

Entanglement in Featureless Mott Insulators

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Outline

- 1 Motivation
- 2 Construction of Honeycomb FBI
- 3 Entanglement Edge of Honeycomb FBI
- 4 Symmetry Protection of Edge

Motivation

Featureless insulators

Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state:

$$E_1 - E_0 \geq \text{const.}$$

Alternate Definition

- Unique ground state on any boundary-less system
- Possibly with 'features' localized to edge of system

Fundamental Result

- Integer charge per unit cell
 - (Lieb, Schultz, Mattis)

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- Gapless modes:

$$E_1 - E_0 \sim \frac{1}{L^z}$$

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Featureless insulators

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- Unique ground state:
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- Spontaneous symmetry breaking:
 $E_1 - E_0 = 0$

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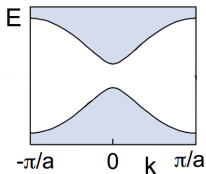
- Topological order:
 $E_1 - E_0 \sim e^{-L/\xi}$
with nontrivial topology

Fundamental Result

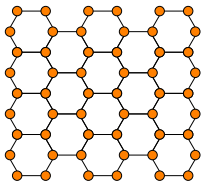
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Free Fermion Featureless Insulators

Classical Insulators

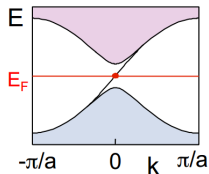


Free fermion band insulator

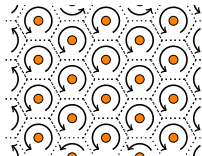


Atomic picture

Topological Insulators



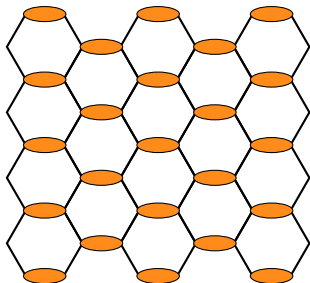
Band insulator with chiral edge ¹



Atomic picture breaks down

Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?

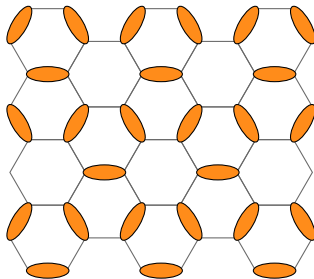


Breaks rotational symmetry

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013)

Honeycomb Bosonic Mott Insulators

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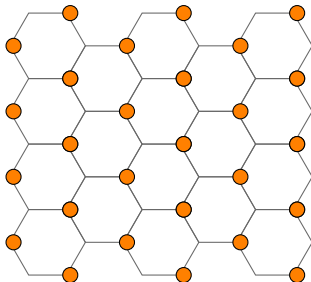


Breaks translationally symmetry, unit cell is 3 times larger

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013)

Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



Breaks point group symmetry D_6 to D_3

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013)

Construction of Honeycomb FBI

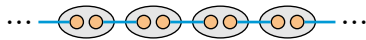
Construction of 1D Featureless Insulators

Classical Insulators



1D Trivial Chain

Topological Insulators



1D Topological Chain

$$\begin{aligned}
 \text{orange dot} - \text{blue line} - \text{orange dot} &= \text{white circle} + \text{orange dot} + \text{orange dot} + \text{white circle} \\
 \text{gray oval with two white circles} &= 0 \\
 \text{gray oval with one white circle and one orange dot} &= 1 \\
 \text{gray oval with two orange dots} &= 2
 \end{aligned}$$

Entangled pairs and projectors used in state construction

Construction of 1D Featureless Insulators

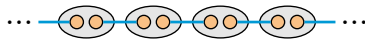
Classical Insulators



1D Trivial Chain

Product state with one boson per site

Topological Insulators

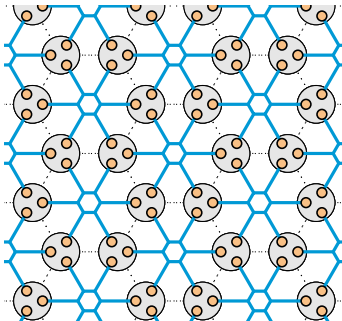


1D Topological Chain

Haldane Insulator Phase
Pollmann et al. (2010)

- Unitarily related to AKLT
- No $SU(2)$ symmetry
- Symmetry protected 2-fold edge degeneracy

Construction of Honeycomb FBI

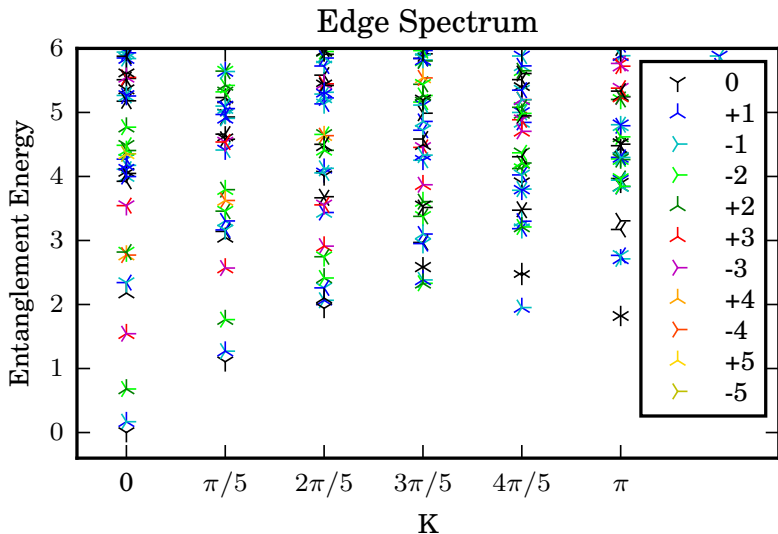


$$|\psi\rangle = \prod_{\hexagon} \left(\sum_{i \in \hexagon} b_i^\dagger \right) |0\rangle$$

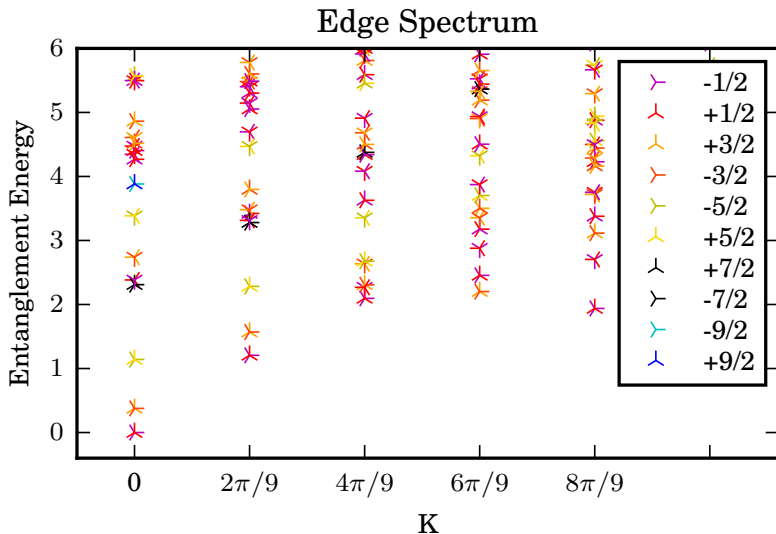
Entanglement Edge of Honeycomb FBI

Edge Geometry

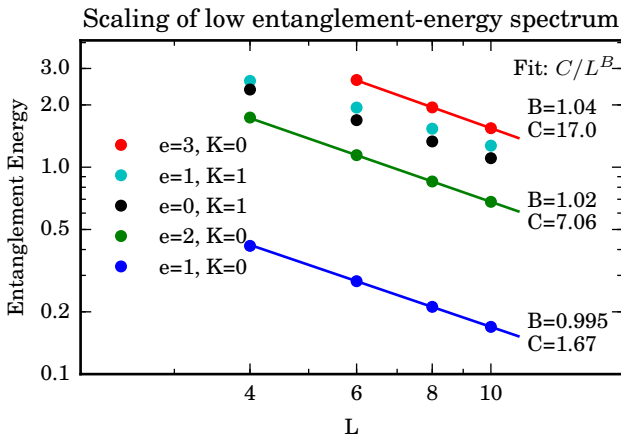
Entanglement Spectrum



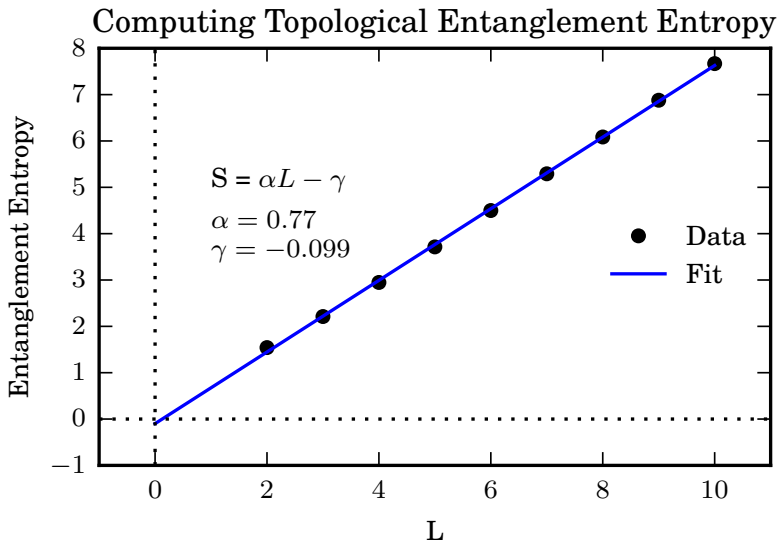
Entanglement Spectrum



Finite Size Analysis

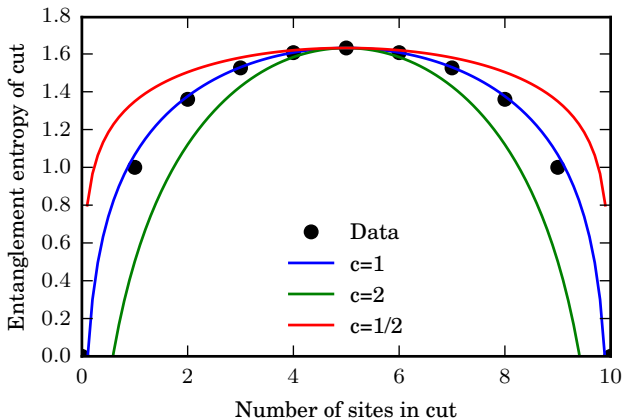


Finite Size Analysis



Identification of Edge CFT

Conformal Charge



$$c = 1$$

Identification of Edge CFT

Conformal Weights

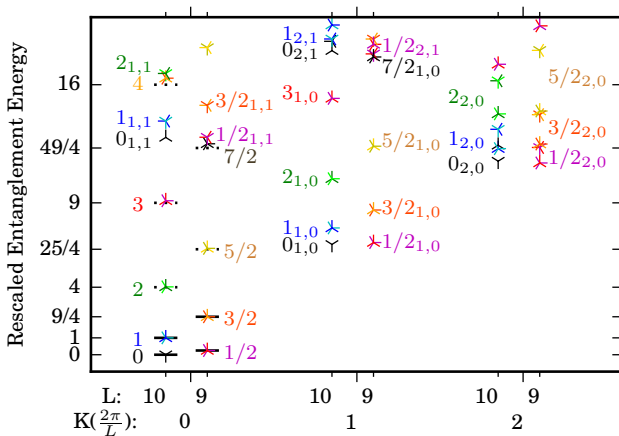
We can match the rescaled entanglement energies to the conformal weights of a free bosonic CFT.

$$\begin{aligned}\mathbf{P} &= \frac{2\pi}{L}(\mathbf{L}_0 - \bar{\mathbf{L}}_0) = \frac{2\pi}{L}(em + n - \bar{n}) \\ \mathbf{H} &= \frac{2\pi}{L}(\mathbf{L}_0 + \bar{\mathbf{L}}_0) = \frac{2\pi}{L}\left(\frac{\kappa e^2}{2} + \frac{m^2}{2\kappa} + \frac{n + \bar{n}}{2}\right)\end{aligned}$$

$$\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})$$

Identification of Edge CFT

Conformal primary identification in entanglement spectra



Symmetry Protection of Edge

Symmetry Protection of Degenerate Edge

Future Work

Entanglement properties with different geometries

- Armchair cylinder edge
- Finite size clusters
- Explain results for arbitrary geometries with tensor network properties, e.g. 'MPO injectivity'

Find a 2D local Hamiltonian and confirm with numerics

$$H_{EBH} = \left(\sum_{\hexagon} \sum_{i,j \in \hexagon} -tb_i^\dagger b_j + V n_i n_j \right) + \mu N?$$

Physical properties of the phase

Can we construct an SU(2) symmetric FI?

Resources

- Hasan, M. Z. and Kane, C. L. (2010). *Colloquium: Topological insulators. Reviews of modern physics*, 82(4):3045–3067.
- Parameswaran, S. A., Kimchi, I., Turner, A. M., Stamper-Kurn, D. M., and Vishwanath, A. (2013). Wannier permanent wave functions for featureless bosonic mott insulators on the $1/3$ -filled kagome lattice. *Phys. Rev. Lett.*, 110:125301.
- Pollmann, F., Turner, A. M., Berg, E., and Oshikawa, M. (2010). Entanglement spectrum of a topological phase in one dimension. *Phys. Rev. B*, 81(6):064439.

Questions?

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Bonus slides