

# Featureless Bosonic Insulators

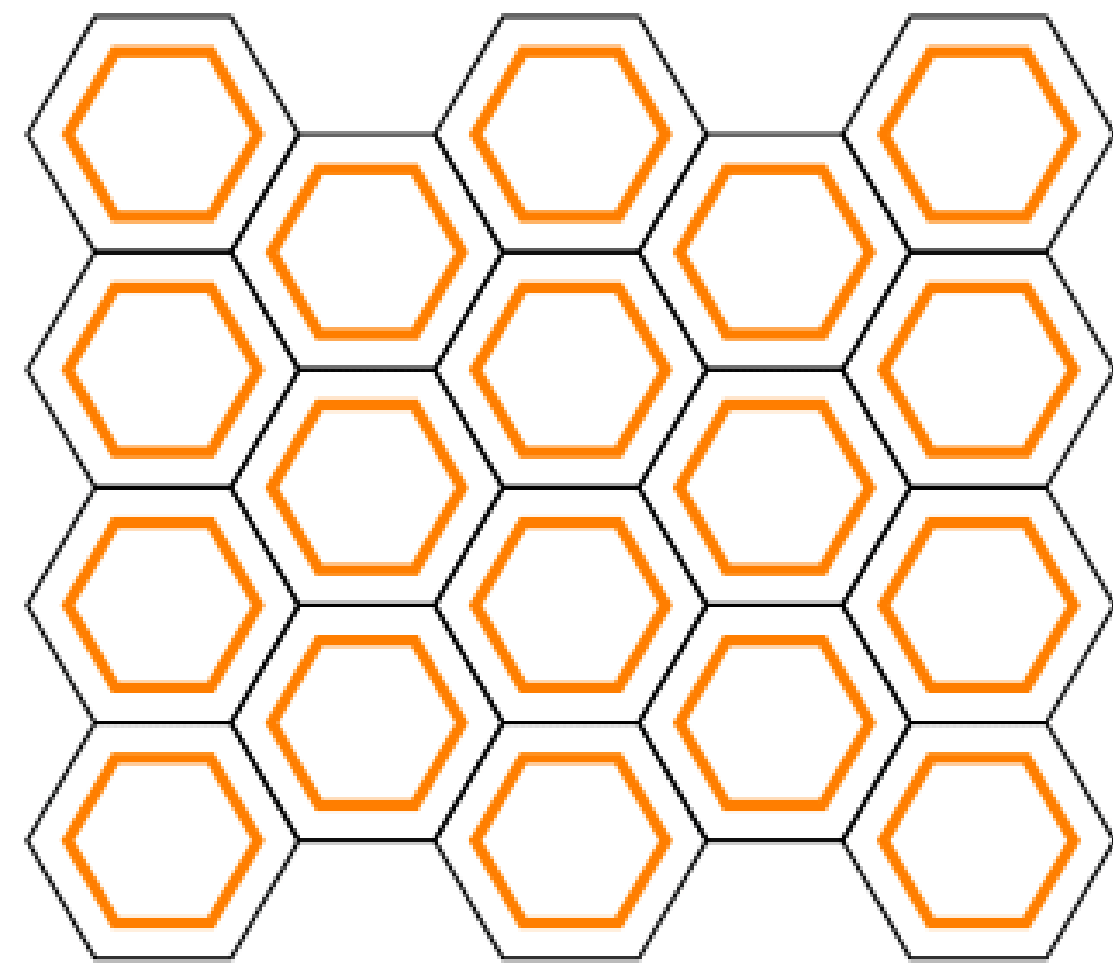
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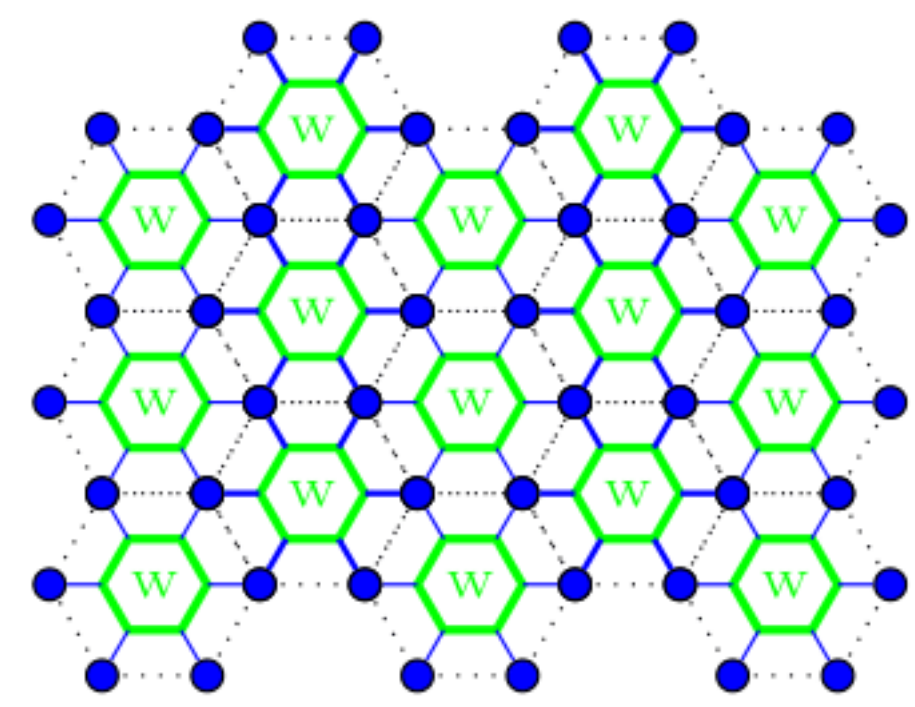
## Honeycomb Lattice Proposed Wavefunction



$$|\psi\rangle = \prod_{\mathbf{O}} \left( \sum_{i \in \mathbf{O}} b_i^\dagger \right) |\mathbf{0}\rangle$$

## PEPS Construction of Honeycomb F.B.I.

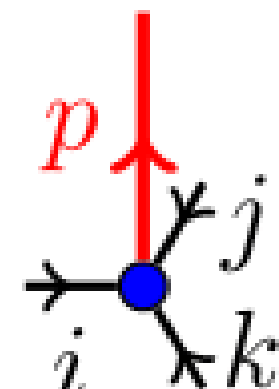
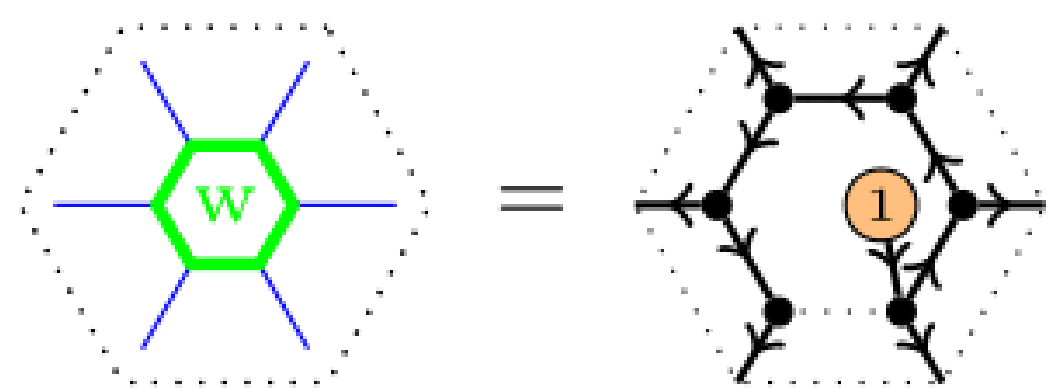
A wavefunction written as a product of local operators acting on a product state can be turned into a tensor network.



Virtual W-state on each plaquette used to synchronize the creation operators in the sum  $\sum_{i \in \mathbf{O}} b_i^\dagger$

$$|W\rangle = |000001\rangle + |000010\rangle + |000100\rangle + |001000\rangle + |010000\rangle + |100000\rangle$$

## PEPS Construction of Honeycomb FBI

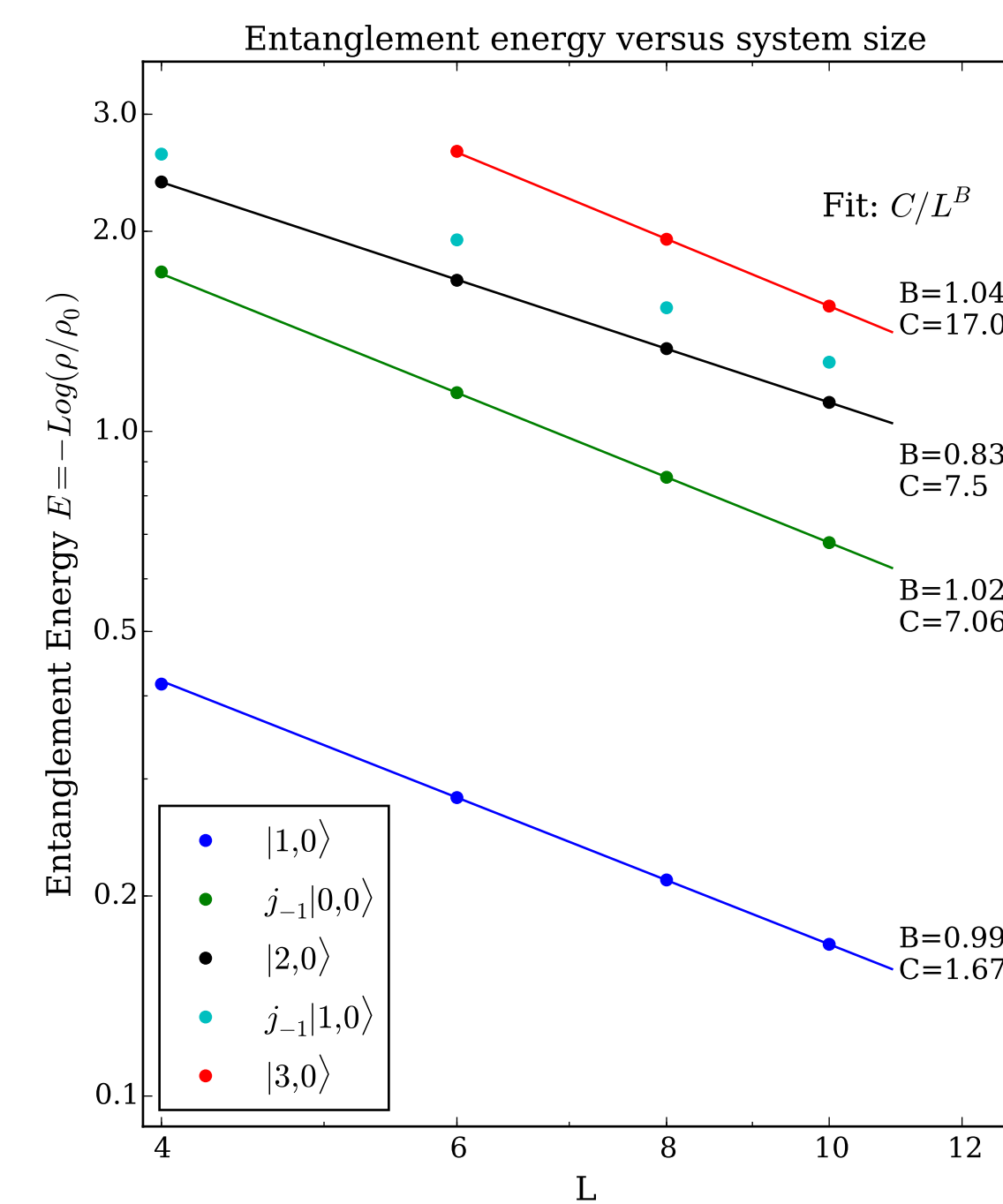


$$|W\rangle = |100\dots\rangle + \dots$$

- W-State can be factored and put on hexagon sites
- Each black directed string has either charge 0 or 1
- Charge conserved

- 'Projects' the three virtual qubits coming into each vertex on to a state in physical Hilbert space
- Physical state is either  $|0\rangle, |1\rangle, |2\rangle, |3\rangle$
- Charge conserved

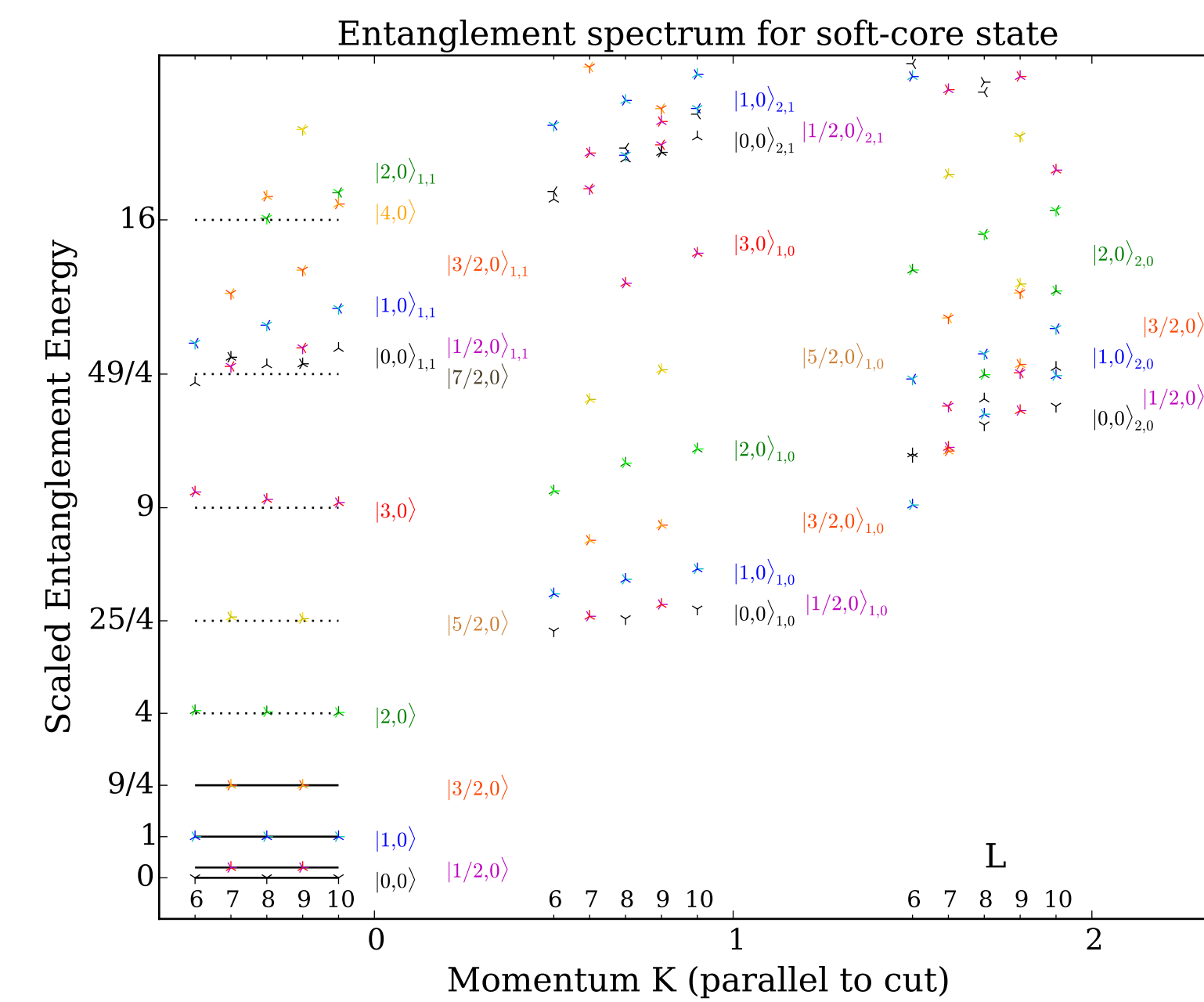
## Finite Size Analysis of Entanglement Spectra



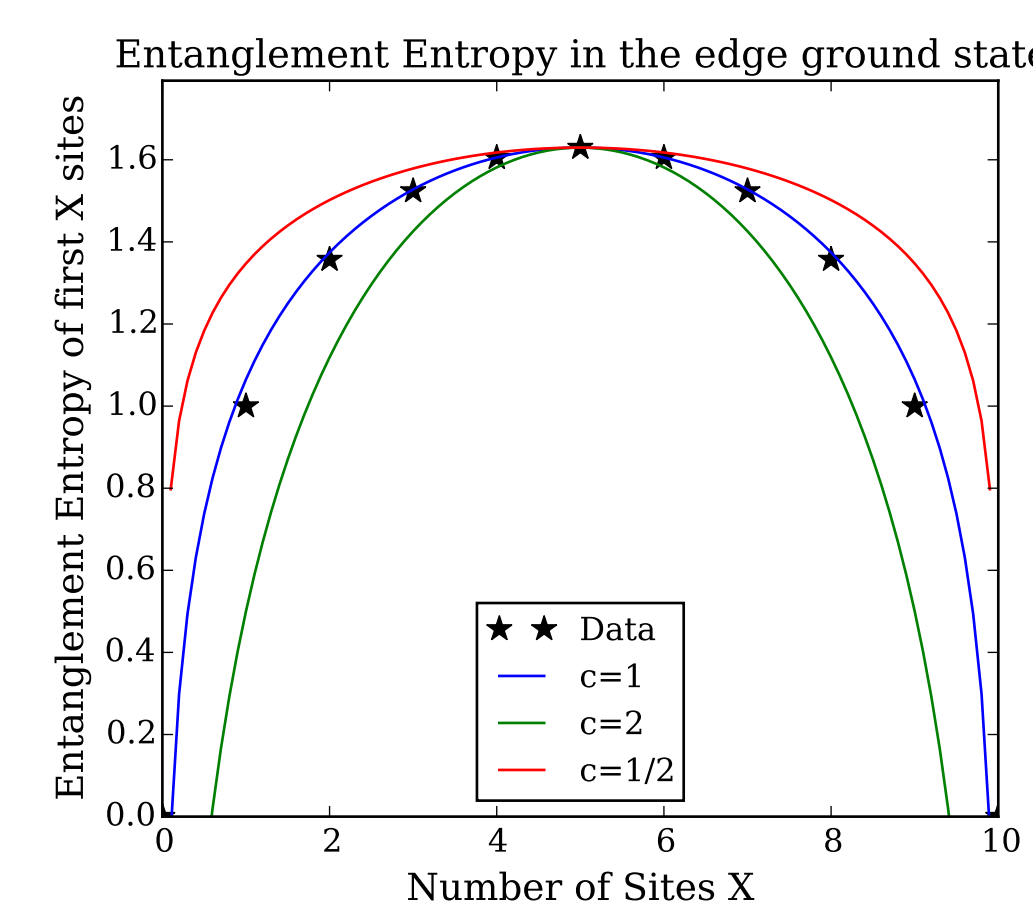
- Fix this to show topological entanglement entropy is 0

- Low energy modes show gapless  $1/L$  behavior

## CFT Identification of Gapless Entanglement Edge



## Conformal Charge



$$c = 1$$

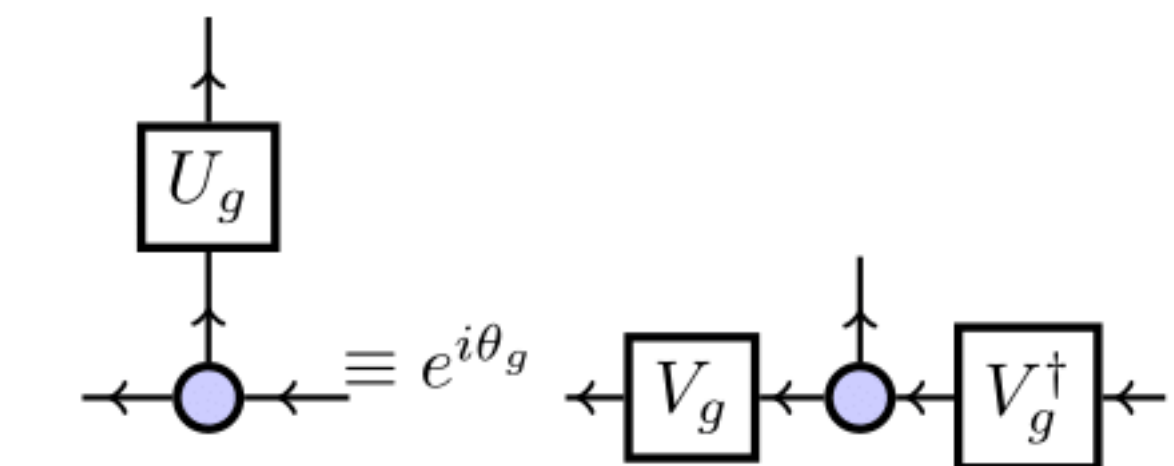
## Conformal Weights

We can match the rescaled entanglement energies to the conformal weights of a free bosonic CFT.

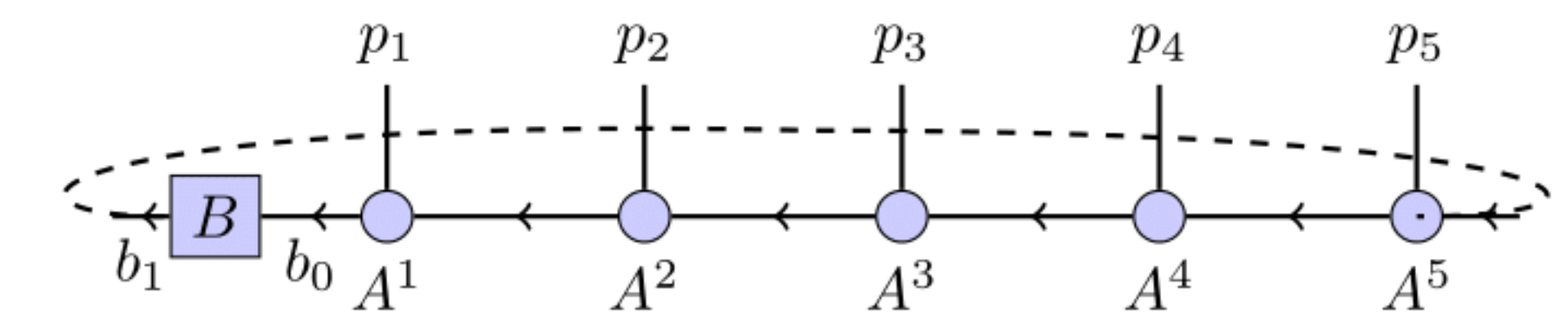
$$\begin{aligned} \mathbf{P} &= \frac{2\pi}{L}(\mathbf{L}_0 - \bar{\mathbf{L}}_0) \\ &= \frac{2\pi}{L}(em + n - \bar{n}) \\ \tilde{\mathbf{P}} &= em + n - \bar{n} \\ \mathbf{H} &= \frac{2\pi}{L}(\mathbf{L}_0 + \bar{\mathbf{L}}_0) \\ &= \frac{2\pi}{L}\left(\frac{\kappa e^2}{2} + \frac{m^2}{2\kappa} + \frac{n + \bar{n}}{2}\right) \\ \tilde{\mathbf{H}} &= \frac{L}{2\pi\kappa}\mathbf{H} \\ &= e^2 + \frac{m^2}{4\kappa^2} + \frac{1}{\kappa}(n + \bar{n}) \end{aligned}$$

## Detecting 1D SPT Order

If  $U_g$  is a global symmetry and  $|\psi\rangle$  is translationally invariant, then the MPS representation satisfies:



Boundary conditions on a MPS can be represented by a matrix  $B$  which acts like:



With PBC ( $B = I$ ), the group action leaves the state invariant. With OBC ( $B = |i\rangle\langle i|$ ), the group action rotates between states that differ only near the boundary; these edge states transform as  $V_g \otimes V_g^\dagger$ .  $V_g$  represents the group projectively. Equivalence classes of projective representations (enumerated by  $H^2(G; U(1))$ ) classify 1D SPT phases.

## Symmetry Protection of the Honeycomb FBI

For the state on a cylinder with odd circumference, and the zig-zag entanglement cut defined in the upper left picture, we have the following:

G	$U_g$	$\theta_g$	$V_g$	$V_g V_g^*$
$U(1)$				
$\pi$				
$\mathcal{I}$				
$\pi\mathcal{I}$				

Since

$$V_{\pi\mathcal{I}} V_{\pi\mathcal{I}}^* = -I \quad \text{or} \quad V_{\pi} V_{\mathcal{I}} = -V_{\mathcal{I}} V_{\pi},$$

the representation is in the nontrivial class of

$$H^2(\mathbb{Z}_2 \times \mathbb{Z}_2^{\mathcal{I}}; U(1)) = \mathbb{Z}_2.$$

## Relation to known 1D physics

- Haldane insulator
  - Unitarily equivalent to the AKLT state
  - Distinct phase under  $\mathbb{Z}_2 \times \mathbb{Z}_2^{\mathcal{I}}$
  - Can be connected adiabatically to  $L = 1$  cylinder FBI
- Two dimensional classification is  $H^3(\mathbb{Z}_2 \times \mathbb{Z}_2^{\mathcal{I}}; U(1)) = \mathbb{Z}_2^4$

## References

- D. W. Kribs, R. Laflamme, D. Poulin, M. Lesosky, Quantum Inf. & Comp. **6** (2006), 383-399.
- P. Zanardi, M. Rasetti, Phys. Rev. Lett. **79**, 3306 (1997).