Complexity Theory 236313 - Homework Assignment #1

Due November 29, 2022. Submit a single PDF file to the course site

November 15, 2022

Question 1. Prove a hierarchy theorem for NSPACE using Immerman's theorem.

Question 2. Let StrongCON = $\{G \mid \text{G is a strongly connected directed graph}\}$. Prove that StrongCON is NL-complete.

Question 3. Prove that if EXP $\neq$ NEXP then NP $\neq$ P.

Question 4. Let L be the language of all tuples (G, s, t, d, k) such that:

- $G = (V, E)$ is an undirected graph,
- $s, t \in V$ are vertices in G,
- $d, k \in \mathbb{N}$ are natural numbers given in unary (e.g. 3 is written as 111),
- There are k paths of length d from s to t in G.

Prove that $L \in \text{DSPACE}\left(\log^2 n\right)$.

Question 5. A one-taped Turing machine M is said to accept x *"on the spot"* if M accepts x and during the execution, M does not access any cell to the right of the input, except (possibly) the first cell on its right.

Prove that $L = \{(\langle M \rangle, x) \mid M \text{ accepts } x \text{ on the spot}\}$ is PSPACE-complete.