

DreamPhysics: Learning Physical Properties of Dynamic 3D Gaussians with Video Diffusion Priors

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Abstract

Dynamic 3D interaction has witnessed great interest in recent works, while creating such 4D content remains challenging. One solution is to animate 3D scenes with physics-based simulation, and the other is to learn the deformation of static 3D objects with the distillation of video generative models. The former one requires assigning precise physical properties to the target object, otherwise the simulated results would become unnatural. The latter tends to formulate the video with minor motions and discontinuous frames, due to the absence of physical constraints in deformation learning. We think that video generative models are trained with real-world captured data, capable of judging physical phenomenon in simulation environments. To this end, we propose DreamPhysics in this work, which estimates physical properties of 3D Gaussian Splatting with video diffusion priors. DreamPhysics supports both image- and text-conditioned guidance, optimizing physical parameters via score distillation sampling with frame interpolation and log gradient. Based on a material point method simulator with proper physical parameters, our method can generate 4D content with realistic motions. Experimental results demonstrate that, by distilling the prior knowledge of video diffusion models, inaccurate physical properties can be gradually refined for high-quality simulation. Codes are released at: <https://github.com/tyhuang0428/DreamPhysics>.

1 Introduction

Dynamic 3D interaction is in highly increased demand for a wide range of applications, *e.g.*, video games, virtual reality, industrial designing, *etc.* Recent research on 3D representation achieves significant progress in both reconstruction [1, 2] and generation [3, 4]. However, there is still a gap between generating static 3D assets and interacting with them in simulation environments [5, 6], where extra properties are required for realistic animation, like material types and physical parameters.

To animate static 3D objects, some works [7, 8] directly injected physical properties into 3D objects, enabling motion predictions in a physics-based simulator. Nonetheless, all these physical parameters are manually assigned to 3D objects, and an inaccurate setting of parameters can easily produce unnatural simulation results (Figure 1(a)). Recently, a series of works [9, 10, 11] attempted to learn time-dependent deformation with the distillation of video generative models [12, 13, 14]. Given input text or image prompts, these methods can automatically generate corresponding dynamic 3D content, while the generated results tend to present minor motions and discontinuous frames. We think it is mainly attributed to the absence of physical constraints in deformation fields, rather than the supervision of video models. Video generators are actually trained with real-world captured video data, which includes physical phenomenon and regulations. To some extent, these pre-trained generators contain physics-based prior knowledge. Therefore, a straightforward question comes out:

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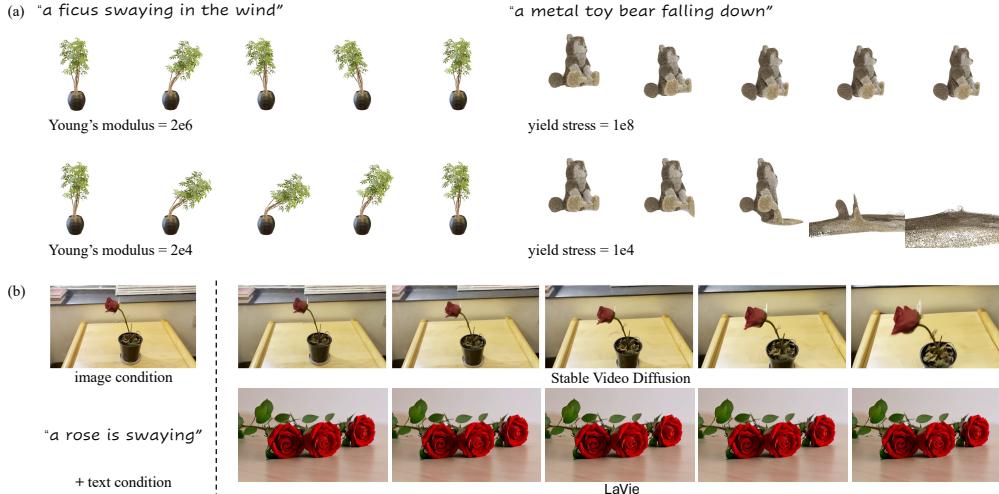


Figure 1: (a): the setting of physical properties will significantly affect the quality of simulated videos; (b) current video diffusion models can hardly control to generate desired results.

Can we incorporate physical constraints into deformation fields, allowing the automatic formulation of appropriate properties with the physical prior of video generative models?

Our concurrent work PhysDreamer [15] proposed to supervise the prediction of physical properties with a video ground-truth generated by image-to-video diffusion models. However, as shown in Figure 1(b), the video generative model can hardly generate the desired results to serve as ground truth, considering its poor control over input prompts. Inspired by those 4D generation works [9, 10, 11], we rethink the usage of video generative models, distilling video prior knowledge to the physical fields. In this way, one can optimize physical parameters with the guidance of Score Distillation Sampling (SDS) [3], instead of the explicit supervision from low-quality generated videos.

To this end, we introduce a new framework, DreamPhysics, to learn physical properties of 3D Gaussian Splatting (GS) [2] with the distillation of video diffusion priors. Specifically, given a static 3D GS scene with corresponding physical properties, a material point method (MPM) [16, 17] system can simulate the time-dependent deformation of each Gaussian kernel. A 3D video is then rendered accordingly, which could exist artifacts due to inaccurate property settings. To optimize these parameters, we adopt SDS loss under the guidance of video diffusion models, predicting diffusion noise from rendered videos. Since both the MPM simulation and the GS rendering are differentiable, the predicted noise can be propagated backward to all the optimizing parameters. These parameters can finally converge to a reasonable range, simulating realistic 4D content. DreamPhysics supports both image- and text-conditioned optimization, with the guidance of image-to-video [18] and text-to-video [14] diffusion models, respectively. We further propose frame interpolation and log gradient optimization to facilitate the training process.

Experimental results demonstrate that DreamPhysics can effectively distill the prior of video diffusion models, assigning proper values to the physical properties. Compared with state-of-the-art 4D generation works, our results enjoy faster training, more realistic motion, and more flexible simulation.

To conclude, our contribution can be summarized as:

- We rethink the existence of physical priors in video generative models, proposing to learn dynamic 3D with the combination of physical constraints and video priors. To this end, we introduce DreamPhysics, optimizing physical parameters by distilling video diffusion priors.
- DreamPhysics allows both image- and text-conditioned optimization. To facilitate the parameter training, we further propose frame interpolation and log gradient.
- Based on an MPM system with optimized physical parameters, DreamPhysics can generate high-quality 4D content. Compared with previous works, our results enjoy more realistic motion simulation.

2 Preliminaries

2.1 Point-Based Representation

Point cloud [19] is an explicit 3D representation, which basically consists of the coordinates for all points. Normal and color information [20, 21] can also be considered to further enrich the feature space of point cloud. Despite the succinct representation, its rendering quality is heavily restricted by the number of points [22, 23]. Derived from NeRF [1], 3D Gaussian Splatting (GS) [2] introduced a point-based explicit radiance field. Points are modeled as a set of Gaussian kernels $\{\mathcal{G}_i\} = \{x_i, \sigma_i, \Sigma_i, C_i\}$, where x_i , σ_i , Σ_i , and C_i denote the center coordinate, opacity, covariance matrix, and spherical harmonic coefficient of the i -th kernel \mathcal{G}_i . To render a 3D GS scene at a specific viewpoint \mathbf{r} , the color can be formulated as,

$$\mathbf{C} = \sum_{i=1}^N T_i \alpha_i C_i, \text{ with } T_i = \prod_{j=1}^{i-1} (1 - \alpha_j), \quad (1)$$

where N is the set of sorted Gaussian kernels related to the pixel and the viewpoint. α_i is the effective opacity given by evaluating a 2D Gaussian with Σ and σ . 3D GS can reconstruct high-fidelity views by real-time rendering, and support explicit interaction and editing.

2.2 Material Point Method

The material point method (MPM) is a numerical simulation mechanic for the analysis of continuum forces. In MPM, the continuum is represented by a set of particles placed in a grid-based space. Different from mesh-based numerical mechanics, MPM can be naturally applied to point-based representation 3D GS. Following PhysGaussian [7], we have a time-dependent state for each Gaussian kernel as:

$$x_i(t) = \Delta(x_i, t), \quad \Sigma_i(t) = F_i(t) \Sigma_i F_i(t)^T, \quad (2)$$

where $\Delta(*, t)$ and $F_i(t)$ are the coordinate deformation and the deformation gradient at timestep t . Considering the continuum rotation $\Omega_i(t)$, the rendering viewpoint also requires adjustment to satisfy the view direction of spherical harmonic coefficient C_i .

2.3 Score Distillation Sampling

The score distillation sampling (SDS) [3, 24] optimization distills the parameters of 3D representation to pre-trained 2D diffusion models, widely-used in 3D generation methods [25, 4, 26, 27]. Recently, SDS has been extended to dynamic 3D generation as SDS-T, where video diffusion models are deployed to supervise the time-dependent deformation of static 3D objects. Specifically, given a video diffusion model ϕ and a camera trajectory $\mathbf{r}(t)$, SDS-T optimizes the rendered 3D video $V_{\mathbf{r}(t)}$ with predicted noise $\hat{\epsilon}_\phi$:

$$\nabla_\theta \mathcal{L}_{\text{SDS-T}}(\theta) = \mathbb{E}_{\mu, \mathbf{r}(t), \epsilon} \left[\omega(\mu) (\hat{\epsilon}_\phi(V_{\mathbf{r}(t)}; \mu, y) - \epsilon) \frac{\partial V_{\mathbf{r}(t)}}{\partial \theta} \right], \quad (3)$$

where μ is the noise timestep and y is the input condition (image and text are both acceptable). θ is the target deformation field.

3 DreamPhysics

3.1 Method Overview

As shown in Figure 2, given a generated object or a reconstructed scene $\{\mathcal{G}_i\}$ represented by 3D GS, DreamPhysics aims to estimate the corresponding physical parameters θ_G for the MPM-based simulator. We can first initialize a series of parameters $\theta_G^{(0)}$ and render a T-length video $V^{(0)} = \{I_1^{(0)}, I_2^{(0)}, \dots, I_T^{(0)}\}$ via MPM simulation. The rendered video may look unrealistic due to the inaccurate initialization of $\theta_G^{(0)}$. Therefore, $V^{(0)}$ is then sent to the SDS-T optimizer, distilling video diffusion priors to $\theta_G^{(1)}$. Similarly, for each training epoch k , we can obtain an optimized $\theta_G^{(k+1)}$ via the distillation of $V^{(k)}$. The final physical parameters θ_G can converge to a reasonable range.

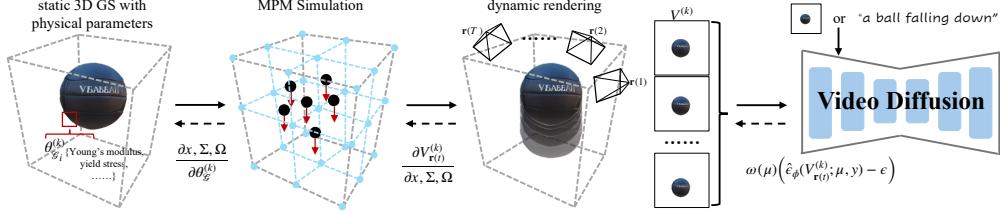


Figure 2: Overview of DreamPhysics. We first initialize a set of physical parameters for a static 3D GS, which is then fed to an MPM simulator. We render a 4D video according to the simulated results and leverage a video diffusion model to optimize the rendered video. The distillation gradients are finally back-propagated to refine the physical parameters.

3.2 Parameter Optimization

With the simulation of MPM [28], we have a time-dependent state $\{x_i(t), \Sigma_i(t), \Omega_i(t)\}$ for each Gaussian kernel \mathcal{G}_i according to Eq. 2. For a timestep t , an image I_t is then rendered at the viewpoint $\mathbf{r}(t)$ as Eq. 1. These two computations are both differentiable but implemented differently. To ensure the gradient flow on our target parameters $\theta_{\mathcal{G}}$, we rewrite the loss in Eq. 3 as,

$$\nabla_{\theta_{\mathcal{G}}^{(k)}} \mathcal{L}_{\text{dis}} = \mathbb{E}_{\mu, \mathbf{r}(t), \epsilon} \left[\omega(\mu) \left(\hat{e}_{\phi}(V_{\mathbf{r}(t)}^{(k)}; \mu, y) - \epsilon \right) \frac{\partial V_{\mathbf{r}(t)}^{(k)}}{\partial x, \Sigma, \Omega} \frac{\partial x, \Sigma, \Omega}{\partial \theta_{\mathcal{G}}^{(k)}} \right]. \quad (4)$$

3.3 Frame Interpolation

As for the sampling timesteps, reducing the interval time can increase the accuracy of simulation results, while current video diffusion models cannot handle excessively high frame rates. Consequently, we keep the simulation process at high frame rates but only extract key frames for video rendering.

However, the MPM simulator is a sequential model, which can easily lead to gradient vanishing or exploding like RNN [29]. We have to conduct truncated back-propagation through time (BPTT), preserving the gradient of key frame simulation only. Truncated BPTT can effectively prevent gradient issues, but the supervision could be limited to specific frames. To ensure that our optimization covers as many video frames as possible, we adopt a frame interpolation. Specifically, given a total number of frames $M \times T$, we can separate them into M groups of frames with equal intervals, *i.e.*, $\{I_i, I_{i+M}, \dots, I_{i+M(T-1)}\}$ for the i -th group. These groups formulate different videos, which are fed into the supervision process alternately.

3.4 Log Gradient

The value range for physical properties can be very large, *e.g.*, the reasonable values for Young's modulus can vary between 1 and 1e7. However, during gradient updates, the same gradient can result in different update granularity across different magnitudes, leading to the parameters easily getting stuck within a specific magnitude range.

To ensure parameters quickly converge to a reasonable range, we propose to perform gradient updates in the logarithmic domain,

$$\log(\theta_{\mathcal{G}}) \leftarrow \log(\theta_{\mathcal{G}}) + t \nabla_{\theta_{\mathcal{G}}}. \quad (5)$$

In this way, the updating granularity across all magnitudes becomes equivalent.

4 Experiments

In this section, we conduct experiments on text-conditioned and image-conditioned optimizations to demonstrate the effectiveness of DreamPhysics. We also compare our method with the concurrent work PhysDreamer [15].



Figure 3: Text-conditioned optimization. (a): if Young’s modulus is set too low, the ficus will excessively tilt to one side; (b): if Young’s modulus is set too high, the oscillation ficus will become too vague. Our method can optimize its value to a reasonable range.

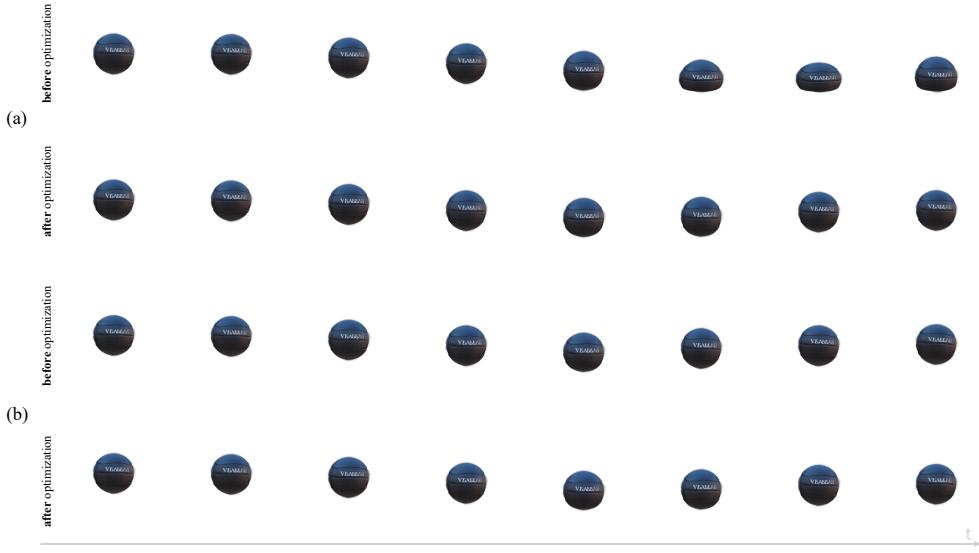


Figure 4: Image-conditioned optimization. (a): if Young’s modulus is set too low, the ball will exhibit excessive deformation; (b): if Young’s modulus is set too high, the ball barely has any deformation. Our method can optimize its value to a reasonable range.

4.1 Text Condition

For text-conditioned optimization, we leverage a text-to-video diffusion model ModelScope [14] with a given text. We select the ficus scene in PhysGaussian [7] and input a text prompt "*ficus swaying in the wind*" to simulate the rotation motion. In Figure 3(a), the ficus will excessively tilt to one side and have difficulty returning to its original position, if its Young’s modulus is set too low. Conversely, if the modulus is too high, the oscillation of ficus will become vague, as shown in Figure 3(b). After the optimization in DreamPhysics, Young’s modulus falls within a normal range and the swaying looks more natural.



Figure 5: Comparison with our concurrent work PhysDreamer.

4.2 Image Condition

We leverage Stable Video Diffusion (SVD) [18] for image-conditioned optimization. Since SVD is an image-to-video diffusion model, the first frame is regarded as the input image. We select a generated ball and try to optimize its dropping process, which is an example of collision motion shown in Figure 4. Similar to the experiments in text-conditioned optimization, we initialize Young’s modulus to be too low in (a) and too high in (b). When hitting the ground, the ball will exhibit excessive deformation or insufficient deformation. DreamPhysics can also effectively adjust the modulus to a reasonable range.

4.3 Comparison with PhysDreamer

PhysDreamer [15] is our concurrent work, which also leverages video diffusion priors to supervise physical properties. The main difference is that PhysDreamer generates a ground-truth video with SVD while our method proposes to distill physics-related priors from video diffusion models.

We selected the telephone scene from PhysDreamer for the comparison, since it hasn’t released the source code for training and corresponding ground-truth videos are hard to generate. As shown in Figure 5, our method can model a swaying telephone line, which is competitive with PhysDreamer.

5 Discussion

DreamPhysics is an early work to study the usage of video priors on physics-based simulation, while some issues remain challenging in this task. In the following, we would like to discuss the potential improvement for future work.

Simulated Motion. Although DreamPhysics supports one more motion (*i.e.*, collision) compared to PhysDreamer, it is still far from enough to simulate real-world events. In fact, each kind of motion depends on independent physical constraints. Current frameworks can hardly combine all the motions into one simulator.

Evaluation Metrics. Current evaluation metrics for video generation are mainly vision-based. However, we find that it is not obvious to distinguish the quality of the simulated videos visually. For example, both videos from Figure 5 look reasonable, and it is hard to tell which is better. A physics-based metric could be helpful for the evaluation of simulation results.

Scene Interaction. Current simulators can only handle the interactions of a few target objects, but environments are dismissed. For example, in both videos of Figure 5, shadows on the wall did not change with the movement of the telephone lines. Besides, more interactions can be considered, such as dragging the mouse or adjusting the weights.

6 Conclusion

In this work, we introduced a new framework DreamPhysics, which learns the physical properties of dynamic 3D Gaussians with video diffusion priors. Based on the physical simulation, DreamPhysics distills the video priors to optimize physical parameters. To facilitate that process, we proposed frame interpolation and log gradient. Our method can produce high-quality 4D content, which is more realistic than previous works.

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