Linear Algebra for MLDS - Homework 3

Analytic Geometry, Determinants

Make sure to read and follow the "Homework Submission Instructions" file

Submit by: May 5, 2022 at 23:59*

Exercise 1: Given the lines: $\ell_1 : (2, -1, -1) + t(3, 2, 0), \ell_2 : (-1, 2, -4) + s(9, 1, 3)$

1. Do the lines ℓ_1, ℓ_2 intersect?

2. Find the equation of the plane that is parallel to the two lines and passes through the point $A(-2, 1, -3)$

3. Find the distance between this plane and the point $P(2, 3, 4)$

Exercise 2: Find an equation of the plane that contains the line $\ell : (-1, 1, 2) + t(3, 2, 4)$ and is perpendicular to the plane $2x + y - 3z + 4 = 0$

Exercise 3: Find the point B on the line $\ell : (7, -5, -2) + t(2, -2, -3)$, that has minimal distance to $A(1, 2, 5)$

Exercise 4: Calculate the following determinants:

1. $\begin{vmatrix} 1 & 0 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

2. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 2 & 0 & 0 & 3 \\ 3 & 0 & 4 & 7 \end{vmatrix}$

Exercise 5: Let A, B be two $n \times n$ matrices, prove the following statements:

1. If A, B are both non-singular, then $\det(AB^{-1}) = \det(A)/\det(B)$

2. If $\det(AB) \neq 0$ then A, B are row-equivalent.

3. If $AB = 0$ and A is non-singular, then $B = 0$

Exercise 6: Find the volumes of the following shapes:

1. The parallelepiped spanned by the vectors $(1, 0, 3), (2, 1, -2), (5, 0, 4)$

2. The triangle with vertices $A(0, 1), B(1, 4), C(3, 3)$

Exercise 7: Solve the following linear system using Cramer's Rule:

$$
\begin{array}{rrrrrrcl}
4x_1 & - & 5x_2 & + & x_3 & = & 0 \\
 & & x_2 & + & x_3 & = & 0 \\
x_1 & + & 2x_2 & + & 3x_3 & = & 1
\end{array}
$$

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*Note that this is independence day, plan your time accordingly