Introduction to control systems
TI0118

# Homework II

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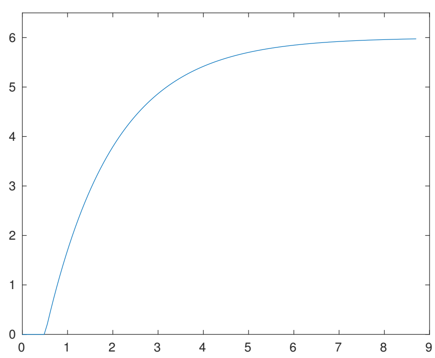
## Exercise 1

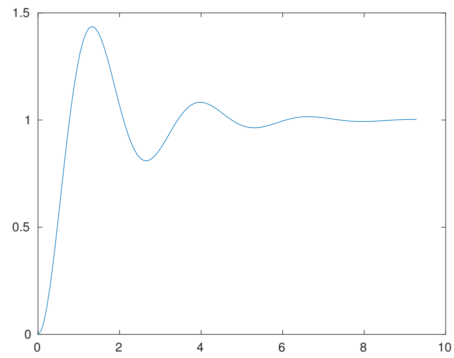
1. Load the data and plot the responses for systems A and B.

   **Answer:** To load the data on MatLab we may use the load function:

```
load HW2_ex1_dataA.txt; #load data A
load HW2_ex1_dataB.txt; #load data B
t1=HW2_ex1_dataA(:,1); #extract the time vector from data A
y1=HW2_ex1_dataA(:,2); #extract the response values from data A
t2=HW2_ex1_dataB(:,1); #extract the time vector from data B
y2=HW2_ex1_dataB(:,2); #extract the response values from data B
figure;
plot(t1,y1); #plot of system's A response
axis([0 9 0 6.5])
figure;
plot(t2,y2); #plot of system's B response
```



(a) Response of the System A



(b) Response of the System B

2. Identify the order of the systems. Based on the plots, estimate the transient response characteristics, such as time constant, settling time, rise time, peak time

and percentage of overshoot. Write the corresponding transfer functions $T_A(s)$ and $T_B(s)$ for the systems A and B, respectively.

**Answer:**

- System A: 1º order

  First Order systems has as principal transient characteristics: time constant, settling time and rise time.

  - Time constant: The time for the step response to rise to 63% of its final value. It may be obtained by $\tau = \frac{1}{a}$ $(c(t) = 1 - e^{-at})$.
  - Settling time: The time for the response to reach, and stay within, 2% of its final value. It may be obtained by $T_s = \frac{4}{a}$.
  - Rise time: The time for the response to go from 0.1 to 0.9 of its final value. It may be obtained by $T_r = \frac{2.2}{a}$

  Using this knowledge about first order systems, we may obtain the transfer function of the system A, since $G(s) = \frac{K}{\tau s + 1}$, where K is the gain of the system. Since we are welling to construct the transfer function from a data set, we may create a function on MatLab to do it. Since the system has time delay, we'll consider it while constructing its transfer function.

```
function [transferFunction]=firstOrderConstruction(t,y)
i=1;
t0=0; %time to the system's response to begin.
while y(i)==0
t0=t(i);
i=i+1;
end
tc=0; %time constant
for i=1:length(t)
    if y(i)<0.63*y(length(y))
        tc=i;
    end
end
if 0.63-(y(tc)/y(length(y)))>(y(tc+1)/y(length(y)))-0.63
    %taking the closest value to 63% of the final value
    tc=tc+1;
end
%a considering the delay time
a =1/(t(tc)-t0);
x=['a=',num2str(a)];
disp(a);
%time constant considering the delay time
tc=t(tc)-t0;
x=['time constant=',num2str(tc)];
disp(x);
%rise time
```

```
risetime = 2.2/a;
x=['rise  time=',num2str(risetime)];
disp(x);
%settling time
setltime=4/a;
x=['settling  time=',num2str(setltime)];
disp(x);
%gain
gain=y(length(y));
x=['gain=',num2str(gain)];
%tf
s=tf('s');
transferFunction=gain*exp(-t0*s)/(tc*s+ 1)
figure
hold on
step(transferFunction)
hold off
end
```

Using this function we obtain:

```
[func]=firstOrderConstruction(t1,y1)
time constant=1.5197
rise  time=3.3433
settling  time=6.0786

transferfFunction =

                        5.975
  exp(-0.484*s) * ----------
                      1.52 s + 1
```

So, we obtain the transfer function:

$$G(s) = \frac{5.975e^{-0.484s}}{1.52s + 1}$$

and its step on 2.

- System B: 2º order (Underdamped)

  Second Order systems has as principal transient characteristics: natural frequency, damping ratio, rise time, peak time, percent overshoot and settling time.

  - Natural frequency ($\omega_n$): Frequency of oscillation of the system without damping.
  - Damping ratio ($\zeta$): The damped oscillation of the system regardless of the time scale, it's: if a system goes into five cycles in its transient response, then it will have the same characteristic, independently of the time if takes going through its cycles, as any that has the same amount of it. We may obtain quantitatively by: $\zeta = \frac{Exponential decay frequency}{Natural frequency\ (rad/second)}$.
  - Rise time ($T_r$): The time for the response to go from 0.1 to 0.9 of its final value.

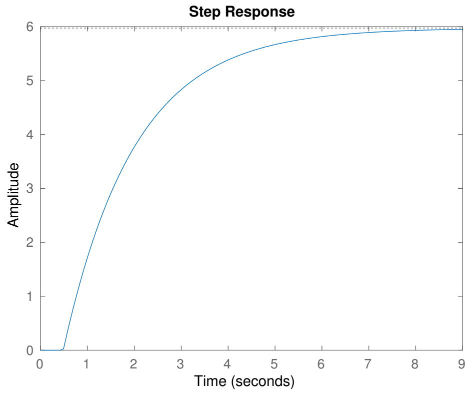



Figure 2: Response of the transfer function obtained

- Peak time ($T_p$): The time required to reach the first peak. It might be obtained by $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$.
- Percent overshoot ($\%OS$): The amount that the waveform overshoots the steady-state. It might be obtained by $\%OS = \dfrac{c_{max} - c_{final}}{c_{final}} \times 100 = e^{-\dfrac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$.
- Settling time ($T_s$): The time for the response to reach, and stay within, 2% of its final value. It may be obtained by $T_s = \dfrac{4}{\zeta\omega_n}$.

Using this knowledge about first order systems, we may obtain the transfer function of the system A, since the general form to second-order transfer function $G(s) = \dfrac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$. Since we are welling to construct the transfer function from a data set, we may create a function on MatLab to do it.

```
function [func]=secondOrderConstruction(t,y)
i=1;
while y(i+1)>y(i) %first peak
    i=i+1;
end
%peak time
tp=t(i);
x=['peak time=',num2str(tp)];
disp(x);
%percent overshoot
OS=(y(i)-y(length(y)))/y(length(y));
x=['%OS=',num2str(OS)];
disp(x);
%damping ratio
```

```
zeta= sqrt((log(OS))^2/((log(OS))^2+pi^2));
x=['zeta=',num2str(zeta)];
disp(x);
%natural frequency
w=pi/(tp*(sqrt(1-zeta^2)));
x=['natural frequency=',num2str(w)];
disp(x);
%settling time
setltime=4/(w*zeta);
x=['settling time=',num2str(setltime)];
disp(x);
%rise time
t01=0;
t09=0;
for i=1:length(t)
    if y(i)<0.1*y(length(y))
        t01=i-1;
    end
    if y(i)>0.9*y(length(y))
        t02=i-1;
        break;
    end
end
tr=t(t02)-t(t01);
x=['rise time=',num2str(tr)];
disp(x);
%transfer function
func=tf([w^2],[1 2*w*zeta w^2])
hold on
step(func)
hold off
end
```

```
func=secondOrderConstruction(t2,y2);
peak time=1.3263
%OS=0.4322
zeta=0.25798
natural frequency=2.4517
settling time=6.3243
rise time=0.58946

func =

          6.011
  ---------------------
  s^2 + 1.265 s + 6.011

Continuous-time transfer function
```

So, we obtain the transfer function:

$$G(s) = \frac{6.011}{s^2 + 1.265s + 6.011}$$

and its step on 3.

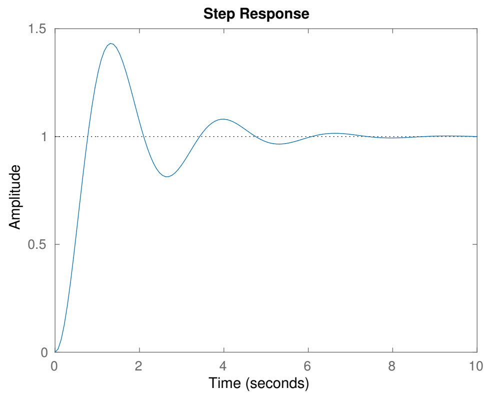



Figure 3: Response of the transfer function obtained

3. Plot and compare the step response of the systems $T_A(s)$ with the data provided in $HW2\_ex1\_dataA.txt$. **Answer:**In order do compare, we may overlap the two response in a single plot:

```
figure;
hold on
plot(t1,y1,'r');
step(func,'--');
axis([0 9 0 6.5])
hold off
```

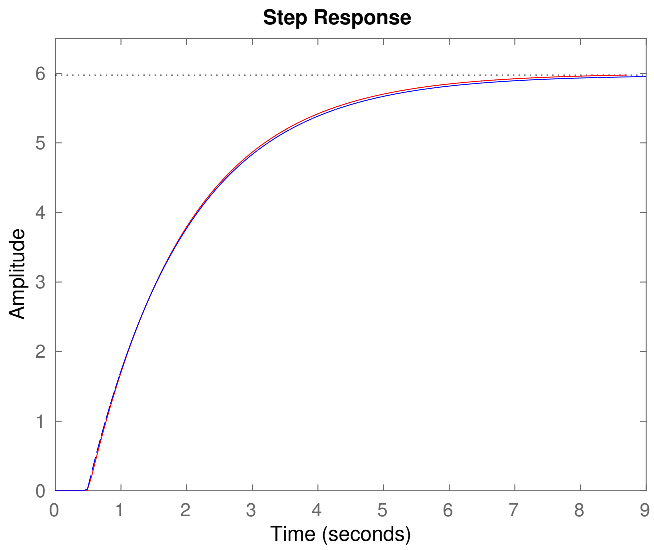



Figure 4: Response of the system and the plot of the data provided

It's possible to see that both system are very similar and there is some differences that maybe were caused by numerical approximation, since we only used data provided.

4. Plot and compare the step response of the systems $T_B(s)$ with the data provided in $HW2\_ex1\_dataB.txt$. **Answer:** In order do compare, we may overlap the two response in a single plot:

```
figure;
hold on
plot(t2,y2,'r');
step(func,'--');
hold off
```

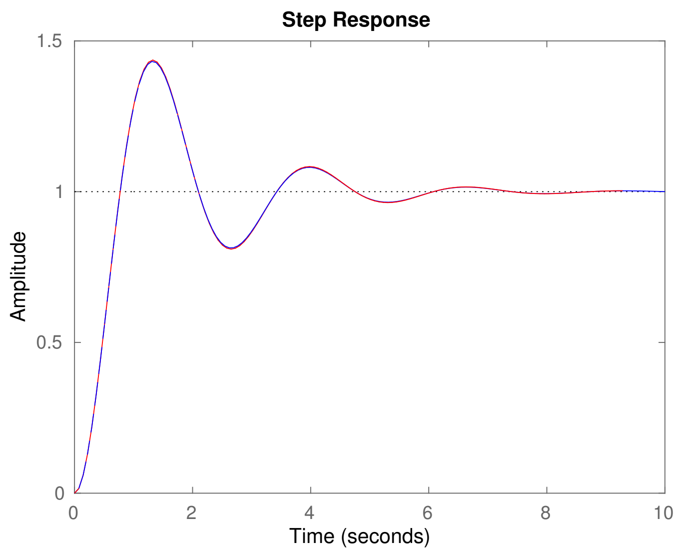



Figure 5: Response of the system and the plot of the data provided

It's possible to see that both system are very similar and there is some differences that maybe were caused by numerical approximation, since we only used data provided. Maybe there are more approximate ways of finding the transfer function, maybe by finding the $T_s$ and using it to find $\zeta$ and $\omega_n$, but it may have differences in the same way.

## Exercise 2

1. Find the transfer function, H(s) = Y (s)/U (s), using the block diagram reduction. **Answer:** There are some "properties" to help the block diagram reduction:

    - Serial blocks: They are equivalent to only one block with the sum of the transfer functions.

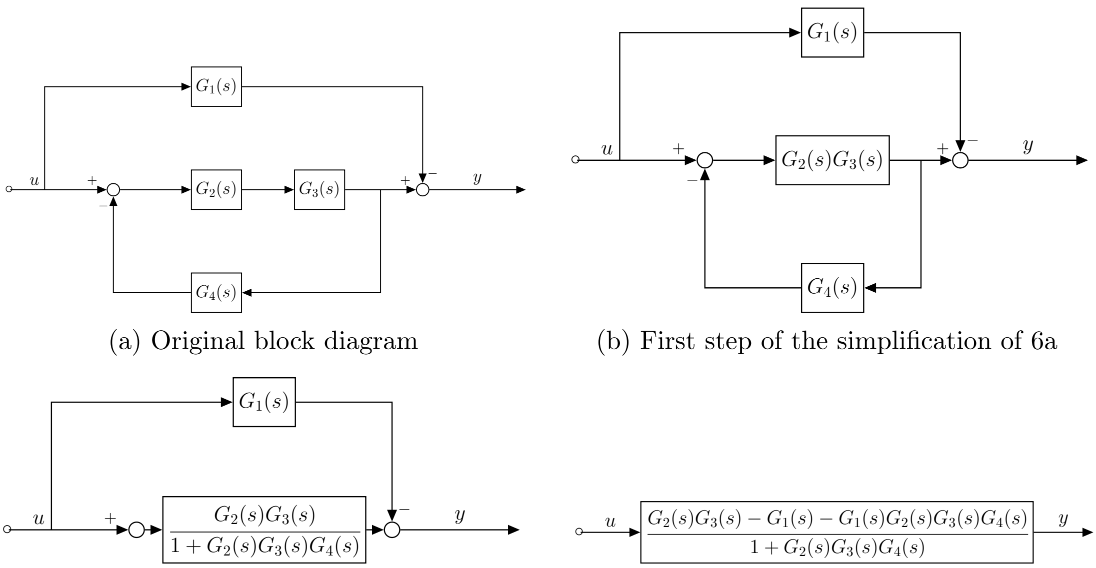
- Two parallel blocks: They are equivalent to $\dfrac{G_1(s)}{1+G_1(s)G_2(s)}$, where $G_1(s)$ is the transfer function of the block in the principal line.

By the application of these properties, we can find:

$$H(s) = \frac{G_2(s)G_3(s) - G_1(s) - G_1(s)G_2(s)G_3(s)G_4(s)}{1+G_2(s)G_3(s)G_4(s)}.$$

The process of reduction can be seen in the process on 6.

Figure 6: Reduction Process




(a) Original block diagram

(b) First step of the simplification of 6a

(c) Second step of the simplification of 6a

(d) Complete simplification of 6a

2. Given the transfer functions in 1, find the expression of H(s).

$$G_1(s) = \frac{1}{s+10}, G_2(s) = \frac{s-1}{s+2}, G_3(s) = \frac{1}{s-1}, G_4(s) = -\frac{8}{s+9} \tag{1}$$

**Answer:** Since we have already calculated the generic transfer function H(s) in (1), we simply need to substitute the given transfer functions.

Generic transfer function:

$$H(s) = \frac{G_2(s)G_3(s) - G_1(s) - G_1(s)G_2(s)G_3(s)G_4(s)}{1+G_2(s)G_3(s)G_4(s)},$$

Substituting the given transfer functions we get:

$$H(s) = \frac{\frac{s-1}{s+2} * \frac{1}{s-1} - \frac{1}{s+10} - \frac{1}{s+10} * \frac{s-1}{s+2} * \frac{1}{s-1} * \frac{-8}{s+9}}{1 + \frac{s-1}{s+2} * \frac{1}{s-1} * \frac{-8}{s+9}}$$

Which leads to:

$$H(s) = \frac{\dfrac{1}{s+2} - \dfrac{1}{s+10} + \dfrac{8}{(s+10)(s+2)(s+9)}}{1 - \dfrac{8}{(s+2)(s+9)}}$$

Thus,

$$H(s) = \frac{\dfrac{(s+10)(s+9) - (s+2)(s+9) + 8}{(s+10)(s+2)(s+9)}}{\dfrac{(s+2)(s+9) - 8}{(s+2)(s+9)}}$$

Simplifying it we obtain:

$$H(s) = \frac{8s + 80}{(s+10)(s^2 + 11s + 10)}$$

what leads to:

$$H(s) = \frac{8s + 80}{s^3 + 21s^2 + 120s + 100}.$$

$$H(s) = \frac{8}{(s+1)(s+10)}$$

3. Analyze the stability of H(s) found in point (2) of this exercise.

   **Answer:** It's possible to analyze the stability of the system by calculating the poles of the system. It's know that a system is stable if and only if every pole of its transfer function lies inside the left-half s-plane. Thus, analyzing the H(s) obtained on (2) the poles of the function are -10 and -1, which are in the lef-half s-plane. As all the poles real zero imaginary part it allow us to affirm that the system is *marginally stable.*

   Another way to check the stability is by the Routh table:

| | | |
|---|---|---|
| $s^2$ | 1 | 10 |
| $s^1$ | 11 | 0 |
| $s^0$ | $-\dfrac{\begin{vmatrix} 1 & 10 \\ 11 & 0 \end{vmatrix}}{11} = 10$ | $-\dfrac{\begin{vmatrix} 1 & 0 \\ 11 & 0 \end{vmatrix}}{11} = 0$ |

   Table 1: Caption

   Since, there's no change of signe in the first column, we may say that the system is stable.

4. Define the gain, the poles and zeros of H(s) found in point (2) of this exercise.

   **Answer:**

- The gain is the constant factor multiplying the transfer function numerator and denominator which has and amplification effect. Gain: 8.
- The poles of the system are the roots of H(s)'s denominator $(s+1)(s+10) \rightarrow$ -10 and -1. Poles: -10 and -1.
- The zeros of the system are thee roots of H(s)'s numerator $\rightarrow$ there are no zeros.

It's possible to see the placement of the poles and zeros on the imaginary plane in the following plot:

```
func=tf([8 80],[1 21 120 100]);
pzmap(func)
axis([-10 1 -2 2])
sgrid
```

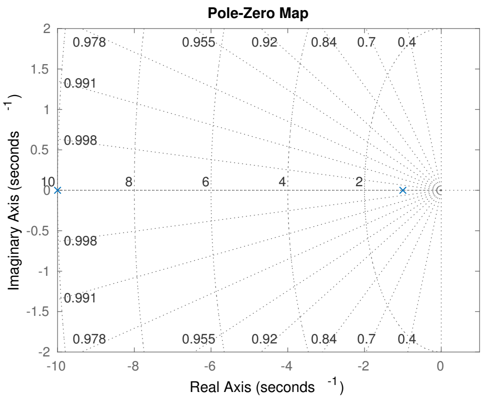



Figure 7: Pole-Zero Map

5. Plot the response of the system to a unit step input. From the plot, identify its initial value, steady-state value and settling time.

   **Answer:** It's possible to plot the response of the system to a unit step input using the following commands: (see on 8)

```
func=tf([8],[1 11 10]);
step(func)
axis([0 8 0 0.85])
```

Thereby, it's possible to define:

- Initial value = 0
- Steady-state = 0.8
- Settling time = 3.8679 seconds (time required for the response to reach and remain within a little error band.)

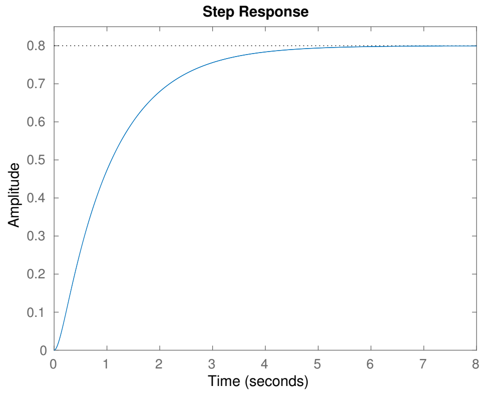



Figure 8: Unit step input response

## Exercise 3

$$G(s) = \frac{K(s+7)}{s(s^3+25s^2+196s+480)}$$

1. Evaluate the system type.

   **Answer:** The system type is the value of n in the denominator, given the transfer function $T(s) = \dfrac{K(s+z_1)(s+z_2)...}{s^n(s+p_1)(s+p_2)...}$, or the number of pure integrations in the forward path. Since, n=1, that is, there is only one pure integrator, the *system type is 1.*

2. Find the value of K to yield a 1% error in the steady-state for an input of 0.1t. **Answer:** By the theorem of the final value, we know that:

$$e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

   . Since we are looking for the value of K for a input of 0.1t and error of 1%, which is equivalent to $\dfrac{0.1}{s^2}$ in the frequency domain, we substitute and calculate it:

$$e(\infty) = \lim_{s\to 0} \frac{s * \dfrac{0.1}{s^2}}{1 + \dfrac{K(s+7)}{s(s^3+25s^2+196s+480)}} = \lim_{s\to 0} \frac{0.1}{s + \dfrac{K(s+7)}{s^3+25s^2+196s+480}},$$

   When, s approaches zero, we have:

$$e(\infty) = \lim_{s\to 0} \frac{0.1}{\dfrac{7K}{480}} = 0.01$$

Thus, it implies that K must be equal $\frac{4800}{7}$ or, equivalently, 685.71.

3. Find the static error constants for the value of K found in point (2) of this exercise.
**Answer:** The static error constant is basically the obtained when s approaches 0 of the function that is left on $\frac{sR(s)}{1+G(s)}$ when we substitue R(s)'s function considering an unitary gain function. Then considering a step input, ramp input and parabolic input, we will get the constant $K_v$ and $K_p$ and $K_a$, respectively.

   - $K_v = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{\frac{4800}{7}(s+7)}{s(s^3+25s^2+196s+480)} = \infty$
   - $K_p = \lim_{s\to 0} sG(s) = \lim_{s\to 0} \frac{\frac{4800}{7}(s+7)}{s^3+25s^2+196s+480} = 10$
   - $K_a = \lim_{s\to 0} s^2G(s) = \lim_{s\to 0} \frac{\frac{4800}{7}(s^2+7s)}{s^3+25s^2+196s+480} = 0$

4. Verify the stability of your system and plot its step response.

**Answer**: In order to analyse the stability of the system for $K = \frac{4800}{7}$, we need to generate the routh table and analyze it using the Routh-Hurwitz Criterion. The forward transfer function of the system is given by $T(s) = \frac{G(s)}{1+G(s)}$, which can be calculated in:

$$T(s) = \frac{\frac{4800}{7}(s+7)}{s^4+25s^3+196s^2+\frac{8160}{7}s+4700)}$$

Generating the Routh table: (2)

Seeing that there's no signal variation in the first column, we can affirm that the system is stable.

Thus, we can plot its step response (9):

| | | | |
|---|---|---|---|
| $s^4$ | $1$ | $196$ | $4800$ |
| $s^3$ | $25$ | $\frac{8160}{7}$ | $0$ |
| $s^2$ | $-\frac{\begin{vmatrix} 1 & 196 \\ 25 & \frac{8160}{7} \end{vmatrix}}{25} = \frac{5228}{35}$ | $-\frac{\begin{vmatrix} 1 & 4800 \\ 25 & 0 \end{vmatrix}}{25} = 4800$ | $0$ |
| $s^1$ | $-\frac{\begin{vmatrix} 25 & \frac{8160}{7} \\ \frac{5228}{35} & 4800 \end{vmatrix}}{\frac{5228}{35}} = 362.35$ | $0$ | $0$ |
| $s^0$ | $-\frac{\begin{vmatrix} \frac{5228}{35} & 4800 \\ 362.35 & 0 \end{vmatrix}}{362.35} = 4800$ | $0$ | $0$ |

Table 2: Caption

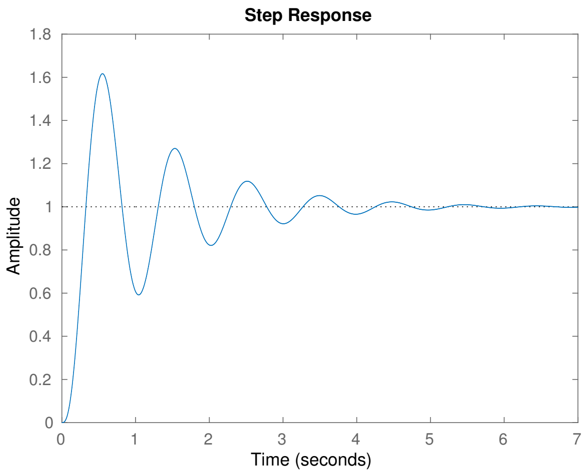



Figure 9: Unit step input response

## Exercice 4

$$G(s) = \frac{5(1+\alpha s)(1+2\frac{0.2}{12}s+\frac{1}{12^2}s^2)(1+2\frac{0.8}{24}s+\frac{1}{24^2}s^2)}{(1+10s)^2(1+0.05s)^2(1+2\frac{0.1}{10}s+\frac{1}{10^2}s^2)}$$

Consider two possibility (a) $\alpha = 8$ and (b) $\alpha = 12$. For both case (a) and case (b):

(a) Considering $\alpha = 8$, we get the following transfer function:

$$G(s) = \frac{5(1+8s)(1+2\frac{0.2}{12}s+\frac{1}{12^2}s^2)(1+2\frac{0.8}{24}s+\frac{1}{24^2}s^2)}{(1+10s)^2(1+0.05s)^2(1+2\frac{0.1}{10}s+\frac{1}{10^2}s^2)}$$

1. Find the poles and zeros of the system. Plot them in a zero-pole map and draw some conclusions.

   **Answer:** It's possible to find zeros by finding the numerator's roots of G(s) and the poles by finding the denominator's roots:

   – Poles
     * $(1+10s)^2 \rightarrow$ roots: $-\frac{1}{10}$, with multiplicity 2.
     * $(1+0.05s)^2 \rightarrow$ roots: $-20$,with multiplicity 2.
     * $10^{-2}s^2 + 2 * 10^{-2}s + 1 \rightarrow$ roots: $-1 + 3\sqrt{11}i$ and $-1 - 3\sqrt{11}i$.

     Poles: $-\frac{1}{10}, -\frac{1}{10}, -20, -20, -1+3\sqrt{11}i$ and $-1-3\sqrt{11}i$

   – Zeros
     * $1+8s \rightarrow$ roots: $-\frac{1}{8}$
     * $1+2\frac{0.2}{12}s+\frac{1}{12^2}s^2 \rightarrow$ roots: $-\frac{12}{5}+\frac{24\sqrt{6}}{5}i$ and $-\frac{12}{5}-\frac{24\sqrt{6}}{5}i$
     * $1+2\frac{0.8}{24}s+\frac{1}{24^2}s^2 \rightarrow$ roots: $-1.92+23.93i$ and $-1.92-23.93i$

     Zeros: $-\frac{1}{8}, -\frac{12}{5}+\frac{24\sqrt{6}}{5}i, -\frac{12}{5}-\frac{24\sqrt{6}}{5}i, -1.92+23.93i$ and $-1.92-23.93i$

   In order to plot the zero-pole map:

2. Find, and justify, a first order aprproximation, $G_a$ of G.

b $\alpha = 12$

1. Find the poles and zeros of the system. Plot them in a zero-pole map and draw some conclusions.
2. Find, and justify, a first order aprproximation, $G_a$ of G.

# Exercice 5

1.