

Entanglement in Featureless Mott Insulators

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Featureless Insulators

Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state:

$$E_1 - E_0 \geq \text{const.}$$

Alternate Definition

- Unique ground state on all boundary-less systems
- Possibly with 'features' localized to edge of system

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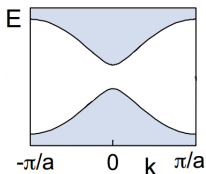
- Gapless modes:
 $E_1 - E_0 \sim \frac{1}{L^z}$

- Spontaneous symmetry breaking:
 $E_1 - E_0 = 0$

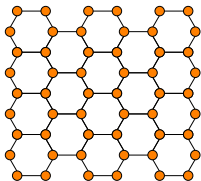
- Topological order:
 $E_1 - E_0 \sim e^{-L/\xi}$

Examples of Featureless Insulators

Classical Insulators

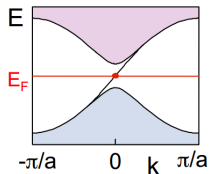


Free fermion band insulator

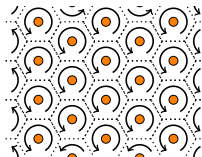


Atomic picture

Topological Insulators



Band insulator with chiral edge



Atomic picture breaks down

Obstructions to Featurelessness

Fundamental Result

A featureless insulator must have an integer charge per unit cell

- (Lieb, Schultz, Mattis 1961)
- (Hastings 2004)

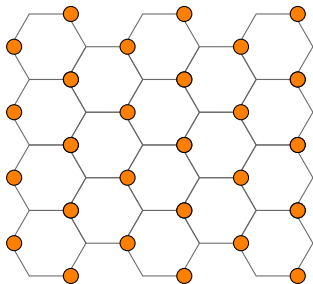
For certain lattices, not all integers are possible

- (Parameswaran 2013)

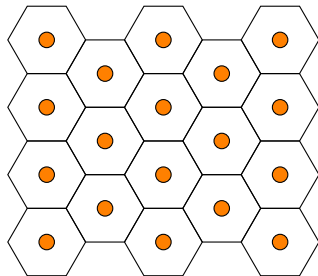
For this talk, we will look at a proposed honeycomb lattice featureless insulator with charge 1 per unit cell.

Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



Breaks rotational symmetry

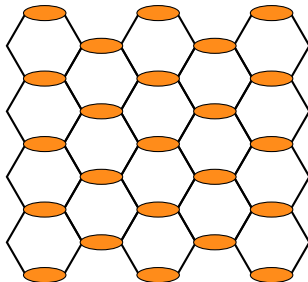


Leaves honeycomb lattice

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013a)

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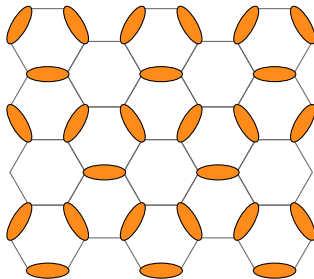


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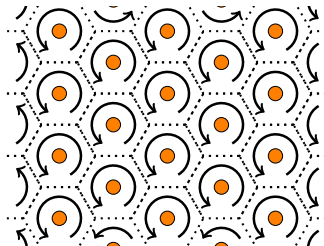
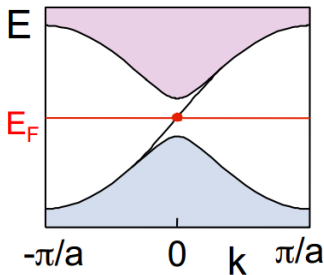


Breaks translationally symmetry, unit cell is 3 times larger

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013a)

Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



Band insulator with chiral edge ¹

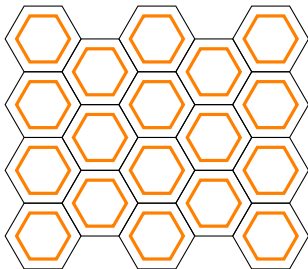
The Haldane Chern insulator is NOT an example. D_6 explicitly broken.

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013a)

¹(Hasan and Kane, 2010)

Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?

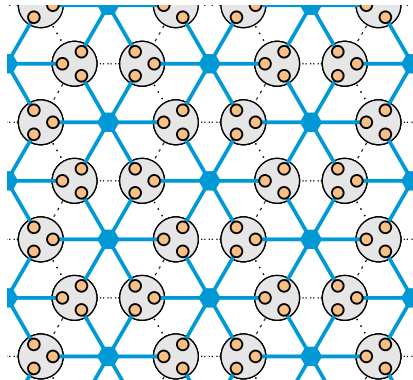


$$|\psi\rangle = \prod_{\hexagon} \left(\sum_{i \in \hexagon} b_i^\dagger \right) |0\rangle$$

Proposed Solution by Kimchi et al. (2013)

Bosons filled into non-orthogonal, plaquette centered orbitals works. Numerics confirm the expected wavefunction properties, but no known parent Hamiltonian has been found.

Computations on Honeycomb FBI



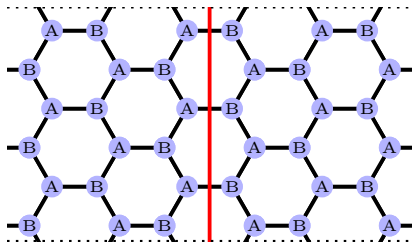
Simple tensor network representation

Cylinder slice treated as single site of an effective 1D system.

Schmidt decomposition computed as in 1D matrix product states.

$$|\psi\rangle = \prod_{\square} \left(\sum_{i \in \square} b_i^\dagger \right) |0\rangle$$

Computations on Honeycomb FBI



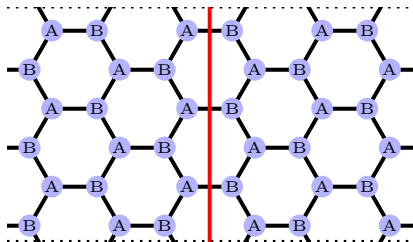
Form of a honeycomb lattice PEPS
on zig-zag cylinder with width $L=3$

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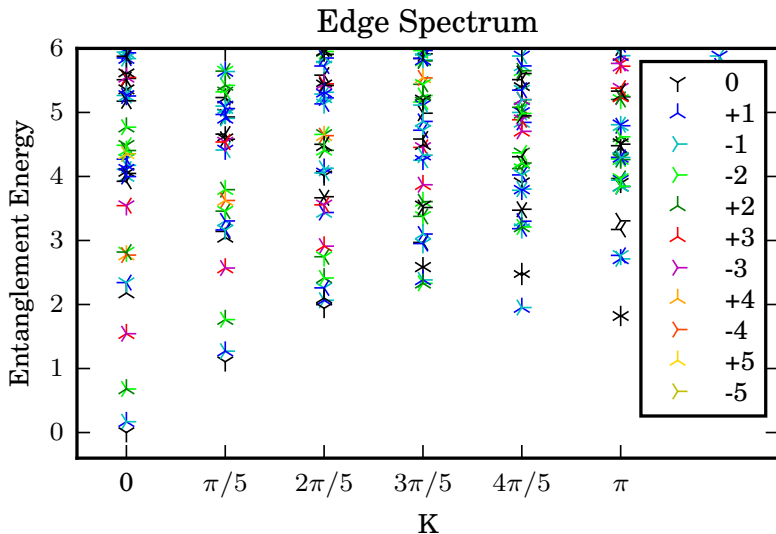
Simple tensor network
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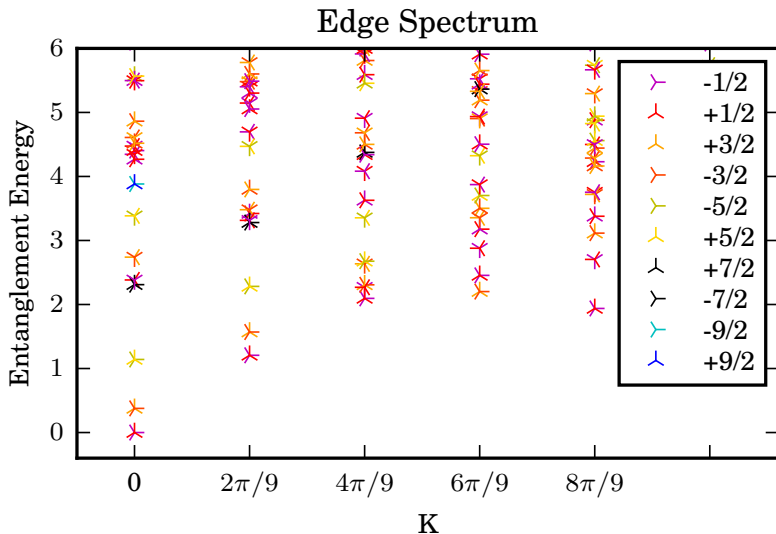
Schmidt decomposition
computed as in 1D matrix
product states.

$$\begin{array}{c}
 \begin{array}{c} \text{...} \uparrow \uparrow \uparrow \text{...} \\ \boxed{\psi} \end{array} = \text{...} \leftarrow \text{diamond} \leftarrow \text{circle } \Gamma \leftarrow \text{diamond} \leftarrow \text{circle } \Gamma \leftarrow \text{diamond} \leftarrow \text{...} \\
 |\psi\rangle = \sum_i |\psi_L^i\rangle \lambda_i |\psi_R^i\rangle
 \end{array}$$

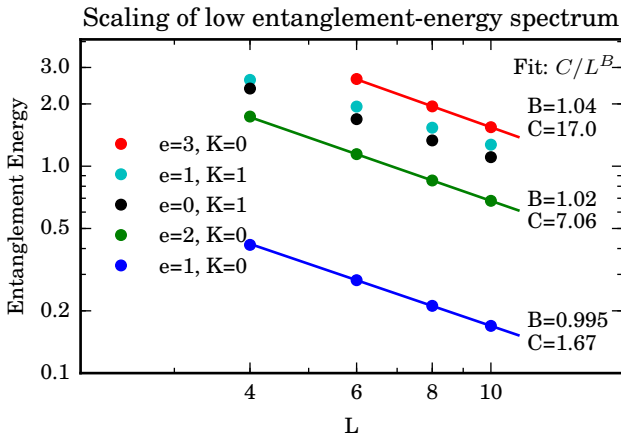
Entanglement Spectrum



Entanglement Spectrum

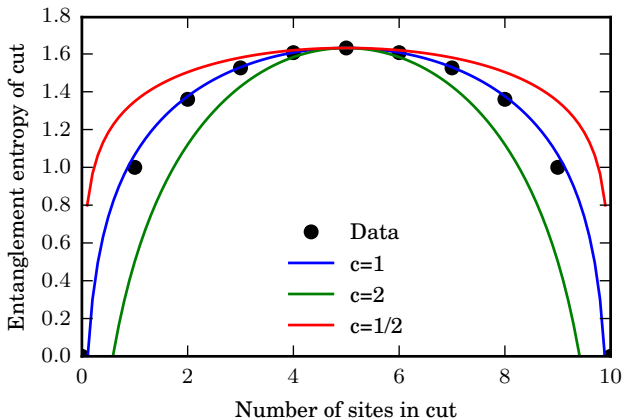


Finite Size Analysis



Identification of Edge CFT

Conformal Charge via 'Nested Entanglement Entropy'



$$c = 1$$

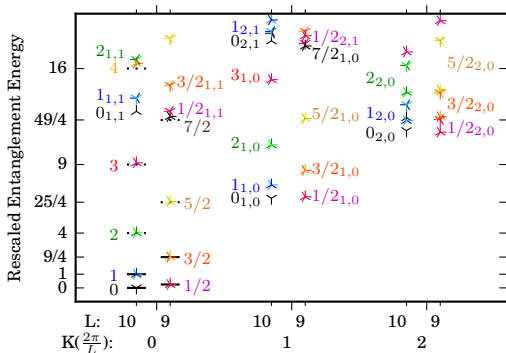
Identification of Edge CFT

Conformal Weights

We can match the rescaled entanglement energies to the conformal weights of a free bosonic CFT.

$$\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})$$

Conformal primary identification in entanglement spectra



Symmetry Protection of Degenerate Edge

$$|\psi\rangle = \sum_i \lambda_i |\psi_L^i\rangle |\psi_R^i\rangle$$

Inversion symmetries map left Schmidt states to right Schmidt states with the same eigenvalue. Combined with the map that matches each Schmidt state on right to its partner on the left, you get an antiunitary action on sets of degenerate Schmidt states.

$$\mathcal{I} \rightarrow V_{\mathcal{I}}$$

$$V_{\mathcal{I}}|e, K\rangle = |-e, K\rangle$$

Symmetry Protection of Degenerate Edge

$$|\psi\rangle = \sum_i \lambda_i |\psi_L^i\rangle |\psi_R^i\rangle$$

On-site symmetries act unitarily on sets of degenerate Schmidt states.

$$\theta \rightarrow V_\theta$$

$$V_\theta |e, K\rangle \rightarrow e^{i\theta} |e, K\rangle$$

$$V_\pi |e, K\rangle (-1)^e |e, K\rangle$$

Symmetry Protection of Degenerate Edge

$$|\psi\rangle = \sum_i \lambda_i |\psi_L^i\rangle |\psi_R^i\rangle$$

Combined inversion and charge parity

Conclusions

For the honeycomb featureless boson insulator:

- Entanglement spectrum reveals a gapless free boson edge
- Edge spectrum points with nonzero charge or nonzero momentum are degenerate
- This degeneracy is protected by combined inversion and charge parity
- Cannot be deformed to trivial state while the bosons are not allowed to live at the hexagon centers
- The representation of the lattice and charge symmetry (size of unit cell and charge per unit cell) matters for classifying featureless insulators

Questions?

Brayden Ware

brayden@physics.ucsb.edu

Bonus slides

Resources

- Hasan, M. Z. and Kane, C. L. (2010). *Colloquium: Topological insulators. Reviews of modern physics*, 82(4):3045–3067.
- Kimchi, I., Parameswaran, S. A., Turner, A. M., Wang, F., and Vishwanath, A. (2013). Featureless and nonfractionalized mott insulators on the honeycomb lattice at $1/2$ site filling. *Proceedings of the National Academy of Sciences*, 110(41):16378–16383.
- Parameswaran, S. A., Kimchi, I., Turner, A. M., Stamper-Kurn, D. M., and Vishwanath, A. (2013a). Wannier permanent wave functions for featureless bosonic mott insulators on the $1/3$ -filled kagome lattice. *Phys. Rev. Lett.*, 110:125301.
- Parameswaran, S. A., Turner, A. M., Arovas, D. P., and Vishwanath, A. (2013b). Topological order and absence of band insulators at integer filling in non-symmorphic crystals. *Nature Physics*, 9(5):–303?
- Pollmann, F., Turner, A. M., Berg, E., and Oshikawa, M. (2010). Entanglement spectrum of a topological phase in one

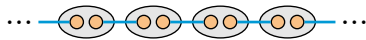
Construction of 1D Featureless Insulators

Classical Insulators



1D Trivial Chain

Topological Insulators



1D Topological Chain

$$\begin{aligned} \text{orange dot} - \text{blue line} - \text{orange dot} &= \text{white circle} + \text{orange dot} + \text{orange dot} + \text{white circle} \\ \text{white circle} - \text{blue line} - \text{white circle} &= 0 \\ \text{white circle} - \text{blue line} - \text{orange dot} &= 1 \\ \text{orange dot} - \text{blue line} - \text{orange dot} &= 2 \end{aligned}$$

Projectors and entangled pairs (PEPS) used in state construction

Construction of 1D Featureless Insulators

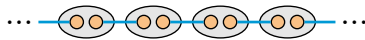
Classical Insulators



1D Trivial Chain

Product state with one boson per site

Topological Insulators



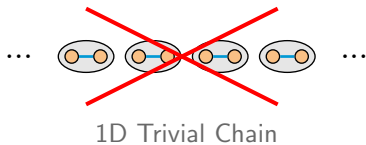
1D Topological Chain

Haldane Insulator Phase
Pollmann et al. (2010)

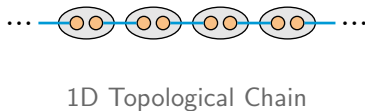
- Unitarily related to AKLT
- No $SU(2)$ symmetry
- Symmetry protected 2-fold edge degeneracy

Construction of 1D Featureless Insulators

Classical Insulators



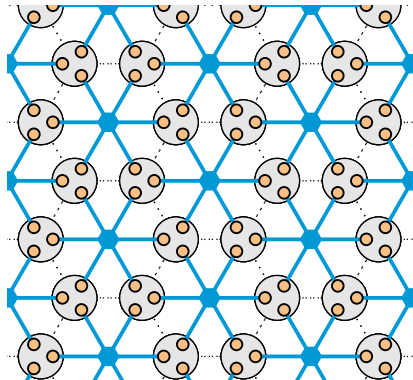
Topological Insulators



$$\begin{aligned}
 \text{two orange dots} &= \text{white circle} - \text{orange dot} - \text{orange dot} - \text{white circle} \\
 \text{two white dots in a circle} &= -\sqrt{2} \\
 \text{one white dot and one orange dot in a circle} &= 0 \\
 \text{two orange dots in a circle} &= +\sqrt{2}
 \end{aligned}$$

Projectors and entangled pairs (PEPS) for $SU(2)$ symmetric state

Tensor Network cut details



$$\text{hexagon with 3 orange dots} = \textcircled{3} \sqrt{3!}$$

$$\text{hexagon with 2 orange dots} = \textcircled{2} \sqrt{2!}$$

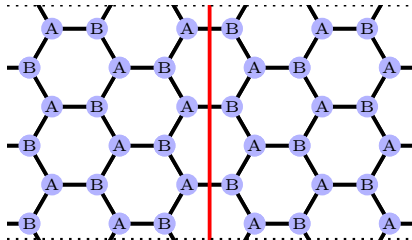
$$\text{hexagon with 1 orange dot} = \textcircled{1}$$

$$\text{hexagon with 0 orange dots} = \textcircled{0}$$

$$\text{blue star tensor} = \text{sum of 6 configurations}$$

$$|\psi\rangle = \prod_{\text{hex}} \left(\sum_{i \in \text{hex}} b_i^\dagger \right) |0\rangle$$

Tensor Network cut details

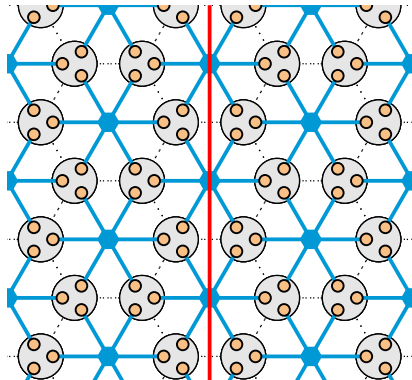


Generic honeycomb lattice PEPS on zig-zag cylinder with $L=3$

In cylindrical geometry:

- Treat state as 1D
- Use MPS techniques
- On-site translational symmetry parallel to cut
- Physical site dimension 4^{2L}

Tensor Network cut details

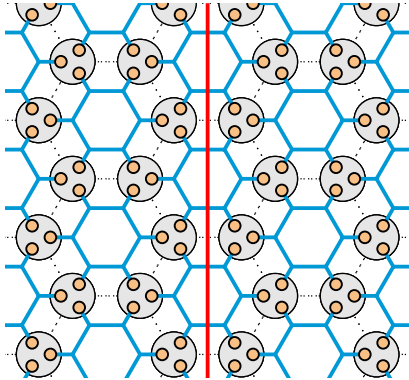


Honeycomb lattice tensor network on zig-zag cylinder with $L=3$

In cylindrical geometry:

- Treat state as 1D
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- On-site translational symmetry parallel to cut
- Physical site dimension 4^{2L}

Tensor Network cut details

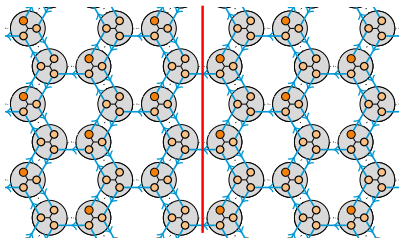


Honeycomb lattice PEPS on zig-zag cylinder with $L=3$, achieved by factoring W-state of plaquette bosons

In cylindrical geometry:

- Treat state as 1D
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- On-site translational symmetry parallel to cut
- Physical site dimension 4^{2L}
- MPS bond dimension = Rank of $\rho_r = 2^L$
- Entanglement spectrum $\{\epsilon_i\}$ defined from eigenvalues $\{\rho_i\}$ of ρ_r via $\epsilon_i = e^{-\rho_i}$
- Charge and Translation represented linearly on edge

Tensor Network cut details

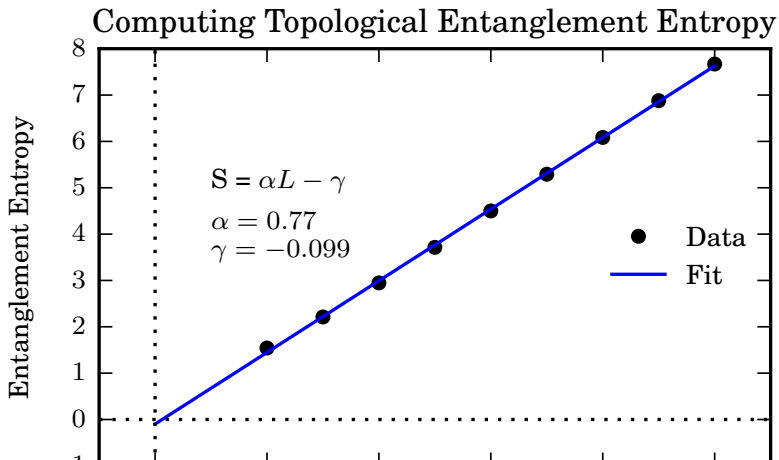


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Topological Entanglement Entropy



Known Results for Honeycomb FBI

Correlations

$$\langle b_i^\dagger b_j \rangle$$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length $\xi/a \sim 3.6$

$$\langle n_i n_j \rangle$$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length $\xi/a \sim 1.6$

Hamiltonian Construction

Try filling plaquette orbitals

$$b_{\hexagon} = \sum_{i \in \hexagon} \frac{1}{\sqrt{6}} b_i^\dagger$$

$$H = \sum_{\hexagon} -\frac{t}{6} b_{\hexagon}^\dagger b_{\hexagon} + V n_{\hexagon} n_{\hexagon}$$

$$= \left(\sum_{\hexagon} \sum_{i,j \in \hexagon} -t b_i^\dagger b_j \right) - \frac{3t}{6} N + V \dots$$

- Fails, gapless modes
- Parent Hamiltonian not known

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$$\langle n_i n_j \rangle$$

- Looks rotationally symmetric
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Hamiltonian Construction

To get a parent Hamiltonian:

- Need symmetric, exponentially localized, orthogonal orbitals
- Such as the Wannier orbitals of a classical band insulator
- Parameswaran et al. (2013a)

Other lattices:

- Need a fixed point of all lattice symmetries
- Fails on nonsymmorphic lattices
- Extension of LSM theorem
- Parameswaran et al. (2013b)

Future Work

Entanglement properties with different geometries

- Armchair cylinder edge
- Finite size clusters
- Explain results for arbitrary geometries with tensor network properties, e.g. 'MPO injectivity'

Find a 2D local Hamiltonian and confirm with numerics

$$H_{EBH} = \left(\sum_{\hexagon} \sum_{i,j \in \hexagon} -tb_i^\dagger b_j + V n_i n_j \right) + \mu N?$$

Physical properties of the phase

Can we construct an SU(2) symmetric FI?