Language Models (and representations)

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Last Time

- Multi-Layer perceptrons

$$f_\theta(\mathbf{x}) = \mathrm{NN}_{\mathrm{MLP2}}(\mathbf{x}) = \mathbf{y}$$

$$\begin{aligned}
\mathbf{h^1} &= g^1(\mathbf{x}\mathbf{W^1} + \mathbf{b^1}) \\
\mathbf{h^2} &= g^2(\mathbf{h^1}\mathbf{W^2} + \mathbf{b^2}) \\
\mathbf{y} &= \mathbf{h^2}\mathbf{W^3}
\end{aligned}$$

Let's talk about sequences

- Predicting how a sequence will continue.

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Language Model

$$p(x_i|x_1, ..., x_{i-1})$$

Language Model: Markov Assumption

$$p(x_i|x_1, ..., x_{i-1}) \approx p(x_i|x_{i-4}, x_{i-3}, x_{i-2}, x_{i-1})$$

Language Model: Markov Assumption

$$p(x_i|x_1, ..., x_{i-1}) \approx p(x_i|x_{i-4}, x_{i-3}, x_{i-2}, x_{i-1})$$

(condition only on last n items)

this is called n-gram language model

Language Model

- LM can also be used to assign a probability to a sequence.

$$
\begin{aligned}
p(x_1, ..., x_n) = & p_{LM}(x_1 | \texttt{*S*}, \texttt{*S*}) \\
& \times p_{LM}(x_2 | \texttt{*S*}, x_1) \\
& \times p_{LM}(x_3 | x_1, x_2) \\
& \times p_{LM}(x_4 | x_2, x_3) \\
& \;\ldots \\
& \times p_{LM}(x_n | x_{n-2}, x_{n-1})
\end{aligned}
$$

Language Model

- Very useful (used in Speech Recognition, Machine Translation.. and many others).
- Does not have to be over natural language.
- Huge research topic. We'll see a neural LM.

Neural LM

$$p(x_k | x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1}) = \mathrm{softmax}(\mathrm{MLP}(\mathbf{x}))$$
$$\mathbf{x} = encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

Neural LM

$$p(x_k|x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1}) = \mathrm{softmax}(\mathrm{MLP}(\mathbf{x}))$$

$$\mathbf{x} = encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

$$\mathrm{softmax}(g(g(\mathbf{x}\mathbf{W^1} + \mathbf{b^1})\mathbf{W^2} + \mathbf{b^2})\mathbf{W^3} + \mathbf{b^3})$$

$$\mathrm{softmax}(\square)$$

$$\uparrow$$

$$\square \mathbf{W^3} + \mathbf{b^3}$$

$$\uparrow$$

$$g(\square \mathbf{W^2} + \mathbf{b^2})$$

$$\uparrow$$

$$g(\square \mathbf{W^1} + \mathbf{b^1})$$

$$\uparrow$$

$$\mathbf{x}$$

$$\uparrow$$

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

$$\mathbb{R}^{d_{out}}$$

$$\text{softmax}(\square)$$

$$\uparrow \qquad \mathbb{R}^{d_{out}}$$

$$\square \mathbf{W^3} + \mathbf{b^3}$$

$$\uparrow \qquad \mathbb{R}^{d_2}$$

$$g(\square \mathbf{W^2} + \mathbf{b^2})$$

$$\uparrow \qquad \mathbb{R}^{d_1}$$

$$g(\square \mathbf{W^1} + \mathbf{b^1})$$

$$\uparrow \qquad \mathbb{R}^{d_{in}}$$

$$\mathbf{x}$$

$$\uparrow$$

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

$\mathbb{R}^{d_{out}}$

$$\mathrm{softmax}(\square)$$

$\mathbb{R}^{d_{out}}$

$$\uparrow$$

$$\square\mathbf{W^3} + \mathbf{b^3}$$

$\mathbb{R}^{d_2}$

$$\uparrow$$

$$g(\square\mathbf{W^2} + \mathbf{b^2})$$

$\mathbb{R}^{d_1}$

$$\uparrow$$

$$g(\square\mathbf{W^1} + \mathbf{b^1})$$

$\mathbb{R}^{d_{in}}$

$$\uparrow$$

$$\mathbf{x}$$

$$\uparrow$$

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

Encoding k elements

$$\text{encode}(x_1, x_2, x_3, x_4)$$

We have k elements in a vocabulary of size |V|

Encoding k elements

$$\text{encode}(x_1, x_2, x_3, x_4)$$

We have k elements in a vocabulary of size |V|

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V={A,B,C,D,E,F,G,H,I,J}

Encoding k elements

$$\text{encode}(D, A, G, C)$$

A= [1,0,0,0,0,0,0,0,0,0]
B= [0,1,0,0,0,0,0,0,0,0]
C= [0,0,1,0,0,0,0,0,0,0]
D= [0,0,0,1,0,0,0,0,0,0]
E= [0,0,0,0,1,0,0,0,0,0]
F= [0,0,0,0,0,1,0,0,0,0]
G= [0,0,0,0,0,0,1,0,0,0]
H= [0,0,0,0,0,0,0,1,0,0]
I= [0,0,0,0,0,0,0,0,1,0]
J= [0,0,0,0,0,0,0,0,0,1]

Encoding k elements

$$\text{encode}(D, A, G, C)$$

A= [1,0,0,0,0,0,0,0,0,0]
B= [0,1,0,0,0,0,0,0,0,0]
C= [0,0,1,0,0,0,0,0,0,0]
D= [0,0,0,1,0,0,0,0,0,0]
E= [0,0,0,0,1,0,0,0,0,0]
F= [0,0,0,0,0,1,0,0,0,0]
G= [0,0,0,0,0,0,1,0,0,0]
H= [0,0,0,0,0,0,0,1,0,0]
I= [0,0,0,0,0,0,0,0,1,0]
J= [0,0,0,0,0,0,0,0,0,1]

$$\mathbf{v}_D + \mathbf{v}_A + \mathbf{v}_G + \mathbf{v}_C$$

[0,0,0,1,0,0,0,0,0,0]
+
[1,0,0,0,0,0,0,0,0,0]
+
[0,0,0,0,0,0,1,0,0,0]
+
[0,0,1,0,0,0,0,0,0,0]
=
[1,0,0,1,0,0,1,0,0,0]

Encoding k elements

$$\text{encode}(D, A, G, C)$$

A= [1,0,0,0,0,0,0,0,0,0]
B= [0,1,0,0,0,0,0,0,0,0]
C= [0,0,1,0,0,0,0,0,0,0]
D= [0,0,0,1,0,0,0,0,0,0]
E= [0,0,0,0,1,0,0,0,0,0]
F= [0,0,0,0,0,1,0,0,0,0]
G= [0,0,0,0,0,0,1,0,0,0]
H= [0,0,0,0,0,0,0,1,0,0]
I= [0,0,0,0,0,0,0,0,1,0]
J= [0,0,0,0,0,0,0,0,0,1]

$$\mathbf{v}_D + \mathbf{v}_A + \mathbf{v}_G + \mathbf{v}_C$$

[0,0,0,1,0,0,0,0,0,0]
+
[1,0,0,0,0,0,0,0,0,0]
+
[0,0,0,0,0,0,1,0,0,0]
+
[0,0,1,0,0,0,0,0,0,0]
=
[1,0,0,1,0,0,1,0,0,0]

what does this miss?

Encoding k elements

$$\text{encode}(D, A, G, C)$$

A= [1,0,0,0,0,0,0,0,0,0]
B= [0,1,0,0,0,0,0,0,0,0]
C= [0,0,1,0,0,0,0,0,0,0]
D= [0,0,0,1,0,0,0,0,0,0]
E= [0,0,0,0,1,0,0,0,0,0]
F= [0,0,0,0,0,1,0,0,0,0]
G= [0,0,0,0,0,0,1,0,0,0]
H= [0,0,0,0,0,0,0,1,0,0]
I= [0,0,0,0,0,0,0,0,1,0]
J= [0,0,0,0,0,0,0,0,0,1]

$$\mathbf{v}_D \circ \mathbf{v}_A \circ \mathbf{v}_G \circ \mathbf{v}_C$$

Encoding k elements

$$\text{encode}(D, A, G, C)$$

$$\mathbf{v}_D \circ \mathbf{v}_A \circ \mathbf{v}_G \circ \mathbf{v}_C$$

[0,0,0,1,0,0,0,0,0,0]
o
[1,0,0,0,0,0,0,0,0,0]
o
[0,0,0,0,0,0,1,0,0,0]
o
[0,0,1,0,0,0,0,0,0,0]
=
[0,0,0,1,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,0,0]

$\mathbb{R}^{d_{out}}$

$\text{softmax}(\square)$

$\mathbb{R}^{d_{out}}$

$\uparrow$

$\square \mathbf{W^3} + \mathbf{b^3}$

$\mathbb{R}^{d_2}$

$\uparrow$

$g(\square \mathbf{W^2} + \mathbf{b^2})$

$\mathbb{R}^{d_1}$

$\uparrow$

$g(\square \mathbf{W^1} + \mathbf{b^1})$

$\mathbb{R}^{d_{in}}$

$\uparrow$

$\mathbf{x}$

$\uparrow$

$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$

$$(\mathbf{v}_D + \mathbf{v}_A + \mathbf{v}_G + \mathbf{v}_C)\mathbf{W}$$

[0,0,0,1,0,0,0,0,0,0]
+
[1,0,0,0,0,0,0,0,0,0]
+
[0,0,0,0,0,0,1,0,0,0]
+
[0,0,1,0,0,0,0,0,0,0]
=
[1,0,0,1,0,0,1,0,0,0]

W

A= [-0.32, 0.09, 0.33,-0.44]
B= [0.29, 0.02,-0.46,-0.39]
C= [-0.46, 0.24,-0.16, 0.08]
D= [-0.15,-0.31, 0.34, 0.00]
E= [-0.10,-0.37, 0.01, 0.40]
F= [-0.28,-0.26,-0.24, 0.31]
G= [-0.32,-0.42,-0.21, 0.18]
H= [-0.09,-0.01, 0.06, 0.14]
I= [0.28,-0.02,-0.39, 0.12]
J= [0.23,-0.22,-0.14, 0.28]

$$(\mathbf{v}_D + \mathbf{v}_A + \mathbf{v}_G + \mathbf{v}_C)\mathbf{W}$$

$$= \mathbf{v}_D \cdot \mathbf{W} + \mathbf{v}_A \cdot \mathbf{W} + \mathbf{v}_G \cdot \mathbf{W} + \mathbf{v}_C \cdot \mathbf{W}$$

[0,0,0,1,0,0,0,0,0,0]
+
[1,0,0,0,0,0,0,0,0,0]
+
[0,0,0,0,0,0,1,0,0,0]
+
[0,0,1,0,0,0,0,0,0,0]
=
[1,0,0,1,0,0,1,0,0,0]

W

A=	[-0.32, 0.09, 0.33,-0.44]
B=	[0.29, 0.02,-0.46,-0.39]
C=	[-0.46, 0.24,-0.16, 0.08]
D=	[-0.15,-0.31, 0.34, 0.00]
E=	[-0.10,-0.37, 0.01, 0.40]
F=	[-0.28,-0.26,-0.24, 0.31]
G=	[-0.32,-0.42,-0.21, 0.18]
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I=	[0.28,-0.02,-0.39, 0.12]
J=	[0.23,-0.22,-0.14, 0.28]

$$(\mathbf{v}_D + \mathbf{v}_A + \mathbf{v}_G + \mathbf{v}_C)\mathbf{W}$$

$$= \mathbf{v}_D \cdot \mathbf{W} + \mathbf{v}_A \cdot \mathbf{W} + \mathbf{v}_G \cdot \mathbf{W} + \mathbf{v}_C \cdot \mathbf{W}$$

sum of rows in **W**

each row corresponds to a certain vocabulary item.

$$(\mathbf{v}_D \circ \mathbf{v}_A \circ \mathbf{v}_G \circ \mathbf{v}_C)\mathbf{W}$$

$$(\mathbf{v}_D \circ \mathbf{v}_A \circ \mathbf{v}_G \circ \mathbf{v}_C)\mathbf{W}$$

still sum of rows in **W**
but **W** has 4x many rows.

[0,0,0,1,0,0,0,0,0,0]

o

[1,0,0,0,0,0,0,0,0,0]

o

[0,0,0,0,0,0,1,0,0,0]

o

[0,0,1,0,0,0,0,0,0,0]

=

[0,0,0,1,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,0,0]

A(-3)= [0.42,-0.15, 0.12, 0.02]
B(-3)= [0.28,-0.15,-0.11, 0.32]
C(-3)= [0.15,-0.24, 0.23, 0.41]
D(-3)= [-0.12,-0.24, 0.12,-0.34]
E(-3)= [-0.42,-0.21, 0.08, 0.40]
F(-3)= [0.20, 0.11,-0.31, 0.33]
G(-3)= [0.07,-0.05, 0.16, 0.23]
H(-3)= [0.28, 0.03, 0.22,-0.49]
I(-3)= [0.08, 0.39,-0.25, 0.27]
J(-3)= [0.10,-0.42,-0.37, 0.35]
A(-2)= [-0.00, 0.41, 0.19, 0.49]
B(-2)= [0.24, 0.48, 0.34,-0.42]
C(-2)= [-0.46, 0.22, 0.24,-0.21]
D(-2)= [-0.11,-0.48, 0.18,-0.22]
E(-2)= [-0.32, 0.10,-0.41,-0.43]
F(-2)= [0.32, 0.02,-0.22, 0.06]
G(-2)= [-0.31,-0.36, 0.09, 0.39]
H(-2)= [0.01,-0.22,-0.09,-0.15]
I(-2)= [0.01, 0.10,-0.16,-0.21]
J(-2)= [-0.24, 0.40,-0.34,-0.13]
A(-1)= [-0.23,-0.38, 0.02, 0.32]
B(-1)= [-0.34, 0.04,-0.18,-0.00]
C(-1)= [0.40,-0.02, 0.10,-0.16]
D(-1)= [0.13,-0.07,-0.19,-0.01]
E(-1)= [0.40, 0.27,-0.33, 0.36]
F(-1)= [0.04,-0.13,-0.43, 0.39]
G(-1)= [0.44, 0.38, 0.03,-0.39]
H(-1)= [0.41,-0.23, 0.33,-0.08]
I(-1)= [-0.50,-0.16,-0.42,-0.27]
J(-1)= [-0.15, 0.41, 0.46,-0.16]
A(+0)= [-0.11, 0.03, 0.20, 0.50]
B(+0)= [0.16,-0.34, 0.20,-0.21]
C(+0)= [0.05,-0.13,-0.23,-0.31]
D(+0)= [0.13,-0.02, 0.38,-0.09]
E(+0)= [0.30, 0.39, 0.10, 0.38]
F(+0)= [-0.16,-0.31,-0.02,-0.34]
G(+0)= [0.06,-0.04, 0.02,-0.32]
H(+0)= [0.25, 0.30, 0.29, 0.24]
I(+0)= [0.40,-0.17,-0.18,-0.19]
J(+0)= [0.27, 0.33,-0.42,-0.07]

$$(\mathbf{v}_D \circ \mathbf{v}_A \circ \mathbf{v}_G \circ \mathbf{v}_C)\mathbf{W}$$

still sum of rows in **W**
but **W** has 4x many rows.

alternatively:

$$= \mathbf{v}_D \cdot \mathbf{W}' + \mathbf{v}_A \cdot \mathbf{W}'' + \mathbf{v}_G \cdot \mathbf{W}''' + \mathbf{v}_C \cdot \mathbf{W}''''$$

$$(\mathbf{v}_D \circ \mathbf{v}_A \circ \mathbf{v}_G \circ \mathbf{v}_C)\mathbf{W}$$

still sum of rows in **W**
but **W** has 4x many rows.

alternatively:

$$= \mathbf{v}_D \cdot \mathbf{W}' + \mathbf{v}_A \cdot \mathbf{W}'' + \mathbf{v}_G \cdot \mathbf{W}''' + \mathbf{v}_C \cdot \mathbf{W}''''$$

$$\mathbf{W} = \mathbf{W}' \circ \mathbf{W}'' \circ \mathbf{W}''' \circ \mathbf{W}''''$$

[0,0,0,1,0,0,0,0,0,0]

o

[1,0,0,0,0,0,0,0,0,0]

o

[0,0,0,0,0,0,1,0,0,0]

o

[0,0,1,0,0,0,0,0,0,0]

A(-3)= [0.42,-0.15, 0.12, 0.02]
B(-3)= [0.28,-0.15,-0.11, 0.32]
C(-3)= [0.15,-0.24, 0.23, 0.41]
D(-3)= [-0.12,-0.24, 0.12,-0.34]
E(-3)= [-0.42,-0.21, 0.08, 0.40]
F(-3)= [0.20, 0.11,-0.31, 0.33]
G(-3)= [0.07,-0.05, 0.16, 0.23]
H(-3)= [0.28, 0.03, 0.22,-0.49]
I(-3)= [0.08, 0.39,-0.25, 0.27]
J(-3)= [0.10,-0.42,-0.37, 0.35]

A(-2)= [-0.00, 0.41, 0.19, 0.49]
B(-2)= [0.24, 0.48, 0.34,-0.42]
C(-2)= [-0.46, 0.22, 0.24,-0.21]
D(-2)= [-0.11,-0.48, 0.18,-0.22]
E(-2)= [-0.32, 0.10,-0.41,-0.43]
F(-2)= [0.32, 0.02,-0.22, 0.06]
G(-2)= [-0.31,-0.36, 0.09, 0.39]
H(-2)= [0.01,-0.22,-0.09,-0.15]
I(-2)= [0.01, 0.10,-0.16,-0.21]
J(-2)= [-0.24, 0.40,-0.34,-0.13]

A(-1)= [-0.23,-0.38, 0.02, 0.32]
B(-1)= [-0.34, 0.04,-0.18,-0.00]
C(-1)= [0.40,-0.02, 0.10,-0.16]
D(-1)= [0.13,-0.07,-0.19,-0.01]
E(-1)= [0.40, 0.27,-0.33, 0.36]
F(-1)= [0.04,-0.13,-0.43, 0.39]
G(-1)= [0.44, 0.38, 0.03,-0.39]
H(-1)= [0.41,-0.23, 0.33,-0.08]
I(-1)= [-0.50,-0.16,-0.42,-0.27]
J(-1)= [-0.15, 0.41, 0.46,-0.16]

A(+0)= [-0.11, 0.03, 0.20, 0.50]
B(+0)= [0.16,-0.34, 0.20,-0.21]
C(+0)= [0.05,-0.13,-0.23,-0.31]
D(+0)= [0.13,-0.02, 0.38,-0.09]
E(+0)= [0.30, 0.39, 0.10, 0.38]
F(+0)= [-0.16,-0.31,-0.02,-0.34]
G(+0)= [0.06,-0.04, 0.02,-0.32]
H(+0)= [0.25, 0.30, 0.29, 0.24]
I(+0)= [0.40,-0.17,-0.18,-0.19]
J(+0)= [0.27, 0.33,-0.42,-0.07]

W'

W''

W'''

W''''

- 1-hot times matrix: row selection
- sum of 1-hot times matrix: row selection + sum
- concat of 1-hot: like using 1-hot from larger vocab

"Embedding Layer"

- Very common in neural network land:
  - associate each vocabulary item with a row in matrix **E** of dense vectors (dim of row << |V|)
  - concat or sum rows of **E** for input.

"Embedding Layer"

$$\text{encode}(D, A, G, C)$$

$$= \mathbf{E}_{[D]} \circ \mathbf{E}_{[A]} \circ \mathbf{E}_{[G]} \circ \mathbf{E}_{[C]}$$

[-0.15,-0.31, 0.34, 0.00,-0.32, 0.09, 0.33,-0.44,-0.32,-0.42,-0.21, 0.18,-0.46, 0.24,-0.16, 0.08]

E

A= [-0.32, 0.09, 0.33,-0.44]
B= [0.29, 0.02,-0.46,-0.39]
C= [-0.46, 0.24,-0.16, 0.08]
D= [-0.15,-0.31, 0.34, 0.00]
E= [-0.10,-0.37, 0.01, 0.40]
F= [-0.28,-0.26,-0.24, 0.31]
G= [-0.32,-0.42,-0.21, 0.18]
H= [-0.09,-0.01, 0.06, 0.14]
I= [0.28,-0.02,-0.39, 0.12]
J= [0.23,-0.22,-0.14, 0.28]

$$
\begin{array}{cc}
 & \mathbb{R}^{d_{out}} \\
\mathrm{softmax}(\square) & \\
\uparrow & \mathbb{R}^{d_{out}} \\
\square\mathbf{W^3} + \mathbf{b^3} & \\
\uparrow & \mathbb{R}^{d_2} \\
g(\square\mathbf{W^2} + \mathbf{b^2}) & \\
\uparrow & \mathbb{R}^{d_1} \\
g(\square\mathbf{W^1} + \mathbf{b^1}) & \\
\uparrow & \mathbb{R}^{d_{in}} \\
\mathbf{E}_{[x_{k-4}]} \circ \mathbf{E}_{[x_{k-3}]} \circ \mathbf{E}_{[x_{k-2}]} \circ \mathbf{E}_{[x_{k-1}]} & \\
\uparrow & \\
encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1}) &
\end{array}
$$

"Embedding Layer"

$$\text{encode}(D, A, G, C)$$

$$= \mathbf{E}_{[D]} \circ \mathbf{E}_{[A]} \circ \mathbf{E}_{[G]} \circ \mathbf{E}_{[C]}$$

[-0.15,-0.31, 0.34, 0.00,-0.32, 0.09, 0.33,-0.44,-0.32,-0.42,-0.21, 0.18,-0.46, 0.24,-0.16, 0.08]

E

A= [-0.32, 0.09, 0.33,-0.44]
B= [0.29, 0.02,-0.46,-0.39]
C= [-0.46, 0.24,-0.16, 0.08]
D= [-0.15,-0.31, 0.34, 0.00]
E= [-0.10,-0.37, 0.01, 0.40]
F= [-0.28,-0.26,-0.24, 0.31]
G= [-0.32,-0.42,-0.21, 0.18]
H= [-0.09,-0.01, 0.06, 0.14]
I= [0.28,-0.02,-0.39, 0.12]
J= [0.23,-0.22,-0.14, 0.28]

how does this relate to what we had before?

what is

$$(\mathbf{E}_{[D]} \circ \mathbf{E}_{[A]} \circ \mathbf{E}_{[G]} \circ \mathbf{E}_{[C]})\mathbf{W}$$

?

"Embedding Layer"

$$(\mathbf{E}_{[D]} \circ \mathbf{E}_{[A]} \circ \mathbf{E}_{[G]} \circ \mathbf{E}_{[C]})\mathbf{W}$$

same row in **E** regardless of position in the sequence.

but **W** will transform this row differently for each position.

$$
\begin{array}{cc}
 & \mathbb{R}^{d_{out}} \\
\operatorname{softmax}(\square) & \\
\uparrow & \mathbb{R}^{d_{out}} \\
\square\mathbf{W^3} + \mathbf{b^3} & \\
\uparrow & \mathbb{R}^{d_2} \\
g(\square\mathbf{W^2} + \mathbf{b^2}) & \\
\uparrow & \mathbb{R}^{d_1} \\
g(\square\mathbf{W^1} + \mathbf{b^1}) & \\
\uparrow & \mathbb{R}^{d_{in}} \\
\mathbf{E}_{[x_{k-4}]} \circ \mathbf{E}_{[x_{k-3}]} \circ \mathbf{E}_{[x_{k-2}]} \circ \mathbf{E}_{[x_{k-1}]} & \\
\uparrow & \\
encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1}) &
\end{array}
$$

what's in **W1**?
what's in **W2**?
what's in **W3**?

$$
\begin{array}{cc}
 & \mathbb{R}^{d_{out}} \\
\text{softmax}(\square) & \\
\uparrow & \mathbb{R}^{d_{out}} \\
\square\mathbf{W^3} + \mathbf{b^3} & \\
\uparrow & \mathbb{R}^{d_2} \\
g(\square\mathbf{W^2} + \mathbf{b^2}) & \\
\uparrow & \mathbb{R}^{d_1} \\
g(\square\mathbf{W^1} + \mathbf{b^1}) & \\
\uparrow & \mathbb{R}^{d_{in}} \\
\mathbf{E}_{[x_{k-4}]} \circ \mathbf{E}_{[x_{k-3}]} \circ \mathbf{E}_{[x_{k-2}]} \circ \mathbf{E}_{[x_{k-1}]} & \\
\uparrow & \\
encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1}) &
\end{array}
$$

Neural LM

$$\text{softmax}(\square)$$

$$\uparrow$$

$$\square\mathbf{W^3} + \mathbf{b^3}$$

$$\uparrow$$

$$g(\square\mathbf{W^2} + \mathbf{b^2})$$

$$\uparrow$$

$$g(\square\mathbf{W^1} + \mathbf{b^1})$$

$$\uparrow$$

$$\mathbf{E}_{[x_{k-4}]} \circ \mathbf{E}_{[x_{k-3}]} \circ \mathbf{E}_{[x_{k-2}]} \circ \mathbf{E}_{[x_{k-1}]}$$

$$\uparrow$$

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

Neural LM

A Neural Probabilistic Language Model

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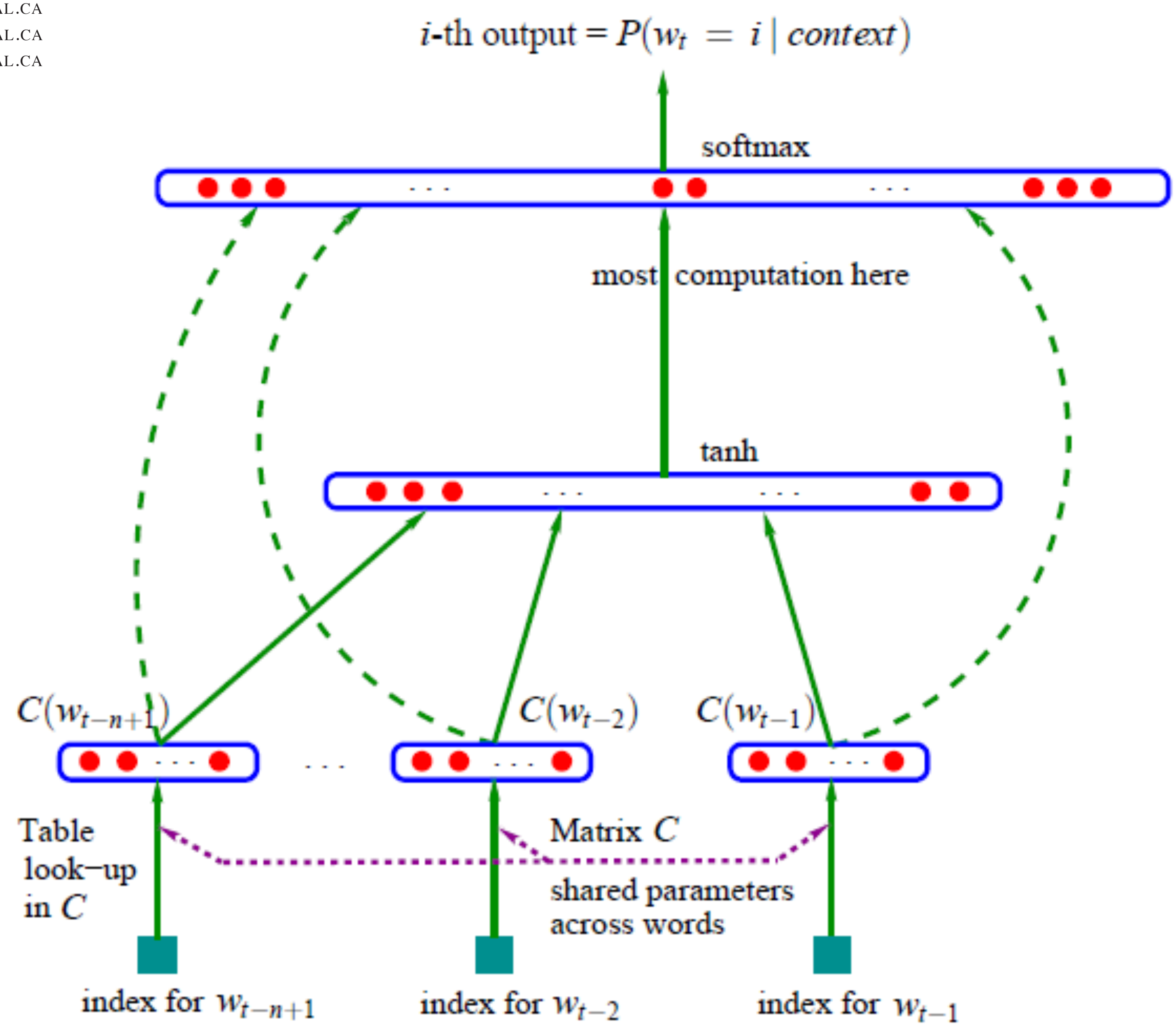
Bengio et al 2003

Pretty much same model,
But fewer layers.



Neural LM

- What is the cost of increasing the history length?

Neural LM

- We can use a trained LM for scoring a given sentence. (how?)

Neural LM

- We can use a trained LM for comparing two given sentences. (how?)

Neural LM

- We can use a trained LM for **generating new sentences**. (how?)

Neural LM

- We can use K trained LMs for **k-class classification**. (how?)

Neural LM

- We can use K trained LMs for **k-class classification**. (how?)

$$p(w_1, w_1, ..., w_n | LM_1)$$

$$p(w_1, w_1, ..., w_n | LM_2)$$

$$p(w_1, w_1, ..., w_n | LM_3)$$

$$\hat{y} = \arg\max_k p(w_1, w_1, ..., w_n | LM_k)$$

sequence prediction tasks

- In LM:
  - predict **word i** based on **k previous words**.
- But we could also predict **label** based on **k items**.
  - which tasks can this be used for?

classification

ACGGCGCTCGATCG V

GTGCGCTCGAACG V

ACGCGTTCTACCG X

tagging

DET	ADJ	NOUN	VERB	PREP	DET	ADJ	NOUN
The	brown	fox	jumped	over	the	lazy	dog

tagging

Window-based approach

NOUN

The brown **fox** jumped over

VERB

brown fox **jumped** over the

PREP

fox jumped **over** the lazy

back to LM

Training a language model

- Set dimensions of **E, W3**, according to vocab size.
- Initial random values for **E, W1, W2, W3, b1, b2, b3**
- For every n-tuple in some text:
    - try to predict last item based on prev n-1
    - use cross-entropy loss.

What happens after training?

- Consider the columns of **W3**.
- Consider the rows of **E**.

$\mathbb{R}^{d_{out}}$

$$\mathrm{softmax}(\square)$$

$\mathbb{R}^{d_{out}}$

$$\uparrow$$

what's in **W3**?

$$\square \mathbf{W^3} + \mathbf{b^3}$$

$\mathbb{R}^{d_2}$

$$\uparrow$$

$$g(\square \mathbf{W^2} + \mathbf{b^2})$$

columns of W3 correspond to vocab items!

$\mathbb{R}^{d_1}$

$$\uparrow$$

$$g(\square \mathbf{W^1} + \mathbf{b^1})$$

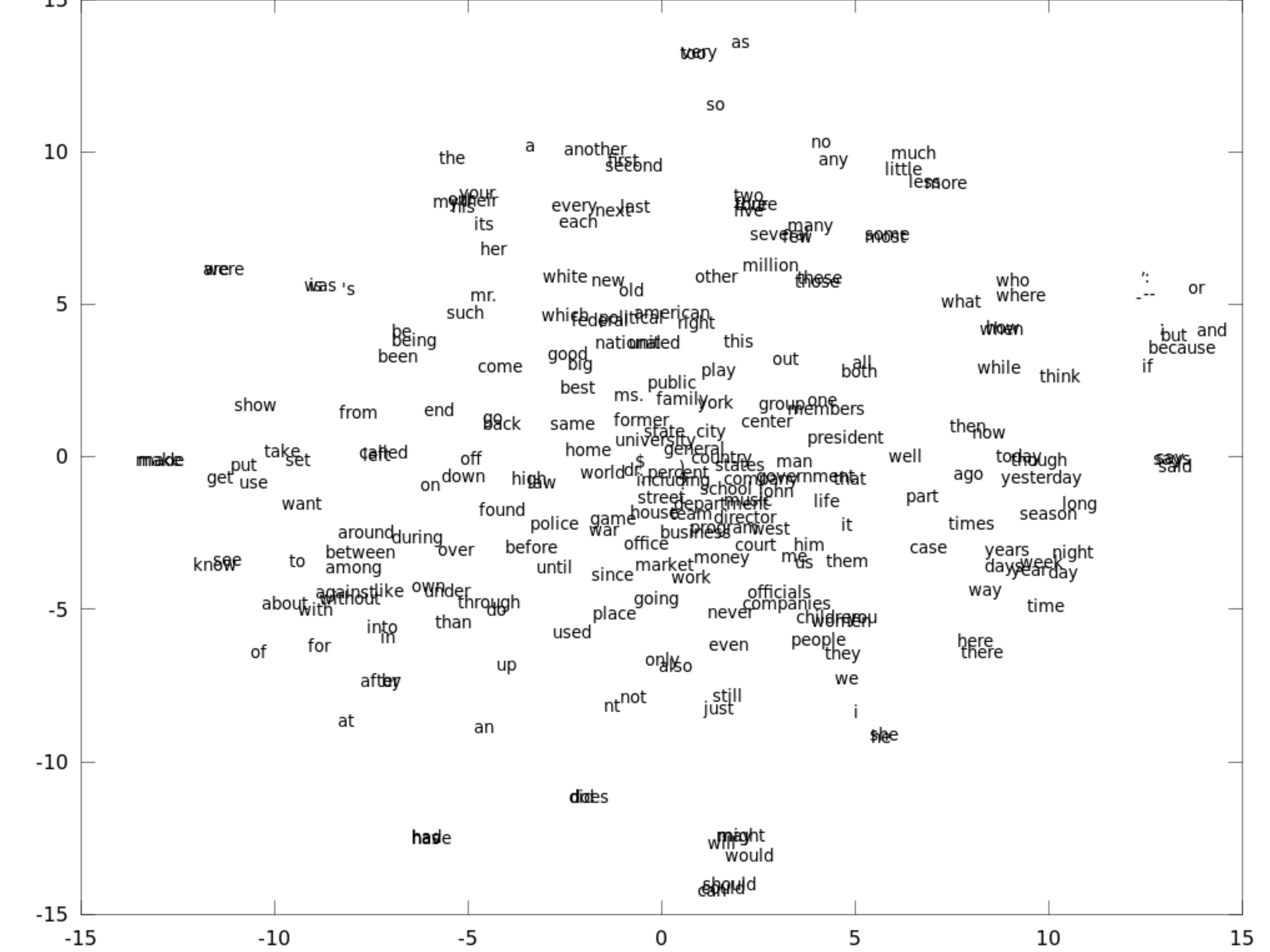
$\mathbb{R}^{d_{in}}$

$$\uparrow$$

$$\mathbf{E}_{[x_{k-4}]} \circ \mathbf{E}_{[x_{k-3}]} \circ \mathbf{E}_{[x_{k-2}]} \circ \mathbf{E}_{[x_{k-1}]}$$

$$\uparrow$$

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$



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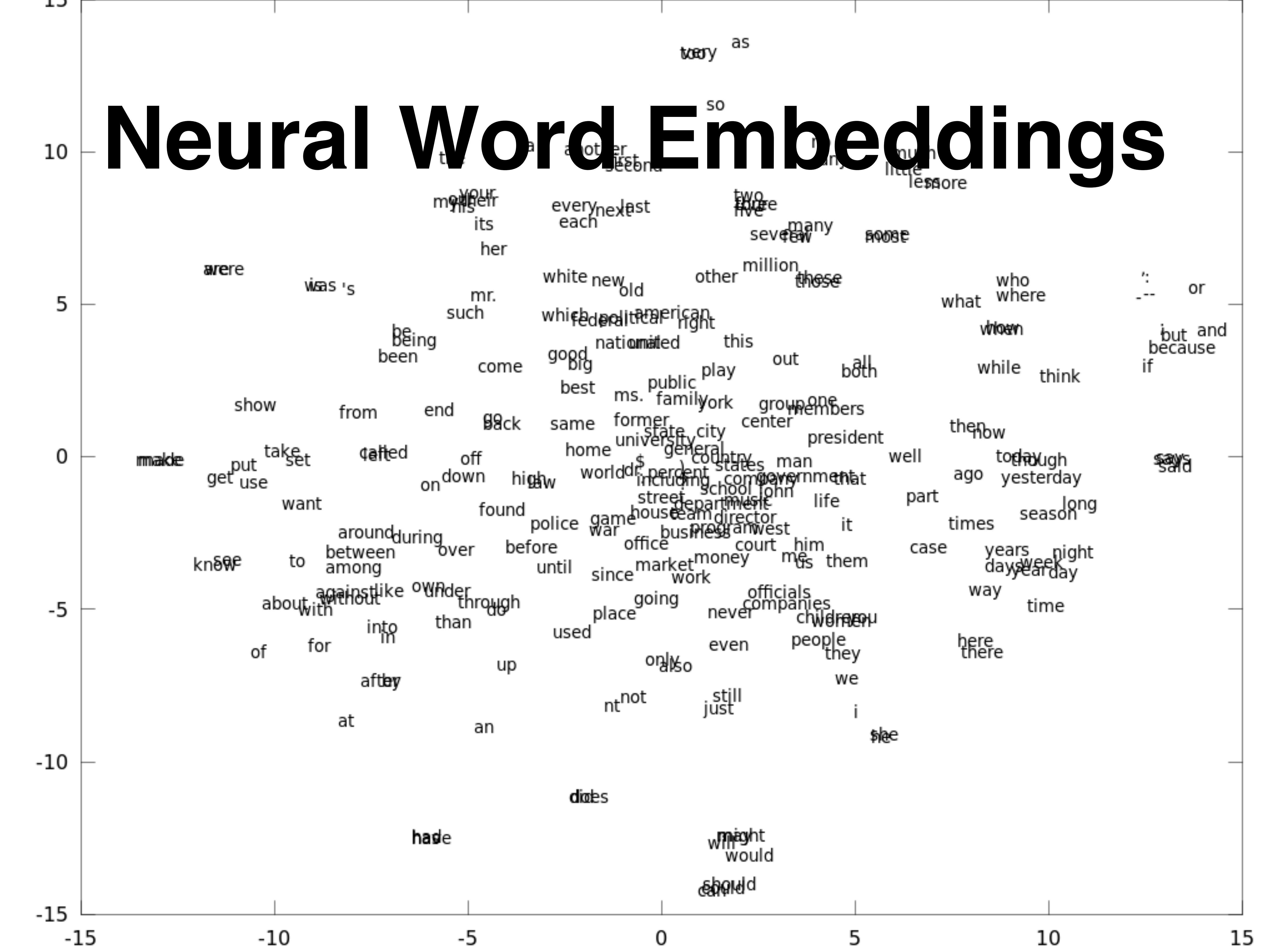
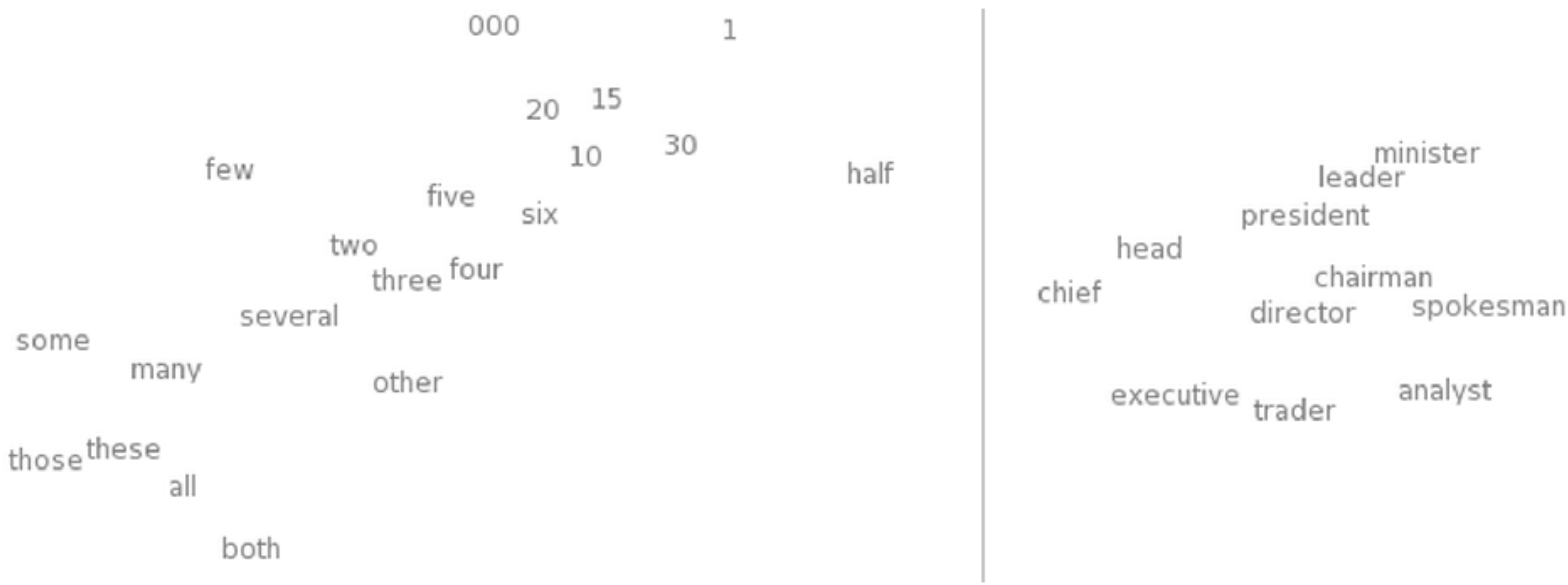
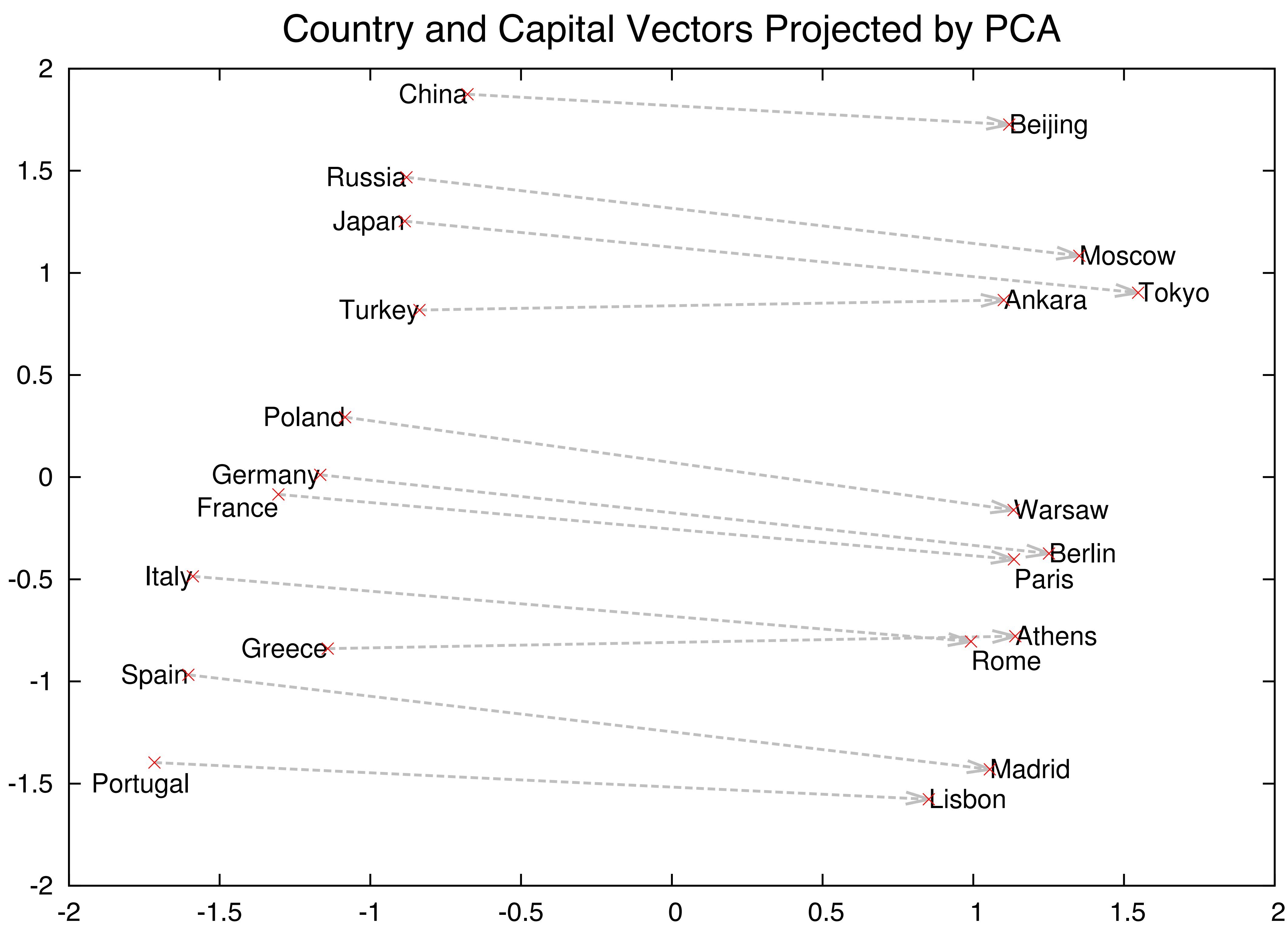
Neural Word Embeddings



Neural Word Embeddings



Neural Word Embeddings



Neural Word Embeddings

Target Word	BoW5
batman	nightwing aquaman catwoman superman manhunter
hogwarts	dumbledore hallows half-blood malfoy snape
turing	nondeterministic non-deterministic computability deterministic finite-state
florida	gainesville fla jacksonville tampa lauderdale
object-oriented	aspect-oriented smalltalk event-driven prolog domain-specific
dancing	singing dance dances dancers tap-dancing

Neural Word Embeddings

- We trained a language model.
- We ended up with **vector representations of words**.
- These representations are useful -- they encode various aspects of word similarity.

tagging + pre-training

we can use the **E** we got from LM training to initialize **E** for the POS tagging task.

(why is that helpful?)

NOUN

The brown **fox** jumped over

VERB

brown fox **jumped** over the

PREP

fox jumped **over** the lazy

pre-training

- This is a sort of **semi-supervised** learning
  or **multi-task** learning.

- We learn from "unannotated" data.

- We then use the representations on tasks with
  annotated data.

Neural Word Embeddings

- We trained a language model.
- We ended up with **vector representations of words**.
- These representations are useful -- they encode various aspects of word similarity.
- **A form of semi-supervised learning.**
- More next week.