

# Entanglement in Featureless Mott Insulators

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# Outline

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- 1 Motivation
- 2 Construction of Honeycomb FBI
- 3 Entanglement Edge of Honeycomb FBI
- 4 Symmetry Protection of Edge

# Motivation

# Featureless insulators

## Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state:

$$E_1 - E_0 \geq \text{const.}$$

## Alternate Definition

- Unique ground state on any boundary-less system
- Possibly with 'features' localized to edge of system

## Fundamental Result

- Integer charge per unit cell
  - (Lieb, Schultz, Mattis)

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- Gapless modes:

$$E_1 - E_0 \sim \frac{1}{L^\nu}$$

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- Unique ground state:

$$E_1 - E_0 \geq \text{const.}$$

- Spontaneous symmetry breaking:

$$E_1 - E_0 = 0$$

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- Topological order:  
 $E_1 - E_0 \sim e^{-L/\xi}$   
with nontrivial topology

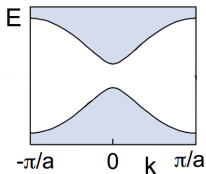
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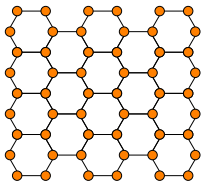


# Free Fermion Featureless Insulators

## Classical Insulators

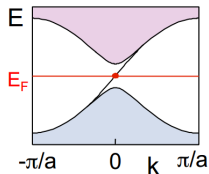


Free fermion band insulator

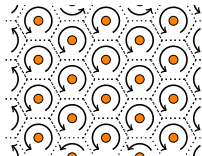


Atomic picture

## Topological Insulators



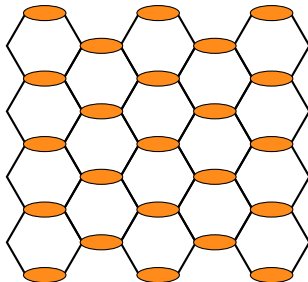
Band insulator with chiral edge <sup>1</sup>



Atomic picture breaks down

# Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?

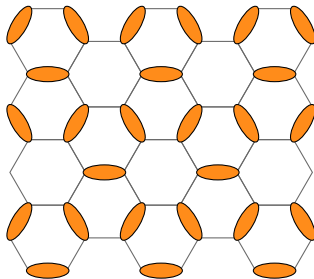


Breaks rotational symmetry

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013)

# Honeycomb Bosonic Mott Insulators

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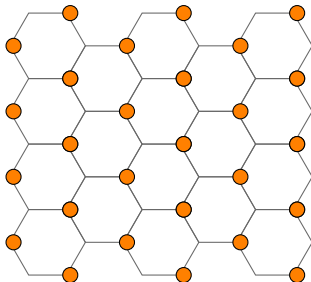


Breaks translationally symmetry, unit cell is 3 times larger

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013)

# Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



Breaks point group symmetry  $D_6$  to  $D_3$

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013)

# Construction of Honeycomb FBI

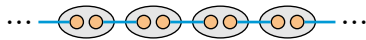
# Construction of 1D Featureless Insulators

## Classical Insulators



1D Trivial Chain

## Topological Insulators



1D Topological Chain

$$\begin{aligned}
 \text{orange dot} - \text{blue line} - \text{orange dot} &= \text{white circle} + \text{orange dot} + \text{orange dot} + \text{white circle} \\
 \text{gray oval with two white circles} &= 0 \\
 \text{gray oval with one white circle and one orange dot} &= 1 \\
 \text{gray oval with two orange dots} &= 2
 \end{aligned}$$

Entangled pairs and projectors used in state construction

# Construction of Honeycomb FBI

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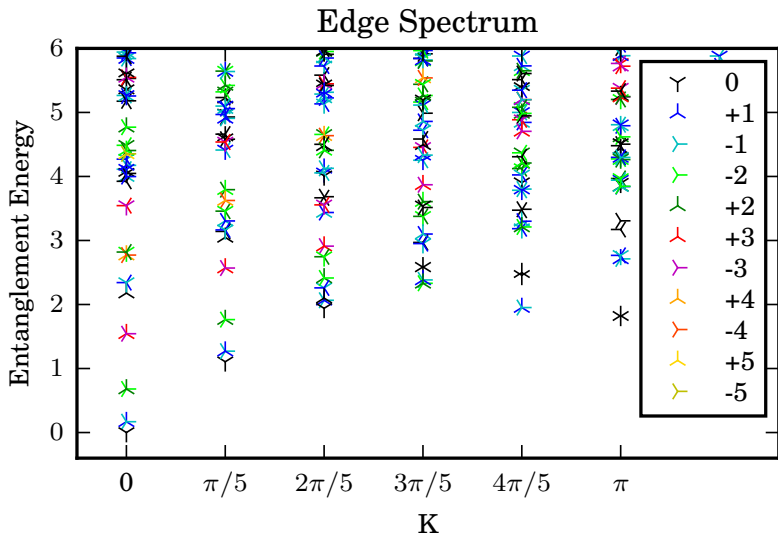
# Entanglement Edge of Honeycomb FBI



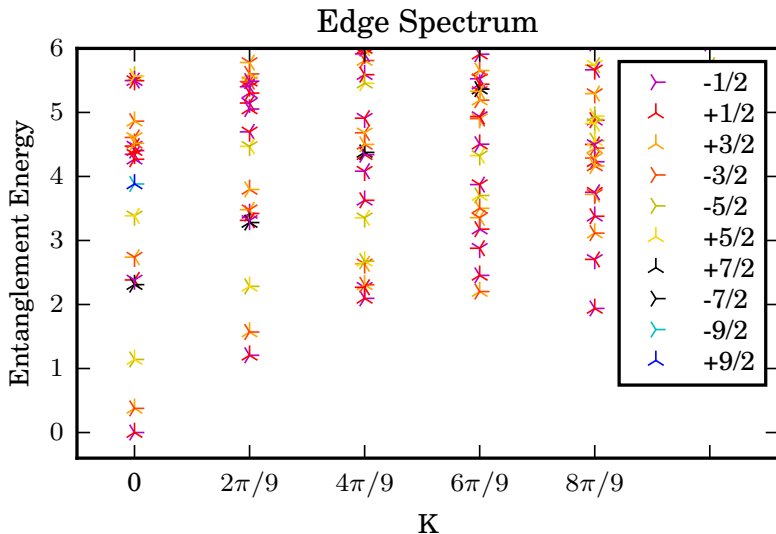
# Edge Geometry

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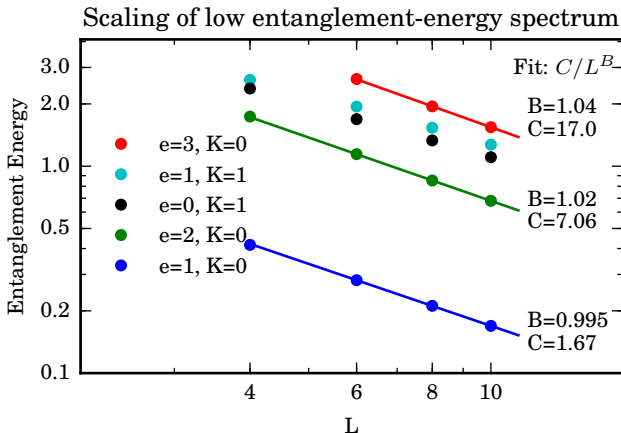
# Entanglement Spectrum



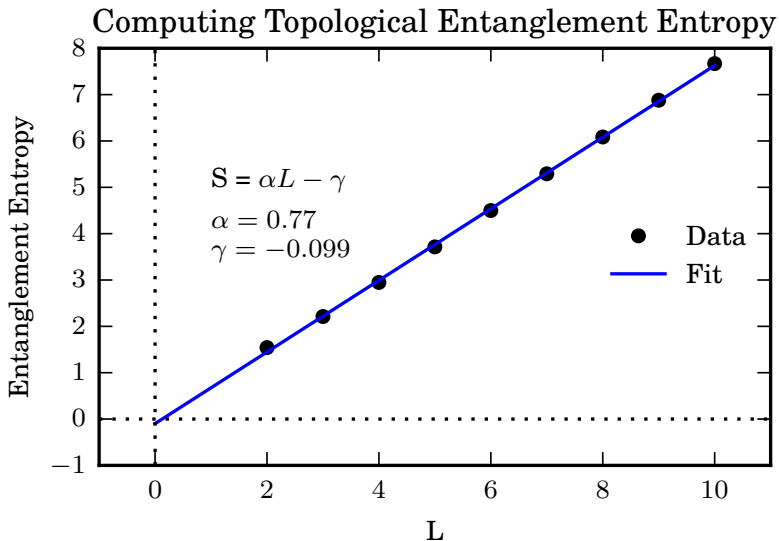
# Entanglement Spectrum



# Finite Size Analysis

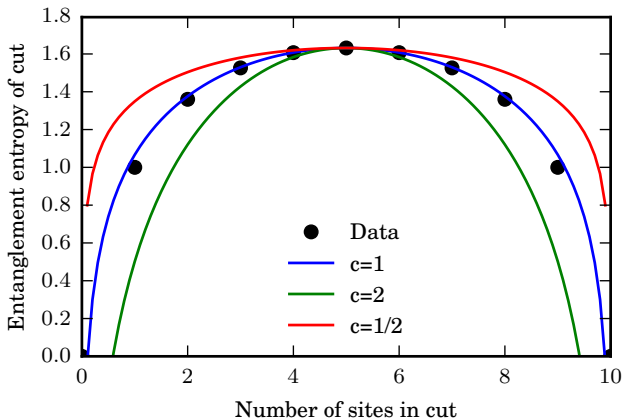


# Finite Size Analysis



# Identification of Edge CFT

## Conformal Charge



$$c = 1$$

# Identification of Edge CFT

## Conformal Weights

We can match the rescaled entanglement energies to the conformal weights of a free bosonic CFT.

$$\begin{aligned}\mathbf{P} &= \frac{2\pi}{L}(\mathbf{L}_0 - \bar{\mathbf{L}}_0) = \frac{2\pi}{L}(em + n - \bar{n}) \\ \mathbf{H} &= \frac{2\pi}{L}(\mathbf{L}_0 + \bar{\mathbf{L}}_0) = \frac{2\pi}{L}\left(\frac{\kappa e^2}{2} + \frac{m^2}{2\kappa} + \frac{n + \bar{n}}{2}\right)\end{aligned}$$

$$\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})$$

## Conformal primary identification in entanglement spectra



# Symmetry Protection of Edge

# Symmetry Protection of Degenerate Edge

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# Future Work

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- Entanglement properties in different geometries
  - Cylinders with different edges
  - Finite size clusters
- Relation to 'MPO Injectivity'
- 
- Numerical testing of parent Hamiltonians
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# Resources

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- Hasan, M. Z. and Kane, C. L. (2010). *Colloquium: Topological insulators. Reviews of modern physics*, 82(4):3045–3067.
- Parameswaran, S. A., Kimchi, I., Turner, A. M., Stamper-Kurn, D. M., and Vishwanath, A. (2013). Wannier permanent wave functions for featureless bosonic mott insulators on the  $1/3$ -filled kagome lattice. *Phys. Rev. Lett.*, 110:125301.

# Questions?

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# Bonus slides