

Extended Kalman Filter Summary

System Model:

$$egin{aligned} x_k &= \mathbf{f}(x_{k-1}, u_k, w_k) \ z_k &= \mathbf{h}(x_k, v_k) \end{aligned}$$

Assumptions:

$$w_k \sim N(0, \mathbf{Q}_k)$$
 $E(w_k w_j^T) = \mathbf{Q}_k \delta_{k-j}$ Not correlated with time $E(v_k v_j^T) = \mathbf{R}_k \delta_{k-j}$ Not correlated with time $E(w_k v_j^T) = \mathbf{Q}_k \delta_{k-j}$ Not correlated with time $E(w_k v_j^T) = \mathbf{Q}_k \delta_{k-j}$ Not correlated with time $E(w_k v_j^T) = \mathbf{Q}_k \delta_{k-j}$ Not correlated with time $E(w_k v_j^T) = \mathbf{Q}_k \delta_{k-j}$ Not correlated with time $E(w_k v_j^T) = \mathbf{Q}_k \delta_{k-j}$ Not correlated with time $E(w_k v_j^T) = \mathbf{Q}_k \delta_{k-j}$

Predict:

$$egin{aligned} \hat{x}_k^- &= f(\hat{x}_{k-1}^+, u_k, 0) \ \mathbf{P}_k^- &=
abla f_x \mathbf{P}_{k-1}^+
abla f_x^T +
abla f_w \mathbf{Q}_k
abla f_w^T \end{aligned}$$

Update:

$$egin{aligned}
u_k &= z_k - h(\hat{x}_k^-, 0) \ S_k &=
abla h_x P_k^-
abla h_x^T +
abla h_v R_k
abla h_v^T \ \mathbf{K}_k &= \mathbf{P}_k^-
abla h_x^T \mathbf{S}_k^{-1} \ \hat{x}_k^+ &= \hat{x}_k^- + \mathbf{K}_k
u_k \ \mathbf{P}_k^+ &= (\mathbf{I} - \mathbf{K}_k
abla h_x) \, \mathbf{P}_k^- \end{aligned}$$



Extended Kalman Filter Summary

Process Model:

$$x_k = \mathbf{f}(x_{k-1}, u_k, w_k)$$

$$\left\|
abla f_x = \left. rac{\partial f}{\partial x}
ight|_{x=\hat{x}_{k-1}^+}$$

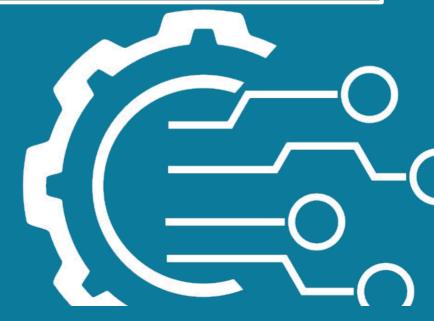
$$oxed{
abla} f_w = rac{\partial f}{\partial w} ig|_{x=\hat{x}_{k-1}^+}$$

Measurement Model:

$$z_k = \mathbf{h}(x_k, v_k)$$

$$\left\|
abla h_x = rac{\partial h}{\partial x}
ight|_{x=\hat{x}_k^-}$$

$$oxed{
abla} h_v = rac{\partial h}{\partial v}ig|_{x=\hat{x}_k^-}$$





Extended Kalman Filter Summary

Discrete - Time Extended Kalman Filter Algorithm

1. The system and measurement equations are given in the following form:

$$egin{aligned} x_k &= \mathbf{f}(x_{k-1}, u_k, w_k) \ z_k &= \mathbf{h}(x_k, v_k) \ w_k &\sim N(0, \mathbf{Q}_k) \ v_k &\sim N(0, \mathbf{R}_k) \end{aligned}$$

2. The filter is initialised as follows:

$$egin{aligned} \hat{x}_0^+ &= E[x_0] \ P_0^+ &= E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] \end{aligned}$$

- 3. For each time step k = 1, 2, ..., perform the follow prediction/update steps:
 - a. Calculate the process model Jacobian Matrices for time k 1 using:

$$egin{align}
abla f_x &= \left. rac{\partial f}{\partial x}
ight|_{x=\hat{x}_{k-1}^+} \
abla f_w &= \left. rac{\partial f}{\partial w}
ight|_{x=\hat{x}_{k-1}^+} \
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abla f$$

b. Perform the Kalman Filter state and covariance prediction step calculations:

$$egin{aligned} \hat{x}_k^- &= f(\hat{x}_{k-1}^+, u_k, 0) \ \mathbf{P}_k^- &=
abla f_x \mathbf{P}_{k-1}^+
abla f_x^T +
abla f_w \mathbf{Q}_k
abla f_w^T \end{aligned}$$



Extended Kalman Filter Summary

Discrete - Time Extended Kalman Filter Algorithm (Cont)

c. Calculate the measurement model Jacobian matrices for time k using:

$$egin{array}{l}
abla h_x &= rac{\partial h}{\partial x}ig|_{x=\hat{x}_k^-} \
abla h_v &= rac{\partial h}{\partial v}ig|_{x=\hat{x}_k^-} \end{array}$$

d. Perform the Kalman Filter state and covariance measurement update step calculations:

$$egin{aligned}
u_k &= z_k - h(\hat{x}_k^-, 0) \ S_k &=
abla h_x P_k^-
abla h_x^T +
abla h_v R_k
abla h_v^T \ \mathbf{K}_k &= \mathbf{P}_k^-
abla h_x^T \mathbf{S}_k^{-1} \ \hat{x}_k^+ &= \hat{x}_k^- + \mathbf{K}_k
u_k \ \mathbf{P}_k^+ &= (\mathbf{I} - \mathbf{K}_k
abla h_x) \, \mathbf{P}_k^- \end{aligned}$$

