Entanglement in Featureless Mott Insulators

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Outline

Motivation

Featureless insulators

Definition of ‘Featureless Insulator’

- Gapped
- Symmetric
- No topological order

- Unique ground state: $E_1 - E_0 \geq const.$

Alternate Definition

- Unique ground state on any boundary-less system
- Possibly with ’features’ localized to edge of system

Fundamental Result

- Integer charge per unit cell
    - (Lieb, Schultz, Mattis)

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- Unique ground state: $E_1 - E_0 \geq const.$
- Gapless modes: $E_1 - E_0 \sim \frac{1}{L^z}$

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Featureless insulators

Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state: $E_1 - E_0 \geq const.$
- Spontaneous symmetry breaking: $E_1 - E_0 = 0$

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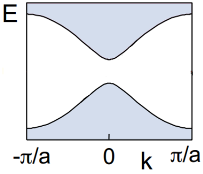
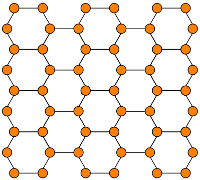
- Unique ground state: $E_1 - E_0 \geq const.$
- Topological order: $E_1 - E_0 \sim e^{-L/\xi}$ with nontrivial topology

Free Fermion Featureless Insulators

Classical Insulators

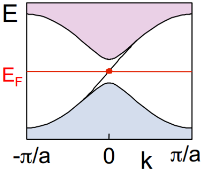


Free fermion band insulator

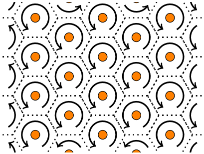


Atomic picture

Topological Insulators



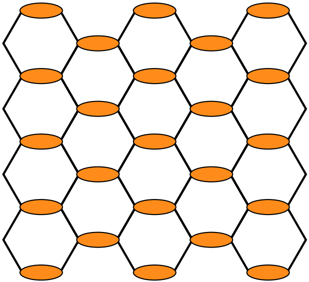
Band insulator with chiral edge [1]



Atomic picture breaks down

Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



Breaks rotational symmetry

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013)

Honeycomb Bosonic Mott Insulators

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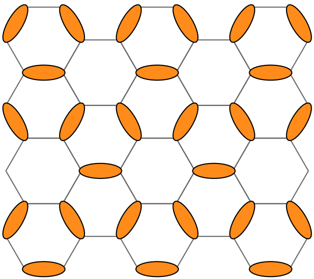

Breaks translationally symmetry, unit cell is 3 times larger

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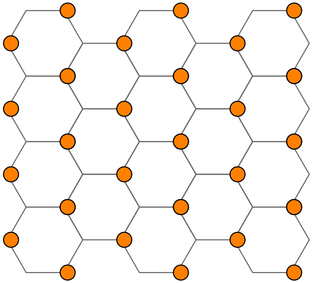


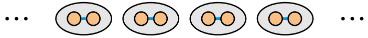
Breaks point group symmetry D_6 to D_3

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013)

Construction of Honeycomb FBI

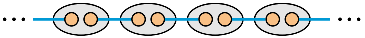
Construction of 1D Featureless Insulators

Classical Insulators

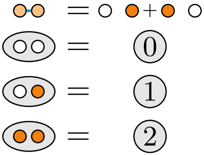


1D Trivial Chain

Topological Insulators



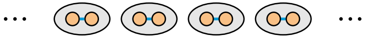
1D Topological Chain



Entangled pairs and projectors used in state construction

Construction of 1D Featureless Insulators

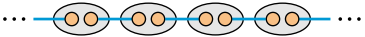
Classical Insulators



1D Trivial Chain

Product state with one boson per site

Topological Insulators



1D Topological Chain

Haldane Insulator Phase
Pollmann et al. (2010)

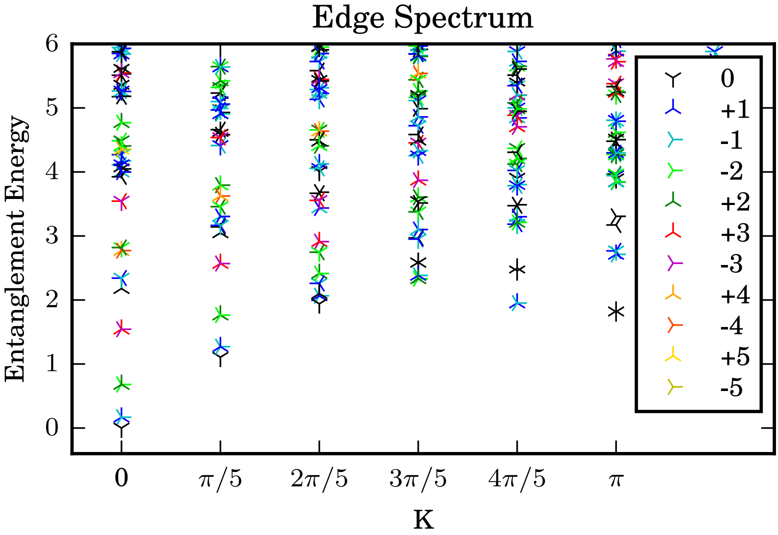
- Unitarily related to AKLT
- No $SU(2)$ symmetry
- Symmetry protected 2-fold edge degeneracy

Construction of Honeycomb FBI

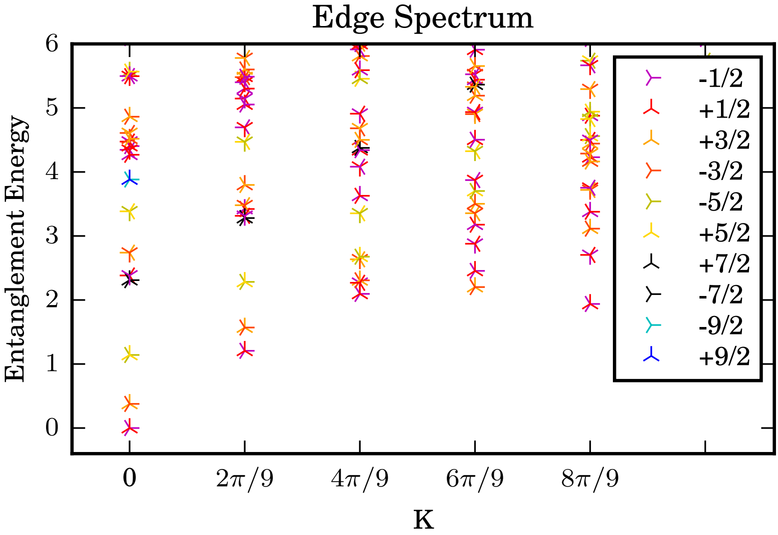
Entanglement Edge of Honeycomb FBI

Edge Geometry

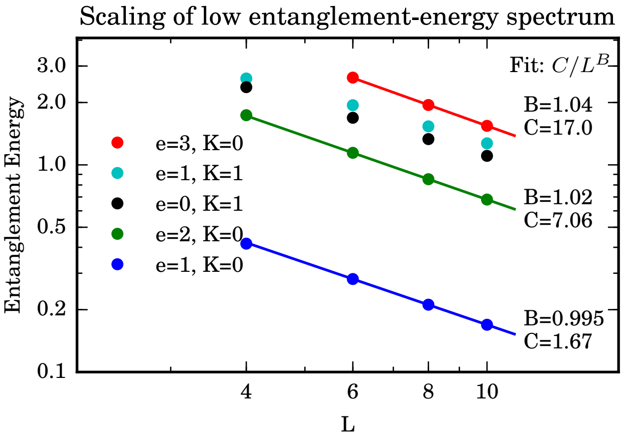
Entanglement Spectrum



Entanglement Spectrum



Finite Size Analysis



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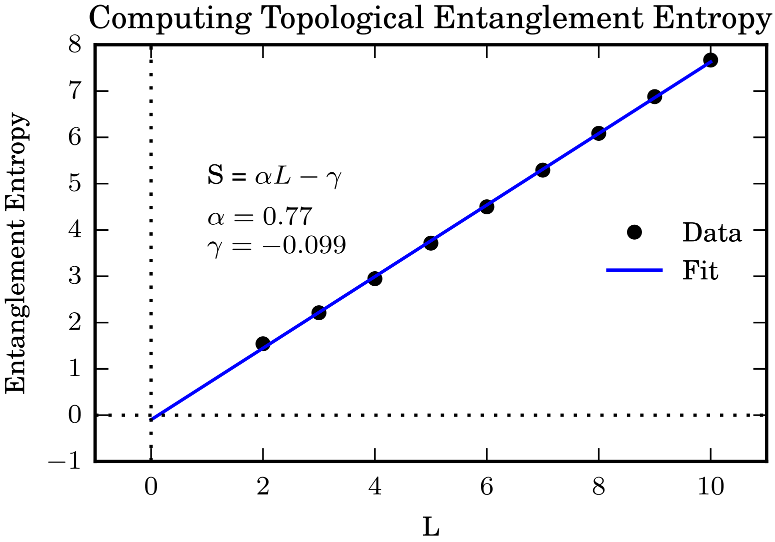
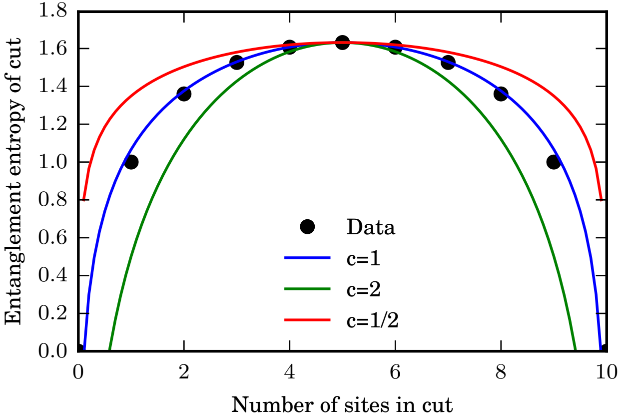


Identification of Edge CFT

Conformal Charge



$$\boxed{c = 1}$$

Identification of Edge CFT

Conformal Weights

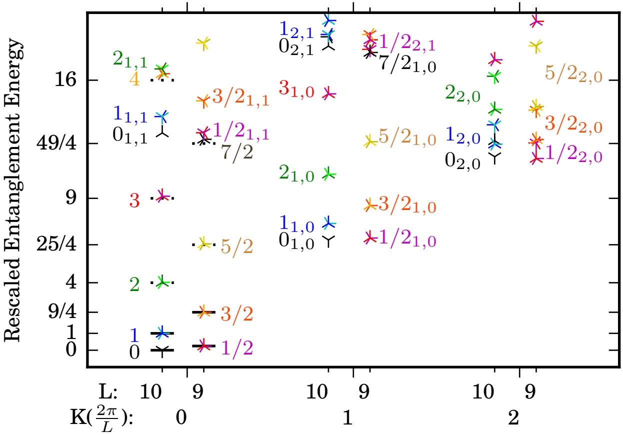
We can match the rescaled entanglement energies to the conformal weights of a free bosonic CFT.

$$\mathbf{P} = \frac{2\pi}{L}(\mathbf{L_0} - \bar{\mathbf{L}}_\mathbf{0}) = \frac{2\pi}{L}(em + n - \bar{n})$$
$$\mathbf{H} = \frac{2\pi}{L}(\mathbf{L_0} + \bar{\mathbf{L}}_\mathbf{0}) = \frac{2\pi}{L}(\frac{\kappa e^2}{2} + \frac{m^2}{2\kappa} + \frac{n + \bar{n}}{2})$$

$$\boxed{\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})}$$

Identification of Edge CFT

Conformal primary identification in entanglement spectra



Symmetry Protection of Edge

Symmetry Protection of Degenerate Edge

Future Work

- Entanglement properties in different geometries
  - Cylinders with different edges
  - Finite size clusters
- Relation to 'MPO Injectivity'
  -
- Numerical testing of parent Hamiltonians
  -

Resources

Hasan, M. Z. and Kane, C. L. (2010). *Colloquium*: Topological insulators. *Reviews of modern physics*, 82(4):3045–3067.

Parameswaran, S. A., Kimchi, I., Turner, A. M., Stamper-Kurn, D. M., and Vishwanath, A. (2013). Wannier permanent wave functions for featureless bosonic mott insulators on the $1/3$-filled kagome lattice. *Phys. Rev. Lett.*, 110:125301.

Pollmann, F., Turner, A. M., Berg, E., and Oshikawa, M. (2010). Entanglement spectrum of a topological phase in one dimension. *Phys. Rev. B*, 81(6):064439.

Questions?

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Bonus slides