Entanglement in Featureless Mott Insulators

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Featureless Insulators

Definition of 'Featureless Insulator'

- Gapped
- Symmetric
- No topological order

- Unique ground state: $E_1 - E_0 \geq const.$

Alternate Definition

- Unique ground state on all boundary-less systems
- Possibly with 'features' localized to edge of system

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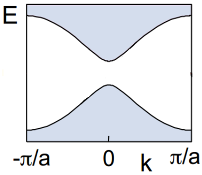
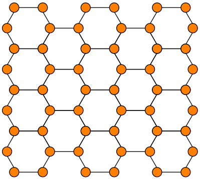
- Unique ground state: $E_1 - E_0 \geq const.$
- Gapless modes: $E_1 - E_0 \sim \frac{1}{L^z}$
- Spontaneous symmetry breaking: $E_1 - E_0 = 0$
- Topological order: $E_1 - E_0 \sim e^{-L/\xi}$

Examples of Featureless Insulators

Classical Insulators

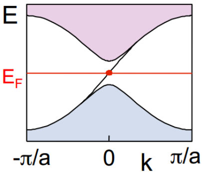


Free fermion band insulator

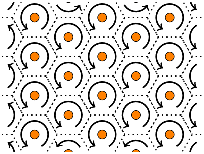


Atomic picture

Topological Insulators



Band insulator with chiral edge



Atomic picture breaks down

Obstructions to Featurelessness

Fundamental Result

A featureless insulator must have an integer charge per unit cell

- (Lieb, Schultz, Mattis 1961)
- (Hastings 2004)

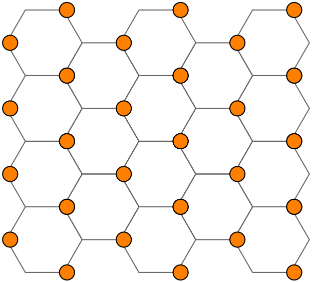
For certain lattices, not all integers are possible

- (Parameswaran 2013)

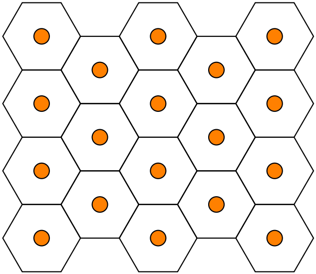
For this talk, we will look at a proposed honeycomb lattice featureless insulator with charge 1 per unit cell.

Honeycomb Bosonic Mott Insulators

Does there exist a featureless bosonic insulator with charge 1 per unit cell on the honeycomb lattice?



Breaks rotational symmetry



Leaves honeycomb lattice

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013a)

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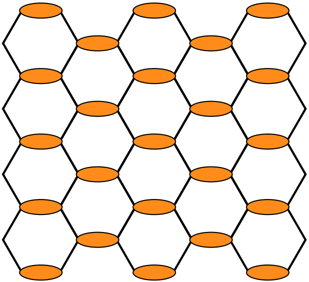

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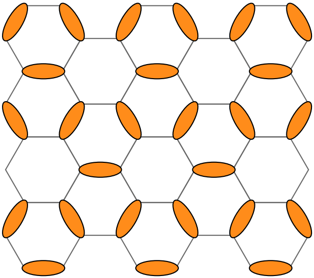

Breaks translationally symmetry, unit cell is 3 times larger

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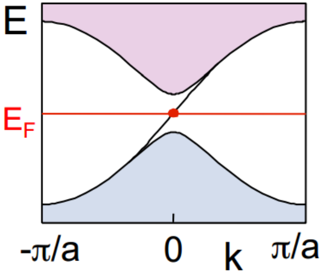
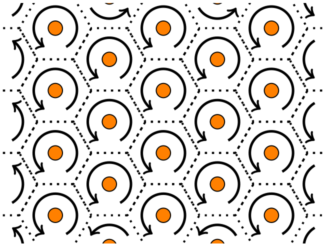


Band insulator with chiral edge [1]



The Haldane Chern insulator is NOT an example. D_6 explicitly broken.

'Classical cartoons and usual tricks' lead to symmetry breaking, as noticed by Parameswaran et al. (2013a)

[1](Hasan and Kane, 2010)

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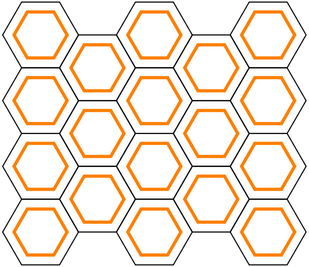
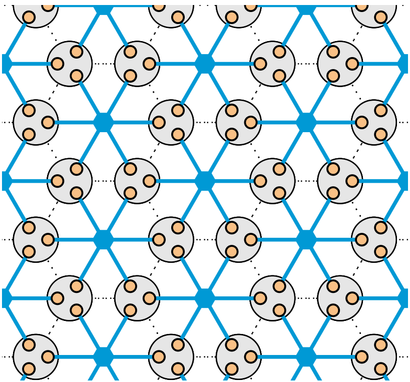


$$|\psi\rangle = \prod_{\hexagon} \left(\sum_{i \in \hexagon} b_i^{\dagger} \right) |\mathbf{0}\rangle$$

Proposed Solution by Kimchi et al. (2013)

Bosons filled into non-orthogonal, plaquette centered orbitals works. Numerics confirm the expected wavefunction properties, but no known parent Hamiltonian has been found.

Computations on Honeycomb FBI



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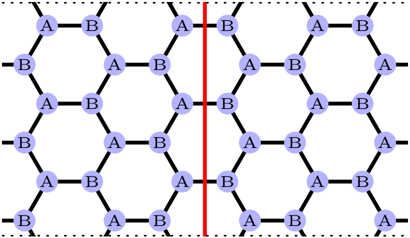
Simple tensor network representation

Cylinder slice treated as single site of an effective 1D system.

Schmidt decomposition computed as in 1D matrix product states.

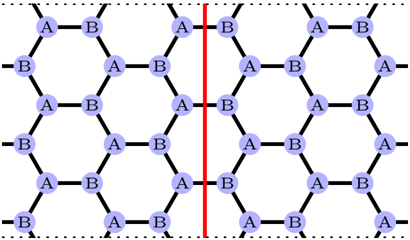
Computations on Honeycomb FBI



Form of a honeycomb lattice PEPS on zig-zag cylinder with width L=3

- Simple tensor network representation
- Cylinder slice treated as single site of an effective 1D system.
- Schmidt decomposition computed as in 1D matrix product states.

Computations on Honeycomb FBI



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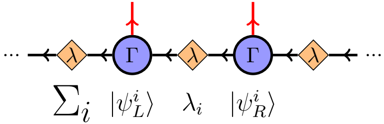
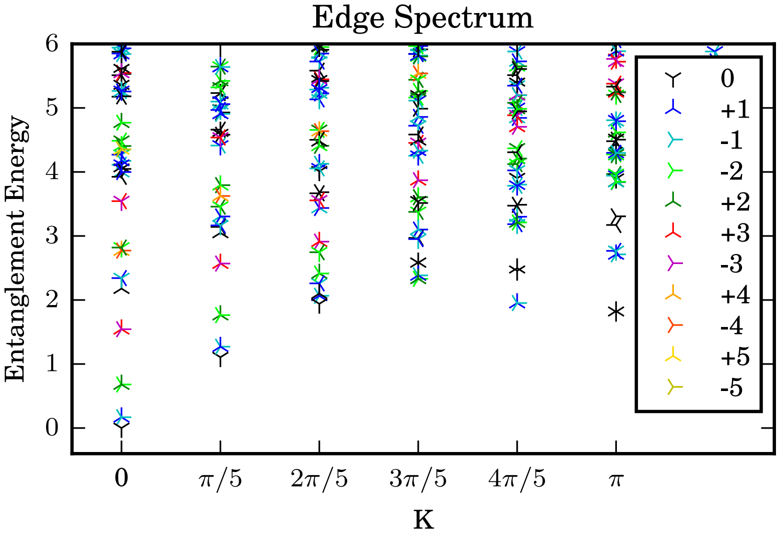


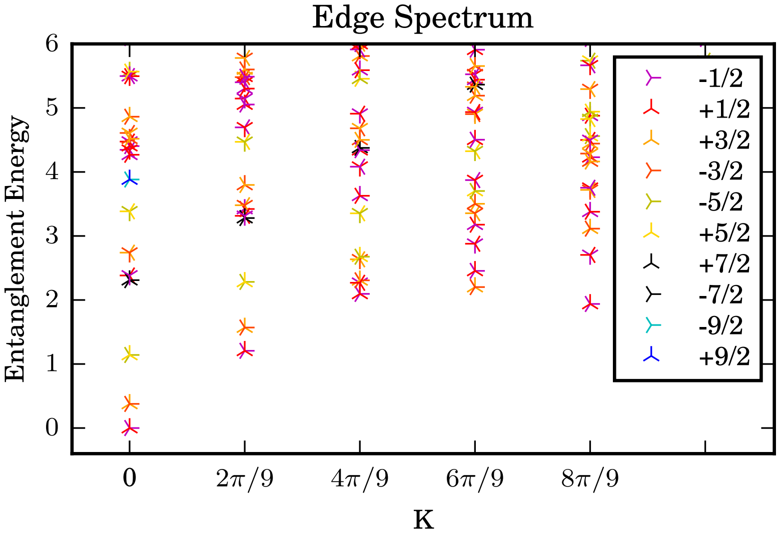
Matrix Product state canonical form

- Simple tensor network representation
- Cylinder slice treated as single site of an effective 1D system.
- Schmidt decomposition computed as in 1D matrix product states.

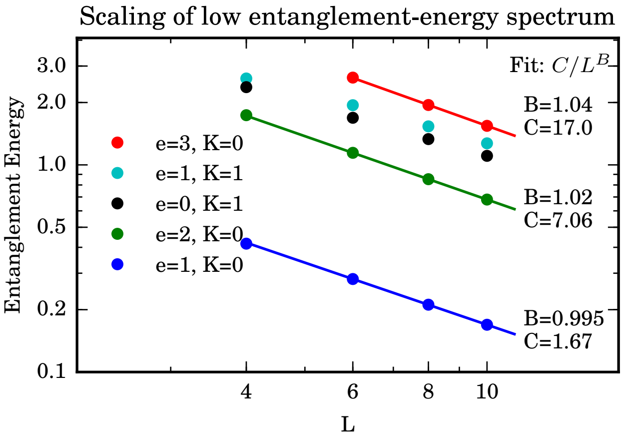
Entanglement Spectrum



Entanglement Spectrum

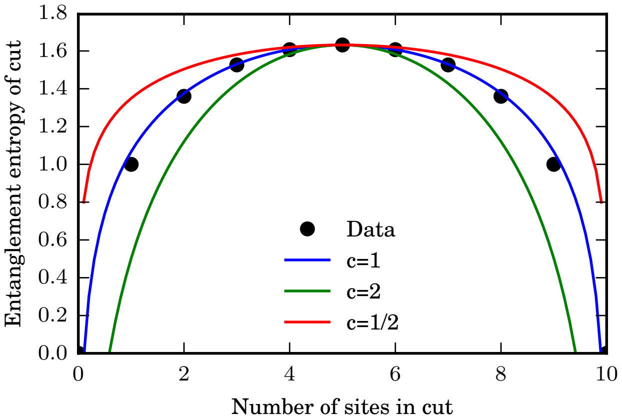


Finite Size Analysis



Identification of Edge CFT

Conformal Charge via 'Nested Entanglement Entropy'



$$\boxed{c = 1}$$

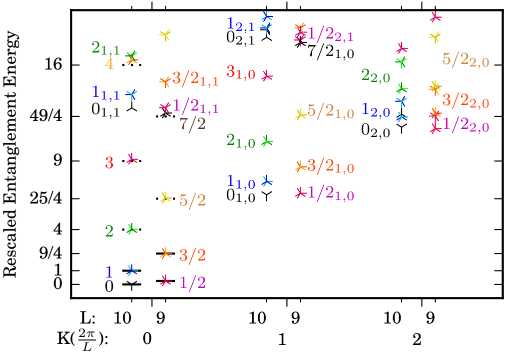
Identification of Edge CFT

Conformal Weights

We can match the rescaled entanglement energies to the conformal weights of a free bosonic CFT.

$$\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})$$

Conformal primary identification in entanglement spectra



Symmetry Protection of Degenerate Edge

$$|\psi\rangle = \sum_i \lambda_i |\psi_L^i\rangle |\psi_R^i\rangle$$

Inversion symmetry $\mathcal{I}$ induces an edge antiunitary action $V_{\mathcal{I}}$

This occurs in two steps:

- $|e, K\rangle_L \to |e, -K\rangle_R$
- $|e, K\rangle_R \to |-e, -K\rangle_L$

Combined:

$$V_{\mathcal{I}}|e, K\rangle \propto |-e, K\rangle$$

Phases work out like this:

$$V_{\mathcal{I}} \sim \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Charge symmetry θ induces an edge unitary action V_θ

For charge parity $\pi \in U(1)$:

$$V_\pi |e, K\rangle = (-1)^e |e, K\rangle$$

$$V_\pi \sim \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Combined antiunitary action $V_{\mathcal{I}\pi}$ satsfies

$$V_{\mathcal{I}\pi} V_{\mathcal{I}\pi}^* = -1$$

Conclusions

For the honeycomb featureless boson insulator:

- Entanglement spectrum reveals a gapless free boson edge
- Edge spectrum points with nonzero charge or nonzero momentum are degenerate
- This degeneracy is protected by combined inversion and charge parity

- Cannot be deformed to trivial state while the bosons are not allowed to live at the hexagon centers
- The representation of the lattice and charge symmetry (size of unit cell and charge per unit cell) matters for classifying featureless insulators

Questions?

Brayden Ware

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Bonus slides

Resources

Hasan, M. Z. and Kane, C. L. (2010). *Colloquium*: Topological insulators. *Reviews of modern physics*, 82(4):3045–3067.

Kimchi, I., Parameswaran, S. A., Turner, A. M., Wang, F., and Vishwanath, A. (2013). Featureless and nonfractionalized mott insulators on the honeycomb lattice at $1/2$ site filling. *Proceedings of the National Academy of Sciences*, 110(41):16378–16383.

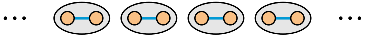
Parameswaran, S. A., Kimchi, I., Turner, A. M., Stamper-Kurn, D. M., and Vishwanath, A. (2013a). Wannier permanent wave functions for featureless bosonic mott insulators on the $1/3$-filled kagome lattice. *Phys. Rev. Lett.*, 110:125301.

Parameswaran, S. A., Turner, A. M., Arovas, D. P., and Vishwanath, A. (2013b). Topological order and absence of band insulators at integer filling in non-symmorphic crystals. *Nature Physics*, 9(5):–303?

Pollmann, F., Turner, A. M., Berg, E., and Oshikawa, M. (2010). Entanglement spectrum of a topological phase in one

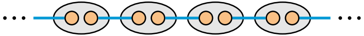
Construction of 1D Featureless Insulators

Classical Insulators



1D Trivial Chain

Topological Insulators



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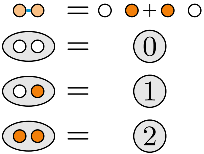

Projectors and entangled pairs (PEPS) used in state construction

Construction of 1D Featureless Insulators

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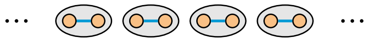
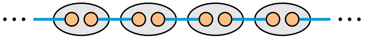


1D Trivial Chain

Product state with one boson per site

Topological Insulators



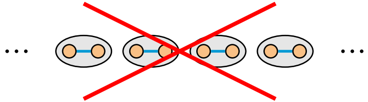
1D Topological Chain

Haldane Insulator Phase
Pollmann et al. (2010)

- Unitarily related to AKLT
- No $SU(2)$ symmetry
- Symmetry protected 2-fold edge degeneracy

Construction of 1D Featureless Insulators

Classical Insulators



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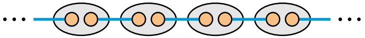

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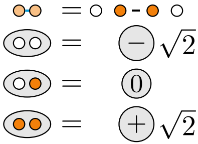
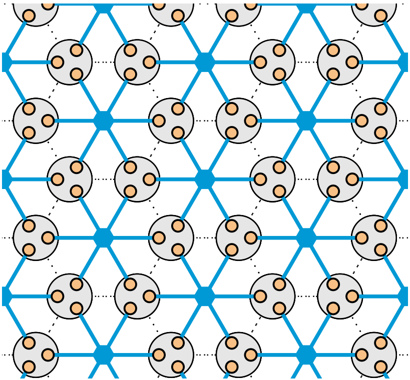
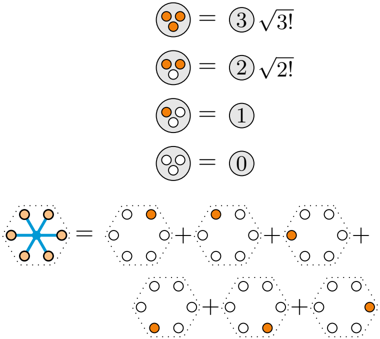


Projectors and entangled pairs (PEPS) for $SU(2)$ symmetric state

Tensor Network cut details





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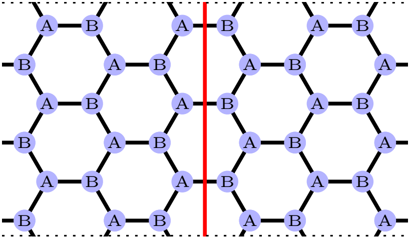
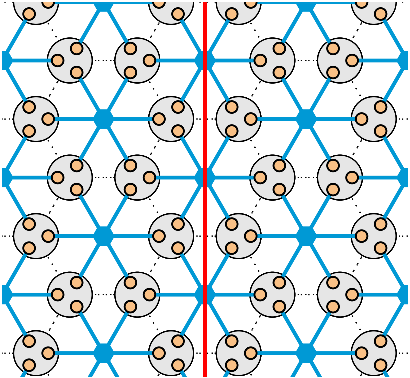


Generic honeycomb lattice PEPS on zig-zag cylinder with L=3

In cylindrical geometry:

- Treat state as 1D
- Use MPS techniques
- On-site translational symmetry parallel to cut
- Physical site dimension 4^{2L}

Tensor Network cut details



Honeycomb lattice tensor network on zig-zag cylinder with L=3

In cylindrical geometry:

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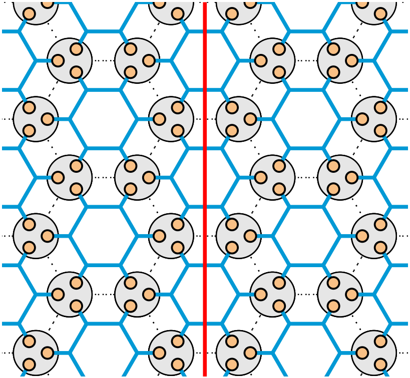
- Physical site dimension 4^{2L}

Tensor Network cut details



Honeycomb lattice PEPS on zig-zag cylinder with L=3, acheived by factoring W-state of plaquette bosons

In cylindrical geometry:

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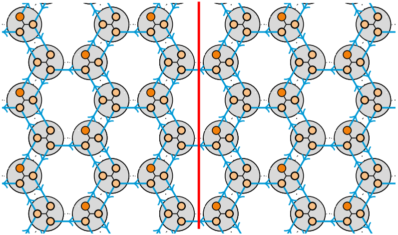
- Physical site dimension 4^{2L}
- MPS bond dimension = Rank of $\rho_r = 2^L$
- Entanglement spectrum $\{\epsilon_i\}$ defined from eigenvalues $\{\rho_i\}$ of ρ_r via $\epsilon_i = e^{-\rho_i}$
- Charge and Translation represented linearly on edge

Tensor Network cut details



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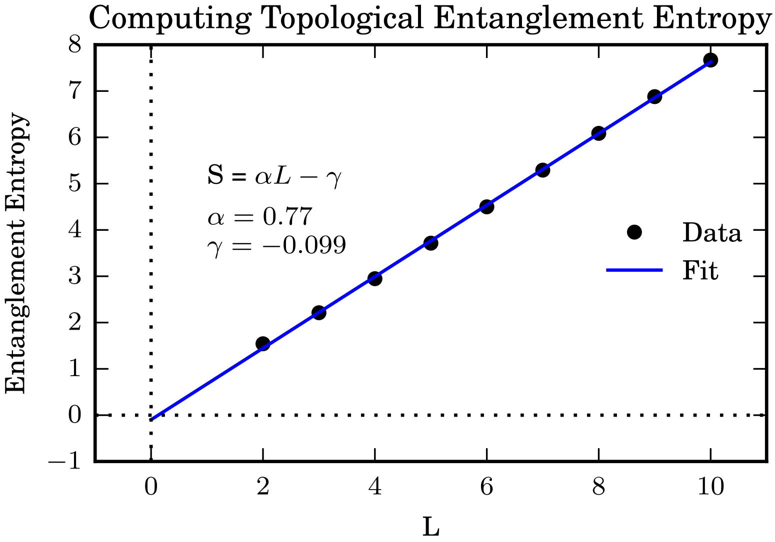
Topological Entanglement Entropy



Known Results for Honeycomb FBI

Correlations

$< b_i^\dagger b_j >$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length $\xi/a \sim 3.6$

$< n_i n_j >$

- Looks rotationally symmetric
- Decays exponentially
- Correlation length $\xi/a \sim 1.6$

Hamiltonian Construction

Try filling plaquette orbitals

$$b_{\hexagon} = \sum_{i \in \hexagon} \frac{1}{\sqrt{6}} b_i^\dagger$$

$$H = \sum_{\hexagon} -\frac{t}{6} b_{\hexagon}^\dagger b_{\hexagon} + V n_{\hexagon} n_{\hexagon}$$

$$= \left(\sum_{\hexagon} \sum_{i,j \in \hexagon} -t b_i^\dagger b_j \right) - \frac{3t}{6} N + V \ldots$$

- Fails, gapless modes
- Parent Hamiltonian not known

Known Results for Honeycomb FBI

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Hamiltonian Construction

To get a parent Hamiltonian:

- Need symmetric, exponentially localized, orthogonal orbitals
- Such as the Wannier orbitals of a classical band insulator
- Parameswaran et al. (2013a)

Other lattices:

- Need a fixed point of all lattice symmetries
- Fails on nonsymmorphic lattices
- Extension of LSM theorem
- Parameswaran et al. (2013b)

Future Work

Entanglement properties with different geometries

- Armchair cylinder edge
- Finite size clusters
- Explain results for arbitrary geometries with tensor network properties, e.g. 'MPO injectivity'

Find a 2D local Hamiltonian and confirm with numerics

$$H_{EBH} = \left(\sum_{\hexagon} \sum_{i,j \in \hexagon} -t b_i^\dagger b_j + V n_i n_j \right) + \mu N?$$

Physical properties of the phase

Can we constructan SU(2) symmetric FI?