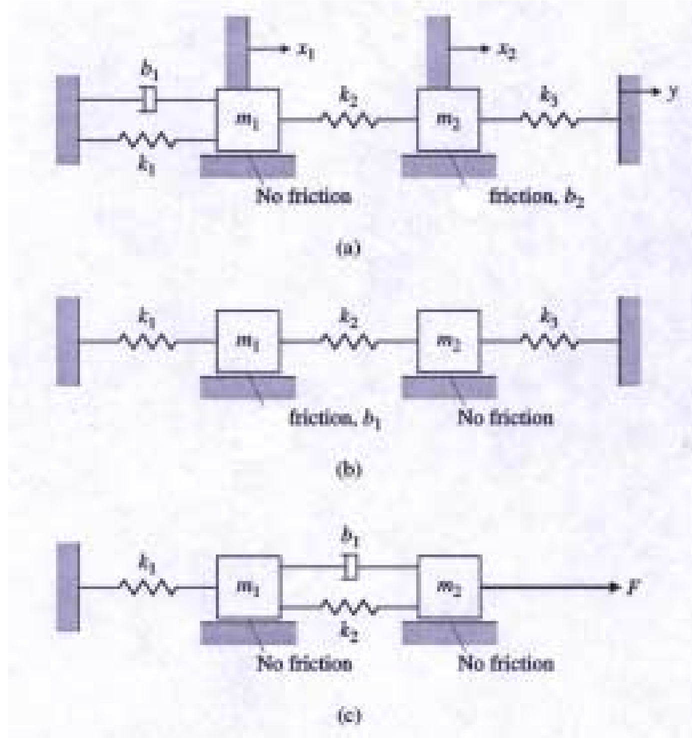
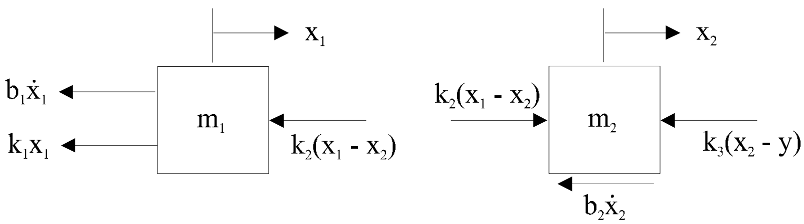
Dynamic Models

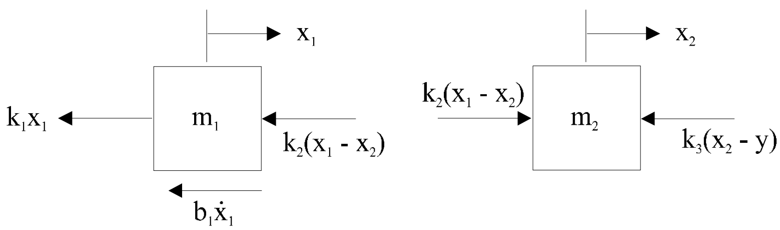
Solution:



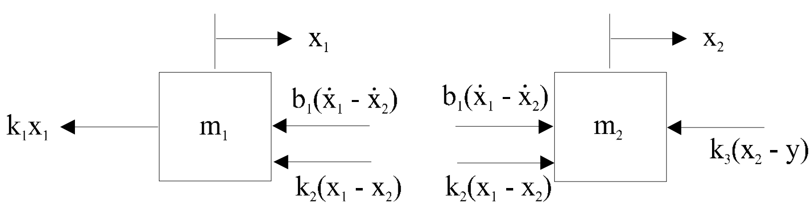
The key is to draw the Free Body Diagram (FBD) in order to keep the signs right. For (a), to identify the direction of the spring forces on the object, let $x_2 = 0$ and fixed and increase x_1 from 0. Then the k_1 spring will be stretched producing its spring force to the left and the k_2 spring will be compressed producing its spring force to the left also. You can use the same technique on the damper forces and the other mass.

(a)
$$\begin{aligned} m_1\ddot{x}_1 &= -k_1x_1 - b_1\dot{x}_1 - k_2(x_1 - x_2) \\ m_2\ddot{x}_2 &= -k_2(x_2 - x_1) - k_3(x_2 - y) - b_2\dot{x}_2 \end{aligned}$$

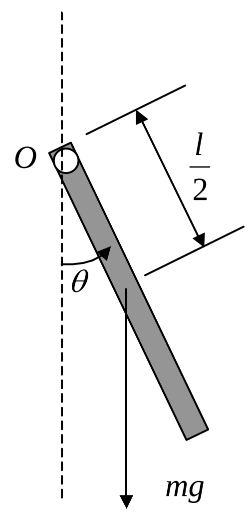




$$\begin{aligned} m_1\ddot{x}_1 &= -k_1x_1 - k_2(x_1 - x_2) - b_1\dot{x}_1 \\ m_2\ddot{x}_2 &= -k_2(x_2 - x_1) - k_3x_2 \end{aligned}$$



$$\begin{aligned} m_1\ddot{x}_1 &= -k_1x_1 - k_2(x_1 - x_2) - b_1(\dot{x}_1 - \dot{x}_2) \\ m_2\ddot{x}_2 &= F - k_2(x_2 - x_1) - b_1(\dot{x}_2 - \dot{x}_1) \end{aligned}$$



$$M = I\alpha,$$

$$\begin{aligned} M_O &= -mg \times \frac{l}{2}\sin\theta = I_O\ddot{\theta} \\ &= \frac{1}{3}ml^2\ddot{\theta} \end{aligned}$$

Moment about point O.

$$\ddot{\theta} + \frac{3g}{2l}\sin\theta = 0$$

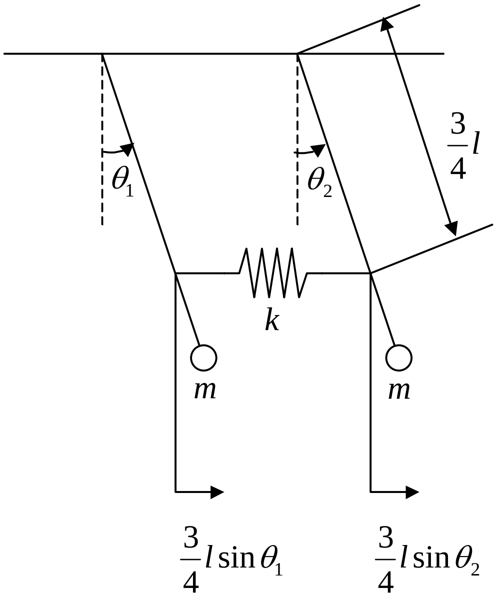
As we assumed θ is small,

$$\ddot{\theta} + \frac{3g}{2l}\theta = 0$$

The frequency only depends on the length of the rod

$$\omega^2 = \frac{3g}{2l}$$

$$\begin{aligned} T &= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2l}{3g}} = 2 \\ l &= \frac{3g}{2\pi^2} = 1.49\,\mathrm{m} \end{aligned}$$



If we write the moment equilibrium about the pivot point of the left pendulem from the free body diagram,

$$M = -mgl\sin\theta_1 - k\frac{3}{4}l\left(\sin\theta_1 - \sin\theta_2\right)\cos\theta_1\frac{3}{4}l = ml^2\ddot{\theta}_1$$

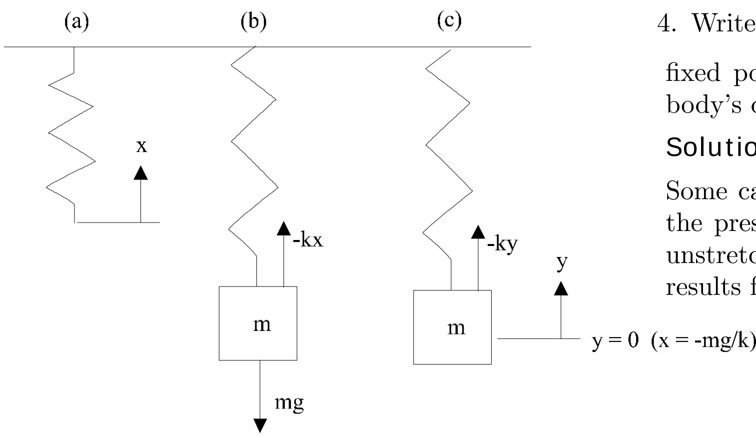
$$ml^2\ddot{\theta}_1 + mgl\sin\theta_1 + \frac{9}{16}kl^2\cos\theta_1\left(\sin\theta_1 - \sin\theta_2\right) = 0$$

Similary we can write the equation of motion for the right pendulem

$$-mgl\sin\theta_2 + k\frac{3}{4}l\left(\sin\theta_1 - \sin\theta_2\right)\cos\theta_2\frac{3}{4}l = ml^2\ddot{\theta}_2$$

As we assumed the angles are small, we can approximate using $\sin\theta_1 \approx \theta_1, \sin\theta_2 \approx \theta_2$, $\cos\theta_1 \approx 1$, and $\cos\theta_2 \approx 1$. Finally the linearized equations of motion becomes,

$$\begin{aligned} ml\ddot{\theta}_1 + mg\theta_1 + \frac{9}{16}kl\left(\theta_1 - \theta_2\right) &= 0 \\ ml\ddot{\theta}_2 + mg\theta_2 + \frac{9}{16}kl\left(\theta_2 - \theta_1\right) &= 0 \end{aligned} \qquad \begin{aligned} \ddot{\theta}_1 + \frac{g}{l}\theta_1 + \frac{9}{16}\frac{k}{m}\left(\theta_1 - \theta_2\right) &= 0 \\ \ddot{\theta}_2 + \frac{g}{l}\theta_2 + \frac{9}{16}\frac{k}{m}\left(\theta_2 - \theta_1\right) &= 0 \end{aligned}$$



4. Write the equations of motion for a body of mass M suspended from a fixed point by a spring with a constant k. Carefully define where the body's displacement is zero.

Solution:

Some care needs to be taken when the spring is suspended vertically in the presence of the gravity. We define $x = 0$ to be when the spring is unstretched with no mass attached as in (a). The static situation in (b) results from a balance between the gravity force and the spring.

From the free body diagram in (b), the dynamic equation results

$$m\ddot{x} = -kx - mg.$$

We can manipulate the equation $\quad m\ddot{x} = -k\left(x + \frac{m}{k}g\right),$

so if we replace x using $y = x + \frac{m}{k}g$,

$$\begin{aligned} m\ddot{y} &= -ky \\ m\ddot{y} + ky &= 0 \end{aligned}$$

The equilibrium value of x including the effect of gravity is at $x = -\frac{m}{k}$ and y represents the motion of the mass about that equilibrium point.

An alternate solution method, which is applicable for any problem involving vertical spring motion, is to define the motion to be with respect to the static equilibrium point of the springs including the effect of gravity, and then to proceed as if no gravity was present. In this problem, we would define y to be the motion with respect to the equilibrium point, then the FBD in (c) would result directly in

$$m\ddot{y} = -ky.$$