# Not so featureless after all: symmetry protected order in an interacting boson state

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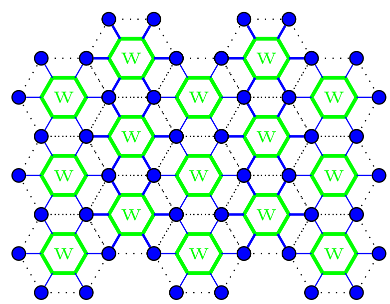
While the Lieb-Schultz-Mattis theorem forbids the existence of fully symmetric quantum paramagnetic phases on lattices with fractional filling of particles per unit cell, such a phase is in principle allowed with certain fractional numbers of particles per site on non-Bravais lattices, including half-filling on the honeycomb lattice. It has been shown that a non-interacting Hamiltonian of spinless fermions or bosons cannot have such a symmetric insulating ground state, and an explicit construction using interactions is challenging. Recently, Kimchi et al. constructed a wavefunction for bosons at half-filling that does not break any symmetries and is not topologically ordered–and in this sense is a featureless insulator in the bulk. Here, however, we reveal that this wavefunction exhibits non-trivial structure at the edge. We apply recently developed techniques based on a tensor network representation of the wavefunction to demonstrate the presence of a gapless entanglement spectrum and a non-trivial action of combined charge-conservation and spatial symmetries on the edge. We will also discuss the possibility of finding a parent Hamiltonian and analyzing the existence of a symmetry-protected topological phase around this state.

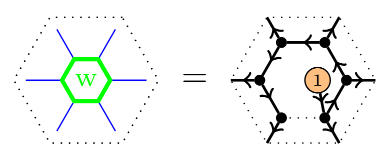
## I. INTRODUCTION

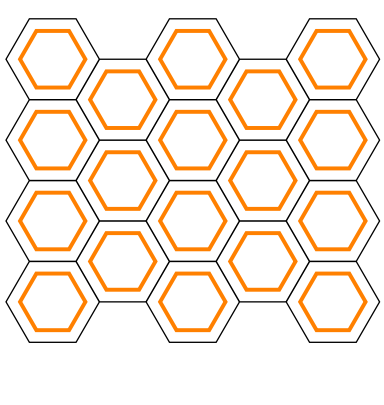
## II. F.B.I. WAVEFUNCTION

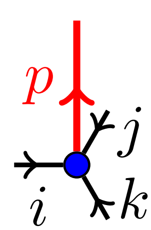
...[1]



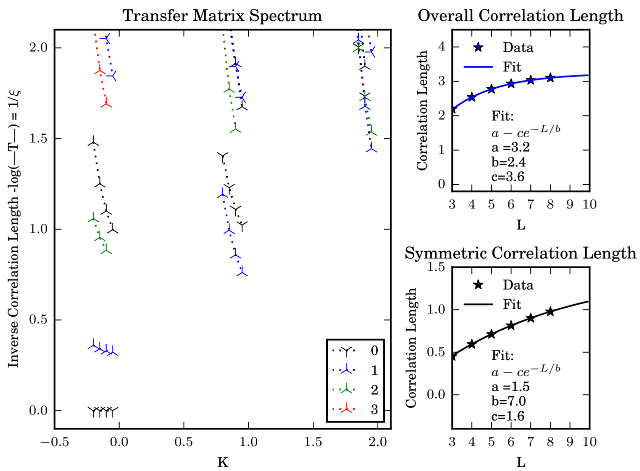

## III. PEPS CONSTRUCTION OF HONEYCOMB F.B.I.





## IV. FEATURELESS CORRELATIONS




## V. ENTANGLEMENT SPECTRUM

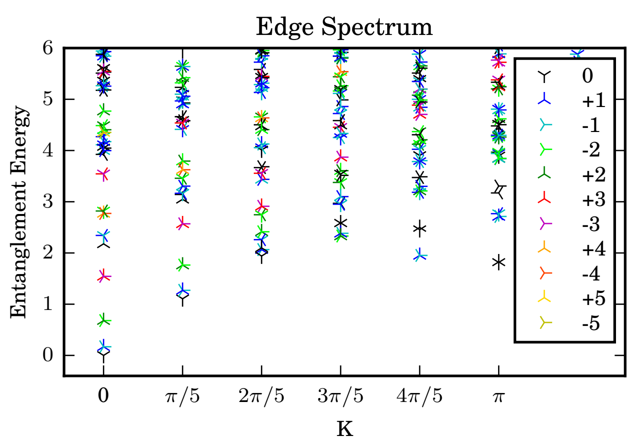
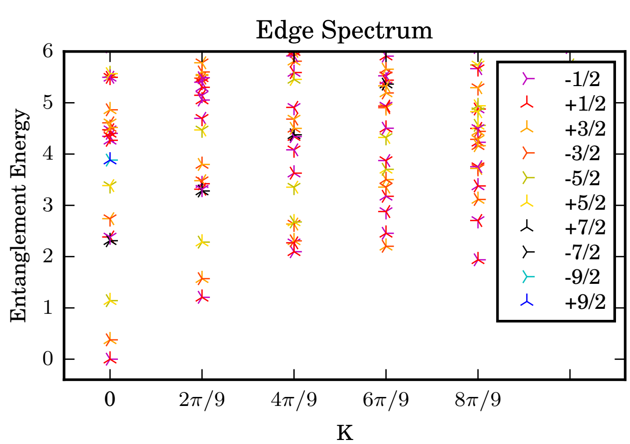



Figure 1. Entanglement spectrum on a zig-zag edge cylinder 10 unit cells in circumference.



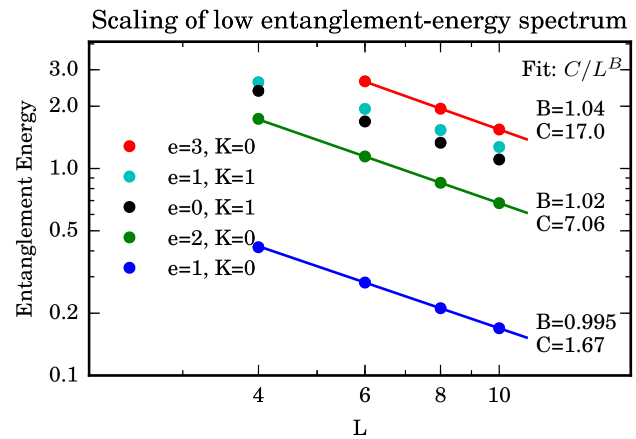
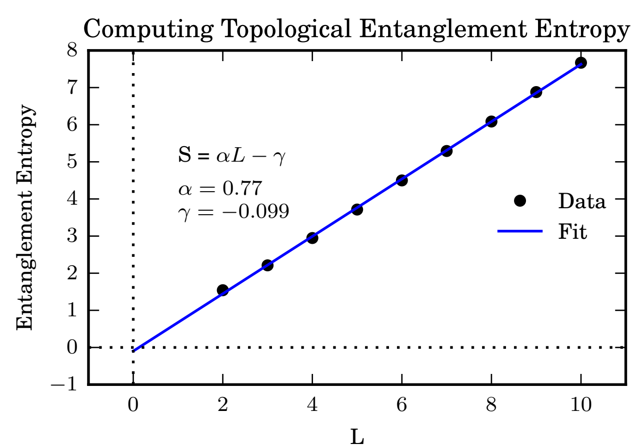




Figure 2. Power law fits for the lowest five states above the ground state in Figure 1. The $1/L$ scaling is a signature of a gapless (entanglement) Hamiltonian.



## VI. IDENTIFICATION OF EDGE CFT

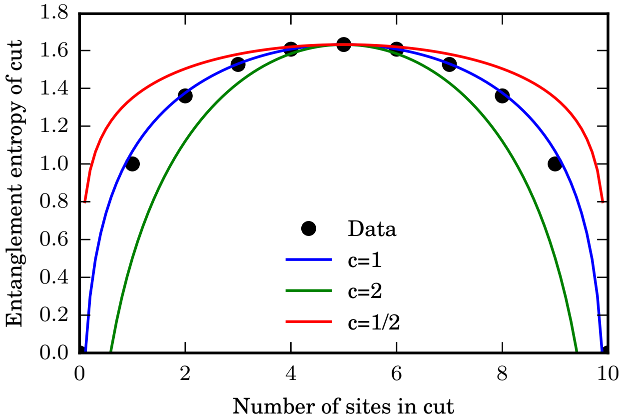



Figure 3. Entanglement entropy within the entanglement ground state of the soft-core boson state on 10 sites. For comparison, the Cardy-Calabrese formula $S(x) = c/3 \log \sin(\pi x/L) + const.$ is shown with $c = \frac{1}{2}, 1$, and 2, with the *const.* fixed by matching the maximum of the entanglement entropy data. $c = 1$ is a good fit.

$$\mathbf{P} = \frac{2\pi}{L}(\mathbf{L_0} - \bar{\mathbf{L}}_\mathbf{0}) = \frac{2\pi}{L}(em + n - \bar{n})$$
$$\mathbf{H} = \frac{2\pi}{L}(\mathbf{L_0} + \bar{\mathbf{L}}_\mathbf{0}) = \frac{2\pi}{L}(\frac{\kappa e^2}{2} + \frac{m^2}{2\kappa} + \frac{n + \bar{n}}{2})$$

$$\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})$$

The degeneracy of level $n, \bar{n}$ states is $Z(n)Z(\bar{n})$.

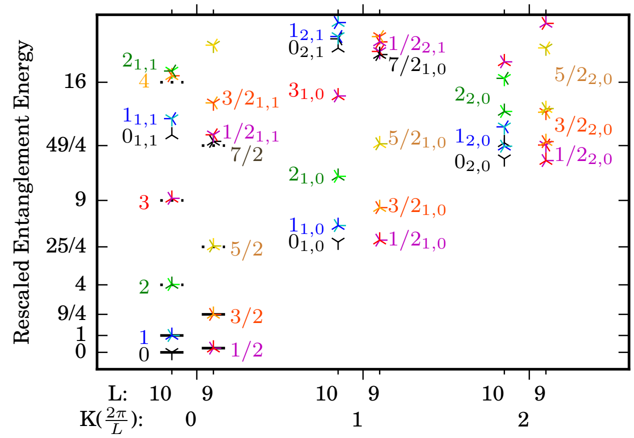
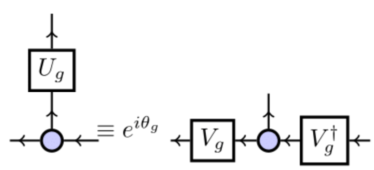
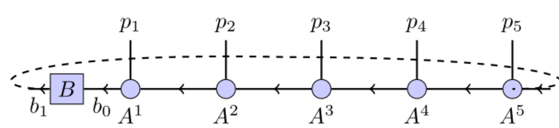



Figure 4. The identification of the states $j_{-n}|e, m = 0\rangle$ in the spectrum of the soft-core boson entanglement Hamiltonian. The label $e$ gives the U(1) charge. The labels $n$, $\bar{n}$ label the levels in the right or left-moving sectors of the Kac-Moody algebra. The best estimate for the Luttinger parameter is $\kappa = 1/6.4$. The label $m$ is 0 for all states shown - however, the primary states $|e, m = \pm 1\rangle$ can be seen centered around momentum $\pi$.

## VII. SYMMETRY PROTECTED TOPOLOGICAL ORDER




| $\mathbf{G}$ | $\mathbf{U_g}$ | $\theta_\mathbf{g}$ | $\mathbf{V_g}$ | $\mathbf{V_g V_g^*}$ |
|---|---|---|---|---|
| $U(1)$ | | | | |
| $\pi$ | | | | |
| $\mathcal{I}$ | | | | |
| $\pi\mathcal{I}$ | | | | |

Since

$$V_{\pi\mathcal{I}} V_{\pi\mathcal{I}}^* = -I \quad \text{or} \quad V_\pi V_\mathcal{I} = -V_\mathcal{I} V_\pi,$$

the representation is in the nontrivial class of

$$H^2(\mathbb{Z}_2 \times \mathbb{Z}_2^\mathcal{I}; U(1)) = \mathbb{Z}_2.$$

## VIII. CONCLUSIONS

## ACKNOWLEDGMENTS

[1] I. Kimchi, S. A. Parameswaran, A. M. Turner, F. Wang, and A. Vishwanath, "Featureless and non-fractionalized mott insulators on the honeycomb lattice at 1/2 site filling," (2012), arXiv:1207.0498 [cond-mat.str-el].