Not so featureless after all: symmetry protected order in an interacting boson state

Brayden Ware
Department of Physics, University of California, Santa Barbara, CA, 93106-6105, USA

Itamar Kimchi
Department of Physics, University of California, Berkeley, CA 94720, USA

S. A. Parameswaran
Department of Physics and Astronomy, University of California, Irvine, CA 92697, USA

Bela Bauer
Station Q, Microsoft Research, Santa Barbara, CA 93106-6105, USA

While the Lieb-Schultz-Mattis theorem forbids the existence of fully symmetric quantum paramagnetic phases on lattices with fractional filling of particles per unit cell, such a phase is in principle allowed with certain fractional numbers of particles per site on non-Bravais lattices, including half-filling on the honeycomb lattice. It has been shown that a non-interacting Hamiltonian of spinless fermions or bosons cannot have such a symmetric insulating ground state, and an explicit construction using interactions is challenging. Recently, Kimchi et al. constructed a wavefunction for bosons at half-filling that does not break any symmetries and is not topologically ordered–and in this sense is a featureless insulator in the bulk. Here, however, we reveal that this wavefunction exhibits non-trivial structure at the edge. We apply recently developed techniques based on a tensor network representation of the wavefunction to demonstrate the presence of a gapless entanglement spectrum and a non-trivial action of combined charge-conservation and spatial symmetries on the edge. We will also discuss the possibility of finding a parent Hamiltonian and analyzing the existence of a symmetry-protected topological phase around this state.

I. INTRODUCTION

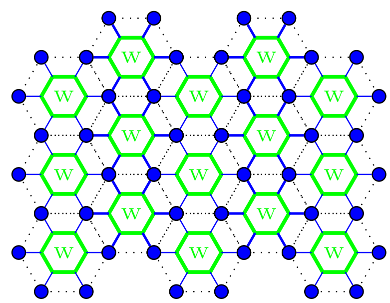
...[1]

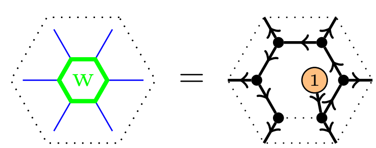
II. F.B.I. WAVEFUNCTION

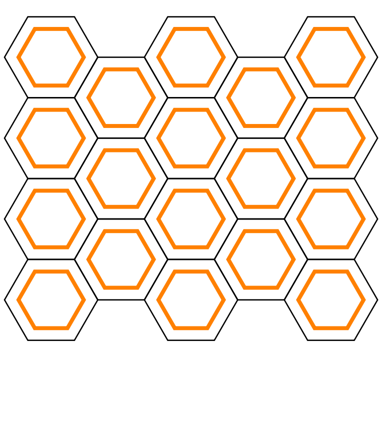
It was argued by Kimchi, et. al.[2] that this state represents a featureless Mott insulating phase of bosons on the honeycomb lattice.

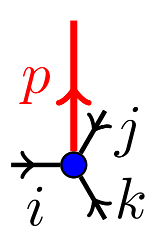
$$|\psi\rangle = \prod_R \sum_{i\in R} b_i^\dagger |0\rangle \tag{1}$$

III. PEPS CONSTRUCTION OF HONEYCOMB F.B.I.

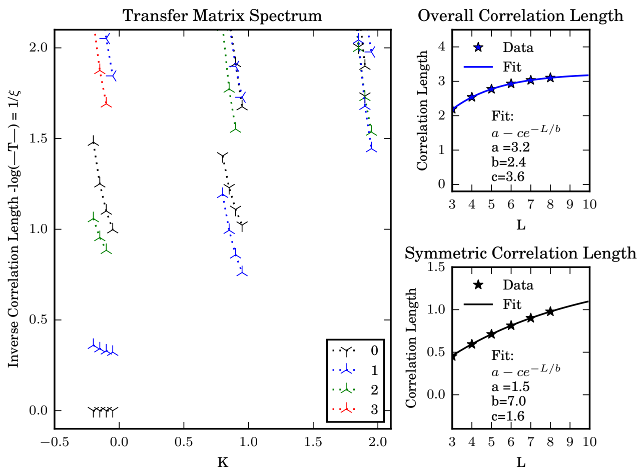








IV. FEATURELESS CORRELATIONS



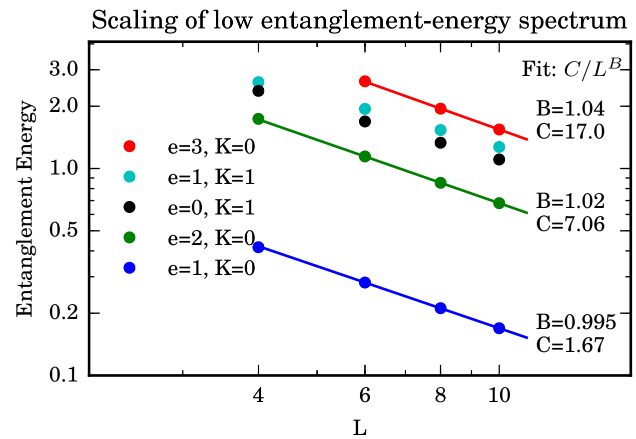


Figure 2. Power law fits for the lowest five states above the ground state in Figure 1. The $1/L$ scaling is a signature of a gapless (entanglement) Hamiltonian.

V. ENTANGLEMENT SPECTRUM

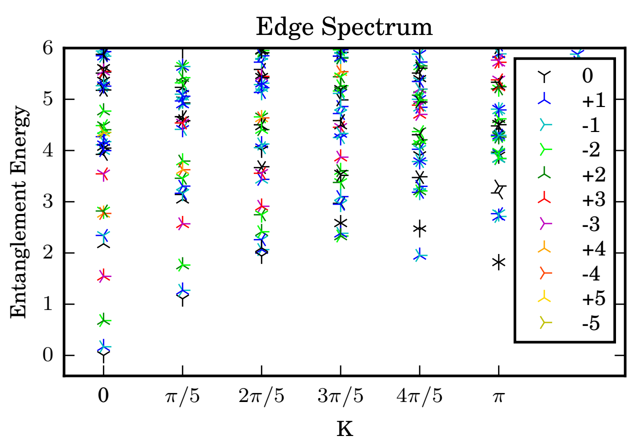
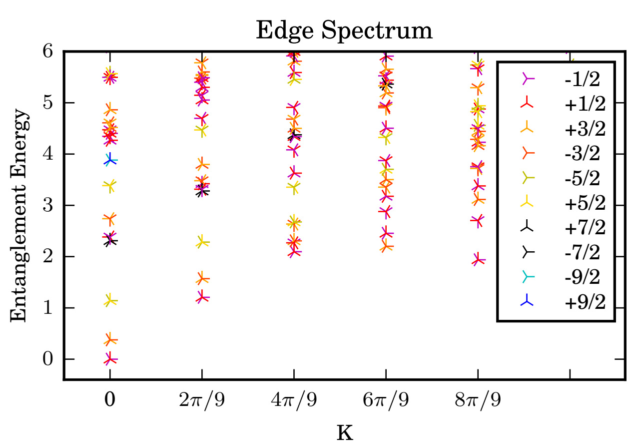
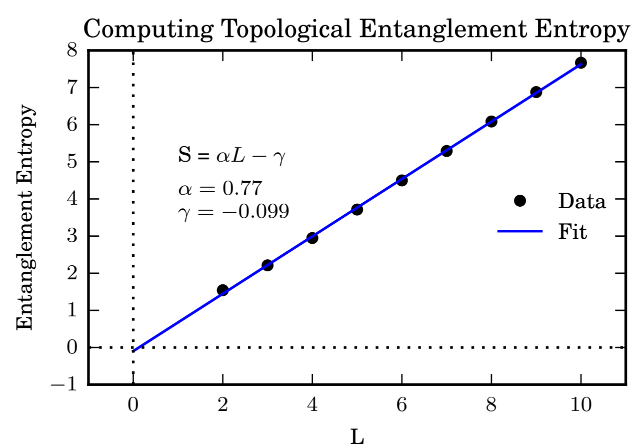


Figure 1. Entanglement spectrum on a zig-zag edge cylinder 10 unit cells in circumference.





VI. IDENTIFICATION OF EDGE CFT

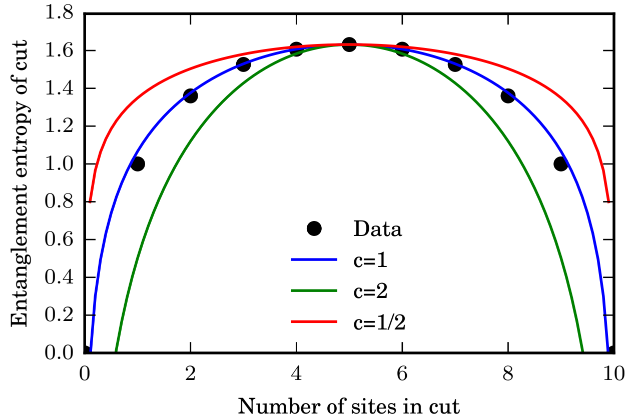


Figure 3. Entanglement entropy within the entanglement ground state of the soft-core boson state on 10 sites. For comparison, the Cardy-Calabrese formula $S(x) = c/3 \log \sin(\pi x/L) + const.$ is shown with $c = \frac{1}{2}, 1$, and 2, with the *const.* fixed by matching the maximum of the entanglement entropy data. $c = 1$ is a good fit.

$$\mathbf{P} = \frac{2\pi}{L}(\mathbf{L_0} - \bar{\mathbf{L}}_\mathbf{0}) \qquad = \frac{2\pi}{L}(em + n - \bar{n})$$
$$\mathbf{H} = \frac{2\pi}{L}(\mathbf{L_0} + \bar{\mathbf{L}}_\mathbf{0}) \qquad = \frac{2\pi}{L}(\frac{\kappa e^2}{2} + \frac{m^2}{2\kappa} + \frac{n + \bar{n}}{2})$$

$$\mathbf{H} \propto e^2 + \frac{m^2}{\kappa^2} + \frac{1}{\kappa}(n + \bar{n})$$

The degeneracy of level $n, \bar{n}$ states is $Z(n)Z(\bar{n})$.

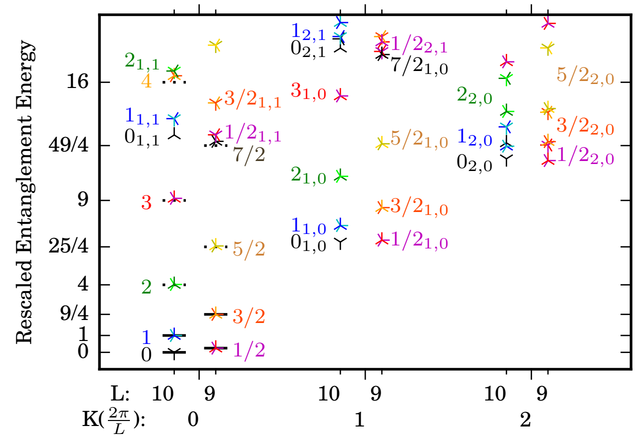
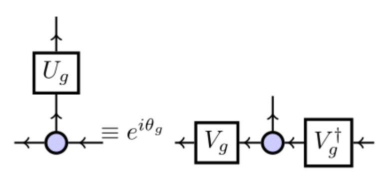
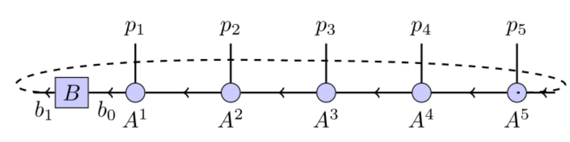


Figure 4. The identification of the states $j_{-n}|e, m = 0\rangle$ in the spectrum of the soft-core boson entanglement Hamiltonian. The label e gives the U(1) charge. The labels n, $\bar{n}$ label the levels in the right or left-moving sectors of the Kac-Moody algebra. The best estimate for the Luttinger parameter is $\kappa = 1/6.4$. The label m is 0 for all states shown - however, the primary states $|e, m = \pm 1\rangle$ can be seen centered around momentum π.

VII. SYMMETRY PROTECTED TOPOLOGICAL ORDER





$\mathbf{G}$	$\mathbf{U_g}$	$\theta_\mathbf{g}$	$\mathbf{V_g}$	$\mathbf{V_g V_g^*}$
$U(1)$				
π				
$\mathcal{I}$				
$\pi\mathcal{I}$				

Since

$$V_{\pi\mathcal{I}} V_{\pi\mathcal{I}}^* = -I \quad \text{or} \quad V_\pi V_\mathcal{I} = -V_\mathcal{I} V_\pi,$$

the representation is in the nontrivial class of

$$H^2(\mathbb{Z}_2 \times \mathbb{Z}_2^\mathcal{I}; U(1)) = \mathbb{Z}_2.$$

VIII. CONCLUSIONS

ACKNOWLEDGMENTS

[1] S. A. Parameswaran, A. M. Turner, D. P. Arovas, and A. Vishwanath, "Topological order and absence of band insulators at integer filling in Non-Symmorphic crystals," (2012), arXiv:1212.0557 [cond-mat.str-el].

[2] I. Kimchi, S. A. Parameswaran, A. M. Turner, F. Wang, and A. Vishwanath, "Featureless and non-fractionalized mott insulators on the honeycomb lattice at 1/2 site filling," (2012), arXiv:1207.0498 [cond-mat.str-el].