

# Fine Tuning The Rotating Skip List

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## Abstract

Skip list [1] is a linked-list-based structure that became an increasingly popular concurrent alternative to search trees due to its logarithmic complexity and local balancing operation. The Rotating skip list [2] is the fastest concurrent skip list to date. In this paper, we investigate, and try to improve upon, the different heuristics used in the Rotating skip list data structure.

## 1 Introduction

### 1.1 Concurrent Skip Lists

Skip lists are probabilistic search structures which provide improved execution time bounds compared with straightforward binary search trees yet are much simpler to implement than any guaranteed- $O(\log n)$  search structure [1]. A skip list comprises multiple levels, each of which is a linked list. Every skip-list node is present at the lowest level, and probabilistically present in each higher level up to some maximum level that is chosen independently and randomly for each node. This maximum is selected using a random number generator with exponential bias: for example, the probability of inserting into level  $x$  is often chosen to be  $2^{-x}$ . Due to its logarithmic complexity, simplicity and local balancing operation, skip list became an increasingly popular concurrent alternative to search trees and the default JDK logarithmic concurrent structures (which is an implementation of the lock-free skip list algorithm suggested by Fraser[3]). Another particularly useful property for parallel skip-list design is that a node can be independently inserted at each level in the list. A node is visible as long as it is linked into the lowest level of the list: insertion at higher levels is necessary only to maintain the property that search time is  $O(\log n)$ . This lead to an improved algorithm [4] that avoids contention hotspots by relying on a maintenance thread to raise, lower and clean-up the towers. This technique lets application threads simply insert an element at the bottom or delete an element

by marking it as logically deleted. Yet, until recently, typical skip list did not exploit multicore platforms as well as trees. This has changed when the Rotating skip list was introduced. The Rotating skip list [2] combines the rotation of trees and the uncontended nature of skip lists into a multicore-friendly data structure. The core algorithmic novelty is the use of wheels instead of the usual towers that are linked together to speedup traversals.

## 1.2 Rotating Skip List Algorithm

In this section, we present an overview of the rotating skip list key-value store implementation. The rotating skip list differs from traditional skip lists in that it is deterministic and uses wheels, its rotations differ from trees rotations in that they execute in constant time to lower the structure. The resulting algorithm is proven to be linearizable and non-blocking (or lock-free). Refer to [2] for more details, figures, pseudo code and correctness proofs.

### 1.2.1 Key-value store

Key-value stores offer the basis for indexes to speed up access to data sets. They support the following operations:

- *put*( $k, v$ ) inserts the key-value pair  $\langle k, v \rangle$  and returns *true* if  $k$  is absent, otherwise return  $\perp$
- *delete*( $k$ ) removes  $k$  and its associated value and returns *true* if  $k$  is present in the store, otherwise return *false*
- *get*( $k$ ) returns the value associated with key  $k$  if  $k$  is present in the store, otherwise return *false*

### 1.2.2 Structure Memory

Locality is achieved by using a rotating array sub-structure, called the wheel, detailed later in a separate section. The structure is a skip list, denoted *sl*, specified with a set of nodes, including two sentinel nodes. The head node is used as an entry point for all accesses to the data structure, it stores a dummy key that is the lowest of all possible keys. A tail node is used to indicate the end of the data structure, its dummy key is strictly larger than any other possible keys. As in other skip lists, node values are ordered in increasing key order from left to right. The global counter ZERO indicates the index of the first level in the wheels, it is set to 0 initially. The node structure contains multiple fields. It first contains a key-value pair denoted by  $\langle k, v \rangle$ . Two special values  $v = \perp$  and  $v = \text{node}$  indicate that the node is logically deleted and physically deleted, respectively. A node is first logically deleted before being physically removed and two logically deleted nodes cannot share the same key  $k$ . The level represents the level of this nodes wheel, similar to the level of the corresponding tower in a traditional skip list, it indicates that the node keeps track of successors indexed

from 0 to level - 1. The *succs* is the nodes wheel, it stores the successor pointer at each level of the node. *next* (resp. *prev*) is a direct pointer to the next (resp. previous) node that contains the smallest key larger than  $k$  (resp. the highest key lower than  $k$ ). Hence the skip list nodes are all linked through a doubly linked list. This doubly linked list allows to backtrack among preceding nodes if the traversal ends up at a deleted node. Finally, the *marker* is a special mark used only during physical removal.

### 1.2.3 Traversal

Each update operation (*put*, *delete*) avoids contention hotspots by localizing the modification to the least contended part of the data structure. All adjustments to the upper levels are sequentially executed by a dedicated maintenance thread, described later, hence allowing a deterministic adjustment of the levels. Any traversal, whether it is for updating or simply searching the structure, is executed from the top of the head node traversing wheels from left to right and levels from top to bottom. Each access looks for the position of some key  $k$  in the skip list by starting from the top level of the head down to the bottom level. The *get* function starts by traversing the structure from the skip list head, namely *sl.head*, till the bottom level. It records the value of *ZERO* at the beginning of the traversal into a local zero variable, sets its starting point to the *set.head* before iterating over each level  $i$  from the top level of the skip list, which is also the top level of the head *set.head.level* - 1, to zero. Once the *get* has reached the bottom level, *node* is actually set to the node with the largest key  $k' < k$ . If this node is physically deleted, the traversal backtracks among deleted nodes and invokes *helpremove* to notify the background thread of the nodes to be removed. Note that the traversal can thus reach a node that is not linked by any *succs* pointer but only one *next* pointer. Then it updates *next* to the immediate next node (*node.next*). Once the right position at the bottom level indicated by *next.k*  $> k$  is reached, the targeted key  $k$  is checked. If it is non logically deleted, the associated value *val* is returned, otherwise  $\perp$  is returned to indicate that no key  $k$  was present in the key-value store. The *put* function is similar in that it first traverses the structure from top to bottom, backtracks from right to left and help remove deleted nodes. The *put* may find that the node with the key it looks for is logically deleted, in which case it simply needs to logically insert it by setting its value to the appropriate one using a CAS. if the node is found as non logically deleted, then *put* is unsuccessful and returns *false*. Finally, if the *put* did not find the targeted key  $k$ , it creates a new node *node* with key  $k$  and value  $v$  that is linked to *next* and inserts it physically using a CAS. The reason why nodes are logically deleted before being physically removed is to minimize contention. The *delete* function marks a node as logically deleted by setting its value to  $\perp$ . A separate maintenance thread is responsible for traversing the bottom level of the skip list to clean up the deleted nodes as described in later section. The *delete* executes like the *put* as it also traverses the structure from top to bottom and backtracks to help remove the deleted nodes. It checks whether a key is absent or if its node is logically

or physically deleted in which case it returns *false*. Otherwise, this function logically deletes the node using a CAS to mark it. **A heuristic helps deciding whether the delete should help removing. This happens only when the ratio of logically deleted nodes over non-logically deleted nodes (as communicated by the background thread) reaches 3 after what it pays off.** Whether it helps physically removing or not, the delete returns *true* if the node was logically deleted.

#### 1.2.4 Wheels instead of towers

The wheel size is adjusted without having to mutate a pointer, simply by over provisioning a static array and using modulo arithmetic to adjust the mutable levels. The modulo arithmetic guarantees that increasing the index past the end of an array wraps around to the beginning of the array. This allows lowering to be done in constant-time by simply increasing the index of the lowest logical level of all the arrays without incurring contention. A global variable ZERO is used to keep track of the current lowest level, and when a lowering occurs the lowest index level is invalidated by incrementing the ZERO variable. This causes other threads to stop traversing the index levels before they reach the previous lowest level of pointers. If ZERO is increased above the length of the array this will not compromise the program since the arrays are accessed with modulo arithmetic. To illustrate how wheels improve locality of reference, consider testing to see if the current node next in the traversal has a key greater than the search key. If the wheels were represented using distinct objects, then *next.k* would need to be changed to *next.node.k*, reflecting the fact that the key-value information is being stored in a distinct node object from the index next references. This extra layer of indirection can hurt performance of the skip list, especially since this redirects all traversals.

#### 1.2.5 Background thread

The background (or maintenance) thread executes a loop where it sleeps for 50 microseconds (**a heuristic parameter**). The maintenance thread raises wheels of non-deleted nodes by calling the *raisebottomlevel* function. This raise is compensated with a constant-time lowering specific to the algorithm. This periodic adaptation makes it unnecessary to use the traditional pseudo-random generators as the lowering and raising (the action of increasing the height of a tower) become deterministic. The *lowerskiplist* function discards, in constant-time, the entire bottom level of the skip list by simply changing the ZERO counter used in the modulo arithmetic, without blocking application threads. Note that the *lowerskiplist* is followed by a *cleanupbottom* level that takes linear time to reclaim the memory of the deleted level, however, all traversals starting after the ZERO increment ignores this level. The *lowerskiplist* function is called, with some heuristic, only if lowering necessary returns true. **The chosen heuristic was if there are 10 times more deleted nodes with height greater than 1 than bottom-level nodes.** The *raisebottomlevel* also cleans up the

skip list by removing the logically deleted nodes. After all logically deleted nodes are discarded, their wheels having been progressively lowered down to a single level, they are garbage collected using an epoch based memory reclamation algorithm discussed later on. The *raisenode* and *raiseindexlevel* simply consist, for each level from bottom to top, of raising each node in the middle of three consecutive non-deleted nodes of the same height. The *helpremove* function called by the *remove* function or by an application thread removes the deleted nodes. It actually only removes nodes that do not have any wheel successors. Nodes with wheels are removed differently by first lowering their levels. Deleted wheels are simply removed later by the background thread within a *helpremove* during maintenance. Note that at the end of the removal the *prev* field can be updated without synchronization as it does not have to be set to the immediate previous node.

### 1.2.6 Memory reclamation

The memory reclamation of our rotating skip list is based on an epoch based garbage collection algorithm similar to the one used in Frasers skip list [3] with some differences. The garbage collector of Frasers skip list is partitioned into sections responsible for managing nodes of a particular skip list level. Partitioning the memory manager like this means that requests to the memory manager regarding different skip list levels do not need to conflict with one another. In contrast, the rotating skip list does no such partitioning of memory management responsibilities, since the rotating skip list uses only one node size for all list elements regardless of their level. This increases the probability of contention in the memory reclamation module when a large number of threads issue memory requests simultaneously.

## 1.3 Our Contribution

We examined and extended the following heuristics:

- Level Hight Heuristics
- Level Delete Heuristics
- Deletion Help Heuristics
- Background Thread Sleeping Heuristics
- MultiSkipList

Detailed descriptions of the changes made, in addition to the evaluations, are included in the prociding sections of the paper.

## 1.4 Code Location

The complete code, including the compiled data structures and the evaluation script, could be found at [github.com/itamartalmon/synchrobench](https://github.com/itamartalmon/synchrobench).

**Note: the code for the Multiple Skip List implementation is located at the `br.ts` branch, all the other implementations are located at the master branch**

## 2 LEVEL HEIGHT HEURISTICS

The skip list maximum level height is considered to be an important factor to its performance. The reasons for that are three fold:

- A maximum level lower than  $\log N$ , where  $N$  is the number of elements in the list, will inevitably hurt the balance properties of the structure and will add to the traversal length
- A common cause of contention is the bottle neck at the top layers of the skip list (similar to the root of a tree structure)
- Due to its random properties, it is possible (and in fact plausible, in some implementations) to have unnecessary high levels that will add some overhead work for each traversal

In all the implementations we encountered the maximum level of the skip list is bound by some constant (usually the word size of the keys, **in the case of the rotating skip list - 20**). We examined the effects of choosing the that property in fixed and dynamic ways.

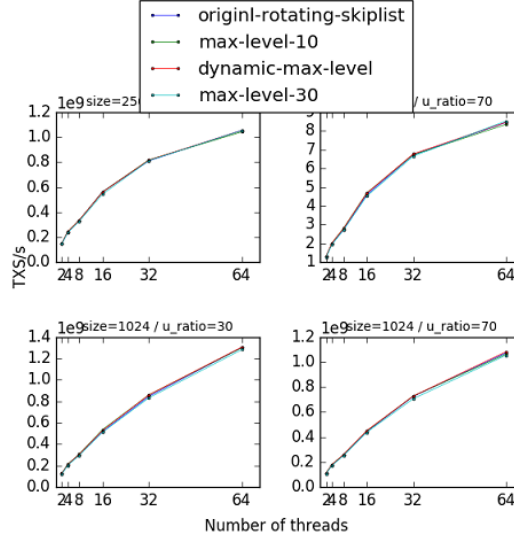
### 2.1 Fixed Max Level

Fixing the Max level to a constant is the simplest way to set it. It is a sensible solution where we know the expected size of our structure and expect it to be relatively stable. We benchmarked the original rotating skip list value of 20, against hardcoded values of 10 and 30. We tested each implementation for 10 runs, 10 seconds each, with initial size of 256 (and 1024), key range of 512 (and 2048) and update operations rate (delete / insert) of 30 (and 70) percent.

### 2.2 Dynamic Max Level

Dynamic Max level is a more sensible solution in cases where we do not know the expected size of the list or that it may change drastically over time. In our implementation we decided to add a dynamic parameter that stop the list from growing above a certain height. This parameter is set to be  $\log N$  in the end of each of the background loop run, where  $N$  is the number of non-deleted

Figure 1: Max Level Heuristics Performance



elements encountered by the background thread. We benchmarked our implementation against the original rotating skip list implementation with the same test properties as above.

### 2.2.1 Evaluation

As can be seen on figure 1, fixing the maximum level to be lower or higher than the original 20 did not cause any significant effect on the rotating skip list performance. This might be due to the deterministic way the background thread increasing the index levels of the nodes - when the traversal encounters 3 neighbouring nodes with similar level - it will increase the middle one. This significantly reduces the chance to have unnecessary high levels since it can only happen in cases where a very large amount of nodes are deleted. Setting the maximum level dynamically did not improve the performance as well, it is possible that the test conditions were not radical enough to expose its effect but since there was empirically no overhead we believe it is safer to use it anyway. Examining the effect of a dynamic maximum level on a skip list implementation with random level-rise remains for future work.

## 3 ROTATION DELETION HEURISTICS

One of the main features of the rotating skip list is the rotation level delete - which efficiently deletes the first index level at once by increasing the level modulu parameter. The motivation was to balance the effect of deletions on the structure balance (in the rotating skip list - raising nodes indexes cannot

cause imbalance due to its deterministic implementation). In the original implementation, the heuristic used to initiate this action is when the background thread encounter more than 10 times deleted high level nodes (referred to as tall-deletions) than the number of non-deleted nodes in a single maintenance loop. The authors stated that it might be interesting to investigate different and more dynamic heuristics. We examined the effects of setting different tall-deletion rate constants and implemented our own heuristic (which is based on the size rate between the bottom and the first level and will be described later with more details).

### 3.1 Tall-Deletions-Size Rate

Since it does not directly relate to the balance properties of the structure - the idea of conditioning the first layer deletion with the number of the deleted high level nodes might sound a bit strange at first, but since this is the only way that a balanced rotating skip list can become imbalanced, it is a sensible and practical solution (as the background thread is the only one which deletes tall nodes it can easily count them with almost no work or implementation complexity overhead). We wanted to check the way in which the tall-deletion threshold rate parameter influence the skip list performance. We benchmarked the original threshold value of 10 against values of 5, 1 and  $1/2$  for 10 runs, 10 seconds each, with initial size of 256 (and 65536), key range of 512 (and 131072) and update operations rate (delete / insert) of 30 (and 60) percent. In addition, we conducted more tests (ratios of 20 and 70, sizes of 1024 and 512) that did not yield any results that shown difference between the different thresholds and chose not to include their plots.

#### 3.1.1 Evaluation

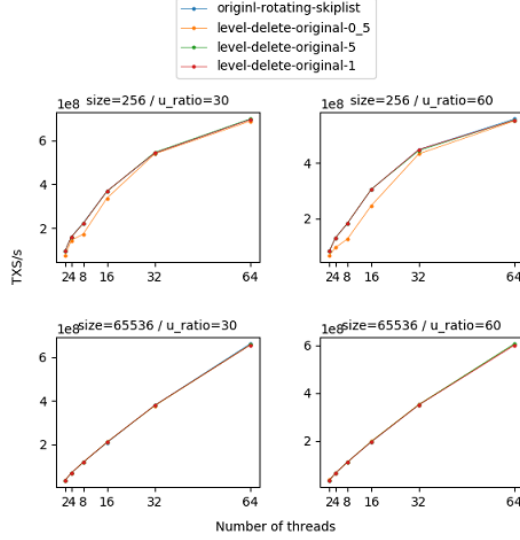
As can be seen on figure 2, somewhat surprisingly, using tall-deletion rate thresholds of different orders of magnitude does not improves or compromises the rotating skip list performance under the benchmarked test conditions (which are broader than what was tested in [2]). This could be a result of the quite extreme condition posed by this heuristic - even for a value of  $1/2$  - we so rarely delete more than third of the number of elements in the list and when we do delete, we usually hurt the performance. This suggests that keeping certain, and even relatively large, number of logically deleted nodes helps the skip list performance (compared to the situation where those are deleted and others raise for rebalancing). The above makes the effect of this feature on the complexity practically negligible.

### 3.2 Relative Level Size

The motivation of using the ratio between the first and bottom level came from the clear insight that it is a quality of the rotating skip list that is both easy to obtain (while the background thread traverse the bottom layer it can just



Figure 2: Tall Deleted By Size Deletion Performance



check the nodes level in  $O(1)$  time) and a good estimate for the imbalance of the structure. As we expect the first level to be around half the size of the bottom level with thought that a good threshold for deleting would be  $3/4$  as it is half the way between the worst situation, where both level are in similar sizes and the optimal  $1/2$  value. We also added a  $2/3$  threshold implementation for the testing in order to better understand the significance of this feature. We benchmarked the level ratios thresholds of  $2/3$  and  $3/4$  against the original (tall-deletion ratio threshold) for 10 runs, 10 seconds each, with initial size of 128 (and 262144), key range of 256 (and 524288) and update operations rate (delete / insert) of 30 (and 60) percent.

### 3.2.1 Evaluation

As can be seen on figure 3, the performance of both of the level ratio delete heuristic ratio was completely similar to the original tall-deletion heuristic implementation under conditions with small range of keys and slightly worse when the key range is large. It is interesting to note that in the large key range tests, the more frequently we use first level delete, the worst the performance gets. This supports the claim from the previous section as it seems that as long we do not have very large number of logically deleted nodes, it is better to keep them than to delete and rebalance.

Figure 10 consists of four line plots arranged in a 2x2 grid, showing TBS/s (Y-axis) versus the Number of threads (X-axis) for different configurations. The legend indicates three methods: originl-rotating-skiplist (blue), level-delete-by-ratio-3\_4 (orange), and level-delete-by-ratio-2\_3 (green).

- Top Left Plot:** size=128 / u\_ratio=30. The Y-axis ranges from 1 to 6 (scaled by 1e8). The X-axis ranges from 2 to 64. All three methods show a similar upward trend, with level-delete-by-ratio-2\_3 (green) performing slightly better at higher thread counts.
- Top Right Plot:** size=128 / u\_ratio=60. The Y-axis ranges from 1 to 4 (scaled by 1e8). The X-axis ranges from 2 to 64. All three methods show a similar upward trend, with level-delete-by-ratio-2\_3 (green) performing slightly better at higher thread counts.
- Bottom Left Plot:** size=262144 / u\_ratio=30. The Y-axis ranges from 1 to 4 (scaled by 1e8). The X-axis ranges from 2 to 64. The originl-rotating-skiplist (blue) and level-delete-by-ratio-3\_4 (orange) methods show a similar upward trend, while level-delete-by-ratio-2\_3 (green) shows a lower performance at higher thread counts.
- Bottom Right Plot:** size=262144 / u\_ratio=60. The Y-axis ranges from 1 to 4 (scaled by 1e8). The X-axis ranges from 2 to 64. The originl-rotating-skiplist (blue) and level-delete-by-ratio-3\_4 (orange) methods show a similar upward trend, while level-delete-by-ratio-2\_3 (green) shows a lower performance at higher thread counts.

#### 4.1 Thread-Num-Size Rate

### 4.1.1 Evaluation

## 5 BACKGROUND THREAD SLEEP HEURISTICS

### 5.1 Thread-Num Rate

### 5.1.1 Evaluation

10

Figure 4: Help-Remove Performance

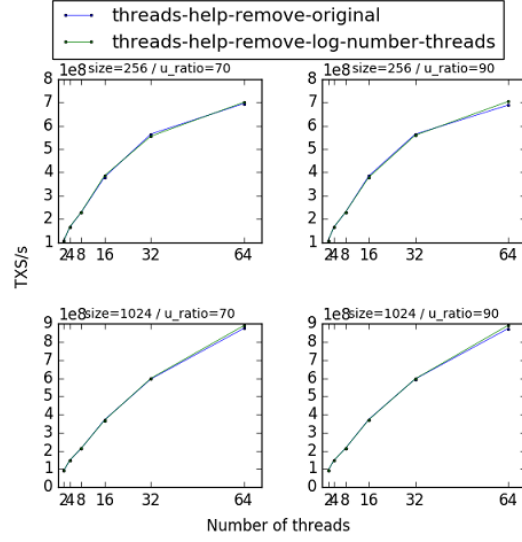


Figure 5: Background Sleep Time Performance

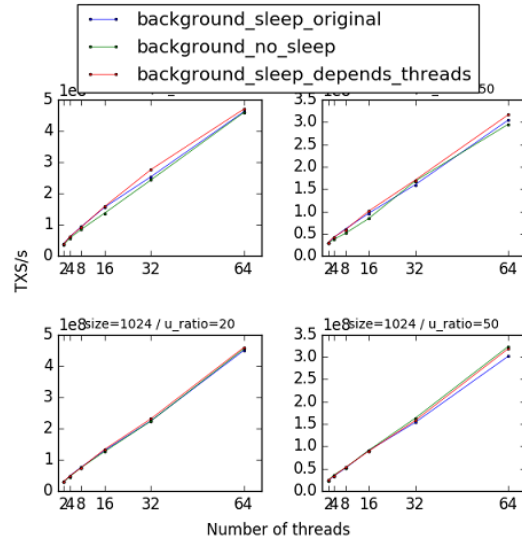
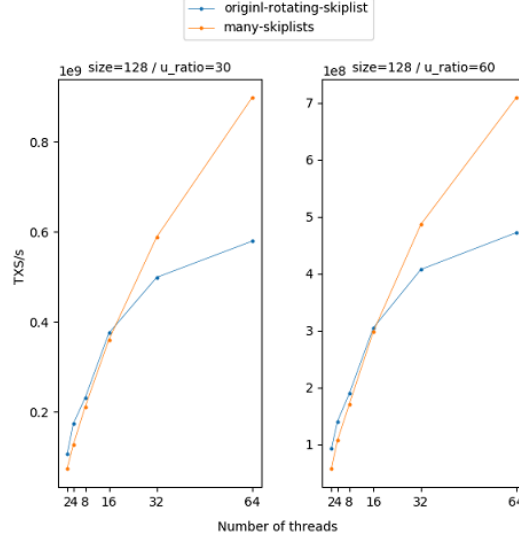


Figure 6: Multi Skip List Performance



## 6 Multiple Skip Lists

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### 6.1 Dynamic Number of Skip List

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#### 6.1.1 Evaluation

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## 7 Experimental Settings

We used multicore machine with 8 AMD x86\_64 Opteron(tm) Processor 6376 each have 8 cores that runs at 2.4 GHz. with L1d cache size of 16K, L1i cache size of 64K, L2 cache size of 2048K and L3 cache size of 6144K.

## 8 Future Research

### 8.0.1 Multiple Background Threads

The amount of maintnance work the background thread need to do is increasing with the size of the list and the number of worker threads and might lead to

imbalanced structure in case too much maintenance need to be done. An obvious solution is to incorporate the multiple lists idea with the background thread idea and have few background threads that work on separate lists and share the work load. This solution keeps the correctness and simplicity of the rotating skip list and will probably increase performance in cases where a single background thread is over loaded. Perhaps the best solution is having worker threads safely 'change into' maintenance threads according to some heuristic. In more general way, it could be interesting designing algorithms that have a dynamic pool of different threads that perform separate tasks in a way that reduces contention.

### 8.0.2 Fine Tuning Probabilistic Skip Lists

In this work, we focused only on extending the rotating skip list design but some of the above extensions could be used (with hopefully stronger effects) on probabilistic skip lists implementations. E.g. in a short testing we performed, it seems that the Fraser skip list (also used in the JDK) tend to have unnecessary higher levels due to its probabilistic nature and it would be interesting to see how maximum level solutions will effect its performance.

### 8.0.3 Heuristics Based on Action Distribution

In some cases, we noticed that some heuristic work better under different update actions distributions (e.g. 50 % deletions and inserts). This leads to possible improvement based on keeping some simple approximation of the action distribution inside the skip list itself (e.g. counters for how many inserts/deletes/searches were performed since the last background loop) and use this estimation to fine tune some thresholds.

## References

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