Reinforcement Learning Quarto

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Invalid Date

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3 Introducción

Este es un libro creado para el curso de **"Markov Decision Processes to Reinforcement Learning with Python"**

4 Tarea 1

4.1 Exercise 1

Exercise 1 Read (Sec 1.1, pp 1-2 Sutton and Barto 2018) and answer the following. Explain why Reinforcement Learning differs for supervised and unsupervised learning.

El aprendizaje reforzado se centra en aprender a qué hacer y cómo hacer situaciones que lleven a maximizar una recompensa, por otro lado el aprendizaje supervisado consiste en aprender de un conjuto de entrenamiento de ejemplos etiquetados dados por un supervisor externo con conocimientos. El objetivo de este aprendizaje es que el sistema extrapole o generalice sus respuestas para actuar correctamente en situaciones que no están presentes en el conjunto de entrenamiento.

4.2 Exercise 2

Exercise 2 See the first Steve Bruton's youtube video about Reinforcement Learning. Then accordingly to its presentation explain what is the meaning of the following expression:

$$V_{\pi}(s) = E\left(\sum_{t} \gamma^{t} r_{t} | s_{0} = s\right)$$

La función mide de una forma que tan buenas son las acciones que se eligen, es decir la función de valor como la recompensa esperada habiendo elegido una política y un estado inicial.

4.3 Exercise 3

Exercise 3 Form (see Sutton and Barto 2018) obtain a time line pear year from 1950 to 2012.

```
library(bibtex)
## Activate the Core Packages
library(tidyverse) ## Brings in a core of useful functions
```

```
-- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
v dplyr     1.1.4     v readr     2.1.5
v forcats   1.0.0     v stringr   1.5.1
v ggplot2   3.5.1     v tibble    3.2.1
v lubridate 1.9.3     v tidyr     1.3.1
v purrr     1.0.2
-- Conflicts ------------------------------------------ tidyverse_conflicts() --
x dplyr::filter() masks stats::filter()
x dplyr::lag()    masks stats::lag()
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to becor
```

```
library(gt)        ## Tables
## Specific packages
library(milestones)
## Initialize defaults
## Initialize defaults
column <- lolli_styles()

data <- read_csv(col_names=TRUE, show_col_types=FALSE, file='rl_time_line.csv')
```

```
Warning: One or more parsing issues, call `problems()` on your data frame for details,
e.g.:
  dat <- vroom(...)
  problems(dat)
```

```
## Sort the table by date
data <- data |>
  arrange(date)

## Build a table
gt(data) |>
  #cols_hide(columns = event) |>
  tab_style(cell_text(v_align = "top"),
            locations = cells_body(columns = date)) |>
  tab_source_note(source_note = "Source: Sutton and Barto (2018)")
```

date	event	referen
1911	Primer idea del ”trial-and-error learning” (TaEL)	Thorno
1927	Aparece el termino ”Reinforcement” en el contexto del aprendizaje animal	Pavlov
1948	Turing describe un diseno para un sistema ”pleasure-pain system”	Turing

1954	Minsky Farley y Clark publican sus investigaciones sobre TaEL.	Farley,
1957	Aparece ”Dynamic Programming” (DP)	Bellma
1957	Introducen los Markov Decision Processes (MDP)	Bellma
1959	Comienza a utilizarse el ”optimal control” (OC)	NA
1959	Se desarrolla extensamente la DP	NA
1960	Aparece el metodo de iteracion de politicas	Howard
1960	Se utilizan por primera vez en ingeniería”Reinforcement” y ”Reinforcement Learning”	Mendel
1960	Se originan los ”Learning Automata”	Tsetlin
1961	Se publica ”Steps Toward Artificial Intelligence”	Minsky
1961	Se describe un sistema ”TaEL” para el tic-tac-toe	Michie
1963	Se desarrolla STeLLA	Andrea
1972	Trabajo de Klopf	Klopf,
1973	Widrow, Gupta y Maitra modifican el LMS.	Widrow
1973	Teoría del aprendizaje de Bush y Mostelle	Bush,
1977	Werbos conecta el OC y PD	Werbos
1978	Sutton desarrolla las ideas de Klopf	Sutton
1986	Introducen los clasificadores	Hollan
1989	Watkins integra los metodos de aprendizaje (MA)	Watkin
1996	Aparece el termino ”Neurodynamic Programming”	Bertsek
2003	”Reinforcement Learning” en economia	Camer
2012	Vision general del Reinforcement Learning y Juegos	Now´e
NA	NA	NA

Source: Sutton and Barto (2018)

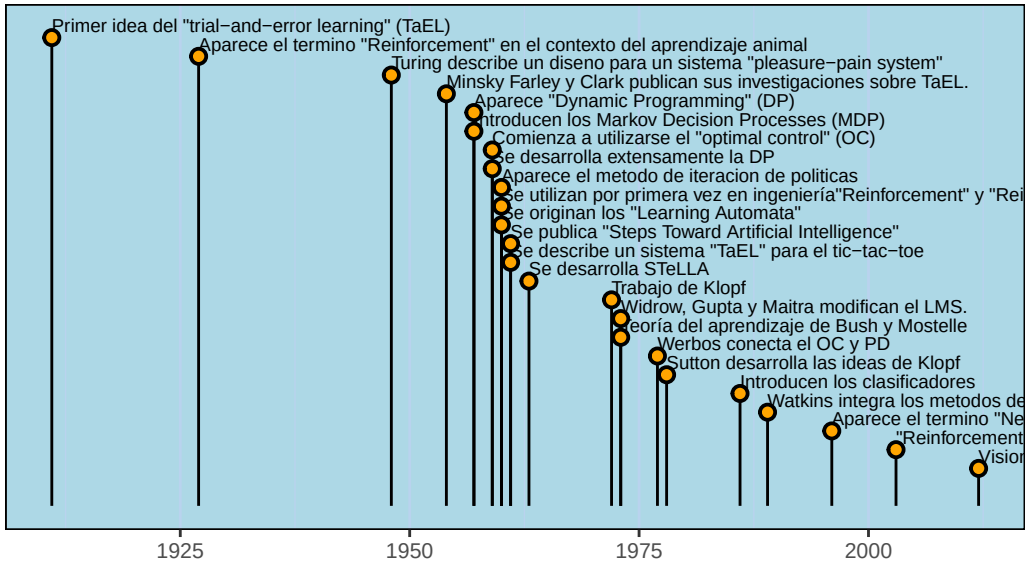
```
## Adjust some defaults
column$color <- "orange"
column$size  <- 15
column$background_color <- "lightblue"
column$text_size <- 2.5
column$source_info <- "Source: Sutton and Barto (2018)"

## Milestones timeline
milestones(datatable = data, styles = column)
```

```
Warning: Removed 1 row containing missing values or values outside the scale range
(`geom_text()`).
```

```
Warning: Removed 1 row containing missing values or values outside the scale range
(`geom_segment()`).
```

```
Warning: Removed 1 row containing missing values or values outside the scale range
(`geom_point()`).
```

```
Warning: Removed 25 rows containing missing values or values outside the scale range
(`geom_segment()`).
```



Source: Sutton and Barto (2018)

4.4 Exercise 4

Exercise 4 Consider the following **consumption-saving** problem with dynamics

$$x_{k+1} = (1+r)(x_k - a_k), \qquad k = 0, 1, ..., N-1,$$

and utility function

$$\beta^N (x_N)^{1-\gamma} + \sum_{k=0}^{N-1} \beta^k (a_k)^{1-\gamma}$$

. Show that the value functions of the DP algorithm take the form

$$J_k(x) = A_k \beta^k x^{1-\gamma},$$

where $A_N = 1$ and for $k = N-1, ..., 0$,

$$A_k = \left[1 + ((1+r)\beta A_{k+1})^{\frac{1}{\gamma}}\right]^{\gamma}$$

Show also that the optimal policies are $h_k(x) = A_k^{-1/\gamma}x$, for $k = N-1, \dots, 0$.

Prueba Procedemos por inducción. Primero comprobamos que se cumple para $n = N-1$, entonces como $J_N(x) = \beta^N (x_N)^{1-\gamma}$, tenemos que

$$J_{N-1} = \min_{a \in A(x)} \{\beta^{N-1}(a)^{1-\gamma} + \beta^N (1+r)^{1-\gamma}(x-a)^{1-\gamma}\},$$

tomando la derivada con respecto a a e igualando a cero.

$$(1-\gamma)\beta^{N-1}a^{-\gamma} - \beta^N(1+r)^{1-\gamma}(x-a)^{-\gamma}$$

$$(1-\gamma)\beta^{N-1}a^{-\gamma} - \beta(1+r)^{1-\gamma}(x-a)^{-\gamma} = 0$$

entonces

$$\left(\frac{x-a}{a}\right)^{\gamma} = \beta(1+r)^{1-\gamma} \implies \frac{x-a}{a} = [\beta(1+r)^{1-\gamma}]^{\frac{1}{\gamma}} \implies a = \frac{x}{[\beta(1+r)^{1-\gamma}]^{\frac{1}{\gamma}} + 1} -$$

Sea a_0 es punto donde se alcanza el mínimo, por tanto

$$J_{N-1}(x) = \beta^{N-1}(a_0)^{1-\gamma} + \beta^N(1+r)^{1-\gamma}(x-a_0)^{1-\gamma}.$$

Desarrollando

$$J_{N-1}(x) = \frac{\beta^{N-1}x^{1-\gamma}}{([\beta(1+r)^{1-\gamma}]^{\frac{1}{\gamma}} + 1)^{1-\gamma}} + \beta^N(1+r)^{1-\gamma}\left[\frac{x[\beta(1+r)^{1-\gamma}]^{\frac{1}{\gamma}}}{\beta(1+r)^{1-\gamma}]^{\frac{1}{\gamma}} + 1}\right]^{1-\gamma}$$

$$J_{N-1}(x) = \beta^{N-1}x^{1-\gamma}\left[[\beta(1+r)^{1-\gamma}]^{\frac{1}{\gamma}} + 1\right]^{\gamma}$$

$$J_{N-1}(x) = A_{N-1}\beta^{N-1}x^{1-\gamma}$$

con

$$A_{N-1} = \left(1 + ((1+r)^{1-\gamma}\beta)^{\frac{1}{\gamma}}\right)^{\gamma}$$

Ahora, supongamos que es válido para $n = k+1$, 4

$$J_{k+1}(x) = A_{k+1}\beta^{k+1}x^{1-\gamma}$$

. De aquí

$$J_k(x) = \min_{a \in (0,x)} \left\{\beta^k a^{1-\gamma} + A_{k+1}\beta^{k+1}(1+r)^{1-\gamma}(x-a)^{1-\gamma}\right\}$$

Encontrando el punto mínimo

$$(1-\gamma)\beta^k a^{-\gamma} - A_{k+1}\beta^{k+1}(1+\gamma)^{1-\gamma}(1-\gamma)(x-a)^{-\gamma} = 0$$

$$(1-\gamma)\beta^k \left[a^{-\gamma} - A_{k+1}\beta(1+r)^{1-\gamma}(x-a)^{-\gamma}\right] = 0$$

$$a^{-\gamma} - A_{k+1}\beta(1+r)^{1-\gamma}(x-a)^{-\gamma} = 0$$
$$\left(\frac{x-a}{a}\right)^{\gamma} = A_{k+1}\beta(1+\gamma)^{1-\gamma}$$
$$a = \frac{x}{[A_{k+1}\beta(1+r)^{1-\gamma}]^{\frac{1}{\gamma}} + 1}$$

De igual forma sea el punto mínimo a_0, se tiene

$$J_k(x) = \frac{\beta^k x^{1-\gamma}}{\left[[A_{k+1}\beta(1+r)^{1-\gamma}]^{\frac{1}{\gamma}} + 1\right]^{1-\gamma}} + \frac{A_{k+1}\beta^{k+1}(1+r)^{1-\gamma}\left(x[A_{k+1}\beta(1+r)^{1-\gamma}]^{\frac{1}{\gamma}}\right)^{1-\gamma}}{\left[[A_{k+1}\beta(1+r)^{1-\gamma}]^{\frac{1}{\gamma}} + 1\right]^{1-\gamma}}$$

Simplificando se concluye que

$$\frac{\beta^k x^{1-\gamma}\left(1 + [A_{k+1}\beta(1+r)^{1-\gamma}]^{\frac{1}{\gamma}}\right)}{\left(1 + [A_{k+1}\beta(1+r)^{1-\gamma}]^{\frac{1}{\gamma}}\right)^{1-\gamma}} = \beta^k x^{1-\gamma} A_k$$

4.5 Exercise 5

Exercise 5 Consider now the infinite–horizon version of the above consumption–saving problem.

1. Write down the associated Bellman equation.
2. Argue why a solution to the Bellman equation should be of the form

$$v(x) = cx^{1-\gamma}$$

, where c is constant. Find the constant and the stationary optimal policy.

Prueba Sea

$$cx^{1-\gamma} = \min\left\{a^{1-\gamma} + \beta c(1+r)^{1-\gamma}(x-a)^{1-\gamma}\right\}$$

Calculando el mínimo

$$(1-\gamma)a^{-\gamma} - \beta c(1+r)^{1-\gamma}(1-\gamma)(x-a)^{-\gamma} = 0$$
$$(1-\gamma)\left[a^{-\gamma} - \beta c(1+r)^{1-\gamma}(x-a)^{-\gamma}\right] = 0$$
$$a^{-\gamma} - \beta c(1+r)^{1-\gamma}(x-a)^{-\gamma} = 0$$
$$\left(\frac{x-a}{a}\right)^{\gamma} = \beta c(1+r)^{1-\gamma}$$
$$x = a\left[\beta c(1+r)^{1-\gamma}\right]^{\frac{1}{\gamma}} + a$$

$$a_0 = a = \frac{x}{[\beta c(1+r)^{1-\gamma}]^{\frac{1}{\gamma}} + 1}$$

Sustitimos a_0

$$cx^{1-\gamma} = \frac{x^{1-\gamma} + \beta c(1+r)^{1-\gamma} x^{1-\gamma} \left[(\beta c(1+r)^{1-\gamma})^{\frac{1}{\gamma}}\right]^{1-\gamma}}{\left[(\beta c(1+r)^{1-\gamma})^{\frac{1}{\gamma}} + 1\right]^{1-\gamma}}$$

$$cx^{1-\gamma} = x^{1-\gamma}\left[1 + [\beta c(1+r)^{1-\gamma}]^{\frac{1}{\gamma}}\right]^{\gamma}$$

así,

$$cx^{1-\gamma} = x^{1-\gamma}\left[1 + [\beta c(1+r)^{1-\gamma}]^{\frac{1}{\gamma}}\right]^{\gamma}$$

$$c = \left[1 + [\beta c(1+r)^{1-\gamma}]^{\frac{1}{\gamma}}\right]^{\gamma}$$

$$c^{\frac{1}{\gamma}} = 1 + [\beta c(1+r)^{1-\gamma}]^{\frac{1}{\gamma}}$$

$$c^{\frac{1}{\gamma}} = \left[1 - \beta^{\frac{1}{\gamma}}(1+r)^{\frac{1-\gamma}{\gamma}}\right]$$

$$c^{\frac{1}{\gamma}} = \frac{1}{1 - \beta^{\frac{1}{\gamma}}(1+r)^{\frac{1-\gamma}{\gamma}}}$$

4.6 Exercise 6

Exercise 6 Let $\{\xi_k\}$ be a sequence of iid random variables such that $E[\xi] = 0$ and $E[\xi^2] = d$. Consider the dynamics

$$x_{k+1} = x_k + a_k + \xi_k, \qquad k = 0, 1, 2, ...,$$

and the discounted cost

$$E \sum \beta^k (a_k^2 + x_k^2).$$

i. Write down the associated Bellman equation.

ii. Conjecture that the solution to the Bellman equation takes the form $v(x) = ax^2 + b$, where a and b are constant.

iii. Determine the constants a and b.

iv. Conjecture that the solution to the Bellman equation takes the form $v(x) = ax^2 + b$, where a y b are constant. Determine the constants a and b. ***Prueba*** Sea $A = a$ y $B = b$, entonces

$$Ax^2 + B = \min_{a \in A(x)} \{a^2 + x^2 + \beta E[A(x + a + \xi)^2 + B]\}$$

$$Ax^2 + B = \min_{a \in A(x)} \{a^2 + x^2 + \beta AE[(x + a + \xi)^2] + \beta B\}$$

$$= \min_{a \in A(x)} \{a^2 + x^2 + A\beta E[x^2 + 2ax + a^2 + 2(x+a)\xi + \xi^2] + \beta B\}$$

$$= \min_{a \in A(x)} \{a^2 + x^2 + A\beta x^2 + 2axA\beta + A\beta a^2 + A\beta d + \beta B\}$$

Encontrando el mínimo con la derivada

$$2a + 2xA\beta + 2A\beta a = 0$$

entonces,

$$a = \frac{-xA\beta}{1 + A\beta}$$

así

$$Ax^2 + B = \frac{(xA\beta)^2}{(a + A\beta)^2} + x^2 + \beta E\left[A\left(\frac{x}{1 + A\beta} + \xi\right)^2\right] + \beta B$$

$$= \frac{x^2 A^2 \beta^2}{(1 + A\beta)^2} + x^2 + A\beta E\left[\frac{x^2}{(1 + A\beta)^2} + \frac{2x\xi}{1 + A\beta} + \xi^2\right] + \beta B$$

$$= \frac{x^2 A\beta(1 + A\beta)}{(1 + A\beta)^2} + x^2 + A\beta d + \beta B$$

$$= x^2\left(1 + \frac{A\beta}{1 + A\beta}\right) + A\beta d + \beta B$$

Por lo que

$$A = 1 + \frac{A\beta}{1 + A\beta}, \qquad B = \frac{A\beta d}{1 - \beta}$$

De esta forma

$$A = \frac{1 + 2A\beta}{1 + A\beta}$$

$$A^2\beta + A(1 - 2\beta) - 1 = 0$$

Obteniendo las soluciones

$$A = \frac{-1 + 2\beta \pm \sqrt{4\beta^2 + 1}}{2\beta}$$

5 Summary

In summary, this book has no content whatsoever.

```
1 + 1
```

```
[1] 2
```

References

Sutton, Richard S., and Andrew G. Barto. 2018. *Reinforcement Learning: An Introduction*. Second. The MIT Press. http://incompleteideas.net/book/the-book-2nd.html.