第4章 正则表达式

1. 正则运算与正则表达式
2. 正则表达式与FA的等价
3. 正则表达式代数定律
4. 正则表达式的应用

4.1正则表达式与正则运算

	arithmetic	theory of computation
objects	numbers	languages
tools	$+, \times$	$\cup, \cdot, *$

Regular operations be used to:

- Design automata to recognize particular languages. (grep in Unix, Perl, text editors)
- Prove that certain other languages are nonregular.

Regular operations

Definition 4.1

Let A and B be languages. We define the regular operations ***union***, ***concatenation***, and ***star*** as follows:

Union: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

Concatenation: $A . B = \{xy \mid x \in A \text{ and } y \in B\}$.

Star: $A^* = \{x_1 x_2 \ldots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$.

Regular Operations

Theorem 4.1 The class of RL is closed under union operation.

Let $N_1=(Q_1, \sum,\delta_1,q_1,F_1)$ recognize A_1,$N_2 =(Q_2, \sum,\delta_2,q_2,F_2)$) recognize A_2, N_1,N_2 is NFA.

Construct $N=(Q, \sum,\delta,q_0,F)$ to recognize A1UA2.

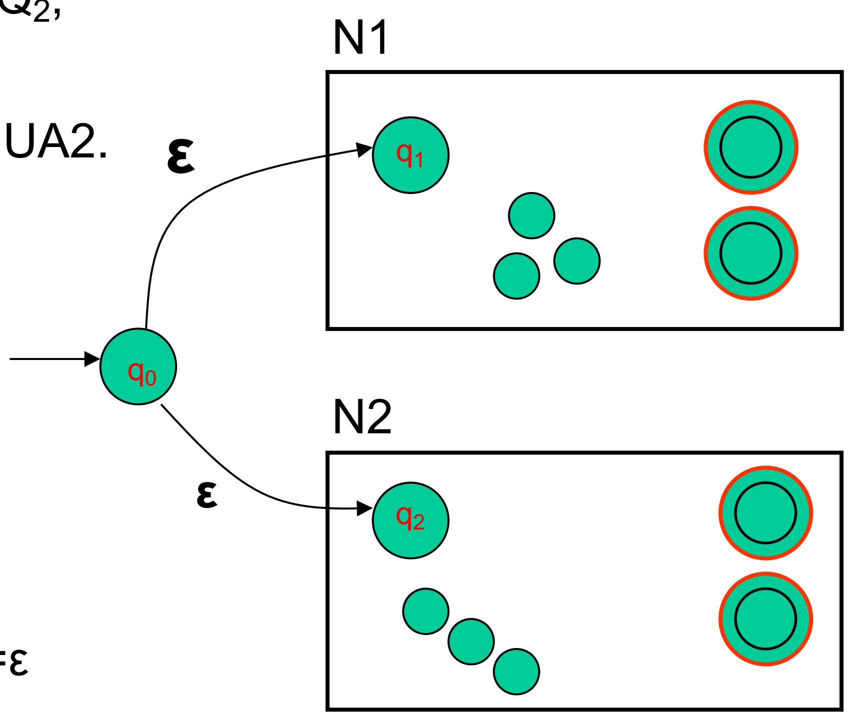
1.$Q=\{q_0\}UQ_1UQ_2$

2.The State q_0 is the start state of N.

3.The accept state $F = F_1 \cup F_2$

4.

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q\in Q_1 \\ \delta_2(q,a) & q \in Q_2 \\ \{q_1,q_2\} & q=q_0 \text{ and } a=\varepsilon \\ \varnothing & q=q_0 \text{ and } a \neq \varepsilon \end{cases}$$

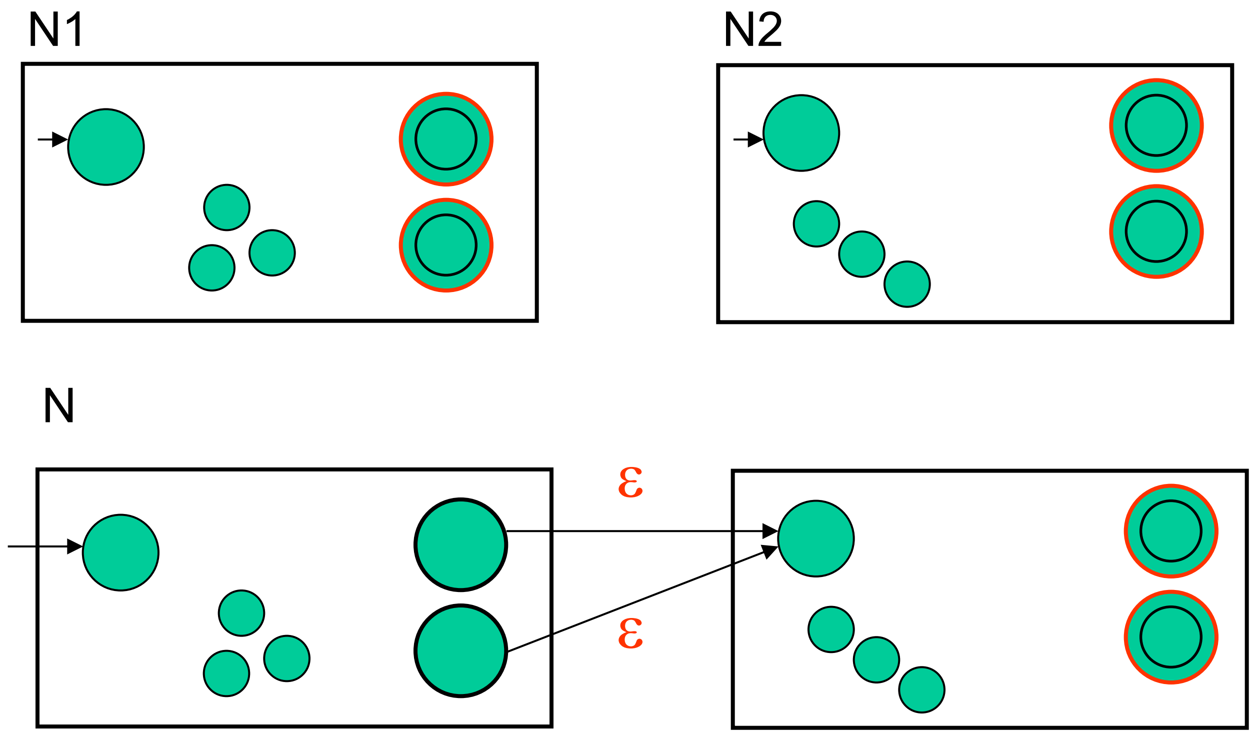


其中：$q \in Q$, $a \in \sum_\varepsilon$

Regular Operations

Theorem 4.2 The class of RL is closed under concatenation operation.

Proof Idea：构造NFA N，N识别A1·A2



Regular Operations

Theorem 4.2 The class of RL is closed under concatenation operation.

PROOF

Let N1 = (Q_1, Σ, δ_1, q_1, F_1) recognize A_1, and N_2 = (Q_2, Σ, δ_2, q_2, F_2) recognize A_2.

Construct N = (Q, Σ, δ, q_1, F_2) to recognize $A_1 . A_2$.

1. $Q = Q_1 \cup Q_2$. The states of N are all the states of N1 and N_2.

2. The state q_1 is the same as the start state of N_1.

3. The accept states F_2 are the same as the accept states of N_2.

4. Define δ so that for any $q \in Q$ and any $a \in \Sigma\varepsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q,a) & q \in F_1 \text{ and } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_2\} & q \in F_1 \text{ and } a = \varepsilon \\ \delta_2(q,a) & q \in Q_2. \end{cases}$$

Regular Operations

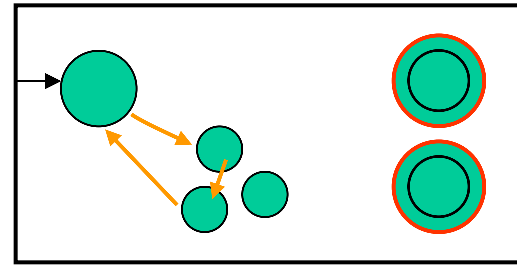
Theorem 4.3 The class of RL is closed under star operation.

Star: $A^* = \{x_1x_2\ldots x_k \mid k>=0 \text{ and each } x_i \in A\}$

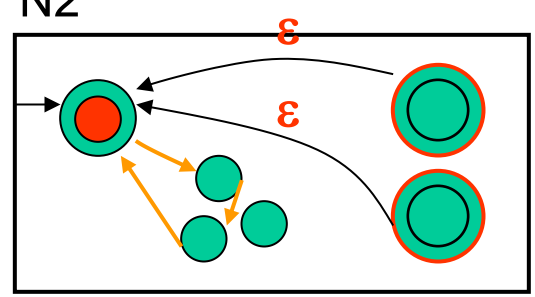
Proof Idea: Construction of N to recognize A*

- A* 可分解成若干个片段x_i，且$x_i \in A$，所以，接受状态应返回起始状态；
- A* 包含 ε ，所以，起始状态起应该是接受状态；
- **问题：N2识别A*吗？**

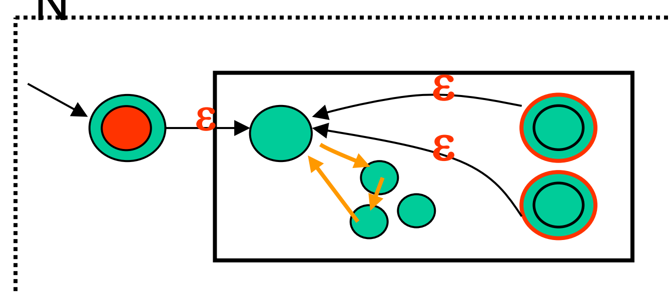
N1



N2



N



Regular Operations

Theorem 4.3 The class of RL is closed under star operation.

PROOF

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1.
Construct $N = (Q,\Sigma,\delta,q_0,F)$ to recognize A_1^*.

1. $Q = \{q_0\} \cup Q_1$.
The states of N are the states of N_1 plus a new start state.

2. The state q_0 is the new start state.

3. $F = \{q_0\} \cup F_1$.
The accept states are the old accept states plus the new start state.

4. Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q,a) & q \in F_1 \text{ and } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon \\ \{q_1\} & q = q_0 \text{ and } a = \varepsilon \\ \emptyset & q = q_0 \text{ and } a \neq \varepsilon. \end{cases}$$

Regular Expressions

Definition 4.2

Given an alphabet Σ, R is a regular expression if

1. $R = a$, with $a \in \Sigma$; denoting the languages $\{a\}$.
2. $R = \varepsilon$; denoting the languages $\{\varepsilon\}$.
3. $R = \varnothing$; denoting the languages $\varnothing$.
4. $R = (R_1+R_2)$, with R_1 and R_2 regular expressions; denoting the languages $L(R_1) \cup L(R_2)$.
5. $R = (R_1 \bullet R_2)$, with R_1 and R_2 regular expressions; denoting the languages L(R1) L(R2).
6. $R = (R_1{}^*)$, with R_1 a regular expression; denoting the languages $L(R_1)^*$.

Regular Expressions

Example 4.1 What is the language defined by r

$$r = (a + b)^* (a + bb).$$

a → { a }, b → { b }

a+b → { a } ∪ { b } = {a, b}

bb → { b } { b } = { bb }

a + bb → {a } ∪ {bb } = { a, bb }

$(a+b)^*$ → $\{ a, b \}^*$

$(a+b)^* (a+bb)$ → $\{ a, b \}^* \{ a, bb \}$

L(r) = { a, bb, aa, abb, ba, bbb, ... }

Regular Expressions

Example 4.2 What is the language defined by r

$$r = (aa)^*(bb)^*b$$

$L(r) = (\{a\}\{a\})^*(\{b\}\{b\})^*\{b\}$

$=(\{aa\})^*(\{bb\})^*\{b\}$

$= \{aa\}^*\{bb\}^*\{b\}$

$= \{a^{2n}b^{2m+1} \mid n \geq 0, m \geq 0\}$

Regular Expressions

Example 4.3 Write a regular expression for the set of strings that consist of alternating 0's and 1's.

Partition :

$010101...0101 \longrightarrow (01)^*$

$101010...1010 \longrightarrow (10)^*$

$0101010...1010 \longrightarrow 0(10)^*$ or $(01)^*0$

$101010...10101 \longrightarrow 1(01)^*$ or $(10)^*1$

The regular expression :

$$(01)^*+(10)^*+0(10)^*+1(01)^* = (\varepsilon+1)(01)^*(\varepsilon+0)$$

Regular Expressions

Example 4.4 Design regular expression for L ,
$L=\{w \mid w\in \{0, 1\}^* \text{ and } w \text{ has no pair of consecutive 0's}\}$.

Partition :

no 0 $\longrightarrow$ 1^*

one 0 $\longrightarrow$ 1^*01^*

more 0's $\longrightarrow$ $(1^*\,011^*)^*\,(0+\varepsilon)$

$r1=(1^*011^*)^*\,(0+\varepsilon)+1^*(0+\varepsilon)$

或 $r2\ =(1+01)^*(0+\varepsilon)$

思考题：

1. r1 = r2?
2. L(r) = {w ∈ Σ*| w has at least one pair of consecutive zeros}. R =？

构造正则表达式的一般性规则

1. **语言只含一个字符串**：字符串作为正则表达式。

   如：正则表达式00和11表示语言{00}和{11}。

2. **语言含多个字符串的连接得到的串**：字符串的连接作为正则表达式。

   如：正则表达式00和11表示语言{00}和{11}，那么，正则表达式0011表示语言{0011}。

3. **语言含零次或多次出现的串**：串的闭包作为正则表达式。

   如：含01的零次或多次出现的串，正则表达式为(01)*。

构造正则表达式的一般性规则

4. **语言中的串有多种可能的形式**：用+运算符表达多种可能性（相当于语言的并）。

   如：(01)*+(10)*+0(10)*+1(01)*。

5. **语言的串中含可有、可无的子串**：用ε和该子串的并与串的其余部分连接。

   如：(ε +1)(01)*(ε +0)

Regular Expressions

Example 4.5 In the following instances, we assume that the alphabet Σ is $\{0,1\}$.

1. $0^*10^* = \{w|$ w contains a single 1$\}$.
2. $\Sigma^*1\Sigma^* = \{w|$ w has at least one 1$\}$.
3. $\Sigma^*001\Sigma^* = \{w|$ w contains the string 001 as a substring$\}$.
4. $1^*(01^+)^* = \{w|$ every 0 in w is followed by at least one 1$\}$.
5. $(\Sigma\Sigma)^* = \{w|$ w is a string of even length$\}$.
6. $(\Sigma\Sigma\Sigma)^* = \{w|$ the length of w is a multiple of 3$\}$.
7. $01 + 10 = \{01, 10\}$.

Regular Expressions

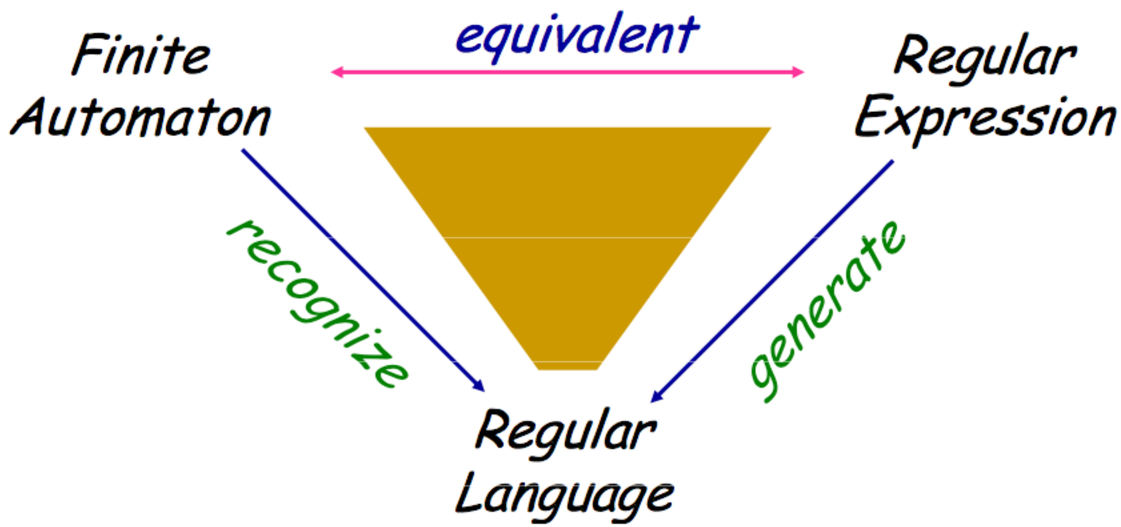
Example 4.5 （续）

8. $0\Sigma^*0 + 1\Sigma^*1 + 0 + 1 = \{w \mid w$ starts and ends with the same symbol$\}$.
9. $(0 + \varepsilon)1^* = 01^* \cup 1^*$.
10. $(0 + \varepsilon)(1 + \varepsilon) = \{\varepsilon, 0, 1, 01\}$.
11. $1^*\emptyset = \emptyset$.
12. $\emptyset^* = \{\varepsilon\}$.
13. $R + \emptyset = R$.
14. $R \bullet \varepsilon = R$.
15. $R \bullet \emptyset = \emptyset$.

思考题： $\emptyset^* = ?$ ， $\emptyset^0 = ?$ ， $\emptyset^i = ?$ ， 其中$i \geq 1$

4.2 正则表达式与FA的等价关系

FA & RE



Equivalence with FA

Theorem 4. 4 **A language is regular if and only if some regular expression describes it.**

Lemma 4.5 **If a language is described by a regular expression, then it is regular.**

（正则表达式表示的语言是正则语言，RE⇒FA）

Lemma 4.6 **If a language is regular, then it is described by a regular expression.**

（正则语言可以用正则表达式表示，FA⇒RE）

Equivalence with FA

Lemma 4.5 If a language is described by a regular expression, then it is regular.

（正则表达式表示的语言是正则语言，RE⇒FA）

Proof Idea:

RE R describes L(R) , L(R) is RL. （求证）

⇕ ⇕

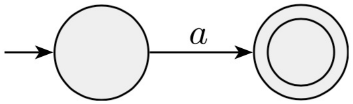
NFA N recognizes L(N) , L(N) is RL. （已知）

How to convert RE R to NFA N ?

Equivalence with FA

Proof1: Let's convert R to NFA N. We consider the six cases in the formal definition of regular expressions.

1. R = a for some a ∈ Σ . Then L(R) = {a}, and the following NFA recognizes L(R).

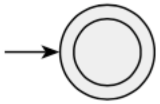


- Note that this machine fits the definition of an NFA but not that of a DFA。
- Formally, N = { $\{q_1, q_2\}$, Σ, δ, q_1, $\{q_2\}$ } , where we describe δ By saying that $\delta(q_1,a)=\{q_2\}$ and that $\delta(r, b)=\phi$ for $r \neq q_1$ or $b \neq a$.

Equivalence with FA

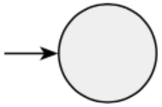
Proof1（续）：

2. R = ε. Then L(R) = {ε}, and the following NFA recognizes L(R).



Formally, $N = \{ \{q_1\}, \Sigma, \delta, q_1, \{q_1\} \}$, where $\delta(r, b) = \phi$ for any r and b.

3. R = ∅. Then L(R) = ϕ, and the following NFA recognizes L(R).



Formally, $N = \{ \{q\}, \Sigma, \delta, q, \phi \}$, where $\delta(r,b) = \phi$ for any r and b.

Equivalence with FA

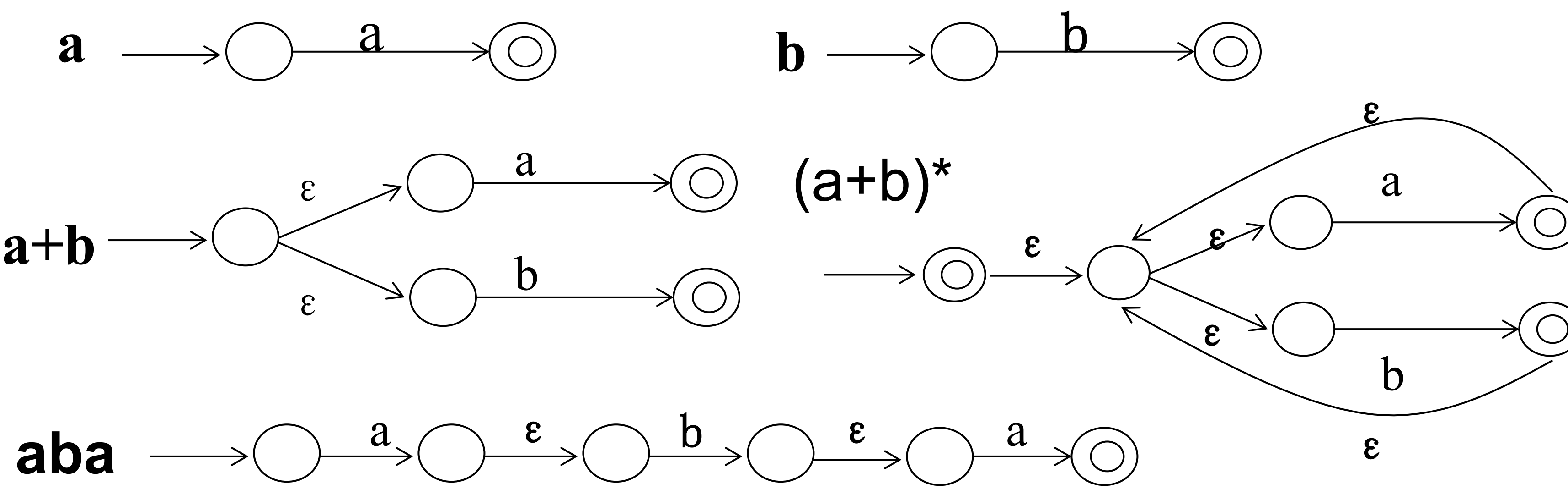
Proof1（续）: Let's convert R into an NFA N. We consider the six cases in the formal definition of regular expressions.

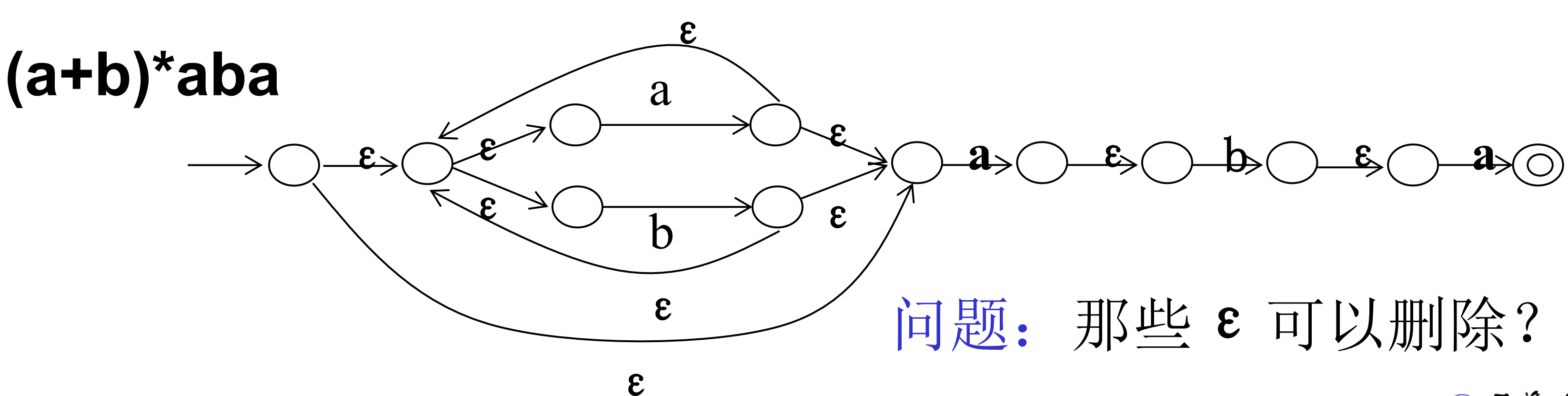
4. $\mathbf{R = R_1 + R_2}$
5. $\mathbf{R = R_1 . R_2}$
6. $\mathbf{R = R_1^*}$

We construct the NFA for R from the NFAs for R_1 and R_2 (or just R_1 in case 6) and the appropriate closure construction. THeorem4.1, 4.2, 4.3.

Equivalence with FA

Example 4.6 将正则表达式（a+b）*aba转化为NFA 。



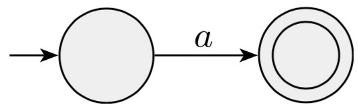


问题：那些 ε 可以删除？

Equivalence with FA

Proof2：对正则表达式的运算符个数n进行归纳证明。

1. n=0，R是单个字母，如R=a, 造自动机识别它



2. 设定理对n=k成立

3. n=k+1时，r有3种情况r_1+r_2, $r_1.r_2$, r_1^*。对最后计算的一个符号分情况（连接，并，星）用定理（**Theorem4.1,4.2 , 4.3**）即得。

Equivalence with FA

Lemma 4.6 If a language is regular, then it is described by a regular expression.

（正则语言可以用正则表达式表示，FA⇒RE）

Proof Idea:

方法一：GNFA法

① RL有DFA M识别，把DFA转化成广义的GNFA。

② 把广义的GNFA转化成正则表达式 RE 。

方法二：R_{ij}^{k} 迭代法

GNFA

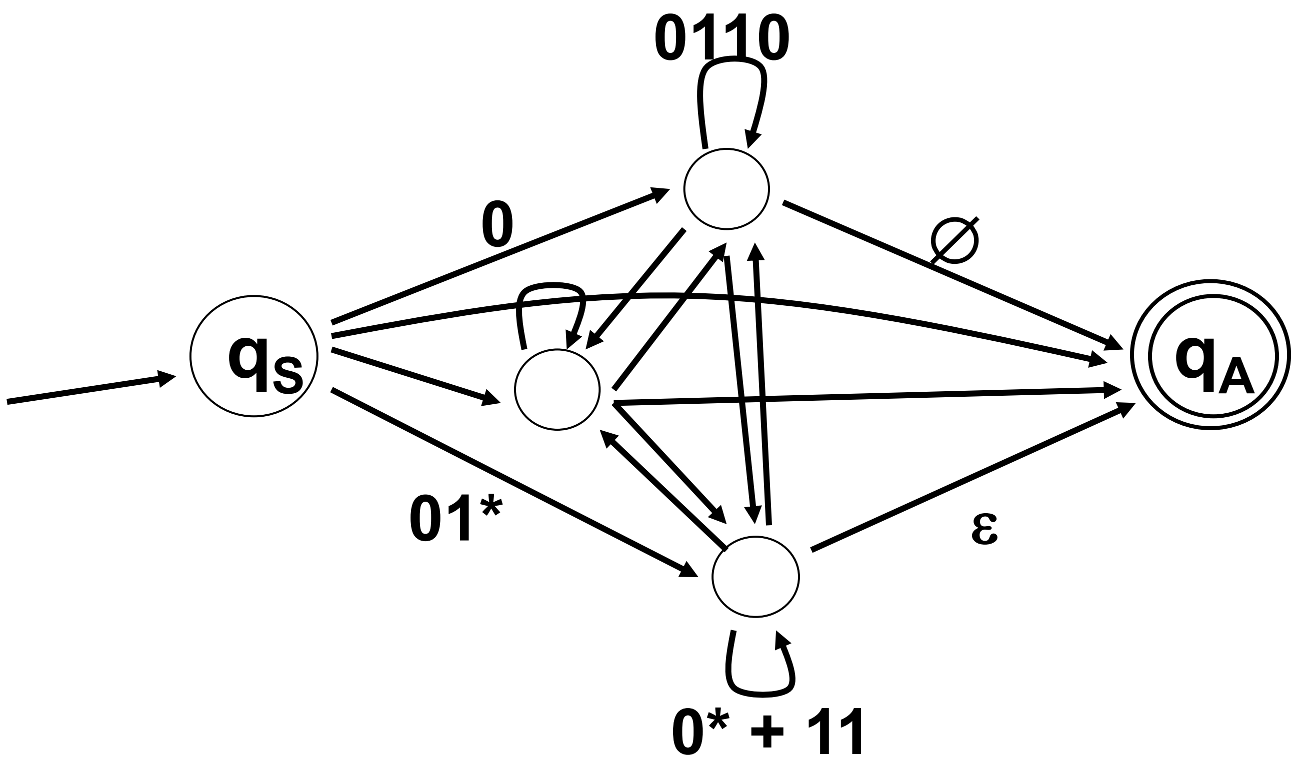
Definition 4.3

A Generalized nondeterministic finite automaton(GNFA) is a 5-tuple$(Q, \Sigma, \delta, q_{start}, q_{accept})$,where

1. Q is the finite states.
2. Σ is the input alphabet.
3. q_{start} is the start state.
4. q_{accept} is the accept state.
5. $\delta:(Q-\{q_{accept}\})\times(Q-\{q_{start}\}) \rightarrow \mathcal{R}$ is the transition function.

- $\mathcal{R}$ is the set of regular expressions over Σ.
- $\delta(q_i, q_j)=R \Leftrightarrow$ from q_i to q_j has the RE R as the label, $R\in\mathcal{R}$.

GNFA



A generalized nondeterministic finite automaton

GNFA的特点

1. The interior $Q-\{q_{accept},q_{start}\}$ is fully connected by δ
2. From q_{start} only 'outgoing transitions'
3. To q_{accept} only 'ingoing transitions'
4. Impossible $q_i \to q_j$ transitions are "$\delta(q_i,q_j) = \emptyset$"
5. $q_{accept} \neq q_{start}$

Observation: This GNFA recognizes the language L(R)

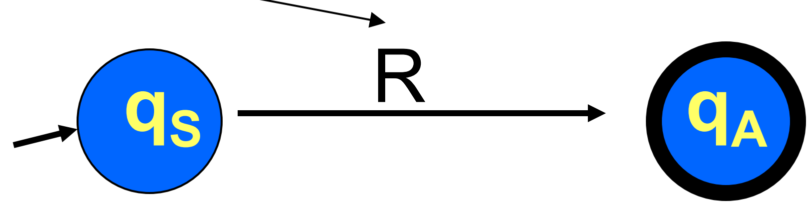
- **GNFA**与一般**NFA**的唯一区别: GNFA的边推广为 RE（子自动机），而不是单个的字母或**ε**。
- 为了简洁易读，通常省略不可能的转移$(q_i,q_j) = \emptyset$。

Equivalence with FA

<u>Proof Idea</u> (given a DFA M):

减少状态（减少跳转标号）， 扩大广义边（子程程序）

1. Construct an equivalent GNFA M’ with k≥2 states
2. Reduce one-by-one the internal states until k=2
   逐个等价地减少状态，把内部的正则表达式变大
3. This GNFA will be of the form



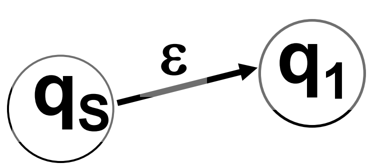
This regular expression R will be such that L(R) = L(M)

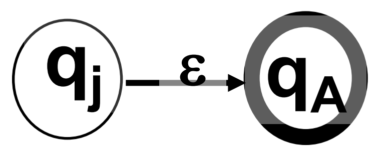
Equivalence with FA

证明（方法一：GNFA法）

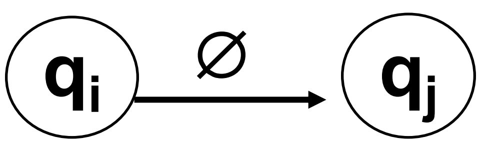
Let M have k states $Q=\{q_1,\ldots,q_k\}$

1. Add two states q_{accept} and q_{start} (加首尾)
   - Connect q_{start} to earlier q_1:
   - Connect old accepting states to q_{accept}

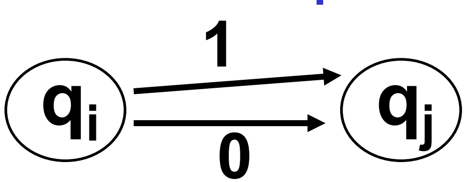




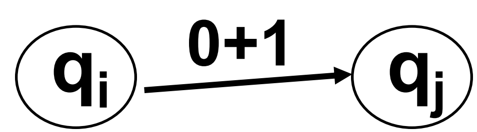
2. Complete missing transitions by $\varnothing$ （补空边）



3. Join multiple transitions (并标号)



becomes



Equivalence with FA

4. Rip the internal states, one by one

① Removing state $q_{rip} \in Q - \{q_{start}\} - \{q_{accept}\}$：

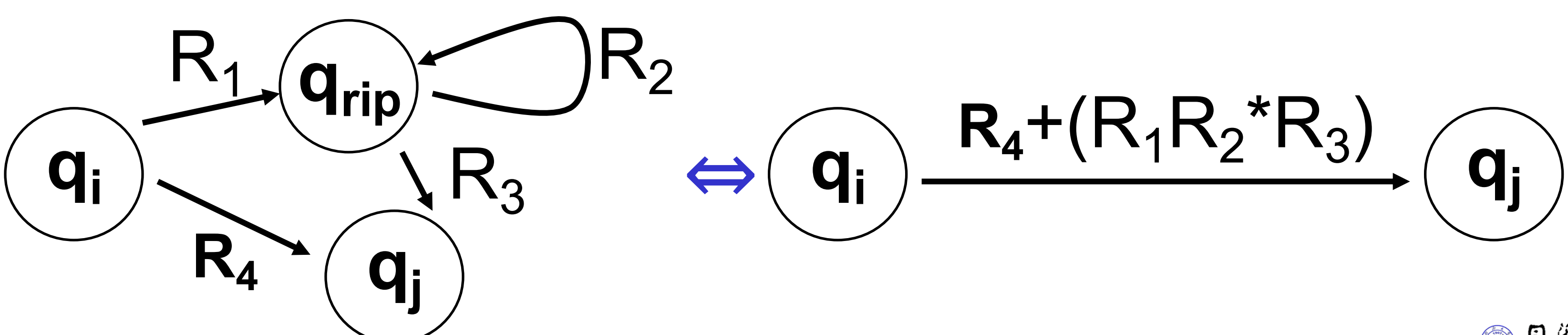
$Q' = Q - \{q_{rip}\}$ 逐步减少内态

① Changing the transition function δ by

$$\delta'(q_i,q_j) = \delta(q_i,q_j) + \delta(q_i,q_{rip})(\delta(q_{rip},q_{rip}))^*\delta(q_{rip},q_j)$$

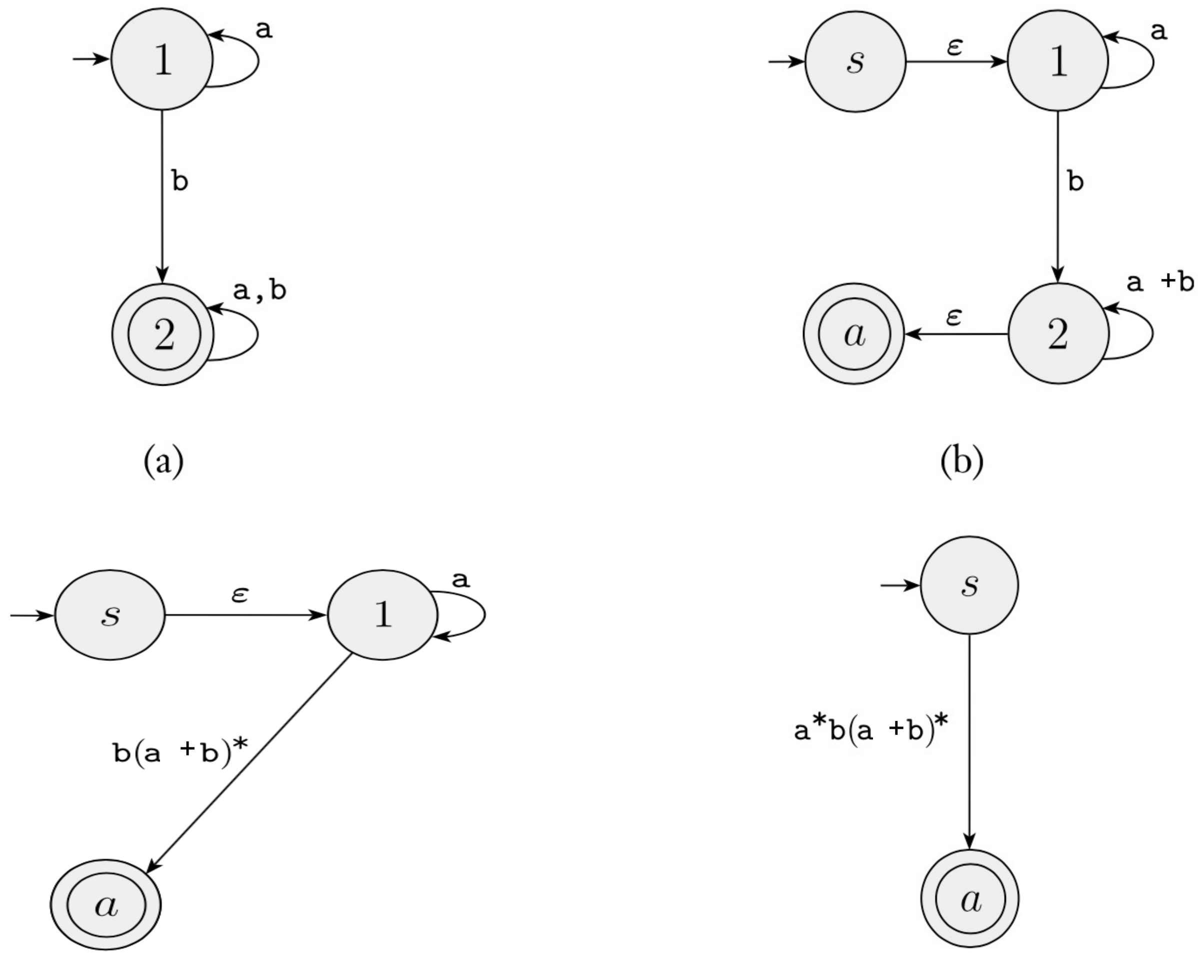
for every $q_i \in Q' - \{q_{accept}\}$ and $q_j \in Q' - \{q_{start}\}$

5. If k > 2 goto 4, else return R



Equivalence with FA

Example 4.7 Converting a two-state DFA to an equivalent regular expression.



(a) (b) (c) (d)

Equivalence with FA

证明（方法二）

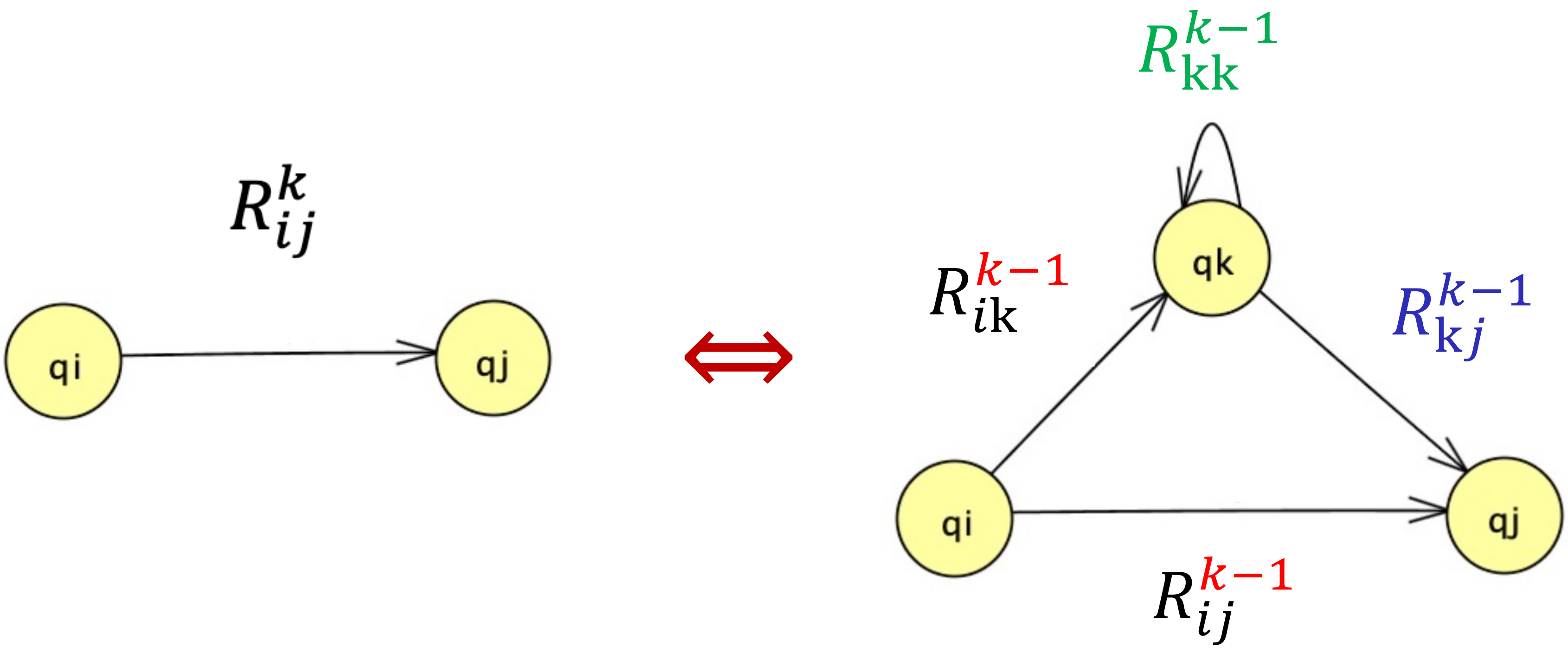
设 DFA M($\{q_1,q_2,\ldots,q_n\}$, $\sum$, δ,q_1,F) 接受语言 L，对M的状态进行了编号。在此基础上，引入记号R_{ij}^k，它是字符串的集合，定义如下：

$R_{ij}^k = \{ x \mid \delta(q_i,x) = q_j$，且中间不经过编号大于k的状态，但i、j可以大于k，$x \in \sum^* \}$ 注意：x是字符串

k的意义是： 对于x的一切非空真前缀y，$\delta(q_i,y) = q_m$，其中$m \le k$。

Equivalence with FA

证明（方法二）　基本思想：引入R_{ij}的递归定义。



$$R_{ij}^{k} = R_{ij}^{k-1} + R_{ik}^{k-1} (R_{kk}^{k-1})^{*} R_{kj}^{k-1}$$

Equivalence with FA

R_{ij}^{k}的递归定义如下：

① $R_{ij}^{0} = \{a \mid a \in \sum, 且\ \delta(q_i,a) = q_j\}$ （i≠j）

② $R_{ij}^{0} = \{a \mid a \in \sum, 且\ \delta(q_i,a) = q_j\} \cup \{\varepsilon\}$ （i = j）

③ $R_{ij}^{k} = R_{ij}^{k-1} + R_{ik}^{k-1} (R_{kk}^{k-1})^{*} R_{kj}^{k-1}$ （k=1,2,…,n）

解释：

① k=0 & i≠j时，q_i到q_j只能一步到达，∴a是单个字符；

② k=0 & i＝j时，除了a外，还包括ε， ∵对于任意状态q_i，都有$\delta(q_i, \varepsilon) = q_i$。

③ R_{ij}^{k}分两种情况讨论：见下一页。

Equivalence with FA

③ R_{ij}^{k}分两种情况讨论（续）

Case 1：若M从q_i出发，读字符串x，到达q_j的过程中，不经过编号大于k-1的任何状态，则x应在R_{ij}^{k-1}中，当然也在R_{ij}^{k}中，所以它应是R_{ij}^{k}的一部分；

Case 2：若经过编号等于k的状态一次或多次，状态变化序列如下，其中“…”处出现的状态编号均小于k。

$$q_i \ldots q_k \ldots q_k \ldots q_k \ldots q_j$$

从q_i …读过x的子串属于R_{ik}^{k-1}，从$q_k \ldots q_k \ldots$读过的x的子串属于$(R_{kk}^{k-1})^*$，从最后一个q_k开始，$q_k \ldots$读过的x的子串属于R_{kj}^{k-1}。因此，x应在$R_{ik}^{k-1}(R_{kk}^{k-1})^*R_{kj}^{k-1}$中。

综合case1、2有：$R_{ij}^{k} = R_{ij}^{k-1} + R_{ik}^{k-1}(R_{kk}^{k-1})^*R_{kj}^{k-1}$

Equivalence with FA

回到原点：Lemma 4.6 If a language is regular, then it is described by a regular expression.

即，已知DFA M识别语言L(M)，如何证明语言L(M)可用正则表达式描述？

$$\mathrm{L}(M) = \bigcup_{q_f \in F} R_{1f}^{n}$$

L(M)就是DFA M识别的语言，其中q_f是可接受状态。

问题：如何将L(M)用正则表达式表示呢？

如果对于任意的R_{1f}^{n}，存在正则表达式r_{1f}^{n}，则即可。

Equivalence with FA

现证，对于任何R_{ij}^k，存在正则表达式r_{ij}^k 代表R_{ij}^k 。

归纳基础： k=0。 R_{ij}^0是一个有穷集合，其中每个元素是$\sum$中的符号或ε， 因此r_{ij}^0可以写成$a_1+a_2+\ldots+a_p$ $(i\neq j)$或$a_1+a_2+\ldots+a_p+\varepsilon$ $(i=j)$的形式。这里，$\{a_1,a_2,\ldots,a_p\}$是使$\delta(q_i,a)=q_j$的一切符号a的集合。

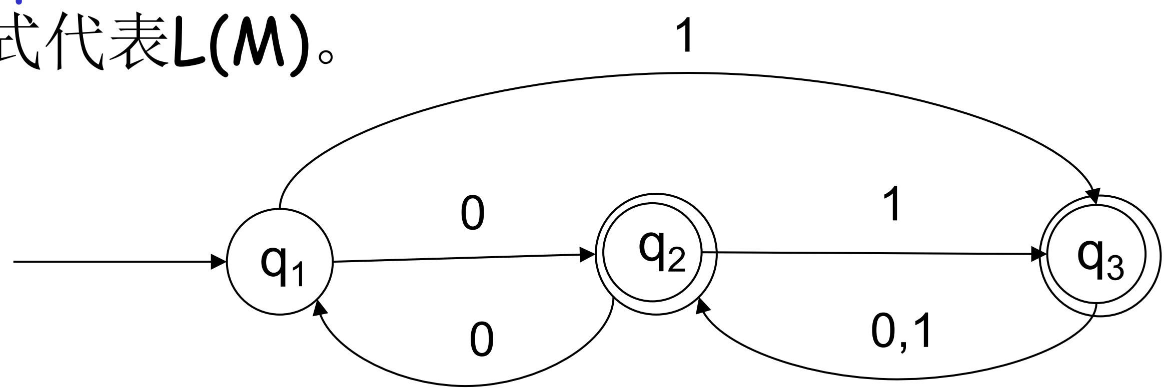
归纳步骤：设对m<k的一切m，都已求出正则表达式r_{ij}^m代表R_{ij}^m， 现在考虑m=k。根据R_{ij}^k递归定义，存在正则表达式

$$r_{ij}^k= r_{ik}^{k-1}(r_{kk}^{k-1})^*r_{kj}^{k-1}+r_{ij}^{k-1}$$ 代表R_{ij}^k 。

证毕。

Equivalence with FA

Example 4.8 给定一个DFA M，按照证明方法二 构造一个正则表达式代表L(M)。



分析：

1. 共3个状态，n=3；k＝0，1，2，3；
2. 根据递归公式求r_{ij}^k，其中k=0，1，2.（见下页）

$$r_{ij}^k = r_{ik}^{k-1}(r_{kk}^{k-1})^* r_{kj}^{k-1} + r_{ij}^{k-1}$$

3. 由状态图可知L(M) = $R_{12}^3 \cup R_{13}^3$，问题是求$r_{12}^3 + r_{13}^3$代表L(M)。

Equivalence with FA

	k=0	k=1	k=2
r_{11}^k	ε	ε	(00)*
r_{12}^k	0	0	0(00)*
r_{13}^k	1	1	0*1
r_{21}^k	0	0	0(00)*
r_{22}^k	ε	ε+00	(00)*
r_{23}^k	1	1+01	0*1
r_{31}^k	∅	∅	(0+1)(00)*0
r_{32}^k	0+1	0+1	(0+1)(00)*
r_{33}^k	ε	ε	ε+(0+1)0*1

$r_{12}^3 = r_{12}^2 + r_{13}^2 (r_{33}^2)^* r_{32}^2$

$r_{13}^3 = r_{13}^2 + r_{13}^2 (r_{33}^2)^* r_{33}^2$

$r = r_{12}^3 + r_{13}^3$

$= \cdots\cdots$

4.3 正则表达式代数定律

主要目的：正则表达式的变换。

基本问题：在定义相同语言的意义下，两个表达式等价。

基本思路：类比算术代数，比如：加法交换律、乘法分配律等

用途：正则表达式化简、变换、推算

4.3 正则表达式代数定律

1. 交换律与结合律

设L、M和N是正则表达式，则：

① L + M = M + L，并的交换率

② (L + M) + N = L + (M + N)，并的结合律

③ (LM)N = L(MN)， 连接的结合律

4.3 正则表达式代数定律

2. 单位元与零元

单位元：当运算符作用于单位元和某个其它值时，结果等于其他值。

零　元：当运算符作用到零元和其它值时，结果是零元。

① $\Phi + L = L + \Phi = L$，　Φ是并运算的单位元

② $\varepsilon L = L\varepsilon = L$,　　　　ε是连接运算的单位元

③ $\Phi L = L\Phi = \Phi$，　　　Φ是连接运算的零元

4.3 正则表达式代数定律

3. 分配律

L(M + N) = LM + LN，连接对于并的左分配律

(M + N)L = ML + NL，连接对于并的右分配律

4. 幂等律

幂等：一个运算符作用到两个相同的参数值，结果还是那个值。

L + L = L，并的幂等律

5. 与闭包有关的定律

① $(L^*)^* = L^*$，表达式闭包的闭包，不改变该语言

② $L(\Phi^*) = L(\varepsilon)=\{\varepsilon\}$，$\Phi$的闭包与ε等价，可以写$\Phi^*$
$=\varepsilon$

③ $\varepsilon^* = \varepsilon$，多个空串连接，还是空串

④ $L^+ = LL^* =L^*L$，L^+表示1个或多个L

⑤ $L^* = L^+ + \varepsilon$

4.4 正则表达式的应用

- UNIX中的正则表达式
- 词法分析
- 查找文本中的模式

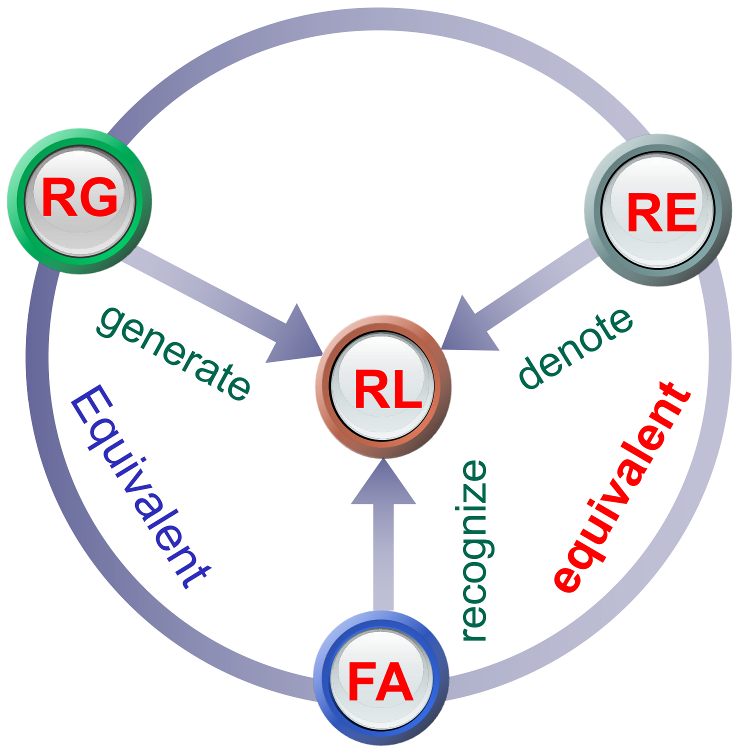
参见：

1. 霍普克罗夫特. 自动机理论、语言和计算导论：第3版[M]. 机械工业出版社，2008. p73-77
2. 陈火旺. 程序设计语言编译原理.第3版[M]. 国防工业出版社，2000. 第三章词法分析

Review

- Review:
  - Language & Regular language
    - A language is a set of strings;
    - language is called a regular language if some finite automaton recognizes it.
  - Regular Operations
    - Theorem 4.1,4.2 , 4.2.
  - Regular Expressions
    - The value of a Regular expression is a language.
    - Theorem 4.4 RL $\leftrightarrow$ RE (describes it)
    - Lemma 4.5 If a language is described by a regular expression , then it is regular
    - Lemma 4.6 If a language is regular, then it is described by a regular expression.

Review



- RL: Regular Language
- RG: Regular Grammar
- RE: Regular Expression
- FA: Finite Automaton