Invited Paper

The Dolinar receiver in an information theoretic framework

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ABSTRACT

Optical communication at the quantum limit requires that measurements on the optical field be maximally informative, but devising physical measurements that accomplish this objective has proven challenging. The Dolinar receiver exemplifies a rare instance of success in distinguishing between two coherent states: an adaptive local oscillator is mixed with the signal prior to photodetection, which yields an error probability that meets the Helstrom lower bound with equality. Here we apply the same local-oscillator-based architecture with an information-theoretic optimization criterion. We begin with analysis of this receiver in a general framework for an arbitrary coherent-state modulation alphabet, and then we concentrate on two relevant examples. First, we study a binary antipodal alphabet and show that the Dolinar receiver's feedback function not only minimizes the probability of error, but also maximizes the mutual information. Next, we study ternary modulation consisting of antipodal coherent states and the vacuum state. We derive an analytic expression for a near-optimal local-oscillator feedback function, and, via simulation, we determine its photon information efficiency (PIE). We provide the PIE versus dimensional information efficiency (DIE) trade-off curve and show that this modulation and the our receiver combination performs universally better than (generalized) on-off keying plus photon-counting, although, the advantage asymptotically vanishes as the bits-per-photon diverges towards infinity.

1. INTRODUCTION

It is well known that the ultimate limits of information transfer using photons as the physical information carrier (i.e., 'optical communication') is determined by the quantum nature of the photons, and approaching these limits requires that measurements on the photons extract the information encoded in their optical states with the highest efficiency. Unfortunately, it is often a difficult and elusive goal to realize measurements that achieve these quantum-mechanical limits. The Dolinar receiver exemplifies a rare instance of success in describing a measurement that achieves the lowest possible error probability in distinguishing between two coherent states.[1,2] Past of shine In the Dolinar receiver, the input coherent-state signal is first mixed with a time-varying local oscillator, and then is shone on an ideal infinite-bandwidth photodetector. The photodetector output up to the present time, which is a counting process of photon-arrival events, is used to determine the local-oscillator complex amplitude that will minimize the probability of incorrectly distinguishing between the two possible input states in the next time increment.[1,3] This incremental optimization algorithm turns out to be also globally optimal, and it achieves the Helstrom error-probability lower bound.

The success of adaptive feedback in binary state discrimination raises the question of whether its use is beneficial in reliable transfer of information for communications. In this paper we investigate the answer to this question. In particular, we consider optical communication using coherent-state modulation, paired with the same adaptive local-oscillator-based receiver architecture shown in Fig. 1. However, our analysis diverges from that of the Dolinar receiver in two significant aspects. First, because the highest rate of reliable communication is determined by the mutual information between the encoding states and the measurement outcome (rather than the probability of error), we modify the feedback objective such that the local oscillator strives to maximize this information-theoretic criterion. Second, we cast the problem in a general framework suitable for studying arbitrary coherent-state constellations (rather than a strictly binary constellation). Within this framework, we show that the globally-optimal local oscillator can be determined by incrementally maximizing the *differential* mutual information in each instant of time, conditioned on the past observations from the photodetector. We

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then proceed to derive a set of equations that must be satisfied by the optimal local oscillator, and we provide an algorithm summary for simulating the optimal receiver.

In addition to studying this receiver architecture in its full generality, we analyze two specific examples. First, we consider binary coherent-state modulation (e.g., binary phase-shift keying). We find, perhaps unsurprisingly, that the Dolinar receiver's feedback function is also the local oscillator function that maximizes mutual information. This result implies that for this adaptive receiver architecture a hard-decision at the end of each symbol is information lossless. This, in turn, implies that binary-phase-shift keying (BPSK) paired with this receiver can achieve only finite bits-per-photon. The second example we consider is a ternary modulation that uses two antipodal coherent states, and the vacuum state. The fact that the vacuum state has no photon cost implies that this modulation scheme can achieve unbounded bits per photon.[4] Indeed, using simulation, we show that this modulation and our optimal receiver architecture yields higher photon information efficiency (PIE) and dimensional information efficiency (DIE) pairs than on-off-keying (OOK) and ideal photon-counting, although the difference asymptotically vanishes as PIE grows larger. Unbounded PIE is not achievable with standard homodyne detection.

Using mutual information as an optimization criterion has been considered previously for homodyne detection, wherein the phase of the local oscillator is adaptively varied to align the receiver with the optimal measurement quadrature.[5] However, the regime of interest in that work is high dimensional constellations that improve primarily the DIE of homodyne detection in the bandwidth-constrained regime. In photon-starved channels, homodyne detection hits a finite PIE asymptote, and therefore its performance falls short in comparison to PIE-efficient modulation and detection schemes, such as OOK with photon-counting.[6,7] In this work we allow feedback to manipulate both phase and amplitude, which encompasses a significantly broader class than the adaptive homodyne receiver, and also overcomes the finite-PIE asymptote suffered by homodyne detection. Furthermore, we analyze two specific examples that are of common interest. Binary phase-shift keying (BPSK), is often employed in coherent (i.e., homodyne or heterodyne) communication systems.[8–10] In addition, BPSK modulation is known to approach the ultimate limits of optical communication set by the Holevo information bound in the photon-starved limit, although, the receiver architecture that accomplishes this is yet to be determined.[6,7] Our second example is a ternary modulation alphabet of antipodal coherent state and the vacuum state. Recently, this alphabet has been utilized in an elegant receiver architecture that converts a sequence of BPSK symbols into a sequence of correlated ternary symbols, and then utilizes an adaptive local-oscillator receiver with a simple feedback algorithm.[11] The analysis framework we develop here not only allows us to determine the optimal feedback algorithm for this scenario, but it also permits us to unambiguously identify the regime in which the particular algorithm used in that work becomes optimal. In addition, because unbounded PIE is achievable if the modulation constellation contains at least one symbol that is costless, this ternary constellation could be of interest for high-PIE communication.

Our paper is organized as follows. In Section 2 we analyze our adaptive receiver architecture in full generality. We express the mutual information between the input symbol and the output as the integral over the differential mutual information gained in each time increment, and we derive a pair of equations that must be satisfied by the optimal local oscillator. We then concentrate on binary modulation in Section 3. We conduct our analysis for binary phase-shift keying (BPSK), and then we show that the mutual information of an arbitrary binary coherent-state constellation is the same as that of the BPSK constellation obtained by subtracting the arithmetic mean of the coherent states. In Section 4 we introduce and study ternary modulation consisting of two antipodal coherent states plus the vacuum state. Finally, in Section 5 we discuss the primary conclusions from our work.

2. GENERAL FORMULATION

Consider the optical detection system shown in Fig. 1. The incoming signal is a time-varying coherent-state optical field whose $\sqrt{\text{photons/s}}$-units complex baseband envelope is denoted by $\alpha(t)$. This signal field is first displaced by a coherent-state local oscillator with $\sqrt{\text{photons/s}}$-units complex baseband amplitude $\alpha_{\text{lo}}(t)$, and subsequently, the displaced field is detected by an ideal photodetector, i.e., one with infinite bandwidth, unity quantum efficiency, and no dark current. The measurement outcome, $N(t)$, is a point-process with arrival events corresponding to photon detections. The observed process is utilized at the receiver to (causally) alter the local

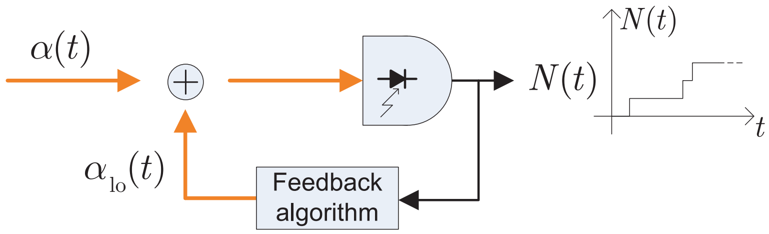


Figure 1. The general structure of an optical feedback receiver, which modifies a local oscillator field, $\alpha_{\rm lo}(t)$, as a function of the photodetector output, $N(t)$. The orange thicker lines indicate optical fields and the thinner black lines represent electrical signals.

oscillator such that an objective function (in this case, the mutual information between the incoming signal and the point process) is maximized.

In accordance with a digital information transmission scenario we assume that $\alpha(t)$ is constant over T-second slots and in each slot it is independently drawn from an alphabet of $\mathcal{K}$ coherent states with complex-valued amplitudes $\{\alpha_k : k = 1, \ldots, \mathcal{K}\}$, and associated *a priori* probabilities $\{q_k : k = 1 \ldots, \mathcal{K}\}$. Therefore, conditioned on $K = k$, $N(t)$ is a Poisson counting process with rate $\lambda_k(t) = |\alpha_k + \alpha_{\rm lo}(t)|^2$ for $t \in (0, T]$.* In our treatment we shall ignore bandwidth, latency and dynamic range limitations in the feedback path and assume that the local oscillator field (amplitude and phase) can be varied instantaneously, based on the counting process output from the photodetector. Note that in our model the receiver enjoys infinite bandwidth, but the transmitter is restricted to have a finite modulation bandwidth.

The maximum rate at which one can transfer information via this system, in bits-per-slot is given by the maximum of the mutual information between the input and the output,

$$C \equiv \max_{\{q_k\}, \alpha_{\rm lo}(t)} I\big(K; \{N(t) : 0 < t \leq T\}\big)\,, \tag{1}$$

where the encoder chooses the optimal encoding distribution over the alphabet, and the receiver chooses the optimal local oscillator. The receiver dynamically adjusts the local oscillator field $\alpha_{\rm lo}(t)$ using the only information it has, *viz*. the observed process $N(\tau)$ during $0 < \tau < t$. With no loss in generality, we assume that $N(0) = 0$ with probability 1. Our approach shall be to first derive the optimal local oscillator given an arbitrary distribution on the input alphabet, and then to maximize the resulting mutual information—with the optimal local oscillator—over the input distribution.

THEOREM 2.1. *Maximizing the mutual information given in Eq.* (1) *is equivalent to incrementally maximizing the mutual information in the time window* $(t, t + \Delta t]$*, as* $\Delta t \to 0$*, conditioned on the observations up to and including time* t.

Proof. Let us begin by expressing Eq. (1) as the following limit,

$$I\big(K; \{N(t) : 0 < t \leq T\}\big) = \lim_{\Delta T \to 0} I\left(K; \{N_{m'}\}_0^{\lfloor T/\Delta T \rfloor}\right)\,, \tag{2}$$

where $N_m \equiv N\big((m+1)\Delta T\big) - N(m\Delta T)$, and $\{N_{m'}\}_j^m \equiv \{N_j, \ldots, N_m\}$ for $j \leq m$. We can now use the chain rule for mutual information to expand the argument of the limit as

$I(X_1, X_2 \ldots, X_n; Y) = \sum_{i=1}^{n} I(X_i; Y | X_{i-1}, X_{i-2}, \ldots X_1)$

$I(X, Y; Z) = I(X; Y) + I(Z; Y | X)$

$I(X; Y, Z) = I(X; Y) + I(X; Z | Y)$

$$I\big(K; \{N_{m'}\}_0^{\lfloor T/\Delta T \rfloor}\big) = \sum_{m=0}^{\lfloor T/\Delta T \rfloor} \Delta T \frac{I\left(K; N_m | \{N_{m'}\}_0^{m-1}\right)}{\Delta T}\,. \tag{3}$$

Because $N(t)$ is a well-defined point-process, the second term inside the summation converges to a limit as ΔT converges to 0.[12] Then, substituting Eq. (3) into Eq. (2) and taking the limit, we obtain

$$I\big(K; \{N(t) : 0 < t \leq T\}\big) = \int_0^T {\rm d}t \lim_{\Delta T \to 0} \frac{I\left(K; N(t + \Delta T) - N(t) | \{N(\tau) : \tau \in (0, t]\}\right)}{\Delta T}\,. \tag{4}$$

*For analytic convenience we have assumed that the slot of interest corresponds to the time window $[0, T)$.

$I(X;Y|Z) = E_{P_Z}\left[I(X;Y|\{Z=z\})\right] = H(X|Z) - H(X|Y,Z)$

The conditional mutual information in the integrand is, by its definition, equal to

$$I\big(K; N(t+\Delta T) - N(t)|\{N(\tau) : \tau \in (0,t]\}\big) = E_N\Big[i\big(K; N(t+\Delta T) - N(t)|\{n(\tau) : \tau \in (0,t]\}\big)\Big], \tag{5}$$

where $E_N[\cdot]$ denotes expectation over the point process $\{N(\tau) : 0 < \tau \leq t\}$, and $i\big(K; N(t+\Delta T) - N(t)|\{n(\tau) : \tau \in (0,t]\}\big)$ is the mutual information conditioned on a *particular realization* of the point process $N(t)$, which we have denoted by the lower case function $n(t)$. Because the local oscillator $\alpha_{\rm lo}(t)$ depends causally on $n(t)$, we arrive at our final expression

and lower case "i"

$$\max_{\alpha_{\rm lo}(t)} I\big(K; N(t) : 0 < t \leq T\big) = \int_0^T \mathrm{d}t\, \mathrm{E}_N\left[\max_{\alpha_{\rm lo}(t)} \lim_{\Delta T \to 0} \frac{i(K; N(t+\Delta T) - N(t)|\{n(\tau) : \tau \in (0,t]\})}{\Delta T}\right]. \tag{6}$$

We refer to the limit inside the expectation as the differential mutual information, as it is a measure of incremental gain in mutual information as a function of time.

Equation (6) shows that the local oscillator that maximizes the mutual information between the input symbol K and the photodetector output $N(t)$ over $0 < t \leq T$ can be chosen at each time instant t, such that it incrementally maximizes the differential mutual information at the next time instant, conditioned on the observations up to and including t. $\square$

In order to evaluate Eq. (6) explicitly, we turn our attention to the numerator in the limit expression. This term is given by

$$\begin{aligned} &i\big(K; N(t+\Delta T) - N(t)|\{n(\tau); \tau \in (0,t]\}\big) = \\ &\qquad H\big(N(t+\Delta T) - N(t)|\{n(\tau); \tau \in (0,t]\}\big) - H\big(N(t+\Delta T) - N(t)|K, \{n(\tau); \tau \in (0,t]\}\big), \end{aligned} \tag{7}$$

where $H(\cdot)$ is the well-known (discrete) entropy function.[13] Using the fact that $N(t+\Delta T) - N(t)$ is a Poisson random variable when conditioned on K and $\alpha_{\rm lo}(t)$, and a compound Poisson random variable when conditioned on $\alpha_{\rm lo}(t)$ alone, we show in Appendix A that the limit expression in Eq. (6) simplifies to

$$\lim_{\Delta T \to 0} \frac{i\big(K; N(t+\Delta T) - N(t)|\{n(\tau); \tau \in (0,t]\}\big)}{\Delta T} = -\overline{\lambda} \log \overline{\lambda} + \sum_{k=1}^{\mathcal{K}} p_k \lambda_k \log \lambda_k\,, \tag{8}$$

where all of the time-dependent variables are evaluated at time t, and we have suppressed showing their time-dependence to simplify our notation. Here $\lambda_k \equiv |\alpha_k + \alpha_{\rm lo}|^2$ for $k \in \{1, 2, \ldots, \mathcal{K}\}$, $p_k \equiv P(K = k|\{n(\tau) : 0 < \tau < t\})$, and $\overline{\lambda} \equiv \sum_k p_k \lambda_k$. Note that $-\lambda \log(\lambda)$ is a concave function of λ, so the right-handside of Eq. (8) is nonnegative, as required from an information metric.

Next, we must maximize the right-hand side of Eq. (8) to find the optimal local oscillator. Denoting $\alpha_{\rm lo} \equiv a \exp(i\phi)$ we find that the critical points of the maximization occur at a and ϕ values that satisfy the system of equations

$$\sum_k p_k \log\left(\frac{\lambda_k}{\overline{\lambda}}\right)(a + |\alpha_k|\cos(\phi - \phi_k)) = 0 \tag{9}$$

$$\sum_k p_k \log\left(\frac{\lambda_k}{\overline{\lambda}}\right)\sin(\phi - \phi_k) = 0\,, \tag{10}$$

where $\phi_k \equiv \angle \alpha_k$ and $a \geq 0$. Finding the analytic solutions to these equations for arbitrary α_k is nontrivial. However, it is easy to verify that if all α_k have common phase up to a π phase shift, then the local oscillator must be either in phase or out-of-phase with the constellation as well, i.e., if all of the constellation points are along a line in the complex plane, then the optimal value of the local oscillator will also reside on this line.

The formalism we have presented in this section provides a system of equations that, in principle, can be solved for an arbitrary constellation to yield the local oscillator that incrementally maximizes the mutual information in the next time instant. The optimal local oscillator not only depends on the constellation points

α_k, but also on the probability of each hypothesis conditioned on the arrival process up to and including t, i.e., $p_k(t) \equiv P(K = k|\{n(\tau) : 0 < \tau \leq t\})$. The evolution equations for the probability of each hypothesis is obtained, via Bayes' rule, as

$$p_k(t + \Delta T) = \frac{P\big(N(t+\Delta T) - N(t)|K = k, \{n(\tau) : \tau \in (0,t]\}\big)}{\sum_k p_k(t) P\big(N(t+\Delta T) - N(t)|K = k, \{n(\tau) : \tau \in (0,t]\}\big)} p_k(t). \tag{11}$$

For $\Delta T \ll 1$, we have that

$$P\big(N(t+\Delta T) - N(t) = m|K = k, \{n(\tau) : \tau \in (0,t]\}\big) = \begin{cases} 1 - \Delta T \lambda_k + o(\Delta T) & \text{for } m = 0 \\ \Delta T \lambda_k + o(\Delta T) & \text{for } m = 1 \\ o(\Delta T) & \text{for } m \geq 2\,, \end{cases} \tag{12}$$

where we refer to a function $f(x)$ as $o(x)$ if $\lim_{x\to 0} \frac{f(x)}{x} = 0$. With some additional manipulation of Eq. (12), we find that $p_k(t)$ evolves continuously except for jump discontinuities at the instances corresponding to photon detections. In particular, if t_i for $i = 1, \ldots, N(T)$ denotes the photon detection epochs, we have

$$\frac{d}{dt} p_k(t) = [\overline{\lambda}(t) - \lambda_k(t)] p_k(t) \tag{13}$$

for $t_i < t \leq t_{i+1}$ and

$$p_k(t_i^+) = \frac{\lambda_k(t_i)}{\overline{\lambda}(t_i)} p_k(t_i)\,, \tag{14}$$

where we have used $p_k(t_i^+)$ to represent the limit of $p_k(t)$ as t approaches t_i from the right. Note that Eqs. (13) and (14) jointly ensure that $\sum_k p_k(t) = 1$ for $0 < t \leq T$.

The solutions to the optimality criteria in Eqs. (9)–(10), together with the evolution equations for the conditional probabilities given in Eqs. (12), (13) and (14), provide a complete description of the adaptive feedback receiver that maximizes the mutual information between the input and the photodetector output for a given *a priori* input distribution. Unfortunately, closed-form analysis of the performance of this receiver is not straightforward in the full generality of this formalism, except for some simple special cases (e.g., see next section). Nonetheless, this receiver can be numerically simulated in full generality utilizing the following algorithm:

I. Initialize:

   i. Determine true hypothesis k', using the *a priori* probability distribution on the signal constellation, $\{q_k : k = 1, \ldots, \mathcal{K}\}$.
   ii. Choose step size ΔT, such that $\max_{k,\alpha_{\mathrm{lo}}} |\alpha_k + \alpha_{\mathrm{lo}}|^2 \Delta T \ll 1$ prevails.[†]
   iii. Set $p_k(0) = q_k$ for $k = 1, \ldots, \mathcal{K}$.

II. Repeat for $m = 1, \ldots, \lfloor T/\Delta T \rfloor$:

   i. Find optimal $\alpha_{\mathrm{lo}}(m\Delta T)$ solving Eqs. (9) and (10), or numerically maximizing the right-hand side of Eq. (8).
   ii. Update $\lambda_k(m\Delta T)$ using the updated local oscillator value from previous step.
   iii. Simulate a Bernoulli random variable according to Eq. (12), with probability of 1 given by $\Delta T \lambda_{k'}(m\Delta T)$ where k' denotes the *true* hypothesis.
   iv. Update conditional probabilities of each hypothesis $p_k(m\Delta T)$ according to Eqs. (13)–(14), and depending on whether an arrival occurs in previous step.
   v. Increase m by one and return to step II.

[†]In our theory $|\alpha_{\mathrm{lo}}|$ can grow unbounded. However, in our simulation $|\alpha_{\mathrm{lo}}|$ is necessarily finite. The maximization is over this finite support for feasible α_{lo} values.

III. END.

In order to develop some insight into the mutual-information maximizing receiver performance, we next concentrate on two examples: a binary and ternary constellation. As we shall see shortly, the former case lends itself to a complete analytical solution, while the latter necessitates numerical evaluation of its performance.

3. BINARY SIGNALING CONSTELLATION

In this section, we first analyze an antipodal (BPSK) signaling scheme, and later show that the performance of any two-element coherent-state alphabet is equivalent to that of an antipodal alphabet derived by subtracting the arithmetic average of the two (complex-valued) coherent states. Consider BPSK with $\{|\alpha\rangle, |-\alpha\rangle\}$ denoting the two real-valued coherent-state field envelopes having units of $\sqrt{\text{photons/s}}$. Because the optimal local oscillator is also real-valued, we concentrate on the solution to Eq. (9), which simplifies to

$$p_+ \left(\frac{\alpha_{\text{lo}}}{\alpha} + 1\right) \log\left(\frac{\lambda_+}{\overline{\lambda}}\right) + (1 - p_+) \left(\frac{\alpha_{\text{lo}}}{\alpha} - 1\right) \log\left(\frac{\lambda_-}{\overline{\lambda}}\right) = 0\,, \tag{15}$$

where we have denoted the parameters related to $|\alpha\rangle$ with the subscript '+' and those related to $|-\alpha\rangle$ with the subscript '$-$.' The solution to this equation is given by

$$\alpha_{\text{lo}} = \frac{\alpha}{1 - 2p_+}\,. \tag{16}$$

The solution in Eq. (16) implies that when $p_+ > 1/2$ the local oscillator becomes negative so that it is attenuating the signal that is more likely in each incremental step and, consequently, amplifying that which is less likely. Thus, if the receiver has deemed the true hypothesis as the more likely one, the probability of an arrival gradually decreases in subsequent increments, whereas if the receiver's guess is incorrect the probability of an arrival increases.

Substituting the solution into Eqs. (13) and (14), we obtain the evolution equations

$$\frac{d}{dt} p_+(t) = -4\alpha^2 \frac{\big(1 - p_+(t)\big)p_+(t)}{1 - 2p_+(t)}\,, \tag{17}$$

and

$$p_+(t_i^+) = 1 - p_+(t_i)\,, \tag{18}$$

respectively. From these expressions it is straightforward to show that $p_+(t)\big(1 - p_+(t)\big)$ follows the deterministic trajectory

$$p_+(t)\big(1 - p_+(t)\big) = p_+(0)\big(1 - p_+(0)\big)e^{-4\alpha^2 t}\,, \tag{19}$$

for $0 \le t \le T$, which allows us to write the optimal local oscillator as

$$\alpha_{\text{lo}}(t) = \frac{\alpha(-1)^{N(t)}}{\sqrt{1 - 4p_+(0)\big(1 - p_+(0)\big)e^{-4\alpha^2 t}}}\,, \tag{20}$$

where we have made use of the equality $|1 - 2p_+(t)| = \sqrt{1 - 4p_+(t)(1 - p_+(t))}$. Finally, substituting this local oscillator into Eq. (8) gives

$$-\overline{\lambda}\log\overline{\lambda} + \sum_{k=1}^{\mathcal{K}} p_k \lambda_k \log \lambda_k = \frac{4\alpha^2 p_+(0)\big(1 - p_+(0)\big)e^{-4\alpha^2 t}}{\sqrt{1 - 4p_+(0)\big(1 - p_+(0)\big)e^{-4\alpha^2 t}}} \log\left(\frac{1 + \sqrt{1 - 4p_+(0)\big(1 - p_+(0)\big)e^{-4\alpha^2 t}}}{1 - \sqrt{1 - 4p_+(0)\big(1 - p_+(0)\big)e^{-4\alpha^2 t}}}\right), \tag{21}$$

which has no dependence on $N(t)$.

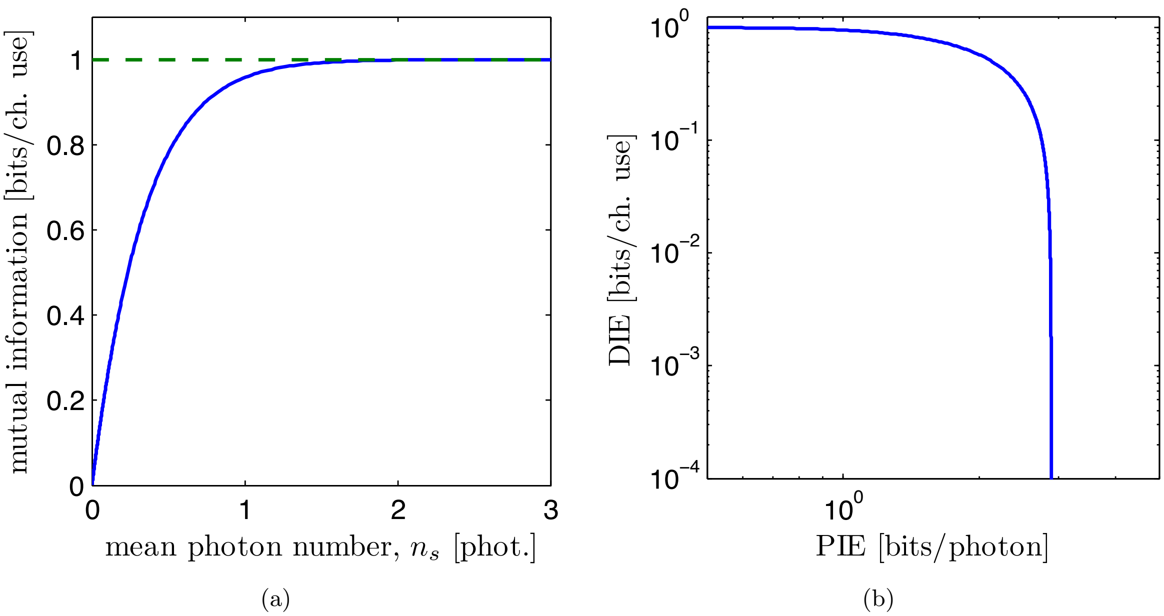


Figure 2. The capacity and efficiency trade-off curves for a BPSK coherent-state constellation and the adaptive local-oscillator receiver, which is achieved when $p_+(0) = p_-(0) = 0.5$. The receiver is equivalent to the Dolinar receiver. (a) The capacity as a function of the mean detected photon number n_S. (b) The photon-information efficiency (PIE) versus the dimensional information efficiency (DIE).

We are now able to state our end result, i.e., the mutual information between the input and the output. Substituting Eq. (21) into Eq. (4) and evaluating the integral yields

$$I\big(K; \{N(t) : 0 < t \leq T\}\big) = h_2\big(p_+(0)\big) - h_2(P_{\rm er})\,, \tag{22}$$

where $h_2(p) \equiv -p\log(p) - (1-p)\log(1-p)$ is the binary entropy function, and

$$P_{\rm er} \equiv \frac{1}{2}\left(1 - \sqrt{1 - 4p_+(0)\big(1-p_+(0)\big)e^{-4\alpha^2 T}}\right) \tag{23}$$

is the probability of symbol error from a hard decision at the end of the observation interval. Because mutual information can be expressed as

$$I\big(K; \{N(t) : 0 < t \leq T\}\big) = H(K) - H\big(K|\{N(t) : 0 < t \leq T\}\big)\,, \tag{24}$$

and $H(K) = h_2\big(p_+(0)\big)$, we conclude that entropy of the input conditioned on the output is

$$H\big(K|\{N(t) : 0 < t \leq T\}\big) = h_2(P_{\rm er}). \tag{25}$$

This equality holds for an *arbitrary* input distribution. It follows from the observation that if the '+' hypothesis is assigned to 0 and the '−' is assigned to 1, then the modulo-two sum of the input and the maximum-likelihood output hypothesis is independent of $N(t)$ for $0 < t \leq T$.[3]

The maximum of Eq. (22) over the input distribution is achieved at $p_+(0) = p_-(0) = 1/2$, and results in the capacity

$$C = \max_{p_+(0)}\big(K; \{N(t) : 0 < t < T\}\big) = 1 - h_2(P_{\rm er}) \text{ bits/channel use}\,. \tag{26}$$

Figure 2(a) shows this capacity as a function of the mean photon number of the input signaling constellation, $n_s \equiv \alpha^2 T$. As the mean photon number increases the probability of error decreases and the capacity approaches the entropy of the input, i.e., 1. In Figure 2(b) we have plotted the PIE versus DIE trade-off curve for this

BPSK plus adaptive-feedback receiver architecture, where the PIE is defined as C/n_s in bits-per-photon, and the DIE is defined as C, in bits-per-channel use (or bits-per-slot). Here we see that the photon efficiency of this architecture is bounded to a finite PIE of $2/\ln(2) = 2.885$ bits/photon.

At this juncture we revisit the local oscillator function we derived in Eq. (16) and note that it is *identical* to the local oscillator function employed in the well-known Dolinar receiver.[1] The Dolinar receiver is an adaptive feedback receiver of the form shown in Fig. 1, in which the local oscillator is chosen such that the receiver can make a minimum probability of error decision between two BPSK symbols. The Dolinar receiver has earned its fame for achieving the Helstrom bound, i.e., the minimum error probability that is quantum-mechanically permissible in distinguishing these two non-orthogonal states.[2] The analysis in this section shows that the Dolinar receiver is also optimal in maximizing the mutual information between the input symbols and the photodetector output for a binary coherent-state constellation. Therefore we conclude that there is *no* soft-information available in the Fig. 1 feedback receiver architecture when the input is from a binary coherent-state constellation. However, because an upper bound on the highest achievable mutual information with single-symbol measurements is not known,[3,14] our result does not eliminate the possibility that a receiver architecture outside of that considered herein could achieve higher mutual information.

We have thus far considered BPSK signaling, but the results extend trivially to arbitrary binary coherent-state constellations with arbitrary *a priori* probability distributions. Suppose we have $\{|\alpha_1\rangle, |\alpha_2\rangle\}$, where $\alpha_1, \alpha_2 \in \mathbb{C}$. Then the optimal local oscillator function α_{lo} must be

$$\alpha_{\text{lo}} = -\frac{1}{2}(\alpha_1 + \alpha_2) + e^{j\theta}\alpha'_{\text{lo}}\,, \tag{27}$$

where $\theta \equiv \angle(\alpha_1 - \alpha_2)$, and α'_{lo} is the optimal local oscillator for the real-valued and antipodal constellation

$$\{\big|-|\alpha_1 - \alpha_2|/2\big\rangle, \big||\alpha_1 - \alpha_2|/2\big\rangle\}. \tag{28}$$

In other words, a fixed offset and rotation will transform two arbitrary coherent states into real-valued and antipodal coherent states. Since both of these operations are achievable with a local oscillator, finding the optimal local oscillator for a real-valued BPSK constellation is sufficient to determine that for any binary coherent-state alphabet.

4. TERNARY SIGNALING CONSTELLATION

In this section we consider a ternary constellation $\{|-\alpha\rangle, |0\rangle, |\alpha\rangle\}$, where $|\pm\alpha\rangle$ are antipodal real-valued coherent states, and $|0\rangle$ is the vacuum state. Once again, because all constellation points are real-valued, the optimal local oscillator is also real-valued. Thus we need only find a solution to Eq. (9), which simplifies in this case to

$$p_-\left(\frac{\alpha_{\text{lo}}}{\alpha} - 1\right)\log\left(\frac{\lambda_-}{\overline{\lambda}}\right) + p_+\left(\frac{\alpha_{\text{lo}}}{\alpha} + 1\right)\log\left(\frac{\lambda_+}{\overline{\lambda}}\right) + (1 - p_- - p_+)\frac{\alpha_{\text{lo}}}{\alpha}\log\left(\frac{\lambda_0}{\overline{\lambda}}\right) = 0\,. \tag{29}$$

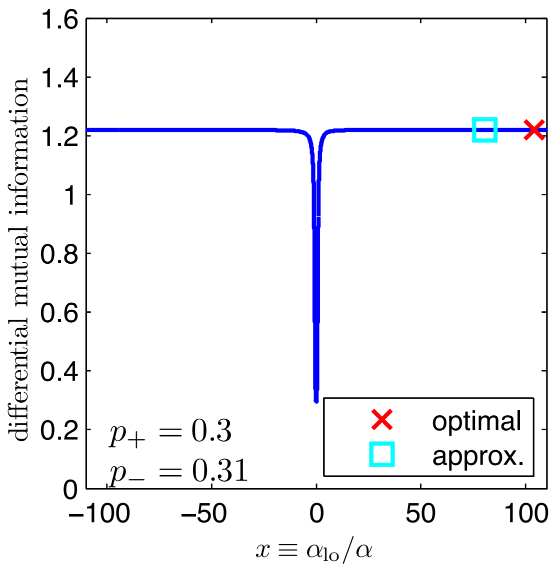
Here the subscripts $\{-, 0, +\}$ refer to the inputs in the same order that they are listed above, and we have suppressed the time dependence to reduce notation clutter.

The analytic solution to this equation is intractable, although it can be solved numerically. In lieu of an optimal solution, let us propose a heuristic local oscillator function that—as we shall show shortly—performs almost as well as the optimal local oscillator. Let us derive inspiration from the Dolinar receiver local oscillator function whose magnitude evolves deterministically. Suppose we choose a local oscillator as

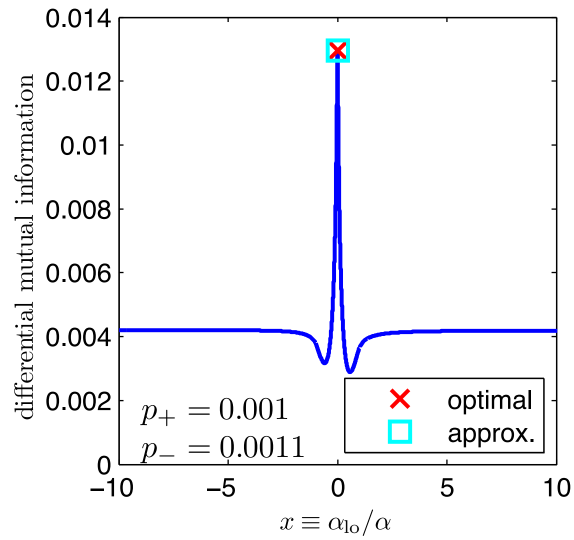
$$\alpha_{\text{lo}} = \begin{cases} \alpha\frac{1+p_-+p_+}{2(p_--p_+)} & I > -(p_- + p_+)\log(p_- + p_+) \\ 0 & \text{otherwise,} \end{cases} \tag{30}$$

where

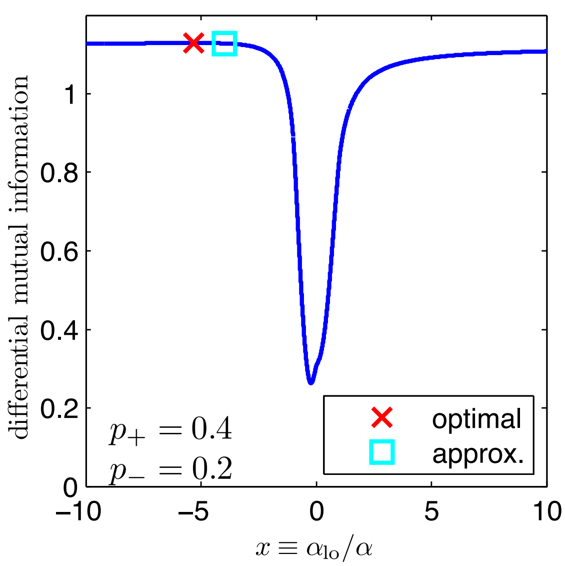
$$I = \left[p_+(x_0+1)^2 - p_-(x_0-1)^2\right]\log\left(\frac{x_0+1}{x_0-1}\right) + \left[1 - p_+ - p_-\right]\log\left(\frac{x_0^2}{x_0^2-1}\right) \tag{31}$$



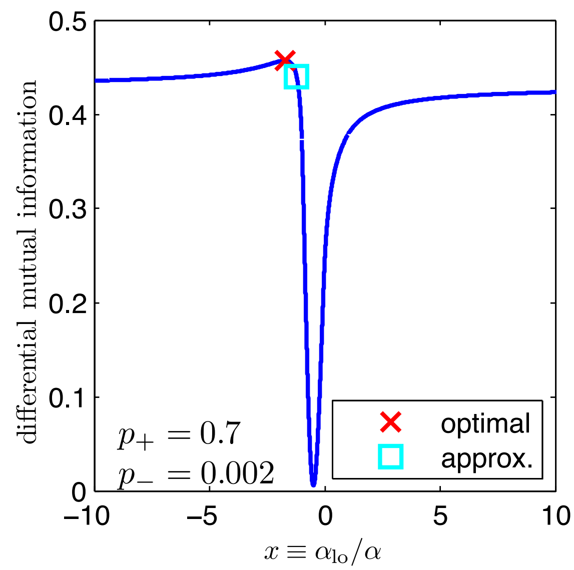
(a) $p_+ = 0.3$, $p_- = 0.31$



(b) $p_+ = 0.001$, $p_- = 0.0011$



(c) $p_+ = 0.4$, $p_- = 0.2$



(d) $p_+ = 0.7$, $p_- = 0.002$

Figure 3. The objective function is plotted as a function of the normalized local oscillator $x \equiv \alpha_{\mathrm{lo}}/\alpha$ for several probability distributions on the three hypotheses. The optimal value of the local oscillator is shown with a cross ($\times$), and the analytic approximation given in Eq. (30) is shown with a square ($\Box$). While the analytic approximation may not be very close to the optimal value, the difference in the differential mutual information is small in all instances shown here.

where $x_0 = (1 + p_- + p_+)/(2p_- - 2p_+)$. The first case in Eq. (30) is the solution that results in

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\prod_{k=\{-,0,+\}} p_k(t)\right) = -5\alpha^2 \left(\prod_{k=\{-,0,+\}} p_k(t)\right) \tag{32}$$

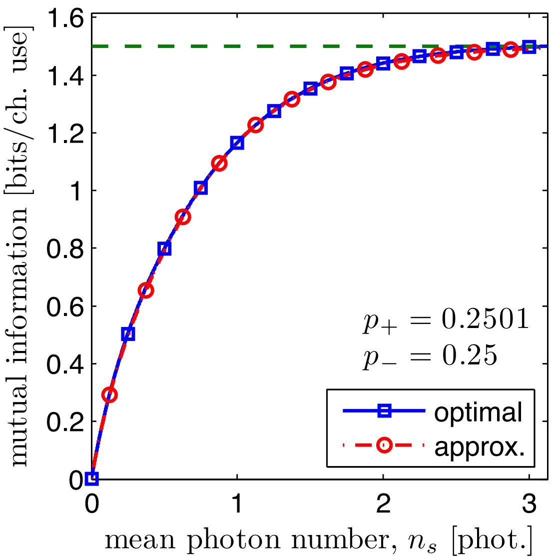
in between photon detection epochs, i.e., the product of the probabilities for the three input states evolves deterministically between photon detection events.[‡] Furthermore, when $p_0(0) = 0$, the local oscillator reduces to that derived in Section 3. On the other hand, the second case in Eq. (30), i.e., $\alpha_{\mathrm{lo}} = 0$, is the optimal solution when the vacuum state has a high-enough likelihood. In Fig. 3 we compare the performance of this heuristic local oscillator to that of the optimal local oscillator, for several different probability distributions. We see that in the cases where the heuristic local oscillator notably differs from the optimal value, the objective function is rather flat around the maximum, which results in a minor performance penalty. On the other hand, when the differential mutual information is peaked around its maximum, the Eq. (30) local oscillator agrees well with the optimal value. This agreement is also evident in the Fig. 4 plots, where we have numerically evaluated and plotted the mutual information obtained with the optimal local oscillator and that obtained with the heuristic approximation we have provided in Eq. (30). The performance with the two local oscillators demonstrates very good agreement in all instances.

In Fig. 5 we have empirically estimated the PIE versus DIE trade-off curve for the ternary alphabet plus the optimal adaptive-feedback receiver combination studied in this section, by taking the convex hull of the trade-off curves attained with numerous input probability distributions of the form $p_+(0) = p_-(0)$. In addition, we have plotted the trade-off curves for other known modulation and receiver pairs, as well as the ultimate limit determined by the Holevo information bound. Figure 5 shows that the ternary alphabet and adaptive feedback receiver pair universally attains higher PIE and DIE than both OOK plus photon-counting, and BPSK plus the Dolinar receiver. In addition, the (PIE, DIE) pairs attained with the ternary modulation plus adaptive receiver asymptotically approaches that attained with OOK plus photon-counting. These are not surprising results. The ternary modulation alphabet encompasses both coherent-state OOK and coherent-state BPSK, and the adaptive feedback receiver encompasses the Dolinar receiver (which is optimal for both the OOK and BPSK modulations). So, the performance with a ternary alphabet and the optimal adaptive feedback receiver should encompass the performance achievable with the other two modulation and receiver pairs. Furthermore, high PIE is achieved in the very low mean photon-number regime. In this regime, the added information from two antipodal coherent states is not significantly larger than that provided by the 'on' versus 'off' states. Hence the performance of OOK plus photon-counting and the ternary modulation plus adaptive feedback receiver should indeed converge at the high PIE limit.

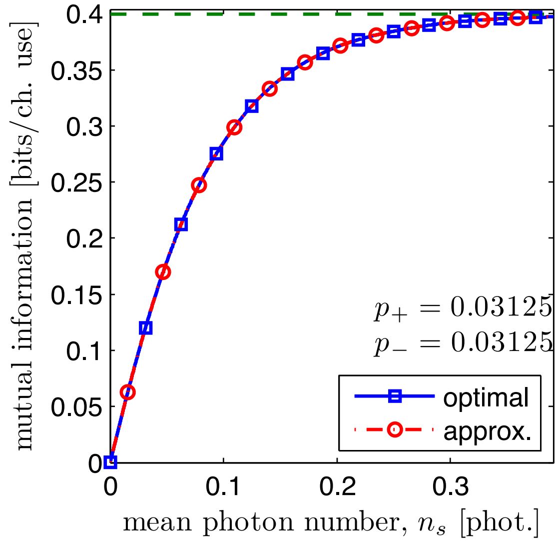
It is well known that the OOK plus photon-counting curve asymptotically approaches the ultimate limit set by the Holevo bound (see curve marked 'Gauss. + ult.' in Fig. 5). Because the ternary modulation plus adaptive feedback receiver lies between these two curves, it too asymptotically approaches the Holevo bound. Unfortunately, however, this receiver closes only a small fraction of the gap that exists between photon-counting and the ultimate limit at finite PIE or DIE.

Before we end this section, let us revisit the local oscillator function that has been used in the quantum joint-detection receiver (JDR) of Guha *et al.*[11] In that work, a receiver of the form we have presented here is used to distinguish the same ternary alphabet we have analyzed, with the input distribution $p_-(0) = p_+(0) = p$ and $p_0(0) = 1 - 2p$ for some $p \in [0, 0.5)$. Their feedback algorithm sets the local oscillator to 0 until the first photodetection event occurs. If a photon is detected then the local oscillator is instantaneously set to that of the Dolinar receiver for the remainder of the observation window. If no photon is registered, then the local oscillator remains 0 until the end of the observation window and the decision is the vacuum state. One might inquire about the optimality of this intuitive strategy, which we can now unambiguously answer using the formalism we have developed herein. Using the approximate local oscillator solution in Eq. (30) and comparing I to $-2p\log(2p)$, we find that $\alpha_{\mathrm{lo}}(0) = 0$ is optimal when $p < 0.5e^{-2} \approx 0.068$. Furthermore, as long as no arrivals are detected

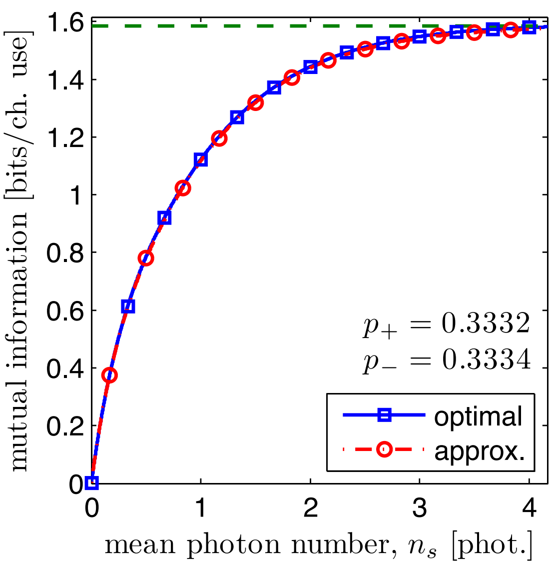
[‡]A photon detection event at t_i yields $\prod_{k=\{-,0,+\}} p_k(t_i^+) = \frac{x^2}{x^2-1} \prod_{k=\{-,0,+\}} p_k(t_i)$, in terms of $x \equiv \alpha_{\mathrm{lo}}/\alpha$, so unlike the binary case a photon detection results in a discontinuity in the product of the probabilities for all finite x.



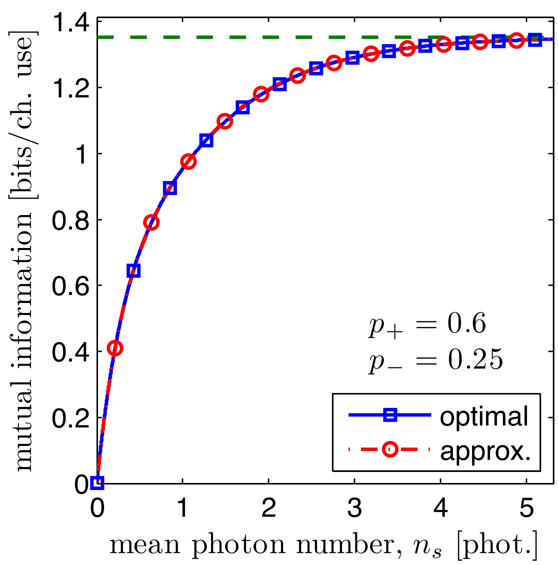
(a) $p_+ = 0.2501$, $p_- = 0.25$



(b) $p_+ = 0.03125$, $p_- = 0.03125$



(c) $p_+ = 0.3332$, $p_- = 0.3334$



(d) $p_+ = 0.6$, $p_- = 0.25$

Figure 4. Simulation results for the mutual information between K and $N(t)$ as a function of the mean photon number n_S, for various input distributions on the three hypotheses. The mutual information attained with the optimal local oscillator is shown with the solid line interspersed with square markers. That attained with the Eq. (30) approximation to the local oscillator is shown with the dash-dotted line interspersed with circle markers. In all instances, there is excellent agreement between the two cases. The dashed horizontal line is the entropy of K, which is the high-n_S asymptote for the mutual information.

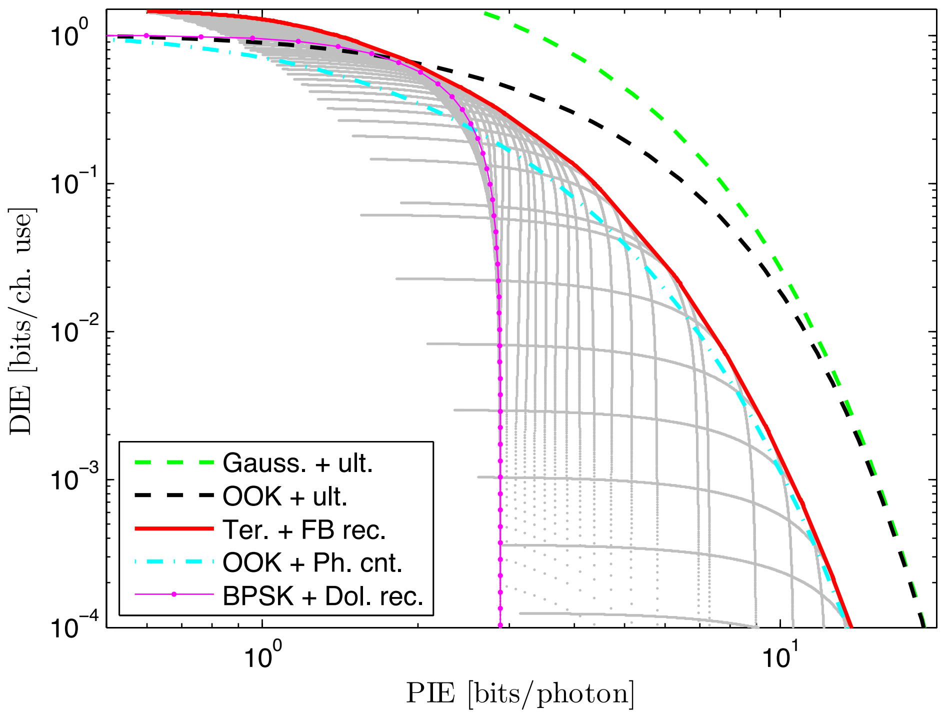


Figure 5. The empirically-derived PIE versus DIE tradeoff curve (red) for the ternary modulation $\{|-\alpha\rangle, |0\rangle, |\alpha\rangle\}$, and the adaptive local-oscillator receiver studied in Section 4, compared against the ultimate limit attained with isotropic Gaussian modulation over coherent-states (green dashed), that attained with generalized on-off keying modulation (OOK) modulation with ideal photon-counting (black dashed), and that attained with binary phase-shift keying (BPSK) and the Dolinar receiver (magenta dotted line). The simulation data that is the basis for the ternary-modulation trade-off curve is also shown for completeness (gray thin lines).

$p_+(t)$ and $p_-(t)$ continue to decrease (see Eq. (13)), so, the optimal local oscillator remains $\alpha_{\rm lo}(t) = 0$ until an arrival is registered. If an arrival is registered at t_1, then via Eq. (14) we conclude that

$$p_0(t_1^+) = 0, \tag{33}$$

$$p_+(t_1^+) = \frac{p_+(t_1)}{p_+(t_1) + p_-(t_1)}, = \frac{1}{2} \tag{34}$$

$$p_-(t_1^+) = \frac{p_-(t_1)}{p_+(t_1) + p_-(t_1)} = \frac{1}{2} \tag{35}$$

prevails, i.e., the probability of a vacuum-state input vanishes and the local oscillator simplifies to that of the Dolinar receiver studied in the previous section, with the input probabilities given by Eqs. (34) and (35). We therefore conclude that the strategy utilized by Guha *et al.*[11] is optimal only if the *a priori* probabilities of the antipodal coherent-states are each no greater than $0.5e^{-2} \approx 0.068$.[§] We have empirically observed in plotting Fig. 5 that this condition is satisfied for PIE¿ 2.5 bits/photon.

5. CONCLUSIONS

In this paper we have studied the class of optical communication receivers that utilize local-oscillator-based optical feedback within one symbol in order to maximize the mutual information between the coherent-state input symbol α_K and the observed photon-detection counting process $N(t)$ for $0 < t \leq T$. We first developed a general framework to study the receiver architecture that we introduced in Fig. 1. We showed that the local

[§]Note that a hybrid pulse-position-modulation (PPM)-BPSK alphabet with PPM order ≥ 8 satisfies this condition.

oscillator function that maximizes the mutual information between the input symbol and the output counting process can be determined incrementally, i.e., it is optimal to choose the local oscillator such that it maximizes the differential mutual information in the next instant of time, given the entire history of the photon-counting process up to the current time. We then derived a system of two equations that determine the critical points of the objective function. This showed that the optimal local oscillator is a function of the signal alphabet $\{\alpha_k : k = 1, \ldots, \mathcal{K}\}$, and the probability of each hypothesis at the current time *conditioned* on the arrival process up to that time, $\{n(\tau) : 0 \leq \tau < t\}$. We concluded our general formulation by deriving the evolution equations for these conditional probabilities. Collectively, this provided a complete description of the receiver with the optimal feedback function, whose performance could be simulated with the algorithm we presented as part of Section 2.

The two sections following this general framework were devoted to studying two cases of interest. First, in Section 3 we considered BPSK modulation. Perhaps not too surprisingly, the optimal local oscillator was identical to that of the Dolinar receiver, which achieves the Helstrom lower bound in the error probability of distinguishing between the two input states. Thus, we concluded from our analysis that, with binary coherent-state modulation, there is no soft information in the output process from the photodetector, $N(t)$. In other words, making a hard decision on the input symbol at the end of the observation window is information lossless. In Section 4 we expanded the binary constellation of Section 3 to include the vacuum state, thereby including a state in the alphabet that costs no photons to transmit. This constellation has been of interest recently in relation to the performance of hybrid modulations, and that of an optical-communication receiver architecture that detects symbols jointly over multiple channel uses. We proposed a heuristic local oscillator function in analytic form—which was inspired by the Dolinar receiver's local oscillator function—and showed that its performance was very close to that of the optimal local oscillator (which had to be solved for numerically). In addition, we showed that the local oscillator function employed in the joint-detection receiver of Guha *et al.* is the optimal strategy if and only if the energy-containing symbols each have *a priori* probabilities less than approximately 0.068. This occurs when the PIE exceeds approximately 2.5 bits-per-photon.

Before we conclude this article, it is worthwhile to briefly address the bandwidth assumptions of our work. Throughout our analysis we have assumed that the transmitter is bandwidth-limited, such that each channel use corresponds to T seconds during which the transmitted symbol remains unchanged. The receiver on the other hand enjoys infinite bandwidth in several respects. First, the photodetector has infinite bandwidth such that individual photon detection instants are resolvable. Second the feedback path enjoys zero delay and infinite bandwidth such that the local oscillator can be adjusted instantaneously. Of course, both of these are unrealistic assumptions and would need to be addressed in a practical implementation. However, we have employed these assumptions because they greatly simplify the analysis and the results are informative of the ultimate gains that are offered by local-oscillator-based optical feedback at the receiver.

In summary, we have analyzed the mutual information that can be obtained with an optical-communications receiver employing ideal local-oscillator optical feedback. We have provided a general formulation that can be used to analyze arbitrary signal constellations, and we have treated two simple cases of interest, highlighting the utility of our theoretical analysis framework.

APPENDIX A. DERIVATION OF THE DIFFERENTIAL CONDITIONAL MUTUAL INFORMATION

From Eq. (6), we need to evaluate

$$\lim_{\Delta T \to 0} \frac{i\big(K; N(t+\Delta T) - N(t) | \{n(\tau) : \tau \in (0,t]\}\big)}{\Delta T}. \tag{36}$$

The numerator inside the limit can be written as

$$\begin{aligned} i\big(K; N(t+\Delta T) - N(t) | \{n(\tau) : \tau \in (0,t]\}\big) &= \\ H\big(N(t+\Delta T) - N(t) | \{n(\tau) : \tau \in (0,t]\}\big) &- H\big(N(t+\Delta T) - N(t) | K, \{n(\tau) : \tau \in (0,t]\}\big), \end{aligned} \tag{37}$$

where $H(X) \equiv -E_X[\log(p(X))]$ is the well-known discrete entropy function for a random variable X having the probability mass function $p(x)$. Because the arrival process $n(\tau)$ for $\tau \in [0,t)$ and the *a priori* probability distribution $\{p_k(0)\}$ completely determines the local oscillator $\alpha_{\rm lo}(t)$, $N(t+\Delta T) - N(t)$ conditioned on $K=k$ and $n(\tau)$ for $\tau \in [0,t)$ is a Poisson random variable with mean value $\Delta T \lambda_k(t)$, where

$$\lambda_k(t) \equiv |\alpha_k + \alpha_{\rm lo}(t)|^2 \tag{38}$$

for small ΔT, such that $\int_t^{t+\Delta T} {\rm d}\tau \lambda_k(\tau) \approx \Delta T \lambda_k(t)$. Using tight bounds on the entropy of a Poisson random variable,[15] and dropping the time dependence to avoid notation clutter, we can express

$$H\big(N(t+\Delta T) - N(t)|K, \{n(\tau) : \tau \in (0,t]\}\big) = E_K\left[-\Delta T \lambda_K \log(\Delta T \lambda_K) + \Delta T \lambda_K\right] + o(\Delta T) \tag{39}$$

$$= -\Delta T \sum_{k=1}^{\mathcal{K}} p_k \lambda_k \log(\lambda_k) - \Delta T \log(\Delta T)\overline{\lambda} + \Delta T \overline{\lambda} + o(\Delta T)\,, \tag{40}$$

where $p_k \equiv P(K=k|\{n(\tau) : 0 \le \tau < t\})$ refers to the conditional probability of α_k, and where $\overline{\lambda} \equiv \sum_{k=1}^{\mathcal{K}} p_k \lambda_k$. Let us now turn to the first term in Eq. (37), noting that $N(t+\Delta T) - N(t)$ conditioned on $\{n(\tau) : \tau \in (0,t]\}$ is a compound Poisson random variable with a probability mass function

$$p_M(m) = \sum_{k=1}^{\mathcal{K}} p_k \frac{\lambda_k^m e^{-\lambda_k}}{m!}\,, \tag{41}$$

where we have temporarily defined $M \equiv N(t+\Delta T) - N(t)|\{n(\tau) : \tau \in (0,t]\}$ for convenience. The entropy of this random variable is

$$H(M) = -E_M[\log\big(p_M(M)\big)] = E_M[\log(M!)] - f(\Delta T)\,, \tag{42}$$

where

$$f(\Delta T) \equiv E_M\left[\log\left(\sum_{k=1}^{\mathcal{K}} p_k (\Delta T \lambda_k)^M e^{-\Delta T \lambda_k}\right)\right]. \tag{43}$$

Using iterated expectations and the fact $M|K$ is a Poisson random variable one can show that $E_M[\log(M!)] = o(\Delta T)$,[15] so that $H(M) = -f(\Delta T) + o(\Delta T)$. Therefore the final step is to find $f(\Delta T)$. We first express $f(\Delta T)$ as

$$f(\Delta T) = \sum_{m=0}^{\infty} \frac{1}{m!}\left[\sum_{k=1}^{\mathcal{K}} p_k (\Delta T \lambda_k)^m e^{-\Delta T \lambda_k}\right] \log\left(\sum_{\ell=1}^{\mathcal{K}} p_\ell (\Delta T \lambda_\ell)^m e^{-\Delta T \lambda_\ell}\right)\,, \tag{44}$$

then, utilizing Taylor series expansions, we obtain

$$f(\Delta T) = -\Delta T \overline{\lambda} + \Delta T \overline{\lambda} \log\left(\Delta T \overline{\lambda}\right) + o(\Delta T)\,. \tag{45}$$

Thus, the entropy of the random variable M is given by

$$H(M) = -\Delta T \overline{\lambda} \log \overline{\lambda} - \Delta T \log(\Delta T)\overline{\lambda} + \Delta T \overline{\lambda} + o(\Delta T)\,. \tag{46}$$

Finally, substituting Eq. (40) and (46) into Eq. (37), and then taking the limit in Eq. (36), we arrive at

$$\lim_{\Delta T \to 0} \frac{i(K; N(t+\Delta T) - N(t)|\{n(\tau) : \tau \in (0,t]\})}{\Delta T} = -\overline{\lambda}\log(\overline{\lambda}) + \sum_{k=1}^{\mathcal{K}} p_k \lambda_k \log(\lambda_k)\,, \tag{47}$$

as we have reported in Eq. (8).

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