Exercise 3

1. In order to track drug smugglers, customs use sniffing dogs when passengers enter Israel. When a sniffing dog barks at a passenger they are arrested and searched. It is known that sniffing dogs bark at 5% of all passengers, and that 40% from passengers being searched do not carry drugs. It is estimated that 4% of passengers arriving to Israel carry drugs.
   a. What is the percentage of drug smugglers that are not arrested?
   b. A passenger arrived and is not arrested, what is the probability that he is a drug smuggler?
   c. A passenger that carrying drugs arrives, what is the probability that he is arrested?
   d. What is the percentage of errors in using sniffing dogs?
   Solve the questions using a tree diagram and (2X2) table.

2. In a certain population 1 in 1000 people has AIDS. There is an innovative test that diagnoses people with AIDS as such in 100% of the cases. If a person does not have AIDS, the test diagnoses him as such in 99% of the cases.
   A random person was tested and diagnosed with AIDS.
   a. Intuitively try to guess whether the probability of the person being with AIDS is greater than 90%.
   b. Now calculate the probability that the person is indeed with AIDS.

3. A mouse is at the origin of the axes. Every time it jumps one unit to the right with probability p , or one unit to the left with probability $1-p$.
   At the point $(2,0)$ there is a slice of cheese and at the point $(-2,0)$ there is a trap.
   What is the probability that the mouse eats the cheese before it falls into the trap?
   Instruction: condition on the first two jumps.

4. 18 soldiers in a certain platoon randomly line up in 6 columns of triplets (equivalent to 3 rows of 6 soldiers each).

   a. What is the size of the sample space?
   b. What is the probability of soldier Schweik standing in the front row?
   c. What is the probability of soldier Schweik standing next to (in the same row) soldier Jaime?
   d. If it is known that soldier Schweik is standing next to (in the same row) soldier Jaime, what is the probability that soldier Schweik is also standing next to (in the same row) private Ryan?
   e. If the platoon has 10 new immigrants. What is the probability that there is at least one triplet that consists of 3 new immigrants?

5. Given a complete numbered and undirected graph $G=(V,E)$ such that $|V|=n$.

   We define a subgraph of G to be $G_s=(V,E_s)$ such that $E_s\subseteq E$.

   Out of all the subgraphs of G one graph is randomly chosen.

   a. What is the probability that a subgraph that is a tree was chosen?
      Instruction: read in Wikipedia about "**Prüfer Sequence**".
      Relying on the fact that Prüfer Sequence is an injective function between the set of numbered trees with n vertices and the collection of vectors of size $n-2$ consisting the natural numbers between 1 to n, prove first that the number of numbered trees with n vertices is n^{n-2}.
   b. It is known that if a vertex i in a numbered tree has a degree d, it will appear $d-1$ times in a Prüfer Sequence. Using this fact, calculate the probability that a subgraph with exactly one vertex of degree 3 and the remaining vertices having a degree less than 3 was chosen if it is known that a subgraph that is a tree was chosen.
   c. For $n=7$, what is the probability that a subgraph with no vertex of degree 2 was chosen if it is known that a subgraph that is a tree was chosen?

6. There is no connection between the two items.

   a. Prove using induction that $P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1)P(A_2 \mid A_1)\cdot\ldots\cdot P\left(A_n \mid \bigcap_{i=1}^{n-1} A_i\right)$.

   b. Let (Ω, F, P) be a probability space and suppose that $B \in F$ is a specific event such that $P(B) > 0$.

      We define a function $Q : F \to \mathbb{R}$ as follows

      $\forall A \in F \quad Q(A) = P(A \mid B)$

      Prove that Q is a probability function, i.e. prove that Q satisfies the 3 conditions that must hold for a probability function.