שיעורי בית 3

Ex 1:

1) $l_1 = (2,-1,-1) + t(3,2,0)$

$l_2 = (-1,2,-4) + s(9,1,3)$

$2 + 3t = -1 + 9s$

$-1 + 2t = 2 + s \quad \Rightarrow 3s = 3$

$-1 = -4 + 3s \quad \Rightarrow \underline{s = 1}$

$\Rightarrow -1 + 2t = 2 + 1 \Rightarrow 2t = 4 \Rightarrow t = 2$

point of intersection:

$x = 2 + 2 \cdot 3 = 8$

$y = -1 + 2 \cdot 2 = 3 \quad \Rightarrow \quad P(8,3,-1)$

$z = -1$

2) the parametric representation of the plane

is: $(-2,1,-3) + t(3,2,0) + s(9,1,3)$

המשך בדף הבא!

המשך תרגיל (Ex 1,2)

$R_1 \quad x = -2 + 3t + 9s \Rightarrow 2x = -4 + 6t + 18s$

$R_2 \quad y = 1 + 2t + s \Rightarrow 3y = 3 + 6t + 3s$

$R_3 \quad z = -3 + 3s \Rightarrow \frac{z+3}{3} = s$

$R_1 - R_2 = \quad 2x - 3y = -7 + 15s$

$$\Rightarrow 2x - 3y = -7 + 15\left(\frac{z+3}{3}\right)$$

$$2x - 3y = -7 + 5z + 15$$

$$2x - 3y - 5z = 8$$

$Ex(1,3)$: $\dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$ $\quad P(2,3,4)$

$$\frac{|2 \cdot 2 - 3 \cdot 3 - 5 \cdot 4 - 8|}{\sqrt{2^2 + (-3)^2 + (-5)^2}} = \frac{33}{\sqrt{38}}$$

Ex 2: $\lambda : (-1,1,2) + t(3,2,4)$

plane : $2x + y - 3z + 4 = 0$

We use the normal vector of the given plane + the directional vector of the line as directional vectors of the plane + $(-1,1,2)$ as a point on the plane thus:

$$\pi = (-1,1,2) + t(3,2,4) + s(2,1,-3)$$

$$x = -1 + 3t + 2s$$
$$y = 1 + 2t + s \quad \Rightarrow \quad ① \; 2y - x = 3 + t$$
$$z = 2 + 4t - 3s \quad \Rightarrow \quad ② \; 3y + z = 5 + 10t$$

$$\Rightarrow 3y + z = 5 + 10(2y - x - 3)$$

$$= 3y + z + 5 + 20y - 10x - 30$$

$$\Rightarrow 10x - 17y + z + 25 = 0$$

Ex3: $d: (7,-5,-2)+t(2,-2,-3)$

$A(1,2,5)$

let's B be a point on the line thus:

$B: (7+2t, -5-2t, -2-3t)$

$\vec{BA} = \Big(1-(7+2t), 2-(-5-2t), 5-(-2-3t)\Big)$

$\vec{BA} = (-6-2t, 7+2t, 7+3t)$

the shortest distance will be when $\vec{BA}$ perpendicular to the line, so:

$$\begin{pmatrix} -6-2t \\ 7+2t \\ 7+3t \end{pmatrix} \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} = 0$$

$-12-4t-14-4t-21-9t=0$

$-17t = 47 \Rightarrow t = -\frac{47}{17}$

$x = 7+2\left(-\frac{47}{17}\right) = \frac{25}{17}$

$y = -5-2\left(\frac{-47}{17}\right) = \frac{9}{17}$

$z = -2-3\left(\frac{-47}{17}\right) =$

$B = \left(\frac{25}{17}, \frac{9}{17}, \frac{107}{17}\right)$

Ex 4:

1.
$$\begin{vmatrix} 1 & 0 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$= 1 \cdot \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 0 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$$

$$= 5 \times 9 - 6 \times 8 + 3(4 \times 8 - 7 \times 5) = 45 - 48 + 96 - 105 = -12$$

2.
$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 2 & 0 & 0 & 3 \\ 3 & 0 & 4 & 7 \end{vmatrix} \xrightarrow{R_2 \to R_2 - 3R_1} \Rightarrow \begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 0 & -2 & -4 \\ 2 & 0 & 0 & 3 \\ 3 & 0 & 4 & 7 \end{vmatrix}$$

$$= (-1)^{1+2} \cdot 2 \cdot \begin{vmatrix} 2 & -2 & -4 \\ 2 & 0 & 3 \\ 3 & 4 & 7 \end{vmatrix} =$$

$$(-1)^{1+2} \cdot 2 \left((-1)^{1+2} \cdot (-2) \cdot \begin{vmatrix} 2 & 3 \\ 3 & 7 \end{vmatrix} + (-1)^{2+2} \cdot 0 \cdot \begin{vmatrix} 2 & -4 \\ 3 & 7 \end{vmatrix} + (-1)^{3+2} \begin{vmatrix} 2 & -4 \\ 2 & 3 \end{vmatrix} \right)$$

$$(-1)^{1+2} \cdot 2 \left(2(2 \times 7 - 3 \times 3) - 4(2 \times 3 + 4 \times 2) \right) = -1 \cdot 2 \cdot (-46) = 92$$

Exs:

1) $\det(AB^{-1}) = \det A \cdot \det B^{-1}$

$= \det A \cdot \left(\frac{1}{\det B}\right) = \frac{\det A}{\det B}$

2) $\det(AB) = \det A \cdot \det B \neq 0 \Rightarrow$

$\det A \neq 0$ and $\det B \neq 0 \Rightarrow A$ and B are non-singular which mean that they are row equivalent to I and from I it is possible to get to A or B by element row operations

so:

$$A \rightleftarrows I \rightleftarrows B$$

3) $AB = 0$ $\quad$ A has A^{-1}

$\Rightarrow A^{-1} \cdot A \cdot B = A^{-1} \cdot 0 \Rightarrow I \cdot B = 0 \Rightarrow B = 0$

Ex 6:

1. lets create a matrix A defined by the vector as such : $A = \begin{array}{cc} \begin{pmatrix} 1, & 0, & 3 \\ 2 & 1 & -2 \\ 5 & 0 & 4 \end{pmatrix} & \begin{array}{c} \vec{v} \\ \vec{u} \\ \vec{w} \end{array} \end{array}$

The abs det A is the oriented volume of the parallelepiped defined by the row/columns of A.

$$\left\| \begin{matrix} 1 & 0 & 3 \\ 2 & 1 & -2 \\ 5 & 0 & 4 \end{matrix} \right\| = \left| (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ 5 & 4 \end{vmatrix} \right| = 4 - 15 = |-11| = 11$$

2. $\vec{AB} = (1,3)$ $\quad \vec{AC}\ (3,2)$ $\quad A = \begin{pmatrix} 3 & 2 \\ 1 & 3 \end{pmatrix}$

the abs det $\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix}$ divided by half will be the area of the triangle (half the oriented area of the parallelogram defined by the row of the matrix

$\det A = 3\times3 - 2\times1 = 7$

the triangle area $= \frac{7}{2} = 3.5$

Ex 7:

$$4x_1 - 5x_2 + x_3 = 0$$
$$x_2 + x_3 = 0$$
$$x_1 + 2x_2 + 3x_3 = 1$$

$$\Rightarrow \begin{array}{c} \begin{array}{cc} \quad A & \qquad b \end{array} \\ \left|\begin{array}{ccc|c} 4 & -5 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 3 & 1 \end{array}\right| \end{array}$$

$$|A| = (-1)^{1+1} \cdot 4 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + (-1)^{3+1} \begin{vmatrix} -5 & 1 \\ 1 & 1 \end{vmatrix} = 4(3-2) + (-5-1) = -2$$

$$|A_1| = \begin{vmatrix} 0 & -5 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 1^{3+1} \begin{vmatrix} -5 & 1 \\ 1 & 1 \end{vmatrix} = (-5-1) = -6$$

$$|A_2| = \begin{vmatrix} 4 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 3 \end{vmatrix} = (-1)^{2+3} \cdot \begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} = -1(4-0) = -4$$

$$|A_3| = \begin{vmatrix} 4 & -5 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix} = (-1)^{3+3} \begin{vmatrix} 4 & -5 \\ 0 & 1 \end{vmatrix} = 4 - 0 = 4$$

$$x_1 = \frac{|A_1|}{|A|} = \frac{-6}{-2} = 3, \quad x_2 = \frac{|A_2|}{|A|} = \frac{-4}{-2} = 2, \quad x_3 = \frac{|A_3|}{|A|} = \frac{4}{-2} = -2$$