Distinct edge phases with the same bulk Abelian Quantum-Hall state

Eugeniu Plamadeala

in collaboration with Jennifer Cano, Meng Cheng, Michael Mulligan, Chetan Nayak, and Jon Yard

Department of Physics, University of California Santa Barbara   Microsoft Station Q

Quantum Hall Basics

A 2DEG in a strong magnetic field develops Landau levels. The low-energy sector is described a Chern-Simons theory

$$S = \int d^3x \left(\frac{1}{4\pi}\epsilon^{\mu\nu\rho}K_{IJ}a^I_\mu\partial_\nu a^J_\rho + \frac{1}{2\pi}\epsilon^{\mu\nu\rho}t_I A_\mu\partial_\nu a^I_\rho\right)$$

Associated lattice $\Lambda = \{m_I\mathbf{e}_I | m_I \in \mathbb{Z}\}$ such that $\mathbf{e}_I \cdot \mathbf{e}_J = K_{IJ}$

Edge action - Luttinger liquid

$$S = \int dxdt\frac{1}{4\pi}\left(K_{IJ}\partial_t\phi^I\partial_x\phi^J - V_{IJ}\partial_x\phi^I\partial_x\phi^J\right) + \frac{1}{2\pi}t_I\epsilon_{\mu\nu}\partial_\mu\phi^I A_\nu$$

Example of Bulk Topological Phase with Two Distinct Edge Phases

A theory with $K = \sigma_x/\sigma_z$ (+ appropriate backscattering term) describes a pair of trivial gapped bosonic/fermionic modes.

Let

$$K_1 = \begin{pmatrix}1 & 0\\ 0 & 7\end{pmatrix} \quad K_2 = \begin{pmatrix}2 & 1\\ 1 & 4\end{pmatrix} \tag{1}$$

Consider a theory with $K = K_1 \oplus \sigma_z$ and backscattering terms:

$$S' = \int dx\,dt\,u'\cos(\phi_3 + \phi_4)$$
$$S'' = \int dx\,dt\,u''\cos(\phi_1 + 7\phi_2 + \phi_3 + 3\phi_4)$$

A basis change $W \in \mathrm{GL}(4,\mathbb{Z})$ shows that

$$K_2 \oplus \sigma_z = W^T\left(K_1 \oplus \sigma_z\right)W$$
$$\phi^I = W^I_J\phi^{J'}$$
$$\phi_4' - \phi_3' = \phi_1 + 7\phi_2 + \phi_3 + 3\phi_4$$

K_1 and K_2 describe physically distinct edge theories: their spectrum of operator scaling dimensions differs. Therefore, Quantum Point Contact tunnelling exponents will differ as well.

Stable-Equivalence and Bulk-Edge Correspondence

A bulk Abelian quantum Hall state associated with K_1 has more than one distinct chiral edge phase if there exists $GL(N,\mathbb{Z})$-inequivalent K_2, an appropriate unimodular lattice L, and $W \in \mathrm{GL}(N+2,\mathbb{Z})$ such that

$$K_2 \oplus L = W^T(K_1 \oplus L)W$$

K_1 and K_2 are said to be *stably equivalent.*

Even unimodular lattices $\equiv$ IQH states of bosons, exist only for $c = 8k$.

Central Charge	8	16	24	32 ...	
Number of lattices	1	2	24	$> 10^9$	...

Odd unimodular lattices $\equiv$ IQH states of fermions, exist for $\forall c \in \mathbb{N}^+$

Central Charge	1-8	9-11	12-13	14	15	...
Number of lattices	1	2	3	4	5	...

The set of stably equivalent K-matrices (or lattices) are said to form a *genus*. **How do we tell if two K-matrices are in the same genus?**

The topologically distinct quasiparticles of an Abelian CS theory and their fusion rules are captured by the *discriminant group* $\mathcal{A} = \Lambda^*/\Lambda$. Given $\mathbf{v}, \mathbf{v}' \in \mathcal{A}$

$$S_{[\mathbf{v}],[\mathbf{v}']} = \frac{1}{\sqrt{\mathcal{A}}}e^{-2\pi i\mathbf{v}\cdot\mathbf{v}'}$$
$$T_{[\mathbf{v}],[\mathbf{v}']} = e^{-\frac{2\pi i}{24}c_-}\,\delta_{[\mathbf{v}],[\mathbf{v}']}\;\theta_{[\mathbf{v}]}$$

where $\theta_{[\mathbf{v}]}$ are the topological twists. (In the case of fermions, only T^2 is well-defined).

Thus if two even K-matrices have the same $(\mathcal{A}, T, S, c_+, c_-)$, they are in the same genus. (Extra complications for fermions.)

A bulk *bosonic* topological phase corresponds to a genus of even lattices, while its edge phases correspond to the lattices in this genus.

A bulk *fermionic* topological phase corresponds to a genus of odd lattices, while its edges correspond to the lattices in the genus, and, in some cases, to the lattices in an associated genus of even lattices.

For the example provided, det $K_1 = 7$.

$$\Lambda = \mathbb{Z}\times\mathbb{Z}, \Lambda^* = \mathbb{Z}\times\mathbb{Z}/7 \implies \mathcal{A} = Z_7$$

generated by $(0,4)^T$, for example. Then, the quasiparticles in the theory are labelled

$$\psi_j \equiv (0,4j),\ j = 0,1,...,6$$

Giving the S matrix

$$S_{jj'} = \frac{1}{7}\exp\left(-\frac{32\pi i}{7}jj'\right)$$

Similarly for K_2, a valid generator is $(0,1)^T$, giving quasiparticles ψ_j' and S matrix

$$S'_{jj'} = \frac{1}{7}\exp\left(-\frac{4\pi i}{7}jj'\right)$$

Finally, identifying $\psi_j \leftrightarrow \psi_j'$ proves consistent with $S_{jj'} = S'_{jj'}$

How many distinct edge phases terminate the same bulk? Using the Smith-Siegel-Minkowski mass formula one can, indirectly, determine how many distinct lattices there are in a given genus.

$$\sum_{\Lambda\in g}\frac{1}{|\mathrm{Aut}(\Lambda)|} = m(K)$$

In particular, all chiral Abelian quantum Hall states with central charge $c > 10$ have multiple distinct edge phases.

References

[1] X.G. Wen & Phys. Rev. B **43**, 11025 (1991).

[2] N. Read, Phys. Rev. Lett **65**, 1502(1990).

[3] E. Plamadeala, M. Mulligan, C. Nayak, Phys. Rev. B **88**, 045151 (2013)

[4] J. Cano, M. Cheng, M. Mulligan, C. Nayak, E.Plamadeala, J.Yard, arXiv:str-el/1310.5708

[5] J. Fröhlich, E. Thiran, Journal of stat. phys. **76** (1994), 209-283

[6] V.V. Nikulin, Math. USSR Izv. **14**, 103 (1980)

[7] D. Belov, G.W. Moore, arXiv:hep-th/0505235

[8] S.D. Stirling, Ph.D. thesis, UT Austin, arXiv:0807.2857