Entanglement in Featureless Mott Insulators

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March 6th 2014

Featureless Insulators

Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state: $E_1 - E_0 \geq const.$

Alternate Definition

- Unique ground state on all boundary-less systems
- Possibly with 'features' localized to edge of system

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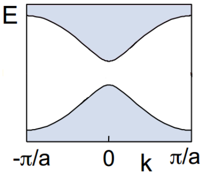
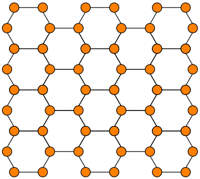
- Unique ground state:
  $E_1 - E_0 \geq const.$
- Gapless modes:
  $E_1 - E_0 \sim \frac{1}{L^z}$
- Spontaneous symmetry breaking:
  $E_1 - E_0 = 0$
- Topological order:
  $E_1 - E_0 \sim e^{-L/\xi}$

Examples of Featureless Insulators

Classical Insulators

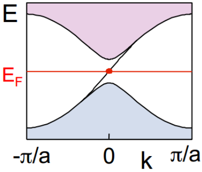


Free fermion band insulator

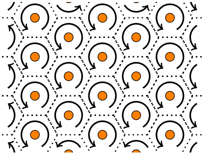


Atomic picture

Topological Insulators



Band insulator with chiral edge



Atomic picture breaks down

Obstructions to Featurelessness

Fundamental Result

A featureless insulator must have an integer charge per unit cell

- (Lieb, Schultz, Mattis 1961)
- (Hastings 2004)

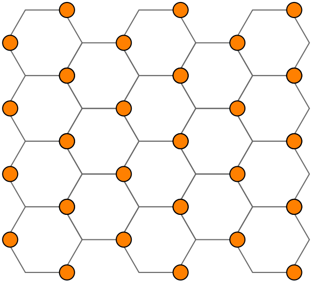
For certain lattices, not all integers are possible

- (Parameswaran 2013)

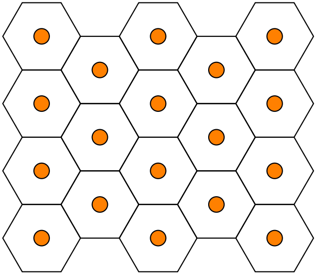
For this talk, we will look at a proposed honeycomb lattice featureless insulator with charge 1 per unit cell.

Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



Breaks rotational symmetry



Leaves honeycomb lattice

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by **?**

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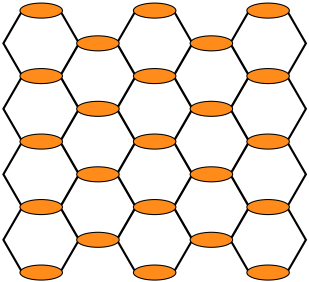

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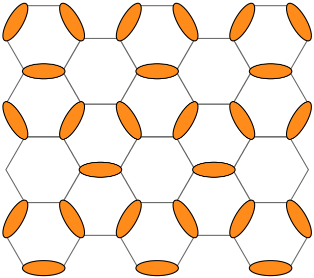

Breaks translationally symmetry, unit cell is 3 times larger

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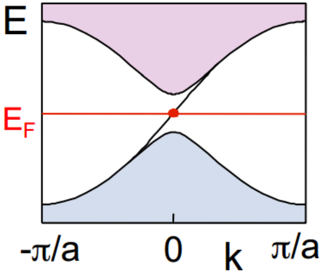
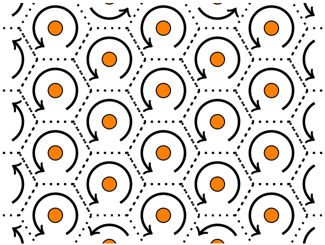


Band insulator with chiral edge [1]



The Haldane Chern insulator is NOT an example. D_6 explicitly broken.

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by **?**

[1] (**?**)

Honeycomb Bosonic Mott Insulators

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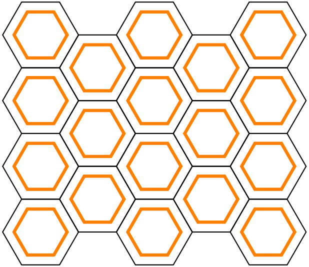
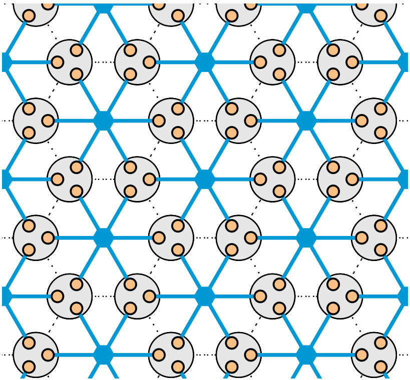


$$|\psi\rangle = \prod_{\hexagon} \left(\sum_{i \in \hexagon} b_i^\dagger \right) |\mathbf{0}\rangle$$

Proposed Solution by **?**

Bosons filled into non-orthogonal, plaquette centered orbitals works. Numerics confirm the expected wavefunction properties, but no known parent Hamiltonian has been found.

Computations on Honeycomb FBI



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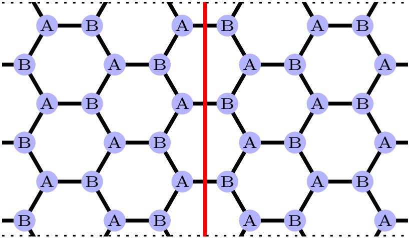
Simple tensor network representation

Cylinder slice treated as single site of an effective 1D system.

Schmidt decomposition computed as in 1D matrix product states.

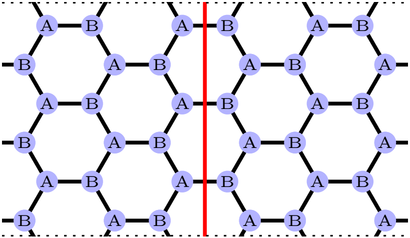
Computations on Honeycomb FBI



Form of a honeycomb lattice PEPS on zig-zag cylinder with width L=3

- Simple tensor network representation
- Cylinder slice treated as single site of an effective 1D system.
- Schmidt decomposition computed as in 1D matrix product states.

Computations on Honeycomb FBI



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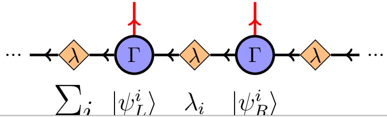
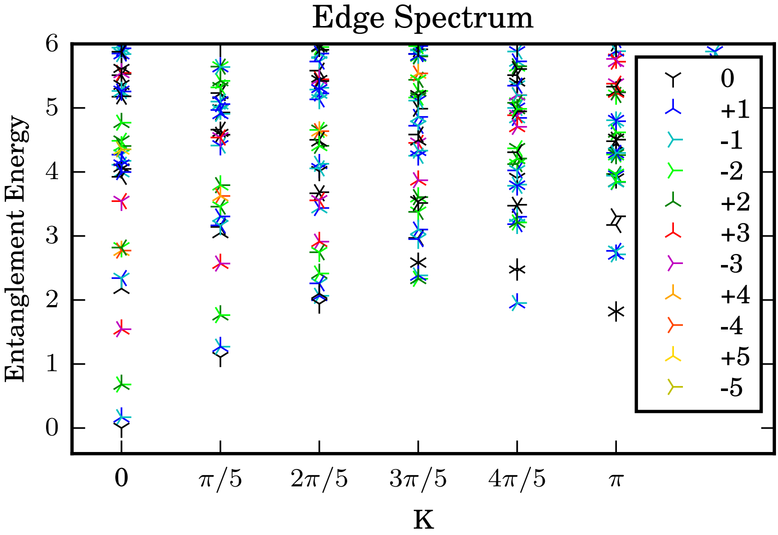
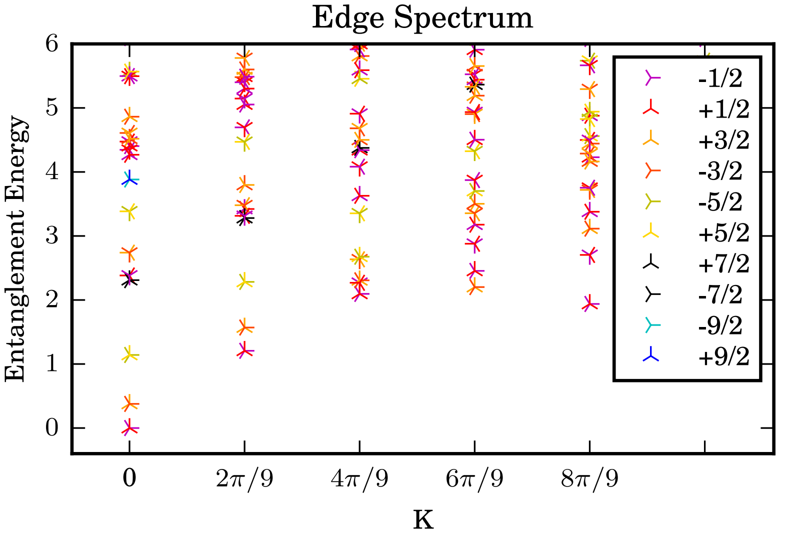


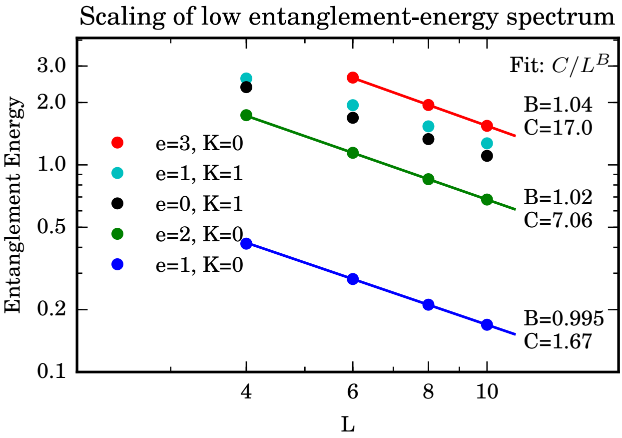
Entanglement Spectrum



Entanglement Spectrum

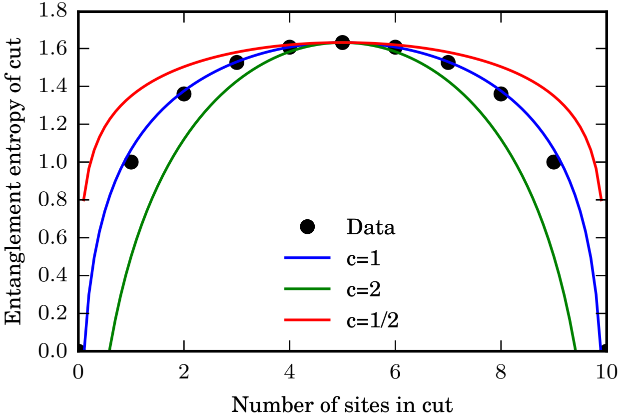


Finite Size Analysis



Identification of Edge CFT

Conformal Charge via 'Nested Entanglement Entropy'



$$\boxed{c = 1}$$

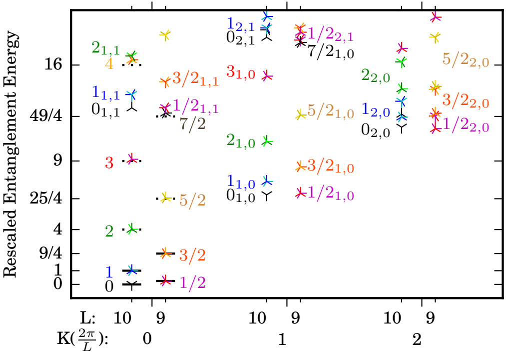
Identification of Edge CFT

Conformal Weights

We can match the rescaled entanglement energies to the conformal weights of a free bosonic CFT.

$$\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n+\bar{n})$$

Conformal primary identification in entanglement spectra



Symmetry Protection of Degenerate Edge

$$|\psi\rangle = \sum_i \lambda_i |\psi_L^i\rangle |\psi_R^i\rangle$$

Inversion symmetry $\mathcal{I}$ induces an edge antiunitary action $V_{\mathcal{I}}$

This occurs in two steps:

- $|e, K\rangle_L \to |e, -K\rangle_R$
- $|e, K\rangle_R \to |-e, -K\rangle_L$

Combined:

$$V_{\mathcal{I}}|e, K\rangle \propto |-e, K\rangle$$

Phases work out like this:

$$V_{\mathcal{I}} \sim \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Charge symmetry θ induces an edge unitary action V_θ

For charge parity $\pi \in U(1)$:

$$V_\pi |e, K\rangle = (-1)^e |e, K\rangle$$

$$V_\pi \sim \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Combined antiunitary action $V_{\mathcal{I}\pi}$ satsfies

$$V_{\mathcal{I}\pi} V_{\mathcal{I}\pi}^* = -1$$

Conclusions

For the honeycomb featureless boson insulator:

- Entanglement spectrum reveals a gapless free boson edge
- Edge spectrum points with nonzero charge or nonzero momentum are degenerate
- This degeneracy is protected by combined inversion and charge parity

- Cannot be deformed to trivial state while the bosons are not allowed to live at the hexagon centers
- The representation of the lattice and charge symmetry (size of unit cell and charge per unit cell) matters for classifying featureless insulators

Questions?

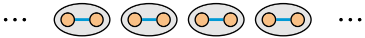
Brayden Ware
brayden@physics.ucsb.edu

Bonus slides

Resources

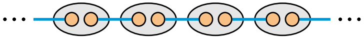
Construction of 1D Featureless Insulators

Classical Insulators

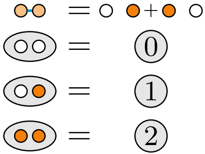


1D Trivial Chain

Topological Insulators



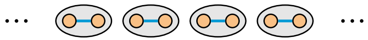
1D Topological Chain



Projectors and entangled pairs (PEPS) used in state construction

Construction of 1D Featureless Insulators

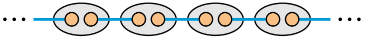
Classical Insulators



1D Trivial Chain

Product state with one boson per site

Topological Insulators



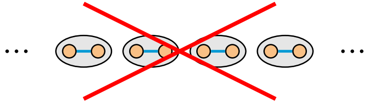
1D Topological Chain

Haldane Insulator Phase **?**

- Unitarily related to AKLT
- No $SU(2)$ symmetry
- Symmetry protected 2-fold edge degeneracy

Construction of 1D Featureless Insulators

Classical Insulators



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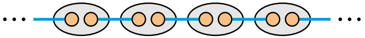

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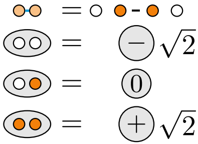
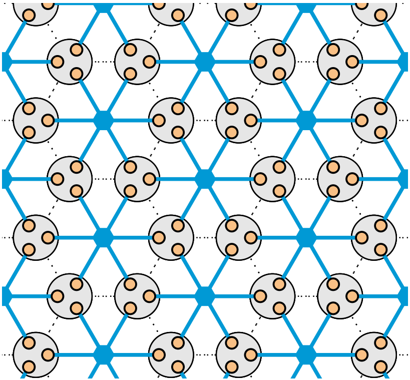
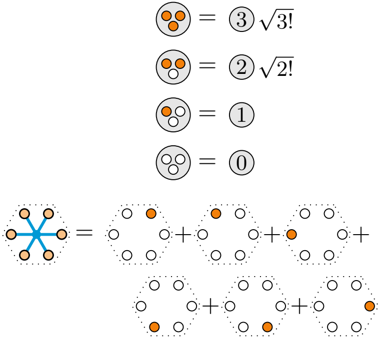


Projectors and entangled pairs (PEPS) for $SU(2)$ symmetric state

Tensor Network cut details





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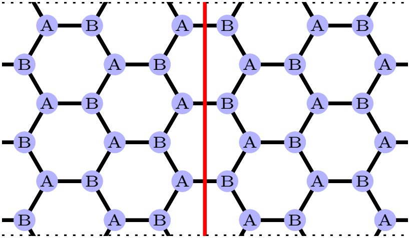
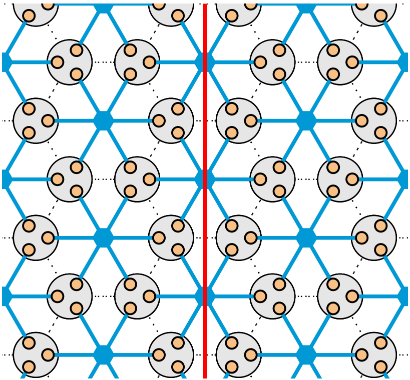


Generic honeycomb lattice PEPS on zig-zag cylinder with L=3

In cylindrical geometry:

- Treat state as 1D
- Use MPS techniques
- On-site translational symmetry parallel to cut
- Physical site dimension 4^{2L}

Tensor Network cut details



Honeycomb lattice tensor network on zig-zag cylinder with L=3

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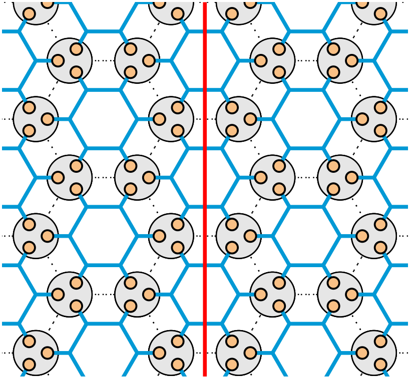
Tensor Network cut details



Honeycomb lattice PEPS on zig-zag cylinder with L=3, acheived by factoring W-state of plaquette bosons

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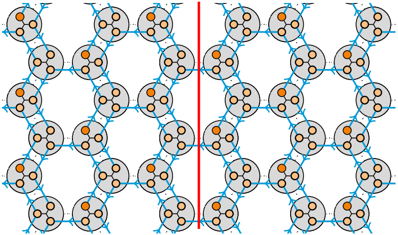
- MPS bond dimension = Rank of $\rho_r = 2^L$
- Entanglement spectrum $\{\epsilon_i\}$ defined from eigenvalues $\{\rho_i\}$ of ρ_r via $\epsilon_i = e^{-\rho_i}$
- Charge and Translation represented linearly on edge

Tensor Network cut details



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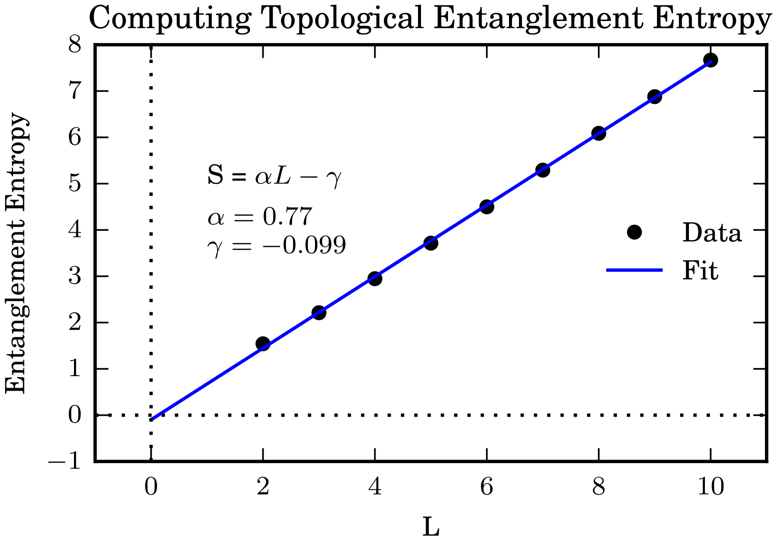
Topological Entanglement Entropy



Known Results for Honeycomb FBI

Correlations

$$< b_i^\dagger b_j >$$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length $\xi/a \sim 3.6$

$$< n_i n_j >$$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length $\xi/a \sim 1.6$

Hamiltonian Construction

Try filling plaquette orbitals

$$b_{\hexagon} = \sum_{i \in \hexagon} \frac{1}{\sqrt{6}} b_i^\dagger$$

$$H = \sum_{\hexagon} -\frac{t}{6} b_{\hexagon}^\dagger b_{\hexagon} + V n_{\hexagon} n_{\hexagon}$$

$$= \left(\sum_{\hexagon} \sum_{i,j \in \hexagon} -t b_i^\dagger b_j \right) - \frac{3t}{6} N + V \ldots$$

- Fails, gapless modes
- Parent Hamiltonian not known

Known Results for Honeycomb FBI

Correlations

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Hamiltonian Construction

To get a parent Hamiltonian:

- Need symmetric, exponentially localized, orthogonal orbitals
- Such as the Wannier orbitals of a classical band insulator
- **?**

Other lattices:

- Need a fixed point of all lattice symmetries
- Fails on nonsymmorphic lattices
- Extension of LSM theorem
- **?**

Future Work

Entanglement properties with different geometries

- Armchair cylinder edge
- Finite size clusters
- Explain results for arbitrary geometries with tensor network properties, e.g. 'MPO injectivity'

Find a 2D local Hamiltonian and confirm with numerics

$$H_{EBH} = \left(\sum_{\hexagon} \sum_{i,j \in \hexagon} -t b_i^\dagger b_j + V n_i n_j \right) + \mu N?$$

Physical properties of the phase

Can we constructan SU(2) symmetric FI?