**Lab 3 Report**
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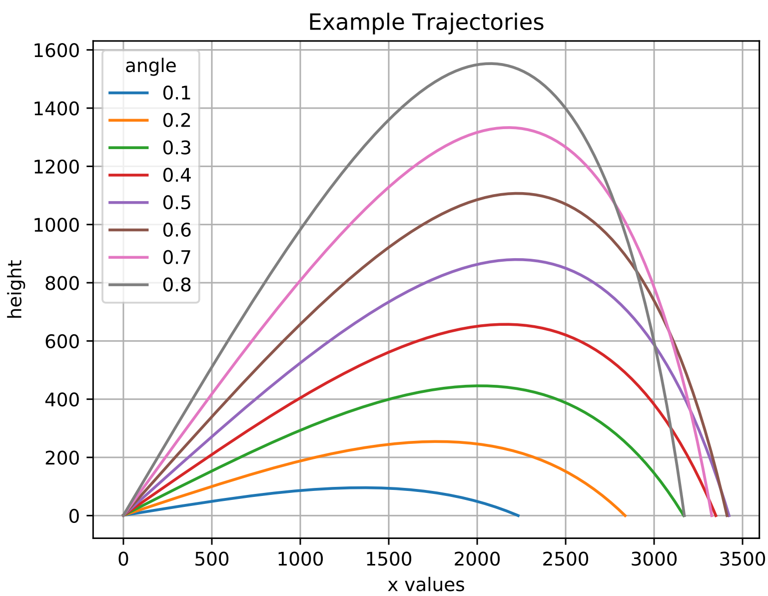
## Part 1

The pdf's of the desired log-log plots are in the doc directory.
On all three graphs the error has a minimum around step size = $10^{-4}$. Below this threshold the algorithm is dominated by round-off error (for small values of step size). Above the $10^{-4}$ threshold the algorithms are dominated by the approximation error, since the step size is too large to give a good approximation of the derivative. The slope of the graph is consistent since we expect the round-off error to become less severe as step size increases (producing a negative slope before the minimum), and we expect the approximation error to become more severe as step size increases (producing a positive slope after the minimum).

## Part 2

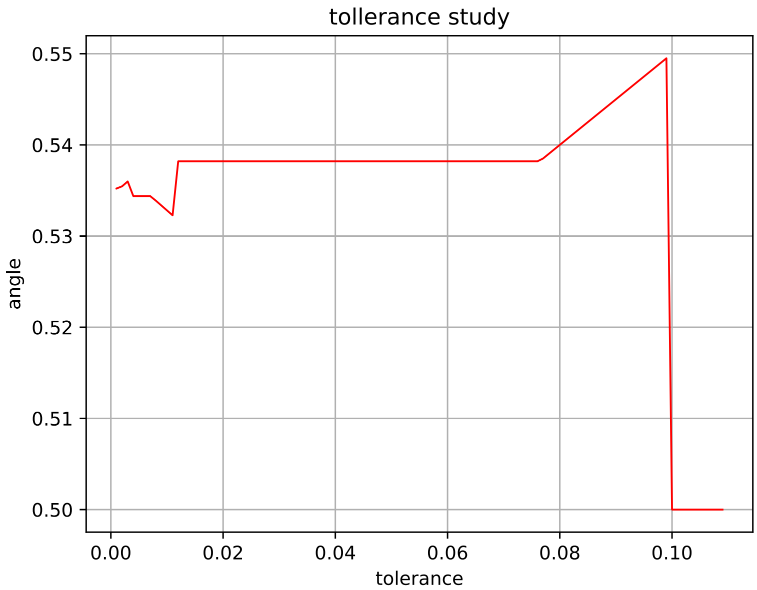
First, I chose the RK4 algorithm with a time step of $10^{-4}$ s. This time step ensures that the RK4 algorithm has reasonable accuracy while maintaining low relative errors for any derivatives that need to be taken along the path. Once the trajectory builder is working well, a graph of several example trajectories can be made:




Now we can make a rudimentary bracket, since we can see that trajectories with angles between 0.3 and 0.8 have larger ranges than the others. The bracket can be further refined by plotting the maximum as a function of the angle and looking at where the maximum appears to be. Doing this puts the maximum firmly between angle = 0.5 and angle = 0.6. To estimate the tolerance, I ran a loop that averages the fractional change of the maximum range near the true maximum (so within the previously defined bracket). If R is the maximum range for a given angle, then:

$$\epsilon = \frac{\sum_{i=1}^{N} \frac{(R_i - R_{i-1})}{R_i}}{N} \tag{1}$$

The tolerance that is used for Brent's method is set to be $\sqrt{\epsilon}$. For my calculations, this corresponds to a 6% fractional change (tol = 0.06). Examine the behavior of the output of Brent's method as a function of the tolerance:




Notice that the algorithm fails to converge for tol $> 0.1$, instead the algorithm just outputs one of the endpoints of the bracket. In the range of the chosen tolerance (0.06) the behavior of Brent's method is stable, and below this range the algorithm becomes more volatile. This justifies my calculation of the tolerance value.
The output of the angle optimization procedure gives $\frac{\pi}{5.837}$ or about 31 degrees.