

# Entanglement in Featureless Mott Insulators

Brayden Ware <sup>1</sup>

Itamar Kimchi <sup>2</sup>   Siddarth Parameswaran <sup>3</sup>   Bela Bauer <sup>4</sup>

<sup>1</sup>UC Santa Barbara   <sup>2</sup>UC Berkeley   <sup>3</sup>UC Irvine   <sup>4</sup>Microsoft Station Q

March 6th 2014

# Outline

---

- 1 Motivation
- 2 Construction of Honeycomb FBI
- 3 Entanglement Edge of Honeycomb FBI
- 4 Symmetry Protection of Edge

# Motivation

# Featureless insulators

## Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state:

$$E_1 - E_0 \geq \text{const.}$$

## Alternate Definition

- Unique ground state on any boundary-less system
- Possibly with 'features' localized to edge of system

## Fundamental Result

- Integer charge per unit cell
  - (Lieb, Schultz, Mattis)

# Featureless insulators

## Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order
- Unique ground state:  
 $E_1 - E_0 \geq \text{const.}$

## Alternate Definition

- Unique ground state on any boundary-less system
- Possibly with 'features' localized to edge of system

## Fundamental Result

- Integer charge per unit cell
  - (Lieb, Schultz, Mattis)

# Featureless insulators

## Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state:

$$E_1 - E_0 \geq \text{const.}$$

- Gapless modes:

$$E_1 - E_0 \sim \frac{1}{L^z}$$

## Alternate Definition

- Unique ground state on any boundary-less system
- Possibly with 'features' localized to edge of system

## Fundamental Result

- Integer charge per unit cell
  - (Lieb, Schultz, Mattis)

# Featureless insulators

## Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state:  
 $E_1 - E_0 \geq \text{const.}$

- Spontaneous symmetry breaking:  
 $E_1 - E_0 = 0$

## Alternate Definition

- Unique ground state on any boundary-less system
- Possibly with 'features' localized to edge of system

## Fundamental Result

- Integer charge per unit cell
  - (Lieb, Schultz, Mattis)

# Featureless insulators

## Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state:  
 $E_1 - E_0 \geq \text{const.}$

## Alternate Definition

- Unique ground state on any boundary-less system
- Possibly with 'features' localized to edge of system

- Topological order:  
 $E_1 - E_0 \sim e^{-L/\xi}$   
with nontrivial topology

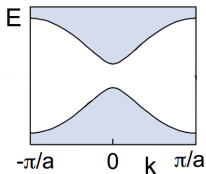
## Fundamental Result

- Integer charge per unit cell
  - (Lieb, Schultz, Mattis)

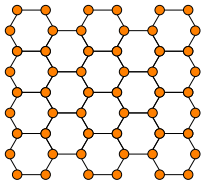


# Free Fermion Featureless Insulators

## Classical Insulators

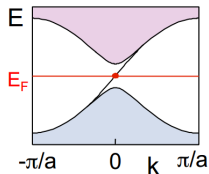


Free fermion band insulator

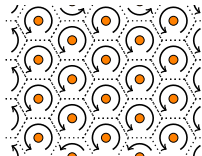


Atomic picture

## Topological Insulators



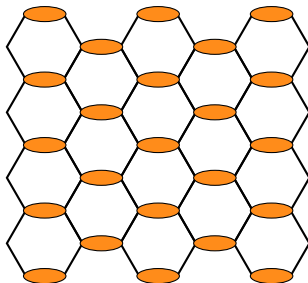
Band insulator with chiral edge <sup>1</sup>



Atomic picture breaks down

# Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?

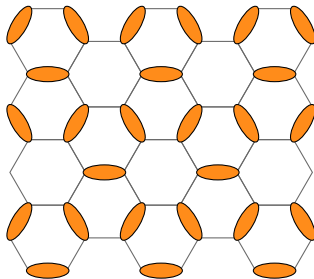


Breaks rotational symmetry

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013)

# Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?

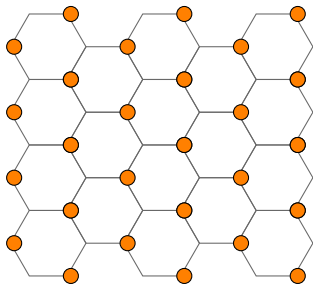


Breaks translationally symmetry, unit cell is 3 times larger

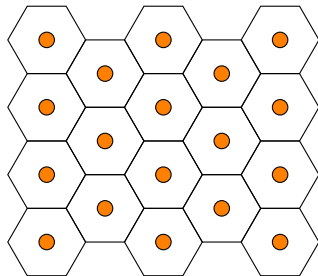
'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013)

# Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



Breaks rotational symmetry



Leaves honeycomb lattice

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013)

# Construction of Honeycomb FBI

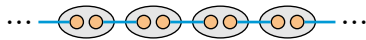
# Construction of 1D Featureless Insulators

## Classical Insulators



1D Trivial Chain

## Topological Insulators



1D Topological Chain

$$\begin{aligned}
 \text{orange dot} - \text{blue line} - \text{orange dot} &= \text{white dot} + \text{orange dot} + \text{orange dot} + \text{white dot} \\
 \text{white dot} - \text{blue line} - \text{white dot} &= 0 \\
 \text{white dot} - \text{blue line} - \text{orange dot} &= 1 \\
 \text{orange dot} - \text{blue line} - \text{orange dot} &= 2
 \end{aligned}$$

Entangled pairs and projectors used in state construction

# Construction of 1D Featureless Insulators

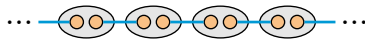
## Classical Insulators



1D Trivial Chain

Product state with one boson per site

## Topological Insulators



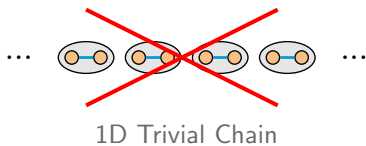
1D Topological Chain

Haldane Insulator Phase  
Pollmann et al. (2010)

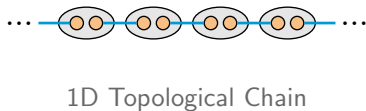
- Unitarily related to AKLT
- No  $SU(2)$  symmetry
- Symmetry protected 2-fold edge degeneracy

# Construction of 1D Featureless Insulators

## Classical Insulators



## Topological Insulators

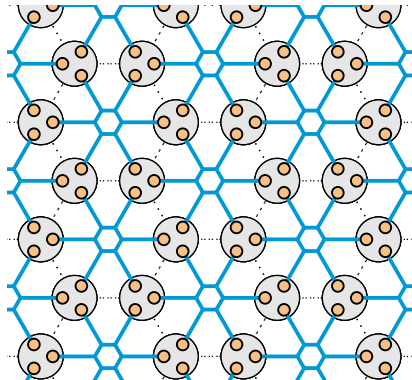


$$\begin{aligned}
 \bullet\bullet &= \circ\bullet - \bullet\circ \\
 \circ\circ &= -\sqrt{2} \\
 \circ\bullet &= 0 \\
 \bullet\bullet &= +\sqrt{2}
 \end{aligned}$$

Entangled pairs and projectors for  $SU(2)$  symmetric state



# Construction of Honeycomb FBI



$$\text{3 orange dots in a circle} = \textcircled{3} \sqrt{3!}$$

$$\text{2 orange dots in a circle} = \textcircled{2} \sqrt{2!}$$

$$\text{1 orange dot in a circle} = \textcircled{1}$$

$$\text{0 orange dots in a circle} = \textcircled{0}$$

$$\text{Central hexagon with 6 dots} = \text{Sum of 6 hexagons with 1 dot each}$$

$$|\psi\rangle = \prod_{\text{hex}} \left( \sum_{i \in \text{hex}} b_i^\dagger \right) |0\rangle$$

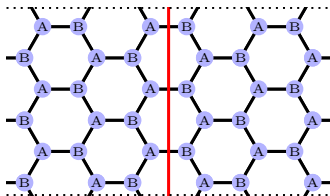
Wavefunction proposed by  
Kimchi et al. (2013)

# Known Results for Honeycomb FBI

---

# Entanglement Edge of Honeycomb FBI

# Edge Geometry

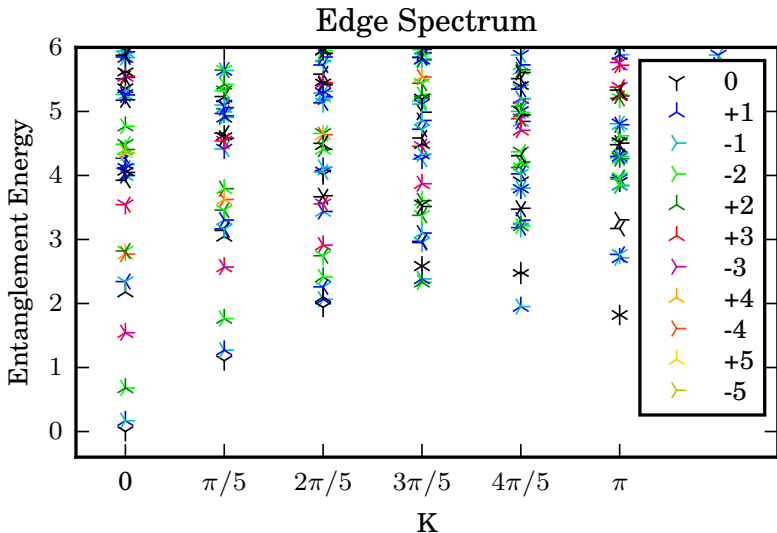


Generic honeycomb lattice PEPS on zig-zag cylinder with  $L=3$

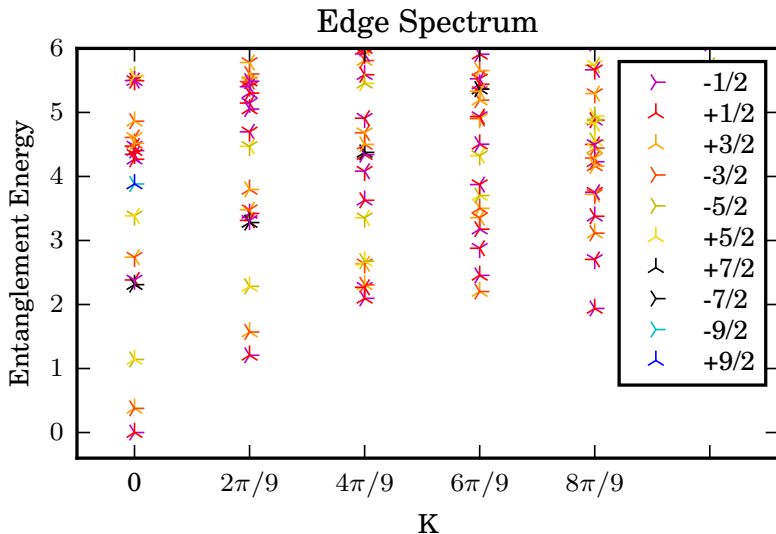
In cylindrical geometry:

- Treat state as 1D
- Use MPS techniques
- On-site translational symmetry parallel to cut
- Physical site dimension  $2^{2L}$
- MPS bond dimension = Rank of  $\rho_r = 2^L$
- Entanglement spectrum  $\{\epsilon_i\}$  defined from eigenvalues  $\{\rho_i\}$  of  $\rho_r$  via  $\epsilon_i = e^{-\rho_i}$
- Charge and Translation represented linearly on edge

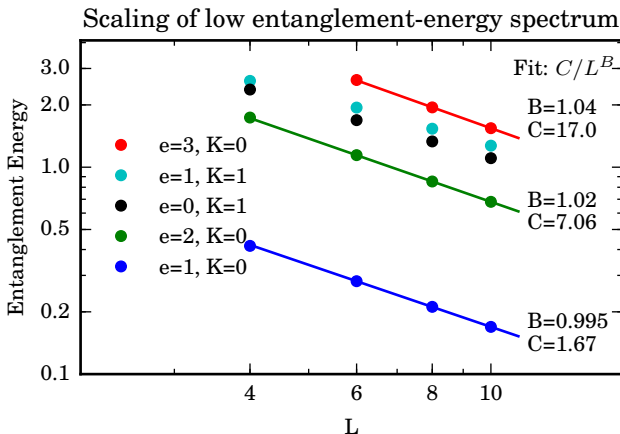
# Entanglement Spectrum



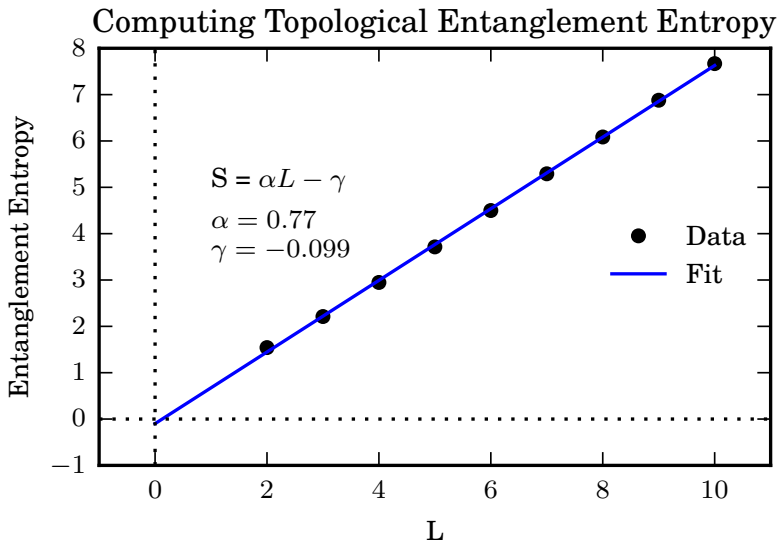
# Entanglement Spectrum



# Finite Size Analysis



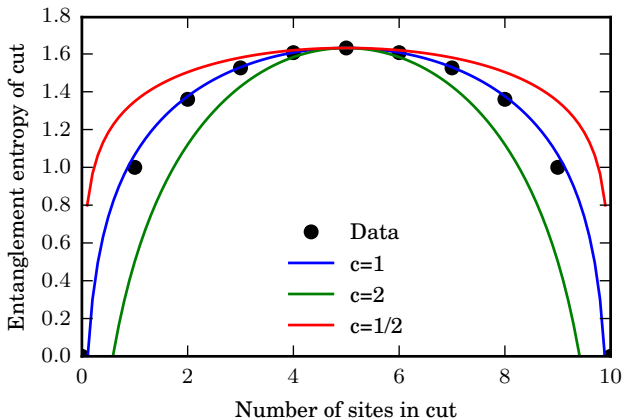
# Finite Size Analysis





# Identification of Edge CFT

## Conformal Charge



$$c = 1$$

# Identification of Edge CFT

## Conformal Weights

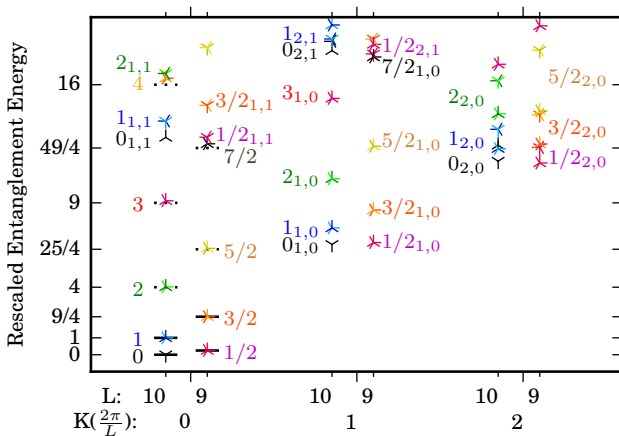
We can match the rescaled entanglement energies to the conformal weights of a free bosonic CFT.

$$\begin{aligned}\mathbf{P} &= \frac{2\pi}{L}(\mathbf{L}_0 - \bar{\mathbf{L}}_0) = \frac{2\pi}{L}(em + n - \bar{n}) \\ \mathbf{H} &= \frac{2\pi}{L}(\mathbf{L}_0 + \bar{\mathbf{L}}_0) = \frac{2\pi}{L}\left(\frac{\kappa e^2}{2} + \frac{m^2}{2\kappa} + \frac{n + \bar{n}}{2}\right)\end{aligned}$$

$$\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})$$

# Identification of Edge CFT

Conformal primary identification in entanglement spectra



# Symmetry Protection of Edge

# Symmetry Protection of Degenerate Edge

# Future Work

Entanglement properties with different geometries

- Armchair cylinder edge
- Finite size clusters
- Explain results for arbitrary geometries with tensor network properties, e.g. 'MPO injectivity'

Find a 2D local Hamiltonian and confirm with numerics

$$H_{EBH} = \left( \sum_{\hexagon} \sum_{i,j \in \hexagon} -tb_i^\dagger b_j + V n_i n_j \right) + \mu N?$$

Physical properties of the phase

Can we construct an SU(2) symmetric FI?

# Resources

---

- Hasan, M. Z. and Kane, C. L. (2010). *Colloquium: Topological insulators. Reviews of modern physics*, 82(4):3045–3067.
- Kimchi, I., Parameswaran, S. A., Turner, A. M., Wang, F., and Vishwanath, A. (2013). Featureless and nonfractionalized mott insulators on the honeycomb lattice at  $1/2$  site filling. *Proceedings of the National Academy of Sciences*, 110(41):16378–16383.
- Parameswaran, S. A., Kimchi, I., Turner, A. M., Stamper-Kurn, D. M., and Vishwanath, A. (2013). Wannier permanent wave functions for featureless bosonic mott insulators on the  $1/3$ -filled kagome lattice. *Phys. Rev. Lett.*, 110:125301.
- Pollmann, F., Turner, A. M., Berg, E., and Oshikawa, M. (2010). Entanglement spectrum of a topological phase in one dimension. *Phys. Rev. B*, 81(6):064439.

# Questions?

Brayden Ware

[brayden@physics.ucsb.edu](mailto:brayden@physics.ucsb.edu)



# Bonus slides