Entanglement in Featureless Mott Insulators

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Outline

1. Motivation
   - Featureless Insulators
   - Topological Band Insulators
   - Bosonic Band Insulators?

2. Featureless Boson Mott Insulators
   - Honeycomb Featureless Insulator Proposal

3. Entanglement Spectra
   - CFT Identification

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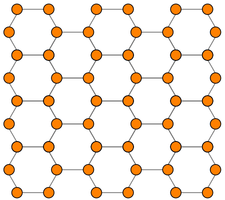
   - CFT Identification

Featureless Insulators

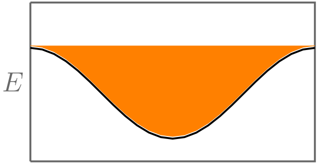
Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order
- Integer charge per unit cell
- Unique ground state with P.B.C.

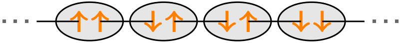
Examples:



Bosonic Mott insulator
with integer filling



Band Insulator



Heisenberg AF Spin-1 chain

Free Fermion Band Insulators

- Crystaline, 0T insulators (including semiconductors)
- Tight-binding Hamiltonian

$$\mathcal{H}_{FF} = \sum_{<ij>} \sum_{\alpha,\beta} -t_{\alpha,\beta} c_i^{\alpha\dagger} c_j^{\beta} - \mu \sum_{i,\alpha} N_i^{\alpha}$$

- Bloch Wavefunctions $|u_{\mathbf{k}}^{\alpha}\rangle$
- Massive Dirac Hamiltonian

$$\mathcal{H}_D(\mathbf{k}) = \mathbf{k}_x \sigma_x + \mathbf{k}_y \sigma_y + m_* \sigma_z$$

- Atomic-insulating like Wannier basis

Semiconductor GaN

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Band Theory

Free Fermion Band Insulators

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Dirac Band Theory

Free Fermion Band Insulators

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- Bloch Wavefunctions $|u_{\mathbf{k}}^{\alpha}\rangle$
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- Atomic-insulating like Wannier basis

Wannier Function

Motivating Questions

Existence

Are there **constraints** on the existence of featureless insulators?
Can we **construct** featureless insulators when possible?

Given a lattice Λ, and an integer m, is there a featureless insulating phase of matter with m particles per unit cell?

Uniqueness

How can we **distinguish** different classes of featureless insulators?
Can we **enumerate** all such classes?

Frame Title

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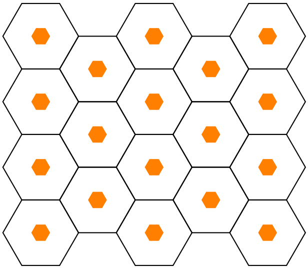
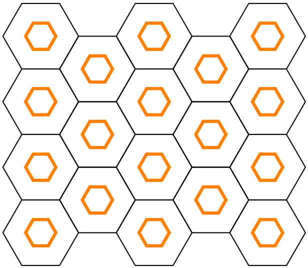
Honeycomb Featureless Boson Insulator

Proposed Wavefunction



Honeycomb Featureless Boson Insulator

Proposed Wavefunction



Honeycomb Featureless Boson Insulator

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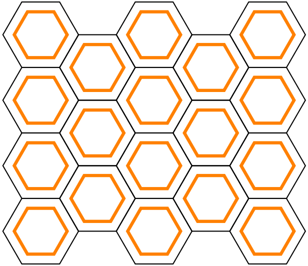
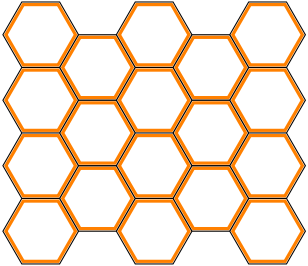


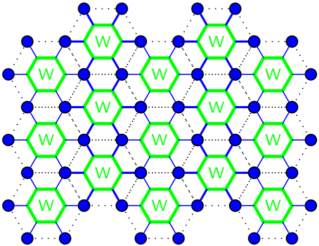
Honeycomb Featureless Boson Insulator

Proposed Wavefunction



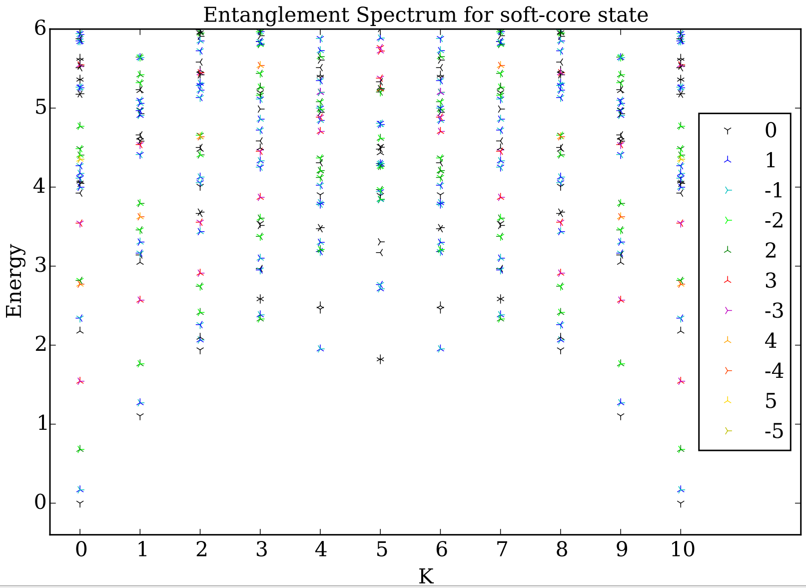
Honeycomb Featureless Boson Insulator

A wavefunction written as a product of local operators acting on a product state can simply be turned into a tensor network.



Outline

Entanglement Spectra



Finite Size Analysis of Spectra

- Topological entanglement entropy is 0
- Low energy modes show gapless $1/L$ behavior

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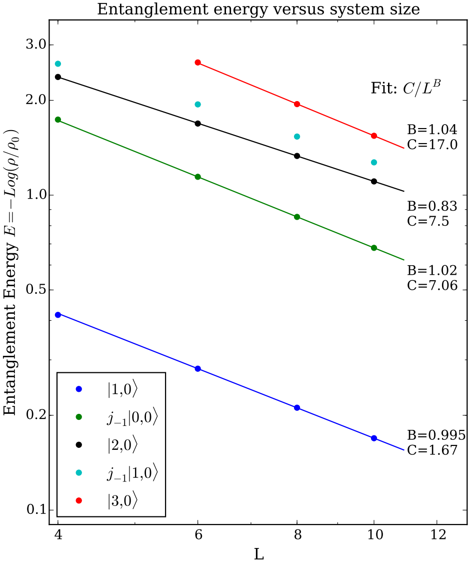
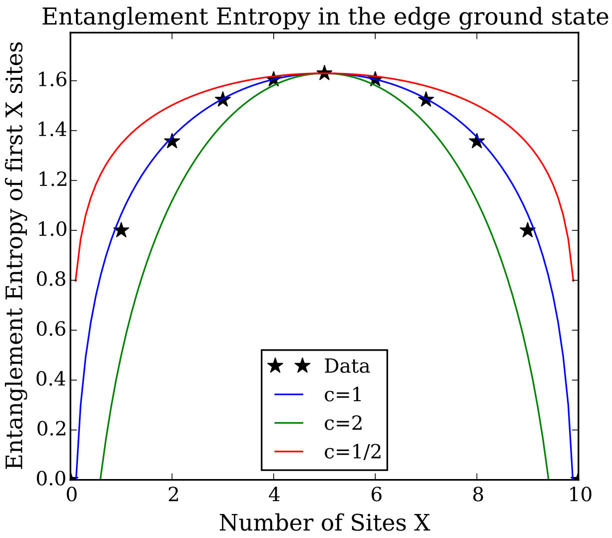
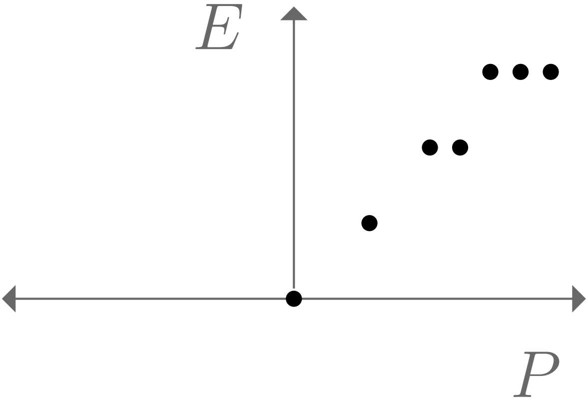


Identifying CFTs: Measuring c



Conformal Tower



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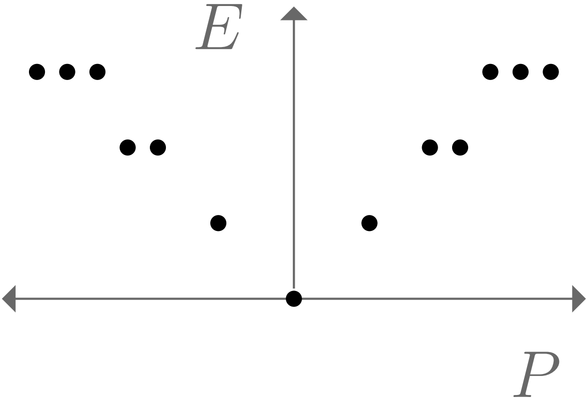

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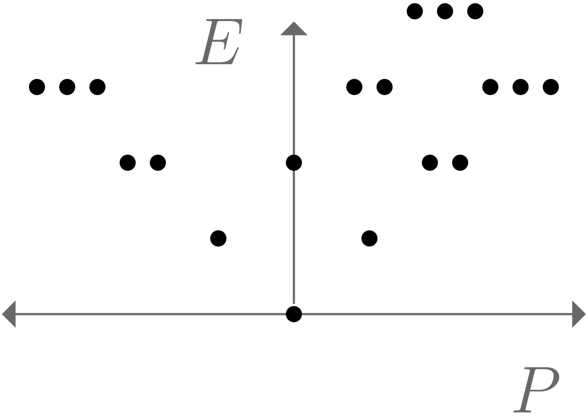

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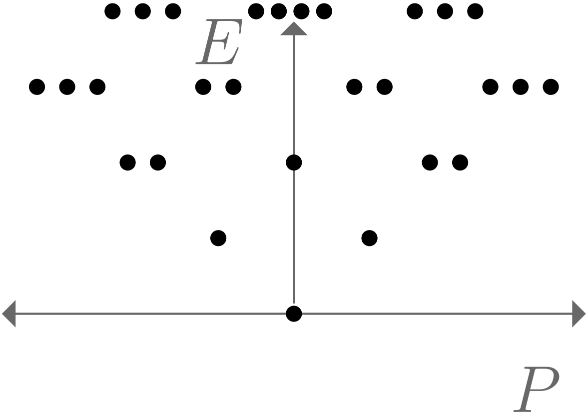


Level identification in CFT spectra

To make a precise comparison with the free-boson CFT, we'll need to solve for (or look up) the solution of this model.
The free-boson CFT is created from the Lagrangian

$$\mathfrak{L} = \frac{g}{2} \int dt \int_0^L dx \left(\frac{1}{v^2}(\partial_t \phi)^2 - (\partial_x \phi)^2\right)$$

and with the compatified field identification

$$\phi \equiv \phi + 2\pi R$$

and placed on the circle of circumference L with periodic boundary conditions

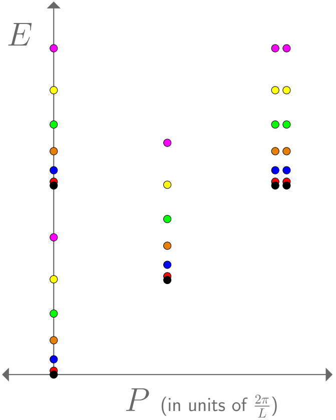
$$\phi(x) \equiv \phi(x + L).$$

Level identification in CFT spectra

$\mathbf{L_0}$	$2\pi g(\frac{e}{4\pi gR} + \frac{mR}{2})^2 + n$
$\mathbf{\bar{L}_0}$	$2\pi g(\frac{e}{4\pi gR} - \frac{mR}{2})^2 + \bar{n}$
$\mathbf{P} = \frac{2\pi v}{L}(\mathbf{L_0} - \mathbf{\bar{L}_0})$	$\frac{2\pi v}{L}(em + n - \bar{n})$
$\mathbf{H} = \frac{2\pi v}{L}(\mathbf{L_0} + \mathbf{\bar{L}_0})$	$\frac{2\pi v}{L}(\frac{e^2}{4\pi gR^2} + \pi g m^2 R^2 + n + \bar{n})$
$\tilde{\mathbf{H}} = \frac{L}{2\pi v\kappa}\mathbf{H}$	$e^2 + \frac{m^2}{4\kappa^2} + \frac{1}{\kappa}(n + \bar{n})$

Eigenvalues of states $|e, m\rangle_{n,\bar{n}}$. The rescaled Hamiltonian $\tilde{\mathbf{H}}$ has eigenvalues that depend on only one free-parameter, $\kappa = 1/(4\pi g R^2)$.(Note: A common convention is to set $g = 1/4\pi$ and describe the system using $R = \sqrt{1/\kappa}$.)

Level identification in CFT spectra



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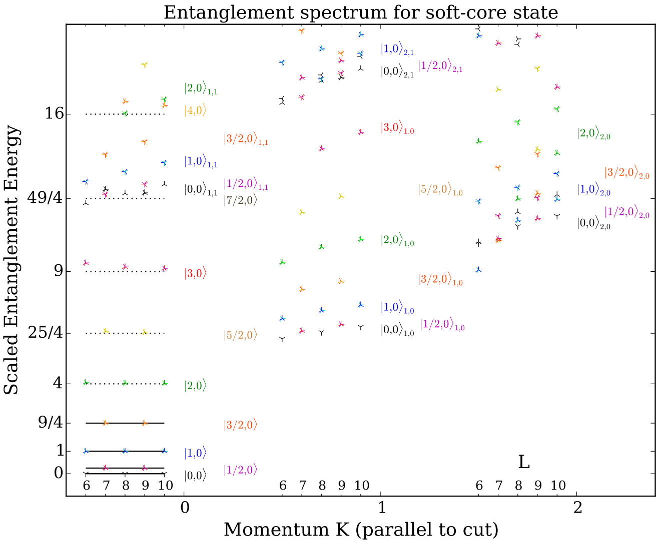

Open questions and speculation

Correspondence between edge and bulk physics In a RG-fixed point tensor network state (such as toric code): Can

Resources

A Practical Introduction to Tensor Networks

Orus, R. A Practical Introduction to Tensor Networks: Matrix Product States and Projected Entangled Pair States. arXiv [cond-mat.str-el] (2013). at <http://arxiv.org/abs/1306.2164>

Questions?

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Bonus slides

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