# Entanglement in Featureless Mott Insulators


Brayden Ware [1]

Itamar Kimchi [2] Siddarth Parameswaran [3] Bela Bauer [4]

[1]UC Santa Barbara [2]UC Berkeley [3]UC Irvine [4]Microsoft Station Q


March 6th 2014

# Featureless Insulators

## Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state: $E_1 - E_0 \geq const.$

## Alternate Definition

- Unique ground state on all boundary-less systems
- Possibly with 'features' localized to edge of system

# Featureless Insulators

### Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

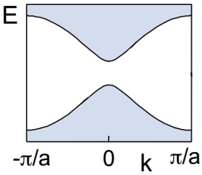
### Alternate Definition

- Unique ground state on all boundary-less systems
- Possibly with 'features' localized to edge of system

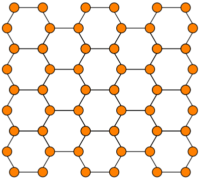
- Unique ground state: $E_1 - E_0 \geq const.$
- Gapless modes: $E_1 - E_0 \sim \frac{1}{L^z}$
- Spontaneous symmetry breaking: $E_1 - E_0 = 0$
- Topological order: $E_1 - E_0 \sim e^{-L/\xi}$

# Examples of Featureless Insulators

## Classical Insulators

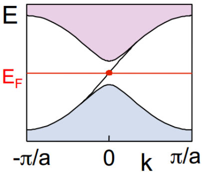



Free fermion band insulator

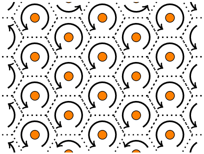


Atomic picture

## Topological Insulators




Band insulator with chiral edge



Atomic picture breaks down

# Obstructions to Featurelessness

## Fundamental Result

A featureless insulator must have an integer charge per unit cell

- (Lieb, Schultz, Mattis 1961)
- (Hastings 2004)

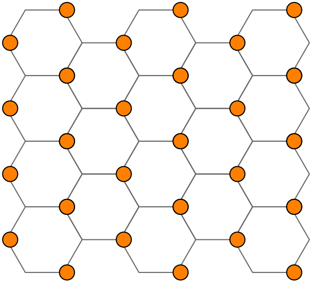
For certain lattices, not all integers are possible

- (Parameswaran 2013)

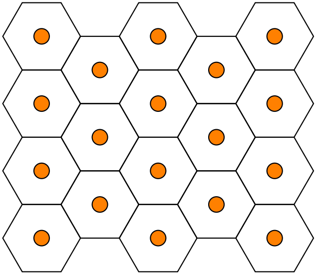
For this talk, we will look at a proposed honeycomb lattice featureless insulator with charge 1 per unit cell.

# Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



Breaks rotational symmetry

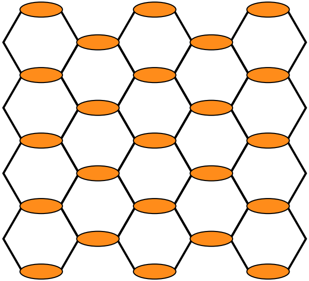


Leaves honeycomb lattice

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013a)

# Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?

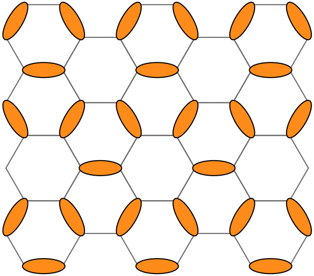


Breaks rotational symmetry

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013a)

# Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?

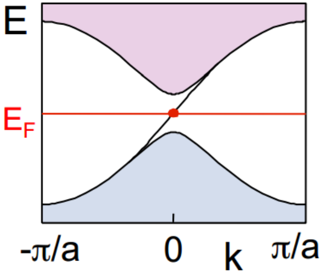


Breaks translationally symmetry, unit cell is 3 times larger

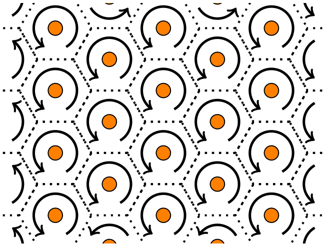
'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013a)

# Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?




Band insulator with chiral edge [1]



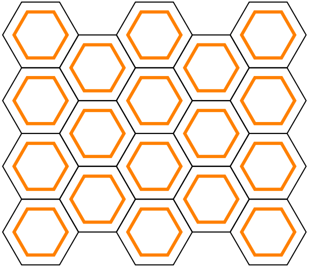
The Haldane Chern insulator is NOT an example. $D_6$ explicitly broken.

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013a)

[1](Hasan and Kane, 2010)

# Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?

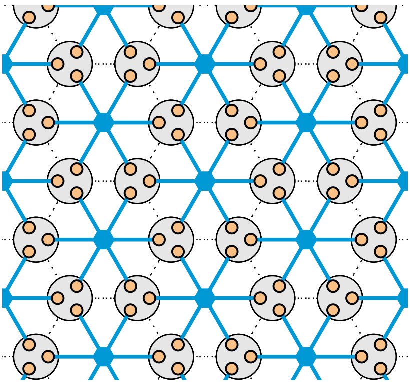


$$|\psi\rangle = \prod_{\hexagon} \left( \sum_{i \in \hexagon} b_i^\dagger \right) |\mathbf{0}\rangle$$

Proposed Solution by Kimchi et al. (2013)

Bosons filled into non-orthogonal, plaquette centered orbitals works. Numerics confirm the expected wavefunction properties, but no known parent Hamiltonian has been found.

# Computations on Honeycomb FBI



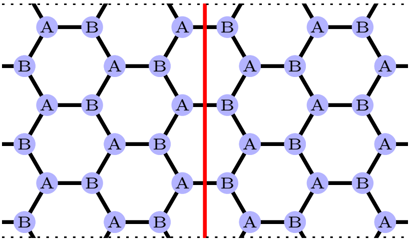
$$|\psi\rangle = \prod_{\hexagon} \left( \sum_{i \in \hexagon} b_i^\dagger \right) |\mathbf{0}\rangle$$

Simple tensor network representation

Cylinder slice treated as single site of an effective 1D system.

Schmidt decomposition computed as in 1D matrix product states.

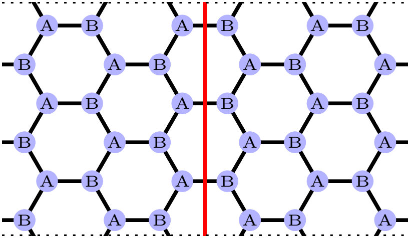
# Computations on Honeycomb FBI




Generic honeycomb lattice PEPS on zig-zag cylinder with L=3

- Simple tensor network representation
- Cylinder slice treated as single site of an effective 1D system.
- Schmidt decomposition computed as in 1D matrix product states.

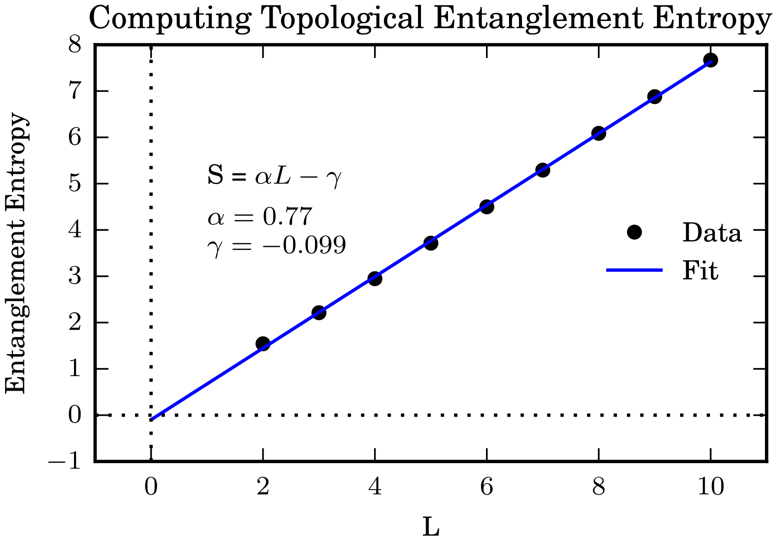
# Computations on Honeycomb FBI



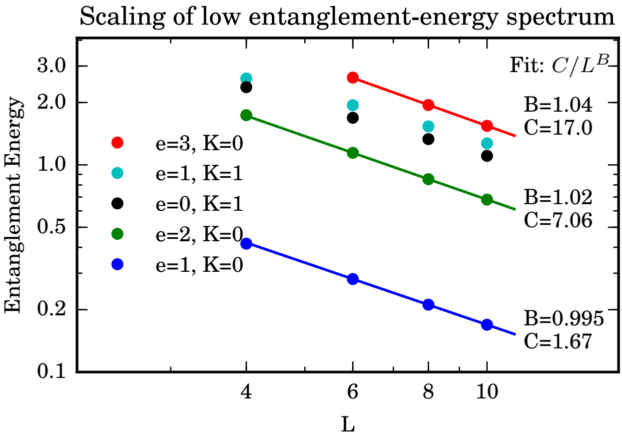

Generic honeycomb lattice PEPS on zig-zag cylinder with L=3

- Simple tensor network representation
- Cylinder slice treated as single site of an effective 1D system.
- Schmidt decomposition computed as in 1D matrix product states.

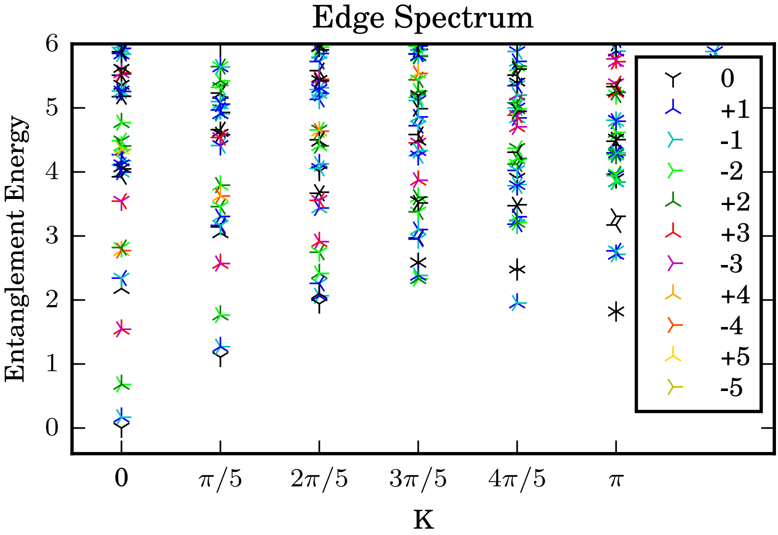
# Finite Size Analysis



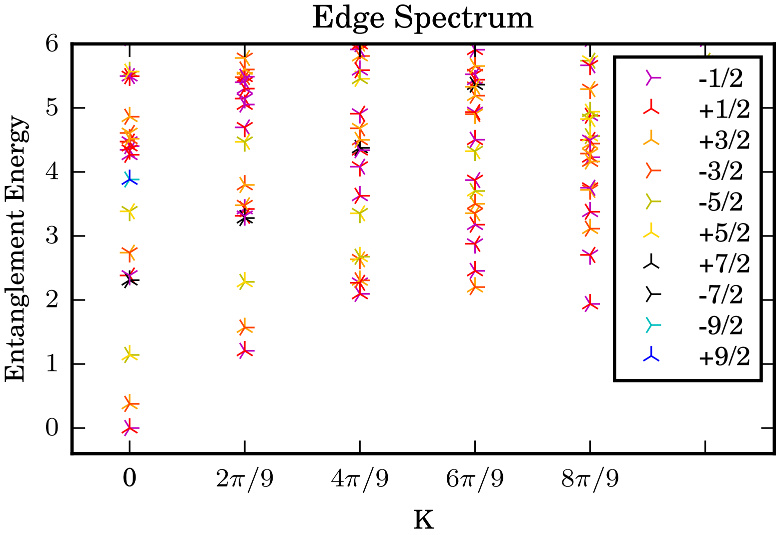
# Finite Size Analysis



# Entanglement Spectrum

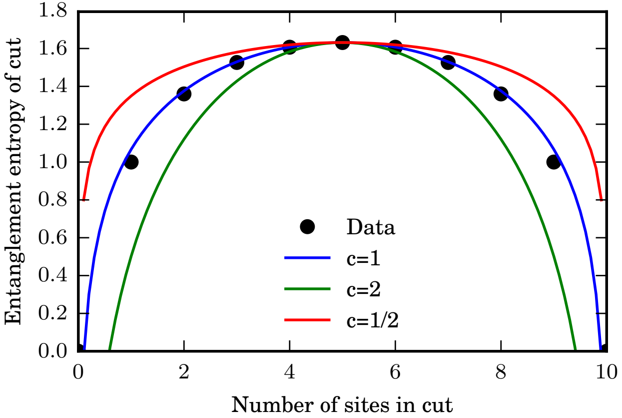


# Entanglement Spectrum



# Identification of Edge CFT

## Conformal Charge




$$\boxed{c = 1}$$

# Identification of Edge CFT

## Conformal Weights

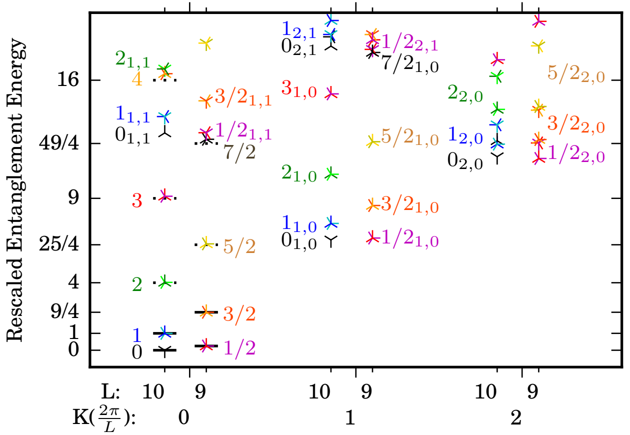
We can match the rescaled entanglement energies to the conformal weights of a free bosonic CFT.

$$
\begin{aligned}
\mathbf{P} = \quad & \frac{2\pi}{L}(\mathbf{L_0} - \bar{\mathbf{L}}_\mathbf{0}) = & \frac{2\pi}{L}(em + n - \bar{n}) \\
\mathbf{H} = \quad & \frac{2\pi}{L}(\mathbf{L_0} + \bar{\mathbf{L}}_\mathbf{0}) = & \frac{2\pi}{L}\left(\frac{\kappa e^2}{2} + \frac{m^2}{2\kappa} + \frac{n + \bar{n}}{2}\right)
\end{aligned}
$$

$$
\boxed{\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})}
$$

# Identification of Edge CFT

## Conformal primary identification in entanglement spectra

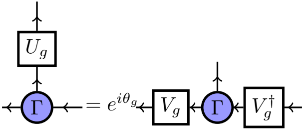


# Symmetry Protection of Degenerate Edge

## 1D Symmetry Protection

On-site symmetries $g$ come with projective representation $V_g$

- $V_g$ acts on sets of degenerate Schmidt states
- Charge and translation represented linearly on edge

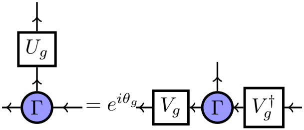


# Symmetry Protection of Degenerate Edge

## 1D Symmetry Protection

Time reversal symmetry $\tau$ represented by antiunitary $V_\tau K$ on the edge

- $\tau^2 = +1$ on this edge



# Symmetry Protection of Degenerate Edge

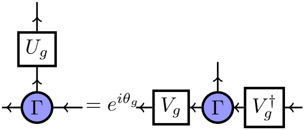
## 1D Symmetry Protection

Inversion $\mathcal{I}$

- $\mathcal{I}$ in combination with swapping Schmidt states represented by antiunitary operation $V_{\mathcal{I}}K$ on the edge
- $\mathcal{I}^2 = V_{\mathcal{I}}V_{\mathcal{I}}^* = 1$

Inversion $\mathcal{I}$ combined with $\pi = e^{i\pi N}$

- $\pi\mathcal{I}$ represented antiunitarily on the edge by $V_{\pi\mathcal{I}}K$
- $(\pi\mathcal{I})^2 = 1$ but $V_{\pi\mathcal{I}}V_{\pi\mathcal{I}}^* = -1$



# Future Work

Entanglement properties with different geometries

- Armchair cylinder edge
- Finite size clusters
- Explain results for arbitrary geometries with tensor network properties, e.g. 'MPO injectivity'

Find a 2D local Hamiltonian and confirm with numerics

$$H_{EBH} = \left( \sum_{\hexagon} \sum_{i,j \in \hexagon} -t b_i^\dagger b_j + V n_i n_j \right) + \mu N?$$

Physical properties of the phase

Can we constructan SU(2) symmetric FI?

# Resources

Hasan, M. Z. and Kane, C. L. (2010). *Colloquium*: Topological insulators. *Reviews of modern physics*, 82(4):3045–3067.

Kimchi, I., Parameswaran, S. A., Turner, A. M., Wang, F., and Vishwanath, A. (2013). Featureless and nonfractionalized mott insulators on the honeycomb lattice at $1/2$ site filling. *Proceedings of the National Academy of Sciences*, 110(41):16378–16383.

Parameswaran, S. A., Kimchi, I., Turner, A. M., Stamper-Kurn, D. M., and Vishwanath, A. (2013a). Wannier permanent wave functions for featureless bosonic mott insulators on the $1/3$-filled kagome lattice. *Phys. Rev. Lett.*, 110:125301.

Parameswaran, S. A., Turner, A. M., Arovas, D. P., and Vishwanath, A. (2013b). Topological order and absence of band insulators at integer filling in non-symmorphic crystals. *Nature Physics*, 9(5):–303?

Pollmann, F., Turner, A. M., Berg, E., and Oshikawa, M. (2010). Entanglement spectrum of a topological phase in one

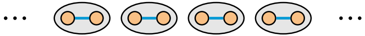
# Questions?

Brayden Ware
brayden@physics.ucsb.edu

# Bonus slides

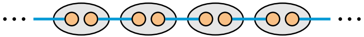
# Construction of 1D Featureless Insulators

## Classical Insulators

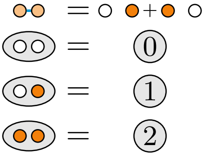


1D Trivial Chain

## Topological Insulators



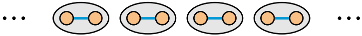
1D Topological Chain




Projectors and entangled pairs (PEPS) used in state construction

# Construction of 1D Featureless Insulators

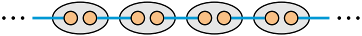
## Classical Insulators



1D Trivial Chain

Product state with one boson per site

## Topological Insulators



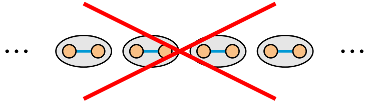
1D Topological Chain

Haldane Insulator Phase
Pollmann et al. (2010)

- Unitarily related to AKLT
- No $SU(2)$ symmetry
- Symmetry protected 2-fold edge degeneracy

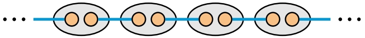
# Construction of 1D Featureless Insulators

## Classical Insulators

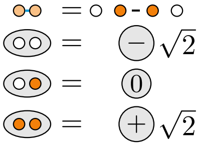


1D Trivial Chain

## Topological Insulators

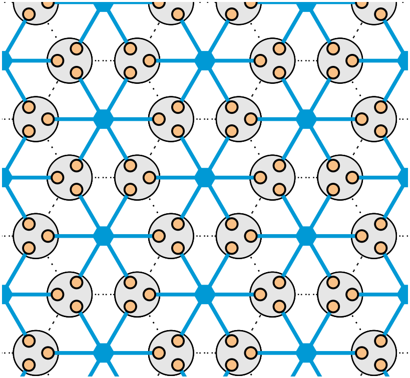


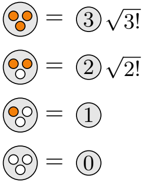
1D Topological Chain

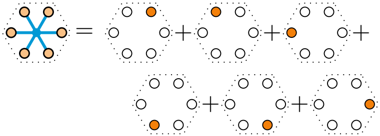



Projectors and entangled pairs (PEPS) for $SU(2)$ symmetric state

# Tensor Network cut details

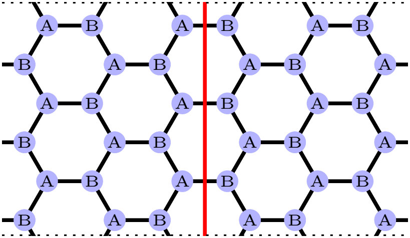





$$|\psi\rangle = \prod_{\hexagon} \left( \sum_{i \in \hexagon} b_i^\dagger \right) |\mathbf{0}\rangle$$

# Tensor Network cut details

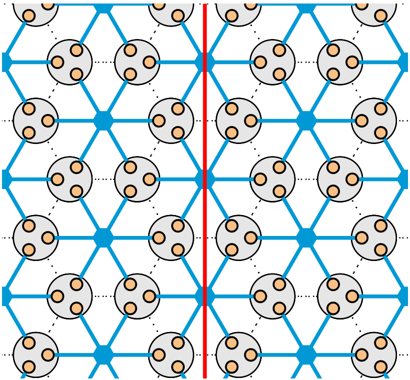



Generic honeycomb lattice PEPS on zig-zag cylinder with L=3

In cylindrical geometry:

- Treat state as 1D
- Use MPS techniques
- On-site translational symmetry parallel to cut
- Physical site dimension $4^{2L}$

# Tensor Network cut details

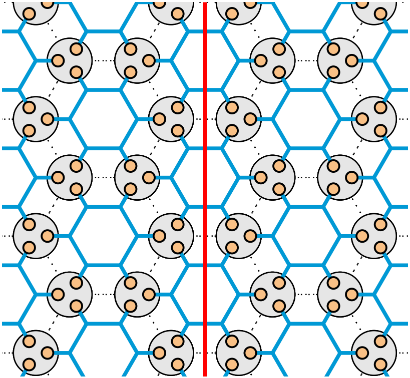


Honeycomb lattice tensor network on zig-zag cylinder with L=3

In cylindrical geometry:

- Treat state as 1D
- Use MPS techniques
- On-site translational symmetry parallel to cut
- Physical site dimension $4^{2L}$

# Tensor Network cut details

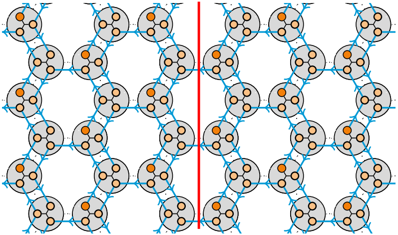


Honeycomb lattice PEPS on zig-zag cylinder with L=3, acheived by factoring W-state of plaquette bosons

In cylindrical geometry:

- Treat state as 1D
- Use MPS techniques
- On-site translational symmetry parallel to cut
- Physical site dimension $4^{2L}$
- MPS bond dimension = Rank of $\rho_r = 2^L$
- Entanglement spectrum $\{\epsilon_i\}$ defined from eigenvalues $\{\rho_i\}$ of $\rho_r$ via $\epsilon_i = e^{-\rho_i}$
- Charge and Translation represented linearly on edge

# Tensor Network cut details



Honeycomb lattice PEPS on zig-zag cylinder with L=3, acheived by factoring W-state of plaquette bosons

In cylindrical geometry:

- Treat state as 1D
- Use MPS techniques
- On-site translational symmetry parallel to cut
- Physical site dimension $4^{2L}$
- MPS bond dimension = Rank of $\rho_r = 2^L$
- Entanglement spectrum $\{\epsilon_i\}$ defined from eigenvalues $\{\rho_i\}$ of $\rho_r$ via $\epsilon_i = e^{-\rho_i}$
- Charge and Translation represented linearly on edge

# Known Results for Honeycomb FBI

## Correlations

$< b_i^\dagger b_j >$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length $\xi/a \sim 3.6$

$< n_i n_j >$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length $\xi/a \sim 1.6$

## Hamiltonian Construction

Try filling plaquette orbitals

$$b_{\hexagon} = \sum_{i \in \hexagon} \frac{1}{\sqrt{6}} b_i^\dagger$$

$$H = \sum_{\hexagon} -\frac{t}{6} b_{\hexagon}^\dagger b_{\hexagon} + V n_{\hexagon} n_{\hexagon}$$

$$= \left( \sum_{\hexagon} \sum_{i,j \in \hexagon} -t b_i^\dagger b_j \right) - \frac{3t}{6} N + V \ldots$$

- Fails, gapless modes
- Parent Hamiltonian not known

# Known Results for Honeycomb FBI

## Correlations

$< b_i^\dagger b_j >$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length $\xi/a \sim 3.6$

$< n_i n_j >$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length $\xi/a \sim 1.6$

## Hamiltonian Construction

To get a parent Hamiltonian:

- Need symmetric, exponentially localized, orthogonal orbitals
- Such as the Wannier orbitals of a classical band insulator
- Parameswaran et al. (2013a)

Other lattices:

- Need a fixed point of all lattice symmetries
- Fails on nonsymmorphic lattices
- Extension of LSM theorem
- Parameswaran et al. (2013b)