

# Not so featureless after all: symmetry protected order in an interacting boson state

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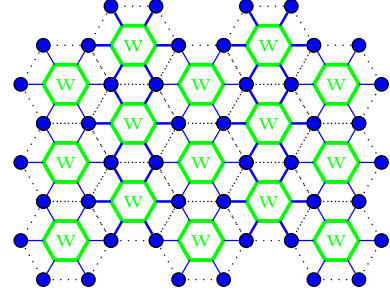
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While the Lieb-Schultz-Mattis theorem forbids the existence of fully symmetric quantum paramagnetic phases on lattices with fractional filling of particles per unit cell, such a phase is in principle allowed with certain fractional numbers of particles per site on non-Bravais lattices, including half-filling on the honeycomb lattice. It has been shown that a non-interacting Hamiltonian of spinless fermions or bosons cannot have such a symmetric insulating ground state, and an explicit construction using interactions is challenging. Recently, Kimchi et al. constructed a wavefunction for bosons at half-filling that does not break any symmetries and is not topologically ordered—and in this sense is a featureless insulator in the bulk. Here, however, we reveal that this wavefunction exhibits non-trivial structure at the edge. We apply recently developed techniques based on a tensor network representation of the wavefunction to demonstrate the presence of a gapless entanglement spectrum and a non-trivial action of combined charge-conservation and spatial symmetries on the edge. We will also discuss the possibility of finding a parent Hamiltonian and analyzing the existence of a symmetry-protected topological phase around this state.

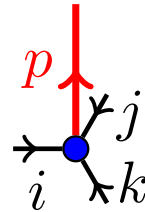
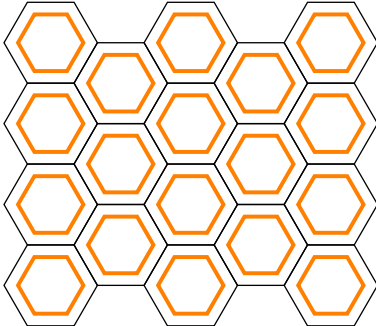
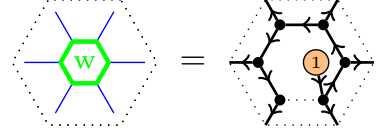
## I. INTRODUCTION

## II. F.B.I. WAVEFUNCTION

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## III. PEPS CONSTRUCTION OF HONEYCOMB F.B.I.



#### IV. ENTANGLEMENT SPECTRUM

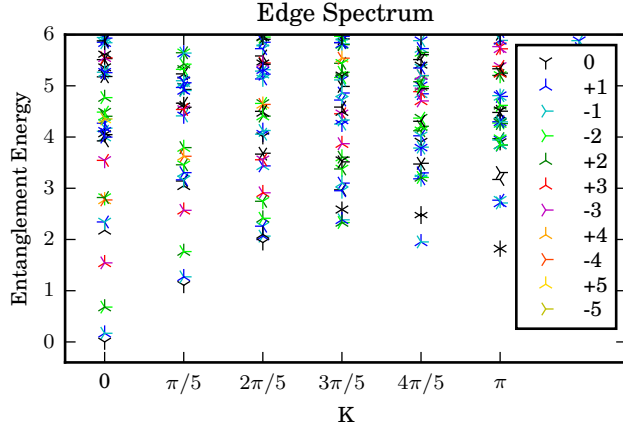


Figure 1. Entanglement spectrum on a zig-zag edge cylinder 10 unit cells in circumference.

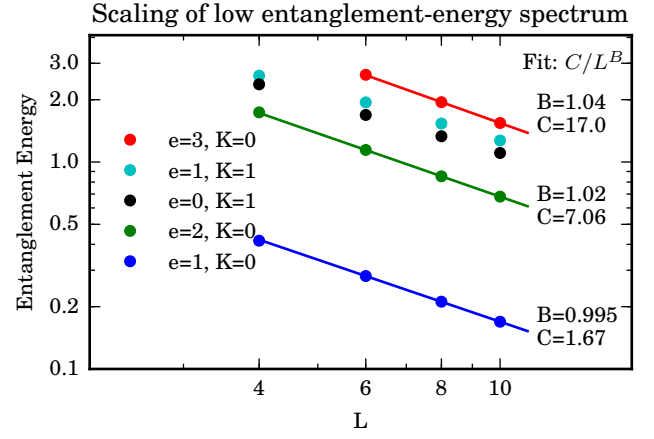
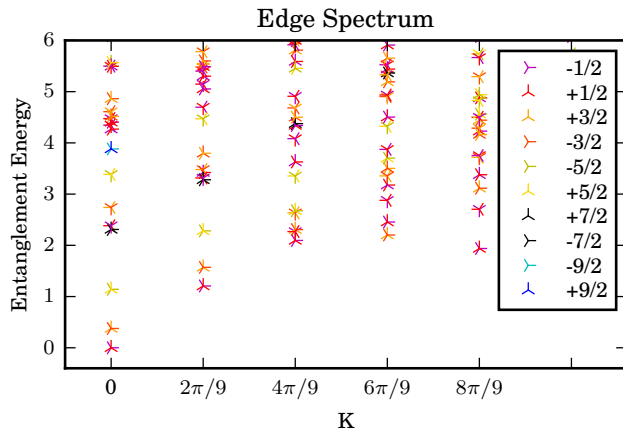
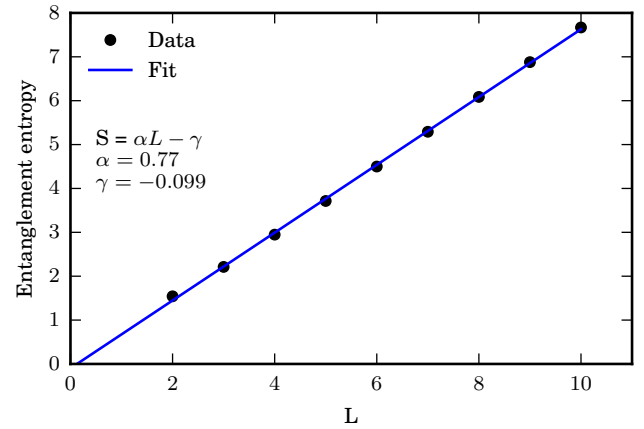


Figure 2. Power law fits for the lowest five states above the ground state in Figure 1. The  $1/L$  scaling is a signature of a gapless (entanglement) Hamiltonian.



## V. IDENTIFICATION OF EDGE CFT

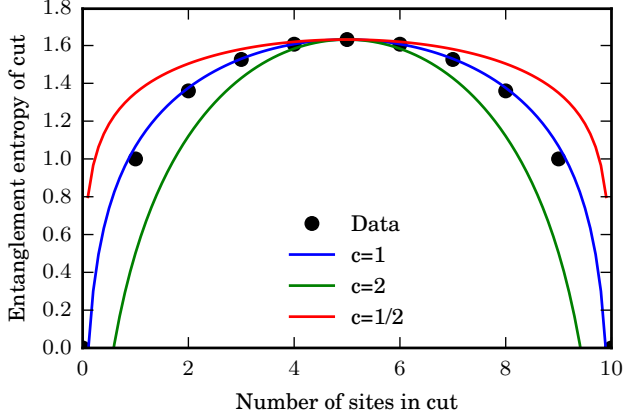


Figure 3. Entanglement entropy within the entanglement ground state of the soft-core boson state on 10 sites. For comparison, the Cardy-Calabrese formula  $S(x) = c/3 \log \sin(\pi x/L) + \text{const.}$  is shown with  $c = \frac{1}{2}, 1$ , and 2, with the *const.* fixed by matching the maximum of the entanglement entropy data.  $c = 1$  is a good fit.

$$\begin{aligned} \mathbf{P} &= \frac{2\pi}{L}(\mathbf{L}_0 - \bar{\mathbf{L}}_0) = \frac{2\pi}{L}(em + n - \bar{n}) \\ \mathbf{H} &= \frac{2\pi}{L}(\mathbf{L}_0 + \bar{\mathbf{L}}_0) = \frac{2\pi}{L}\left(\frac{\kappa e^2}{2} + \frac{m^2}{2\kappa} + \frac{n + \bar{n}}{2}\right) \end{aligned}$$

$$\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})$$

The degeneracy of level  $n, \bar{n}$  states is  $Z(n)Z(\bar{n})$ .

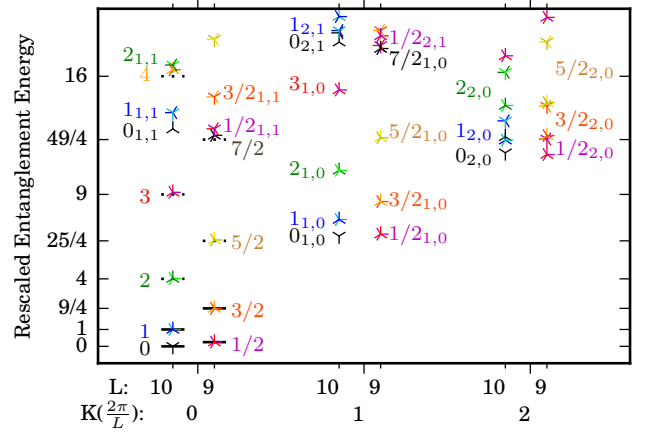
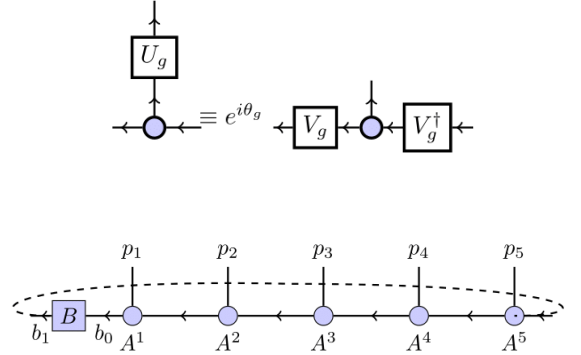


Figure 4. The identification of the states  $j_{-n}|e, m=0\rangle$  in the spectrum of the soft-core boson entanglement Hamiltonian. The label  $e$  gives the  $U(1)$  charge. The labels  $n, \bar{n}$  label the levels in the right or left-moving sectors of the Kac-Moody algebra. The best estimate for the Luttinger parameter is  $\kappa = 1/6.4$ . The label  $m$  is 0 for all states shown - however, the primary states  $|e, m = \pm 1\rangle$  can be seen centered around momentum  $\pi$ .

## VI. SYMMETRY PROTECTED TOPOLOGICAL ORDER



$\mathbf{G}$	$\mathbf{U}_g$	$\theta_g$	$\mathbf{V}_g$	$\mathbf{V}_g \mathbf{V}_g^*$
$U(1)$				
$\pi$				
$\mathcal{I}$				
$\pi\mathcal{I}$				

Since

$$V_{\pi\mathcal{I}}V_{\pi\mathcal{I}}^* = -I \quad \text{or} \quad V_{\pi}V_{\mathcal{I}} = -V_{\mathcal{I}}V_{\pi},$$

the representation is in the nontrivial class of

$$H^2(\mathbb{Z}_2 \times \mathbb{Z}_2^{\mathcal{I}}; U(1)) = \mathbb{Z}_2.$$

## VII. CONCLUSIONS

## ACKNOWLEDGMENTS

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<sup>1</sup> I. Kimchi, S. A. Parameswaran, A. M. Turner, F. Wang, and A. Vishwanath, “[Featureless and non-fractionalized mott insulators on the honeycomb lattice at 1/2 site filling,](#)” (2012), [arXiv:1207.0498 \[cond-mat.str-el\]](#).