Machine Learning
机器学习

Lecture11：Dimension reduction & Feature selection
降维和特征选择

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维度灾难问题
Curse of dimensionality

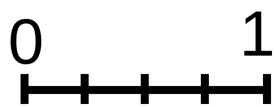
- Many explored domains have hundreds to tens of thousands of **variables/features** with many **irrelevant** and **redundant** ones!
- In domains with many features the underlying probability distribution can be very complex and very hard to estimate (e.g. dependencies between variables) !
- Irrelevant and redundant features can confuse learners!

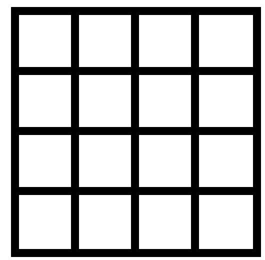
- Limited training data!
- Limited computational resources!
- **Curse of dimensionality**!

维度灾难问题
Curse of dimensionality

– Spatial sampling

4 units for each dimension, 10 samples per unit





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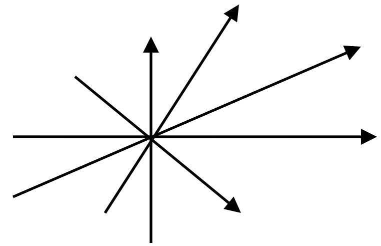

total sample：

1-D： 2-D： 10-D：

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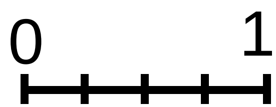
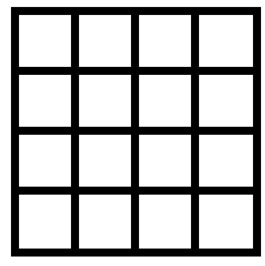
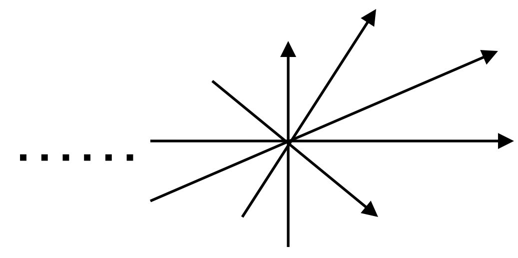






total sample：

1-D：4

~40

2-D：4*4=16

~160

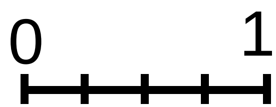
10-D：4^10=1048576

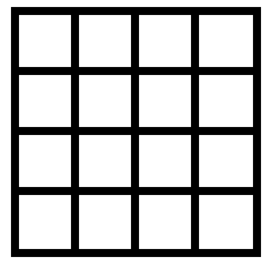
~10M

维度灾难问题
Curse of dimensionality

– Spatial sampling

4 units for each dimension, 10 samples per unit





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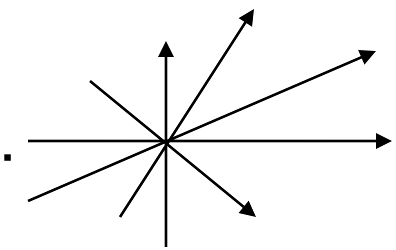

total sample：

1-D：4

~40

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~160

10-D：4^10=1048576

~10M

– Sample sparsity

total sample：1000

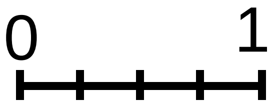
partition of each dimension：4

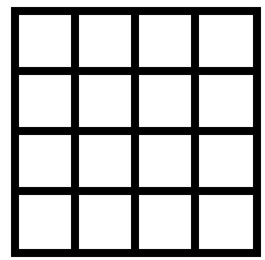
Samples per unit：

维度灾难问题
Curse of dimensionality

– Spatial sampling

4 units for each dimension, 10 samples per unit





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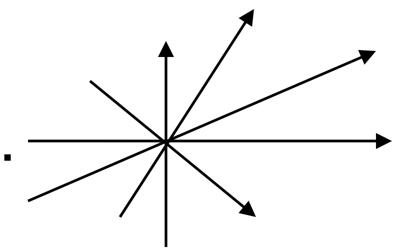


total sample：

1-D：4 ~40

2-D：4*4=16 ~160

10-D：4^10=1048576 ~10M

– Sample sparsity

total sample：1000

partition of each dimension：4

Samples per unit：

1D：1000/4 = 250

2D：1000/(4*4)= 62.5

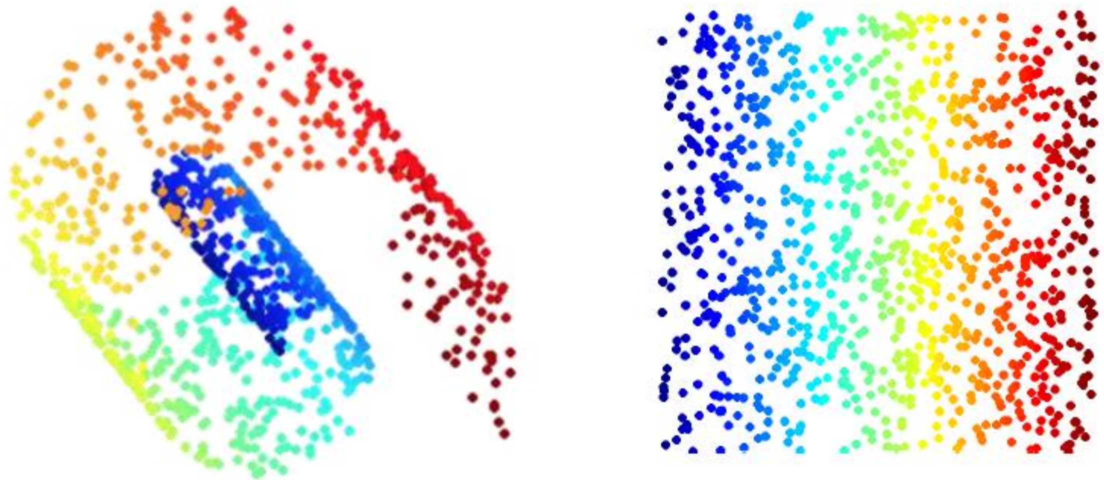
10D：1000/(4^10)= 0.001

噪声影响
Noise influence

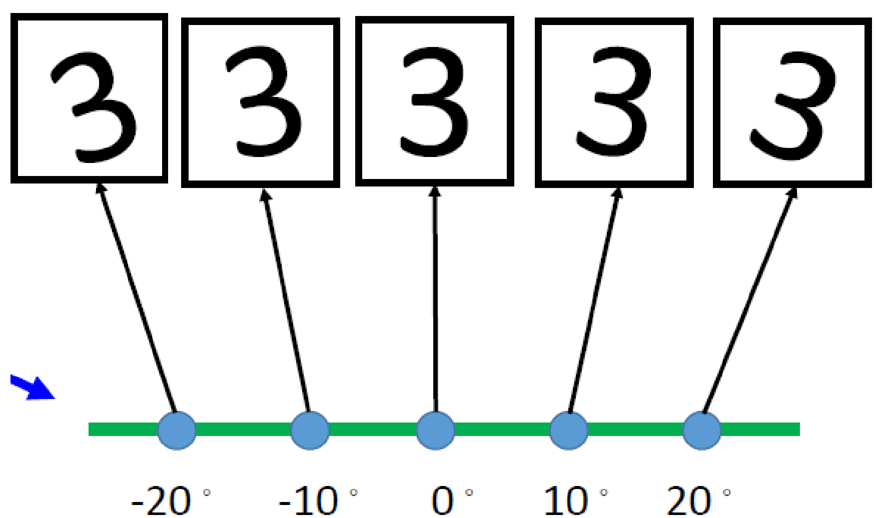
- Feature space ： 101D
- The distance between the positive and negative samples at the first dimension: 1
- The noise of the sample in its residual dimension ： 10%
- “Noise distance” ： $\sqrt{100 \times 0.1^2} = 1$
- Even if the noise is only 10%, the noise distance in high dimensional space is enough to mask the essential difference between positive and negative samples.

低维嵌入
Low dimensional embedding

Looks like 3-D,
Actually, 2-D



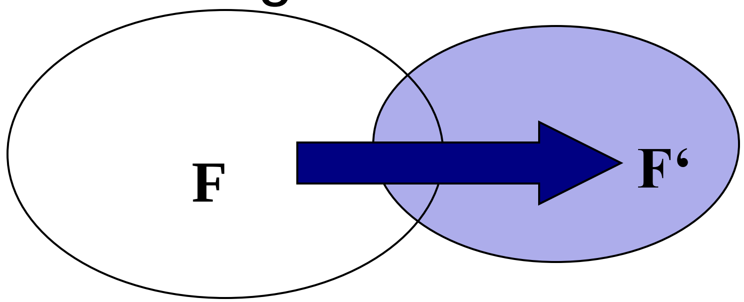
28x28 D,
Actually, 1-D



降维与特征选择

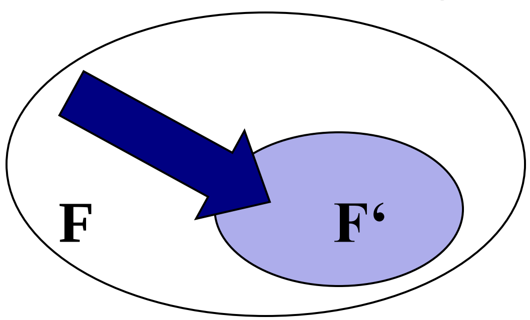
Dmensionality reduction& Feature selection

- Dimensionality reduction:creating a subset of new features by combinations of the existing features



$$\{f_1,\ldots,f_i,\ldots,f_n\}\xrightarrow[f.extraction]{}\{g_1(f_1,\ldots,f_n),\ldots,g_j(f_1,\ldots,f_n),\ldots,g_m(f_1,\ldots,f_n)\}$$

- Feature Selection: choosing a subset of all the features



$$\{f_1,\ldots,f_i,\ldots,f_n\}\xrightarrow[f.selection]{}\{f_{i_1},\ldots,f_{i_j},\ldots,f_{i_m}\}$$

$$i_j\in\{1,\ldots,n\};\ j=1,\ldots,m$$

$$i_a=i_b\Rightarrow a=b;\ \ a,b\in\{1,\ldots,m\}$$

Machine Learning Problems

	Supervised Learning	*Unsupervised Learning*
Discrete	classification or categorization	clustering
Continuous	regression	dimensionality reduction

降维方法 Dmensionality reduction Method

- Principal Component Analysis (PCA)
- Multidimensional Scaling (MDS)
- Locally Linear Embedding (LLE)
- Laplacian Eigenmaps (LE）
- Isometric Mapping(Isomap)
- T-distributed Stochastic Neighbor Embedding (t-SNE)
- Auto-encoders
- Non-negative Matrix Factorization(NMF)
- Canonical Correlation Analysis(CCA)
- Independent Component Analysis (ICA)
- Probabilistic PCA
- Kernel PCA
- (Linear Discriminant Analysis(LDA)-supervised)
- …

举例
Dimensionality Reduction Example

- Given 53 blood and urine samples (features) from 65 people.
- How can we visualize the measurements?

Matrix format (65x53)

	H-WBC	H-RBC	H-Hgb	H-Hct	H-MCV	H-MCH	H-MCHC
A1	8.0000	4.8200	14.1000	41.0000	85.0000	29.0000	34.0000
A2	7.3000	5.0200	14.7000	43.0000	86.0000	29.0000	34.0000
A3	4.3000	4.4800	14.1000	41.0000	91.0000	32.0000	35.0000
A4	7.5000	4.4700	14.9000	45.0000	101.0000	33.0000	33.0000
A5	7.3000	5.5200	15.4000	46.0000	84.0000	28.0000	33.0000
A6	6.9000	4.8600	16.0000	47.0000	97.0000	33.0000	34.0000
A7	7.8000	4.6800	14.7000	43.0000	92.0000	31.0000	34.0000
A8	8.6000	4.8200	15.8000	42.0000	88.0000	33.0000	37.0000
A9	5.1000	4.7100	14.0000	43.0000	92.0000	30.0000	32.0000

Instances

Features

Difficult to see the correlations between the features...

举例
Dimensionality Reduction Example

- Given 53 blood and urine samples (features) from 65 people.
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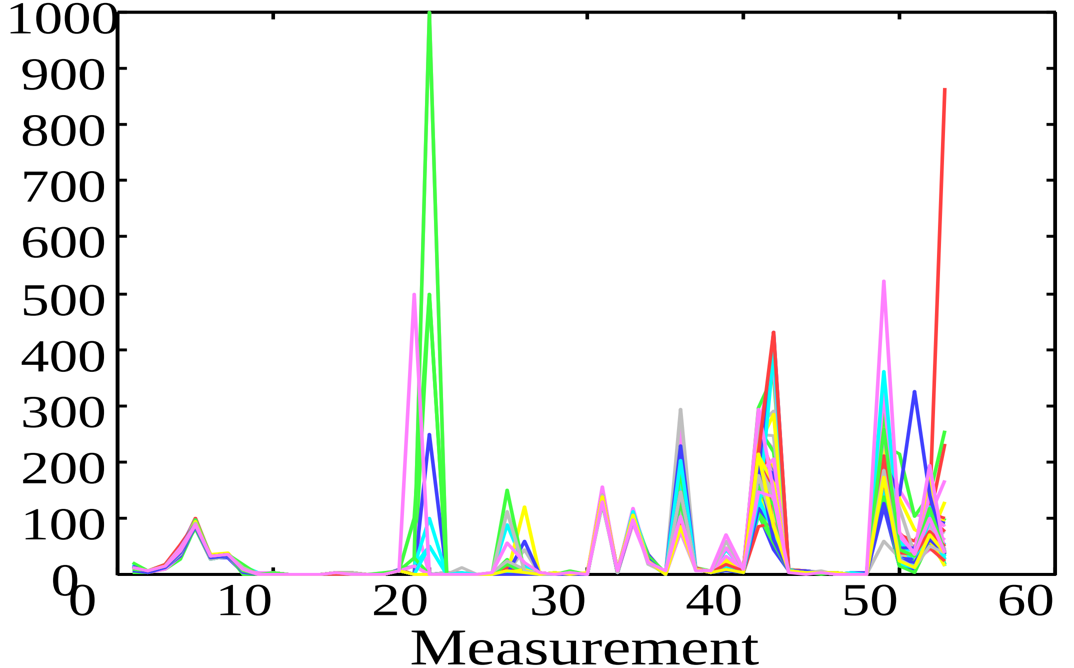
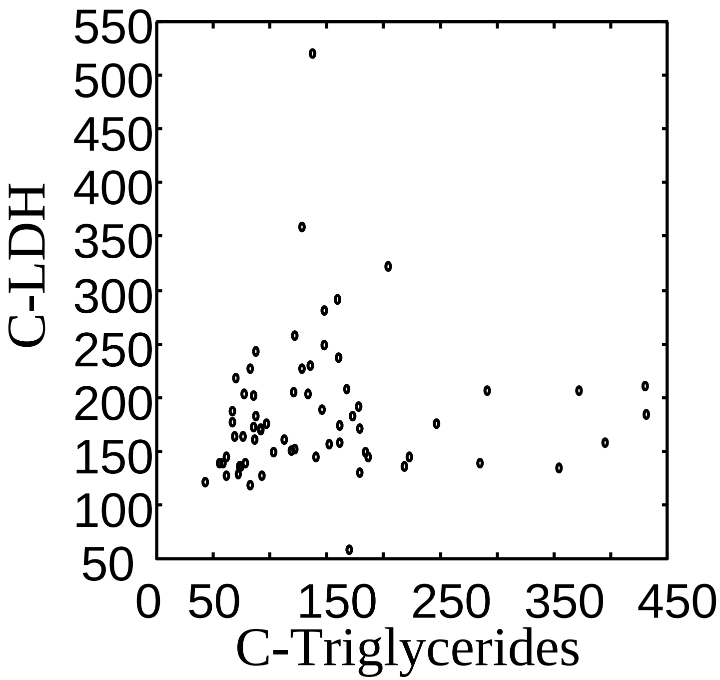
Matrix format (65x53)



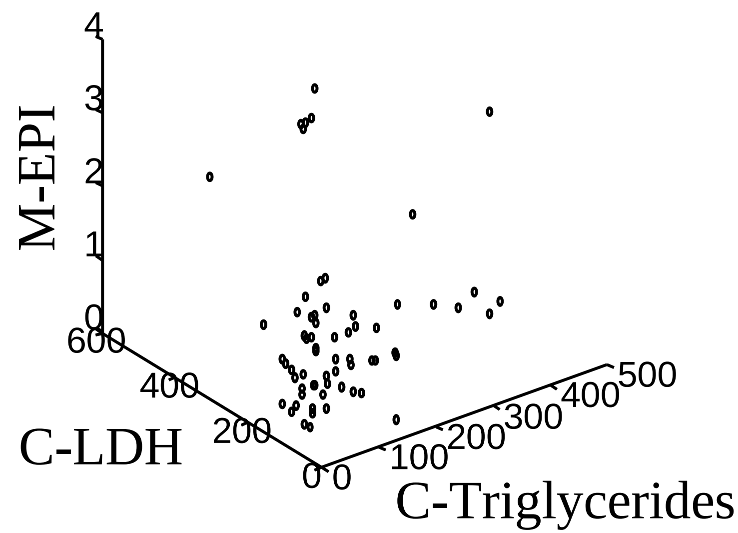
Difficult to compare the different patients...

举例
Dimensionality Reduction Example

Bi-variate



Tri-variate



How can we visualize the other variables???

… difficult to see in 4 or higher dimensional spaces...

举例
Dimensionality Reduction Example

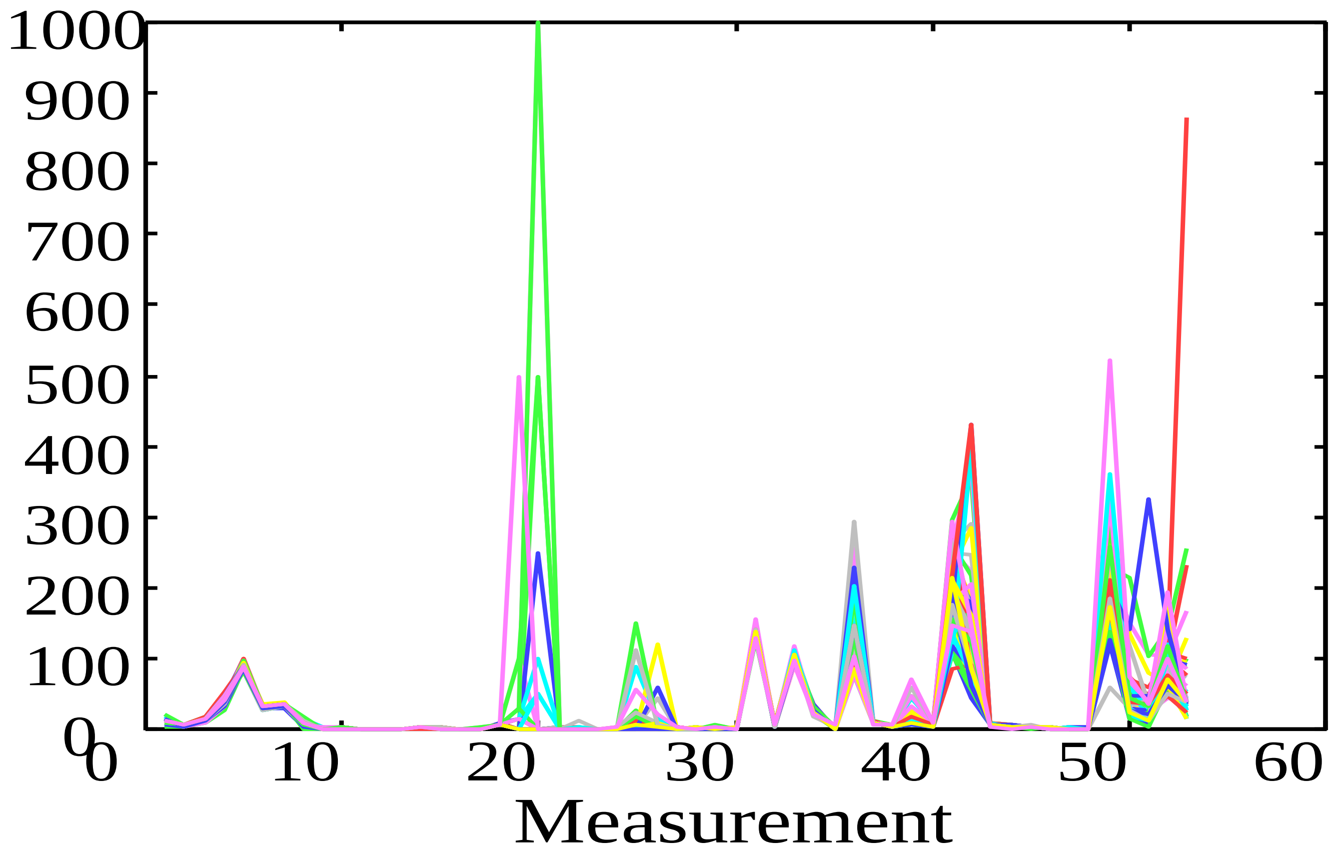
- Is there a representation better than the coordinate axes?
- Is it really necessary to show all the 53 dimensions?

举例
Dimensionality Reduction Example

- Is there a representation better than the coordinate axes?
- Is it really necessary to show all the 53 dimensions?
  - discard low significance dimensions
  - Get compact description (what if there are strong correlations between the features？)

举例
Dimensionality Reduction Example

- Spectral format (65 pictures, one for each person)



Difficult to compare the different subjects...

举例
Dimensionality Reduction Example

- Is there a representation better than the coordinate axes?
- Is it really necessary to show all the 53 dimensions?
  - discard low significance dimensions
  - Get compact description (what if there are strong correlations between the features？)

- How could we find
  the *smallest* subspace of the 53-D space that
  keeps the *most information* about the original data?

举例
Dimensionality Reduction Example

- Is there a representation better than the coordinate axes?
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- How could we find
  the *smallest* subspace of the 53-D space that
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- A solution: Principal Component Analysis

举例
Dimensionality Reduction Example

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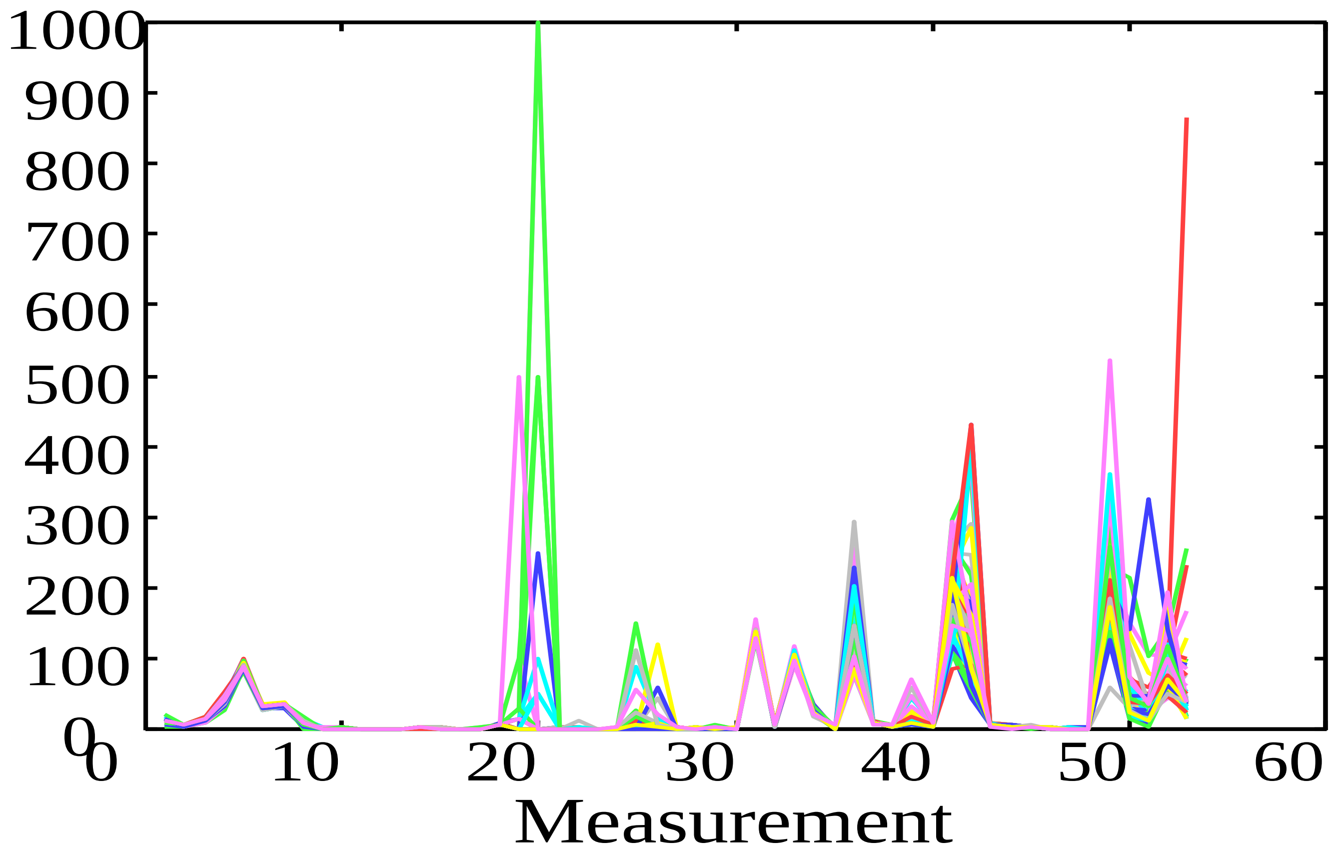
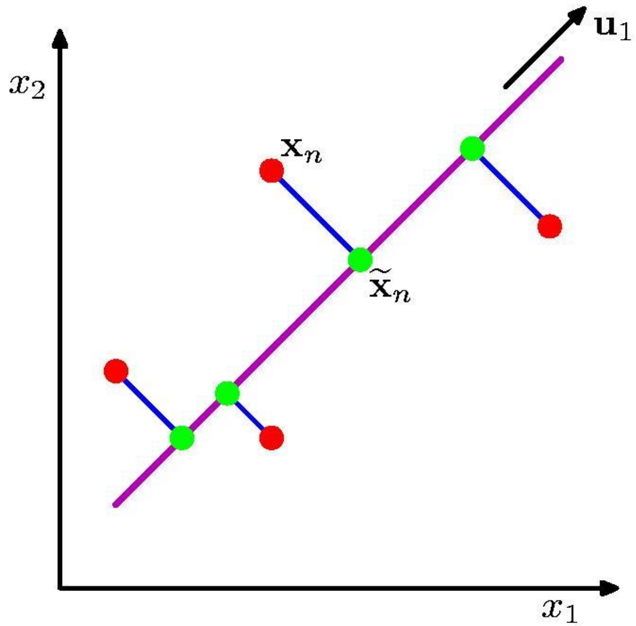


Difficult to compare the different subjects...

主成分分析
Principle Component Analysis

PCA:



Orthogonal projection of data onto lower-dimension linear space that...

- maximizes variance of projected data (purple line)
- minimizes mean squared distance between data point and projections (sum of blue lines)

主成分分析
Principle Component Analysis

- **Data Preparation**:
  - Standardize the data if the features have different units or scales. (centering, scaling)

- **Compute and Select Principal Components**:
  - Iterative Algorithms(For very large datasets, can find the main components without computing the full covariance matrix)
  - Covariance Matrix (data dimensions are not excessively high)
  - Singular Value Decomposition (can handle matrices of any size)

- **Transform the Original Dataset**:
  - Project the original dataset into the new space formed by the selected principal components to obtain the reduced-dimensionality data representation.

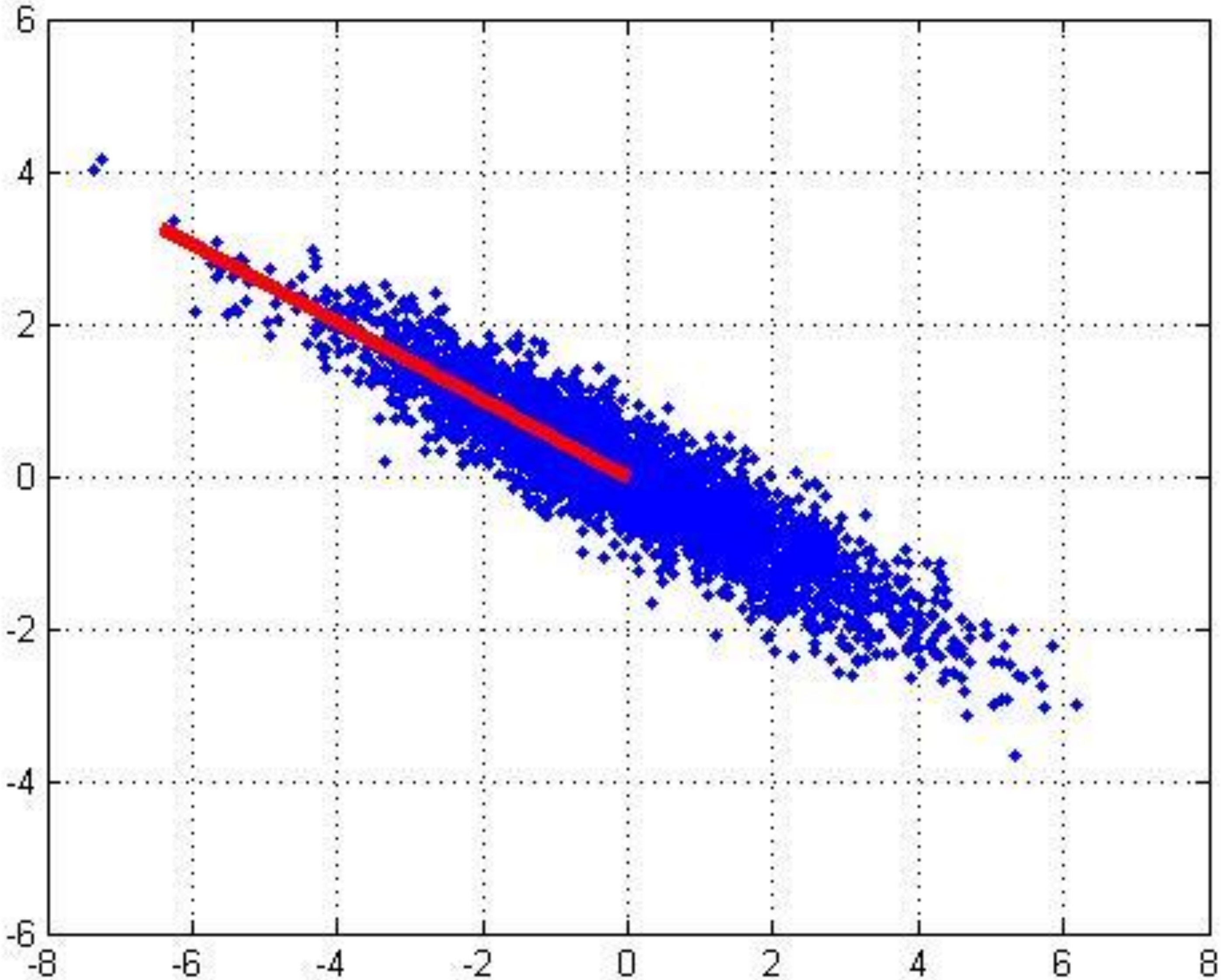
主成分分析算法一（顺序求解）
PCA algorithm I (sequential)

- **Vectors** originating from the center of mass

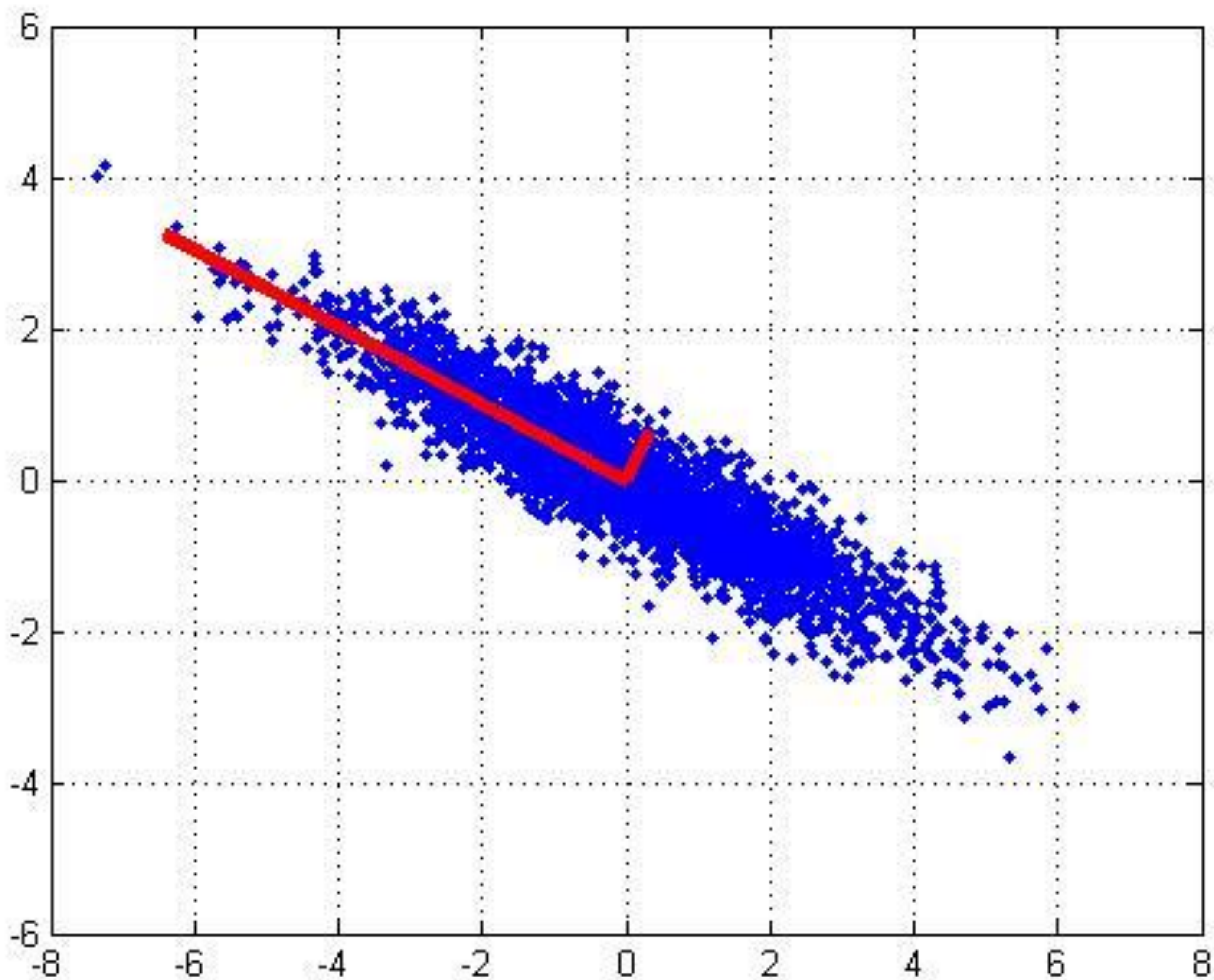
- Principal component #1 points in the direction of the **largest variance**.

- Each subsequent principal component…
  - is **orthogonal** to the previous ones, and
  - points in the directions of the **largest variance of the residual subspace**

1st PCA axis



2^{nd} PCA axis



主成分分析算法一（顺序求解）
PCA algorithm I (sequential)

Given the **centered** data $\{\mathbf{x}_1, \ldots, \mathbf{x}_m\}$, compute the principal vectors:

$$\mathbf{w}_1 = \arg\max_{\|\mathbf{w}\|=1} \frac{1}{m} \sum_{i=1}^{m} \{(\mathbf{w}^T \mathbf{x}_i)^2\}$$

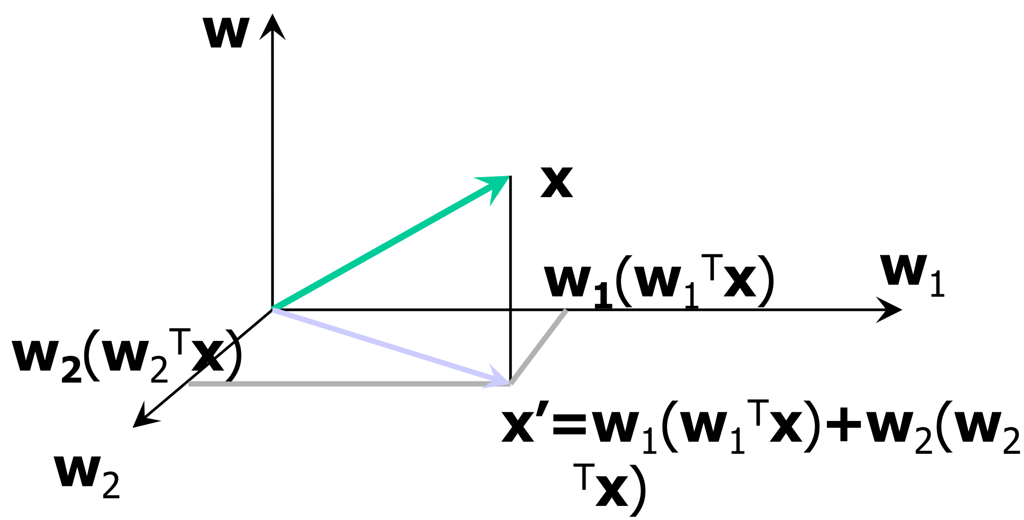
1st PCA vector

$$\mathbf{w}_k = \arg\max_{\|\mathbf{w}\|=1} \frac{1}{m} \sum_{i=1}^{m} \{[\mathbf{w}^T (\mathbf{x}_i - \underbrace{\sum_{j=1}^{k-1} \mathbf{w}_j \mathbf{w}_j^T \mathbf{x}_i}_{\mathbf{x}' \text{ PCA reconstruction}})]^2\}$$

kth PCA vector

can be obtained by gradient ascent

We maximize the variance of the projection in the residual subspace



主成分分析算法二（特征值分解）PCA algorithm II(sample covariance matrix)

- Given data $\{\mathbf{x}_1, \ldots, \mathbf{x}_m\}$, compute covariance matrix $\boldsymbol{\Sigma}$

$$\boxed{\Sigma = \frac{1}{m}\sum_{i=1}^{m}(\mathbf{x}_i - \overline{\mathbf{x}})(\mathbf{x} - \overline{\mathbf{x}})^T}$$

where

$$\boxed{\overline{\mathbf{x}} = \frac{1}{m}\sum_{i=1}^{m}\mathbf{x}_i}$$

(Optional) Standardization

- Eigenvalue decomposition $\Sigma = U \Lambda U^T$

- PCA basis vectors = the eigenvectors of Σ

- Larger eigenvalues $\Rightarrow$ more important eigenvectors

主成分分析算法二（特征值分解）PCA algorithm II(sample covariance matrix)

PCA algorithm($\mathbf{X}$, k): top k eigenvalues/eigenvectors

% $\mathbf{X}$ = N × m data matrix, each data point $\mathbf{x}_i$ = column vector, i=1..m

- $\mathbf{X} \leftarrow$ subtract mean $\overline{X}$ from each column vector $\mathbf{x}_i$ in $\mathbf{X}$
- $\mathbf{\Sigma} \leftarrow \mathbf{X}\mathbf{X}^T$ …compute covariance matrix of $\mathbf{X}$
- $\{ \lambda_i, \mathbf{u}_i \}_{i=1..N}$ = eigenvectors/eigenvalues of $\mathbf{\Sigma}$
  ... Sort the eigenvalues in descending order $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{N'}$
- Return $\{ \lambda_i, \mathbf{u}_i \}_{i=1..k}$
  % top k principle components

主成分分析算法三（奇异值分解）
PCA algorithm III (SVD of the data matrix)

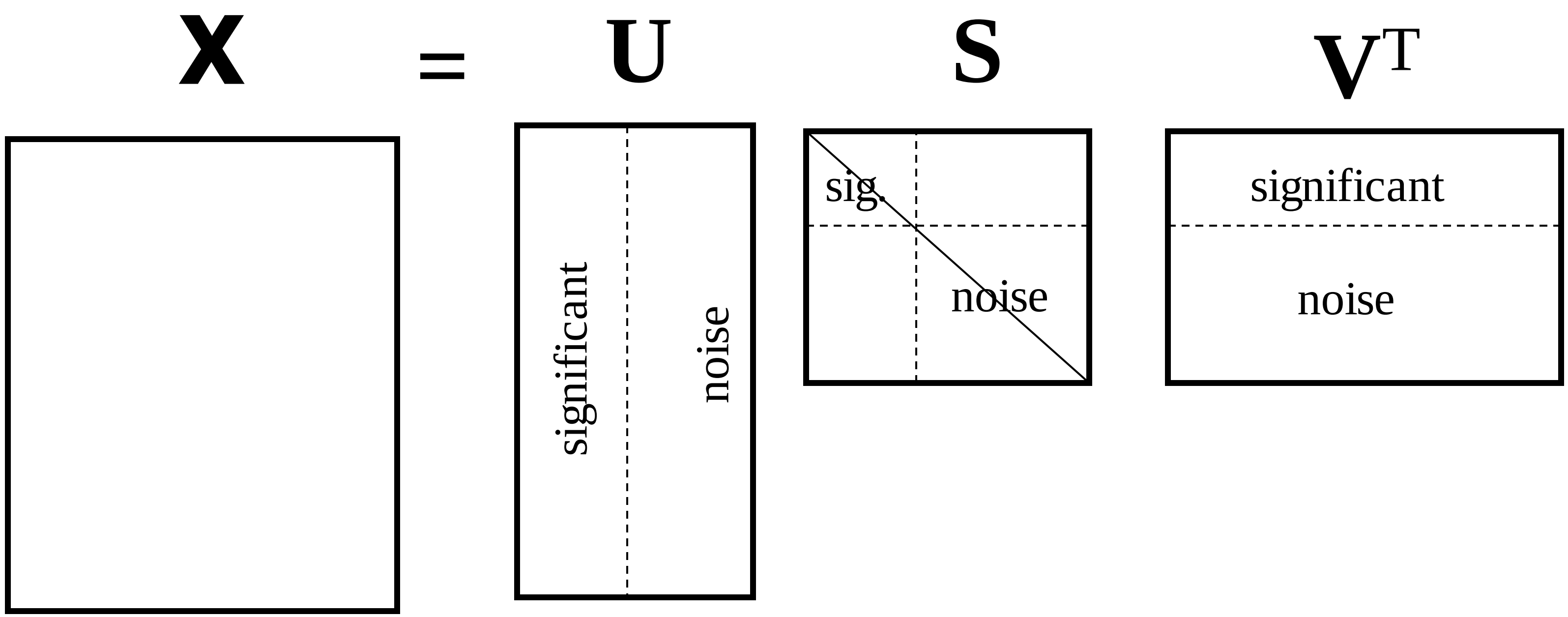
Singular Value Decomposition of the **centered** data matrix **X**.

$$\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_m] \in \mathbb{R}^{N\times m},$$

m: number of instances,

N: dimension

$$X_{N\times m} = U_{N\times N} S_{N\times m} V_{m\times m}^T \approx U_{N\times k} S_{k\times k} V_{k\times m}^T$$



主成分分析算法三（奇异值分解）PCA algorithm III (SVD of the data matrix)

- **Columns of U**
  - the principal vectors, { $\mathbf{u}^{(1)}$, …, $\mathbf{u}^{(k)}$ }
  - orthogonal and has unit norm – so $U^TU = I$
  - Can reconstruct the data using linear combinations of { $\mathbf{u}^{(1)}$, …, $\mathbf{u}^{(k)}$ }
- **Matrix S**
  - Diagonal
  - Shows importance of each eigenvector
- **Columns of V^T**
  - The coefficients for reconstructing the samples

主成分个数
How many PCA components

Problem: How many to keep?

Many criteria.

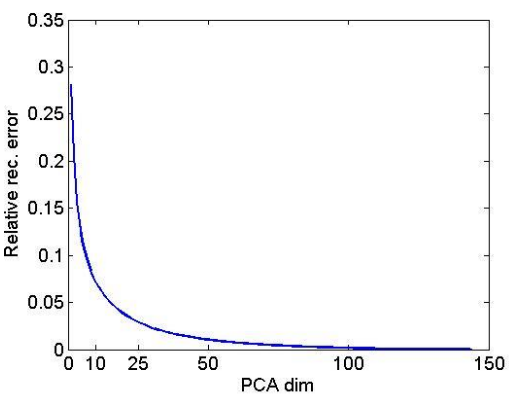
e.g. % total data variance:

$$\max(m) \ni \frac{\sum_{i=(m+1):n} \lambda_i}{\sum_{i=1:n} \lambda_i} < \varepsilon$$

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
ratio	0.45	0.18	0.13	0.12	0.07	0.04

Using 4 components is good enough

L_2 error and PCA dim



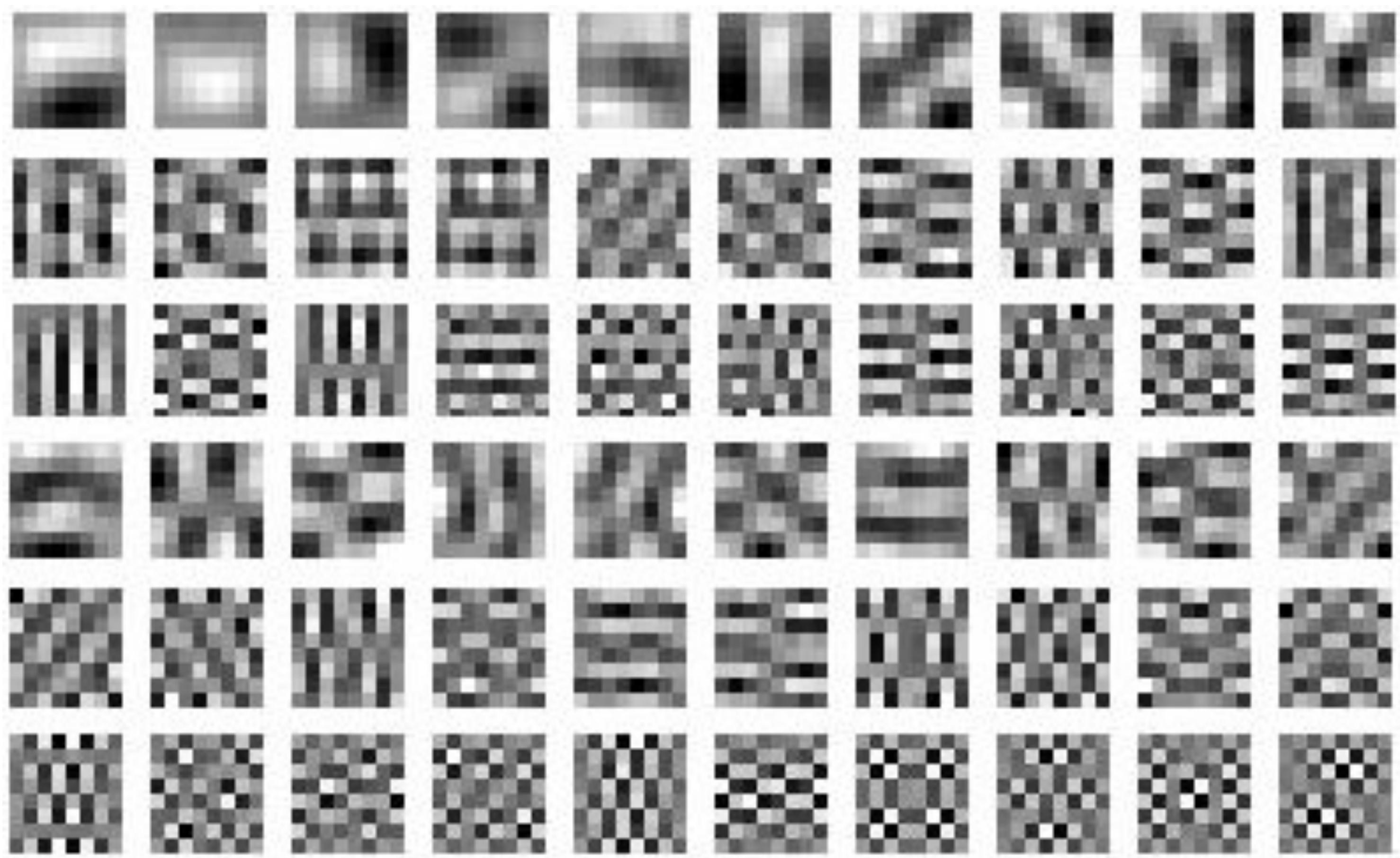
主成分分析用于图像压缩
PCA application: Image Compression

Original Image

- Divide the original 372x492 image into patches:
  - Each patch is an instance that contains 12x12 pixels on a grid
- View each as a 144-D vector

主成分分析用于图像压缩
PCA application: Image Compression

60 most important eigenvectors：



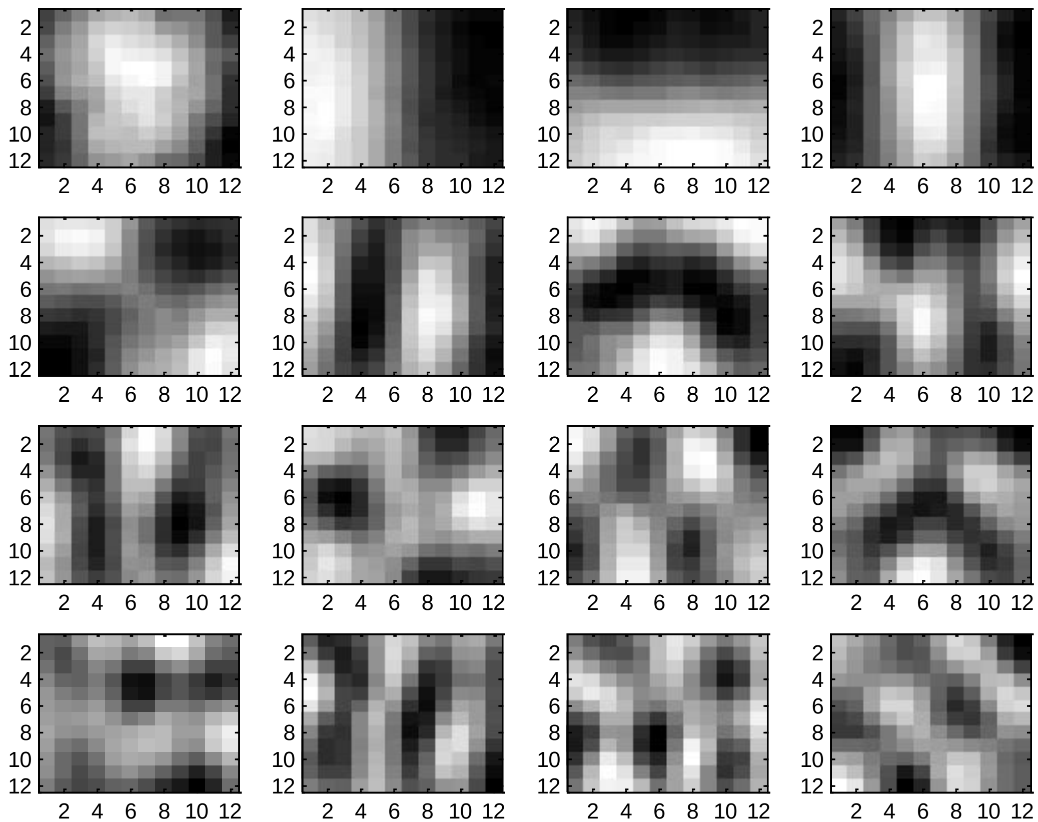
Looks like the discrete cosine bases of JPG!...

主成分分析用于图像压缩
PCA application: Image Compression

PCA compression: 144D) 60D

主成分分析用于图像压缩
PCA application: Image Compression

16 most important eigenvectors：

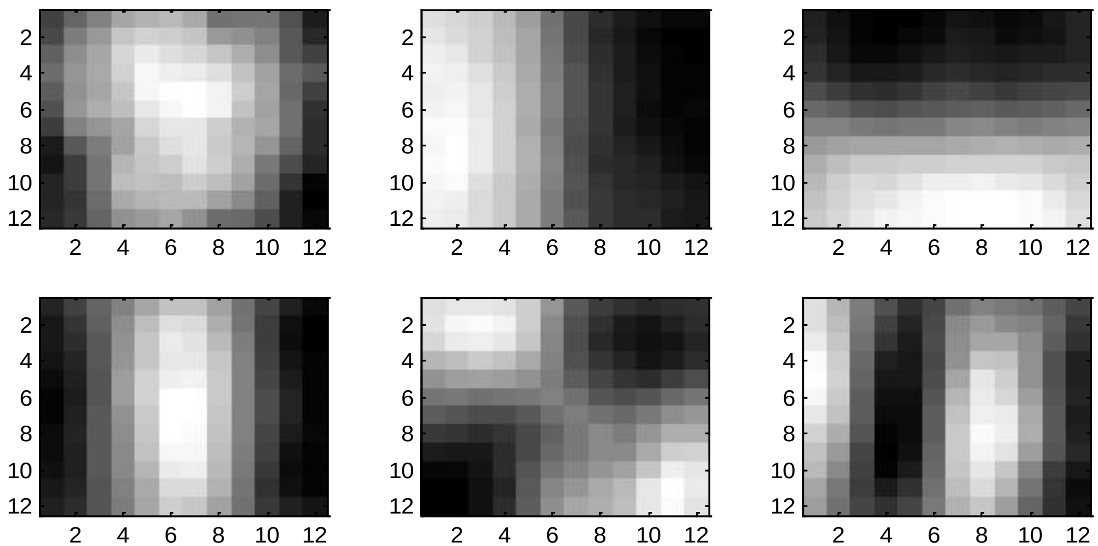


主成分分析用于图像压缩
PCA application: Image Compression

PCA compression: 144D) 16D

主成分分析用于图像压缩
PCA application: Image Compression

6 most important eigenvectors：

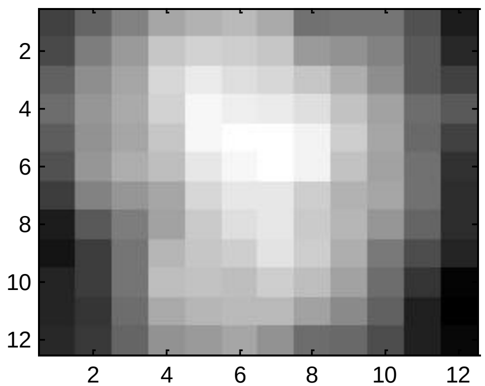


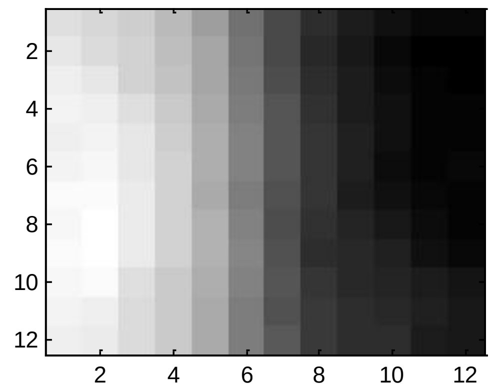
主成分分析用于图像压缩
PCA application: Image Compression

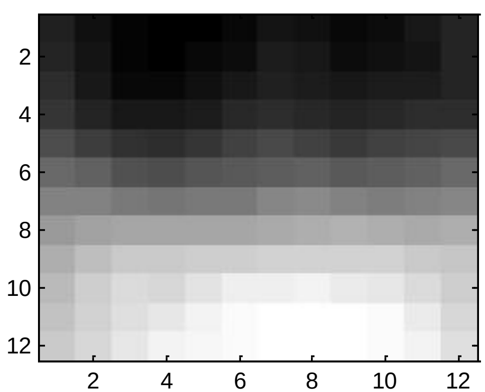
PCA compression: 144D) 6D

主成分分析用于图像压缩
PCA application: Image Compression

3 most important eigenvectors：





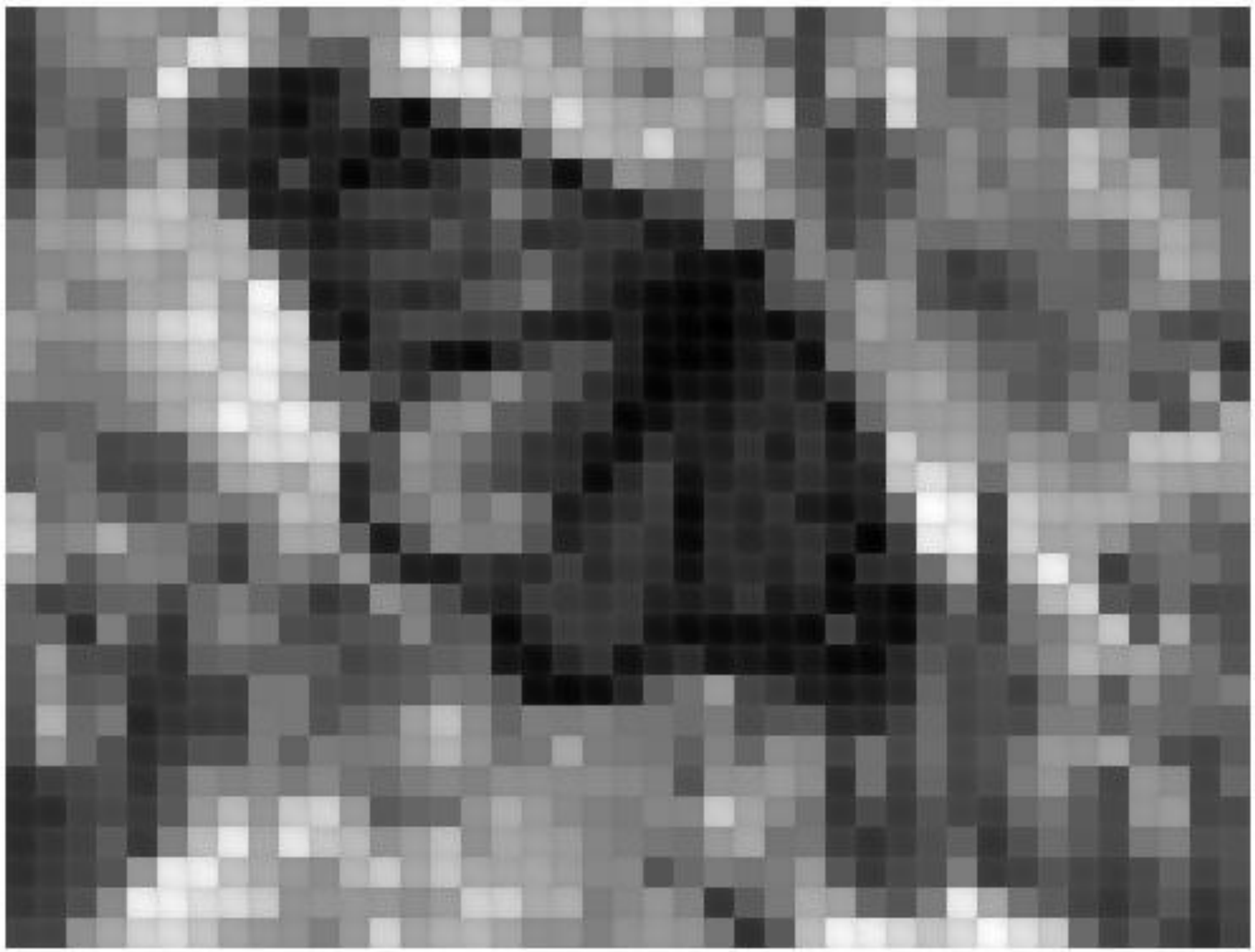


主成分分析用于图像压缩
PCA application: Image Compression

PCA compression: 144D) 3D

主成分分析用于图像压缩
PCA application: Image Compression

PCA compression: 144D) 1D



主成分分析用于滤除噪声
PCA application: Noise Filtering

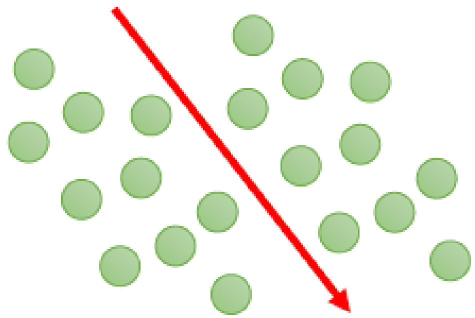
Noisy image

主成分分析用于滤除噪声
PCA application: Noise Filtering

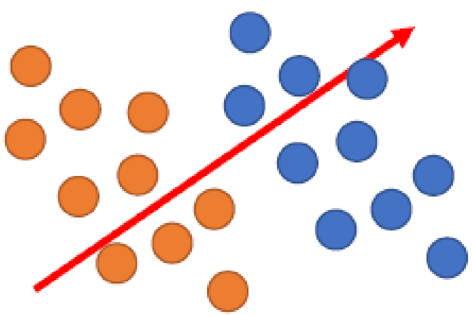
Denoised image using 15 PCA components

主成分分析弱点
Weakless of PCA

Unsupervised

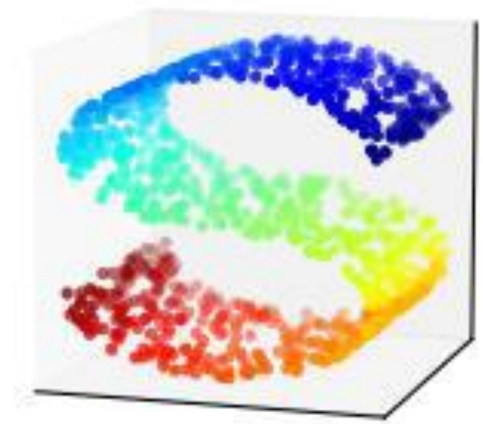


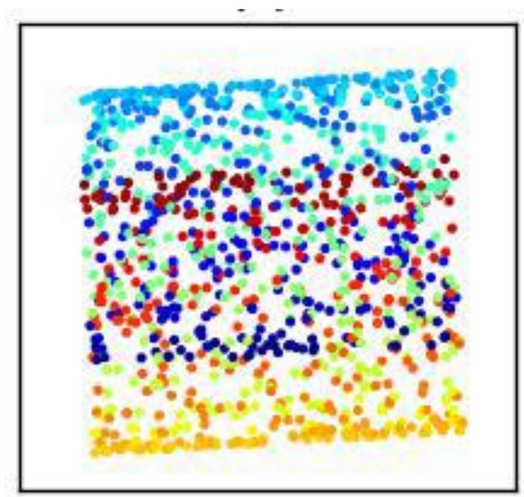
PCA



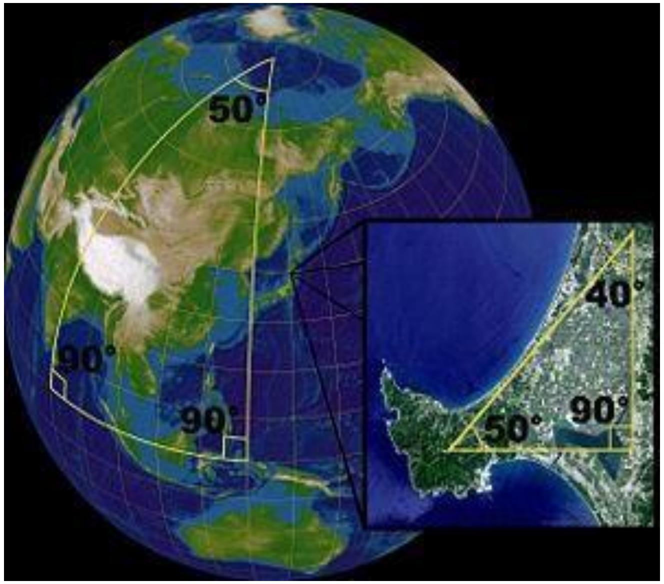
LDA

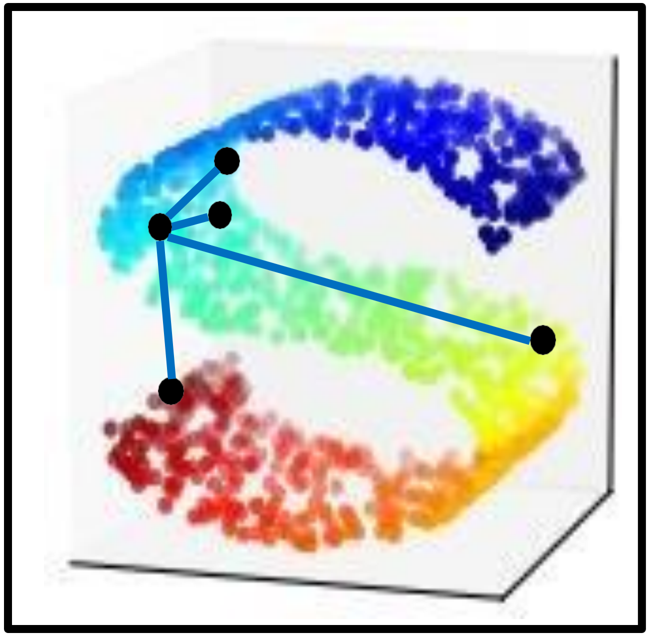
Linear:

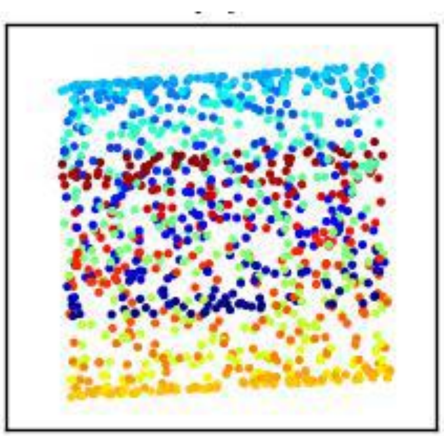




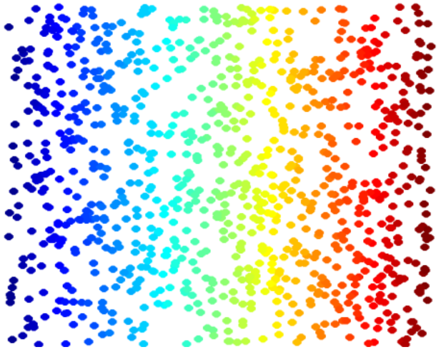
流形学习
Manifold Learning







Intrinsic Space



t−分布领域嵌入算法(t-SNE)
T-distributed Stochastic NeighborEmbedding

$$x \longrightarrow z$$

Compute similarity between all pairs of x: $S\left(x^i, x^j\right)$

$$P\left(x^j | x^i\right) = \frac{S\left(x^i, x^j\right)}{\sum_{k \neq i} S(x^i, x^k)}$$

Compute similarity between all pairs of z: $S'\left(z^i, z^j\right)$

$$Q\left(z^j | z^i\right) = \frac{S'\left(z^i, z^j\right)}{\sum_{k \neq i} S'(z^i, z^k)}$$

Find a set of z making the two distributions as close as possible

$$L = \sum_i KL\left(P\left(* | x^i\right) || Q\left(* | z^i\right)\right)$$

$$= \sum_i \sum_j P\left(x^j | x^i\right) log \frac{P\left(x^j | x^i\right)}{Q(z^j | z^i)}$$

t−分布领域嵌入算法(t-SNE)

T-distributed Stochastic NeighborEmbedding

x ⟶ z

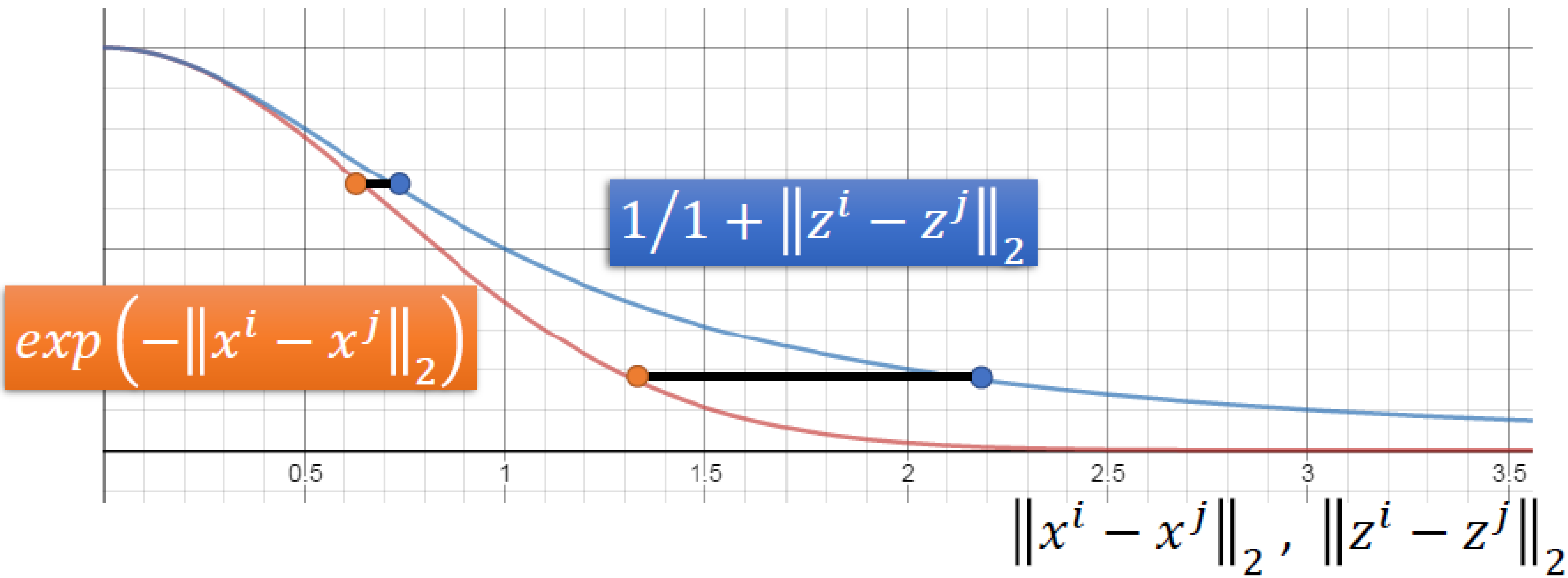
$$S(x^i, x^j) = exp\left(-\|x^i - x^j\|_2\right)$$

SNE:

$$S'(z^i, z^j) = exp\left(-\|z^i - z^j\|_2\right)$$

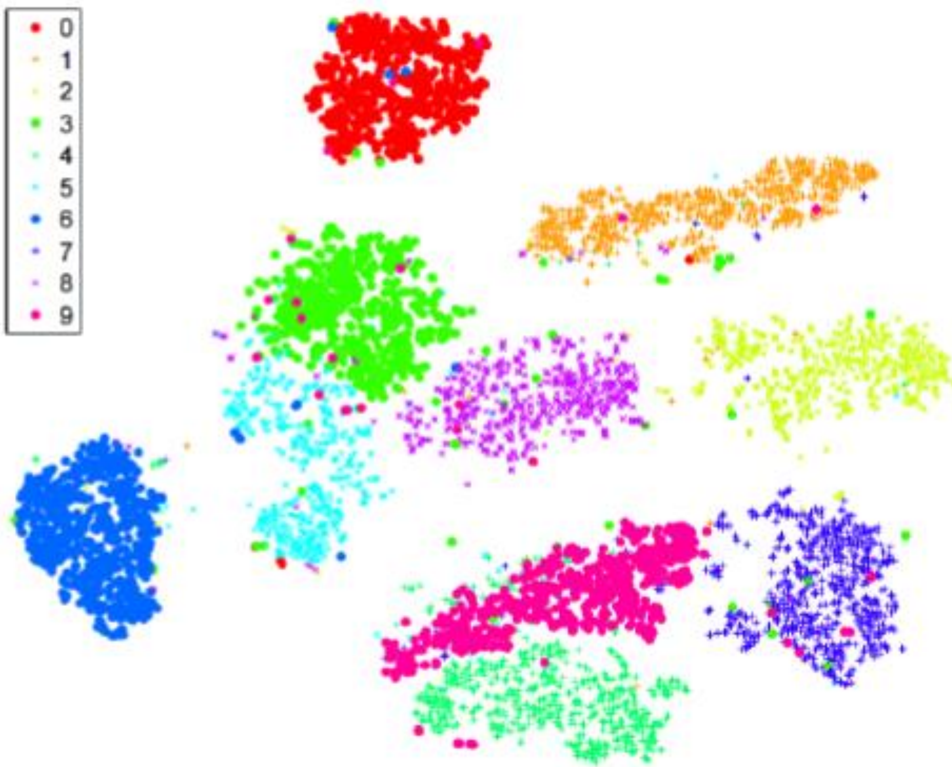
t-SNE:

$$S'(z^i, z^j) = 1/1 + \|z^i - z^j\|_2$$



t−分布领域嵌入算法(t-SNE)
T-distributed Stochastic NeighborEmbedding

- Good at visualization

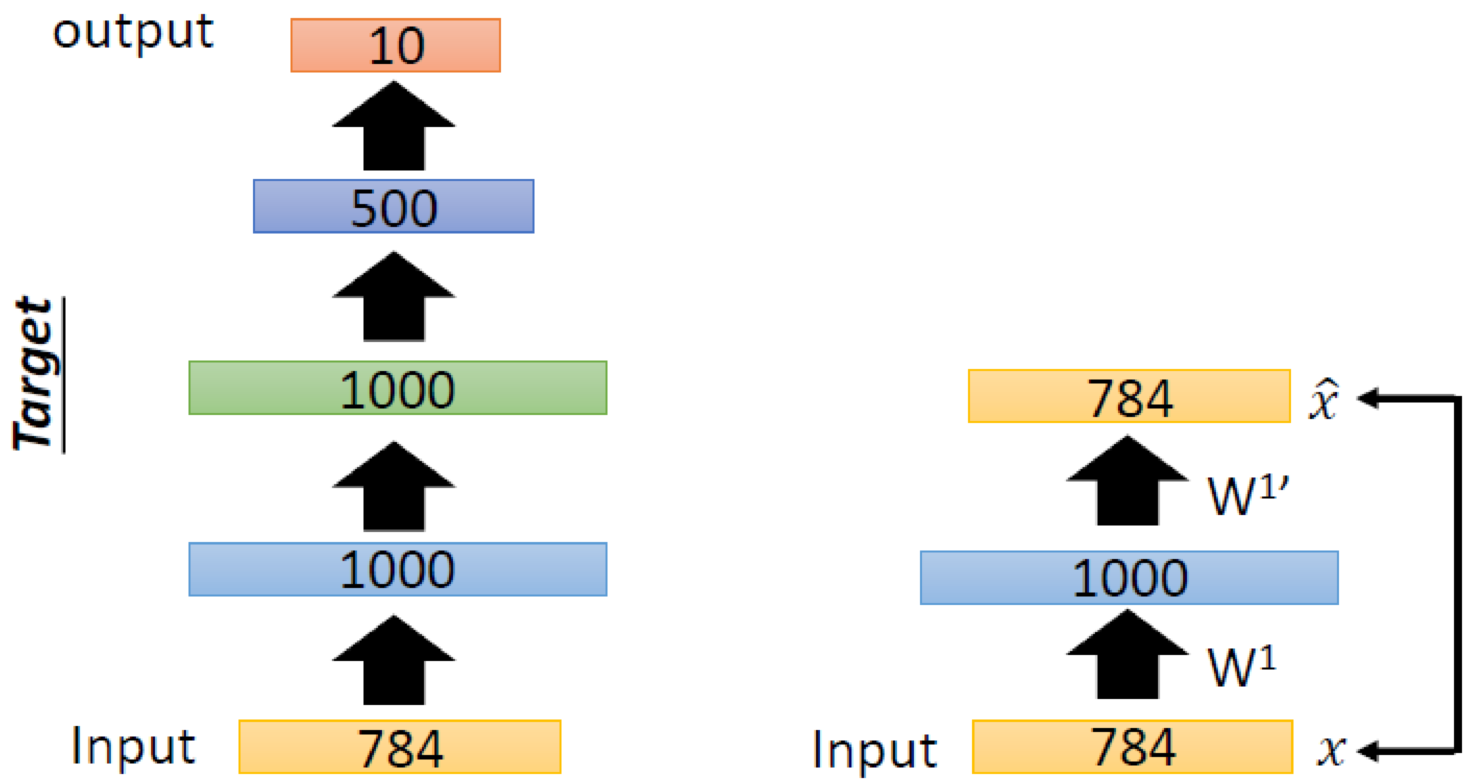


t-SNE on MNIST

t-SNE on COIL-20

自编码器
Auto-encoder

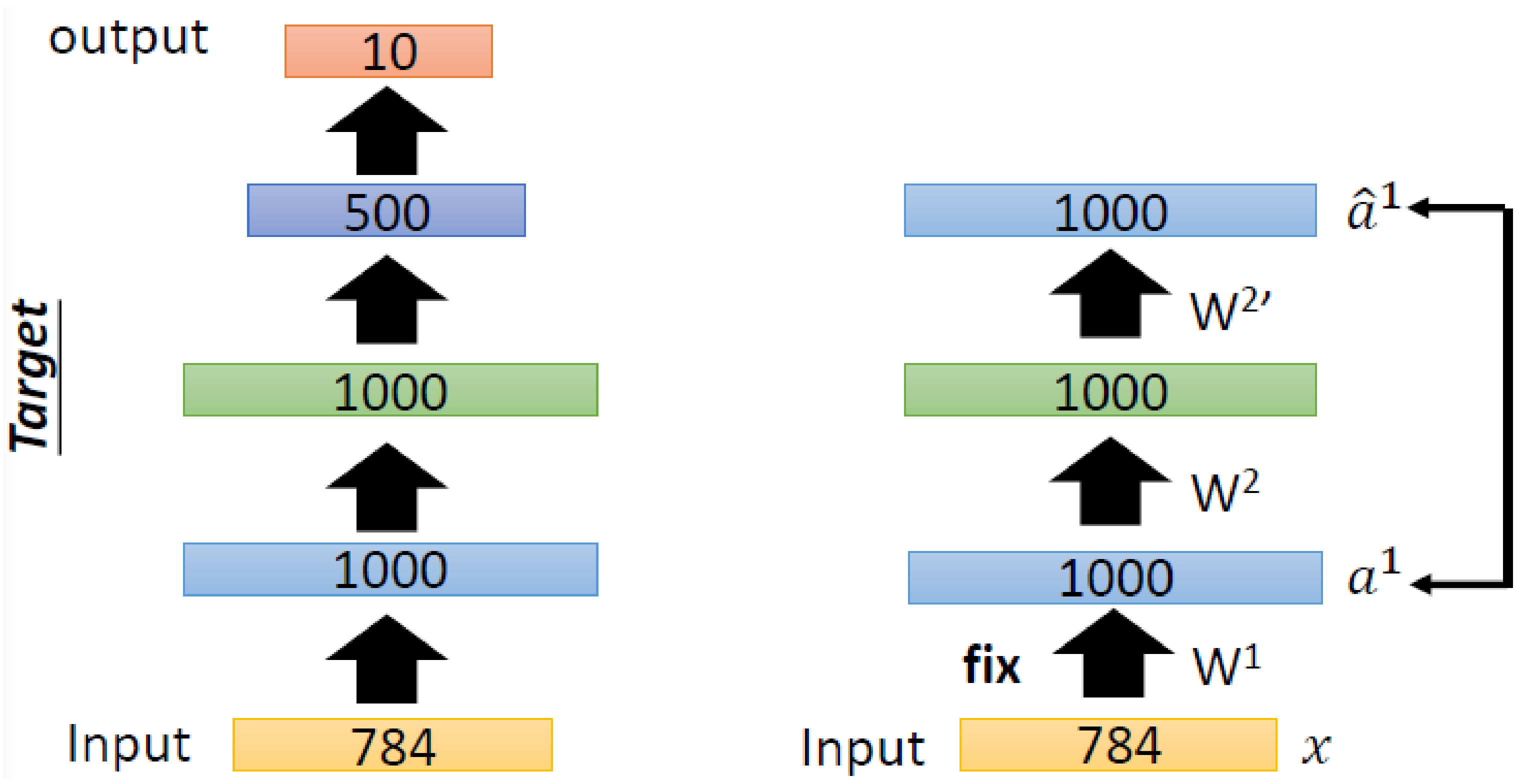
– Greedy Layer-wise Pre-training again



Reference: Hinton, Geoffrey E., and Ruslan R. Salakhutdinov. "Reducing the dimensionality of data with neural networks." *Science* 313.5786 (2006): 504-507

自编码器
Auto-encoder

– Greedy Layer-wise Pre-training again



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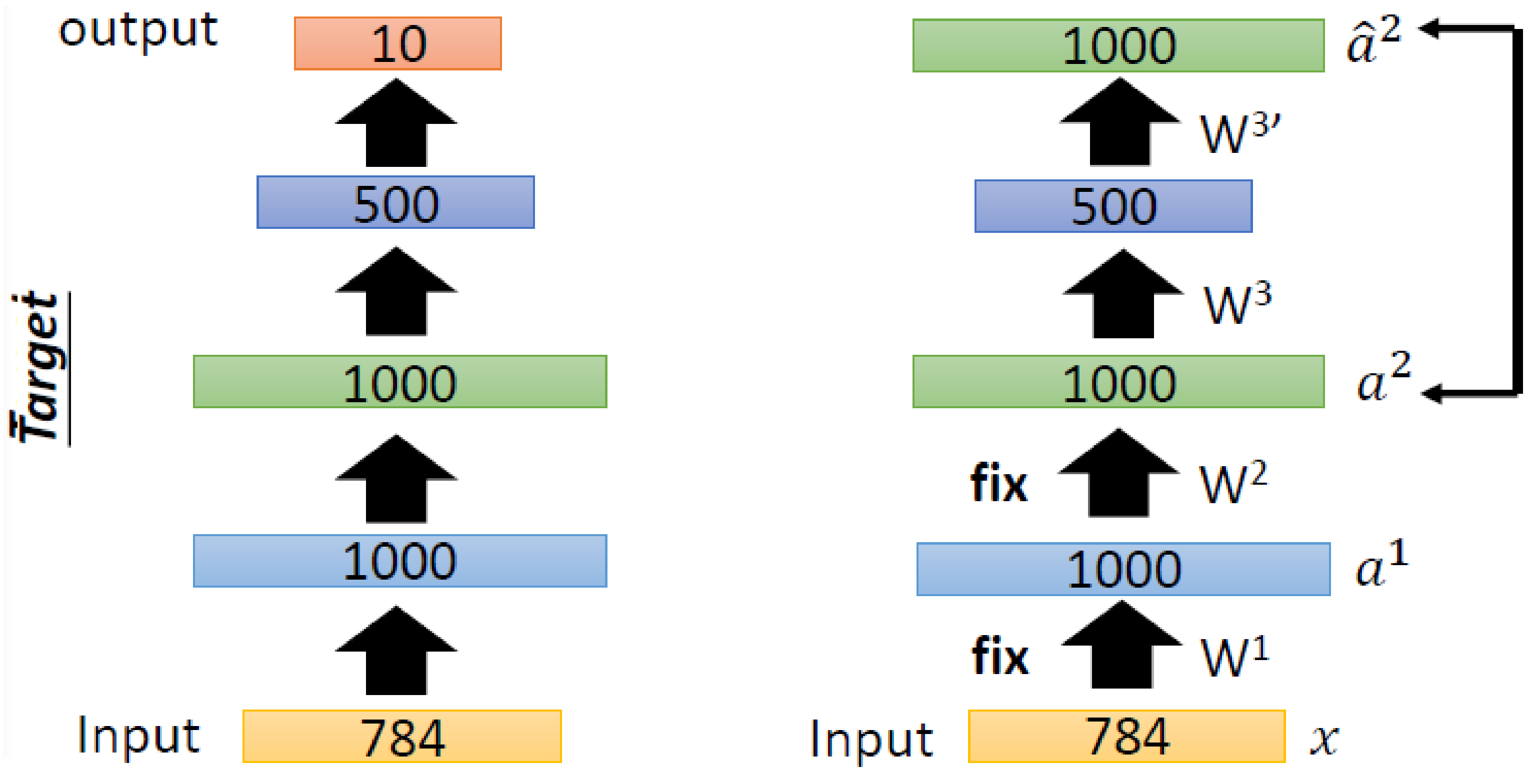
自编码器
Auto-encoder

– Greedy Layer-wise Pre-training again



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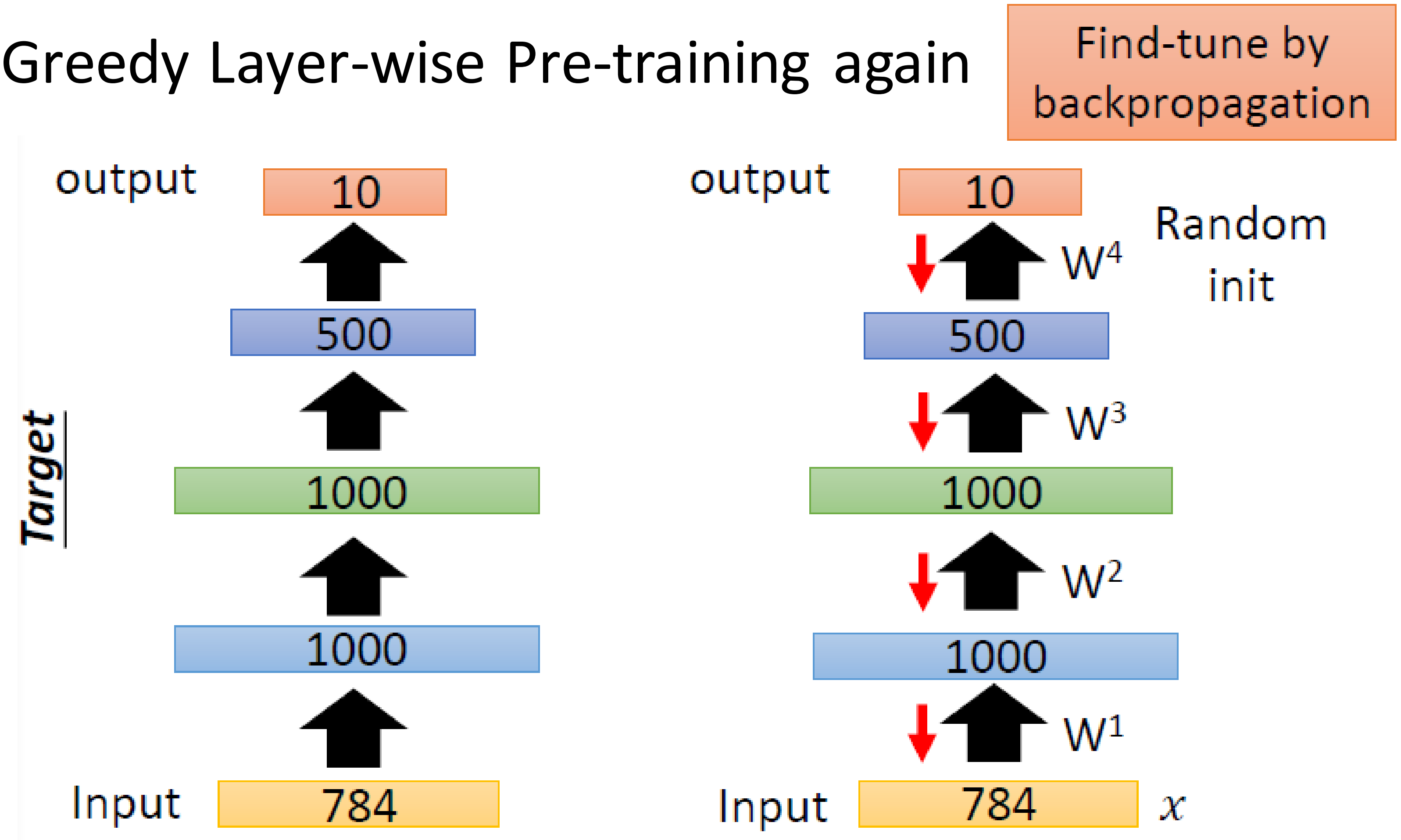
自编码器
Auto-encoder

– Greedy Layer-wise Pre-training again

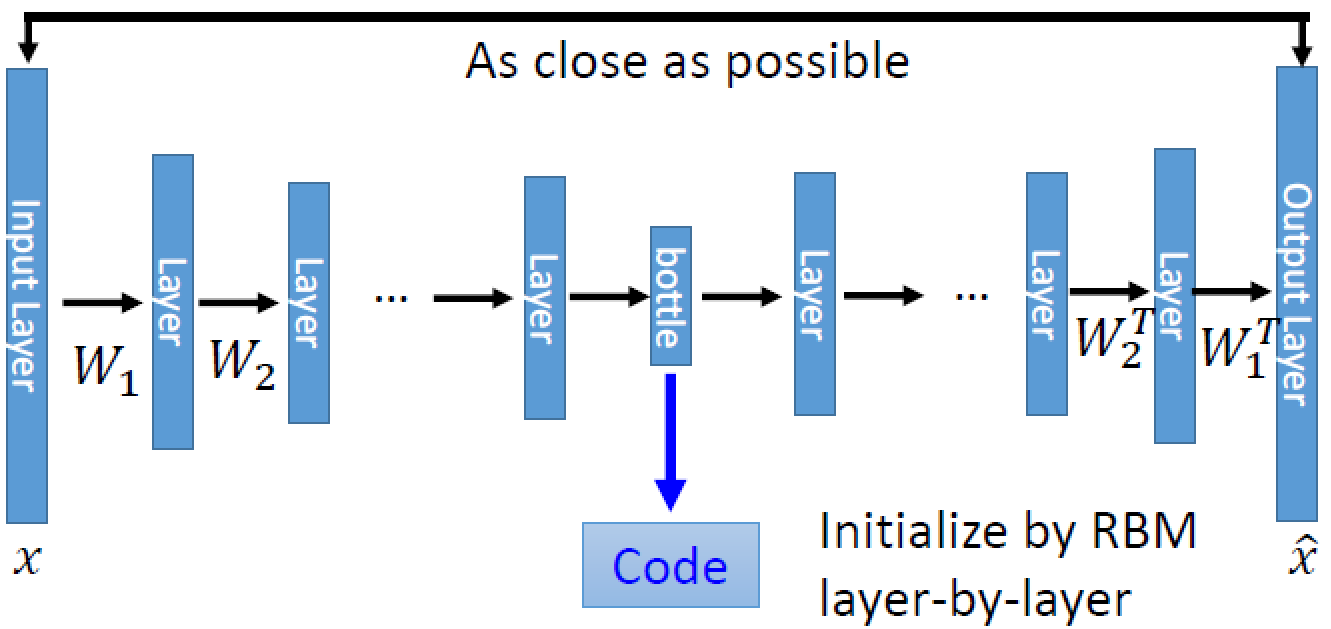


Reference: Hinton, Geoffrey E., and Ruslan R. Salakhutdinov. "Reducing the dimensionality of data with neural networks." *Science* 313.5786 (2006): 504-507

自编码器
Auto-encoder

– An auto-encoder is an artificial neural net used for unsupervised learning of efficient codings.

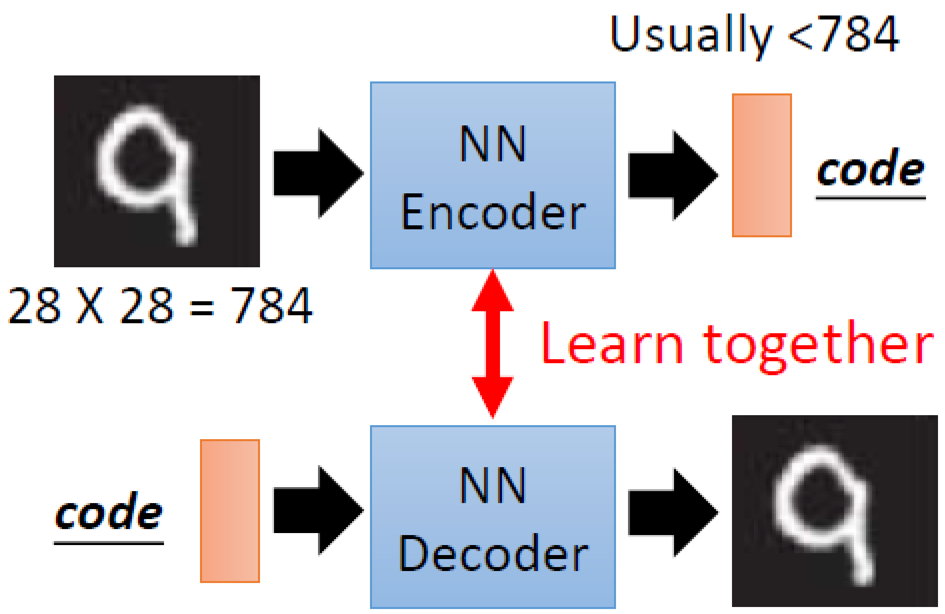
– Greedy Layer-wise Pre-training again



use neural networks to recover the data

Reference: Hinton, Geoffrey E., and Ruslan R. Salakhutdinov. "Reducing the dimensionality of data with neural networks." *Science* 313.5786 (2006): 504-507

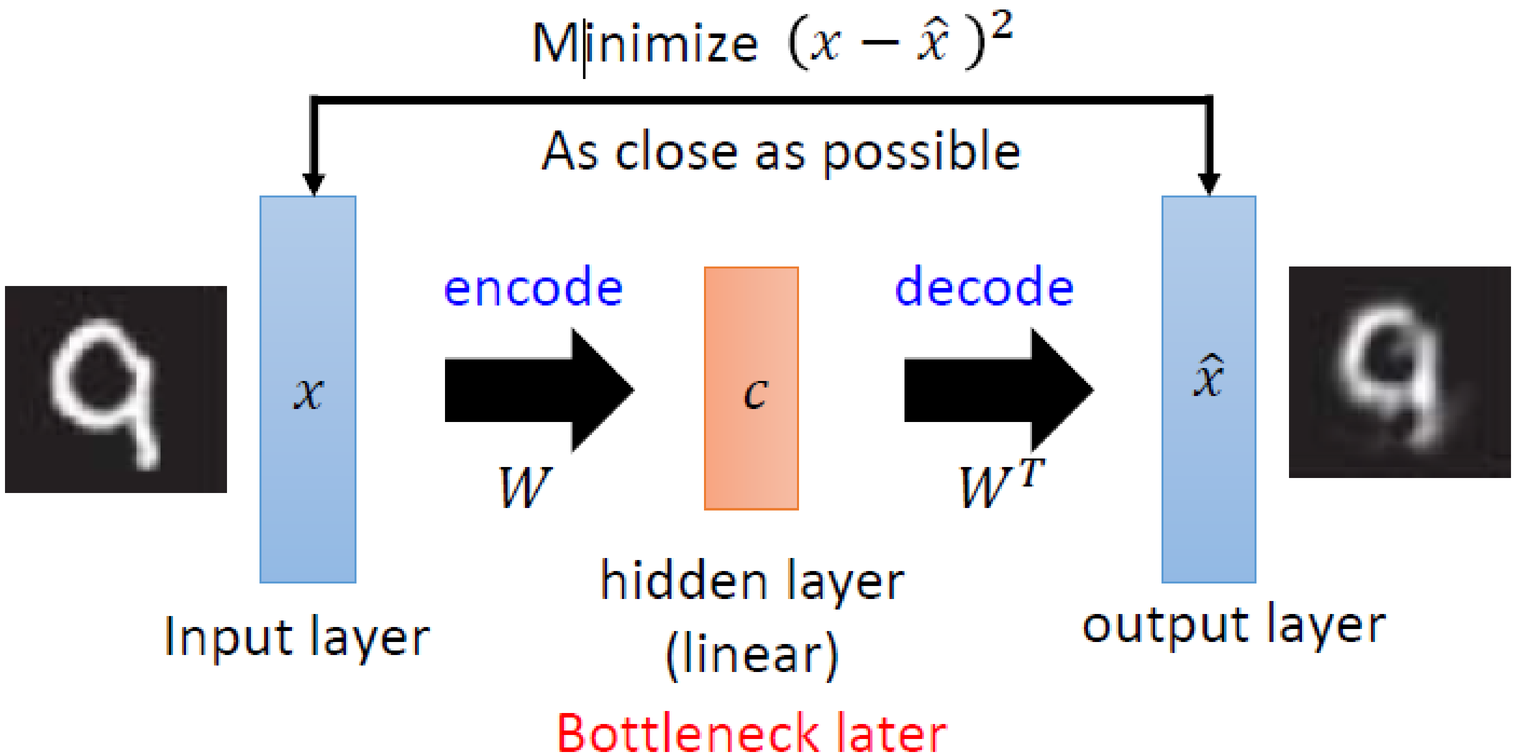
学习自编码器
Learning Auto-encoder



Compact representation of the input object

Can reconstruct the original object

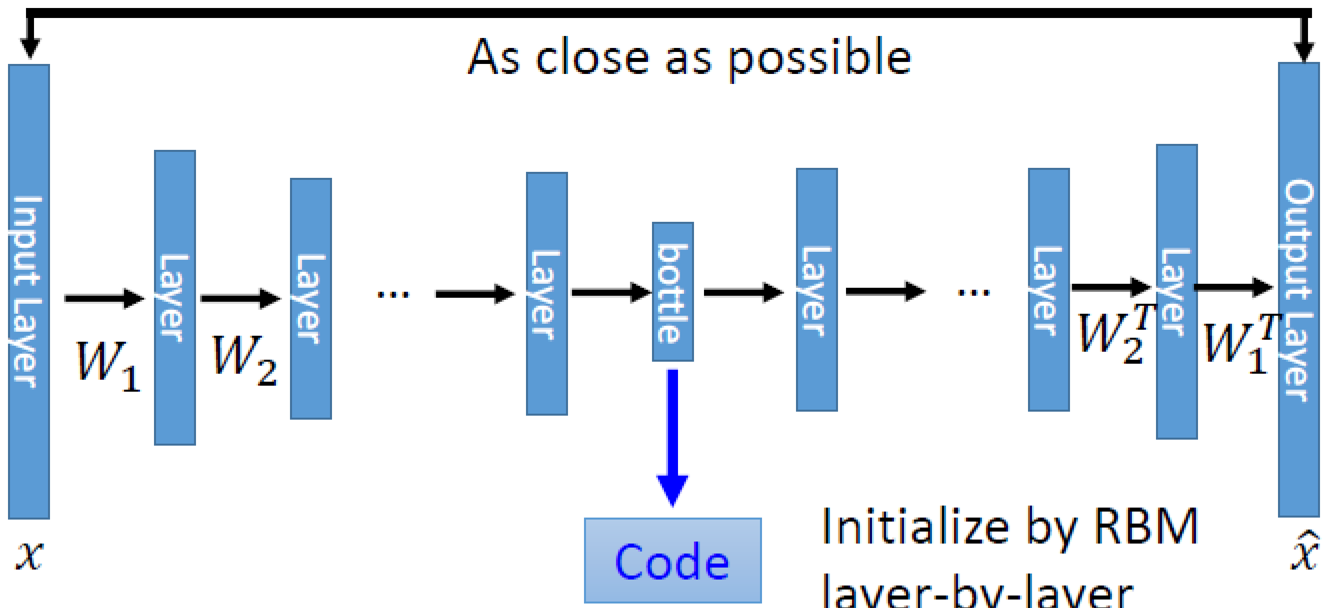
学习PCA
Learning PCA



Output of the hidden layer is the code

深度自编码器
Deep Auto-encoder

The auto-encoder can be deep

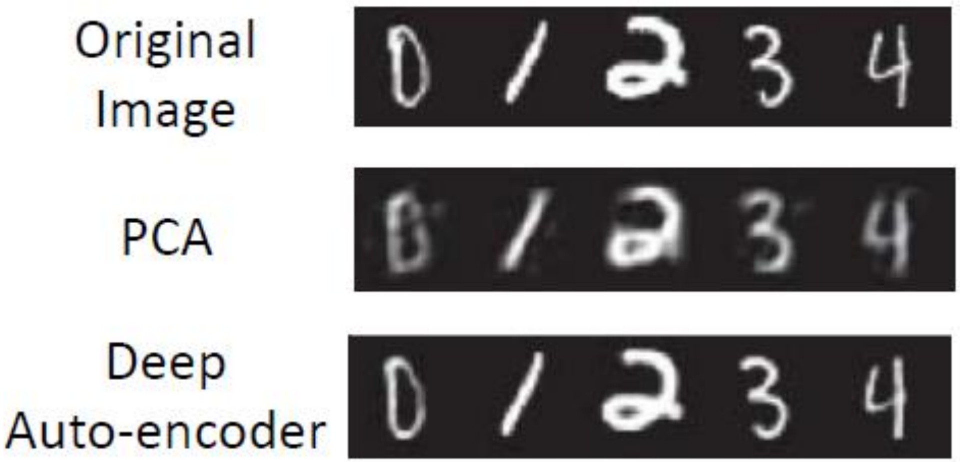


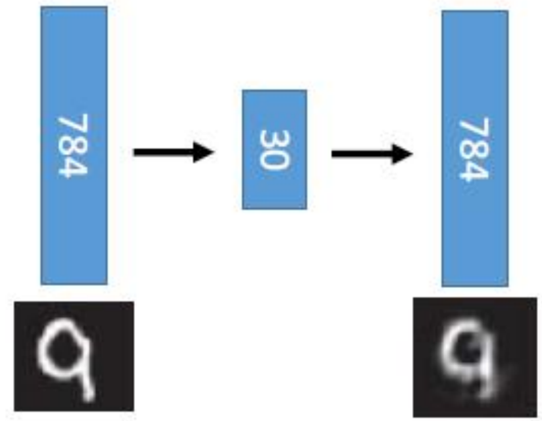
Initialize by RBM layer-by-layer

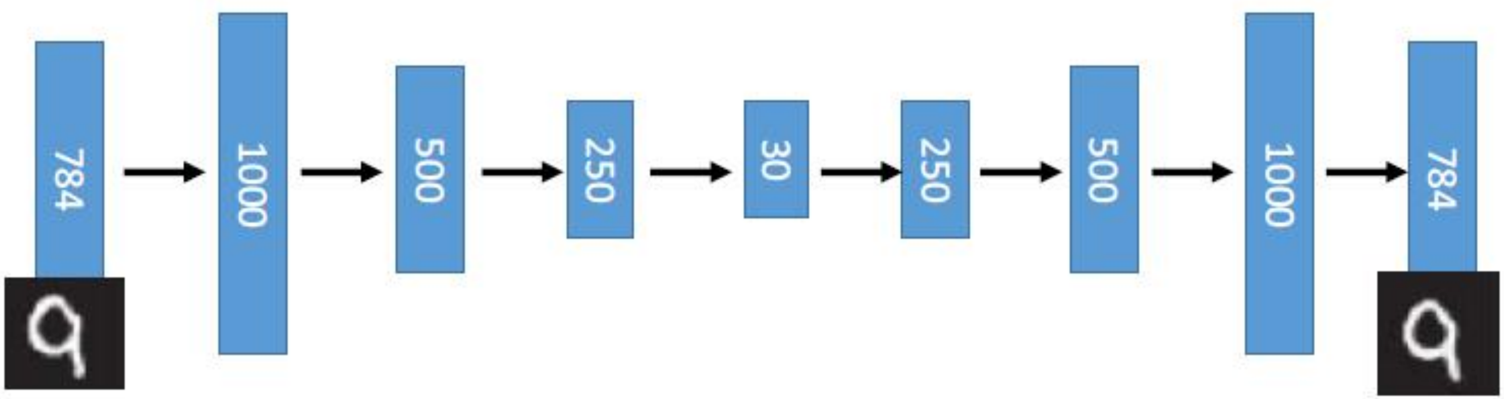
use neural networks to recover the data

Reference: Hinton, Geoffrey E., and Ruslan R. Salakhutdinov. "Reducing the dimensionality of data with neural networks." *Science* 313.5786 (2006): 504-507

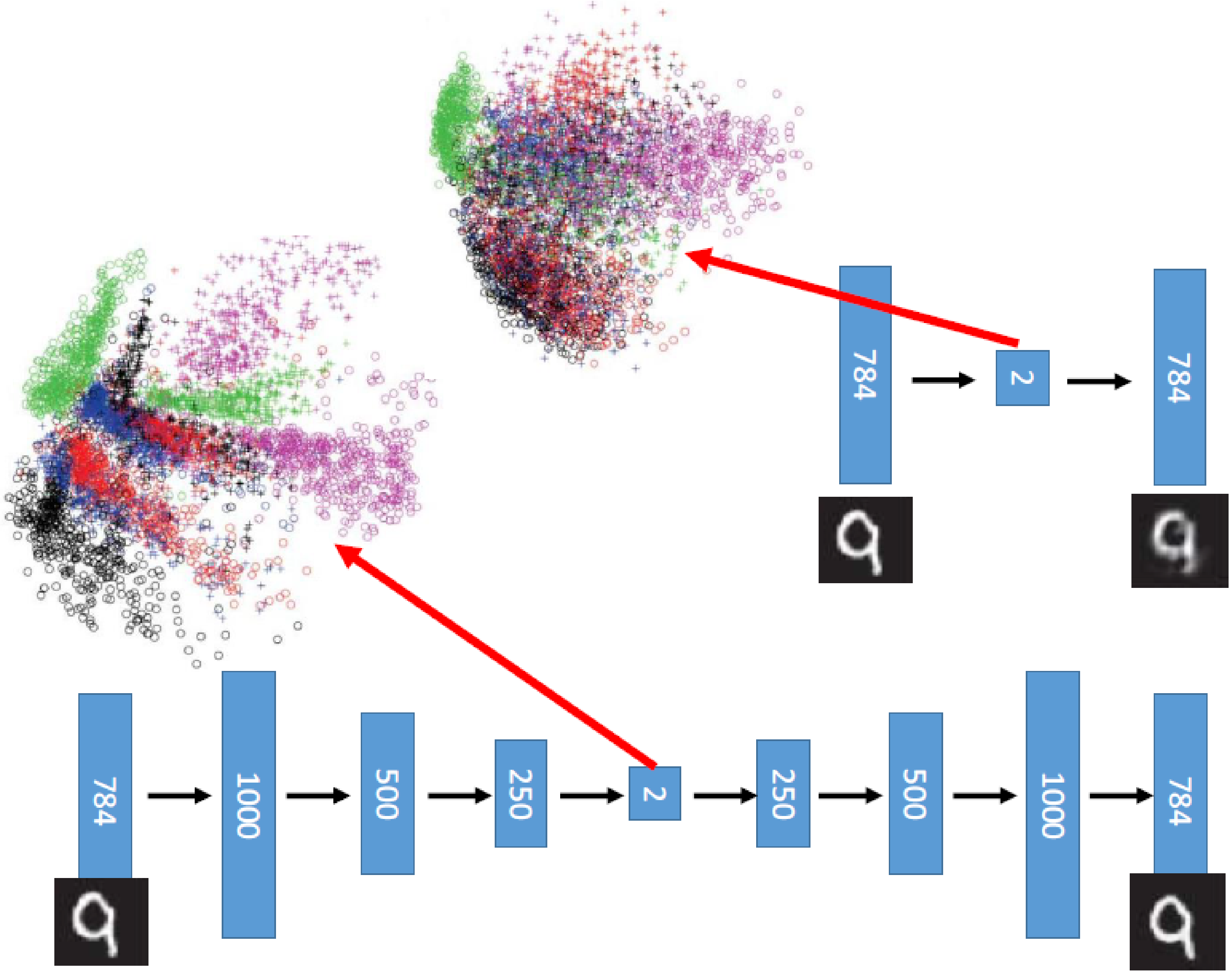
深度自编码器
Deep Auto-encoder





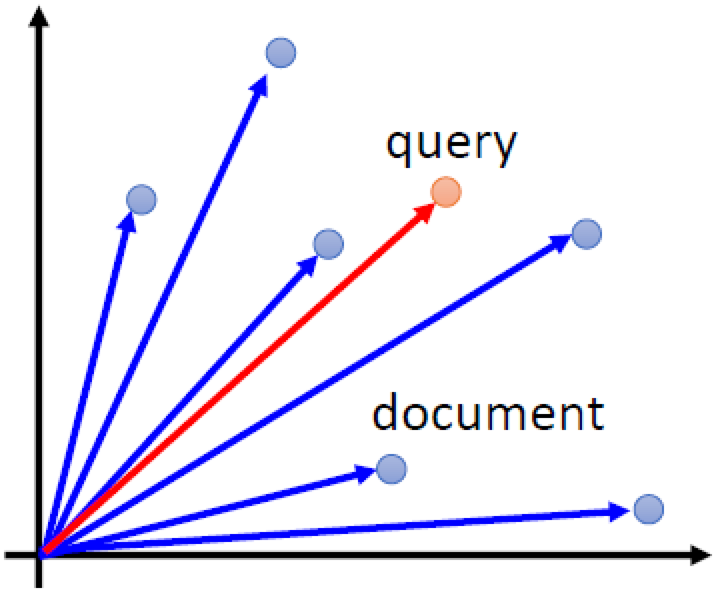


深度自编码器
Deep Auto-encoder

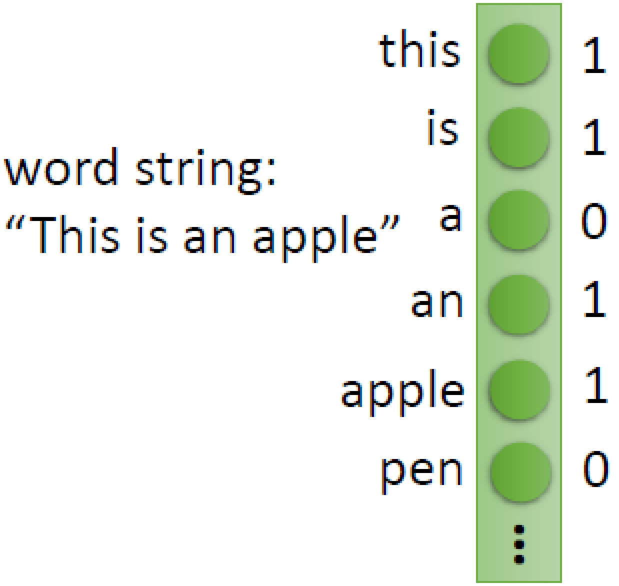


深度自编码器用于文本检索
Auto-encoder for Text Retrieval

Vector Space Model



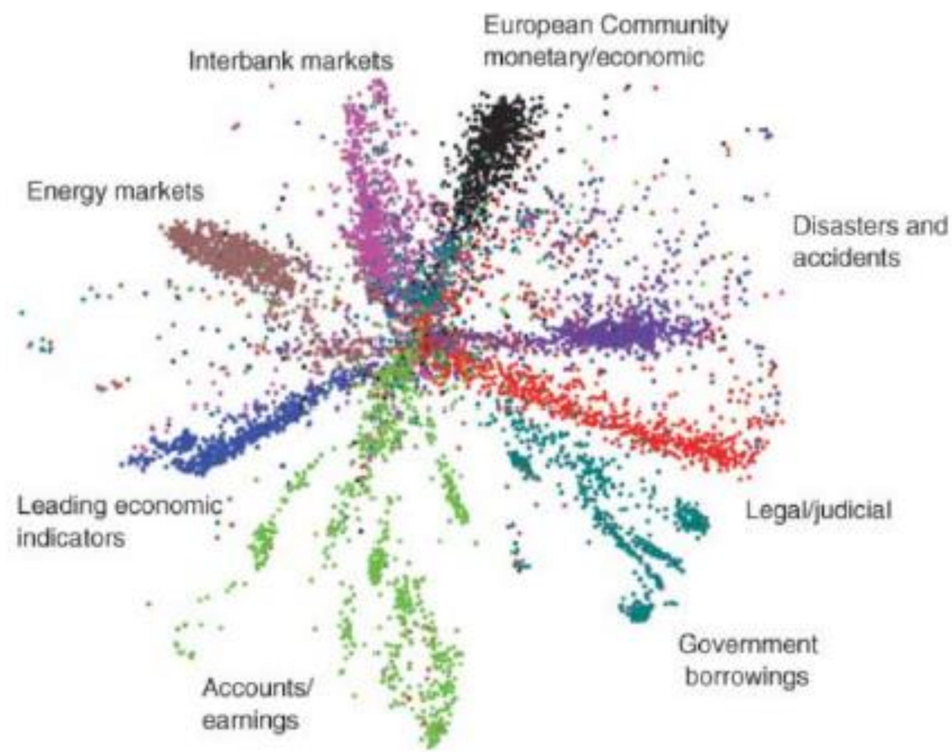
Bag-of-word



Semantics are not considered

深度自编码器用于文本检索
Auto-encoder for Text Retrieval

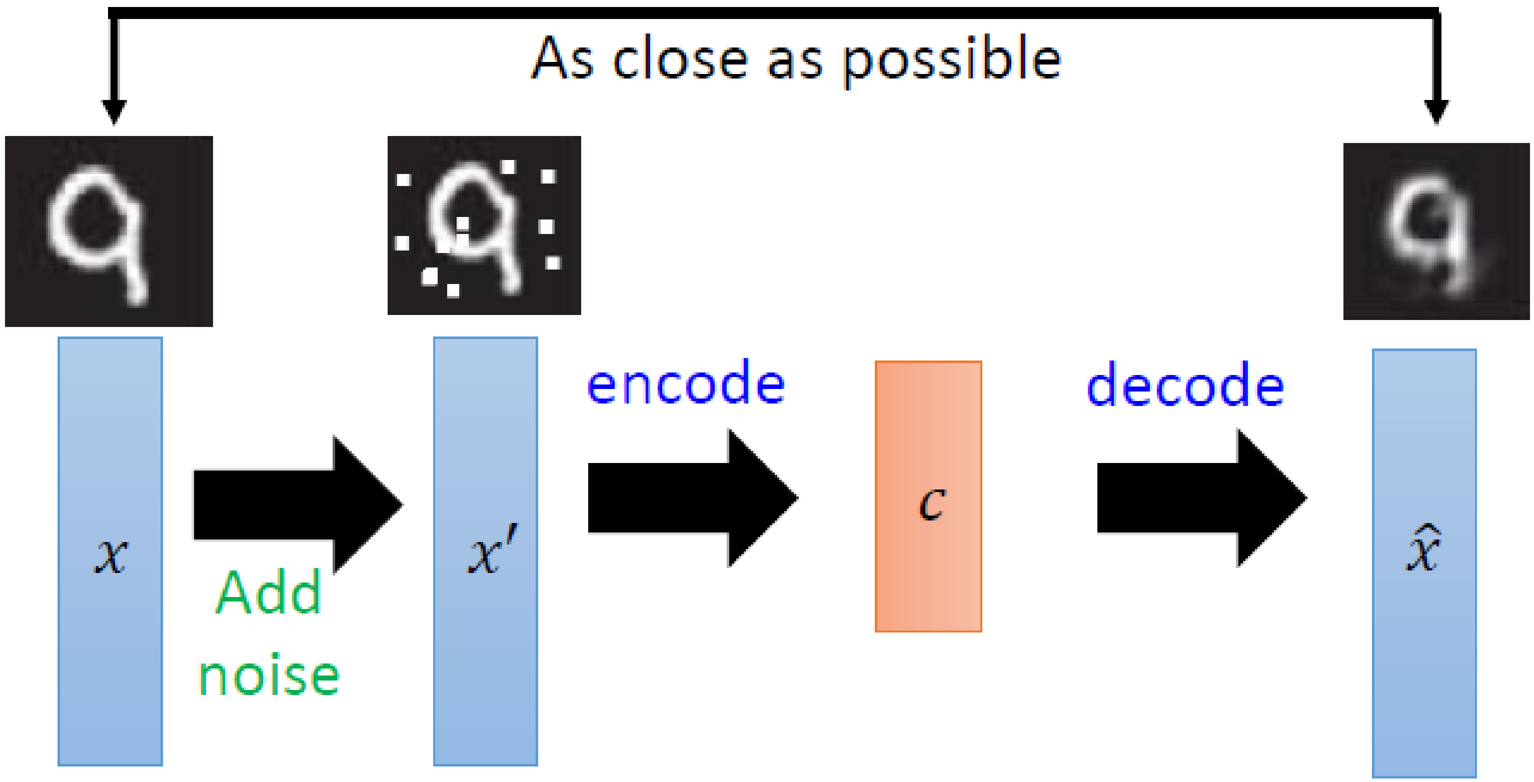
Latent semantic analysis
based on PCA



A 2000- 500-250-125-2 autoencoder
Trained by DBN

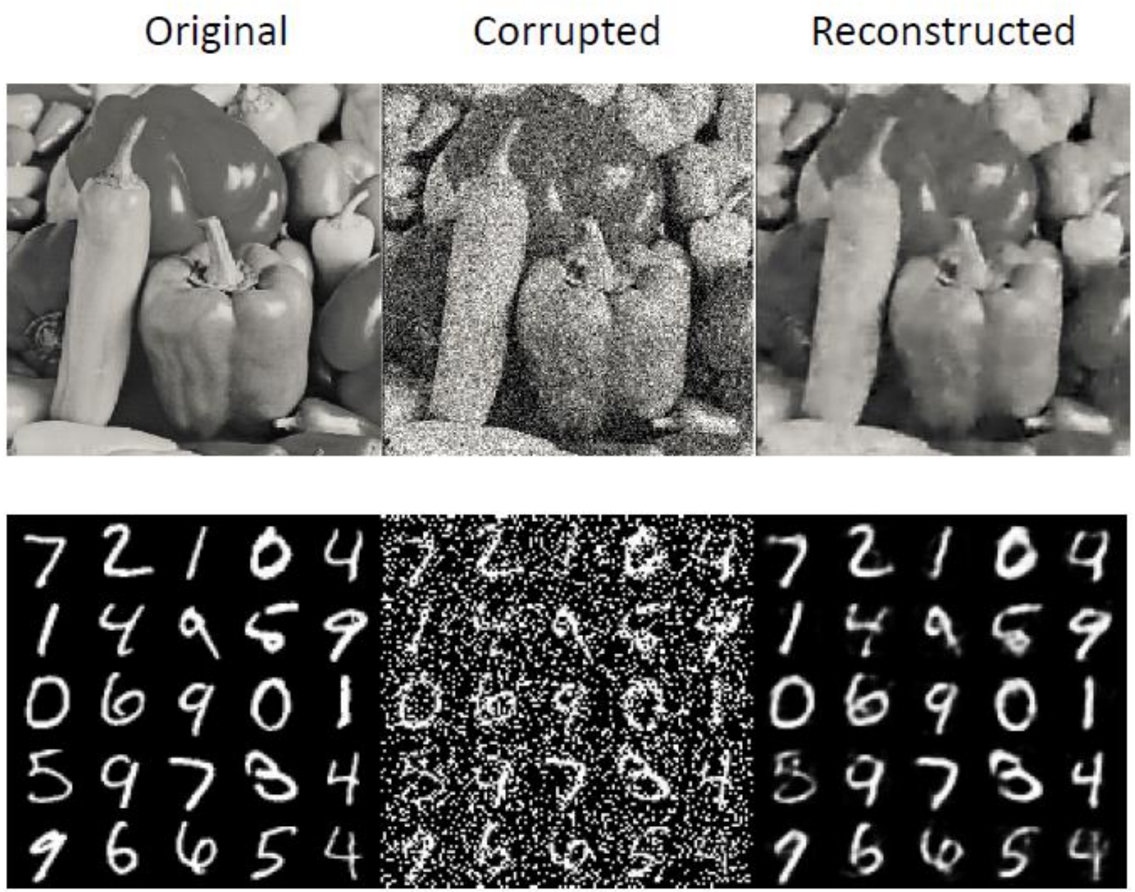
Hinton, Geoffrey E., and Ruslan R. Salakhutdinov. "Reducing the dimensionality of data with neural networks." *science* 313.5786 (2006): 504-507.

深度自编码器用于去噪
Denoising Auto-Encoder Examples

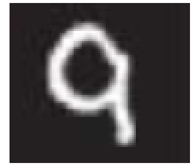


Vincent, Pascal, et al. "Extracting and composing robust features with denoising autoencoders." *ICML*, 2008.

深度自编码器用于去噪声
Denoising Auto-Encoder Examples



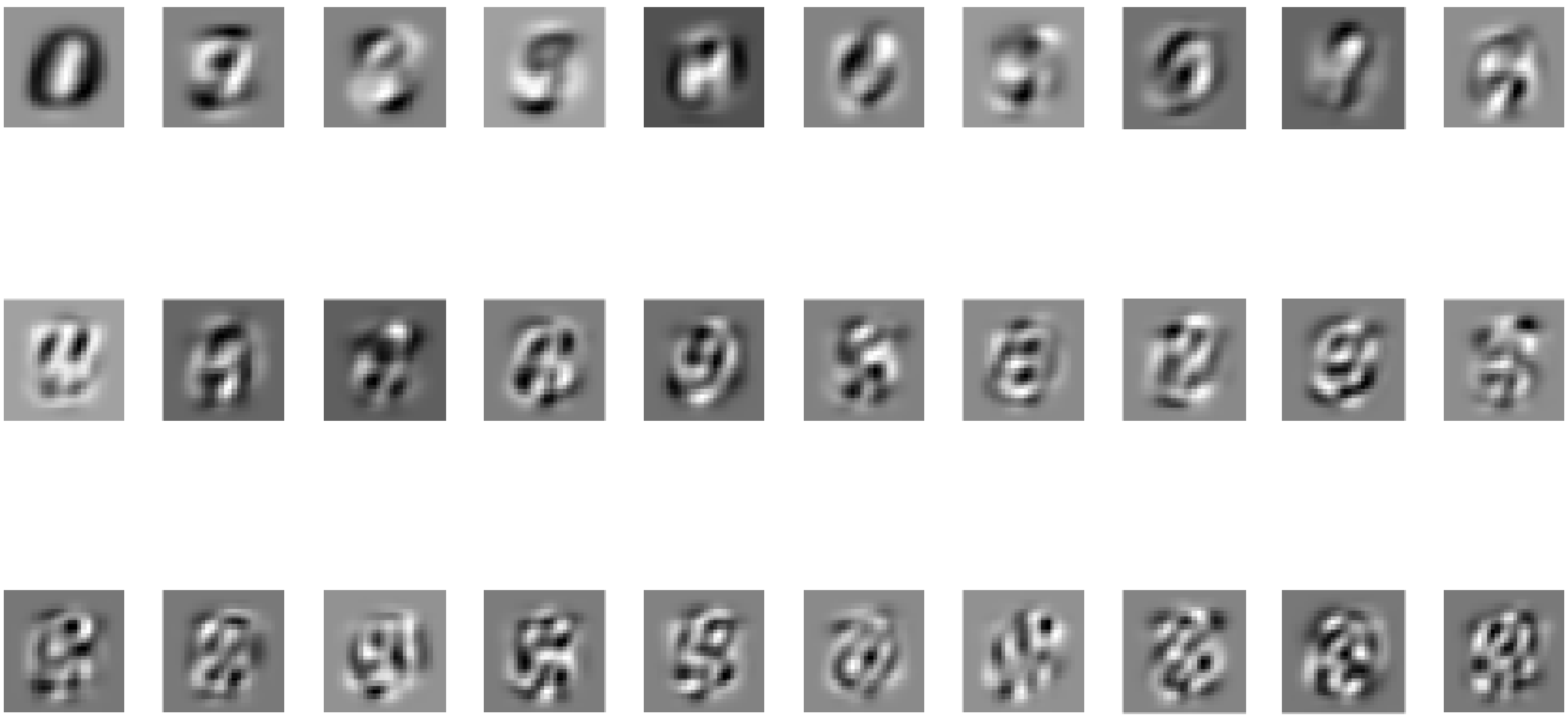
PCA人脸分析
PCA on Face



$$= a_1 w^1 + a_2 w^2 + \cdots$$

images

30 components:



Eigen-digits

PCA人脸分析
PCA on Face

30 components:

http://www.cs.unc.edu/~lazebnik/research/spring08/assignment3.html

Eigen-face

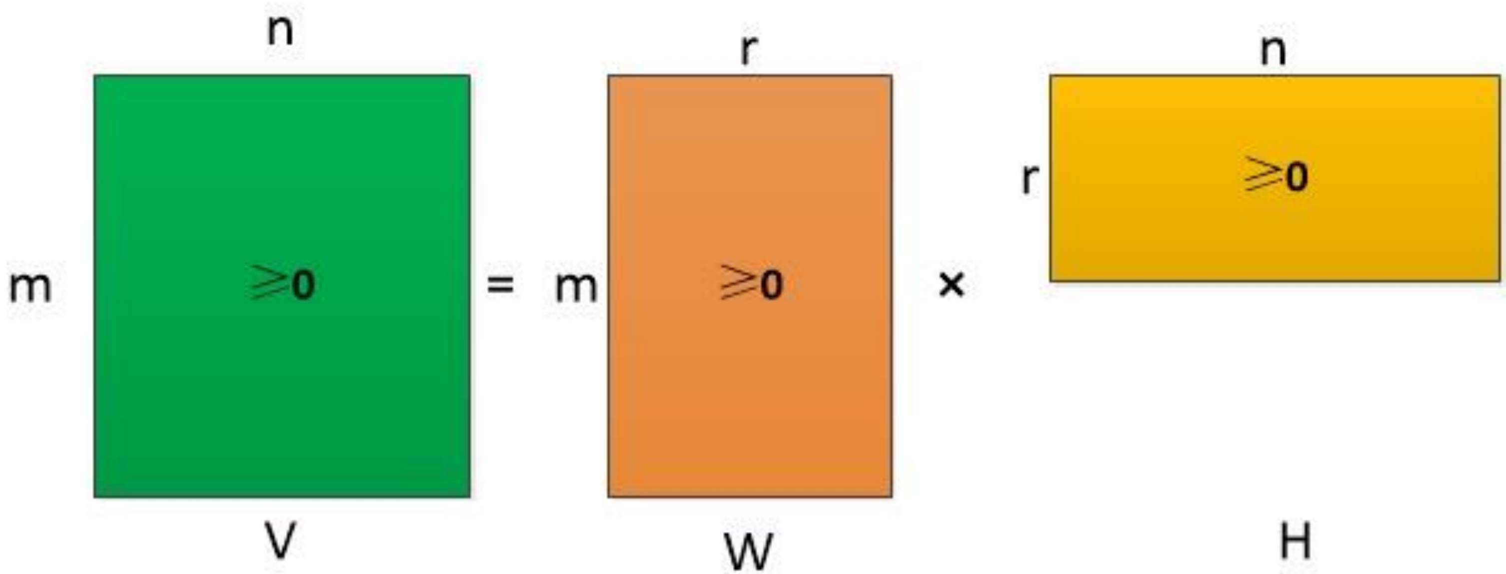
PCA结果
What happens to PCA

$$= \underline{a_1} w^1 + \underline{a_2} w^2 + \cdots$$

Can be any real number

- PCA involves adding up and subtracting some components (images)
  - Then the components may not be “parts of digits”
- Non-negative matrix factorization (NMF)
  - Forcing a_1, a_2 ...... be non-negative
    - additive combination
  - Forcing w^1, w^2...... be non-negative
    - More like “parts of digits”
- Ref: Daniel D. Lee and H. Sebastian Seung. "Algorithms for non-negative matrix factorization." *Advances in neural information processing systems*. 2001.

非负矩阵分解（ NMF ）
Non-negative Matrix Factorization

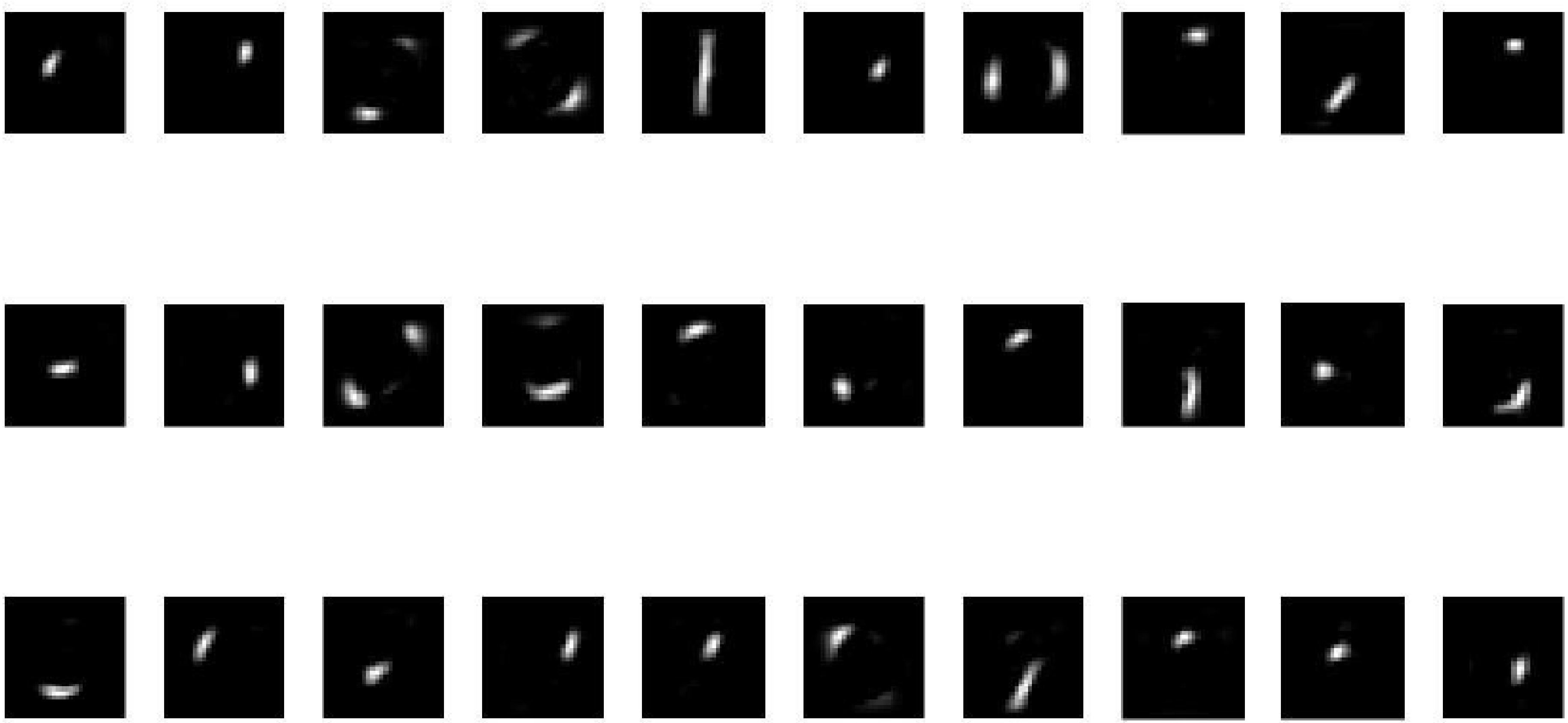


Non-negativity constraint offers physical and real-world interpretability!

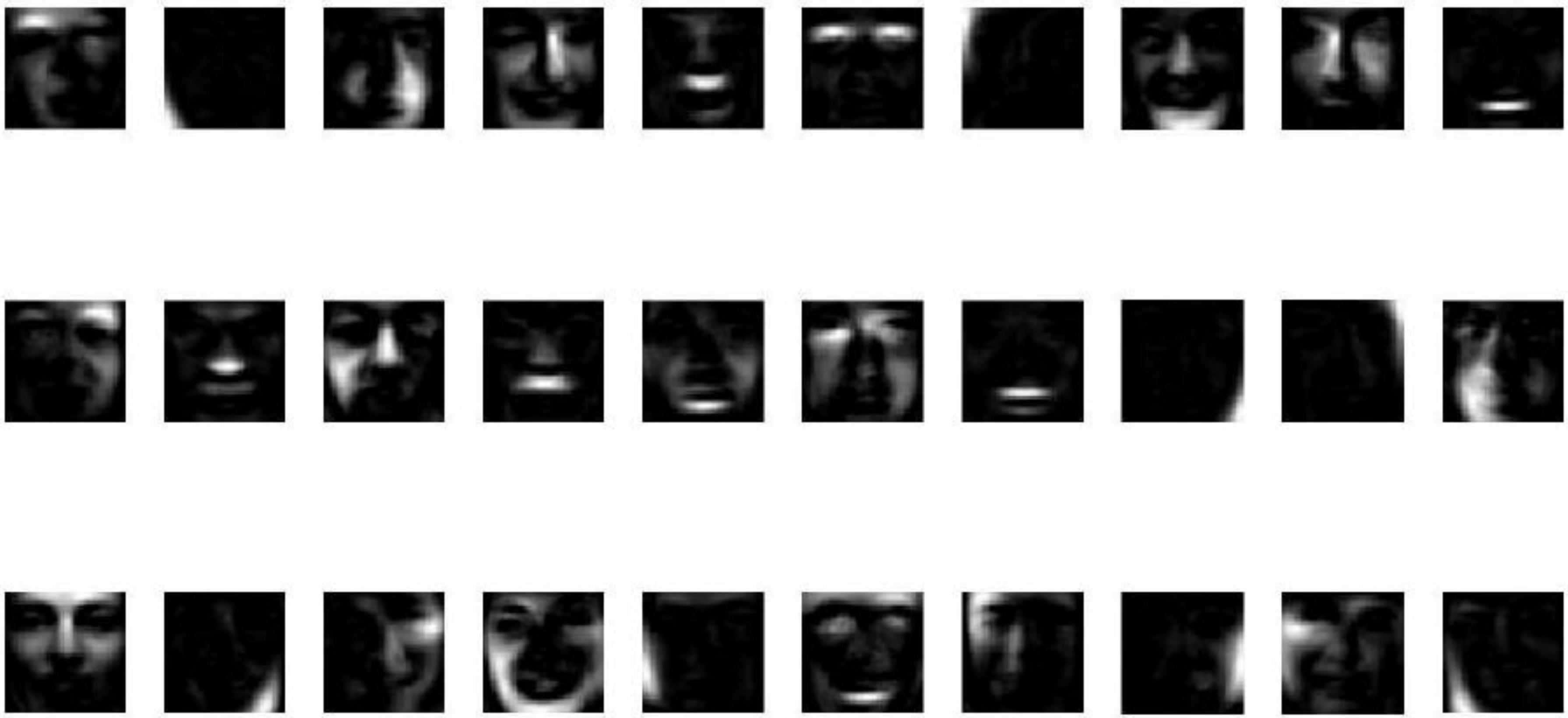
$$\boldsymbol{argmin}\frac{1}{2}\|X - WH\|^2 = \frac{1}{2}\sum_{i,j}(X_{ij} - WH_{ij})^2$$

$$\boldsymbol{argmin}\, J(W,H) = \sum_{ij}\left(X_{ij} ln\frac{X_{ij}}{WH_{ij}} - X_{ij} + WH_{ij}\right)$$

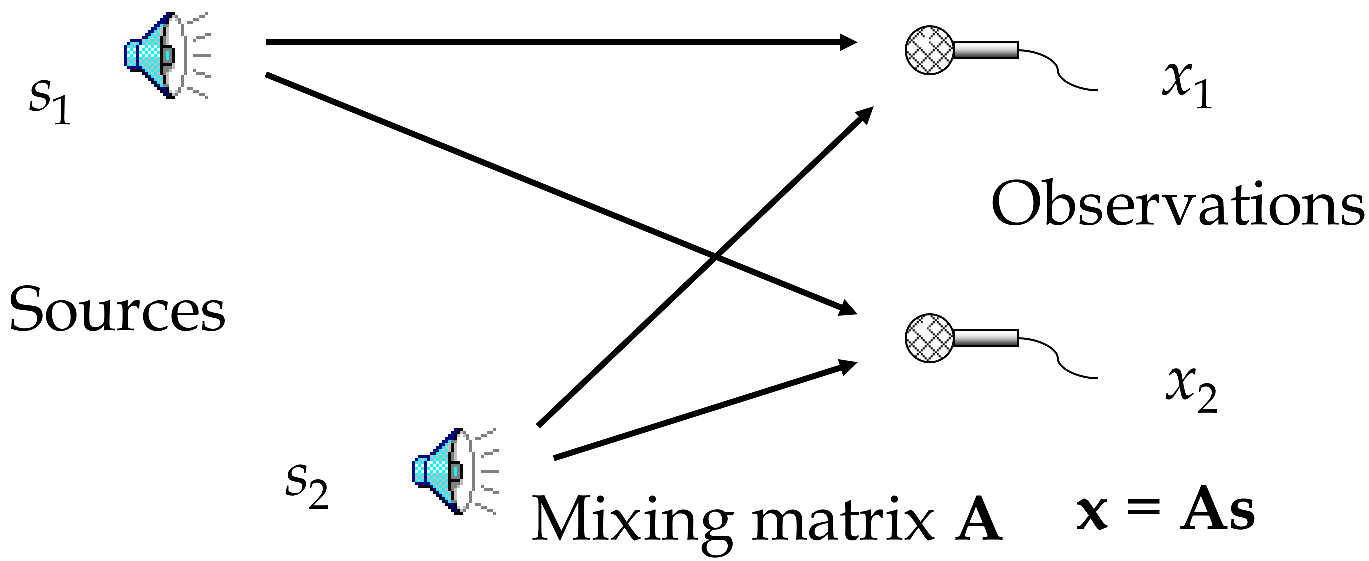
非负矩阵分解应用
NMF on MNIST



非负矩阵分解应用
NMF on Face



鸡尾酒会问题
Cocktail-party problem



鸡尾酒会问题
Cocktail-party problem

Model

$$x_1(t) = a_{11}s_1(t) + a_{12}s_2(t)$$
$$x_2(t) = a_{21}s_1(t) + a_{22}s_2(t)$$

We observe

$$\begin{pmatrix} x_1(1) \\ x_2(1) \end{pmatrix}, \begin{pmatrix} x_1(2) \\ x_2(2) \end{pmatrix}, \ldots, \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

We want

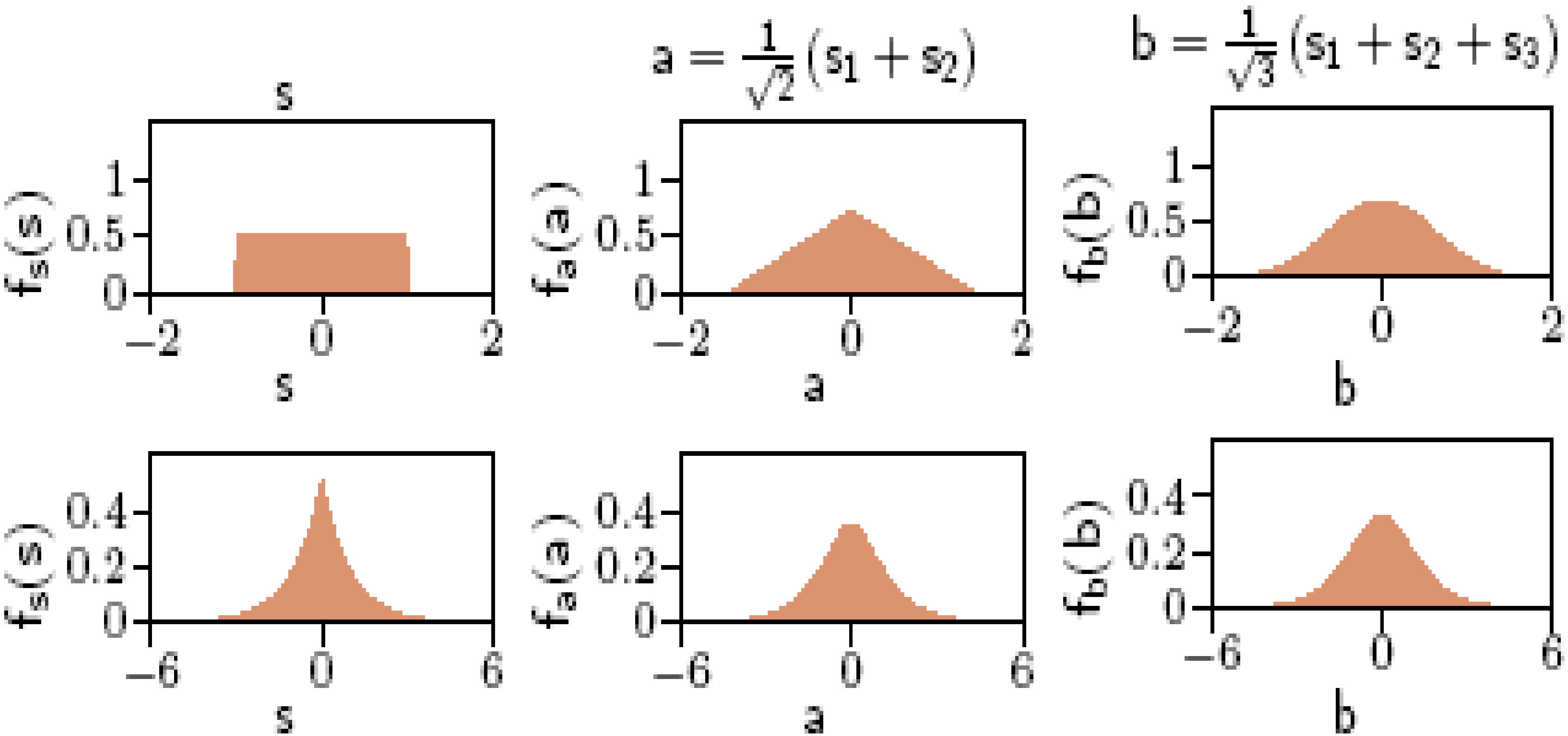
$$\begin{pmatrix} s_1(1) \\ s_2(1) \end{pmatrix}, \begin{pmatrix} s_1(2) \\ s_2(2) \end{pmatrix}, \ldots, \begin{pmatrix} s_1(t) \\ s_2(t) \end{pmatrix}$$

But we don't know $\{a_{ij}\}$, nor $\{s_i(t)\}$

Goal: Estimate $\{s_i(t)\}$, (and also $\{a_{ij}\}$)

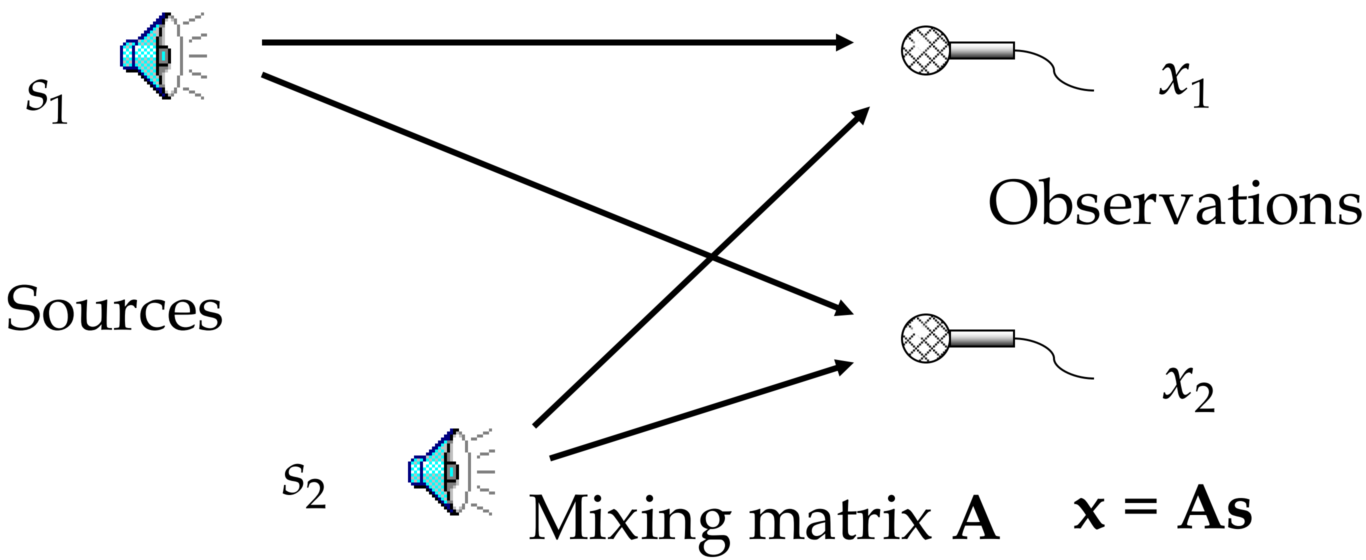
中心极限定理
Central Limit Theorem

The sum of independent variables converges to the normal distribution



Figs borrowed from Ata Kaban

ICA 假设
ICA Assumptions



1. $\mathbf{s}_1,\ldots\mathbf{s}_K$ statistical Independence
2. Nongaussianity
   (at most one component is Gaussian)
3. A is column full-rank

ICA 原则
ICA Principal (Non-Gaussian is Independent)

$$y = w^T x = w^T As = z^T s$$

- Key to estimating A is non-gaussianity

- y is a linear combination of s_i, with weights given by z_i.
- $z^T s$ is more gaussian than either of s_i. AND becomes least gaussian when its equal to one of s_i.
- So we could take ***w*** as a vector which <u>maximizes the non-gaussianity</u> of $\boldsymbol{w}^T\boldsymbol{x}$.
- Such a ***w*** would correspond to a ***z*** with only one non zero comp. So we get back the s_i.

非高斯性衡量
Measures of Non-Gaussianity

- Kurtotis : gauss=0 (sensitive to outliers)

$$kurt(y) = E\{y^4\} - 3(E\{y^2\})^2$$

Subgaussian: negative
Supergaussian: positive

- Entropy : gauss=largest

$$H(y) = -\int f(y) \log f(y) dy$$

- Neg-entropy : gauss = 0 (difficult to estimate)

$$J(y) = H(y_{gauss}) - H(y)$$

- Approximations

$$J(y) = \frac{1}{12} E\{y^2\}^2 + \frac{1}{48} kurt(y)^2$$

$$J(y) \approx [E\{G(y)\} - E\{G(v)\}]^2$$

where v is a standard gaussian random variable and :

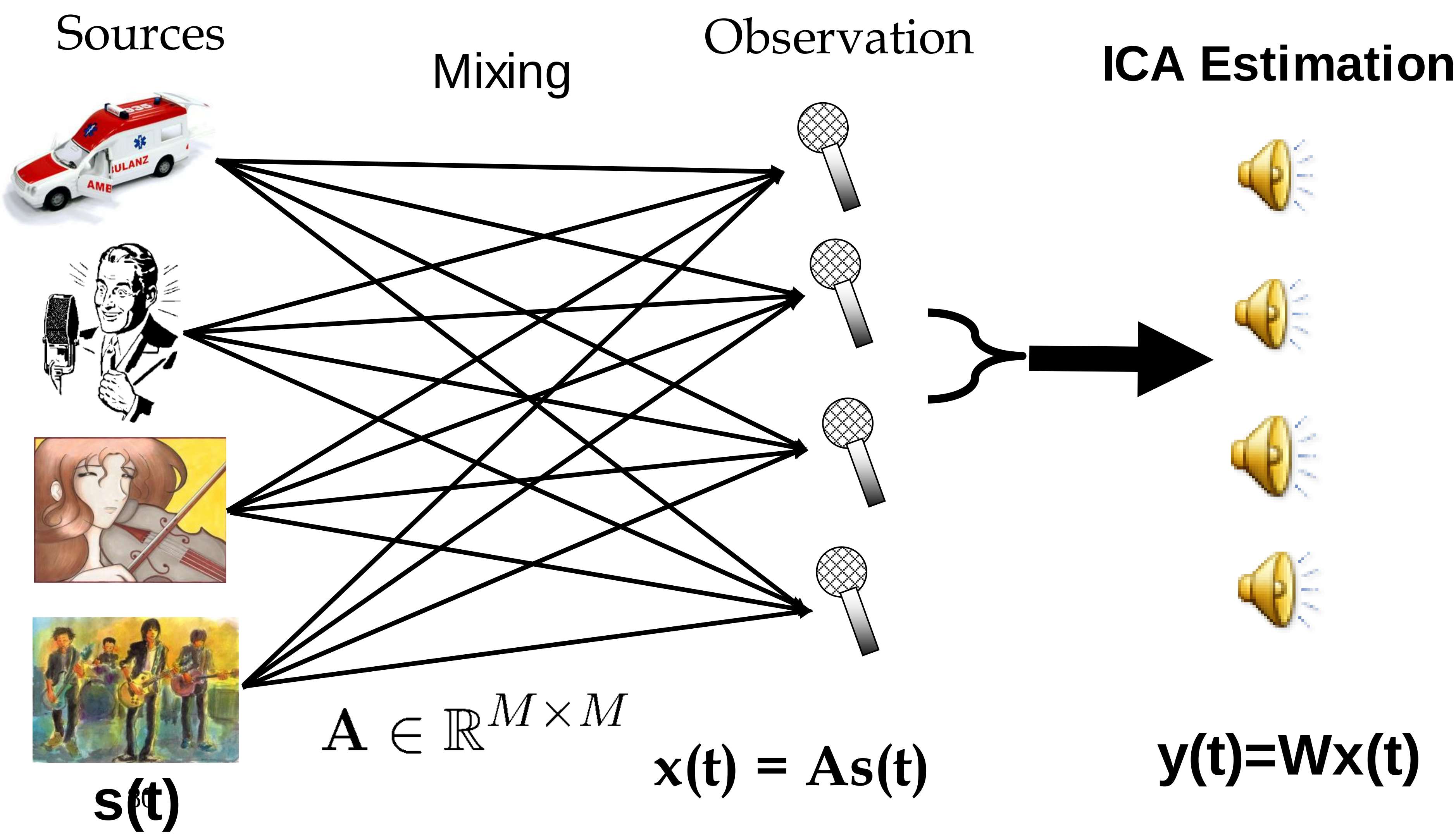
$$G(y) = \frac{1}{a} \log\cosh(a.y)$$

$$G(y) = -\exp(-a.u^2/2)$$

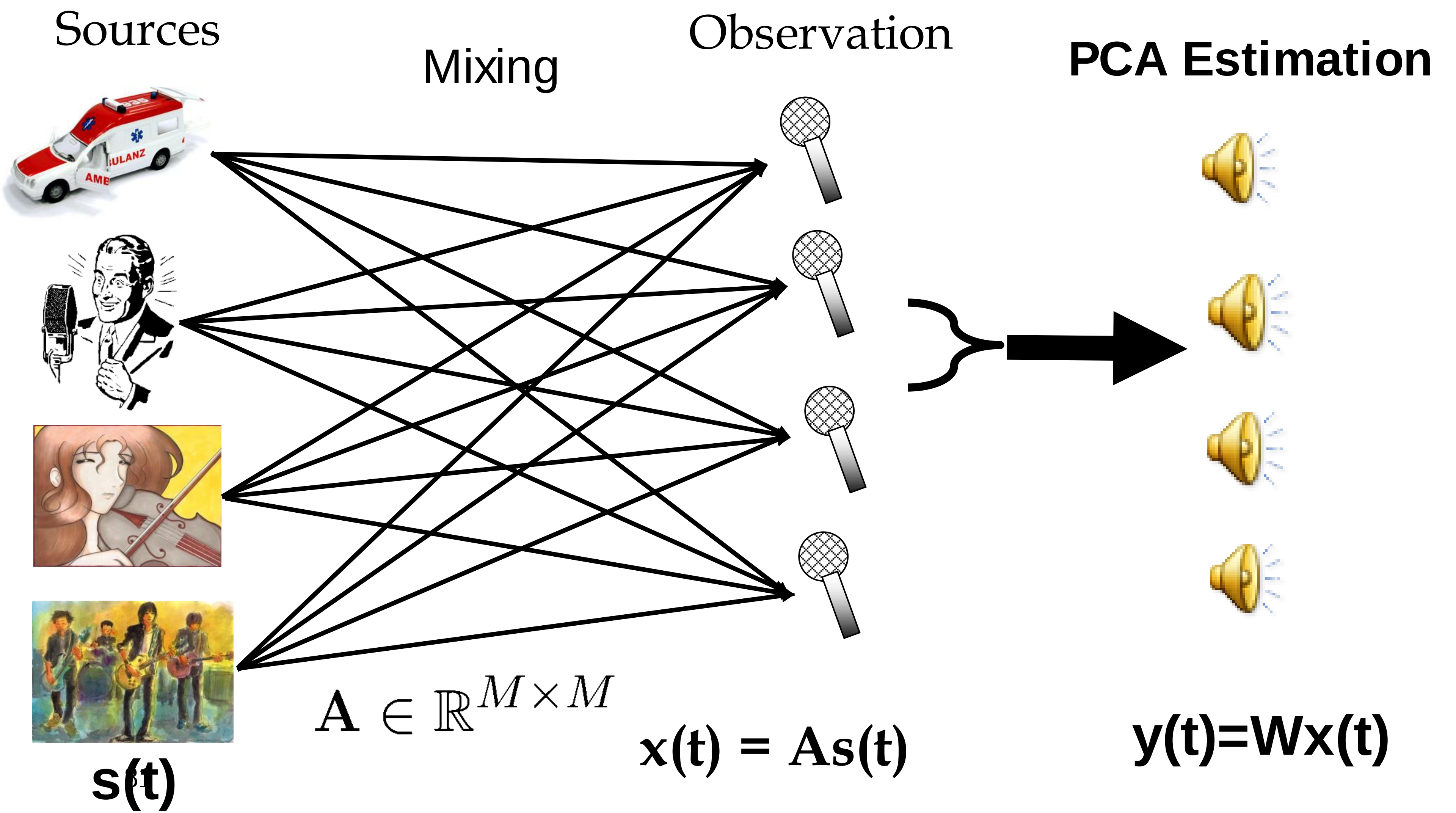
FastICA算法
FastICA Algorithm

- Based on: w
- Basic form: $J(y) \approx \left[E\{G(y)\} - E\{G(v)\}\right]^2$
  - Choose an initial (e.g. Random) weight vector
  - Let $w = w^+ / \|w^+\|$
  - Let $w^+ = E\{xg(w^T x)\} - E\{g'(w^T x)\}w$
  - If not converged, go back to step 2
- For several units: decorrelation
  - Let $w_{p+1} = w_{p+1} - \sum_{j=1}^{p} w_{p+1}^T w_j w_j$
  - Let $w_{p+1} = w_{p+1} / \sqrt{w_{p+1}^T w_{p+1}}$

鸡尾酒会问题ICA求解
The Cocktail Party Problem Solving with ICA

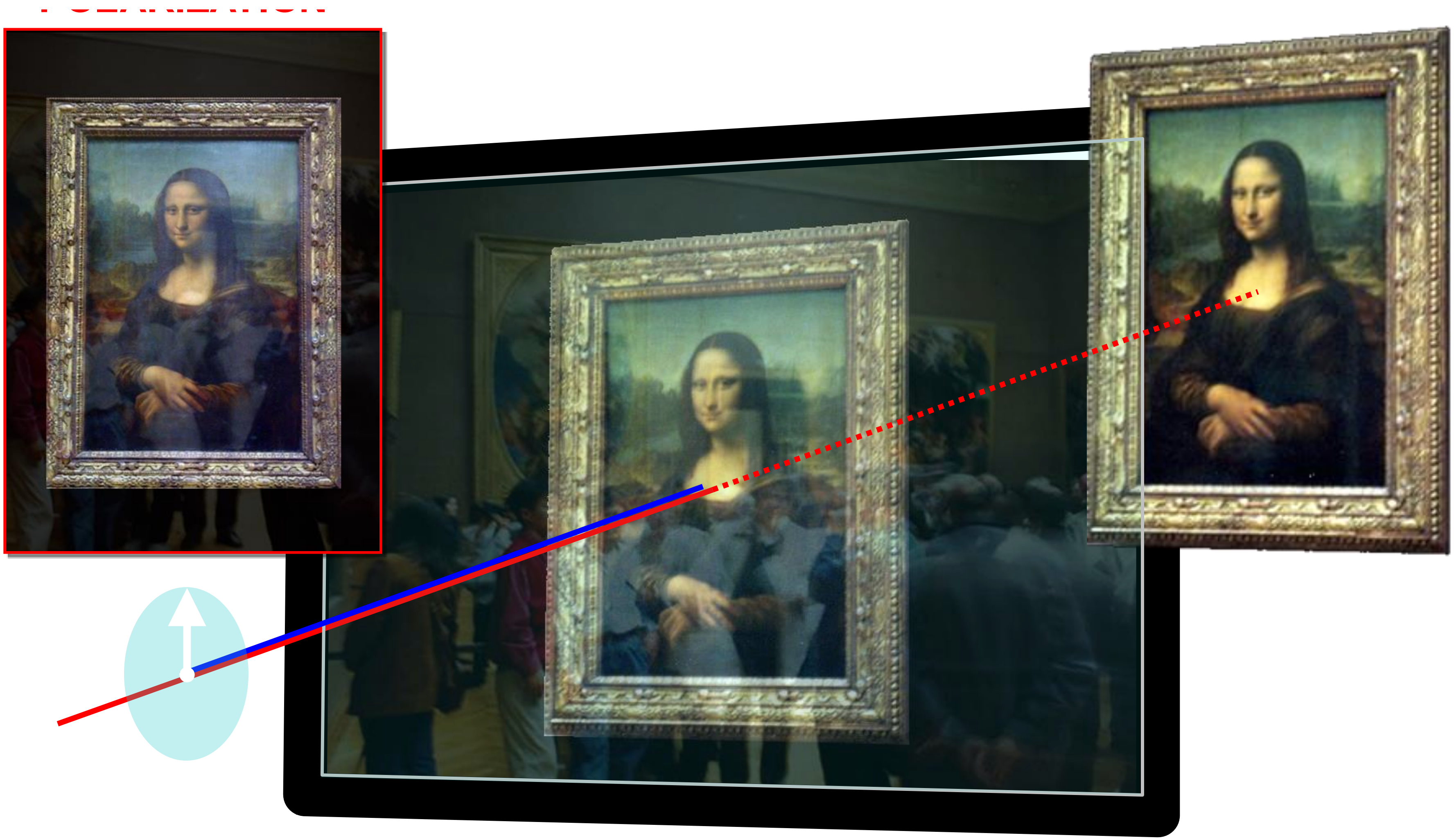


鸡尾酒会问题ICA求解
The Cocktail Party Problem Solving with PCA

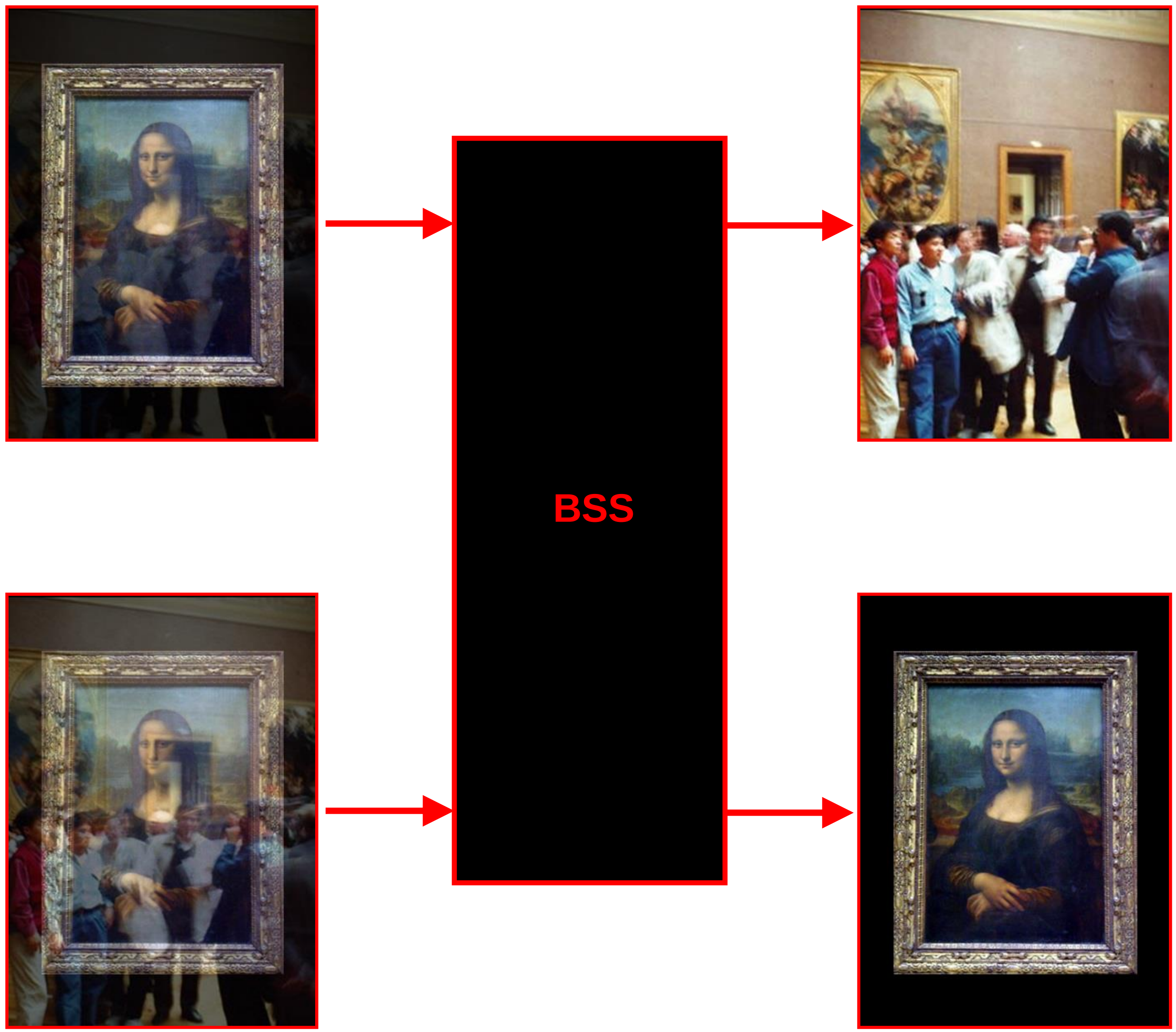


图像问题ICA求解
ICA Application on image

图像问题ICA求解
ICA Application on image

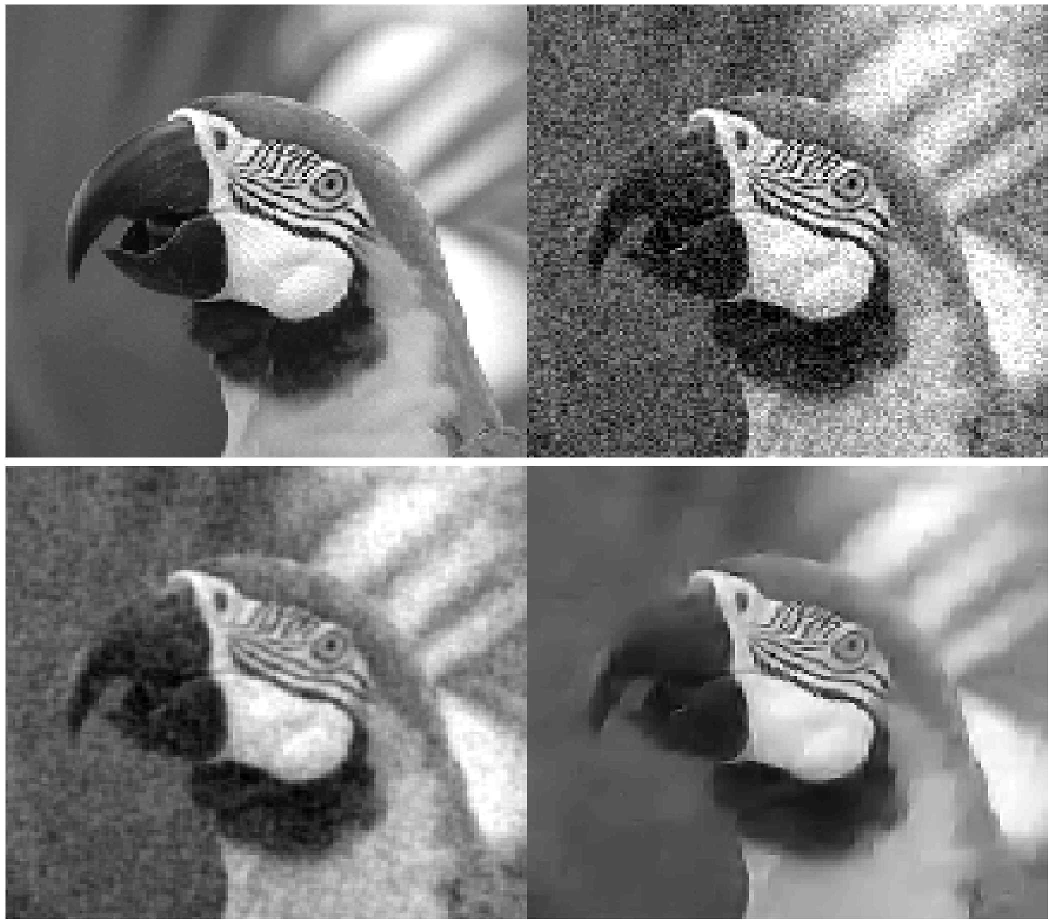


图像问题ICA求解
ICA Application on image



图像问题ICA求解
ICA Application on image

Original image



Noisy image

Wiener filtering

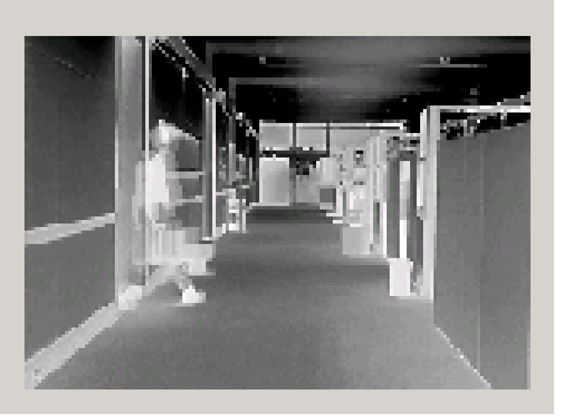
ICA filtering

视频问题ICA求解

ICA Application on Video

Source images

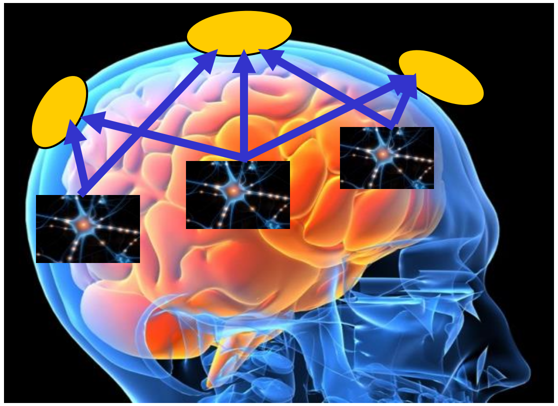
视频问题ICA求解
ICA Application on Video



ICs

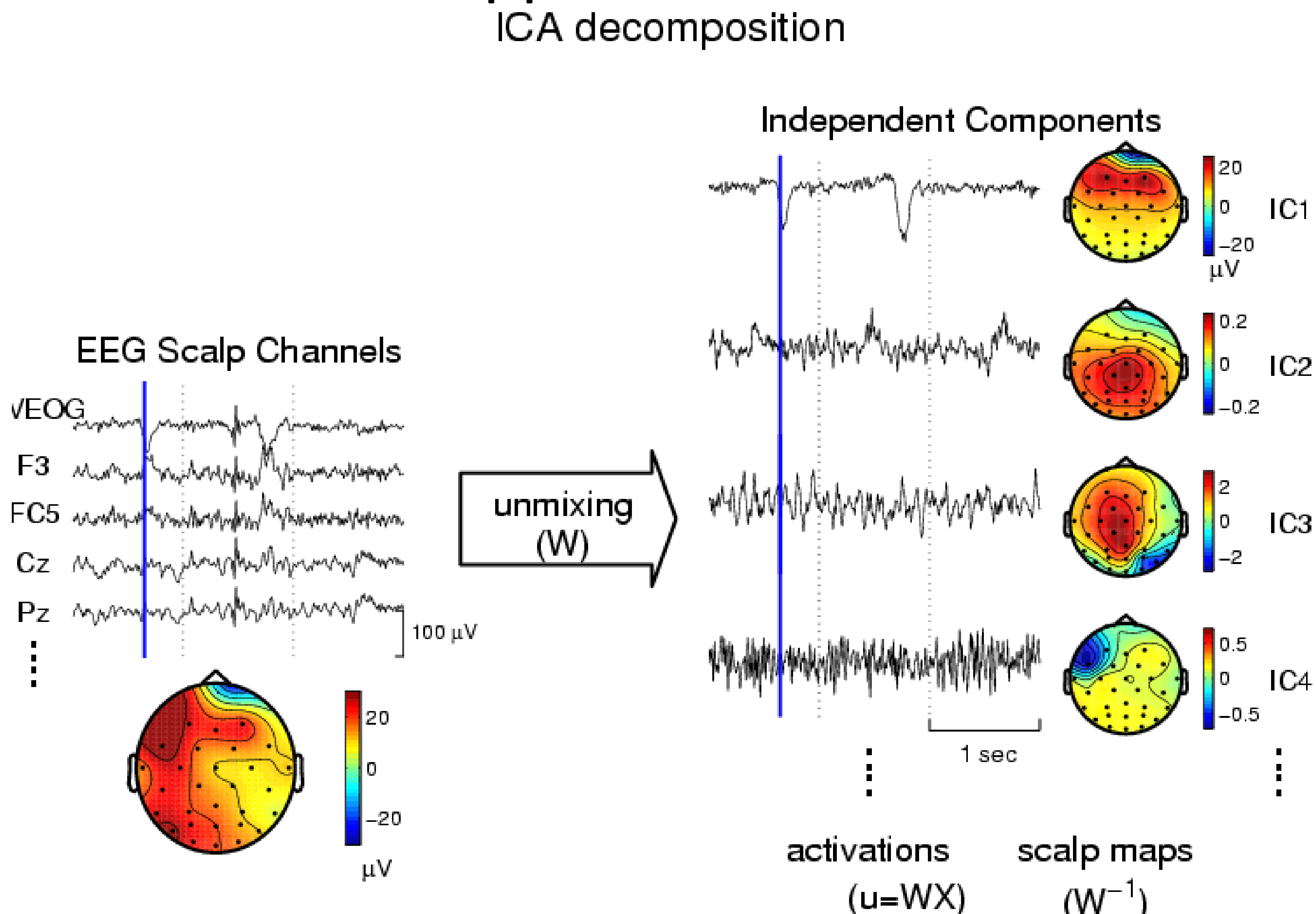
EEG问题ICA求解
ICA Application on EEG

- EEG ~ *Neural cocktail party*
- Severe ***contamination*** of EEG activity by
  - eye movements
  - blinks
  - muscle
  - heart, ECG artifact
  - vessel pulse
  - electrode noise
  - line noise, alternating current (60 Hz)

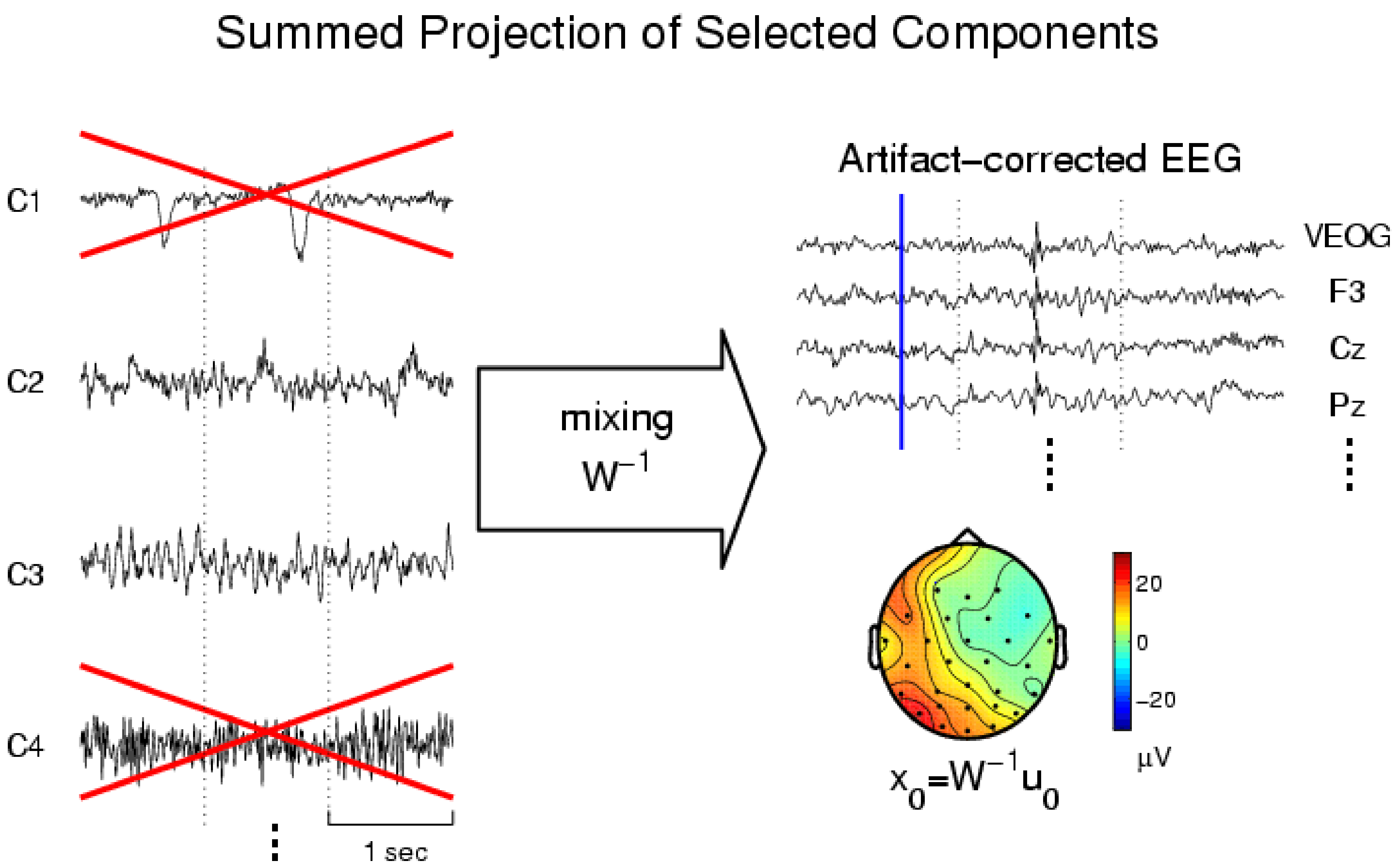


- ICA can improve signal
  - effectively ***detect, separate and remove*** activity in EEG records from a wide variety of artifactual sources.
    (Jung, Makeig, Bell, and Sejnowski)
- ICA weights help find **location** of sources

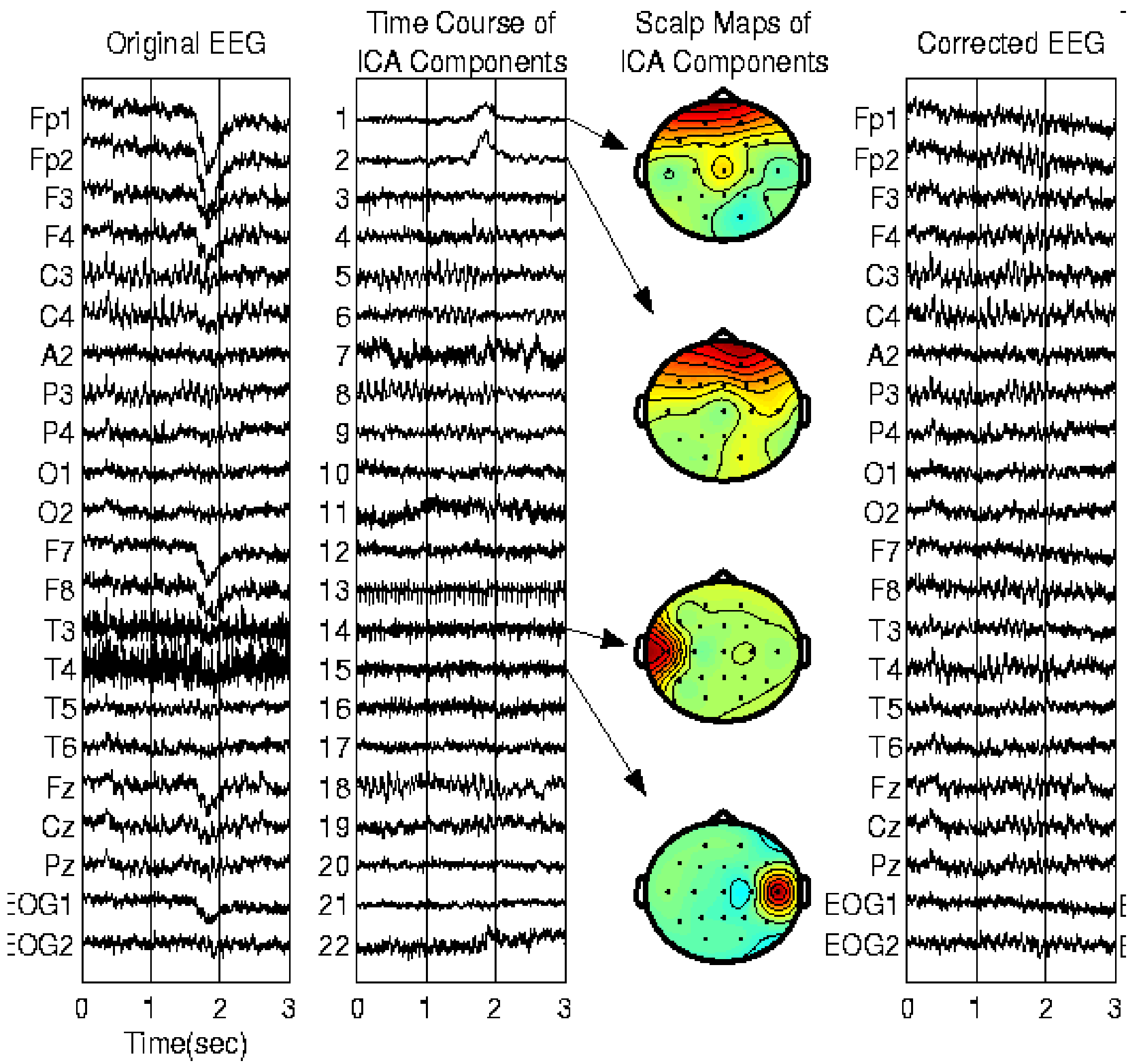
EEG问题ICA求解
ICA Application on EEG



EEG问题ICA求解
ICA Application on EEG

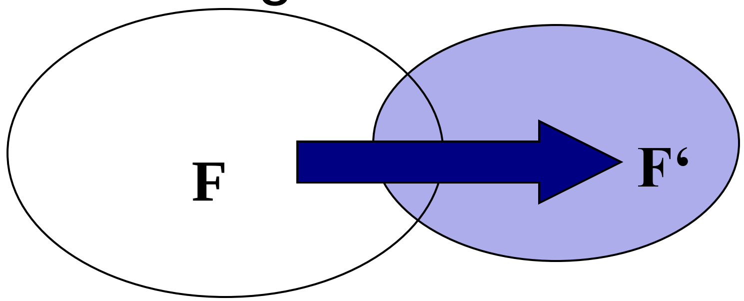


EEG问题ICA求解
ICA Application on EEG



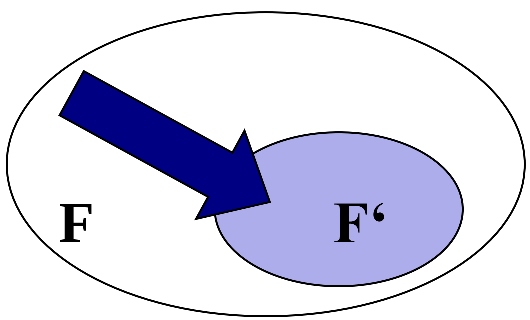
降维与特征选择
Dimensionality reduction& Feature selection

- Dimensionality reduction:creating a subset of new features by combinations of the existing features



$$\{f_1,\dots,f_i,\dots,f_n\}\xrightarrow[f.extraction]{}\{g_1(f_1,\dots,f_n),\dots,g_j(f_1,\dots,f_n),\dots,g_m(f_1,\dots,f_n)\}$$

- Feature Selection: choosing a subset of all the features



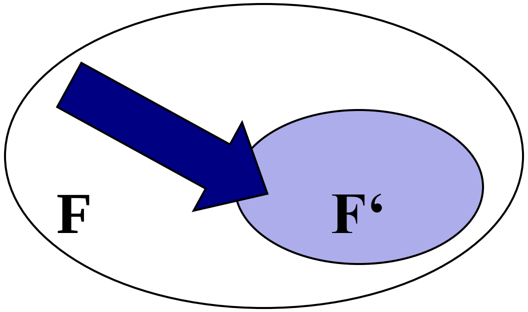
$$\{f_1,\dots,f_i,\dots,f_n\}\xrightarrow[f.selection]{}\{f_{i_1},\dots,f_{i_j},\dots,f_{i_m}\}$$

$$i_j\in\{1,\dots,n\};\ j=1,\dots,m$$

$$i_a=i_b\Rightarrow a=b;\ \ a,b\in\{1,\dots,m\}$$

特征选择
Feature selection

- Find the optimal feature subset.



  – Filter Methods
  – Wrapper Methods
  – Embedded Methods

过滤法
Filter Methods

Select subsets of variables as a pre-processing step, independently of the used classifier!!

Features could be evaluated by

- Distance
- Information
- dependency(correlation)
- …

过滤法
Filter Methods

- Relief (Relevant Features)

$$\delta^j = \sum_i -\text{diff}(x_i^j, x_{i,\text{nh}}^j)^2 + \text{diff}(x_i^j, x_{i,\text{nm}}^j)^2$$

- Fischer's ratio.

$$\frac{|S_b|}{|S_w|} = \frac{(\mu_1 - \mu_2)^2}{\sigma_1^2 + \sigma_2^2}$$

- information gain

$$\text{Gain}(A) = \text{Ent}(D) - \sum_{v=1}^{V} \frac{|D^v|}{|D|} \text{Ent}(D^v)$$

- coefficient scores

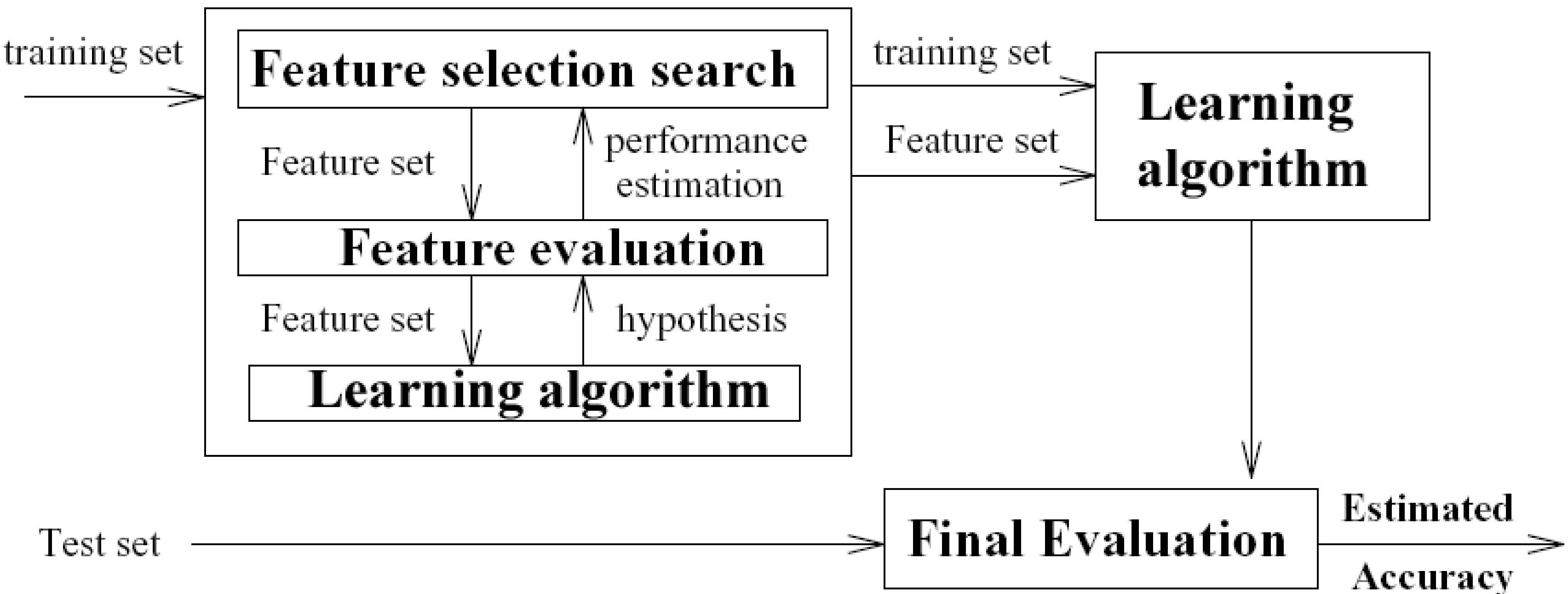
$$\Re(i) = \frac{\text{cov}(X_i, Y)}{\sigma(X_i) \times \sigma(Y)}$$

- …

过滤法
Filter Methods

- usually fast
- provide generic selection of features
- feature set not optimized for used classifier
- sometimes used as a preprocessing step for other methods

包裹法
Wrapper Methods



Features could be evaluated by classifier error rate (the classifier themselves).

包裹法
Wrapper Methods

- The problem of finding the optimal subset is NP-hard!
- A wide range of heuristic search strategies can be used. Two different classes:
  - **Forward selection**
    (start with empty feature set and add features at each step)
  - **Backward elimination**
    (start with full feature set and discard features at each step)
- predictive power is usually measured on a validation set or by cross-validation
- By using the learner as a black box wrappers are universal and simple!
- Criticism: a large amount of computation is required.

嵌入法
Embedded Methods

- Specific to a given learning machine!
- Performs feature selection (implicitly) in the process of training
- E.g. LASSO [Tibshirani, 1996]

$$\min_{\boldsymbol{w}} \sum_{i=1}^{m} (y_i - \boldsymbol{w}^\top \boldsymbol{x}_i)^2 + \lambda \|\boldsymbol{w}\|_1$$

- Random Forest and Gradient Boosting

问题：简述你所了解到的特征降维的几种方法及其特点