Assignment

Question: Is set of odd Number with binary operation (+) i.e, <O,+> an abelian group ? If not explain the reasons with necessary notations.

Answer: No, the set of odd numbers under the binary operation of addition, denoted as <O,+>, is not an abelian group. It fails to satisfy two of the fundamental axioms required for a structure to be considered a group.

Reasons for failure:

Let the set of odd numbers be denoted by
$O = \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$. For <O,+> to be a group, it must satisfy several properties. It fails on the following:

1. Failure of the closure Property ×

The closure property states that for any two elements a and b in the set, the result of their operation, a+b, must also be in the set, this is not true for the set of odd Numbers

P.T.O

* Simple Example : Consider the old numbers 3 and 5. Both are is the set of O. However, their sum is $3+5=8$. The numbers 8 is an even number and is therefore not in the set O.

* General Proof : Any old number can be written in the form $2k+1$, where k is an integer. Let's take two arbitrary old numbers.

$a = 2k_1 + 1$ and $b = 2k_2 + 1$

Their sum is :

$$a+b = (2k_1+1) + (2k_2+1)$$
$$= 2k_1 + 2k_2 + 2$$
$$\therefore a+b = 2(k_1+k_2+1)$$

The result, is a multiple of 2, which, by definition is an even number. Since the sum of any two old num is always, even the set O is not closed under addition.

2. Absence of an Identity Element *

A Group must contain an identity element (e), which is an element that leaves any other element unchanged when the operation of addition, the identify.

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0 (since $a+0=a$)

However, 0 is an even number, so it is not and element of the set of all number 0, Since the identity element for additional is not in the set, this axim, is not satisfied

Since $\langle O,+\rangle$ fails to meet the closure and identity axiomy, it, is not a group.

Consequently, it cannot be an abelian group.