Booth Algorithm

In the last few sections we have discussed the method of multiplying two positive operands using different techniques. The subsequent sections will be discussing on the method of multiplying two operands when one of them is positive and the other one is negative. One of such method is **Booth algorithm.**

Booth's algorithm is specific have two advantages:

1. one of them is it treats both multiplier and multiplicand in 2's compliment system equally and generates the product without having to exchange multiplicand and multiplier.

2. Another advantage which doesn't hold true all the time but works well for few cases were in it tries to reduce the number of 1's in summand bit by recoding the multiplier and the technique is called as **Booth recoding of multiplier. (Steps for the same will be discussed subsequently).** Because of this there will be less number of addition operations to be performed.

Number representation here uses 2's compliment as it is signed number representation.

Steps for multiplying two operands using Booth Algorithm:

1. The first step is similar to the one that is discussed for the multiplication of positive numbers were in you have to select the number of bits for the multiplication. Consider the largest of two numbers in terms of its magnitude irrespective of sign, represent it in binary and even represent the other operand equal to number of bits of the largest operand. For example, if one of the operand is -13 and the other +5, here 13 is considered largest.

2. Next consider the multiplier and recode the multiplier according to Booth's recoding technique to get a new multiplier. (Booths recoding technique is discussed after this)

3. Next do the multiplication of multiplicand and recoded multiplier. In recoded multiplier, bit 1 is the multiplicand value and bit -1 is the 2's compliment of the multiplicand value. In deriving each and every partial products, extend the partial products up to 2n bits were in remaining bits are extended with the value of sign bits. For example if sign bit is 0, extend remaining bits with 0 and if sign bit is 1, extend remaining bits with 1.

4. Finally add the partial products to get the product of 2n bits.

Booth multiplier recoding technique Consider n bits of multiplier 0 0 1 1 1 1 0 (30) as an example and lets see how to recode this multiplier:

1. First represent the multiplier as it is and add an extra bit to the right of LSB of the multiplier as shown in the figure below.

```
0  0  1  1  1  1  0  [0] <-- added zero
```

2. In the next step, group bits of multiplier in pair from the right most bit which is an added bit 0 as shown below:

```
0  0  1  1  1  1  0  [0]
```

3. Next to get the recoded bit, subtract left bit from right bit in the group, for example in the first group do 0 – 0, in second group do 0 – 1 , third group 1 – 1 etc. as shown below :

```
0    0    1    1    1    1    0   [0]
   0   +1    0    0    0   -1    0
```

For n bit multiplier, there will be n recoded bits. Here +1 means multiplicand value and -1 means 2’s compliment of the multiplicand.

Example: This example of multiplication operation +13 X -6 solved according to booth algorithm gives a clear picture on its steps:

```
   0 1 1 0 1   (+13)                      0  1  1  0  1
 X 1 1 0 1 0   (-6)      ==>              0 -1 +1 -1  0
 -----------                     -------------------------
                         0  0  0  0  0  0  0  0  0  0
                         1  1  1  1  1  0  0  1  1
                         0  0  0  0  1  1  0  1
                         1  1  1  0  0  1  1
                         0  0  0  0  0  0
                        ------------------------------
                         1  1  1  0  1  1  0  0  1  0   (-78)
```

Steps:

1. First represent +13 = 01101 in binary 2’s compliment which is largest and then represent -6 in 5 bits which is equal to 11010 in 2’s compliment system.

2. Next recode the multiplier according to booth technique which yields 0 -1 +1 -1 0 as recoded multiplier.

3. Perform the multiplication as shown above. In each partial product, extend bit value to 2n bits. The unfilled places are extended to sign bits of the partial product. If the bit value is 1 add down the multiplicand. If it is -1 add 2’s compliment of multiplicand.

Similarly solve all other problems.

Fast Multiplication

In this section, let us look into the fast multiplication technique that implements the multiplication in a faster way as compared to that of traditional methods. There are two methods under this:

1. Bit Pair recoding of the multipliers

2. Carry save addition method

1. Bit Pair recoding of the multiplier: a very efficient technique compared to that of Booth algorithm. Even this method implements the recoding of the multiplier but the recoding technique reduces the multiplier bits from n to n/2 of ceil function. For example if there are 5 bits there will be 5/2 = 3 bits in the recoded multiplier. Technique of bit pair recoding is discussed in subsequent section.

Steps for multiplying two operands using Bit Pair Recoding method:

1. The first step is similar to the one that is discussed for the multiplication of positive numbers were in you have to select the number of bits for the multiplication. Consider the largest of two numbers in terms of its magnitude irrespective of sign, represent it in binary and even represent the other operand equal to number of bits of the largest operand. For example, if one of the operand is -13 and the other +5, here 13 is considered largest.

2. Next consider the multiplier and recode the multiplier according to Bit Pair recoding technique to get a new multiplier. (Bit Pair recoding technique is discussed after this)

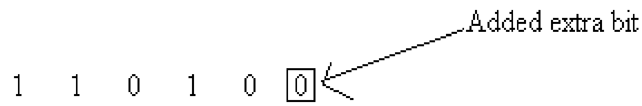
3. Next do the multiplication of multiplicand and recoded multiplier. In recoded multiplier, bit 1 is the multiplicand value and bit -1 is the 2's compliment of the multiplicand value, bit 2 is 2 times the multiplicand and bit -2 is 2 times the 2's compliment of multiplicand. In deriving each and every partial products, extend the partial products up to 2n bits were in remaining bits are extended with the value of sign bits. For example if sign bit is 0, extend remaining bits with 0 and if sign bit is 1, extend remaining bits with 1.

4.Apart from these one more important thing is each partial product is shifted two times to the left instead of one time.

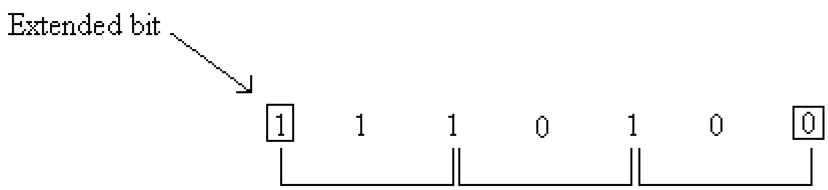
5. Finally add the partial products to get the product of 2n bits.

Bit Pair recoding technique Consider n bits of multiplier 1 1 0 1 0 (-6) as an example and lets see how to recode this multiplier:

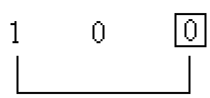
1. First represent the multiplier as it is and add an extra bit to the right of LSB of the multiplier as shown in the figure below.



2. In the next step, group 3 bits each of multiplier from the right most bit which is an added bit 0. If there is a shortage for the last group, extend the MSB value of multiplier to that bit. The operation is shown below.



3. Next to get the recoded bit, grouped bits are divided into bit **i, bit i+1 and bit i-1** in which middle one in the group is i, left most in the group is i+1 and rightmost in the group is i-1. For example for the group sown below:



Here i=0(middle), i+1 =1(leftmost) and i-1(rightmost) = 0

Then the recoded value is extracted according to the 3 bit truth table shown below:

Multiplier bit-pair $i+1$	Multiplier bit-pair i	Multiplier bit on the right $i-1$	Multiplicand selected at position i
0	0	0	0 × M
0	0	1	+ 1 × M
0	1	0	+ 1 × M
0	1	1	+ 2 × M
1	0	0	− 2 × M
1	0	1	− 1 × M
1	1	0	− 1 × M
1	1	1	0 × M

(b) Table of multiplicand selection decisions

Here in this table,

i. 0 means add 0 to the partial product.

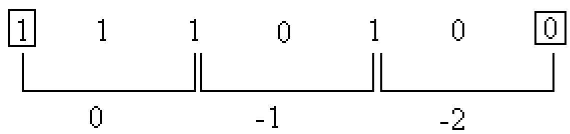
ii. 1 means add multiplicand

iii. -1 means add 2’s compliment of the multiplicand

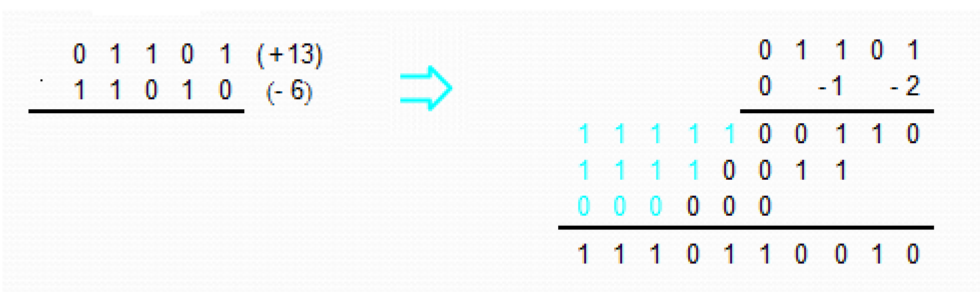
iv. 2 means add 2 times multiplicand. To get 2 times of any binary value, add zero to the right of LSB, it doubles value. For example 101 is 5 and if you add 0 to the LSB, it becomes 1010 which is 10 and is double of 5.

v. -2 means add 2 times 2’s compliment of the multiplicand.

According to this table, recoded bit for our multiplier is:



Example Problem: Consider the multiplication of +13X-6 according to Bitpair recoding technique.



Steps:

1. First represent +13 = 01101 in binary 2’s compliment which is largest and then represent -6 in 5 bits which is equal to 11010 in 2’s compliment system.

2. Next recode the multiplier according to bit pair recoding technique which yields 0 -1 -2 as recoded multiplier.

3. Perform the multiplication as shown above. In each partial product, extend bit value to 2n bits. The unfilled places are extended to sign bits of the partial product. If the bit value is 1 add down the multiplicand. If it is -1 add 2’s compliment of multiplicand. For the bit value -2, add 2X2’s

compliment of multiplicand which is equal to 100110. One more important thing to be kept in mind is that for every partial product, make left shift of 2 bits as shown in the above problem.

Note: Different problems have been solved in the class. Please go through it

Note: For Carry Save addition, go through the problems that is solved in the class which is sufficient for the time being

☺ It's never too late until it is done ☺

****************ALL THE BEST****************