


*Extrema:*  
Correctly Calculating min and max

◀.....▶

Walter E. Brown, Ph.D.  
< webrown.cpp @ gmail.com >




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A little about me

- B.A. (math's); M.S., Ph.D. (computer science).
- Professional programmer for over 50 years, programming in C++ since 1982.
- Experienced in industry, academia, consulting, and research:
  - Founded a Computer Science Dept.; served as Professor and Dept. Head; taught and mentored at all levels.
  - Managed and mentored the programming staff for a reseller.
  - Lectured internationally as a software consultant and commercial trainer.
  - Retired from the Scientific Computing Division at Fermilab, specializing in C++ programming and in-house consulting.
- **Not dead — still doing training & consulting. (Email me!)**




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Emeritus participant in C++ standardization

- Written ~170 papers for WG21, proposing such now-standard C++ library features as `gcd/lcm`, `cbegin/cend`, `common_type`, and `void_t`, as well as all of headers `<random>` and `<ratio>`.
- Influenced such core language features as *alias templates*, *contextual conversions*, and *variable templates*; recently worked on *requires-expressions*, `operator<=>`, and more!
- Conceived and served as Project Editor for *Int'l Standard on Mathematical Special Functions in C++* (ISO/IEC 29124), now incorporated into `<cmath>`.
- Be forewarned: Based on my training and experience, I hold some rather strong opinions about computer software and programming methodology — these opinions are not shared by all programmers, but they should be! 😊



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Description of this talk

- The C++ standard library long ago selected `operator<` as its ordering primitive.
- This brief talk will explain why `operator<` must be used with care, in even such seemingly simple algorithms as `max` and `min`.
- We will also discuss the use of `operator<` in other order-related algorithms, showing how easy it is to **make mistakes** when using the `operator<` primitive directly.

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The intuitive approach ①

- As C-style macros:
  - `#define MIN(a,b) ((a)<(b)?(a):(b))`
  - `#define MAX(a,b) ((b)<(a)?(a):(b))`
- As simple functions:
  - `int min(int a, int b) { return a < b ? a : b; }`
  - `int max(int a, int b) { return b < a ? a : b; }`
- **Lifted**, now as (C++20) function templates:
  - `auto min(auto a, auto b) { return a < b ? a : b; }`
  - `auto max(auto a, auto b) { return b < a ? a : b; }`

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The intuitive approach ②

- But those C++ templates ...
  - `auto min(auto a, auto b) { return a < b ? a : b; }`
  - `auto max(auto a, auto b) { return b < a ? a : b; }`

... have several issues:

- ✗ The **by-value parameter passage** can be expensive (e.g., for large `string` arg's).
- ✗ When the arguments have distinct types, it's **unclear** what the **return type** should be. (It's even non-obvious how to compare them generically — e.g., consider **signed** vs. **unsigned**!)
- ✗ Major concern: are the algorithms **correct for all values**?

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The cures are mostly straightforward

- ✓ Enforce consistent types via a **named parameter type**.
- ✓ Avoid expensive copies via call/return by ref-to-const.
- After these adjustments we have:
  - `template< class T >`  
`T const &`  
`min( T const & a, T const & b ) { return a < b ? a : b; }`
  - `template< class T >`  
`T const &`  
`max( T const & a, T const & b ) { return b < a ? a : b; }`

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Alas, none of the code I've shown so far is right!

- Can you identify the misbehaviors?
  - `template< class T >`  
`T const &`  
`min ( T const & a, T const & b ) { return a < b ? a : b; }`
  - `template< class T >`  
`T const &`  
`max( T const & a, T const & b ) { return b < a ? a : b; }`
- Did you notice that each returns `b` when `a == b`?
  - Why should `max` and `min` of the same two arguments ever give the same result?
  - ("It took Stepanov 15 years to get `min` and `max` right.")

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In other words, ...

- ... these algorithms mishandle the case of `a == b`!
  - "[At] CppCon 2014, Committee member Walter Brown mentioned that `max` returns the wrong value [when] both arguments have an equal value. ...
  - "Why should it matter which value is returned?"
- Many programmers have made similar observations:
  - That equal values are indistinguishable, so ...
  - It ought not matter which is returned, so ...
  - This case is uninteresting and not worth even discussing.
- Alas, for `min` and `max` algorithms, such opinions are superficial and, in general, are incorrect!

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Many types do distinguish equal values

- Example:
  - `struct student {`  
`string name; int id;`  
`inline static int registrar = 0;`  
`S( string n ) : name{ n }, id{ registrar++ } { } // c'tor`  
`friend bool` // hidden friend  
`operator < ( student s1, student s2 )`  
`{ return s1.name < s2.name; } // id not salient`  
`};`
- Since each `student` variable has a unique `id` number, it matters greatly which one is returned by `min/max`!

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An important insight

- Given two values `a` and `b`, in that order:
  - Unless we find a reason to the contrary, ...
  - `min` should prefer to return a, and ...
  - `max` should prefer to return b.
- Never should `max` and `min` return the same value:
  - When values `a` and `b` are in order,  
`min` should return `a` / `max` should return `b`; ...
  - When values `a` and `b` are out of order,  
`min` should return `b` / `max` should return `a`.

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Even more succinctly stated

- We should always prefer algorithmic stability ...
  - ... especially when it costs nothing to provide it!
- Recall what we mean by stability:
  - An algorithm dealing with items' order is stable ...
  - If it keeps the original order of equal items.
- I.e., a stable algorithm ensures that:
  - For all equal items `a` and `b`, ...
  - `a` will precede `b` in its output ...
  - Whenever `a` preceded `b` in its input.

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Therefore, I recommend ...

- For min:
 

```
... { return out_of_order(a, b) ? b : a; } // in_order ? a : b
```
- For max:
 

```
... { return out_of_order(a, b) ? a : b; } // in_order ? b : a
```
- Where:
 

```
bool out_of_order( ... x, ... y ) { return y < x; }
bool in_order( ... x, ... y ) { return not out_of_order(x, y); }
```

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Analogous logic also applies elsewhere ①

- template< input\_iterator In, output\_iterator<In> Out >
 

```
Out merge( In b1, In e1 // 1st range
            , In b2, In e2 // 2nd range
            , Out to ) {

                while( b1 != e1 and b2 != e2 )
                {
                    if( out_of_order(*b1, *b2) ) *to++ = *b1++;
                    else *to++ = *b2++;

                    while( b1 != e1 ) *to++ = *b1++;
                    while( b2 != e2 ) *to++ = *b2++;

                    return to;
                }
            }
```

"Prefer the 1st range. Must have a reason to take from the 2nd."

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Analogous logic also applies elsewhere ②

- template< class T >
 

```
void sort2( T & a, T & b ) {
                if( out_of_order(a, b) ) { if( in_order(a, b) ) return;
                swap(a, b);
            } // postcondition: in_order(a, b)
```
- template< class T >
 

```
void sort3( T & a, T & b, T & c ) {
                if( sort2(a, b); in_order(b, c) ) return;
                if( swap(b, c); in_order(a, b) ) return;
                swap(a, b);
            }
```
- (Did you recognize bubble sort?)

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Algorithm logic from stackoverflow — is this correct?

- template< class T >
 

```
void sort3( T & a, T & b, T & c ) {
                if( a < b ) {
                    if( b < c ) return;
                    else if( a < c ) swap(b, c);
                    else { /* rotate right into order c, a, b */ }
                }
                else {
                    if( a < c ) swap(a, b);
                    else if( c < b ) swap(a, c);
                    else { /* rotate left into order b, c, a */ }
                }
            }
```

Algorithm does more work than necessary: operator < is no substitute for in\_order!

Algorithm isn't stable: operator < is no substitute for in\_order!

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Our main takeaways

By itself, operator < is **not** sufficient to tell us whether its operands are **in order**.

By itself, operator < is sufficient to tell us only whether its **reversed** operands are **out of order**.

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Bonus algorithm: minmax

- Suppose you need both extrema:
 

```
template< class T >
pair<T const &, T const &>
minmax( T const & a, T const & b )
{
    return { min(a,b), max(a,b) };
}
```
- But it's cheaper to make one call to out\_of\_order than the two made via separate calls to min and to max:
 

```
return out_of_order(a, b) ? { b, a }
                        : { a, b };
```

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Finally, a modest programming challenge

- If you've never considered the generalized `minmax`:
  - `template< forward_iterator F >`  
`pair<F, F>`  
`minmax( F from, F upto ); // let N = distance(from, upto)`
  - It returns `m` and `M`, iterators in `[from, upto)`, such that `m` is the first iterator whose `*m` is smallest, and `M` is the last iterator whose `*M` is largest.
- Separate calls to `min` then `max` functions would lead to  $\mathcal{O}(N + N = 2N)$  calls to `out of order`:
  - But Pohl's `minmax` needs only  $3N/2$  calls to `out of order`.
  - (This is `std::minmax_element` in `<algorithm>`.)


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FIN

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< webrown.cpp @ gmail.com >



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