Linear Algebra for MLDS - Homework 2

Linear Systems, Matrix Inverse

Make sure to read and follow the "Homework Submission Instructions" file

Submit by: April 7, 2022 at 23:59

Exercise 1: Solve the following Linear Systems:

1. $$\begin{aligned} x_1 + 3x_3 &= 7 \\ 2x_2 - 4x_3 &= -8 \\ 3x_1 + 2x_2 + 10x_3 &= 23 \end{aligned}$$

2. $$\begin{aligned} x_2 + 3x_3 &= 0 \\ 6x_1 - 3x_3 &= 4 \\ -2x_1 + 2x_2 &= 1 \end{aligned}$$

3. $$\begin{aligned} -4x_1 + 6x_2 &= -12 \\ x_1 - x_2 &= 1 \\ 2x_1 + x_2 &= 17 \\ 6x_1 - 8x_2 &= 0 \end{aligned}$$

Exercise 2: Let A be a 3×5 matrix such that the solution set of its homogeneous system $A\bar{x} = \bar{0}$ has two free variables, show that for any $\bar{b} \in \mathbb{R}^3$, the system $A\bar{x} = \bar{b}$ has a solution.

Exercise 3: Let A be an $m \times n$ matrix, $\bar{b} \in \mathbb{R}^n$. Prove or disprove the following claims regarding: $A\bar{x} = \bar{b}$

1. If $m > n$ then $A\bar{x} = \bar{b}$ has no solutions.
2. If $m < n$ then $A\bar{x} = \bar{b}$ has infinite solutions.
3. If $m < n$ and $\bar{b} = \bar{0}$ then $A\bar{x} = \bar{b}$ has infinite solutions.
4. If $m > n$ and $\bar{b} = \bar{0}$ then $A\bar{x} = \bar{b}$ has a unique solution.

Exercise 4: Check whether the following matrices are invertible, and if so, find their inverse.

1. $$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

2. $$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & 2 \\ 2 & 1 & 2 \end{pmatrix}$$

Exercise 5: Let A be an $n \times n$ matrix

1. Show that if $A^2 = 0$ then A is singular
2. Show that if $A^2 - 2A + I = 0$ then A is non-singular, find A^{-1}

continues on the next page →

Exercise 6: Let A, B be non-singular matrices with: $A^{-1} = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ $B^{-1} = \begin{pmatrix} 0 & 3 & 1 \\ 3 & 1 & 3 \\ 1 & 3 & 0 \end{pmatrix}$, without explicitly calculating the matrices A, B, do the following:

1. Calculate $(BA^2)^{-1}$
2. Find a vector $\bar{x}$ such that $A\bar{x} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$
3. Prove/Disprove: A, B are row equivalent.
4. Find the number of solutions to $A^2\bar{x} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$

Exercise 7:

1. Find the parametric representation of the line passing through the point $(-1, 2, 3)$, parallel to the vector $\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$
2. Does this line pass through the points $(2, 2, 1), (-10, 2, 0)$?
3. Find the intersection of this line with the xy plane.