Linear Algebra for MLDS - Homework 1

Vectors and Matrices

Make sure to read and follow the "Homework Submission Instructions" file

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Exercise 1: Given the vectors $\bar{a} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \bar{b} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$

Calculate $\bar{a} + \bar{b}, \|\bar{a} + \bar{b}\|, (\bar{a} - 2\bar{b}) \cdot (3\bar{a} + \bar{b})$

Exercise 2: Given the vectors $\bar{a} = \begin{pmatrix} t \\ 3 \end{pmatrix}, \bar{b} = \begin{pmatrix} 1 \\ t \end{pmatrix}$ where $t \in \mathbb{R}$ is a parameter.

Find all values for t for which the following occur, for each draw an example.

1. $\bar{a}$ and $\bar{b}$ are parallel.
2. $\bar{a}$ and $\bar{b}$ are perpendicular.
3. The angle between $\bar{a}$ and $\bar{b}$ is acute.
4. The angle between $\bar{a}$ and $\bar{b}$ is obtuse.

Exercise 3: Prove the following statement for two vectors $\bar{a}, \bar{b} \in \mathbb{R}^n$:
$\bar{a} + \bar{b}$ is perpendicular to $\bar{a} - \bar{b}$ if and only if $\bar{a}$ and $\bar{b}$ have the same length.

Exercise 4: Given the following matrices: $A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 & -3 \\ 2 & 5 & 4 \\ 0 & 4 & 9 \end{pmatrix}$

Calculate $A - 2B, AB, B^t A^t, A^2$

try to observe some relation between two of these.

Exercise 5: Prove the following statements:

1. If A and B are $m \times n$ skew-symmetric matrices and $\alpha \in \mathbb{R}$ then $A + \alpha B$ is skew-symmetric.
2. If A is any matrix then $\frac{1}{2}(A + A^t)$ is symmetric and $\frac{1}{2}(A - A^t)$ is skew-symmetric.
3. Deduce from item (2) that any matrix A can be written as a sum of a symmetric matrix and a skew-symmetric matrix.
4. Demonstrate item (3) for $A = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 4 & 3 \\ 0 & 2 & 2 \end{pmatrix}$

Exercise 6: Prove or Disprove the following claims, for $n \times n$ matrices A, B, C:

1. If A, B are upper triangular then $AB = BA$
2. If A is a scalar matrix then $AB = BA$
3. If $A^2 = 0$ then $A = 0$
4. If $A^2 - I = 0$ then $A = I$ or $A = -I$
5. If $AB = BA$ and $AC = CA$ then $A(B + C^3) = (B + C^3)A$