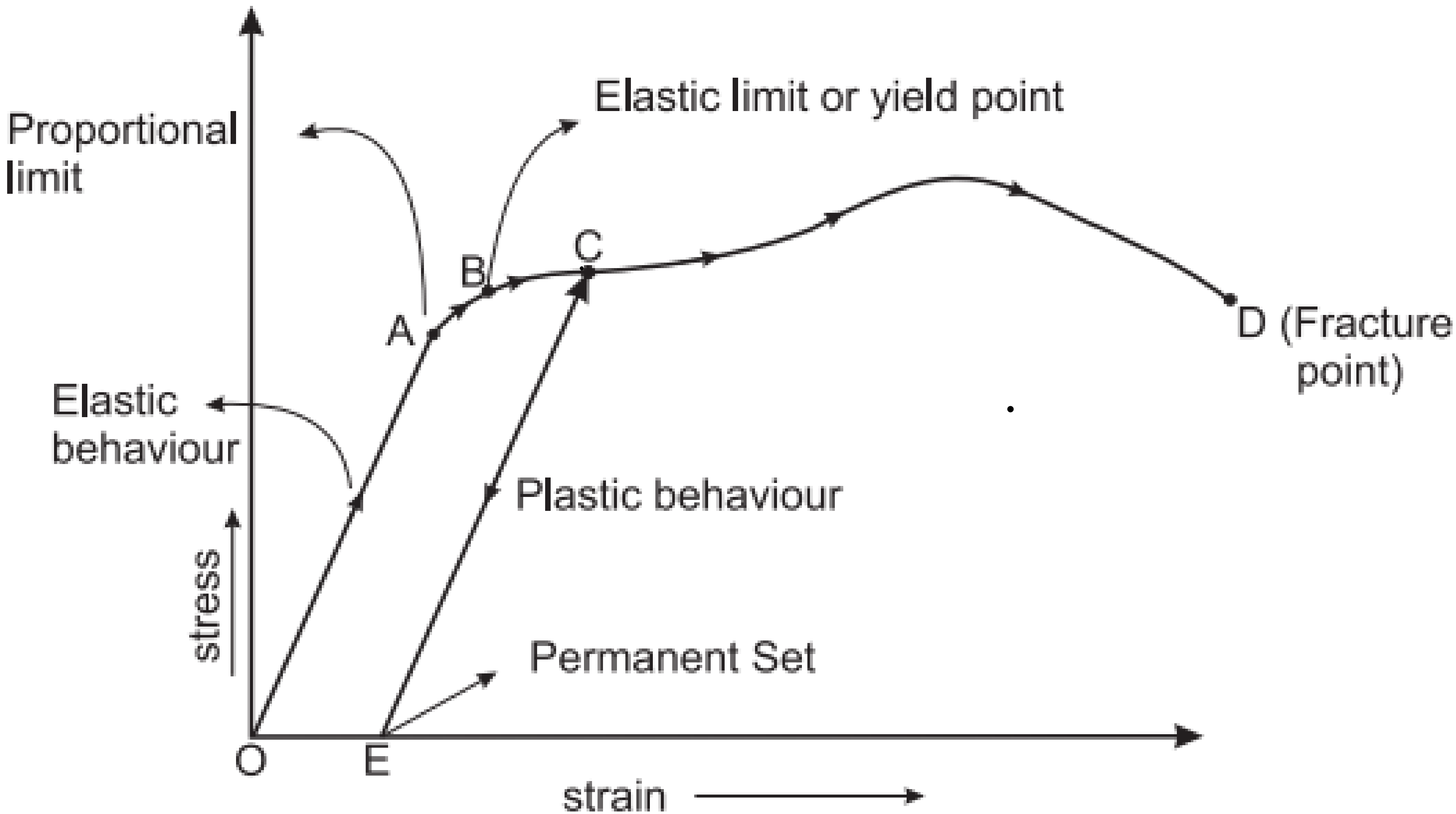
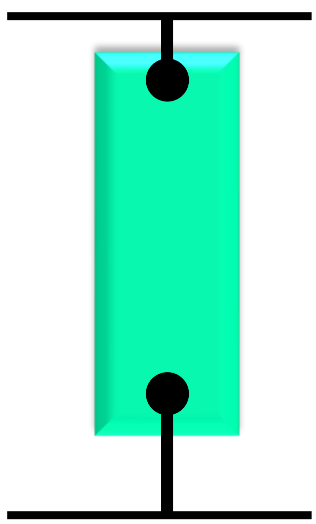
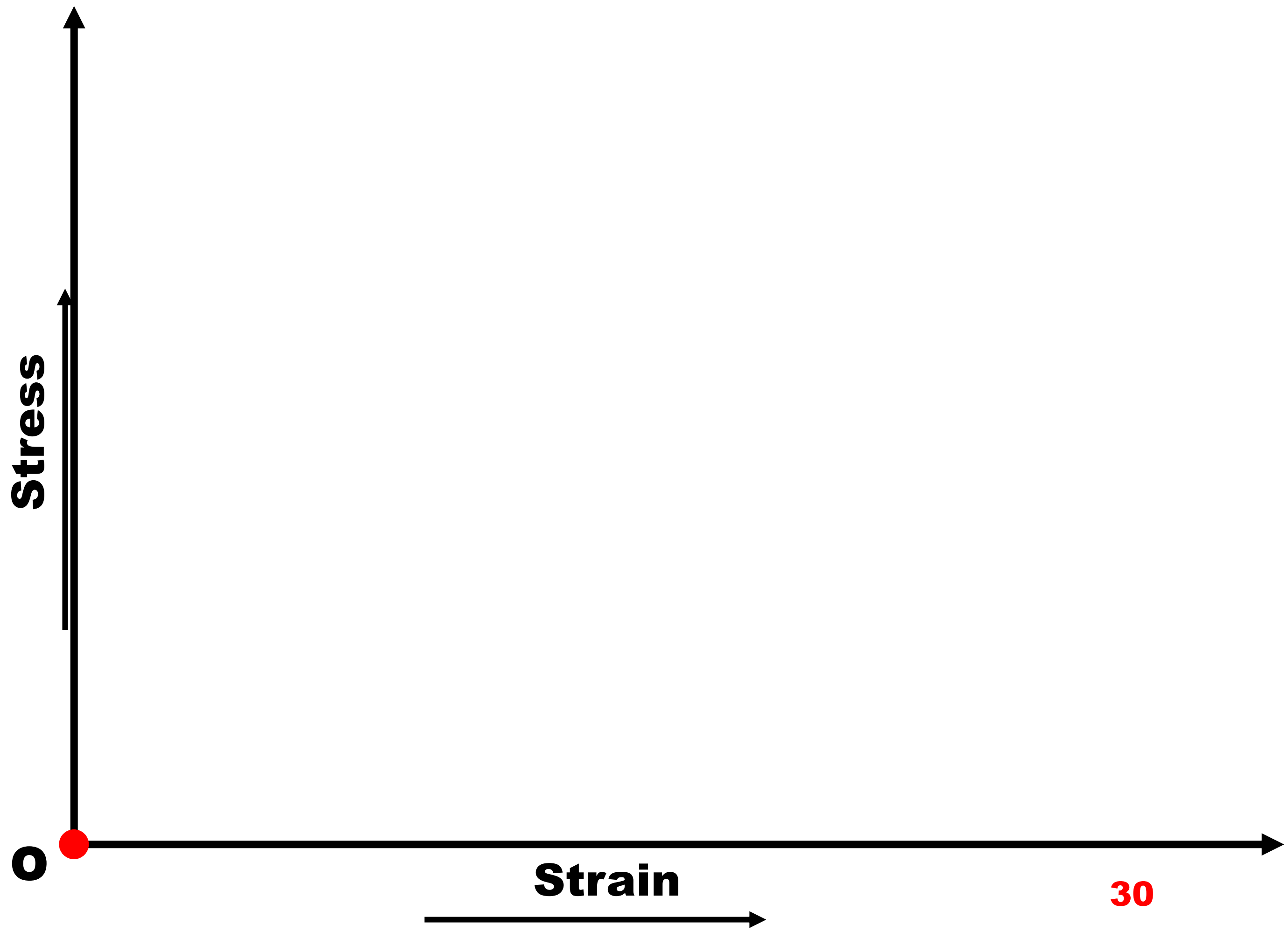


Fig. 6.7 : stress-strain curve.

6.6 Stress-Strain Curve





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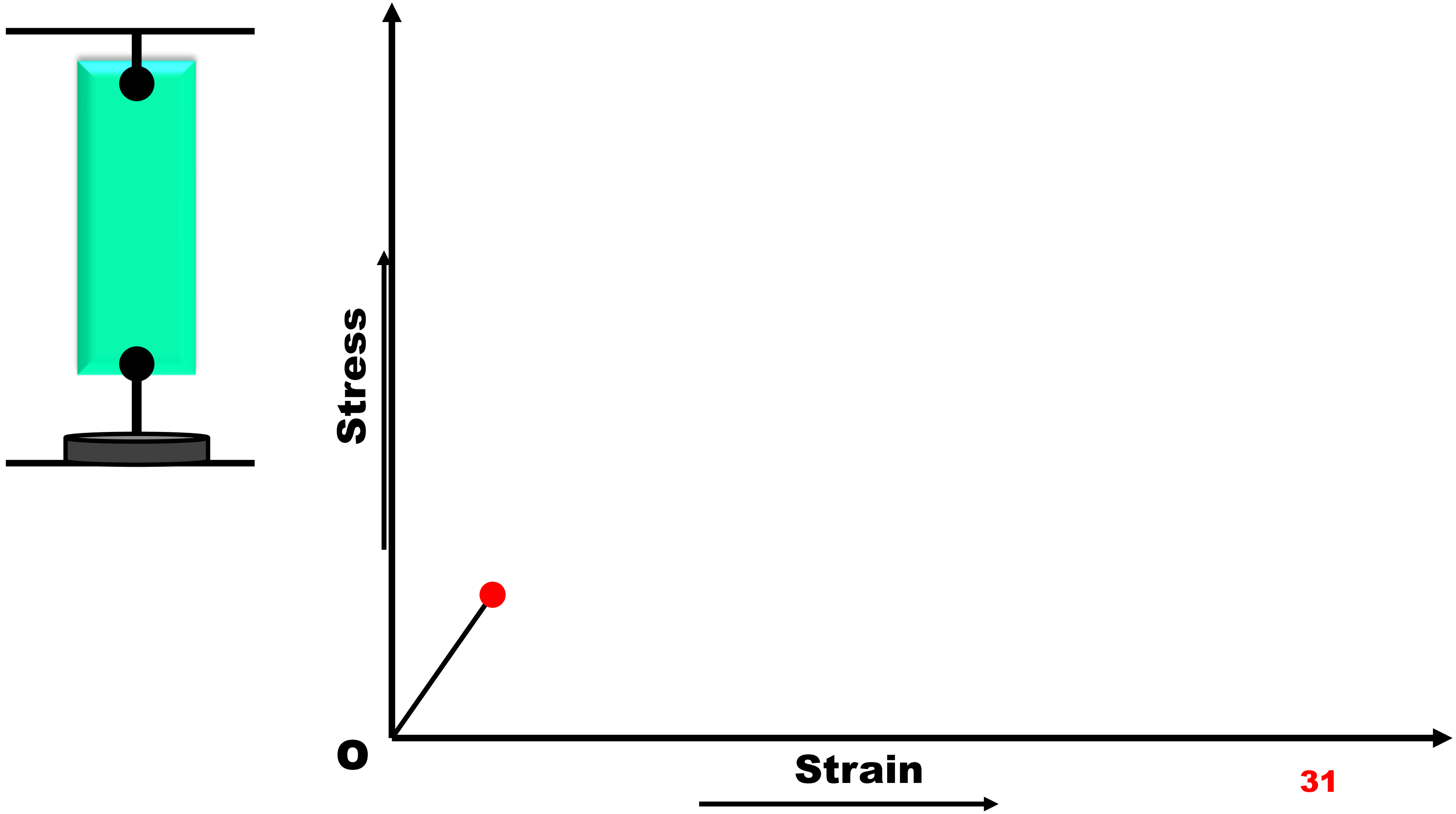

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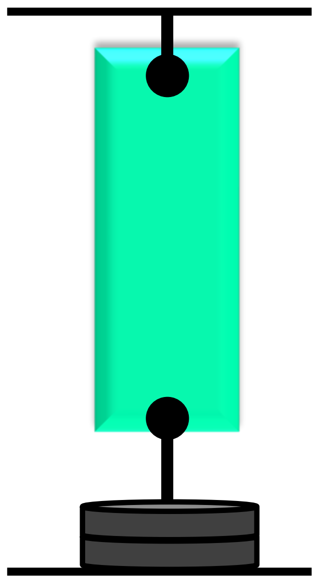

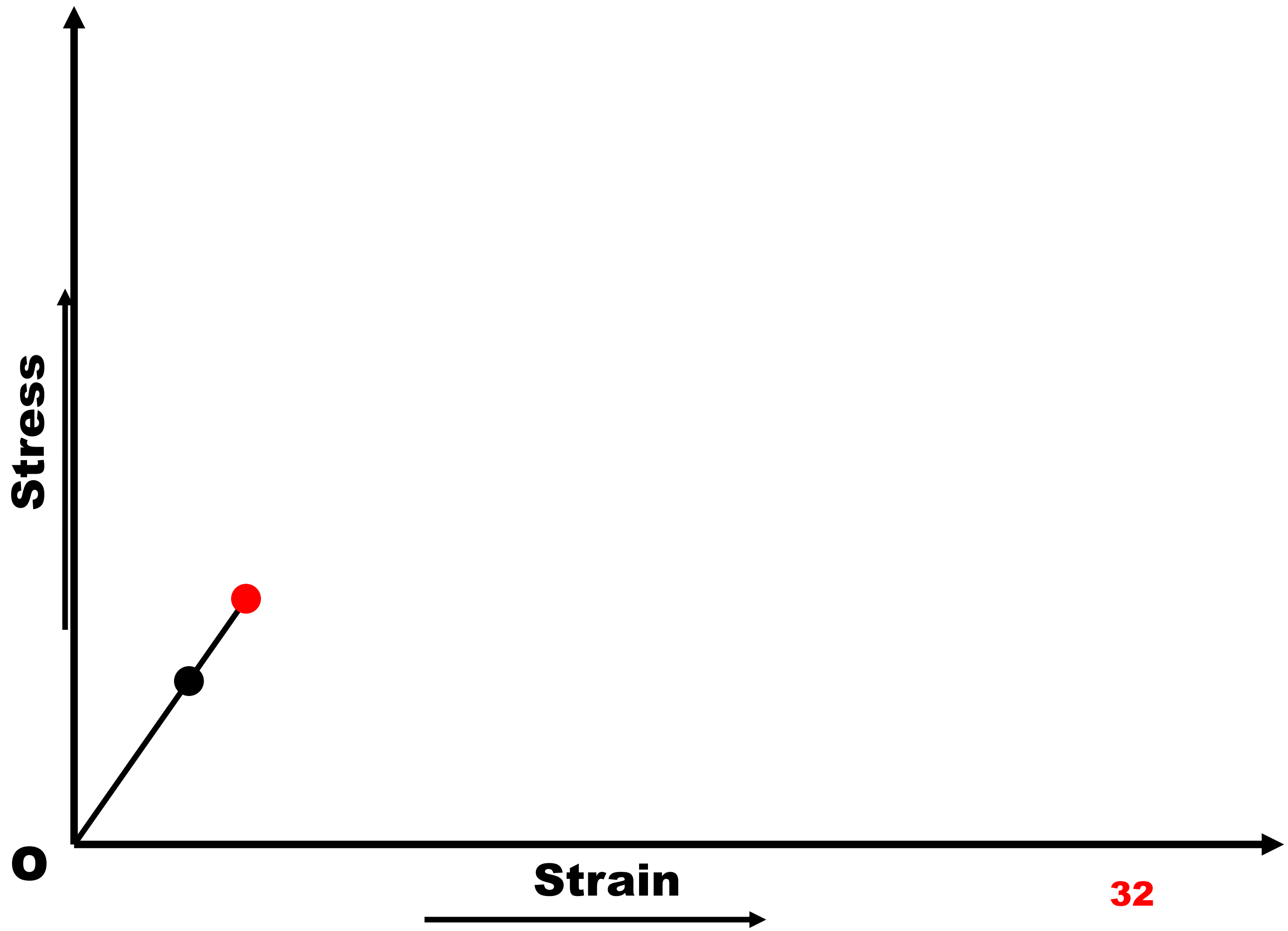

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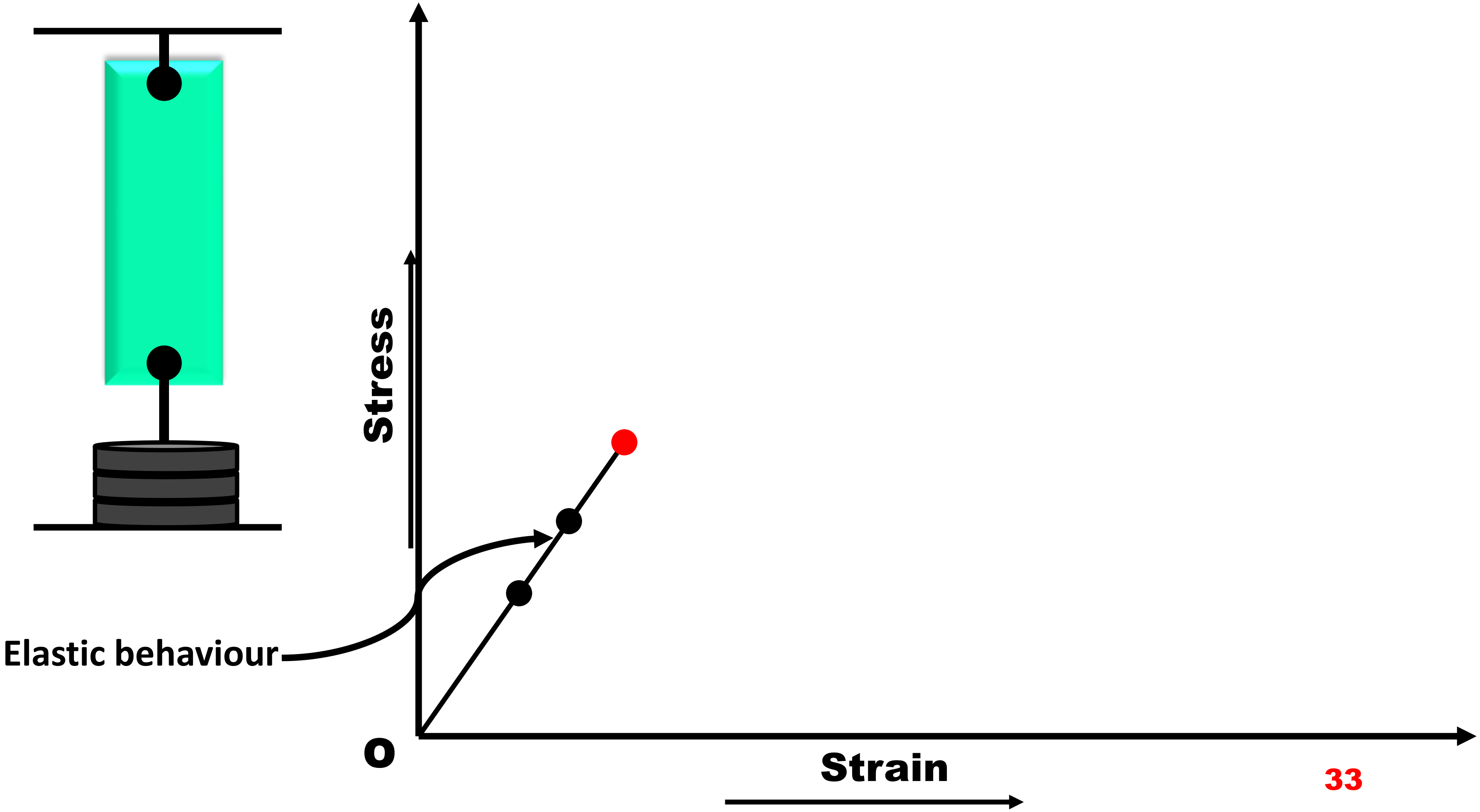
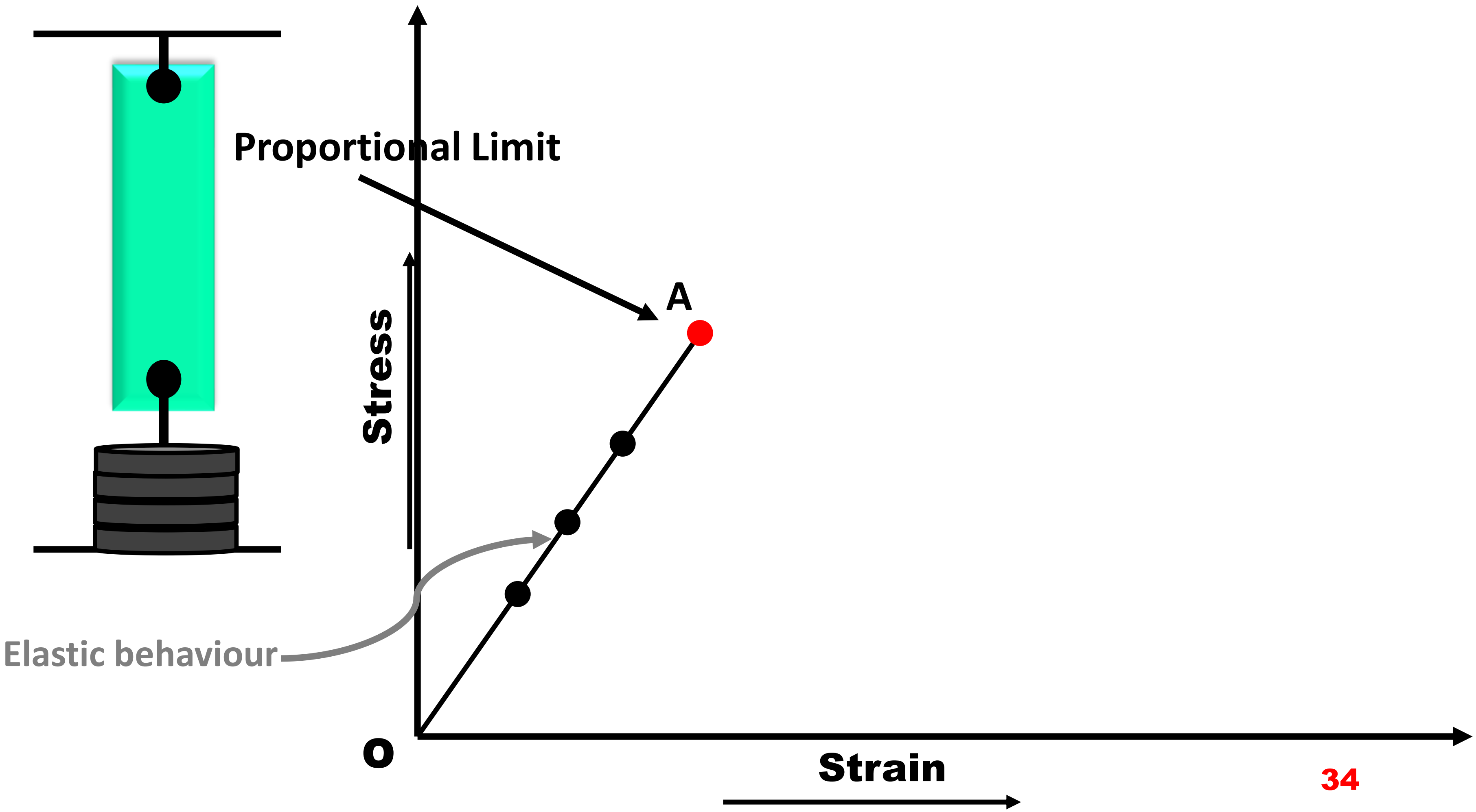
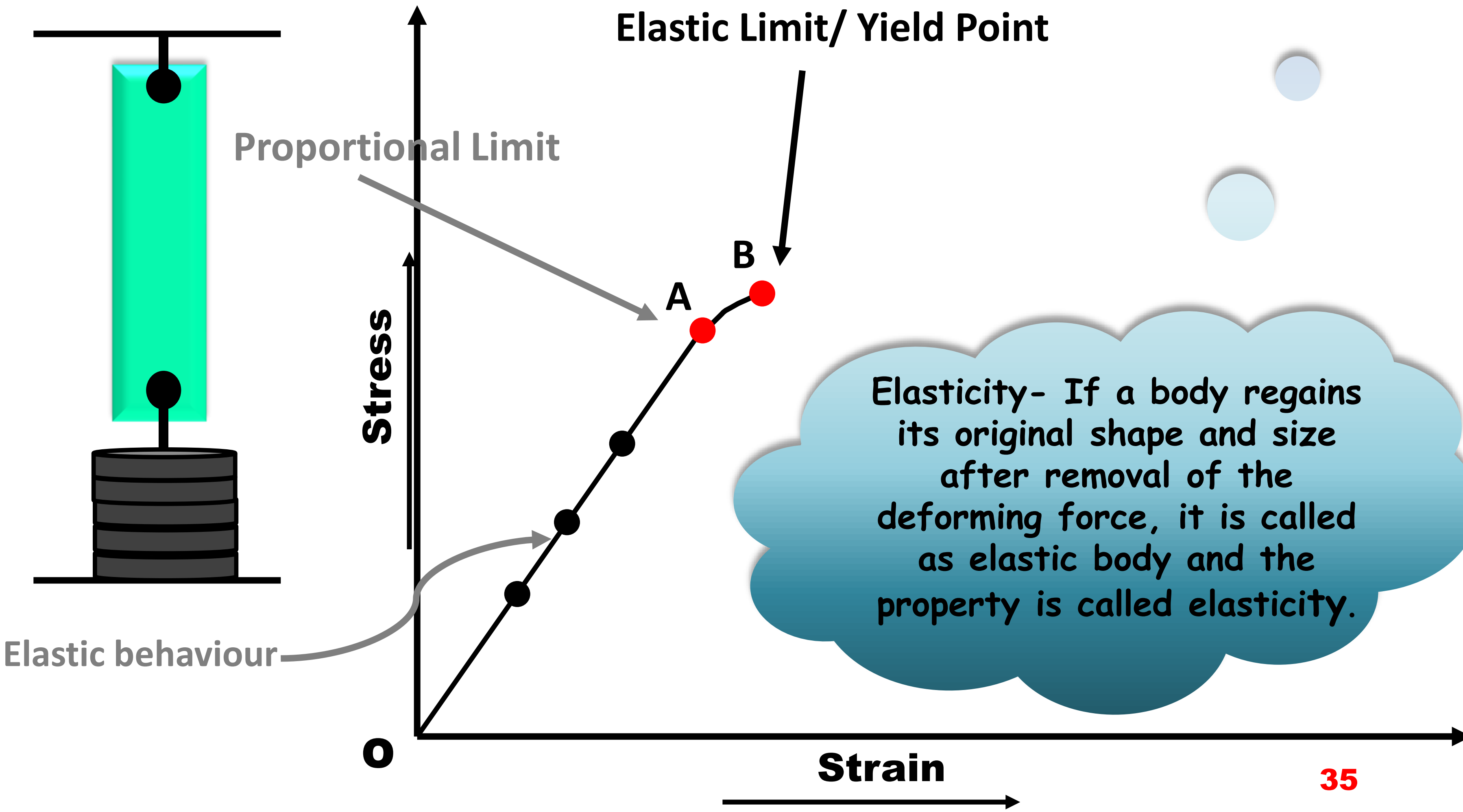


6.6 Stress-Strain Curve

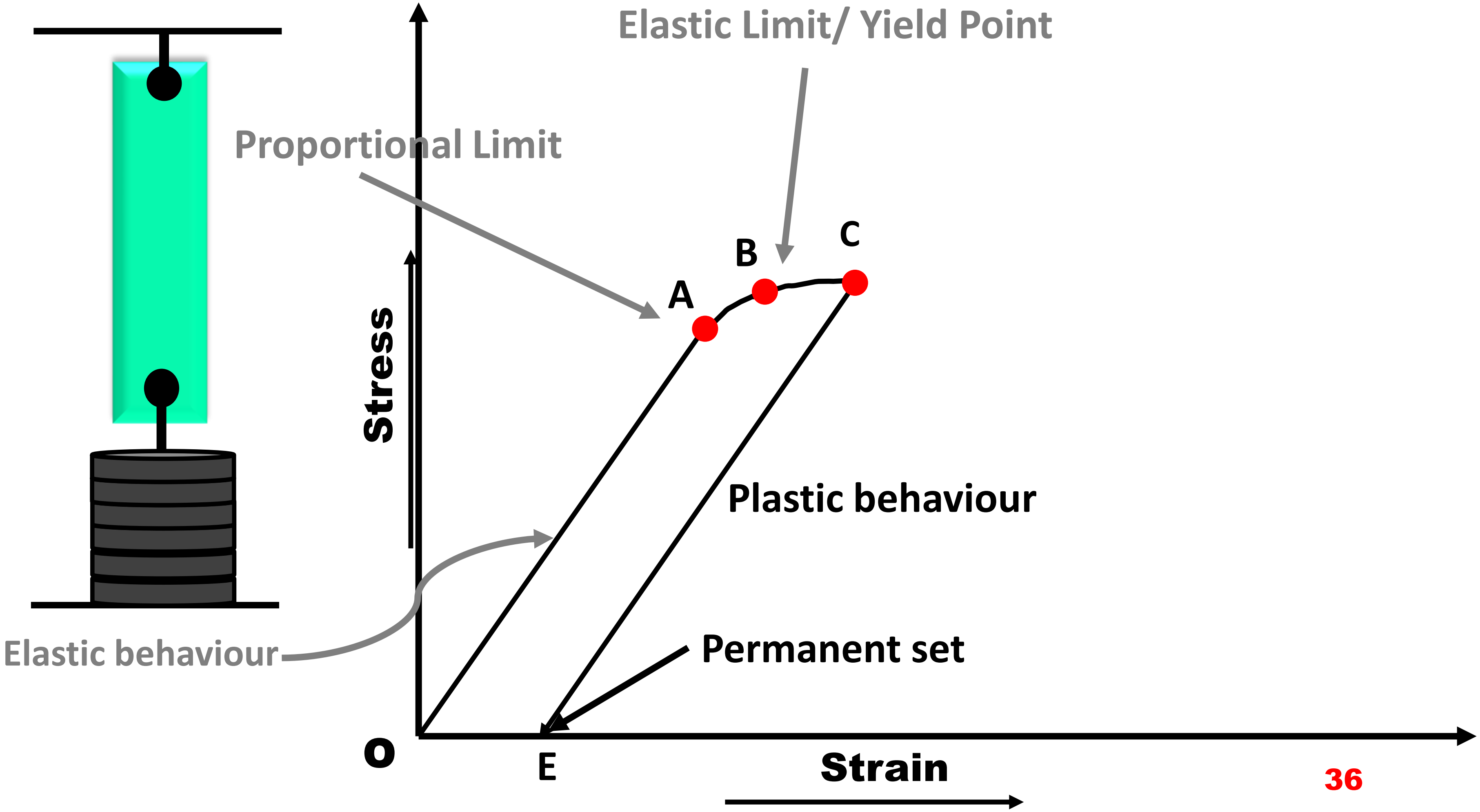


6.6 Stress-Strain Curve

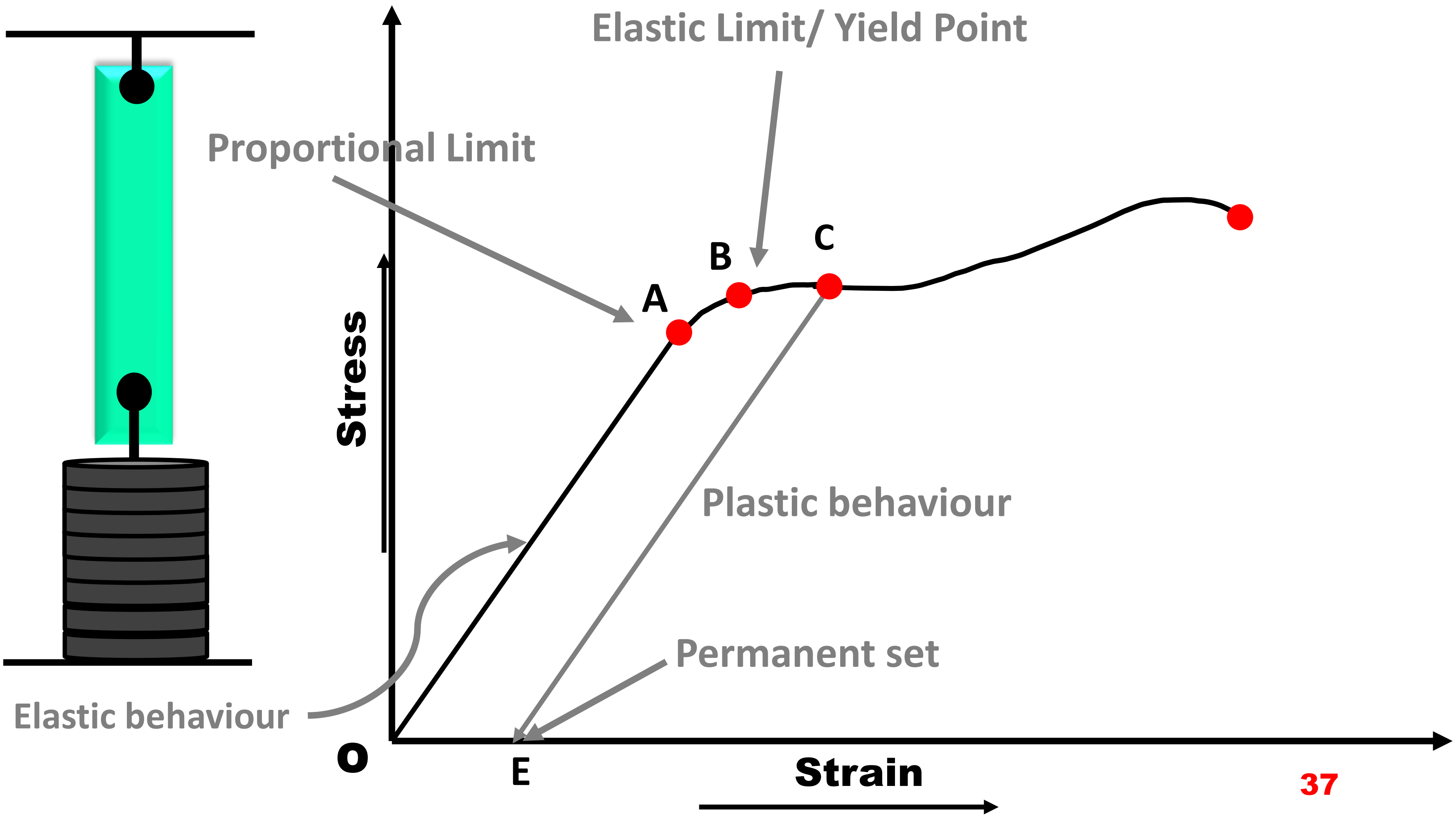


Elasticity- If a body regains its original shape and size after removal of the deforming force, it is called as elastic body and the property is called elasticity.

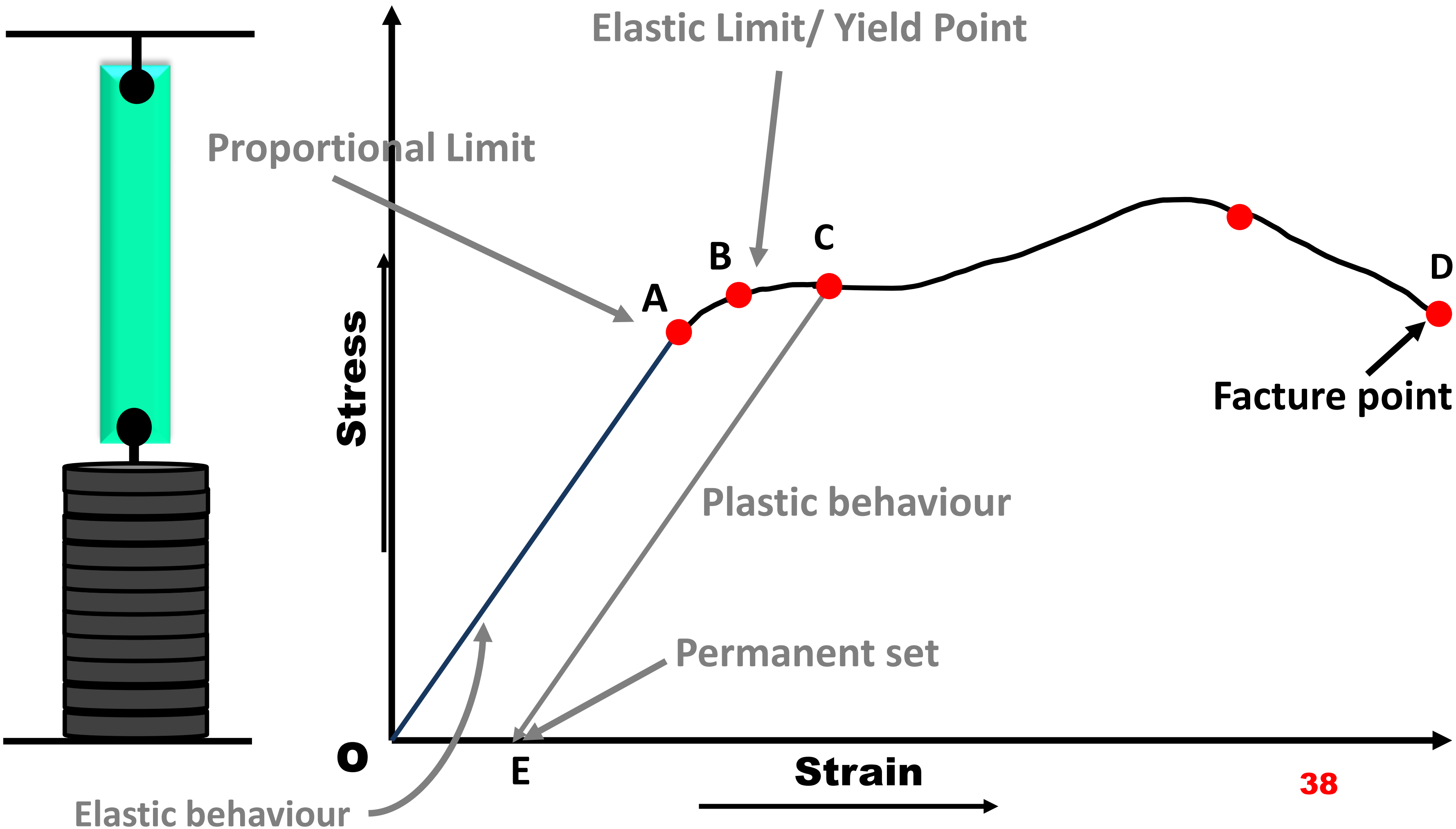
6.6 Stress-Strain Curve



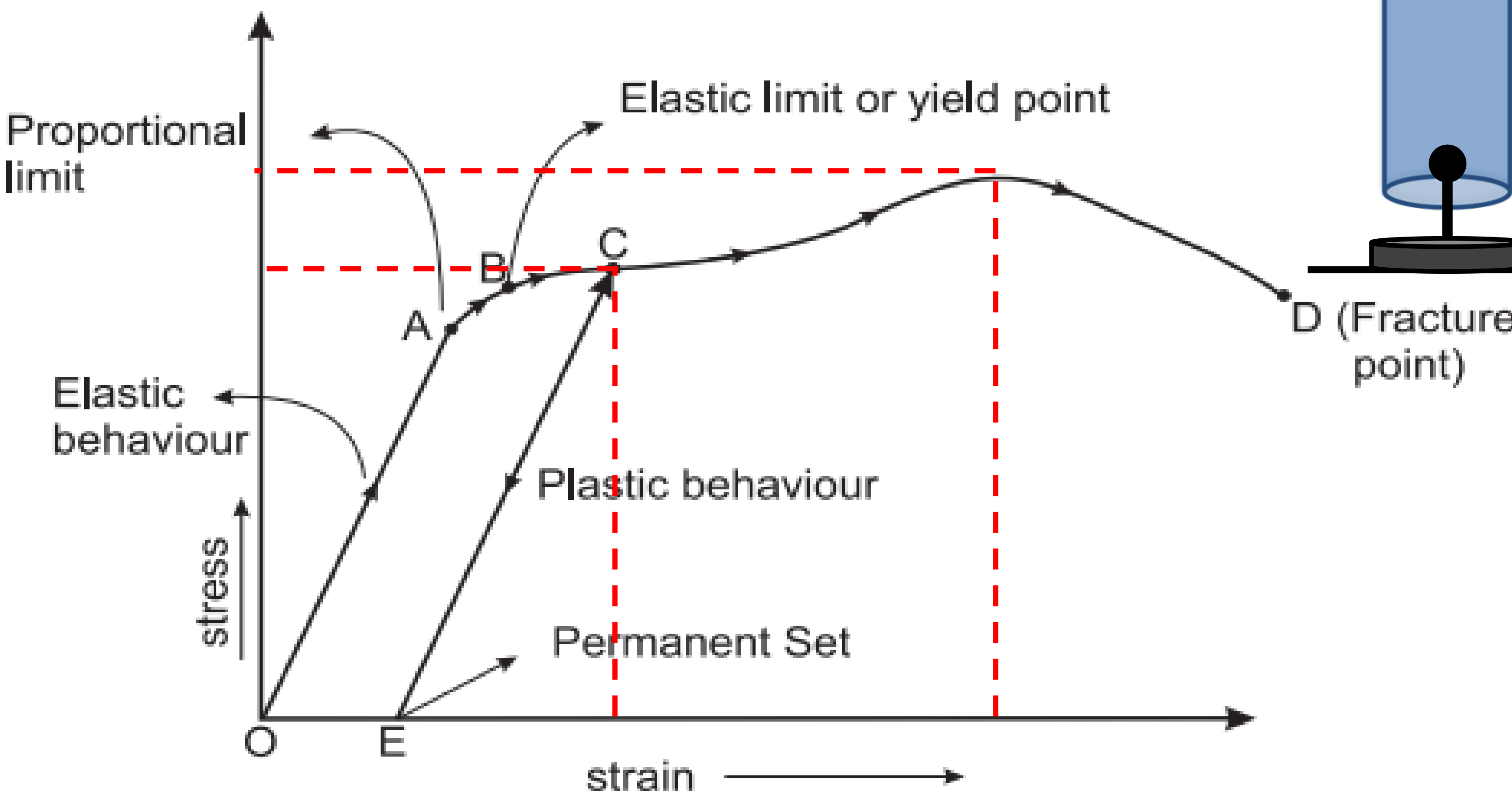
6.6 Stress-Strain Curve



6.6 Stress-Strain Curve



6.6 Stress-Strain Curve



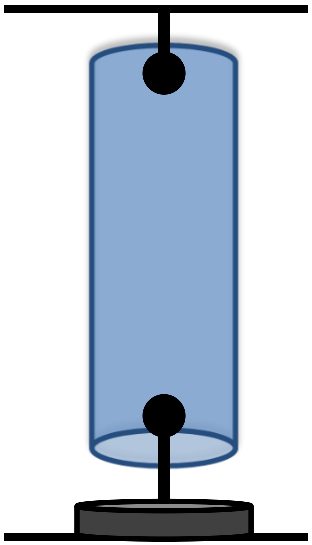


Fig. 6.7 : stress-strain curve.

Stress Strain Graph & Classification of Material

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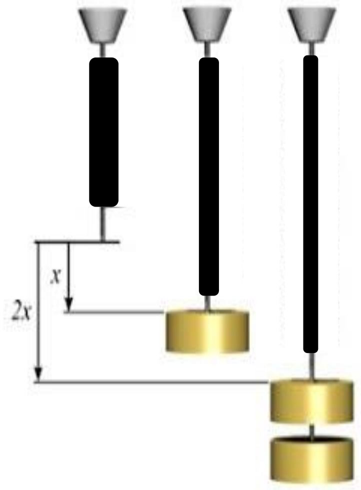
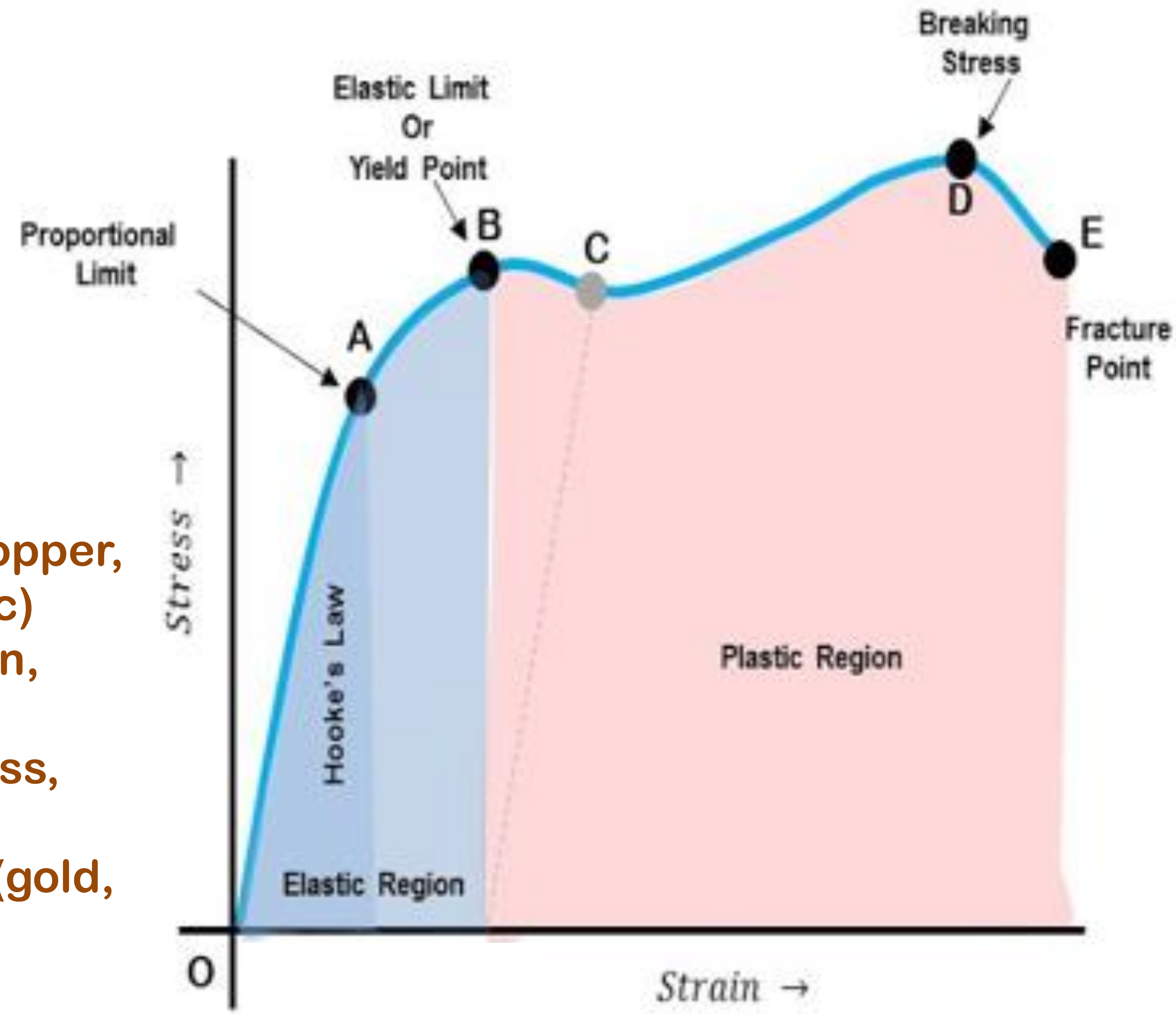
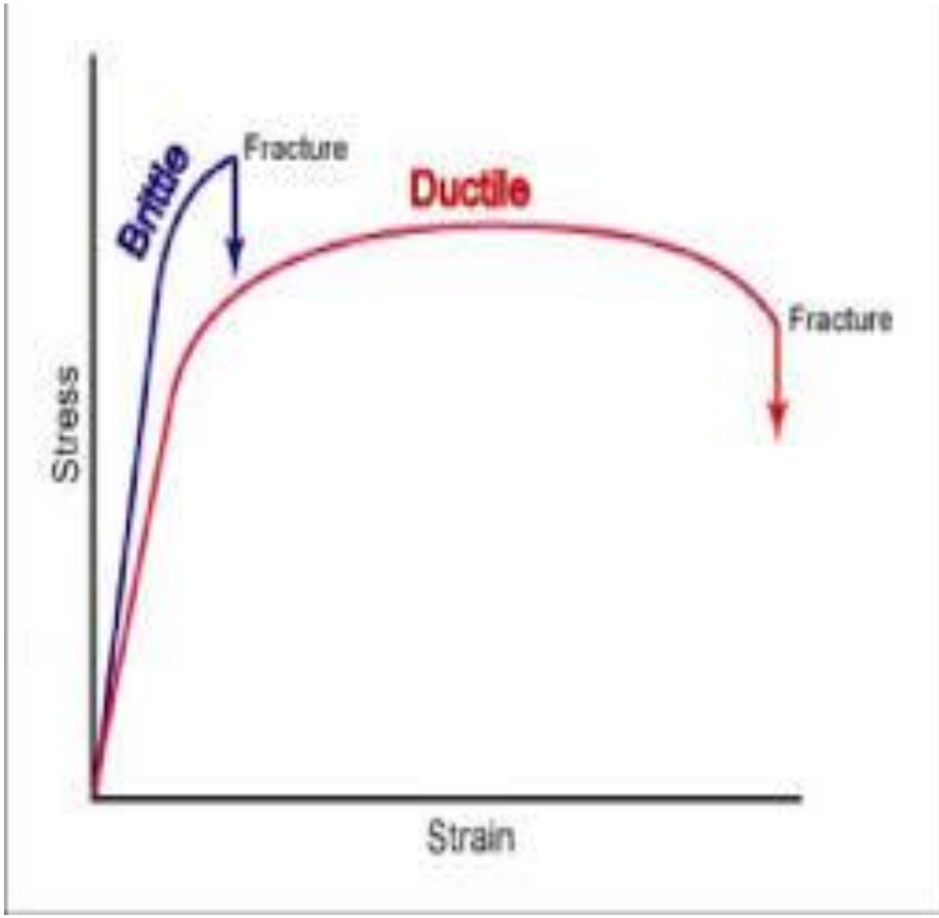


1. Elastic Materials (copper, aluminium , silver etc)
2. Ductile Materials(iron, copper etc)
3. Brittle Materials (glass, ceramics etc)
4. Malleable Materials (gold, silver)
5. Elastomers

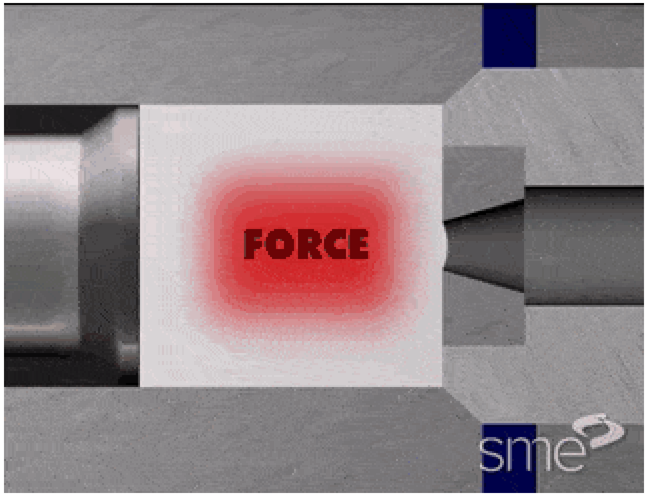


Stress and strain curve (ductile and brittle)

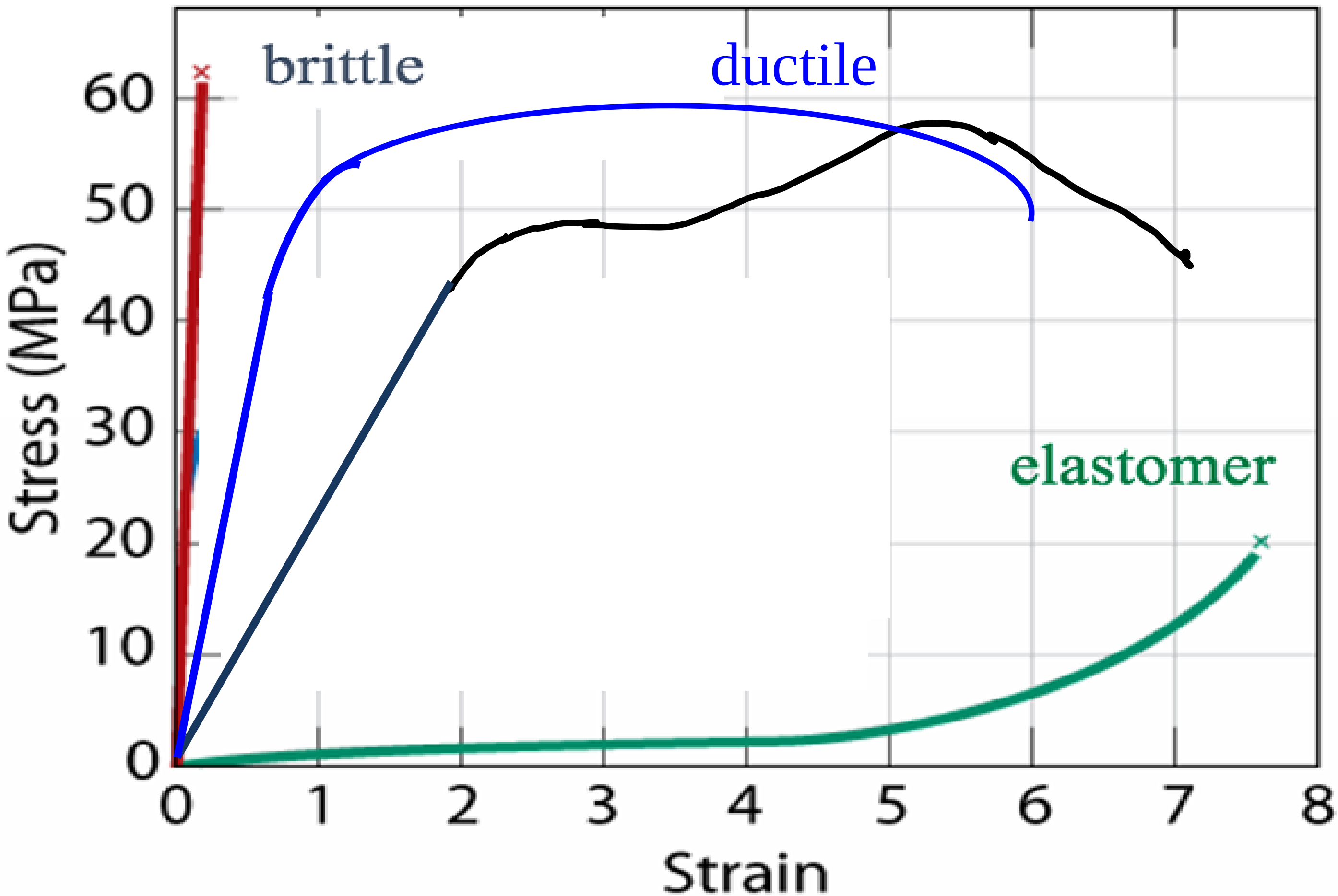


Materials break within elastic limit are called **Brittle**.

Material which can lengthen considerably and undergo plastic deformation till they break are called **ductile**.



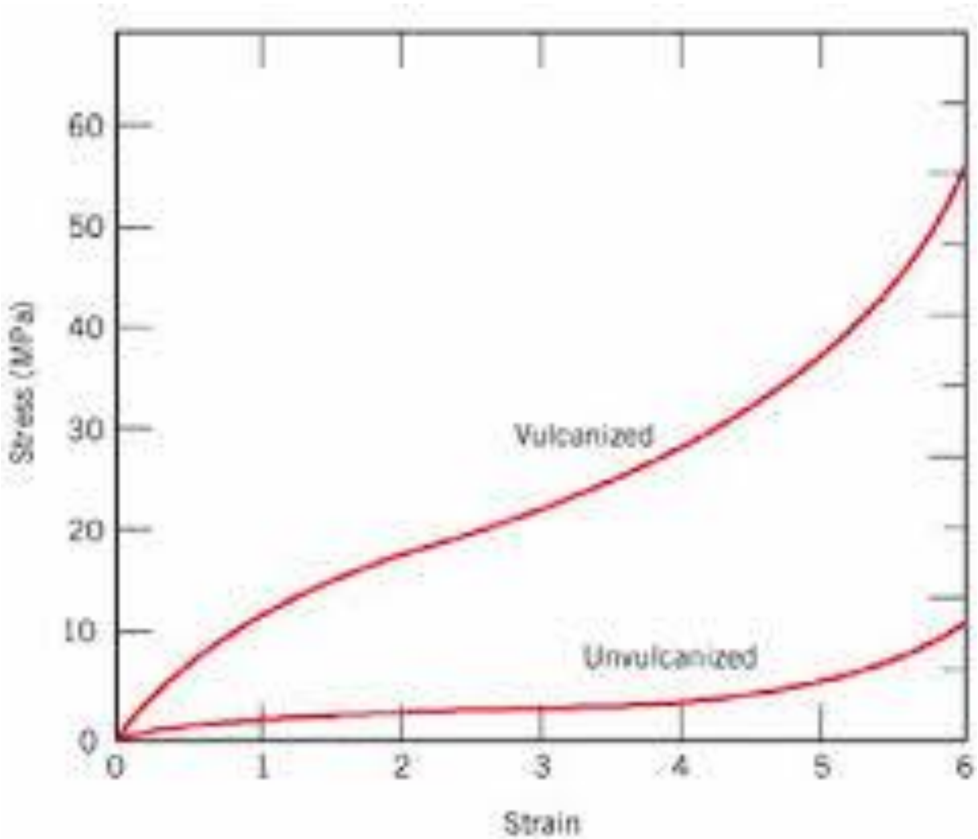
A material that can be stretched to a larger value of strain is called an ELASTOMER

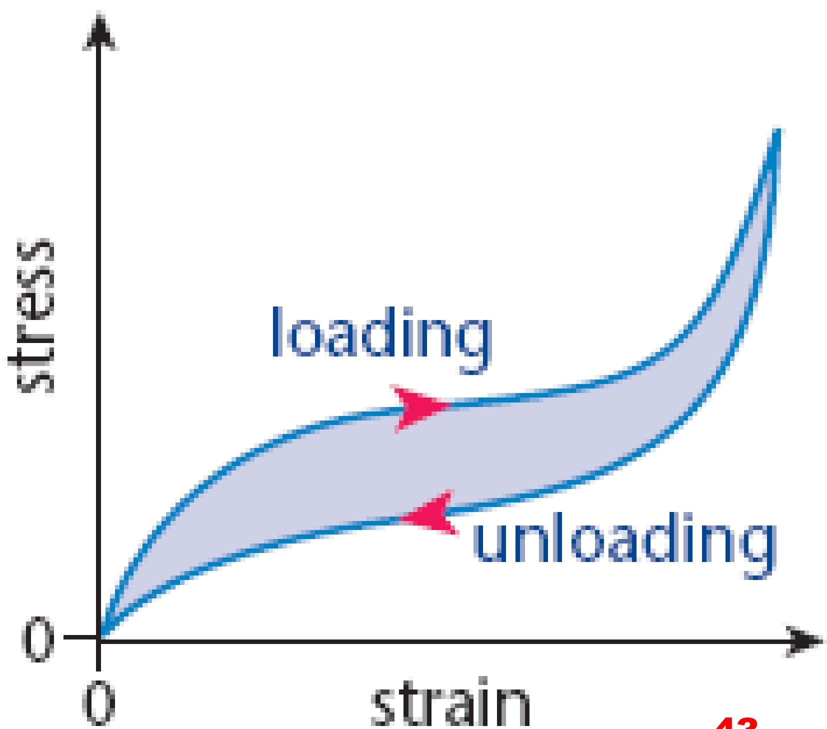


Prof. Ashwini Gonsalves

ELASTOMER

Elastic hysteresis : *Lagging of strain behind the stress*

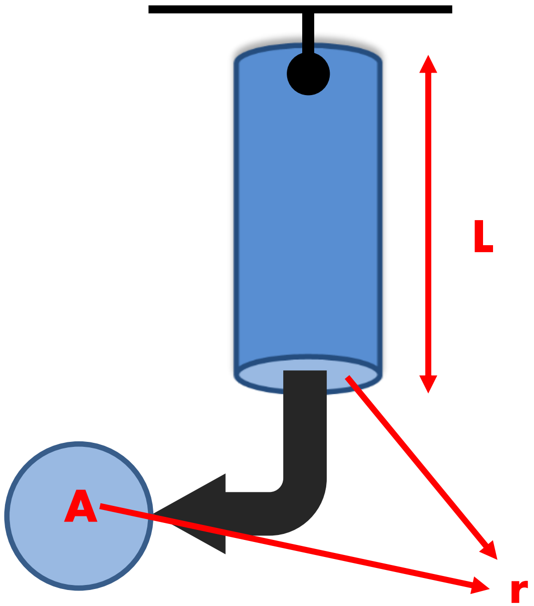




6.7 Strain Energy

The elastic potential energy gained by a wire during elongation by a stretching force is called as strain energy.

Consider a wire of original length L and cross sectional area A stretched by a force F acting along its length. The wire gets stretched and elongation l is produced in it. The stress and the strain increase proportionately.



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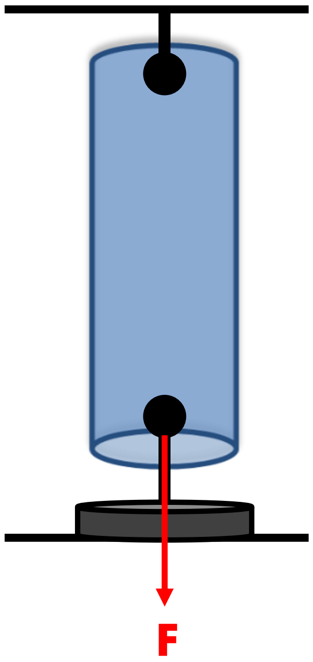

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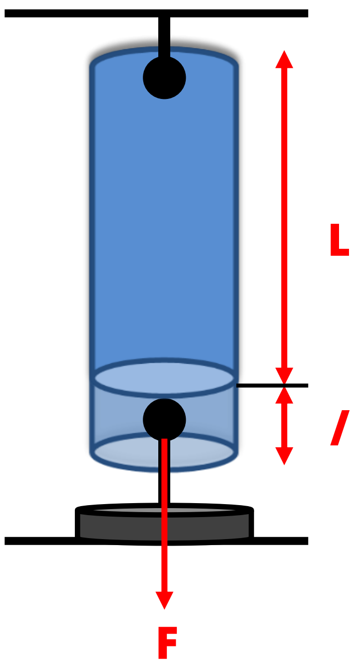

$$\text{Longitudinal stress} = \frac{F}{A}$$

$$\text{Longitudinal strain} = \frac{l}{L}$$

$$\text{Young's modulus} = \frac{\textit{longitudinal stress}}{\textit{longitudinal strain}}$$

$$Y = \frac{\left(\frac{F}{A}\right)}{\left(\frac{l}{L}\right)} = \frac{FL}{Al}$$

$$\therefore F = \frac{YAl}{L} \quad \text{--- (6.19)}$$

6.7 Strain Energy

let ‘f’ be the force applied and ‘x’ be the corresponding extension. The force at this stage is given by Eq. (6.19) as

$$f = \frac{YAx}{L}$$

For further extension dx in the wire, the work done is given by

$$\text{Work} = (\text{force})\cdot(\text{displacement}).$$

$$dW = f\,dx$$

$$\therefore\ dW = \frac{YAx}{L}dx$$

When the wire gets stretched from x = 0 to x = *l*, the total work done is given as

$$W = \int_0^l dW$$

$$\therefore W = \int_0^l \frac{YAx}{L}dx$$

$$\therefore W = \frac{YA}{L}\int_0^l x\,dx$$

$$\therefore W = \frac{YA}{L}\left[\frac{x^2}{2}\right]_0^l$$

$$\therefore W = \frac{YA}{L}\left[\frac{l^2}{2} - \frac{0^2}{2}\right]$$

$$W = \frac{YAl^2}{2L}$$

$$W = \frac{1}{2}\frac{YAl}{L}l \qquad W = \frac{1}{2}Fl$$

6.7 Strain Energy

$$W = \frac{1}{2}\frac{YAl}{L}l$$

$$W = \frac{1}{2}Fl$$

$$\text{Work done} = \frac{1}{2}\ (\text{load})\cdot(\text{extension}) \quad \text{--- (6.20)}$$

This work done by stretching force is equal to energy gained by the wire. This energy is strain energy.

$$\text{Strain energy} = \frac{1}{2}\ (\text{load})\cdot(\text{extension}) \text{ --- (6.21)}$$

Strain energy per unit volume can be obtained by using Eq. (6.20) and various formula of stress, strain and young's modulus.

6.7 Strain Energy

Work done per unit volume

$$= \frac{\textit{work done in streching wire}}{\textit{volume of wire.}}$$

$$= \frac{1}{2}\frac{F.l}{A.L}$$

$$= \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{l}{L}\right)$$

Work done per unit volume

$$= \frac{1}{2}\ (\text{stress})\cdot(\text{strain})$$

As $Y = \frac{\text{stress}}{\text{strain}}$,

$\text{Stress} = Y\cdot\ (\text{strain})$ and

$$\text{strain} = \frac{\text{stress}}{Y}$$

$\therefore$ Strain energy per unit volume

$$= \frac{1}{2}Y\cdot(\text{strain})^2 \qquad \text{--- (6.23)}$$

Also, strain energy per unit volume

$$= \frac{1}{2}\frac{(\text{stress})^2}{Y} \qquad \text{--- (6.24)}$$

Thus Eq. (6.22), (6.23) and (6.24) give strain energy per unit volume in various forms.

iv) **A wire gets stretched by 4mm due to a certain load. If the same load is applied to a wire of same material with half the length and double the diameter of the first wire. What will be the change in its length? [Ans: 0.5mm]**

Solution

Given : Y is same (same material)

$L_2 = \frac{1}{2} L_1$, $l_1 = 4mm$, $r_2 = 2r_1$ $(\because d_2 = 2d_1)$

Formula - $Y = \frac{MgL}{\pi r^2 l_1}$

$Y_1 = Y_2 \Rightarrow \frac{MgL_1}{\pi r_1^2 l_1} = \frac{MgL_2}{\pi r_2^2 l_2}$

$\Rightarrow l_2 = \frac{L_2}{L_1} l_1 \frac{r_1^2}{r_2^2}$

iv) **A wire gets stretched by 4mm due to a certain load. If the same load is applied to a wire of same material with half the length and double the diameter of the first wire. What will be the change in its length? [Ans: 0.5mm]**

Solution

$$\therefore l_2 = \frac{1}{2}\frac{L_1}{L_1} l_1 \frac{r_1^2}{4r_1^2}$$

$$\therefore l_2 = \frac{l_1}{8}$$

$$= \frac{4}{8}$$

$$= 0.5\ mm$$

v) Calculate the work done in stretching a steel wire of length 2m and cross sectional area 0.0225 mm^2 when a load of 100 N is slowly applied to its free end. [Young's modulus of steel= 2×10^{11} N/ m^2]

[Ans: 2.222J]

Solution

Given - $L = 2m$, $F = 100\,N$

$A = 0.0225 \times 10^{-3} m^2$, $Y = 2 \times 10^{11} N/m^2$

formula $\therefore W = \frac{1}{2} F \times l$

$= \frac{1}{2} F \times \frac{FL}{YA}$ $\left(l = \frac{FL}{YA} \right)$

$= \frac{1}{2} \frac{F^2 l}{YA}$

$W = 2.222\,J$

vi) A solid metal sphere of volume 0.31 m^3 is dropped in an ocean where water pressure is 2×10^7 N/ m^2 . Calculate change in volume of the sphere if bulk modulus of the metal is 6.1×10^{10} N/ m^2

[Ans: 10^{-4} m^3]

Solution

Given - $K = 6.1 \times 10^{10}$ N/m^2, $V = 0.31 m^3$

$dp = 2 \times 10^7$, $dv = ?$

formula $k = \frac{V\,dp}{dv}$

$$dv = \frac{V\,dp}{K}$$

$$= 10^{-4} m^3$$

vii) A wire of mild steel has initial length 1.5 m and diameter 0.60 mm is extended by 6.3 mm when a certain force is applied to it. If Young's modulus of mild steel is 2.1 x 10^{11} N/ m^2 , calculate the force applied.

[Ans: 250 N]

Solution

Given : $L = 1.5\,m$, $d = 0.60\,mm = 0.6 \times 10^{-3}\,m$

$= 6 \times 10^{-4}\,m$

$r = 3 \times 10^{-4}\,m$

$l = 6.3\,mm = 6.3 \times 10^{-3}\,m$

$Y = 2.1 \times 10^{11}\,N/m^2$

Formula : $Y = \dfrac{FL}{Al} = \dfrac{FL}{\pi r^2 l}$

$F = \dfrac{Y \pi r^2 l}{L} = 250\,N$

viii) **A composite wire is prepared by joining a tungsten wire and steel wire end to end. Both the wires are of the same length and the same area of cross section. If this composite wire is suspended to a rigid support and a force is applied to its free end, it gets extended by 3.25mm. Calculate the increase in length of tungsten wire and steel wire separately. [Given: Y_{steel} = 2 × 10^{11} N/ m^2 ,$Y_{tungsten}$ = 3.40 × 10^{8} N/ m^2]**

[Ans: extension in tungsten wire = 3.244 mm, extension in steel wire = 0.0052 mm]

Solution

Given : $Y_1 = 3.4 \times 10^{11}$ N/m^2 (Tungsten)

$Y_2 = 2 \times 10^{11}$ N/m^2 (steel)

$l_1 + l_2 = 3.25$ mm —— (1)

Formula : Tungsten   steel

$Y_1 = \frac{FL}{Al_1}$ $Y_2 = \frac{FL}{Al_2}$

$l_1 = \frac{FL}{AY_1}$ & $l_2 = \frac{FL}{AY_2}$ $\Rightarrow$ $\frac{l_1}{l_2} = 0.617$ —— (2)

ix) A steel wire having cross sectional area 1.2 mm^2 is stretched by a force of 120 N. If a lateral strain of 1.455 mm is produced in the wire, calculate the Poisson's ratio. [Ans: 0.291]

Solution

Given $A = 1.2\ mm^2 = 1.2 \times 10^{-6}\ m^2$, $F = 120\ N$

Lateral strain $\frac{d}{D} = 1.455 \times 10^{-3}$

Y for steel $= 2 \times 10^{11}\ N/m^2$

Poisson's Ratio $\sigma = \frac{d/D}{l/L} = \frac{d/D}{\frac{\text{stress}}{Y}} = \frac{d/D}{F/AY}$

$= 0.291$

x)

A telephone wire 125m long and 1mm in radius is stretched to a length 125.25m when a force of 800N is applied. What is the value of Young's modulus for material of wire? [Ans: 1.27×10 11N/ m^2]

Solution

Given : $L = 125\,m$, $r = 1\,mm = 1 \times 10^{-3}\,m$

$l = 0.25\,m$, $F = 800\,N$

Formula : $Y = \frac{FL}{Al}$

$\therefore Y = 1.27 \times 10^{11}\,N/m^2$

xi) **A rubber band originally 30cm long is stretched to a length of 32cm by certain load. What is the strain produced? [Ans: 6.667×10^{-2}]**

Solution

- Given : $L = 30 \times 10^{-2}$ m

$l = 2 \times 10^{-2}$ m

$$\text{Strain} = \frac{l}{L} = 0.0666$$

$$= 6.66 \times 10^{-2}$$

xii) What is the stress in a wire which is 50m long and 0.01 cm^2 in cross section, if the wire bears a load of 100kg? [Ans: 9.8× 108 N/ m^2]

Solution

Given $L = 50\,m$, $A = 0.01\,cm^2 = 0.01 \times 10^{-4}\,m^2$

$= 10^{-6}\,m^2$

$F = 100 \times 9.8\,N$

$\text{Stress} = \frac{F}{A} = 9.8 \times 10^{8}\,N/m^2$

xiii) **What is the strain in a cable of original length 50m whose length increases by 2.5cm when a load is lifted? [Ans: 5× 10^{-4}]**

Solution

Given : $L = 50 m$

$l = 2.5 cm = 2.5 \times 10^{-2} m$

$$Strain = \frac{l}{L} = \frac{2.5 \times 10^{-2}}{50} = \underline{\underline{0.05 \times 10^{-2}}}$$

$$= \underline{\underline{5 \times 10^{-4}}}$$