Lab 4 Report
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Computational Physics

Part 1

Gaussian quadrature converges much more quickly than either the trapezoidal or Simpson's rule for both the exponential and the logarithm. The approximations converged quickly enough that the absolute error was within the machine precision. For these cases, instead of trying to take log(0) I set the error equal to the epsilon value. For the sin function both the trapezoidal and Simpson's rule beat Gaussian quadrature. This is because most of the equation's support is near the peaks of the sin function, where the shape closely approximates a polynomial. In this case, Simpson's attempt to fit a quadratic function to the graph works unusually well–resulting in a better approximation for the sin function than the exponential or the logarithm.

Part 2

The code for the psuedo-Monte Carlo methods takes a couple of hours to run, so I included all the plots in the doc directory. The bias converges to zero pretty quickly with increasing N, implying that–averageing over a large number of trials–the approximation is approaching the exact value. The standard deviation converges less quickly to a value around 9. This convergence was heavily dependent on running a large number of trials, during testing the standard deviation varied more than the other parameters between different trials. The pull distributions were almost always skewed right (this is more evident when looking at the individual pulls rather than the average. But, in the end the pull's look roughly Gaussian with a small (about 1?) standard deviation. It would be good to have some different pull studies for larger numbers of trials, perhaps only for the small-N approximations.

Examining the convergence of the quasi-Monte Carlo method (using the Sobol sequence), notice that changing N from 10^2 to 10^5 gives only two more correct digits to the approximation. This suggests that the approximation does not converge very quickly with increasing N, but gives reasonably good estimates for lower N. So, the results given for the computationally cheap small-N approximations are not that much worse than the expensive large-N approximations.