Chung Capacity Summary

**Detecting a signal over an optical channel with photon counter receiver**

Given an optical channel, a signal on that channel can be described as a coherent state denoted by |s> where s\inC.

Considering a Noise free environment, in order to detect a signal in this type of channel, we use a photon counter receiver. Such a receiver is a direct detection receiver that detects the intensity of the optical signal at any point in time and generates a Poisson process, where the rate of the process holds \Lambda = |s|^2.

Suppose we have two coherent state signals denoted by |s\_0>, |s\_1> and we would like to distinguish between the two binary hypotheses with the corresponding priori probabilities \pi\_0, \pi\_1 respectively under hypotheses \H=0,1 , while holding some transmission cost constraint.

One approach was given by Kennedy who proposed adding a constant additional coherent state signal |l> before feeding the signal’s input to the receiver. Doing so generates a coherent state |s+l> which the receiver in turn outputs a Poisson process with rate |s+l|^2.

An additional approach was given by Dolinar who suggested in addition to Kennedy’s design to replace the constant signal with a controlled \l(t) one where it is chosen adaptively based on the photon arrivals up to that moment, in order to achieve more certainty in the hypothesis choice with time.

Given Dolinar’s updated controlled signal design, we get at a recursive method in which the posteriori probabilities of the two possible hypotheses denoted by \pi\_1(t), \pi\_2(t) are updated after each step of time \Delta to give \pi\_1(t+\Delta), \pi\_2(t+\Delta).

At this point, by making \Delta arbitrarily small we can expect the current Poisson process to return 0 with a very high probability of 1-\lambda\*\Delta or 1 with probability \lambda\*\Delta. This can be thought of as a binary channel.

Since we obtained an approximation of a binary channel we may now ask how should \l(t) be decided in other to maximize the mutual information of the binary channel.



The mutual information is given by:

I(X,Y) = H(Y) – H(Y|X) = H\_b(\Delta\*(\pi\_0\*\lambda\_0 + \pi\_1\*\lambda\_1)) – (\pi\_0\*H\_b(\lambda\_0\*\Delta) + \pi\_1\*H\_b(\lambda\_1\*\Delta))

Since we look to maximize the mutual information, we shall derive it and compare to 0:

d(I(X,Y))/dl = log\_2

either a \0 or \1 at which point the receiver’s controler can thought as a binary channel

Namely, after observing the signal mixture on the receiver during some arbitrarily small positive interval of time ,\Delta, we obtain

photon counter receiver = direct detection receivers: detect the intensity of the optical signal which generates a Poisson process

Noise free environment

Kennedy used this receiver architecture to distinguish between binary hypotheses