HW5 - אלגברה לינארית

[redacted]

Ex. 1 ① $\dim \mathbb{R}^3 = 3$ and B has 3 vectors

so we need to only prove that B is

linearly independent:

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow{R_3 \to R_3 - R_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow{R_3 \to R_3 - R_2}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow$$ HLS has only trivial solution $\Rightarrow$ B is L.I

so B is a basis for $\mathbb{R}^3$

② $$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 1 & 1 & 1 & 4 \end{array}\right) \xrightarrow{R_3 \to R_3 - R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 2 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right) \Rightarrow \begin{matrix} \alpha_1 = 2 \\ \alpha_2 = 3 \\ \alpha_3 = -1 \end{matrix} \Rightarrow [v]_B = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

(3) $[v]_B = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$

$$v = 2\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - 1\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

(4) $$\left(\begin{array}{ccc|c} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 1 & 1 & 1 & z \end{array}\right) \xrightarrow{R_3 \to R_3 - R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 1 & 1 & z-x \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z-x-y \end{array}\right)$$

$$\Rightarrow [v]_B = \begin{pmatrix} x \\ y \\ z-x-y \end{pmatrix}$$

Ex 2 $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$\xrightarrow{R_3 \to R_3 + R_2} \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\text{Row}(A) = \{ (1,2,1), (0,-1,0) \}$

$\text{col}(A) = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \right\}$

$\text{rank}(A) = \dim \text{col}(A) = \dim \text{row}(A) = 2$

$\text{nul}(A) \to$ from the r.e.f above

x_3 is a free variable let's set it as $x_3 = t$

$\Rightarrow \begin{array}{l} x_1 + 2x_2 + t = 0 \\ -x_2 = 0 \\ x_3 = t \end{array} \Rightarrow \begin{array}{l} x_1 = -t \\ x_2 = 0 \\ x_3 = t \end{array}$

$\text{nul}(A) = \text{span} \left\{ \begin{pmatrix} -t \\ 0 \\ t \end{pmatrix}, t \in R \right\}$

the basis of $\text{nul}(A)$ is $\left\{ \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$

nullity $= 1$

: Ex2 [illegible]

$$A = \begin{pmatrix} 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 2 & 4 & -1 & 3 & 1 \end{pmatrix} \xrightarrow{R_3 \to R_3 - 2R_1}$$

$$\begin{pmatrix} 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & -1 & -1 & -1 \end{pmatrix} \xrightarrow{R_3 \to R_3 + R_2} \begin{pmatrix} 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- $Row(A) = \{(1,2,0,2,1), (0,0,1,1,1)\}$

$col(A) = \left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\}$

$rank(A) = 2$

- $nul(A) \Rightarrow$

$x_1 + 2x_2 + 2x_4 + x_5 = 0$

$x_3 + x_4 + x_5 = 0$

x_2, x_4, x_5 are free variables

lets $x_2 = 1 \quad x_4 = 0 \quad x_5 = 0$

$\Rightarrow x_1 = -2$

$x_2 = 1$

$x_3 = 0$

$x_4 = 0$

$x_5 = 0$

$$\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

let's take $x_2 = 0$, $x_4 = 1$ $x_5 = 0$

$$\Rightarrow \begin{pmatrix} -2 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

let's take $x_2 = 0$, $x_4 = 0$, $x_5 = 0$

$$\Rightarrow \begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\text{nul}(A) = \left\{ \begin{pmatrix} -x_5 - 2x_4 - 2x_2 \\ x_2 \\ -x_5 - x_4 \\ x_4 \\ x_5 \end{pmatrix}, \; x_2, x_4, x_5 \in \mathbb{R} \right\}$$

$$\text{basis of nul}(A) = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

nullity $= 3$

Ex3:

1. $F\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^2 \\ x \\ y \end{pmatrix}$

$$F\left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right) = \begin{pmatrix} (x_1+x_2)^2 \\ x_1+x_2 \\ y_1+y_2 \end{pmatrix} = \begin{pmatrix} x_1^2+2x_1x_2+x_2^2 \\ x_1+x_2 \\ y_1+y_2 \end{pmatrix}$$

$$\neq \begin{pmatrix} x_1^2 \\ x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2^2 \\ x_2 \\ y_2 \end{pmatrix}$$

thus F is not a linear transformation

2. $F\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a+d$

$$F\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} w & x \\ y & z \end{pmatrix}\right) = F\begin{pmatrix} a+w & b+x \\ c+y & d+z \end{pmatrix}$$

$$= a+w+d+z = a+d+w+z$$

$$= F\begin{pmatrix} a & b \\ c & d \end{pmatrix} + F\begin{pmatrix} w & x \\ y & z \end{pmatrix}$$

$$F\left(\alpha\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = \alpha a + \alpha d = \alpha(a+d) = \alpha F\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

thus F is L.T.

③ $F\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$F\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} w & x \\ y & z \end{pmatrix}\right) = F\begin{pmatrix} a+w & b+x \\ c+y & d+z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\neq F\begin{pmatrix} a & b \\ c & d \end{pmatrix} + F\begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

thus F is not L.T

other option:

$F\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \neq \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow$ not L.T

Exa: $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y \\ y+z \end{pmatrix}$

$\ker T = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

$= \begin{matrix} x+y=0 \\ y+z=0 \end{matrix} \Rightarrow \begin{matrix} x=-y \\ x=z \end{matrix}$

$\ker(T) = \left\{ \begin{pmatrix} x \\ -x \\ x \end{pmatrix} x \in \mathbb{R} \right\}$

basis of $\ker(T) = \left\{ \begin{matrix} 1 \\ -1 \\ 1 \end{matrix} \right\}$

$\dim \ker(T) = 1$

$$\text{Range}(T) = \left\{ \begin{pmatrix} x+y \\ y+z \end{pmatrix} \middle| \ x, y, z \in \mathbb{R} \right\}$$

can be represented as $\left\{ x \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y \begin{pmatrix} 1 \\ 1 \end{pmatrix} + z \begin{pmatrix} 0 \\ 1 \end{pmatrix} \middle| \ x, y, z \in \mathbb{R} \right\}$

let's check linear dependency

we can see that $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is linear combination of the other two so

basis of range(T) is $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$

$\dim(\text{range}(T)) = 2$

$\ker(T) \neq \{0\}$ so T is not one-to-one

so T is not invertible

Ex4 : $t^{en?}$

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x - 2z \\ y + z \\ x + 2y \\ x + y - z \end{pmatrix}$$

$$\ker(T) = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \;\middle|\; \begin{matrix} x - 2z = 0 \\ y + z = 0 \\ x + 2y = 0 \\ x + y - z = 0 \end{matrix} \right\}$$

HLS

$$\begin{array}{c} \begin{matrix} x & \;\; y & \;\; z \end{matrix} \\ \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & -1 \end{pmatrix} \end{array} \xrightarrow[R_4 \to R_4 - R_1]{R_3 \to R_3 - R_1} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow[R_3 \to R_3 - 2R_2]{R_4 \to R_4 - R_2}$$

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{matrix} x - 2z = 0 \\ y + z = 0 \\ z = t \end{matrix} = \begin{pmatrix} 2t \\ -t \\ t \end{pmatrix}$$

$$\ker T = \left\{ \begin{pmatrix} 2t \\ -t \\ t \end{pmatrix} \;\middle|\; t \in \mathbb{R} \right\} = \operatorname{span}\left\{ \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \right\}$$

$$\dim(\ker(T)) = 1$$

$$\text{Range}(T) = \left\{ \begin{pmatrix} x - 2z \\ y + z \\ x + 2y \\ x + y - z \end{pmatrix} \middle| \ x, y, z \in \mathbb{R} \right\}$$

$$= \text{span} \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \\ -1 \end{pmatrix} \right\}$$

$$-2 \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$ so we can

remove the third vector

thus basis of $\text{range}(T) = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix} \right\}$

$\dim(\text{range}(T)) = 2$

$\dim(\text{range}(T)) + \dim(\ker(T)) = 2 + 1 = 3 = \dim \mathbb{R}^3$

$\ker T \neq \{0\}$ so T is not one to one

$\Rightarrow T$ is not invertible

Ex 5: $F\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ x+y \\ -x \end{pmatrix}$

$$B = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$F(v_1) = F\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$F(v_2) = F\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$F(v_3) = F\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$[F(v_1)]_B = \alpha_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$[F(v_2)]_B = \quad -\text{''}- \qquad -\text{''}- \qquad -\text{''}- \quad = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$[F(v_3)]_B = \quad -\text{''}- \qquad -\text{''}- \qquad -\text{''}- \quad = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{array}{c} \begin{array}{cccccc} \alpha_1 & \alpha_2 & \alpha_3 & v_1 & v_2 & v_3 \end{array} \\ \left(\begin{array}{ccc|c|c|c} 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & 1 & -1 & 0 & -1 \end{array}\right) \end{array} \xrightarrow{R_2 \to R_2 - R_1}$$

$$\left(\begin{array}{ccc|c|c|c} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & -1 \end{array}\right) \xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|c|c|c} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 2 & -2 & -1 & -1 \end{array}\right)$$

$$\xrightarrow[R_1 \to 2R_1 - R_3]{R_2 \to 2R_2 + R_3} \left(\begin{array}{ccc|c|c|c} 2 & 0 & 0 & 4 & 1 & 3 \\ 0 & 2 & 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & -2 & -1 & -1 \end{array}\right)$$

$$[F(v_1)]_B = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

$$[F(v_2)]_B = \begin{pmatrix} 0.5 \\ 0.5 \\ -0.5 \end{pmatrix}$$

$$[F(v_3)]_B = \begin{pmatrix} 1.5 \\ -0.5 \\ -0.5 \end{pmatrix}$$

$$[F]_B^B = A = \begin{pmatrix} 2 & 0.5 & 1.5 \\ 0 & 0.5 & -0.5 \\ -1 & -0.5 & -0.5 \end{pmatrix} =$$

$$\left[F\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}\right]_B = A \cdot \left[\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}\right]_B$$

$$\left[\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}\right]_B = \alpha_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 4 \\ 0 & 1 & 1 & 4 \end{array}\right) \xrightarrow{R_2 \to R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 2 \\ 0 & 1 & 1 & 4 \end{array}\right) \longrightarrow$$

$R_3 \to R_3 - R_2$

$$\xrightarrow{} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 2 & 2 \end{array}\right)$$

$$\begin{cases} \alpha_1 + \alpha_3 = 2 \\ \alpha_2 - \alpha_3 = 2 \\ \alpha_3 = 1 \end{cases} \Rightarrow \begin{cases} \alpha_1 = 1 \\ \alpha_2 = 3 \\ \alpha_3 = 1 \end{cases}$$

$$\left[\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}\right]_B = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$$

- $$A = \left[F\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}\right]_B = \begin{pmatrix} 2 & 0.5 & 1.5 \\ 0 & 0.5 & -0.5 \\ -1 & -0.5 & -0.5 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}$$

$$F\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} = 5\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - 3\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix}$$

- Let's check if F

$$F\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2+4 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix}$$

$$\mathrm{Det}(A) = 2 \cdot (-0{,}25 - 0{,}25) - 1(-0{,}25 - 0{,}75)$$

$\Rightarrow \mathrm{Det}(A) = -1 + 1 = 0 \Rightarrow A$ is singular

$\Rightarrow T$ is not invertible

Ex6:

(1) F is one to one L.T thus

$\dim(\ker(F)) = 0$

$\dim(\ker(F)) + \dim(\text{range}(F)) = \dim \mathbb{R}^3 = 3$

thus $\dim(\text{range}(F)) = 3$ False

(2) F is onto $\mathbb{R}^3$ thus $\text{range}(F) = \mathbb{R}^3$

$\Rightarrow \dim \text{range}(F) = 3$

$\dim(\ker(F)) + \dim(\text{range}(F)) = 5$

thus $\dim(\ker F) = 2$ True

(3) let assume there is L.T as described

$\dim(\ker F) = 0$

$\dim(\ker(F)) + \dim(\text{range}(F)) = 4$

$\dim(\text{range}(F)) = 4 > \dim \mathbb{R}^3 = 3$ which cannot be

thus $\dim(\ker(F)) \neq 0$ and F is not one to one True

④ F is invertible $\Rightarrow$

$\dim(\ker(F)) = 0$ $\Rightarrow$ from the one to one property

$\Rightarrow$ $\dim(\text{range}(F)) = n$

also

$\text{range}(F) = \mathbb{R}^m$ $\Rightarrow$ from the onto property

$\dim(\text{range}(F)) = m$

thus $n = m$ True

⑤ Let $e_1, e_2 \in \mathbb{R}^2$ be bases for $\mathbb{R}^2$

let's define $T: \mathbb{R}^2 \to \mathbb{R}^2$ such

$T_{e_1} = e_2$

$T_{e_2} = 0$

$T(ae_1 + be_2) = aT_{e_1} + bT_{e_2} = ae_2$

$\Rightarrow T(ae_1 + be_2) = 0 \Leftrightarrow a = 0$

thus: $\ker(T) = \{be_2, b \in \mathbb{R}\}$

$\text{range}(T) = \{ae_2, a \in \mathbb{R}\}$

thus $\ker(T) = \text{range}(T)$ False

6. Let $F\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ x \end{pmatrix}$ and $S\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$

both have same ker $= \left\{ \begin{pmatrix} 0 \\ y \end{pmatrix}, y \in \mathbb{R} \right\}$

but thier ranges are obviously different. False