We know that the eigenvectors of T^n are the same as the eigenvectors of T. This is provable:

$$vT^2 = (vT)\,T = \lambda vT = \lambda^2 v$$

Therefore if v is eigenvector of T with eigenvalue λ, we get that is is also an eigenvector of T^n with eigenvalue λ^n.(can be fully proved using induction).

Therefore, v will also be an eigenvector of e^T with eigenvalue e^λ:

$$ve^T = v\sum \frac{1}{n!}T^n = \sum \frac{1}{n!}vT^n = \sum \frac{1}{n!}\lambda^n v = e^\lambda v$$

We have an unknown T, but we know $T^2, T^3, T^4, ...$ Therefore we can construct matrix H:

$$H = \sum_{n=2}^{\infty} \frac{1}{n!}T^n$$

This is equal to:

$$H = \sum_{n=2}^{\infty} \frac{1}{n!}T^n = e^T - T - 1$$

We can easily see that the eigenvector of T are H's eigenvectors too, with eigenvalue $e^\lambda - \lambda - 1$.

Therefore the algorithm would be, construct $H = \sum_{n=2}^{\infty} \frac{1}{n!}T^n$. Find it's eigenvectors and eigenvalues. The eigenvectors are T's eigenvectors too. The eigenvalues will be equal to:

$$\lambda_H = e^\lambda - \lambda - 1$$

Where λ_H is the eigenvalue we have found for H. Solving this equation we get the eigenvalue of T. Doing this for all eigenvalues of H will give us all eigenvalues of T. Then we can construct T simply:

$$T = V\Lambda V^{-1}$$

Because the values of T are bound (stochastic matrix), we can use final number N in the construction of $H = \sum_{n=2}^{N} \frac{1}{n!}T^n$ and assure a certain maximal error (not getting into this now).

We can also use the algorith in a general case if we are missing T^k where $k \in \{n_1, n_2, ...\}$, as in missing for example T^1, T^3, T^{18}. In this case we will construct:

$$H = \sum_{n \notin \{n_1, n_2, ...\}}^{\infty} \frac{1}{n!}T^n = e^T - \sum_{k \in \{n_1, n_2, ...\}} \frac{1}{k!}T^k$$

After finding the eigenvectors we will find the eigenvalues using the equation:

$$\lambda_H = e^{\lambda} - \sum_{k \in \{n_1, n_2, \ldots\}} \frac{1}{k!} \lambda^k$$

Again, because the values of T are bound (stochastic matrix), we can use final number N in the construction of $H = \sum_{n=2}^{N} \frac{1}{n!} T^n$ and assure a certain maximal error (not getting into this now).

Prime orders

We have thought of finding only primes as in T^2, T^3, T^5 and constructing other orders using these. This can maybe give us better approximations and faster results instead of finding T^{21} we can use $\left(T^3\right)^7$ or $\left(T^7\right)^3$