Entanglement in Featureless Mott Insulators

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Featureless insulators

Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state: $E_1 - E_0 \geq const.$

Alternate Definition

- Unique ground state on all boundary-less systems
- Possibly with 'features' localized to edge of system

Fundamental Result

- Integer charge per unit cell
    - (Lieb, Schultz, Mattis)

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- Unique ground state: $E_1 - E_0 \geq const.$
- Gapless modes: $E_1 - E_0 \sim \frac{1}{L^z}$

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Featureless insulators

Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state:
  $E_1 - E_0 \geq const.$
- Spontaneous symmetry breaking:
  $E_1 - E_0 = 0$

Alternate Definition

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- Possibly with 'features' localized to edge of system

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Featureless insulators

Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state: $E_1 - E_0 \geq const.$
- Topological order: $E_1 - E_0 \sim e^{-L/\xi}$ with nontrivial topology

Alternate Definition

- Unique ground state on all boundary-less systems
- Possibly with 'features' localized to edge of system

Fundamental Result

- Integer charge per unit cell
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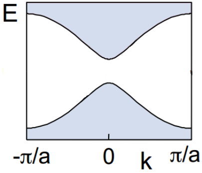
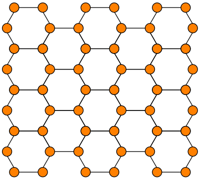
Free Fermion Featureless Insulators

Classical Insulators

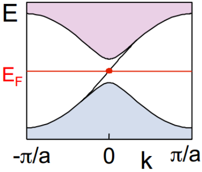


Free fermion band insulator

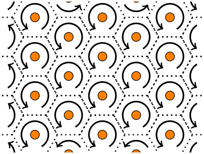


Atomic picture

Topological Insulators



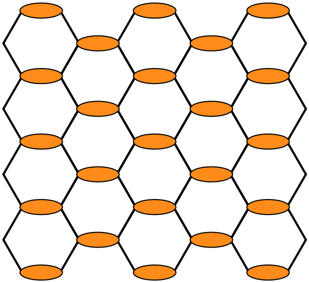
Band insulator with chiral edge [1]



Atomic picture breaks down

Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



Breaks rotational symmetry

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013a)

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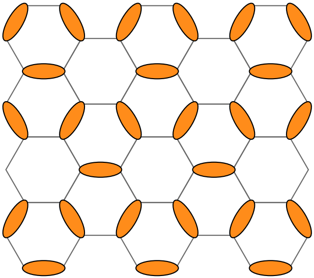

Breaks translationally symmetry, unit cell is 3 times larger

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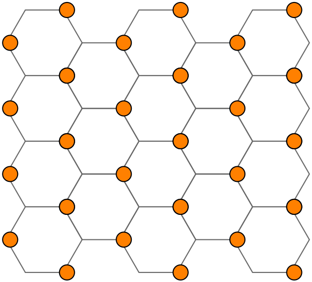
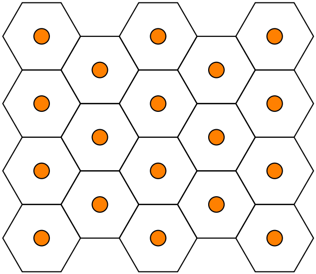


Breaks rotational symmetry

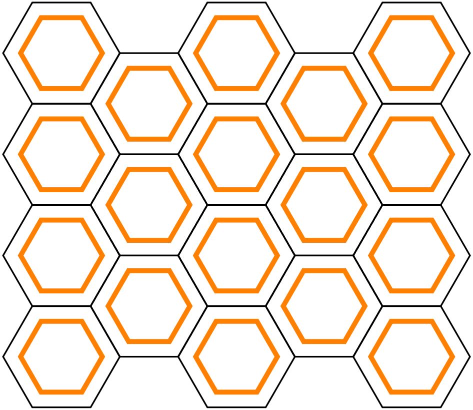


Leaves honeycomb lattice

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013a)

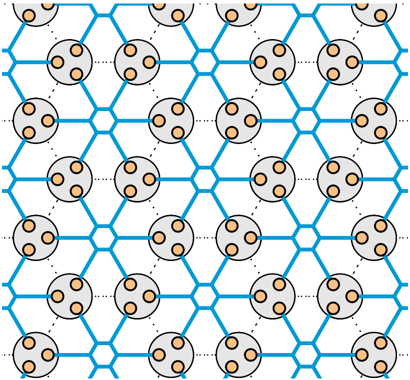
Honeycomb Bosonic Mott Insulators

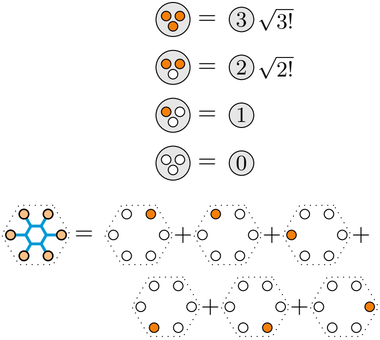
Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



Proposed Solution by Kimchi et al. (2013)

Construction of Honeycomb FBI





$$|\psi\rangle = \prod_{\hexagon} \left(\sum_{i \in \hexagon} b_i^\dagger \right) |\mathbf{0}\rangle$$

Wavefunction proposed by Kimchi et al. (2013)

Known Results for Honeycomb FBI

Correlations

$< b_i^\dagger b_j >$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length $\xi/a \sim 3.6$

$< n_i n_j >$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length $\xi/a \sim 1.6$

Hamiltonian Construction

Try filling plaquette orbitals

$$b_{\hexagon} = \sum_{i \in \hexagon} \frac{1}{\sqrt{6}} b_i^\dagger$$

$$H = \sum_{\hexagon} -\frac{t}{6} b_{\hexagon}^\dagger b_{\hexagon} + V n_{\hexagon} n_{\hexagon}$$

$$= \left(\sum_{\hexagon} \sum_{i,j \in \hexagon} -t b_i^\dagger b_j \right) - \frac{3t}{6} N + V \ldots$$

- Fails, gapless modes
- Parent Hamiltonian not known

Known Results for Honeycomb FBI

Correlations

$< b_i^\dagger b_j >$

- Looks rotationally symmetric
- Decays exponentially
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$< n_i n_j >$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length $\xi/a \sim 1.6$

Hamiltonian Construction

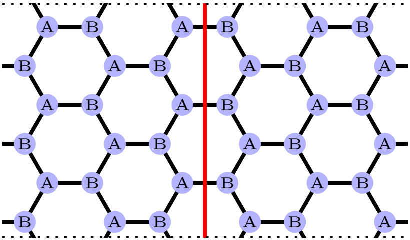
To get a parent Hamiltonian:

- Need symmetric, exponentially localized, orthogonal orbitals
- Such as the Wannier orbitals of a classical band insulator
- Parameswaran et al. (2013a)

Other lattices:

- Need a fixed point of all lattice symmetries
- Fails on nonsymmorphic lattices
- Extension of LSM theorem
- Parameswaran et al. (2013b)

Edge Geometry

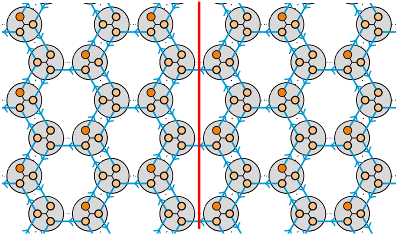


Generic honeycomb lattice PEPS on zig-zag cylinder with L=3

In cylindrical geometry:

- Treat state as 1D
- Use MPS techniques
- On-site translational symmetry parallel to cut
- Physical site dimension 4^{2L}

Edge Geometry

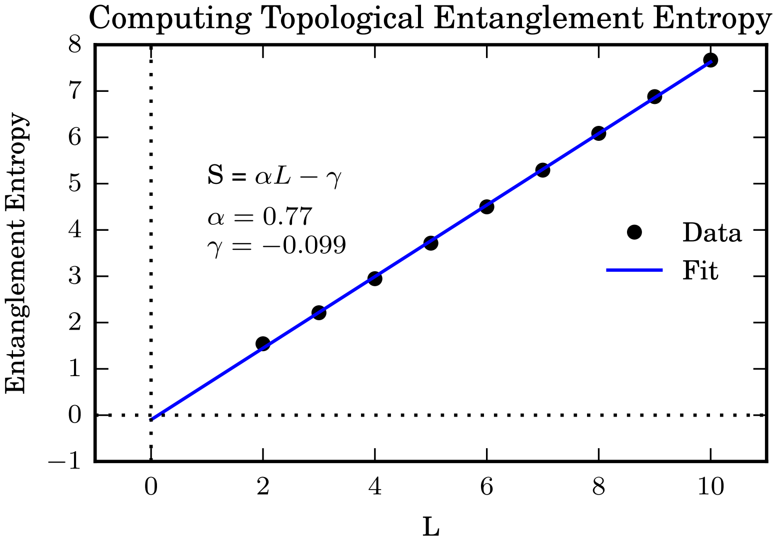


Honeycomb lattice PEPS on zig-zag cylinder with L=3, acheived by factoring W-state of plaquette bosons

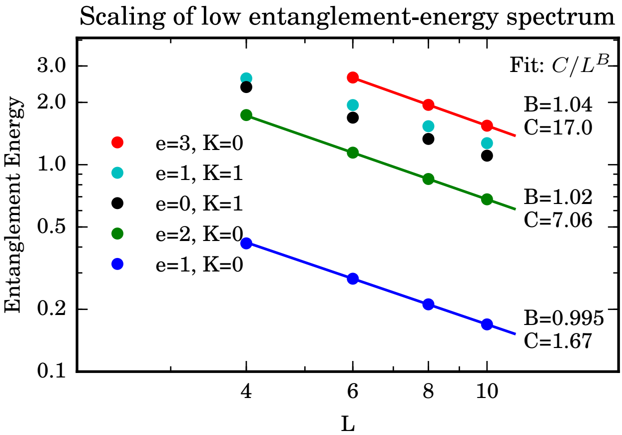
In cylindrical geometry:

- Treat state as 1D
- Use MPS techniques
- On-site translational symmetry parallel to cut
- Physical site dimension 4^{2L}
- MPS bond dimension = Rank of $\rho_r = 2^L$
- Entanglement spectrum $\{\epsilon_i\}$ defined from eigenvalues $\{\rho_i\}$ of ρ_r via $\epsilon_i = e^{-\rho_i}$
- Charge and Translation represented linearly on edge

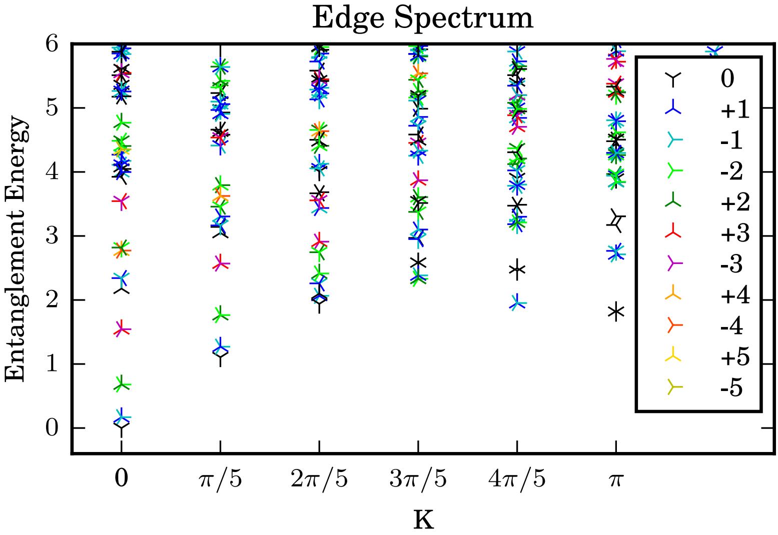
Finite Size Analysis



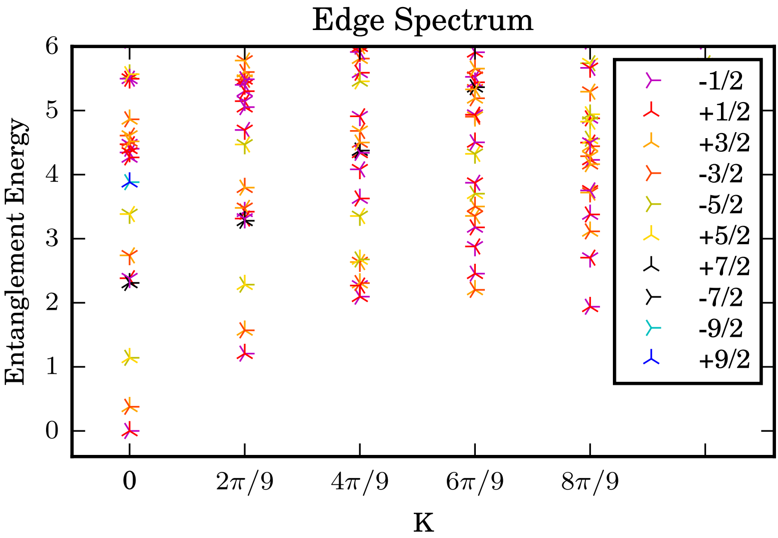
Finite Size Analysis



Entanglement Spectrum

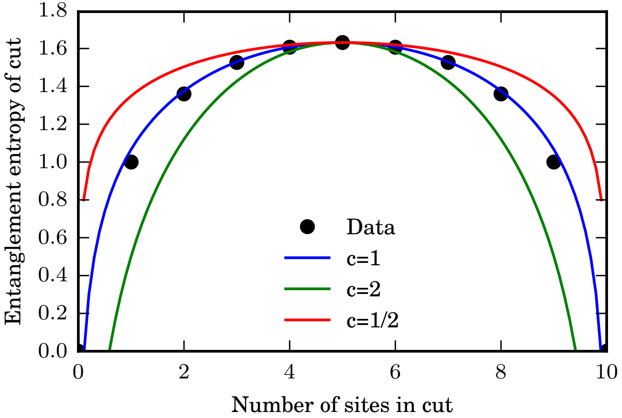


Entanglement Spectrum



Identification of Edge CFT

Conformal Charge



$$\boxed{c = 1}$$

Identification of Edge CFT

Conformal Weights

We can match the rescaled entanglement energies to the conformal weights of a free bosonic CFT.

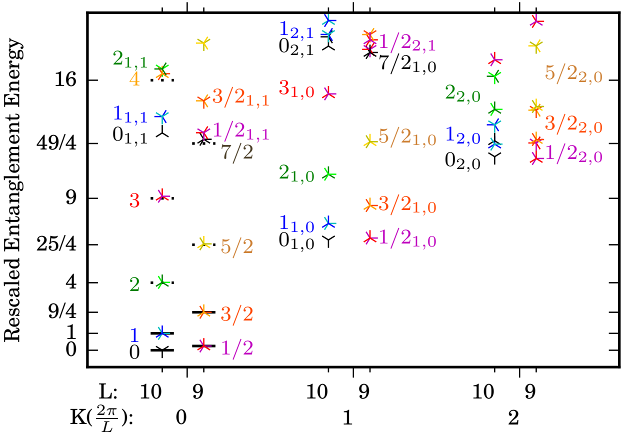
$$\mathbf{P} = \frac{2\pi}{L}(\mathbf{L_0} - \mathbf{\bar{L}_0}) = \frac{2\pi}{L}(em + n - \bar{n})$$

$$\mathbf{H} = \frac{2\pi}{L}(\mathbf{L_0} + \mathbf{\bar{L}_0}) = \frac{2\pi}{L}(\frac{\kappa e^2}{2} + \frac{m^2}{2\kappa} + \frac{n + \bar{n}}{2})$$

$$\boxed{\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})}$$

Identification of Edge CFT

Conformal primary identification in entanglement spectra

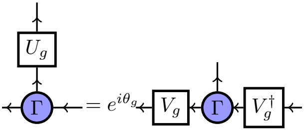


Symmetry Protection of Degenerate Edge

1D Symmetry Protection

On-site symmetries g come with projective representation V_g

- V_g acts on sets of degenerate Schmidt states
- Charge and translation represented linearly on edge

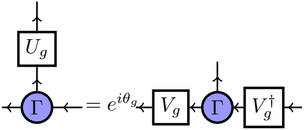


Symmetry Protection of Degenerate Edge

1D Symmetry Protection

Time reversal symmetry τ represented by antiunitary $V_\tau K$ on the edge

- $\tau^2 = +1$ on this edge



Symmetry Protection of Degenerate Edge

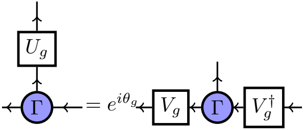
1D Symmetry Protection

Inversion $\mathcal{I}$

- $\mathcal{I}$ in combination with swapping Schmidt states represented by antiunitary operation $V_{\mathcal{I}}K$ on the edge
- $\mathcal{I}^2 = V_{\mathcal{I}}V_{\mathcal{I}}^* = 1$

Inversion $\mathcal{I}$ combined with $\pi = e^{i\pi N}$

- $\pi\mathcal{I}$ represented antiunitarily on the edge by $V_{\pi\mathcal{I}}K$
- $(\pi\mathcal{I})^2 = 1$ but $V_{\pi\mathcal{I}}V_{\pi\mathcal{I}}^* = -1$



Future Work

Entanglement properties with different geometries

- Armchair cylinder edge
- Finite size clusters
- Explain results for arbitrary geometries with tensor network properties, e.g. 'MPO injectivity'

Find a 2D local Hamiltonian and confirm with numerics

$$H_{EBH} = \left(\sum_{\hexagon} \sum_{i,j \in \hexagon} -t b_i^\dagger b_j + V n_i n_j \right) + \mu N?$$

Physical properties of the phase

Can we constructan SU(2) symmetric FI?

Resources

Hasan, M. Z. and Kane, C. L. (2010). *Colloquium*: Topological insulators. *Reviews of modern physics*, 82(4):3045–3067.

Kimchi, I., Parameswaran, S. A., Turner, A. M., Wang, F., and Vishwanath, A. (2013). Featureless and nonfractionalized mott insulators on the honeycomb lattice at $1/2$ site filling. *Proceedings of the National Academy of Sciences*, 110(41):16378–16383.

Parameswaran, S. A., Kimchi, I., Turner, A. M., Stamper-Kurn, D. M., and Vishwanath, A. (2013a). Wannier permanent wave functions for featureless bosonic mott insulators on the $1/3$-filled kagome lattice. *Phys. Rev. Lett.*, 110:125301.

Parameswaran, S. A., Turner, A. M., Arovas, D. P., and Vishwanath, A. (2013b). Topological order and absence of band insulators at integer filling in non-symmorphic crystals. *Nature Physics*, 9(5):–303?

Pollmann, F., Turner, A. M., Berg, E., and Oshikawa, M. (2010). Entanglement spectrum of a topological phase in one

Questions?

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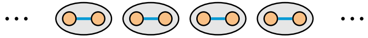
Bonus slides

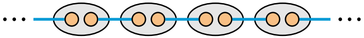
Construction of 1D Featureless Insulators

Classical Insulators

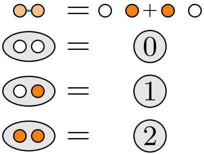


1D Trivial Chain

Topological Insulators



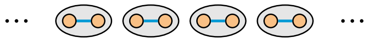
1D Topological Chain



Projectors and entangled pairs (PEPS) used in state construction

Construction of 1D Featureless Insulators

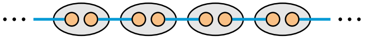
Classical Insulators



1D Trivial Chain

Product state with one boson per site

Topological Insulators



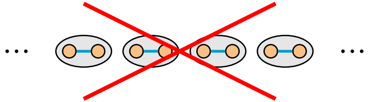
1D Topological Chain

Haldane Insulator Phase
Pollmann et al. (2010)

- Unitarily related to AKLT
- No $SU(2)$ symmetry
- Symmetry protected 2-fold edge degeneracy

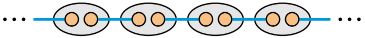
Construction of 1D Featureless Insulators

Classical Insulators



1D Trivial Chain

Topological Insulators



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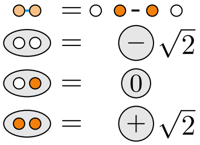

Projectors and entangled pairs (PEPS) for $SU(2)$ symmetric state