

# Entanglement in Featureless Mott Insulators

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# Featureless Insulators

## Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order
- Unique ground state:  
 $E_1 - E_0 \geq \text{const.}$

## Alternate Definition

- Unique ground state on all boundary-less systems
- Possibly with 'features' localized to edge of system

## Fundamental Result

- Integer charge per unit cell
  - (Lieb, Schultz, Mattis)

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- Gapless modes:  
 $E_1 - E_0 \sim \frac{1}{L^z}$

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- Spontaneous symmetry breaking:  
 $E_1 - E_0 = 0$

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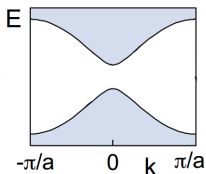
- Topological order:  
 $E_1 - E_0 \sim e^{-L/\xi}$   
with nontrivial topology

## Fundamental Result

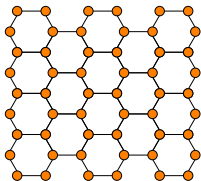
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# Free Fermion Featureless Insulators

## Classical Insulators

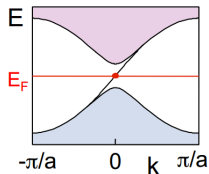


Free fermion band insulator

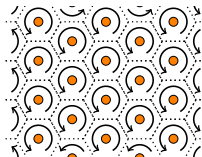


Atomic picture

## Topological Insulators



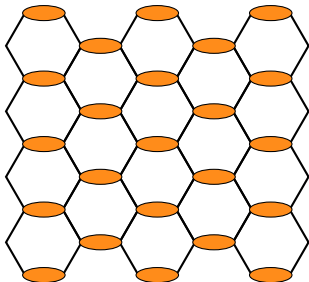
Band insulator with chiral edge <sup>1</sup>



Atomic picture breaks down

# Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



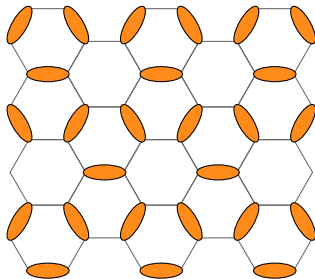
Breaks rotational symmetry

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013a)



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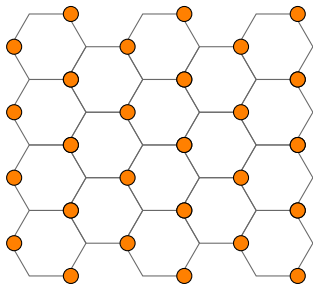


Breaks translationally symmetry, unit cell is 3 times larger

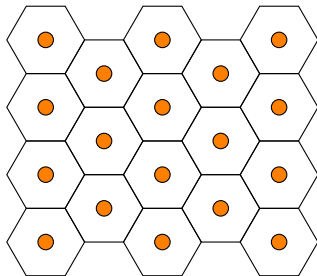
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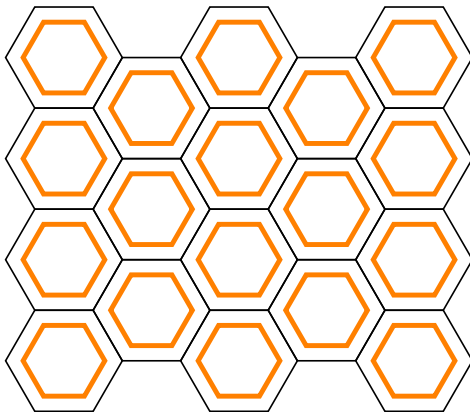


Leaves honeycomb lattice

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013a)

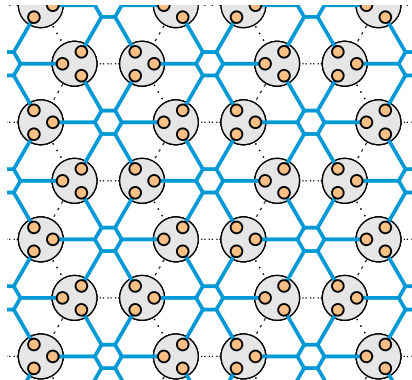
# Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



Proposed Solution by Kimchi et al. (2013)

# Construction of Honeycomb FBI



$$\text{3 dots in circle} = \textcircled{3} \sqrt{3!}$$

$$\text{2 dots in circle} = \textcircled{2} \sqrt{2!}$$

$$\text{1 dot in circle} = \textcircled{1}$$

$$\text{0 dots in circle} = \textcircled{0}$$

$$\text{Central hexagon} = \text{Hexagon 1} + \text{Hexagon 2} + \text{Hexagon 3} + \text{Hexagon 4} + \text{Hexagon 5} + \text{Hexagon 6}$$

$$|\psi\rangle = \prod_{\text{hex}} \left( \sum_{i \in \text{hex}} b_i^\dagger \right) |0\rangle$$

Wavefunction proposed by  
Kimchi et al. (2013)

# Known Results for Honeycomb FBI

## Correlations

$$\langle b_i^\dagger b_j \rangle$$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length  $\xi/a \sim 3.6$

$$\langle n_i n_j \rangle$$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length  $\xi/a \sim 1.6$

## Hamiltonian Construction

Try filling plaquette orbitals

$$b_{\hexagon} = \sum_{i \in \hexagon} \frac{1}{\sqrt{6}} b_i^\dagger$$

$$H = \sum_{\hexagon} -\frac{t}{6} b_{\hexagon}^\dagger b_{\hexagon} + V n_{\hexagon} n_{\hexagon}$$

$$= \left( \sum_{\hexagon} \sum_{i,j \in \hexagon} -t b_i^\dagger b_j \right) - \frac{3t}{6} N + V \dots$$

- Fails, gapless modes
- Parent Hamiltonian not known

# Known Results for Honeycomb FBI

## Correlations

$$\langle b_i^\dagger b_j \rangle$$

- Looks rotationally symmetric
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## Hamiltonian Construction

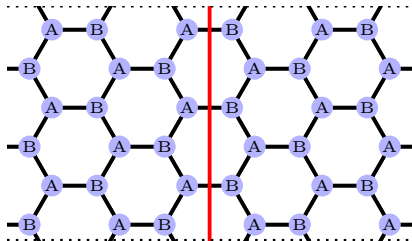
To get a parent Hamiltonian:

- Need symmetric, exponentially localized, orthogonal orbitals
- Such as the Wannier orbitals of a classical band insulator
- Parameswaran et al. (2013a)

Other lattices:

- Need a fixed point of all lattice symmetries
- Fails on nonsymmorphic lattices
- Extension of LSM theorem
- Parameswaran et al. (2013b)

# Edge Geometry

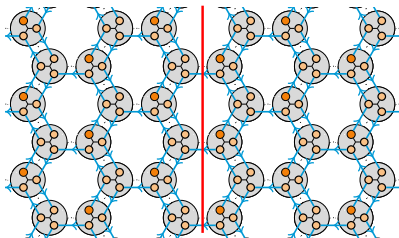


Generic honeycomb lattice PEPS on  
zig-zag cylinder with  $L=3$

In cylindrical geometry:

- Treat state as 1D
- Use MPS techniques
- On-site translational symmetry parallel to cut
- Physical site dimension  $4^{2L}$

# Edge Geometry

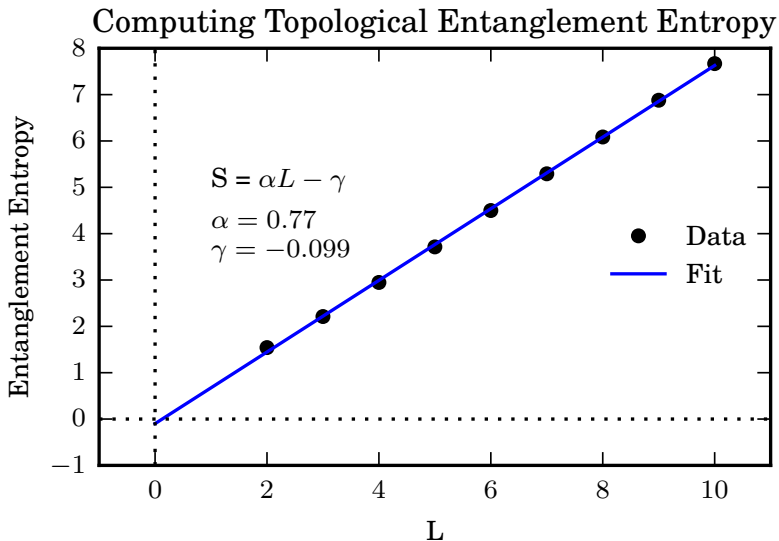


Honeycomb lattice PEPS on zig-zag cylinder with  $L=3$ , achieved by factoring W-state of plaquette bosons

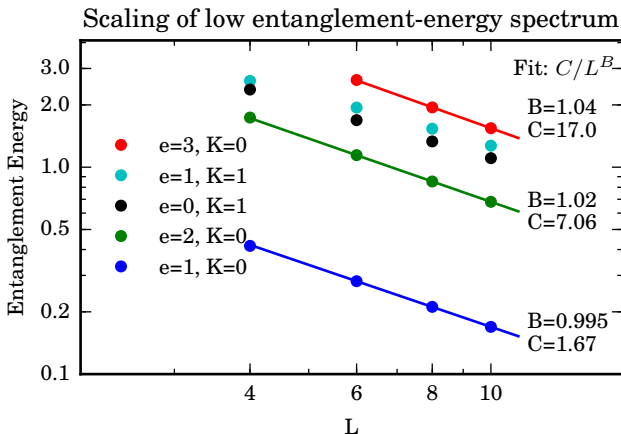
- In cylindrical geometry:
- Treat state as 1D
- Use MPS techniques
- On-site translational symmetry parallel to cut
- Physical site dimension  $4^{2L}$
- MPS bond dimension = Rank of  $\rho_r = 2^L$
- Entanglement spectrum  $\{\epsilon_i\}$  defined from eigenvalues  $\{\rho_i\}$  of  $\rho_r$  via  $\epsilon_i = e^{-\rho_i}$
- Charge and Translation represented linearly on edge



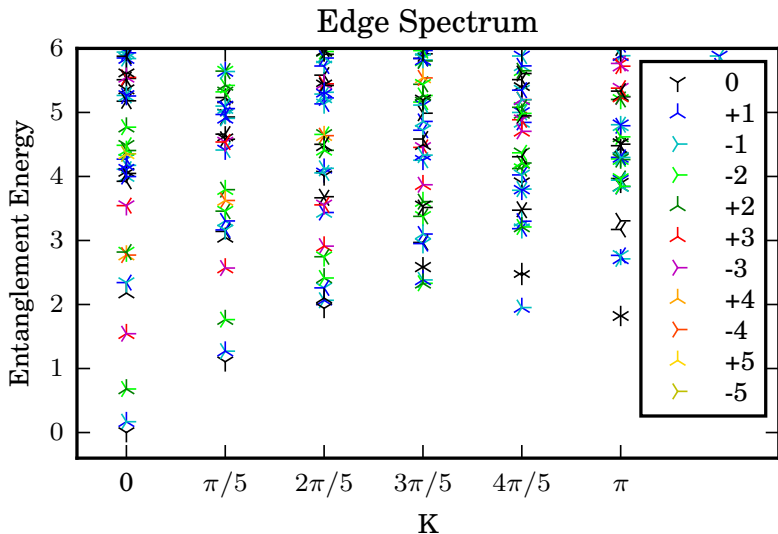
# Finite Size Analysis



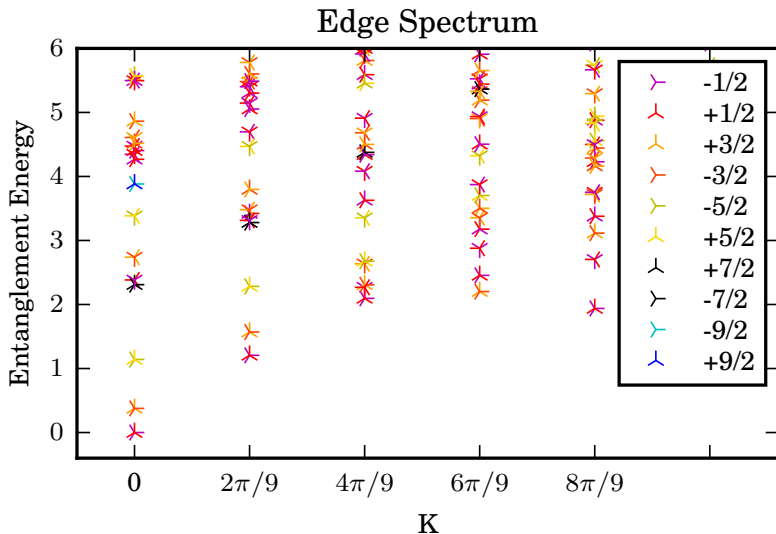
# Finite Size Analysis



# Entanglement Spectrum

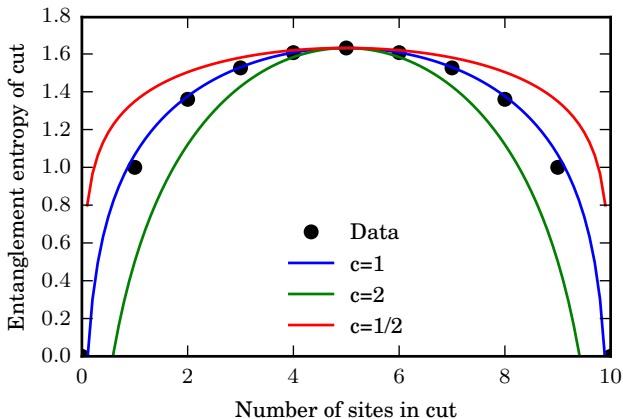


# Entanglement Spectrum



# Identification of Edge CFT

## Conformal Charge



$$c = 1$$

# Identification of Edge CFT

## Conformal Weights

We can match the rescaled entanglement energies to the conformal weights of a free bosonic CFT.

$$\begin{aligned}\mathbf{P} &= \frac{2\pi}{L}(\mathbf{L}_0 - \bar{\mathbf{L}}_0) = \frac{2\pi}{L}(em + n - \bar{n}) \\ \mathbf{H} &= \frac{2\pi}{L}(\mathbf{L}_0 + \bar{\mathbf{L}}_0) = \frac{2\pi}{L}\left(\frac{\kappa e^2}{2} + \frac{m^2}{2\kappa} + \frac{n + \bar{n}}{2}\right)\end{aligned}$$

$$\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})$$

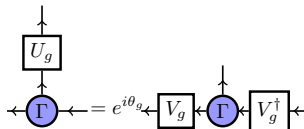
## Conformal primary identification in entanglement spectra

# Symmetry Protection of Degenerate Edge

## 1D Symmetry Protection

On-site symmetries  $g$  come with projective representation  $V_g$

- $V_g$  acts on sets of degenerate Schmidt states
- Charge and translation represented linearly on edge



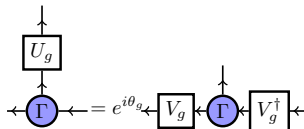


# Symmetry Protection of Degenerate Edge

## 1D Symmetry Protection

Time reversal symmetry  $\tau$   
represented by antiunitary  $V_\tau K$  on  
the edge

- $\tau^2 = +1$  on this edge



# Symmetry Protection of Degenerate Edge

## 1D Symmetry Protection

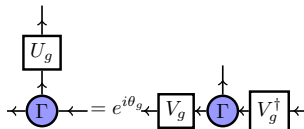
Inversion  $\mathcal{I}$

- $\mathcal{I}$  in combination with swapping Schmidt states represented by antiunitary operation  $V_{\mathcal{I}}K$  on the edge

- $\mathcal{I}^2 = V_{\mathcal{I}}V_{\mathcal{I}}^* = 1$

Inversion  $\mathcal{I}$  combined with  $\pi = e^{i\pi N}$

- $\pi\mathcal{I}$  represented antiunitarily on the edge by  $V_{\pi\mathcal{I}}K$
- $(\pi\mathcal{I})^2 = 1$  but  $V_{\pi\mathcal{I}}V_{\pi\mathcal{I}}^* = -1$



# Future Work

Entanglement properties with different geometries

- Armchair cylinder edge
- Finite size clusters
- Explain results for arbitrary geometries with tensor network properties, e.g. 'MPO injectivity'

Find a 2D local Hamiltonian and confirm with numerics

$$H_{EBH} = \left( \sum_{\hexagon} \sum_{i,j \in \hexagon} -tb_i^\dagger b_j + V n_i n_j \right) + \mu N?$$

Physical properties of the phase

Can we construct an SU(2) symmetric FI?

# Resources

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- Hasan, M. Z. and Kane, C. L. (2010). *Colloquium: Topological insulators. Reviews of modern physics*, 82(4):3045–3067.
- Kimchi, I., Parameswaran, S. A., Turner, A. M., Wang, F., and Vishwanath, A. (2013). Featureless and nonfractionalized mott insulators on the honeycomb lattice at  $1/2$  site filling. *Proceedings of the National Academy of Sciences*, 110(41):16378–16383.
- Parameswaran, S. A., Kimchi, I., Turner, A. M., Stamper-Kurn, D. M., and Vishwanath, A. (2013a). Wannier permanent wave functions for featureless bosonic mott insulators on the  $1/3$ -filled kagome lattice. *Phys. Rev. Lett.*, 110:125301.
- Parameswaran, S. A., Turner, A. M., Arovas, D. P., and Vishwanath, A. (2013b). Topological order and absence of band insulators at integer filling in non-symmorphic crystals. *Nature Physics*, 9(5):–303?
- Pollmann, F., Turner, A. M., Berg, E., and Oshikawa, M. (2010). Entanglement spectrum of a topological phase in one

# Questions?

Brayden Ware

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# Bonus slides

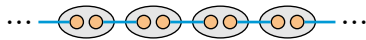
# Construction of 1D Featureless Insulators

## Classical Insulators



1D Trivial Chain

## Topological Insulators



1D Topological Chain

$$\begin{aligned}
 \text{orange dot} - \text{blue line} - \text{orange dot} &= \text{white circle} + \text{orange dot} + \text{orange dot} + \text{white circle} \\
 \text{gray oval with two white dots} &= 0 \\
 \text{gray oval with one white and one orange dot} &= 1 \\
 \text{gray oval with two orange dots} &= 2
 \end{aligned}$$

Projectors and entangled pairs (PEPS) used in state construction

# Construction of 1D Featureless Insulators

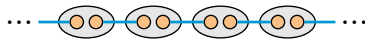
## Classical Insulators



1D Trivial Chain

Product state with one boson per site

## Topological Insulators



1D Topological Chain

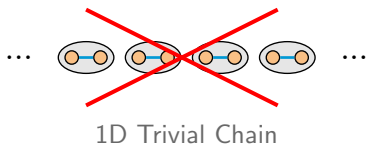
Haldane Insulator Phase  
Pollmann et al. (2010)

- Unitarily related to AKLT
- No  $SU(2)$  symmetry
- Symmetry protected 2-fold edge degeneracy

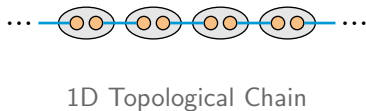


# Construction of 1D Featureless Insulators

## Classical Insulators



## Topological Insulators



$$\begin{aligned}
 \bullet\bullet &= \circ - \bullet\bullet\circ \\
 \bullet\bullet &= -\sqrt{2} \\
 \circ\bullet &= 0 \\
 \bullet\bullet &= +\sqrt{2}
 \end{aligned}$$

Projectors and entangled pairs (PEPS) for  $SU(2)$  symmetric state