Knowledge Inference

A. Alvarez, A. Arruarte, M. Larrañaga
Lenguajes y Sistemas Informáticos
UPV/EHU

eman ta zabal zazu

universidad del país vasco

euskal herriko unibertsitatea

Knowledge Inference

- Also called *Latent Knowledge Estimation*
  - Latent: "not directly measurable"
    - Knowledge that we have that we have not yet harnessed

- Goal
  - Measuring what a student knows at a specific time
  - Measuring what relevant knowledge components a student knows at a specific time

Knowledge Component

- Anything a student can know that is meaningful to the current learning situation
  - Skill
  - Fact
  - Concept
  - Principle
  - …

Why is it useful to measure student knowledge?

- Primary goal in education
  - Enhancing student knowledge

- If you can measure it,
  - You know whether you are making it better
  - You can inform instructors, students, peers… about it
  - You can automated pedagogical decisions

How do we get at latent knowledge?

- We can’t measure it directly
- We can’t look directly into the brain
  – Yet

- But we can look at performance
- And we can look at performance over time
  – More information than performance at one specific moment

Not trivial…

- This is a research problem with a long history…

Some approaches for Latent Knowledge Estimation/Knowledge Inference

- **Bayesian Knowledge Tracing**

  Corbett and Anderson, 1995
- Performance Factor Analysis

  Pavlik *et al.*, 2009
- Item Response Theory

  Baker, 2001
- Q-matrix

  Barnes, 2005
- Others

Bayesian Knowledge Tracing (BKT)

- Classic approach for measuring tightly defined skill in online learning
- Thoroughly articulated and studied by Albert Corbett and John Anderson (1995)
  - Variation of Bayesian calculations proposed by Richard Atkinson in the 1970s

Key goal of BKT

- Measuring how well a student knows a specific skill/knowledge component at a specific time
- Based on their past history of performance with that skill/knowledge component

Skills should be tightly defined

- The goal in not to measure overall skill for a broadly-defined construct
  - Such as arithmetic
- But to measure a specific skill or KC
  - Such as addition of two-digit numbers where no carrying is needed

- Unlike approaches such as Item Response Theory

Typical use of BKT

- Assess a student's knowledge of skill/KC
- Based on a sequence of items that are dichotomously scored
  - E.g. the student can get a score of 0 or 1 on each item
- Where each item corresponds to a single skill
- Where the student can learn on each item due to help, feedback, scaffolding, etc.

Key assumptions of BKT

- Each item must provide a single latent skill
- Each skill has four parameters

- Parameters & Pattern of successes and failures the student has had on each relevant skill we can compute:
  - $P(L_n)$: Latent knowledge
  - P(CORR): Probability the learner will get the item correct

Key assumptions of BKT

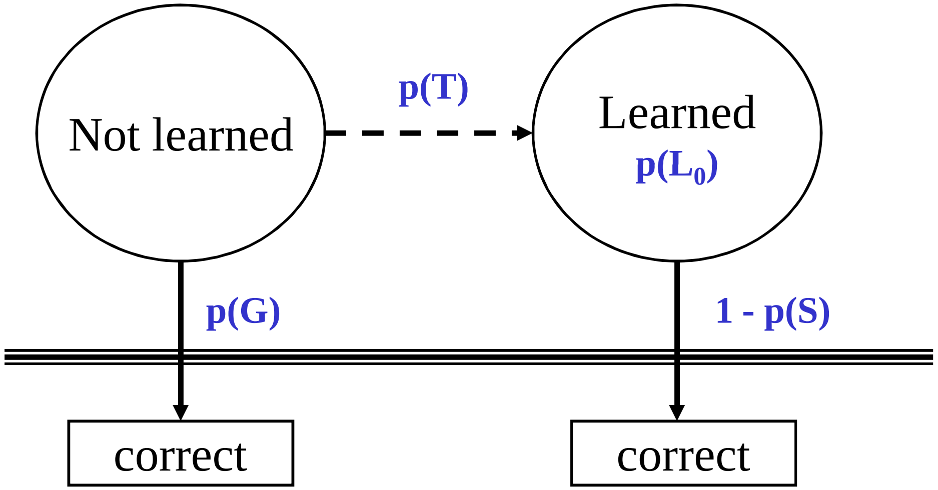
- Two-state learning model
  - Learned or Unlearned
- The student can learn a skill at each opportunity to apply the skill
- A student does not forget a skill, once s/he knows it

Model performance assumptions

- If the student knows a skill
  - There is still some chance the student will **slip** and make a mistake
- If the student does not know a skill
  - There is still some chance the student will **guess** correctly

- So, link between performance and learning it is not a perfect link

Model performance assumptions



Learning parameters

$p(L_0)$: Probability the skill is already known before the first opportunity to use the skill

p(T): Probability the skill will be learned at each opportunity to use the skill

Performance parameters

p(G): Probability the student will guess correctly if the skill is not known

p(S): Probability the student will slip if the skill is known

Predicting current student correctness

- $P(CORR) = P(L_n)*P(\sim S) + P(\sim L_n)*P(G)$

Bayesian Knowledge Tracing

- Whenever the student has an opportunity to use a skill
  - The probability that the student knows the skill is updated
  - Using formulas derived from Bayes' Theorem

Formulas

- $$P(L_{n-1}|Correct_n) = \frac{P(L_{n-1}) * (1 - P(S))}{P(L_{n-1}) * (1 - P(S)) + (1 - P(L_{n-1})) * P(G)}$$

- $$P(L_{n-1}|Incorrect_n) = \frac{P(L_{n-1}) * P(S)}{P(L_{n-1}) * P(S) + (1 - P(L_{n-1})) * (1 - P(G))}$$

- $$P(L_n|Action_n) = P(L_{n-1}|Action_n) + ((1 - P(L_{n-1}|Action_n)) * P(T))$$

Example

- $P(L_0)=0.4$; $P(T)=0.1$; $P(S)=0.3$; $P(G)=0.2$

Action	$P(L_{n-1})$	$P(L_{n-1}\|actual)$	$P(L_n)$
	0.4		

Example

- $P(L_0)=0.4$; $P(T)=0.1$; $P(S)=0.3$; $P(G)=0.2$

Action	$P(L_{n-1})$	$P(L_{n-1}\|actual)$	$P(L_n)$
1	0.4	X	

Example

- $P(L_0)=0.4$; $P(T)=0.1$; $P(S)=0.3$; $P(G)=0.2$

Action	$P(L_{n-1})$	$P(L_{n-1}\|actual)$	$P(L_n)$
1	0.4	$\frac{0.4*0.3}{0.4*0.3+0.6*0.8}$	

Example

- $P(L_0)=0.4$; $P(T)=0.1$; $P(S)=0.3$; $P(G)=0.2$

Action	$P(L_{n-1})$	$P(L_{n-1}\|actual)$	$P(L_n)$
1	0.4	0.2	

Example

- $P(L_0)=0.4$; $P(T)=0.1$; $P(S)=0.3$; $P(G)=0.2$

Action	$P(L_{n-1})$	$P(L_{n-1}\|actual)$	$P(L_n)$
1	0.4	0.2	0.2+0.8*0.1

Example

- $P(L_0)=0.4$; $P(T)=0.1$; $P(S)=0.3$; $P(G)=0.2$

Action	$P(L_{n-1})$	$P(L_{n-1}\|actual)$	$P(L_n)$
1	0.4	0.2	0.28

Example

- $P(L_0)=0.4$; $P(T)=0.1$; $P(S)=0.3$; $P(G)=0.2$

Action	$P(L_{n-1})$	$P(L_{n-1}\|actual)$	$P(L_n)$
1	0.4	0.2	0.28
2	0.28		

Example

- $P(L_0)=0.4$; $P(T)=0.1$; $P(S)=0.3$; $P(G)=0.2$

Action	$P(L_{n-1})$	$P(L_{n-1}\vert actual)$	$P(L_n)$
1	0.4	0.2	0.28
2	0.28	γ	

Example

- $P(L_0)=0.4$; $P(T)=0.1$; $P(S)=0.3$; $P(G)=0.2$

Action	$P(L_{n-1})$	$P(L_{n-1}\|actual)$	$P(L_n)$
1	0.4	0.2	0.28
2	0.28	$\frac{0.28*0.7}{0.28*0.7+(1-0.28)*0.2}$	

Example

- $P(L_0)=0.4$; $P(T)=0.1$; $P(S)=0.3$; $P(G)=0.2$

Action	$P(L_{n-1})$	$P(L_{n-1}\|actual)$	$P(L_n)$
1	0.4	0.2	0.28
2	0.28	0.58	

Example

- $P(L_0)=0.4$; $P(T)=0.1$; $P(S)=0.3$; $P(G)=0.2$

Action	$P(L_{n-1})$	$P(L_{n-1}\|actual)$	$P(L_n)$
1	0.4	0.2	0.28
2	0.28	0.58	0.58+(1-0.58)*0.1

Example

- $P(L_0)=0.4$; $P(T)=0.1$; $P(S)=0.3$; $P(G)=0.2$

Action	$P(L_{n-1})$	$P(L_{n-1}\text{l}actual)$	$P(L_n)$
1	0.4	0.2	0.28
2	0.28	0.58	0.62

A few notes about BKT

- Only uses first problem attempt on each item
  - Throws out information…
  - But used the clearest information…
- Several variants

Conceptual idea behind knowledge tracing

- Knowing a skill generally leads to correct performance
- Correct performance implies that a student knows the relevant skill

- Hence, by looking at whether a student's performance is correct, we can infer whether they know the skill

Parameters constraints

- To avoid *model degeneracy*
  - Model degeneracy is based on violating the conceptual idea behind knowledge tracing
    - When knowing a skill leads to worse performance
    - When getting a skill wrong means you know it

  - Baker, Corbett & Aleven (2008)
    - $P(G)<0.5$, $P(s)<0.5$
  - Corbett & Anderson (1995)
    - $P(G)<0.3$, $P(S)<0.1$

Knowledge tracing

- How do we know if a knowledge tracing model is any good?

- We pick the knowledge tracing parameters that best predict performance
  - Whether a student's action will be correct or wrong at a given time

Three public tools

- BNT-SM: Bayes Net Toolkit
  - http://www.cs.cmu.edu/~listen/BNT-SM/
- Fitting BKT at Scale
  - https://sites.google.com/site/myudelson/projects/
- BKT-BF: BKT-Brute Force (Grid Search)
  - http://www.columbia.edu/~rsb2162/BKT-BruteForce.zip

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