

# Entanglement in Featureless Mott Insulators

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# Outline

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## 1 Motivation

## 2 Entanglement Edge of Honeycomb Insulators

# Motivation

# Featureless insulators

## Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state:

$$E_1 - E_0 \geq \text{const.}$$

## Alternate Definition

- Unique ground state on any boundary-less system
- Possibly with 'features' localized to edge of system

## Fundamental Result

- Integer charge per unit cell
  - (Lieb, Schultz, Mattis)

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- Gapless modes:

$$E_1 - E_0 \sim \frac{1}{L^\nu}$$

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- Unique ground state:  
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- Spontaneous symmetry breaking:  
 $E_1 - E_0 = 0$

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 $E_1 - E_0 \sim e^{-L/\xi}$   
with nontrivial topology

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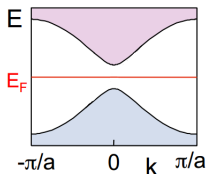
- Topological order:

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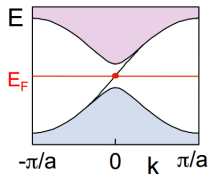
# Examples of Featureless Insulators

## Classical Insulators

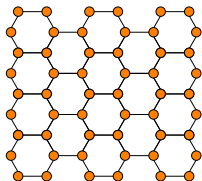


Free fermion band insulator

## Topological Insulators



Band insulator with chiral edge <sup>1</sup>



Bosonic Mott insulator

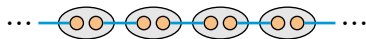
# Examples of Featureless Insulators

## Classical Insulators

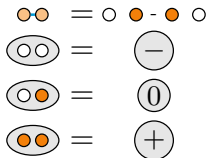


1D Trivial Chain Caricature

## Topological Insulators



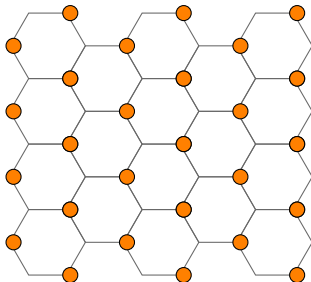
1D Topological Chain Caricature



Entangled pairs and projectors for AKLT state

# Honeycomb Bosonic Mott Insulators

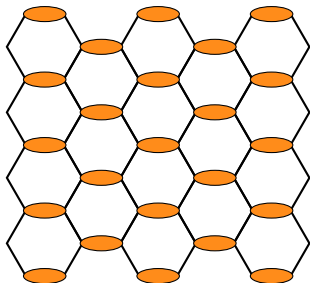
Does there exist a featureless bosonic insulator with filling  $m=1$  on the honeycomb?



Breaks point group symmetry  $D_6$  to  $D_3$

# Honeycomb Bosonic Mott Insulators

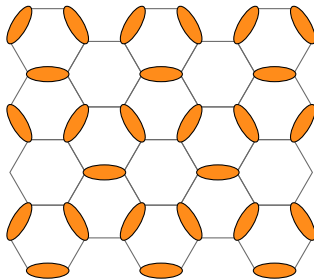
Does there exist a featureless bosonic insulator with filling  $m=1$  on the honeycomb?



Breaks rotational symmetry

# Honeycomb Bosonic Mott Insulators

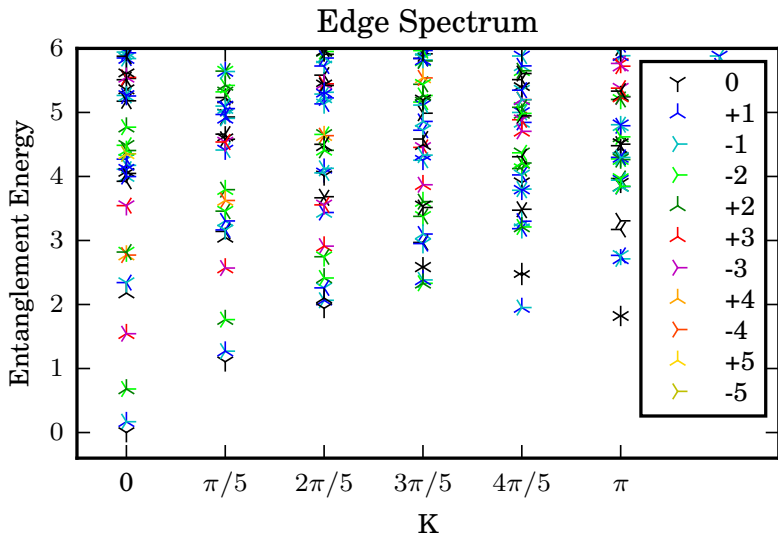
Does there exist a featureless bosonic insulator with filling  $m=1$  on the honeycomb?



Breaks translational symmetry, unit cell is 3 times larger

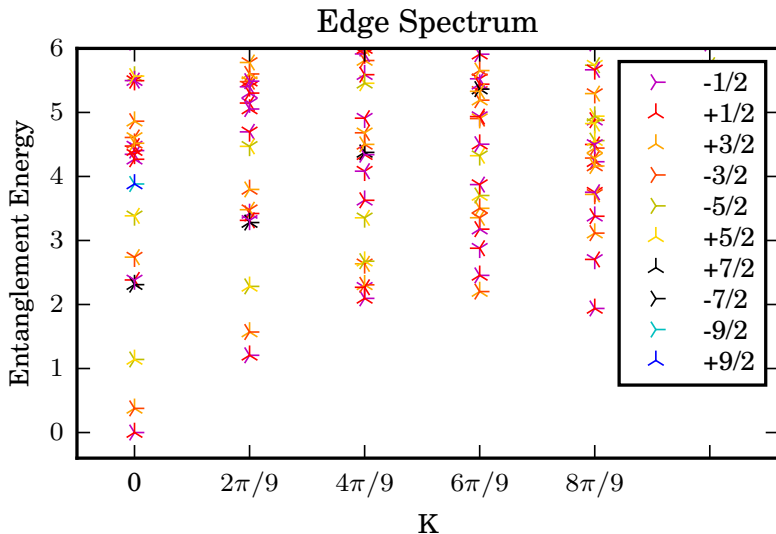
# Entanglement Edge of Honeycomb Insulators

# Entanglement Spectrum

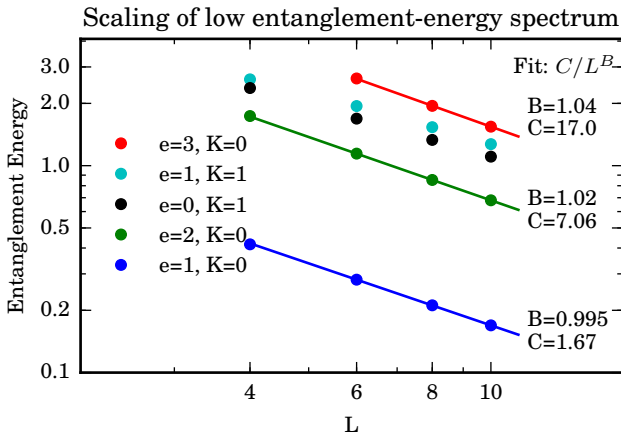




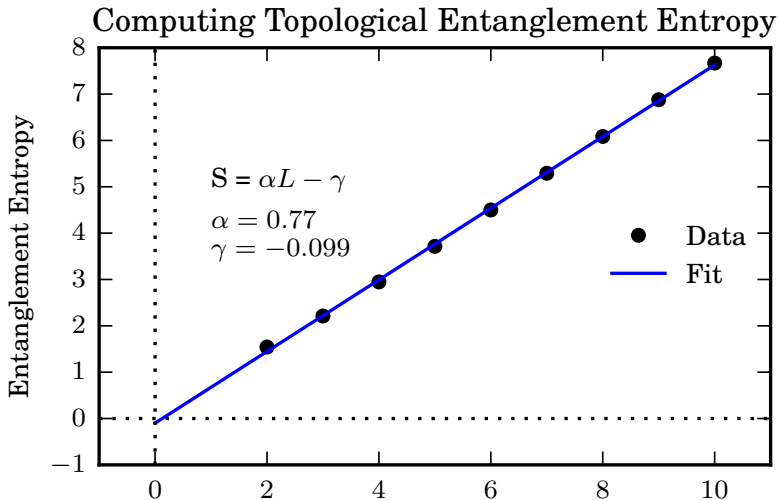
# Entanglement Spectrum



# Finite Size Analysis of Entanglement Spectra

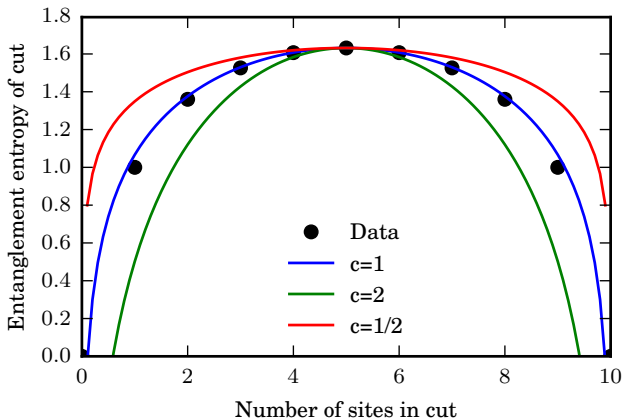


# Finite Size Analysis of Entanglement Spectra



# Identification of Edge CFT

## Conformal Charge



$$c = 1$$

# Identification of Edge CFT

## Conformal Weights

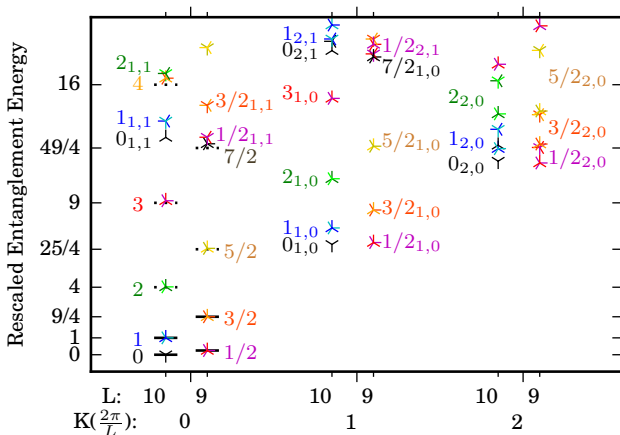
We can match the rescaled entanglement energies to the conformal weights of a free bosonic CFT.

$$\begin{aligned}\mathbf{P} &= \frac{2\pi}{L}(\mathbf{L}_0 - \bar{\mathbf{L}}_0) = \frac{2\pi}{L}(em + n - \bar{n}) \\ \mathbf{H} &= \frac{2\pi}{L}(\mathbf{L}_0 + \bar{\mathbf{L}}_0) = \frac{2\pi}{L}\left(\frac{\kappa e^2}{2} + \frac{m^2}{2\kappa} + \frac{n + \bar{n}}{2}\right)\end{aligned}$$

$$\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})$$

# Identification of Edge CFT

Conformal primary identification in entanglement spectra



# Future Work

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- Entanglement properties in different geometries
  - Cylinders with different edges
  - Finite size clusters
- Relation to 'MPO Injectivity'
  -
- Numerical testing of parent Hamiltonians
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# Resources

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Hasan, M. Z. and Kane, C. L. (2010). *Colloquium: Topological insulators. Reviews of modern physics*, 82(4):3045–3067.



# Questions?

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# Bonus slides