*Extrema:*

# Correctly Calculating min and max

Walter E. Brown, Ph.D.

< webrown.cpp @ gmail.com >



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## A little about me

- B.A. (math's); M.S., Ph.D. (computer science).
- Professional programmer for over 50 years, programming in C++ since 1982.
- Experienced in industry, academia, consulting, and research:
  - Founded a Computer Science Dept.; served as Professor and Dept. Head; taught and mentored at all levels.
  - Managed and mentored the programming staff for a reseller.
  - Lectured internationally as a software consultant and commercial trainer.
  - Retired from the Scientific Computing Division at Fermilab, specializing in C++ programming and in-house consulting.
- Not dead — still doing training & consulting. (Email me!)

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## Emeritus participant in C++ standardization

- Written ~170 papers for WG21, proposing such now-standard C++ library features as gcd/lcm, cbegin/cend, common_type, and void_t, as well as all of headers <random> and <ratio>.
- Influenced such core language features as *alias templates*, *contextual conversions*, and *variable templates*; recently worked on *requires-expressions*, operator<=>, and more!
- Conceived and served as Project Editor for *Int'l Standard on Mathematical Special Functions in C++* (ISO/IEC 29124), now incorporated into <cmath>.
- Be forewarned: Based on my training and experience, I hold some rather strong opinions about computer software and programming methodology — these opinions are not shared by all programmers, but they should be! ☺

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## Description of this talk

- The C++ standard library long ago selected operator< as its ordering primitive.
- This brief talk will explain why operator< must be used with care, in even such seemingly simple algorithms as max and min.
- We will also discuss the use of operator< in other order-related algorithms, showing how easy it is to make mistakes when using the operator< primitive directly.

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## The intuitive approach ①

- As C-style macros:
  - `#define MIN (a, b) ( ((a) < (b)) ? (a) : (b) )`
  - `#define MAX(a, b) ( ((b) < (a)) ? (a) : (b) )`
- As simple functions:
  - `int min ( int a, int b ) { return a < b ? a : b; }`
  - `int max( int a, int b ) { return b < a ? a : b; }`
- Lifted, now as (C++20) function templates:
  - `auto min ( auto a, auto b ) { return a < b ? a : b; }`
  - `auto max( auto a, auto b ) { return b < a ? a : b; }`

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## The intuitive approach ②

- But those C++ templates …
  - `auto min ( auto a, auto b ) { return a < b ? a : b; }`
  - `auto max ( auto a, auto b ) { return b < a ? a : b; }`

… have several issues:

✗ The by-value parameter passage can be expensive (*e.g.*, for large string arg's).

✗ When the arguments have distinct types, it's unclear what the return type should be. (It's even non-obvious how to compare them generically — *e.g.*, consider signed vs. unsigned!)

✗ Major concern: are the algorithms correct for all values?

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## The cures are mostly straightforward

- ✓ Enforce consistent types via a named parameter type.
- ✓ Avoid expensive copies via call/return by ref-to-const.
- After these adjustments we have:
  - template< class T >
    T const &
    min( T const & a, T const & b ) { return a < b ? a : b; }
  - template< class T >
    T const &
    max( T const & a, T const & b ) { return b < a ? a : b; }

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## Alas, none of the code I've shown so far is right!

- Can you identify the misbehaviors?
  - template< class T >
    T const &
    min ( T const & a, T const & b ) { return a < b ? a : b; }
  - template< class T >
    T const &
    max( T const & a, T const & b ) { return b < a ? a : b; }
- Did you notice that each returns b when a == b?
  - Why should max and min of the same two arguments ever give the same result?
  - ("It took Stepanov 15 years to get min and max right.")

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## In other words, …

- … these algorithms mishandle the case of a == b!
  - "[At] CppCon 2014, Committee member Walter Brown mentioned that max returns the wrong value [when] both arguments have an equal value. …
  - "*Why should it matter which value is returned?*"
- Many programmers have made similar observations:
  - That equal values are indistinguishable, so …
  - It ought not matter which is returned, so …
  - This case is uninteresting and not worth even discussing.
- Alas, for min and max algorithms, such opinions are superficial and, in general, are incorrect!

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## Many types do distinguish equal values

- Example:
  - struct student {
    string name; int id;
    inline static int registrar = 0;
    S( string n ) : name{ n }, id{ registrar++ } { } // c'tor
    friend bool // hidden friend
    operator < ( student s1, student s2 )
    { return s1.name < s2.name; } // id *not salient*
    };
- Since each student variable has a unique id number, it matters greatly which one is returned by min/max!

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## An important insight

- Given two values a and b, in that order:
  - Unless we find a reason to the contrary, …
  - **min should prefer to return a,** and …
  - **max should prefer to return b.**
- Never should max and min return the same value:
  - When values a and b are in order, min should return a / max should return b; …
  - When values a and b are out of order, min should return b / max should return a.

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## Even more succinctly stated

- We should always prefer algorithmic stability …
  - … especially when it costs nothing to provide it!
- Recall what we mean by stability:
  - An algorithm dealing with items' order is stable …
  - If it keeps the original order of equal items.
- *I.e.*, a stable algorithm ensures that:
  - For all equal items a and b, …
  - a will precede b in its output …
  - Whenever a preceded b in its input.

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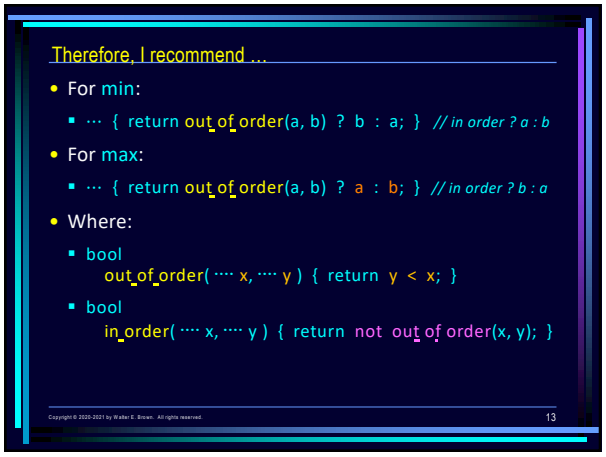


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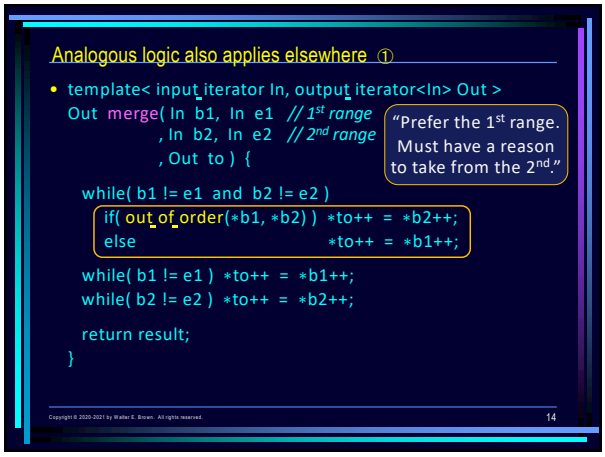


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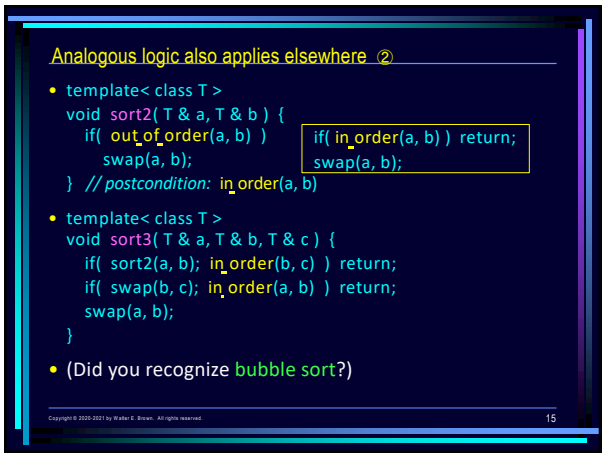


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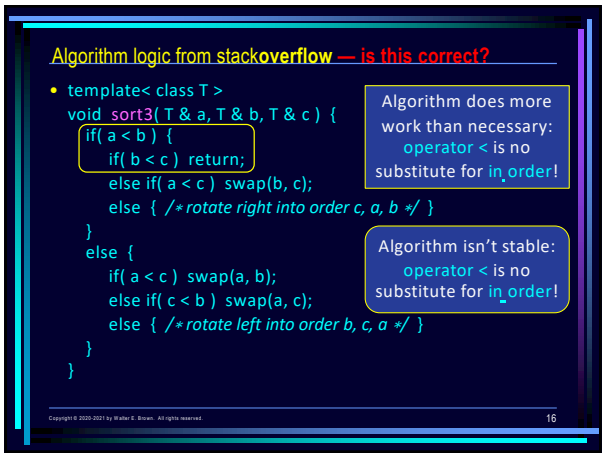


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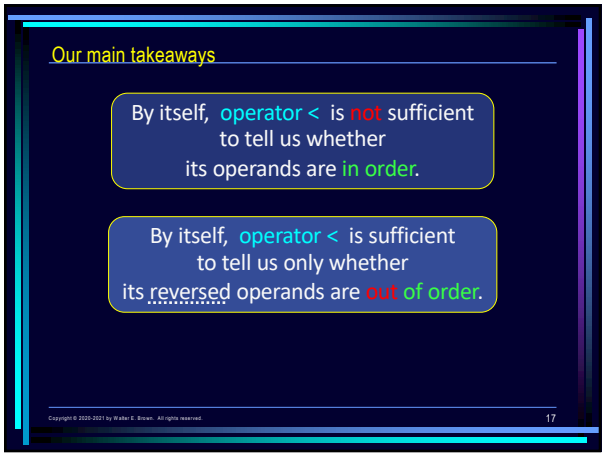


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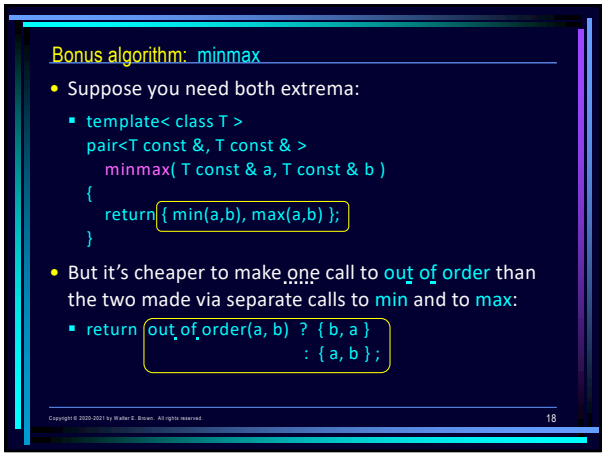


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## Finally, a modest programming challenge

- If you've never considered the generalized minmax:
  - template< forward iterator F >
    pair<F, F>
    minmax( F from, F upto ); // let N = distance(from, upto)
  - It returns m and M, iterators in [from, upto), such that
    m is the first iterator whose *m is smallest, and
    M is the last iterator whose *M is largest.
- Separate calls to min then max functions would lead to $\mathcal{O}(N + N = 2N)$ calls to out of order:
  - But Pohl's minmax needs only $3N/2$ calls to out of order.
  - (This is std::minmax_element in <algorithm>.)


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FIN

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