A Fast Iterative Shrinkage-Thresholding Algorithm for Linear Inverse Problems

Amir Beck and Marc Teboulle

Presented by Itamar Salazar

$$\mathbf{A}\mathbf{x} = \mathbf{b} + \mathbf{w}$$

$$\text{(LS)}: \quad \hat{\mathbf{x}}_{\text{LS}} = \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2.$$

$$\text{(T)}: \quad \hat{\mathbf{x}}_{\text{TIK}} = \underset{\mathbf{x}}{\operatorname{argmin}} \{\|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2 + \lambda \|\mathbf{L}\mathbf{x}\|^2\}.$$

ISTA

$$\min_{\mathbf{x}}\{F(\mathbf{x}) \equiv \|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2 + \lambda\|\mathbf{x}\|_1\},$$

$$\mathbf{x}_{k+1} = \mathcal{T}_{\lambda t}\left(\mathbf{x}_k - 2t\mathbf{A}^T(\mathbf{A}\mathbf{x}_k - \mathbf{b})\right),$$

$$\mathcal{T}_{\alpha}(\mathbf{x})_i = (|x_i| - \alpha)_+ \operatorname{sgn}(x_i).$$

CONTRIBUTION

$$\min_{\mathbf{x}}\{F(\mathbf{x}) \equiv f(\mathbf{x}) + g(\mathbf{x})\},$$

$$\mathbf{x}_{k+1} = \mathcal{T}_{\lambda t}(G(\mathbf{x}_k)),$$

$$\mathbf{x}_{k+1} = \mathcal{T}_{\lambda t}(G(\mathbf{y}_k)),$$

ISTA ALGORITHM

ISTA with constant stepsize

Input: $L := L(f)$ - A Lipschitz constant of ∇f.

Step 0. Take $\mathbf{x}_0 \in \mathbb{R}^n$.

Step k. $(k \geq 1)$ Compute

$$\mathbf{x}_k = p_L(\mathbf{x}_{k-1}). \tag{3.1}$$

ISTA ALGORITHM

ISTA with backtracking

Step 0. Take $L_0 > 0$, some $\eta > 1$, and $\mathbf{x}_0 \in \mathbb{R}^n$.

Step k. $(k \geq 1)$ Find the smallest nonnegative integers i_k such that with $\bar{L} = \eta^{i_k} L_{k-1}$

$$F(p_{\bar{L}}(\mathbf{x}_{k-1})) \leq Q_{\bar{L}}(p_{\bar{L}}(\mathbf{x}_{k-1}), \mathbf{x}_{k-1}). \tag{3.2}$$

Set $L_k = \eta^{i_k} L_{k-1}$ and compute

$$\mathbf{x}_k = p_{L_k}(\mathbf{x}_{k-1}). \tag{3.3}$$

FISTA ALGORITHM

FISTA with constant stepsize

Input: $L = L(f)$ - A Lipschitz constant of ∇f.

Step 0. Take $\mathbf{y}_1 = \mathbf{x}_0 \in \mathbb{R}^n$, $t_1 = 1$.

Step k. $(k \geq 1)$ Compute

$$\mathbf{x}_k = p_L(\mathbf{y}_k), \tag{4.1}$$

$$t_{k+1} = \frac{1 + \sqrt{1 + 4t_k^2}}{2}, \tag{4.2}$$

$$\mathbf{y}_{k+1} = \mathbf{x}_k + \left(\frac{t_k - 1}{t_{k+1}}\right)(\mathbf{x}_k - \mathbf{x}_{k-1}). \tag{4.3}$$

NUMERICAL EXAMPLES

original

blurred and noisy

PLES

ISTA: $F_{100} = 5.44\text{e-}1$

ISTA: $F_{200} = 3.60\text{e-}1$

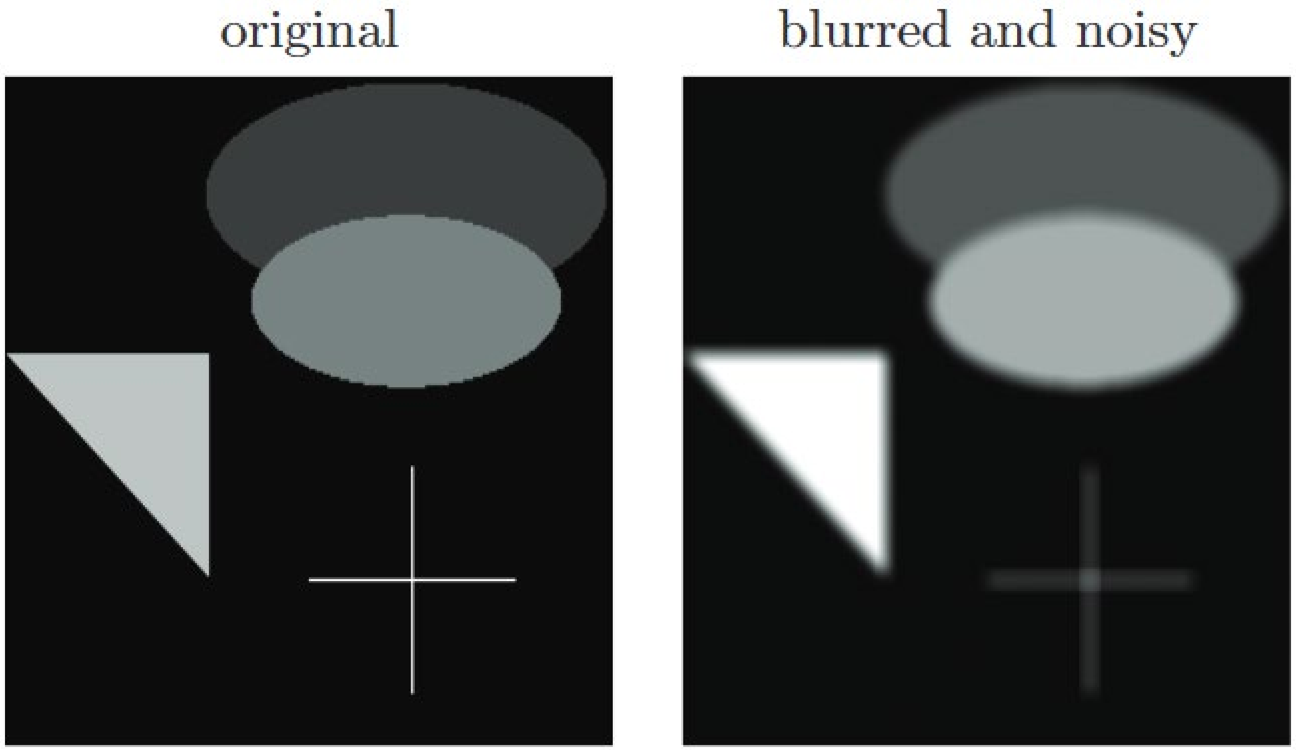
MTWIST: $F_{100} = 3.09\text{e-}1$

MTWIST: $F_{200} = 2.61\text{e-}1$

FISTA: $F_{100} = 2.40\text{e-}1$

FISTA: $F_{200} = 2.28\text{e-}1$

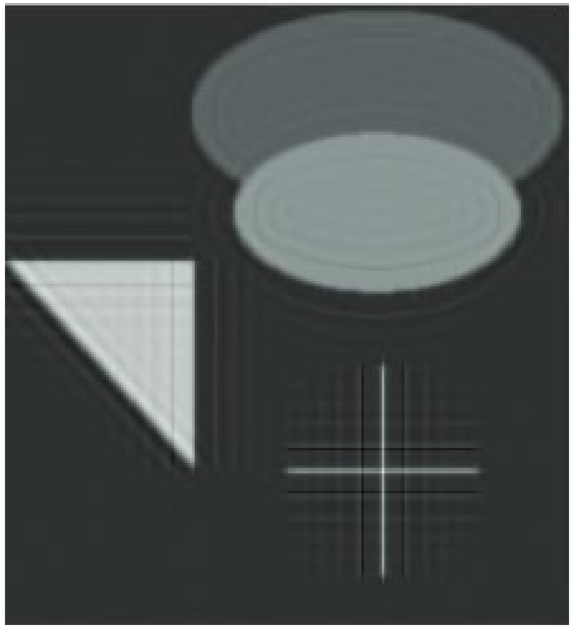
NUMERICAL EXAMPLES



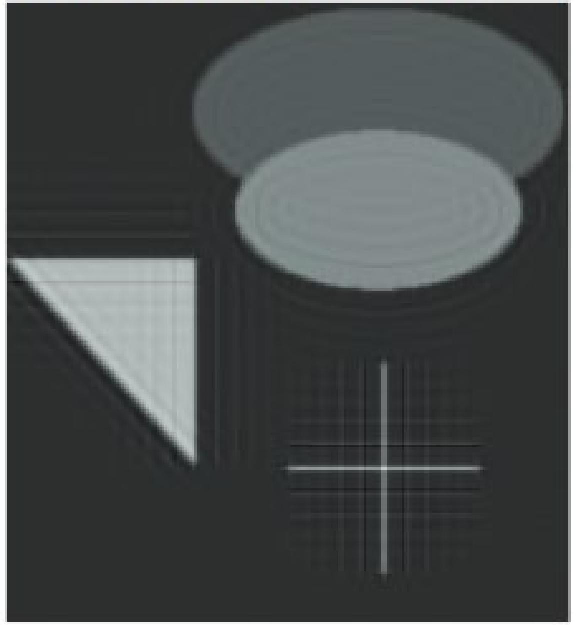
PLES

ISTA: $F_{100} = 5.67\text{e-}1$

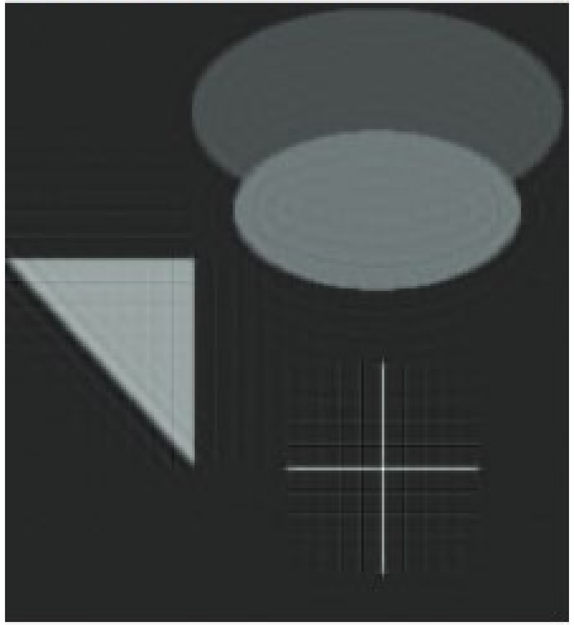
ISTA: $F_{200} = 4.27\text{e-}1$



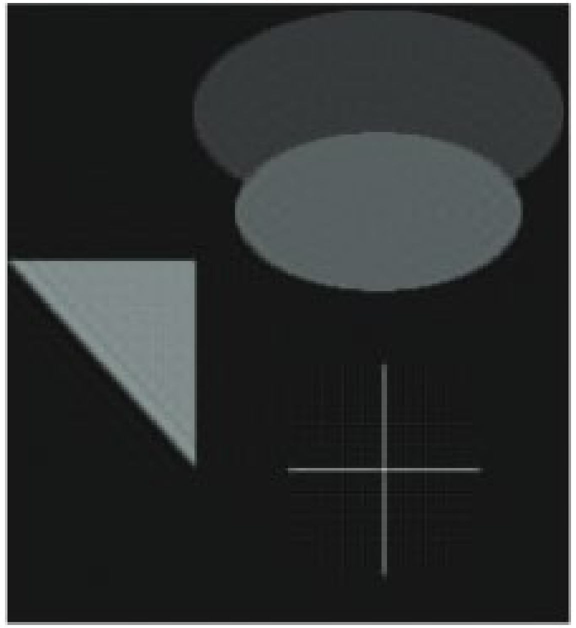
MTWIST: $F_{100} = 3.83\text{e-}1$



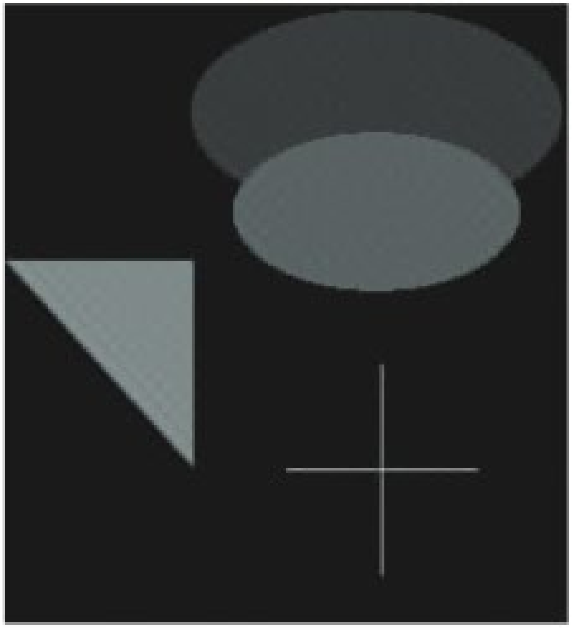
MTWIST: $F_{200} = 3.41\text{e-}1$

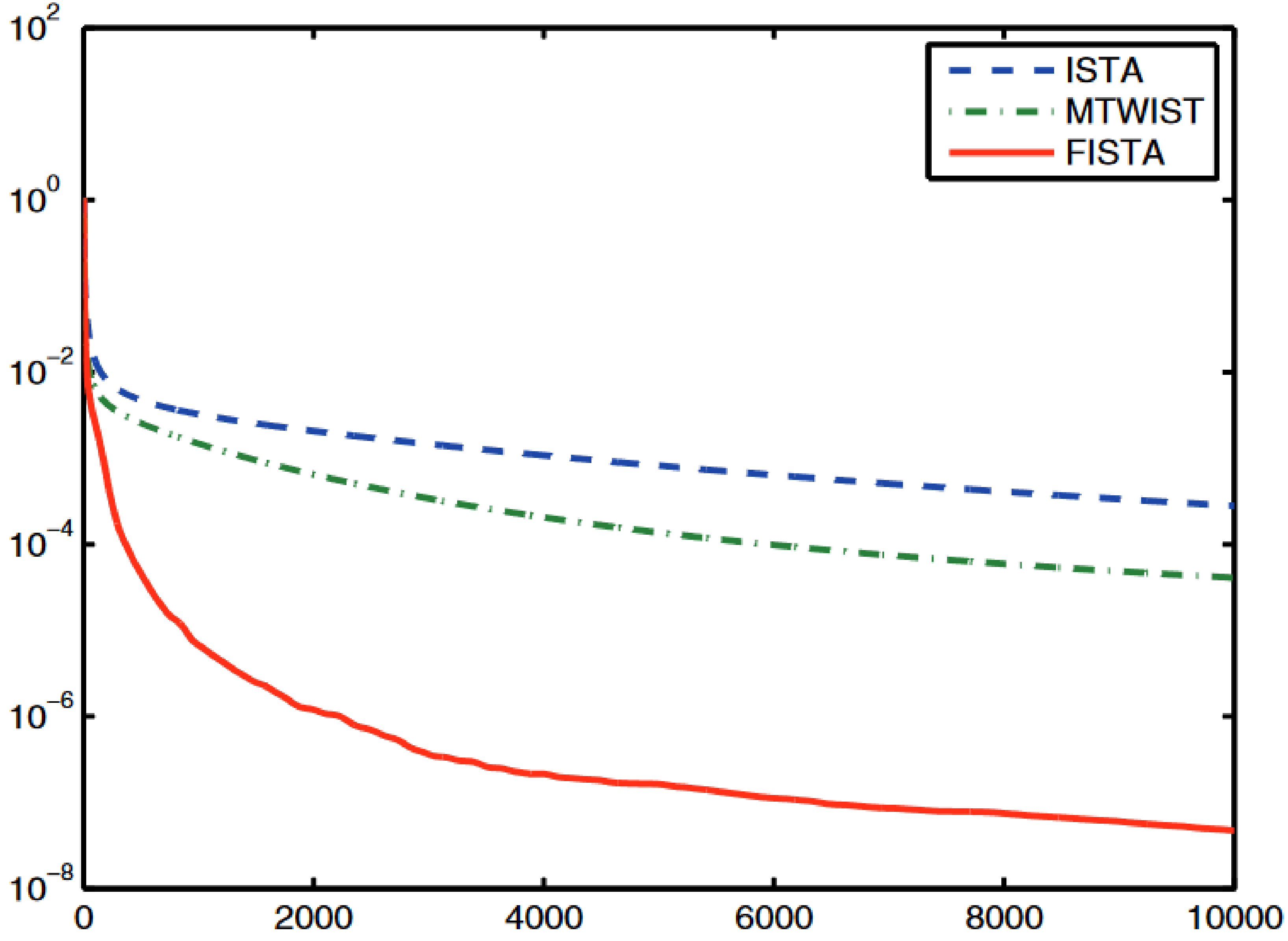


FISTA: $F_{100} = 3.21\text{e-}1$



FISTA: $F_{200} = 3.09\text{e-}1$





10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
2000
4000
6000
8000
10000
ISTA
MTWIST
FISTA