Lab 2 Report
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Answers to part 2 section 4

Q: What is the empirical order of convergence of these algorithms for p = 2 and p = 8?
A: If A is has a higher error than B, then A > B. For p = 2 the energy error (for large Δt) has Euler > EulerCromer > RK2 > RK4 > RK6. But for small Δt the energy error has Euler > EulerCromer > RK2 = RK4 =RK6. Similarly, the coordinate errors have (for large Δt) Euler > RK2 > EulerCromer > RK4 > RK6. And for small Δt Euler > RK2 > EulerCromer > RK4 = RK6. Notice that the RK2 and EulerCromer algorithms switch order when comparing the energy error to the coordinate error.
For p = 8 only the energy error is considered. For small Δt Euler > EulerCromer > RK2 > RK4 = RK6. And for large Δt Euler > EulerCromer > RK2 > RK4 > RK6.

Q: For p = 2, do you reach the same conclusion about the order of convergence from your examinations of coordinate errors and energy errors? If not, what could be the reason for this disagreement?
A: No, the EulerCromer algorithm surpasses the RK2 algorithm when considering the absolute coordinate error (but they are close). However, the RK2 algorithm performs much better than the EulerCromer algorithm when considering the relative energy error. This suggests that the RK2 algorithm produces velocities that compensate for the errors in the position when the relative energy error is computed.

Q: For what Δt value does the precision of the RK4 algorithm become dominated by the round-off errors?
A: It happens around (or a bit before) 10^{-3}, depending on which error is considered and the value of p. For p = 8 energy errors, RK4 starts to become dominated by round-off when $\Delta t \approx 10^{-3.5}$. This is similar to the Δt value for the p = 2 coordinate errors. But for the p = 2 energy errors, the RK4 algorithm doesn't succumb to round-off error until $\Delta t \approx 10^{-3}$