Entanglement in Featureless Mott Insulators

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Outline

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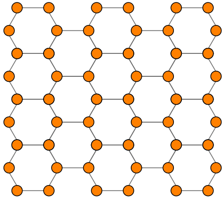
Featureless Insulators

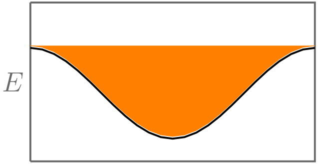
Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No (bulk) fractionalization
- Unique ground state on torus

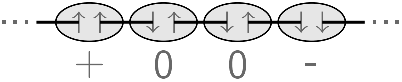


Bosonic Mott insulator with integer filling

Examples:



Band Insulator



Haldane phase of spin-1 Chain (AKLT)

Lieb-Schultz-Mattis Theorem

States with fractional charge per unit cell cannot be featureless.

Theorem: Lieb, Schultz, Mattis (1961)

A spin $1/2$ chain with $SU(2)$ and translational symmetry has a ground state that is either gapless or breaks symmetry.

Extension: Oshikawa (1999)

A particle-number conserving system with a fractional number of particles per unit cell cannot have a fixed energy gap on a torus. The same holds for a $U(1)$-symmetric spin system with total spin J per unit cell and magnetization m per unit cell, with $J - m$ not integer.

Applications of LSM

Spin-$1/2$ XXZ chain

$$H_{XY} = J\sum_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y)$$

Jordan-Wigner transformation maps to half-filled free-fermion band. Remains gapless under small $U(1)$ perserving pertubations:

- hS^z - gapless until $m = \pm 1/2$
- $J_z S_i^z S_{i+1}^z$ - gapless until AFM/FM order
- $J_2 \vec{S}_i \cdot \vec{S}_{i+2}$ - gapless until SSB of translation, unit cell doubles

Applications of LSM

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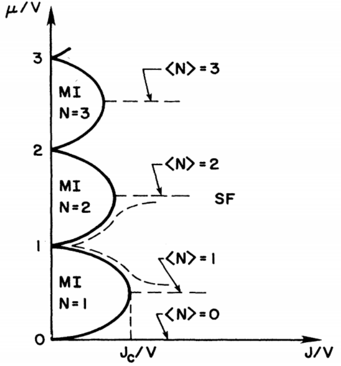
Magnetization/Density Plateaus

Stable featureless insulators form magnetization plateaus

Example Hamiltonians and phase diagrams:

- Bose-Hubbard model

$$H_{BH} = \sum_i J\vec{S}_i \cdot \vec{S}_j + J'(\vec{S}_i \cdot \vec{S}_j)^2 + D(S_i^z)^2$$



Magnetization/Density Plateaus

Stable featureless insulators form magnetization plateaus
Example Hamiltonians and phase diagrams:

- Haldane Phase/ AKLT Model

$$H_{AKLT} =$$

Magnetization/Density Plateaus

Stable featureless insulators form magnetization plateaus
Example Hamiltonians and phase diagrams:

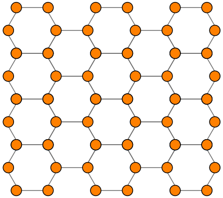
- Spin 3/2 Haldane Phase

$$H_{AKLT} =$$

Outline

Existence of Featureless Insulators

Given a (non-Bravais) lattice and an integer particle number per unit cell, is there always a featureless insulator?
(Diagrams showing naive constructions on honeycomb/Kagome/p4g lattices don't work.)



Bosonic Mott insulator
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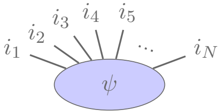
What is entanglement?

When a wavefunction in a product Hilbert space cannot be written as a product state, it is **entangled**.

$$|\psi\rangle = |\psi_L\rangle \otimes |\psi_R\rangle ?$$



$$|\psi\rangle = \sum \psi_{i_1 i_2 \ldots i_N} |i_1 i_2 \ldots i_N\rangle$$

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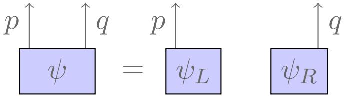

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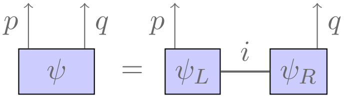
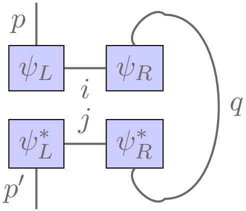


Calculate reduced density matrices

$$\rho_L = Tr_R|\psi\rangle\langle\psi|$$

Diagonalize

$$\rho_L = \sum_\alpha \rho_\alpha |\psi_L^\alpha\rangle\langle\psi_L^\alpha|$$



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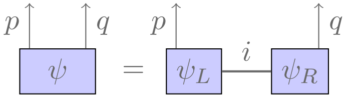
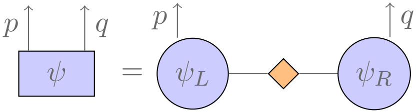


Diagonalize and form the Schmidt decomposition

$$|\psi\rangle = \sum_\alpha \sqrt{\rho_\alpha}|\psi_L^\alpha\rangle \otimes |\psi_R^\alpha\rangle$$



Quantitative measures of entanglement - entropy

$$S_A = -\sum_\alpha \rho_\alpha \log \rho_\alpha$$

What is a matrix product state?

Matrix Product states parameterize Hilbert space, using few parameters for weakly entangled states.
Ground states of gapped quantum Hamiltonians satisfy entanglement area law.

Computing Correlation Functions in MPS

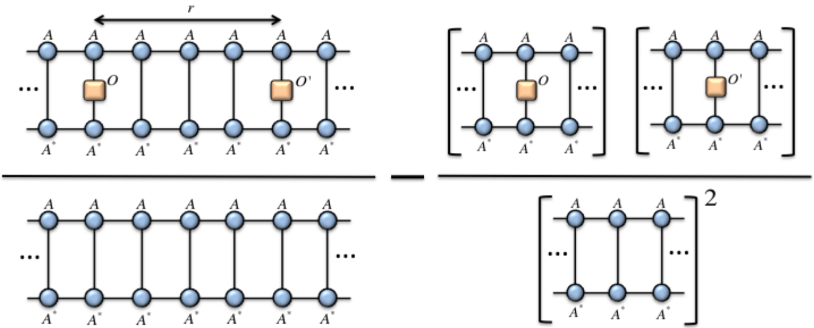


Diagram for $C(r) = \langle O_i O'_{i+r} \rangle - \langle O_i \rangle \langle O'_{i+r} \rangle$

- $\langle O_i O'_{i+r} \rangle = (v_L | T_O T_I^r T_{O'} | v_R)$

How do 'features' show up in MPS?

Transfer matrix becomes degenerate for featured 1D or quasi-1D states.

- Correlation functions at long distance split into multiple sectors.
- Correlations in spontaneous symmetry breaking MPS have extreme senstivity to boundary conditions
- Correlations in topological ordered state (on large enough cylinder) completely insensitive to boundary conditions when operators don't wrap around cylinder.
- Gapless 2D PEPS on cylinder?
- Notes on phase transitions in MPS?

Flux-Threading Argument for LSM

Technical Slide: Threading Flux in a MPS

Outline

Existence of Featureless Insulators

Status of existence question:

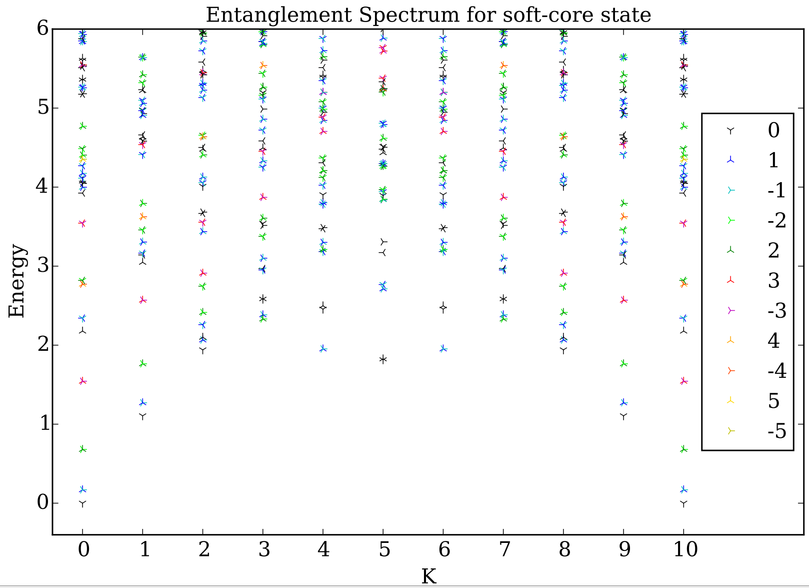
- On *non-symmorphic* lattices, not all integer particle-numbers can be realized.
  - Flux removal doesn't commute with glide-reflections or screw-axes where the the translation vector is not a lattice translation.
- On Kagome lattice, boson insulator with $1/3$ site filling constructed by filling Wannier orbitals of the lowest band of a fermion band insulator.
- On honeycomb lattice, no such band insulator.

- Proposed wavefunction
- Goals:
  - Rule out spontaneous symmetry breaking by computing correlations
  - Rule out topological order by computing topological

Tensor Network Construction of FBI

A wavefunction written as a product of local operators acting on a product state can simply be turned into a tensor network.

Entanglement Spectra



Finite Size Analysis of Spectra

- Topological entanglement entropy is 0
- Low energy modes show gapless $1/L$ behavior

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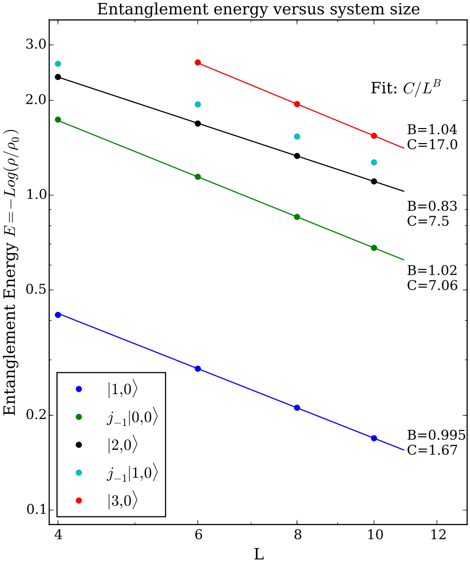
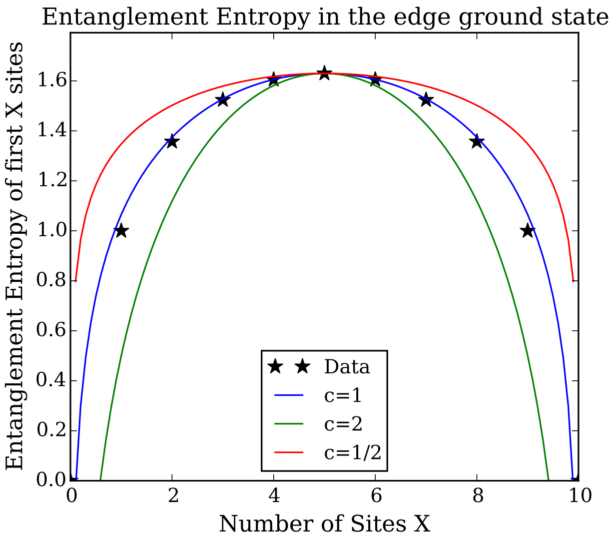
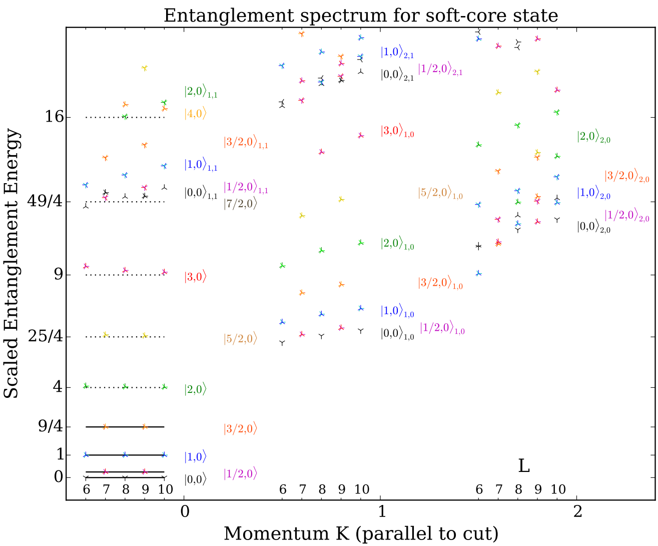


Identifying CFTs: Measuring c



Level identification in CFT spectra



Open questions and speculation

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Resources

A Practical Introduction to Tensor Networks

Orus, R. A Practical Introduction to Tensor Networks: Matrix Product States and Projected Entangled Pair States. arXiv [cond-mat.str-el] (2013). at <http://arxiv.org/abs/1306.2164>

Questions ?

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Bonus slides

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