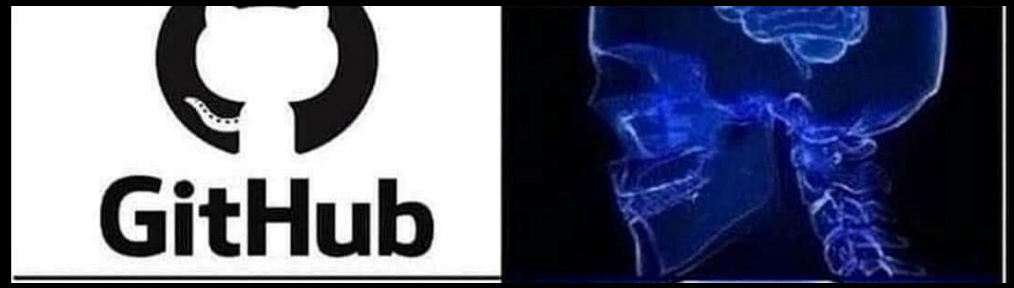
Housekeeping

- Syllabus
- Project
- Feedback so far
  - What’s going well
  - Not so well?
  - What can WE do?

Meme sourced from my sister’s repository



Meme sourced from my sister’s repository

Meme sourced from my sister’s repository

Meme sourced from my sister’s repository

gitolite.engsci.utoronto.ca

Graphs: Searching

Week 3

Content

Today

- Graphs
  - Terminology, examples
  - Notation

Friday

- Graph algorithms
  - Conceptual

Next week

  - Advanced graph algorithm(s)
  - Implementation and code

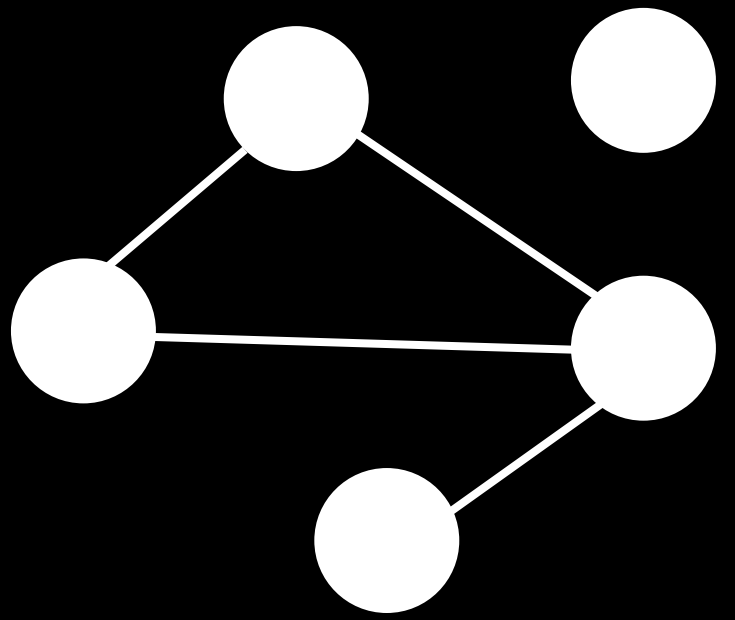
Graphs

What I used to think graphs are

Graphs

What I used to think graphs are

What graphs really are



Graphs

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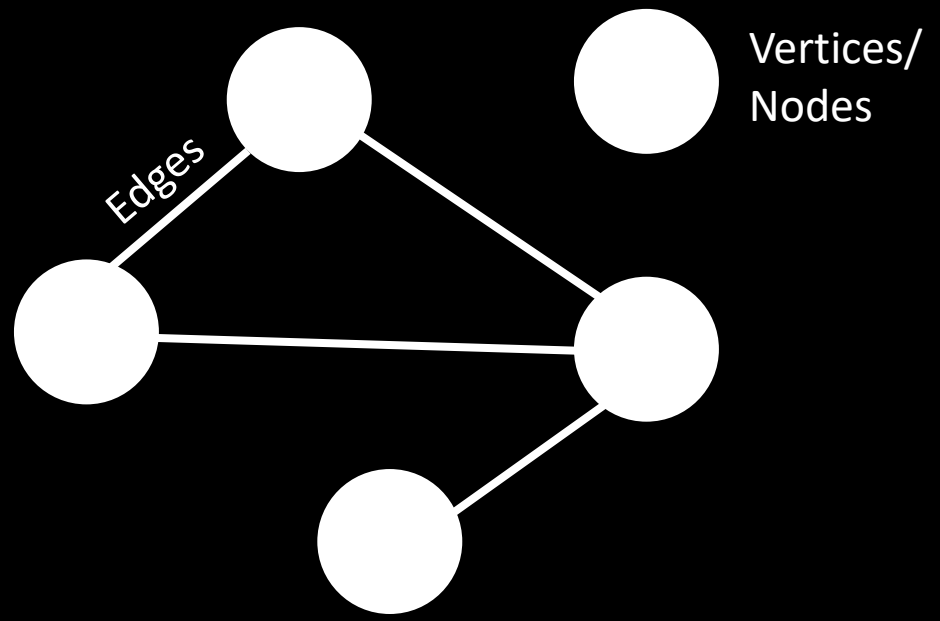

Graphs

- A way to show relationships
- Vertices/nodes store some sort of data
- Edges indicate the relationship

Graphs

- A way to show relationships
- Vertices/nodes store some sort of data
- Edges indicate the relationship
- Trees are a type of graph!
  - Heaps (a type of tree) are also a type of graph

Graphs

An EngSci Social Network (Acebook?)

Graphs

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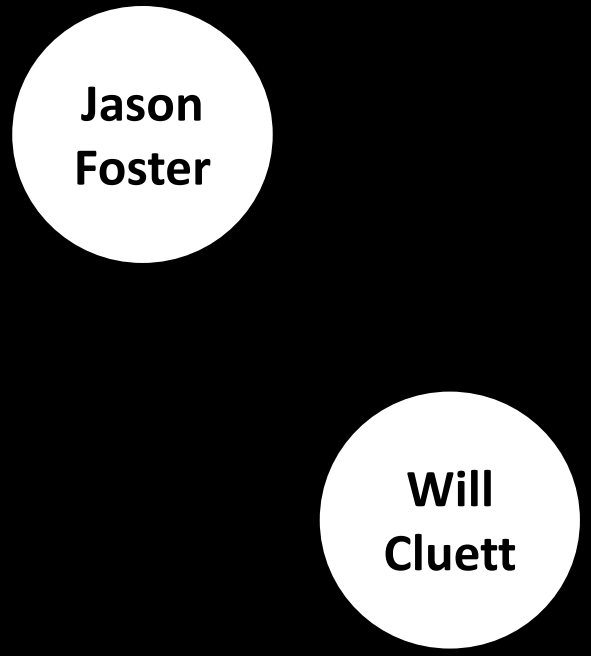

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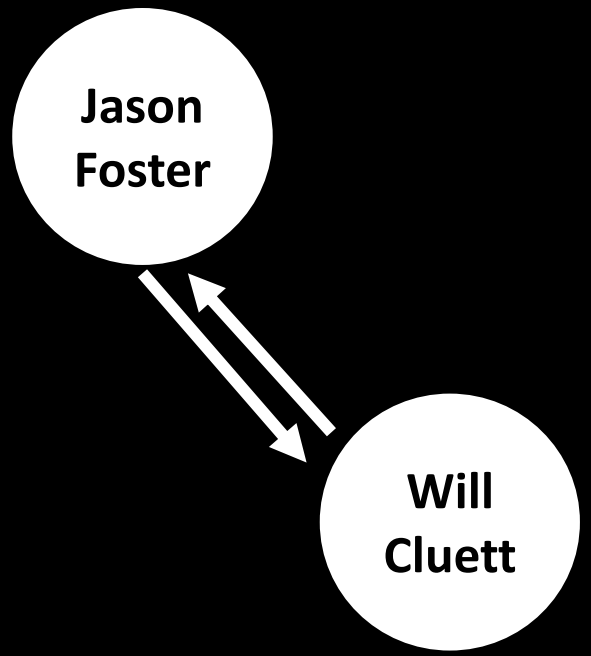

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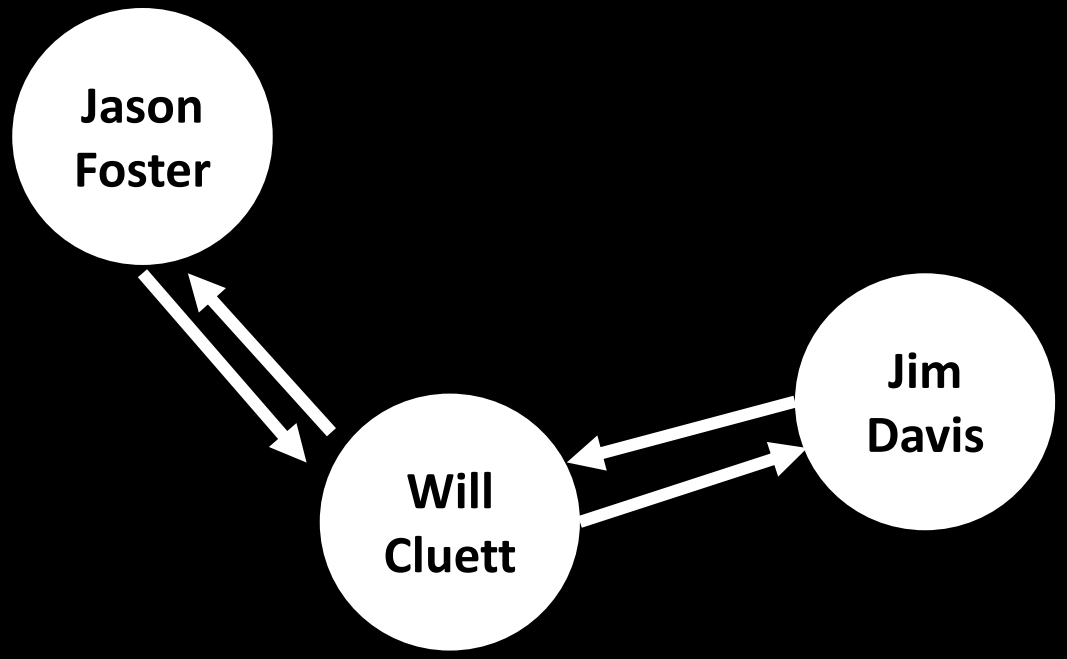

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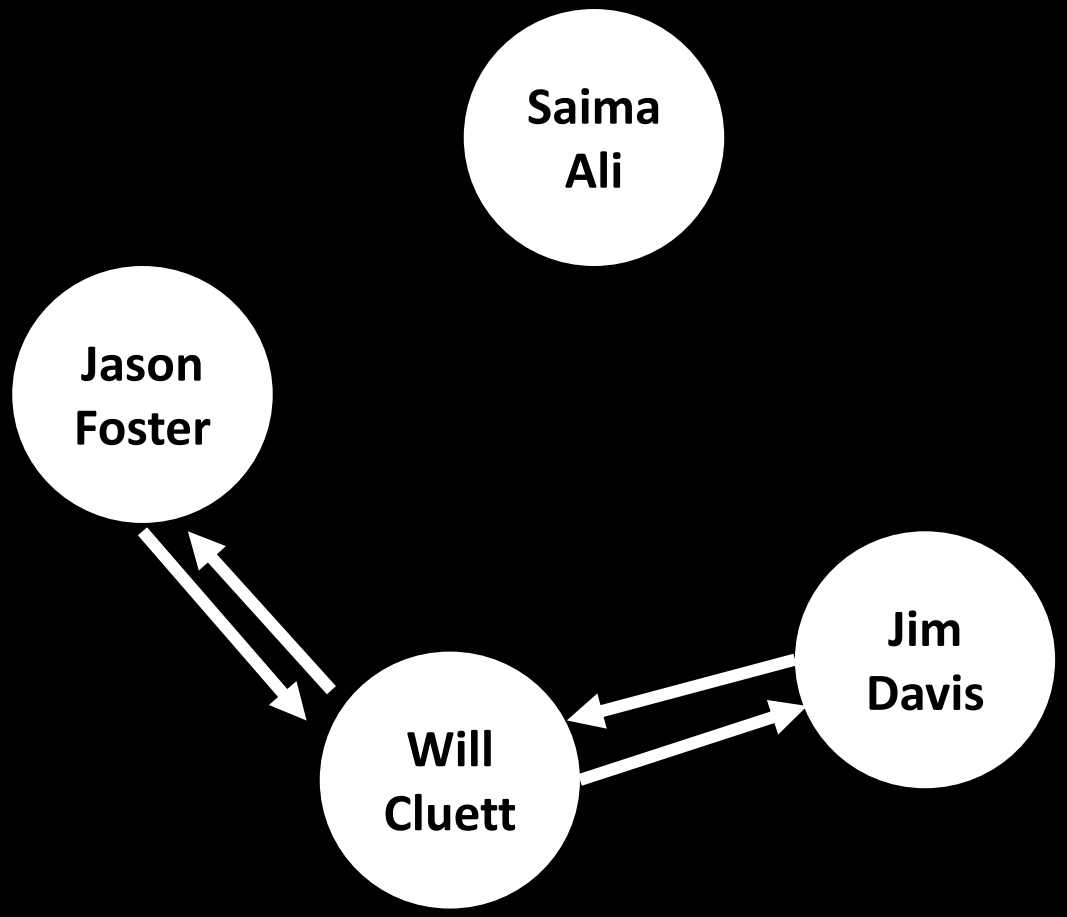

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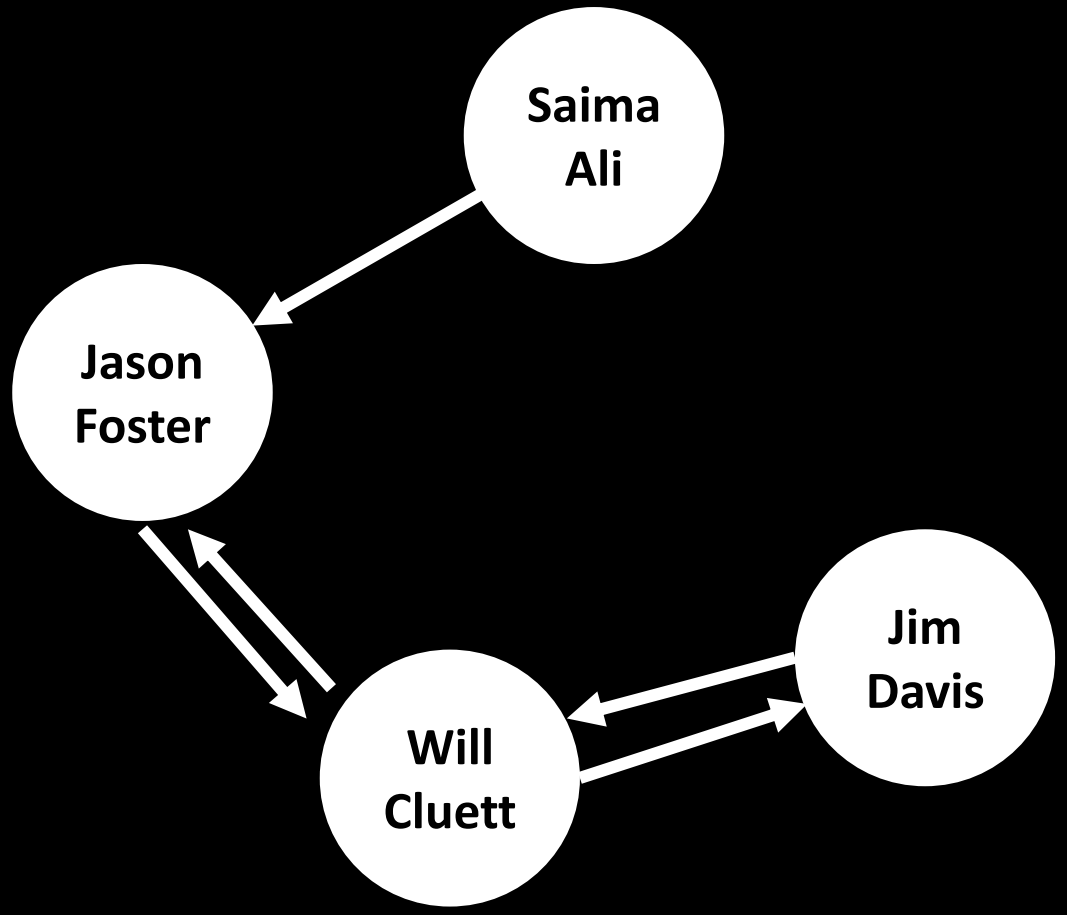

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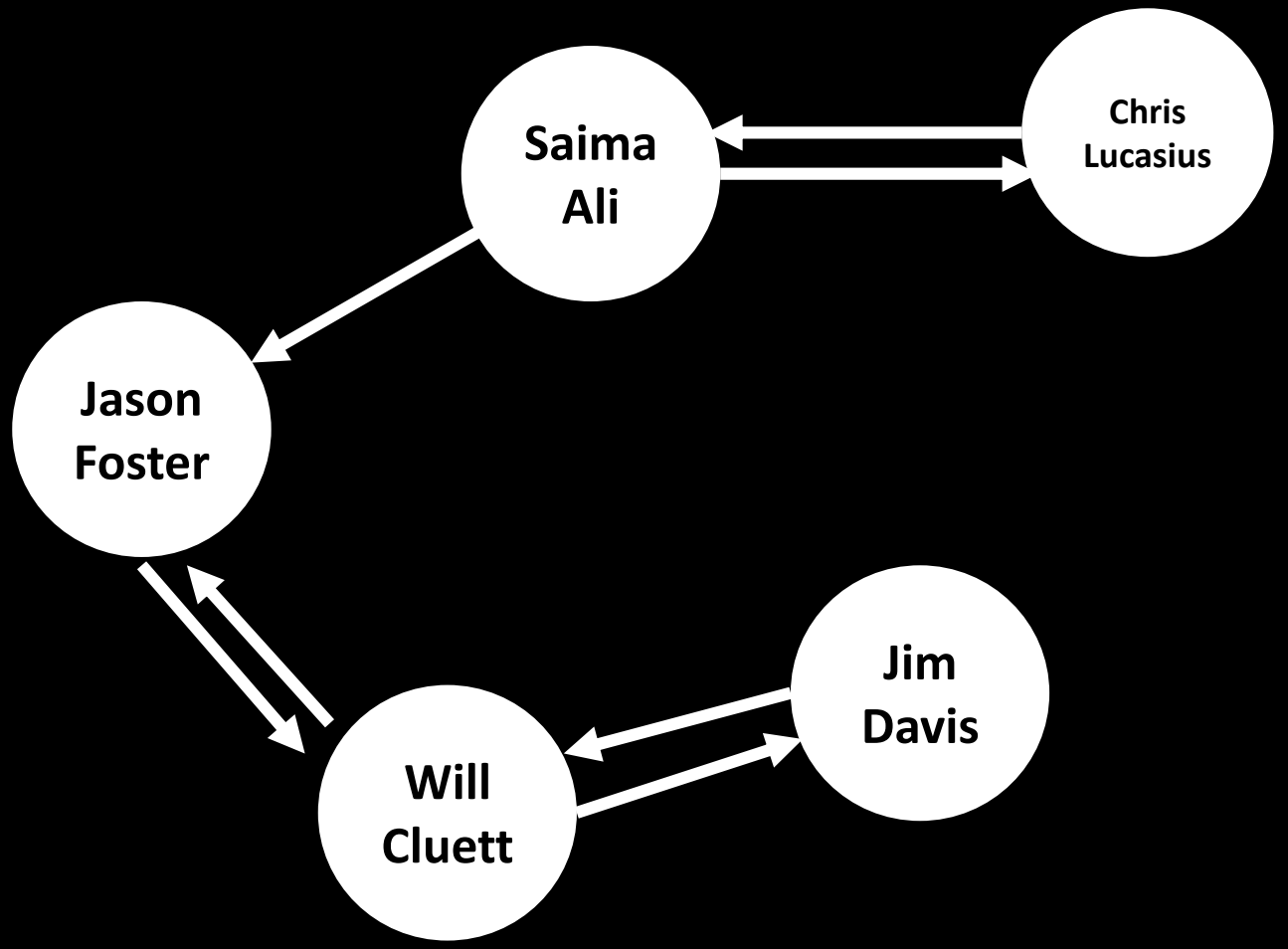

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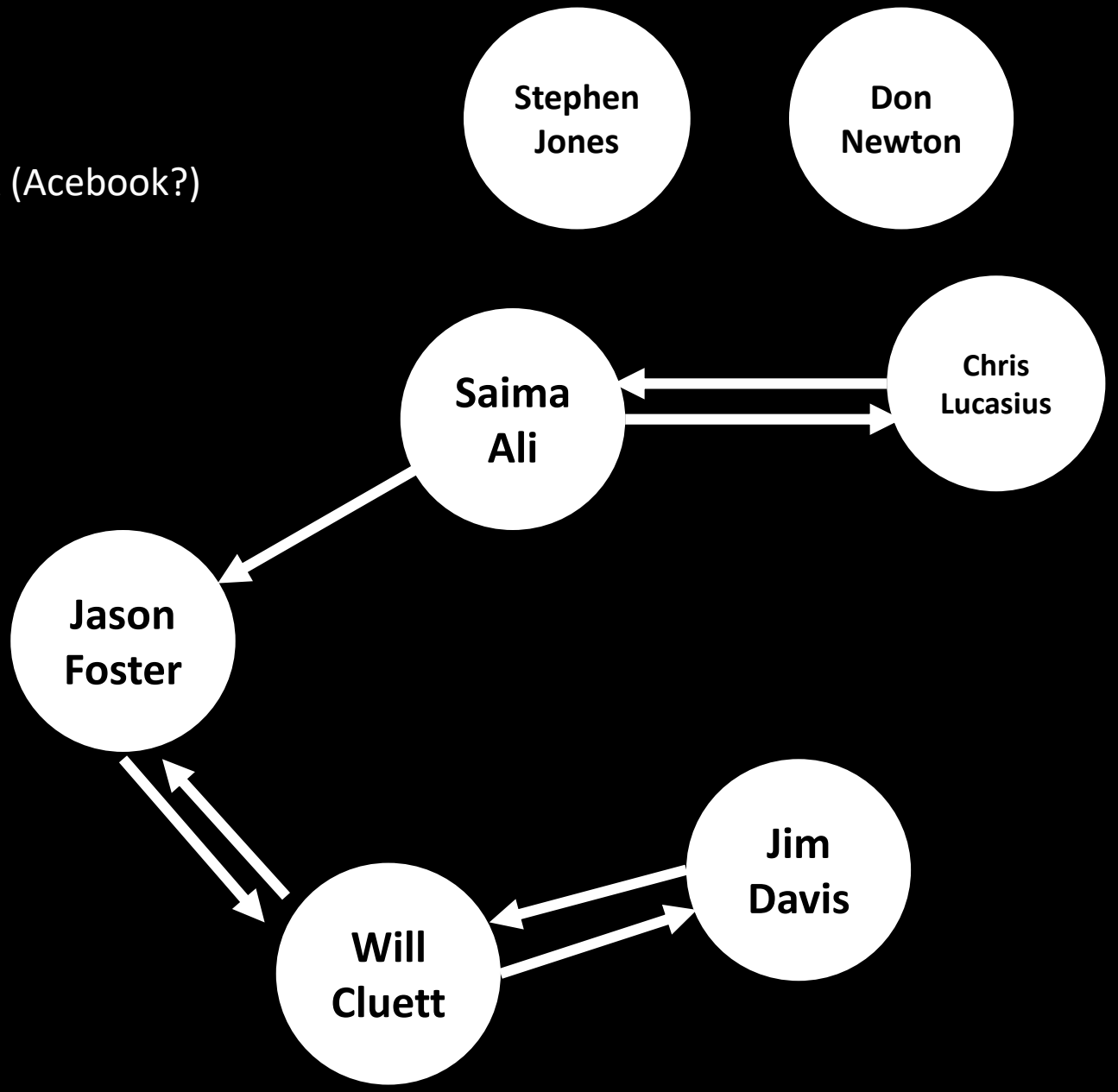

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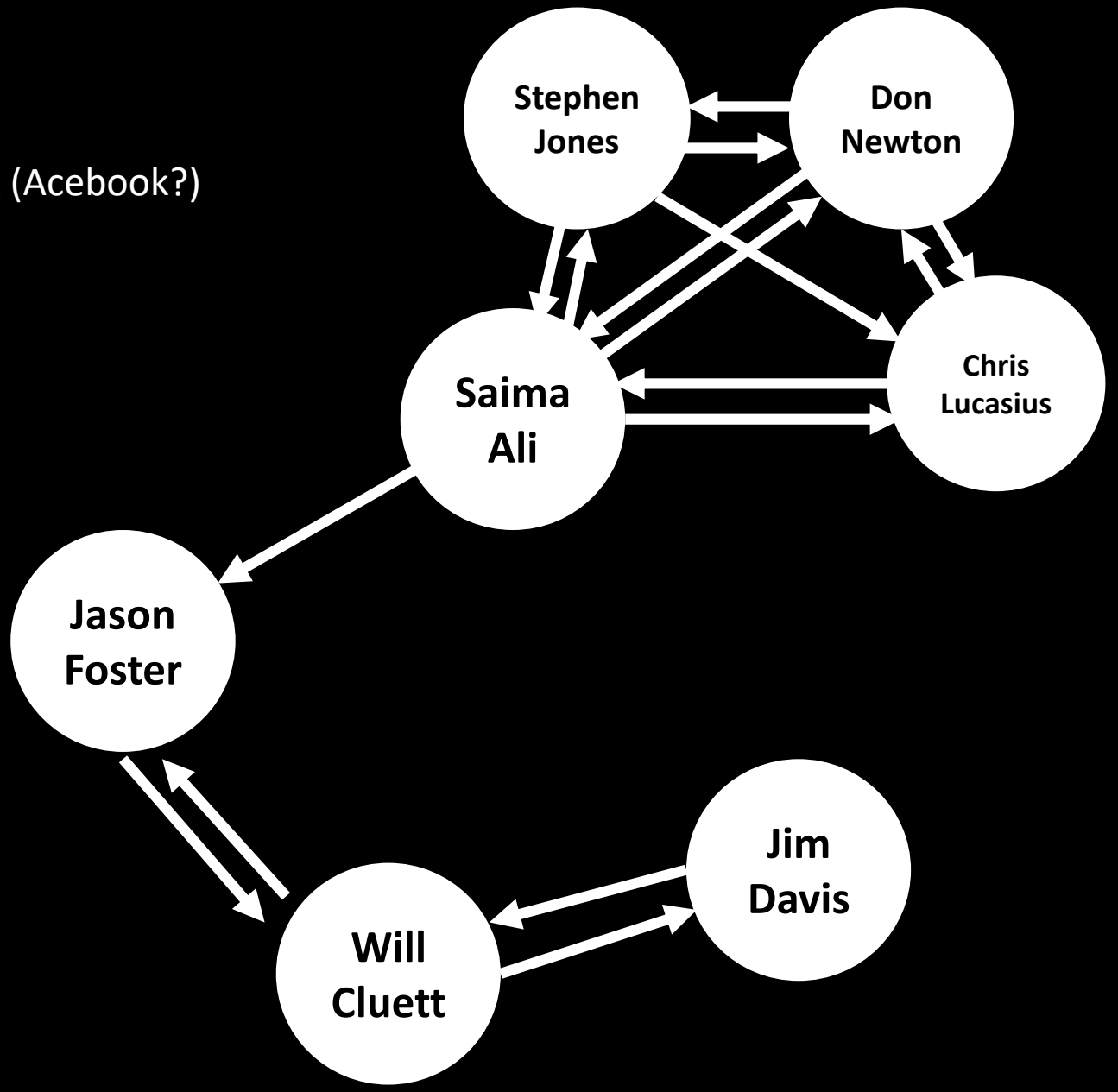

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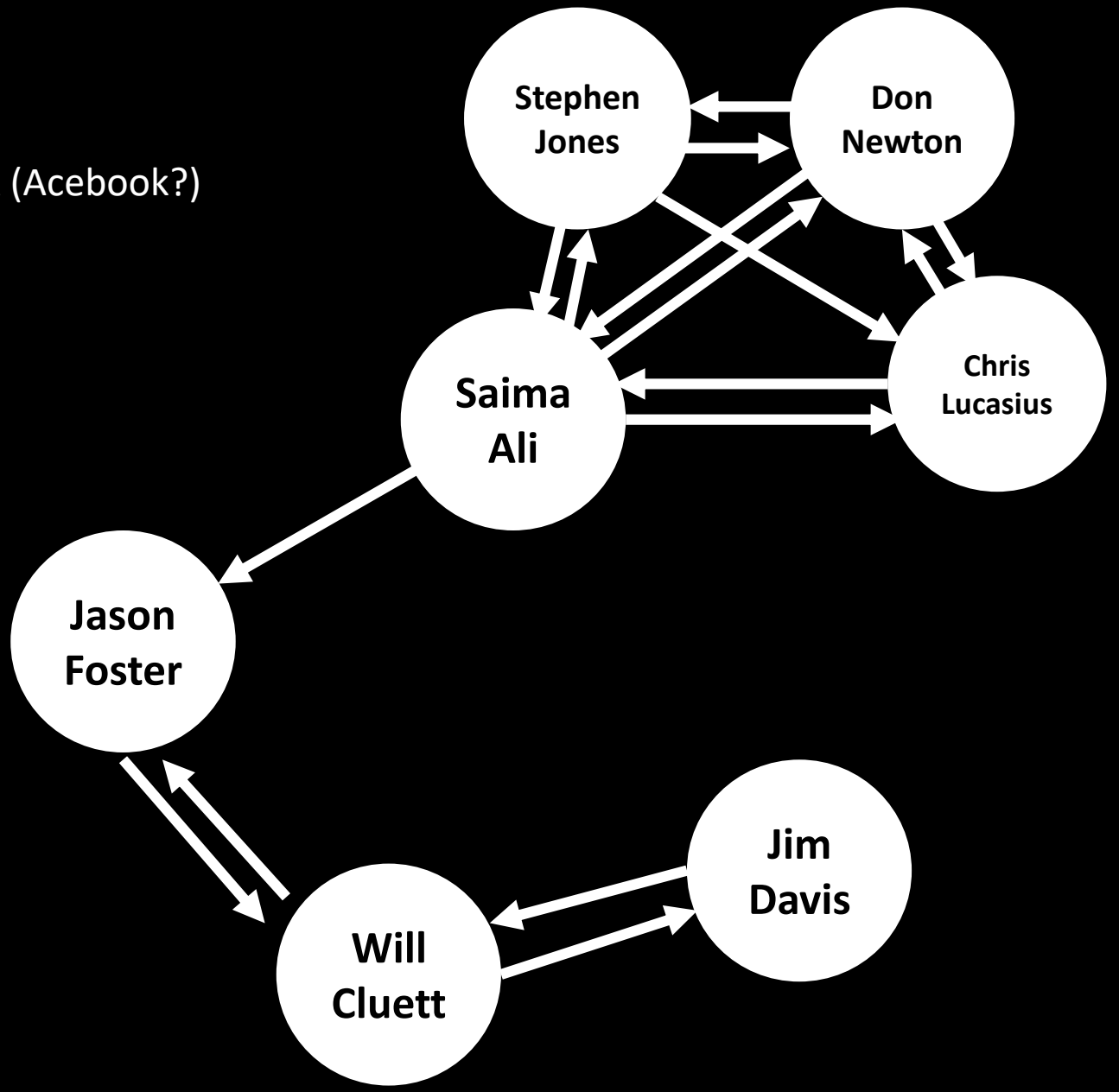

Nodes: People

Edges: Flow of e-mails

Graphs

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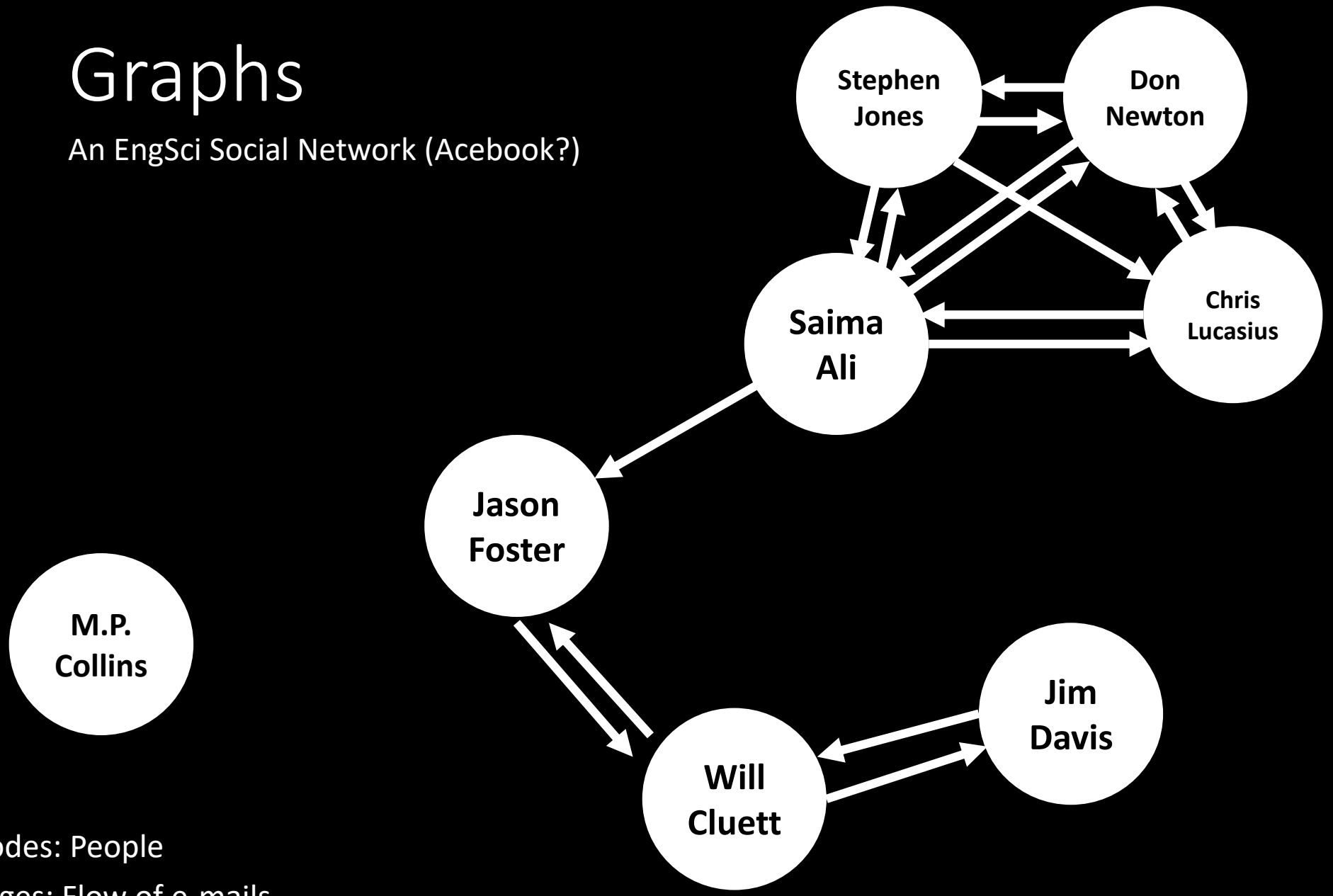

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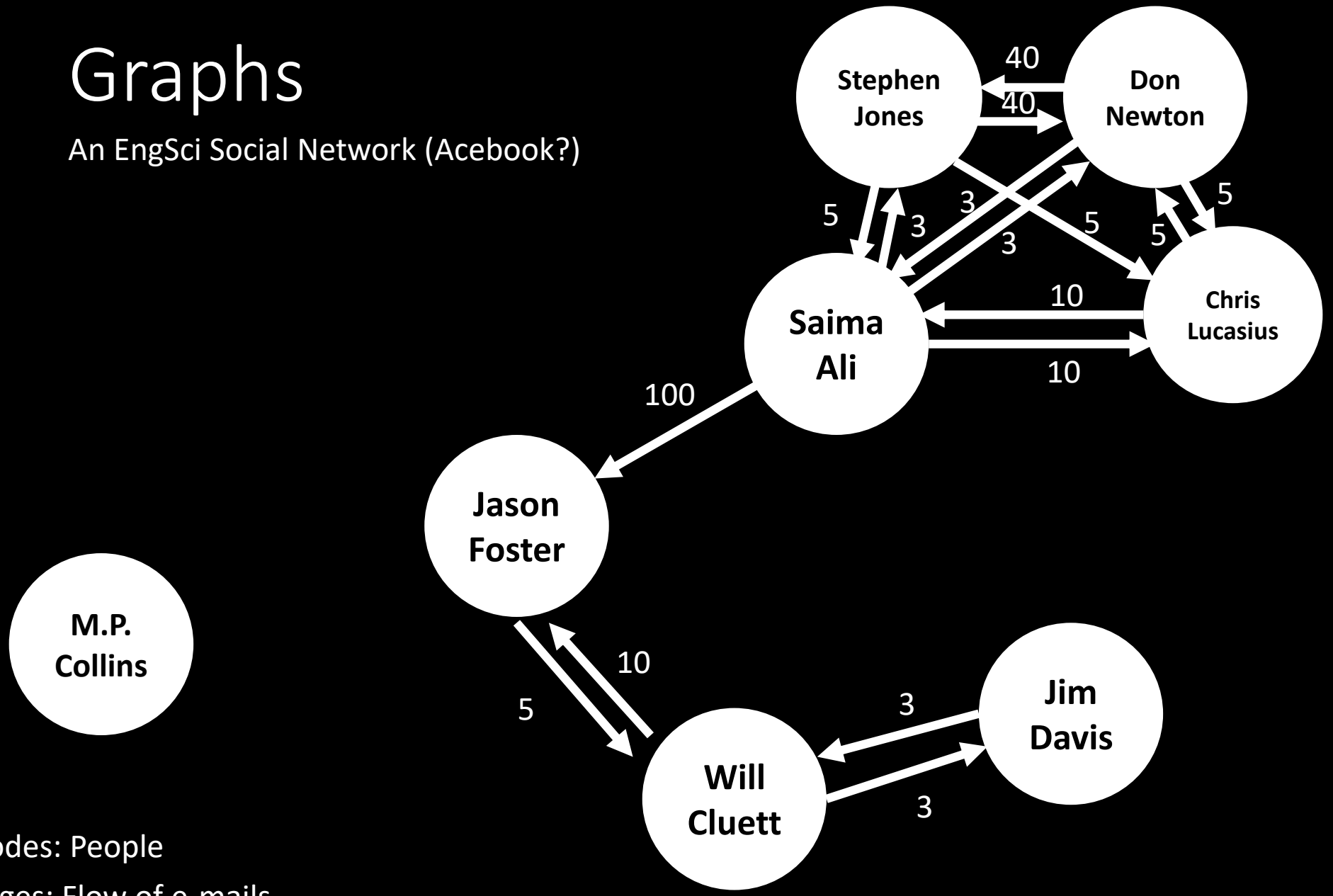

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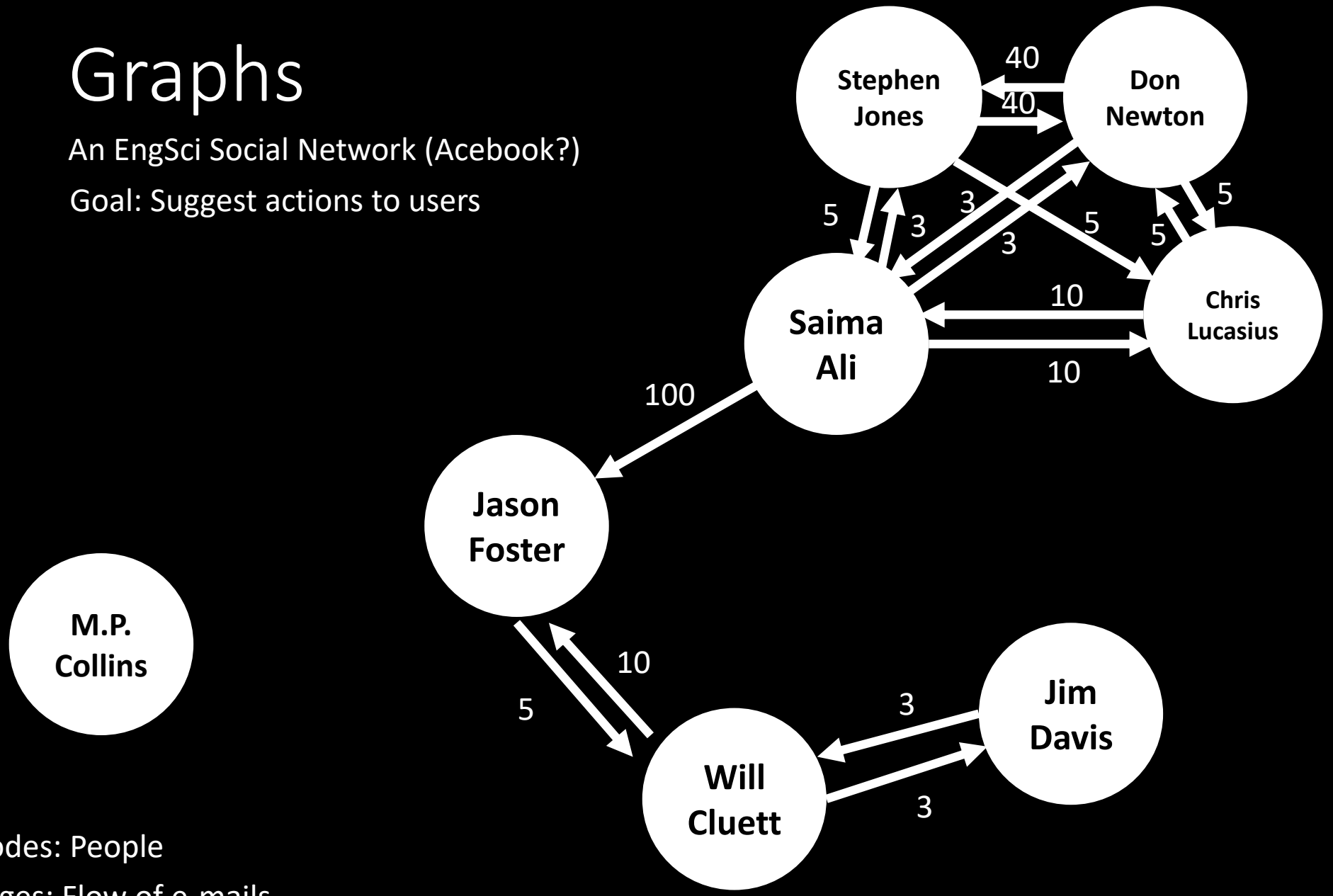
Graphs

An EngSci Social Network (Acebook?)

Goal: Suggest actions to users



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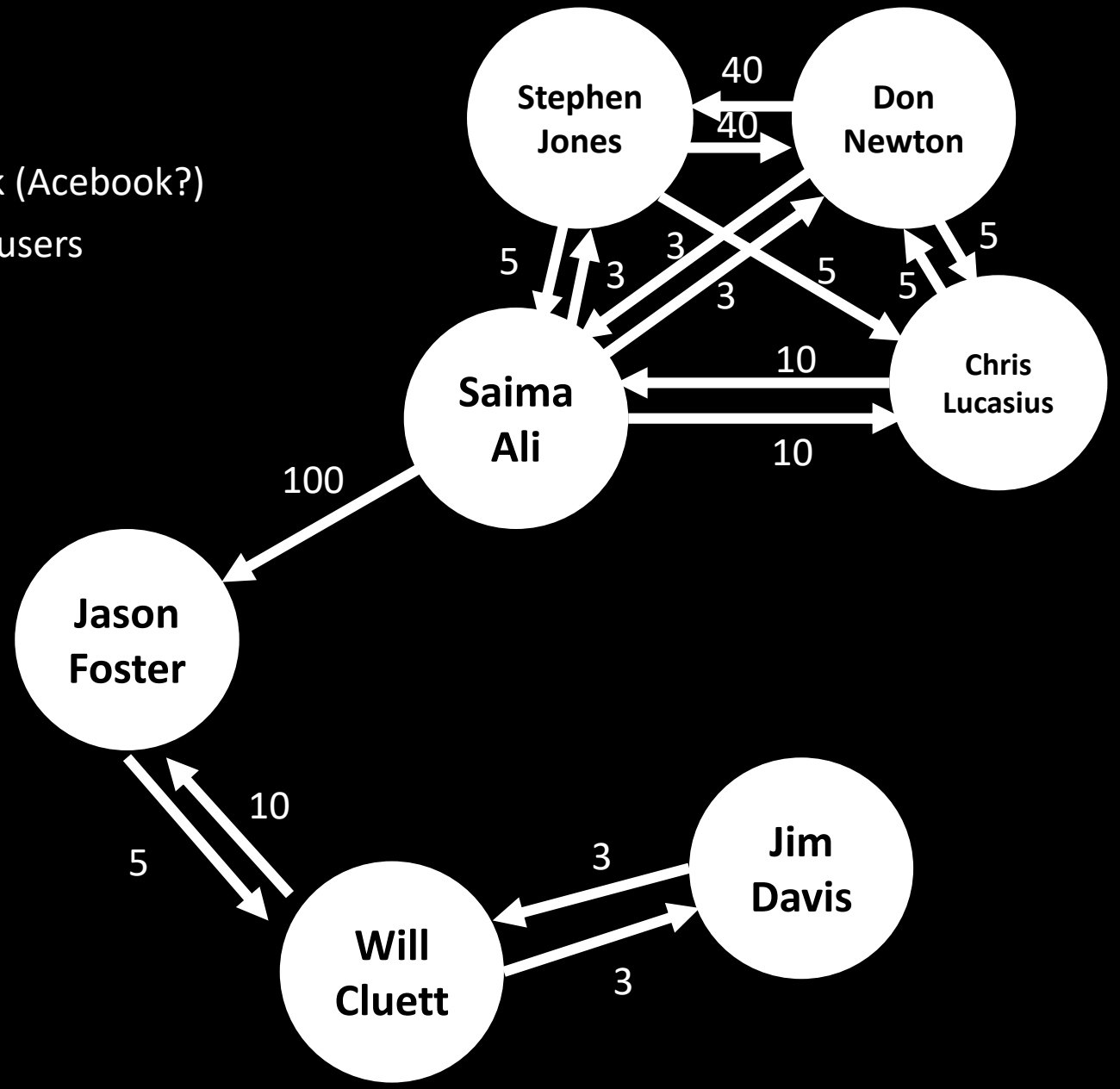

You're missing out.

M.P. Collins

Nodes: People

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Graphs

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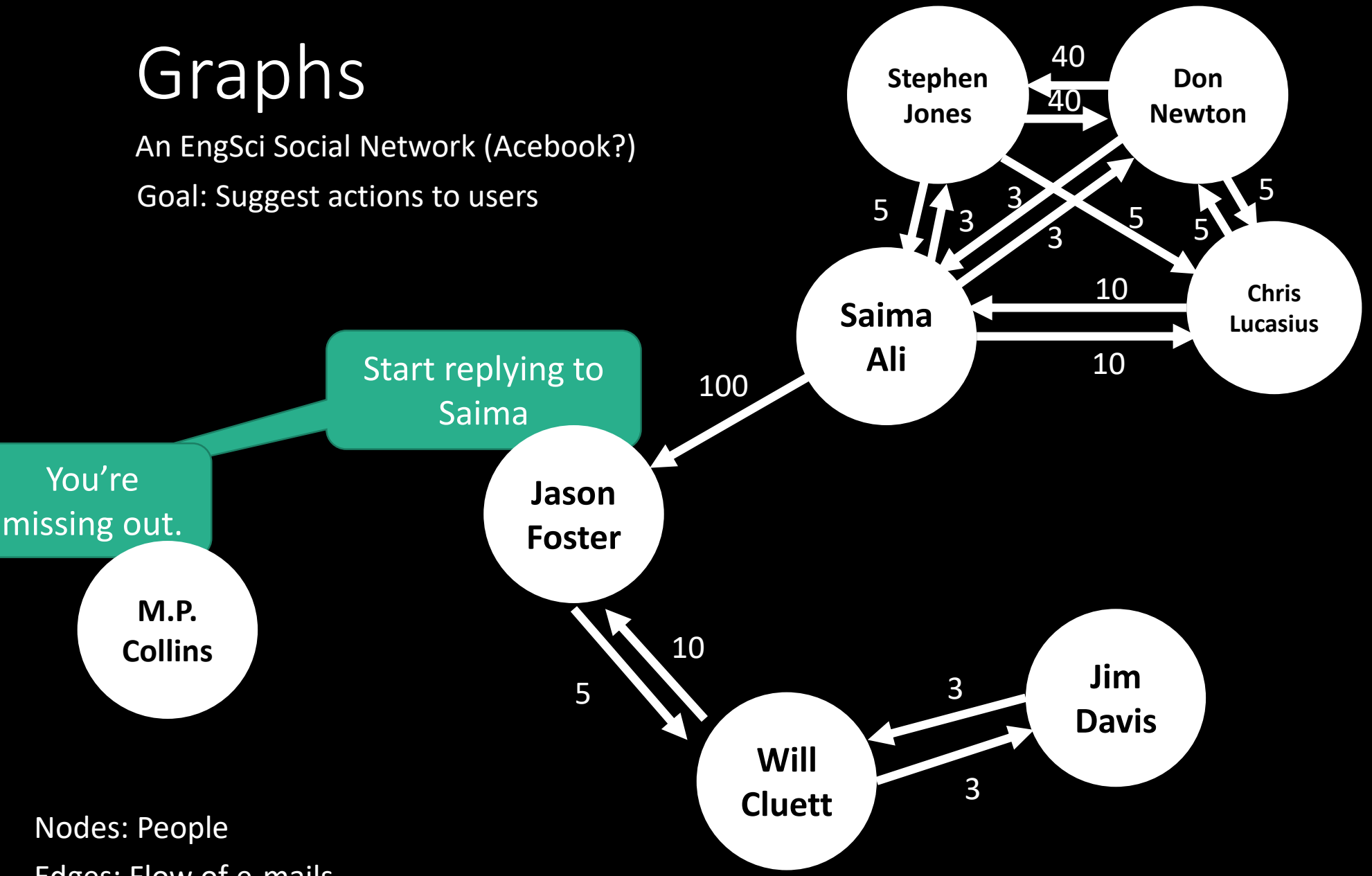

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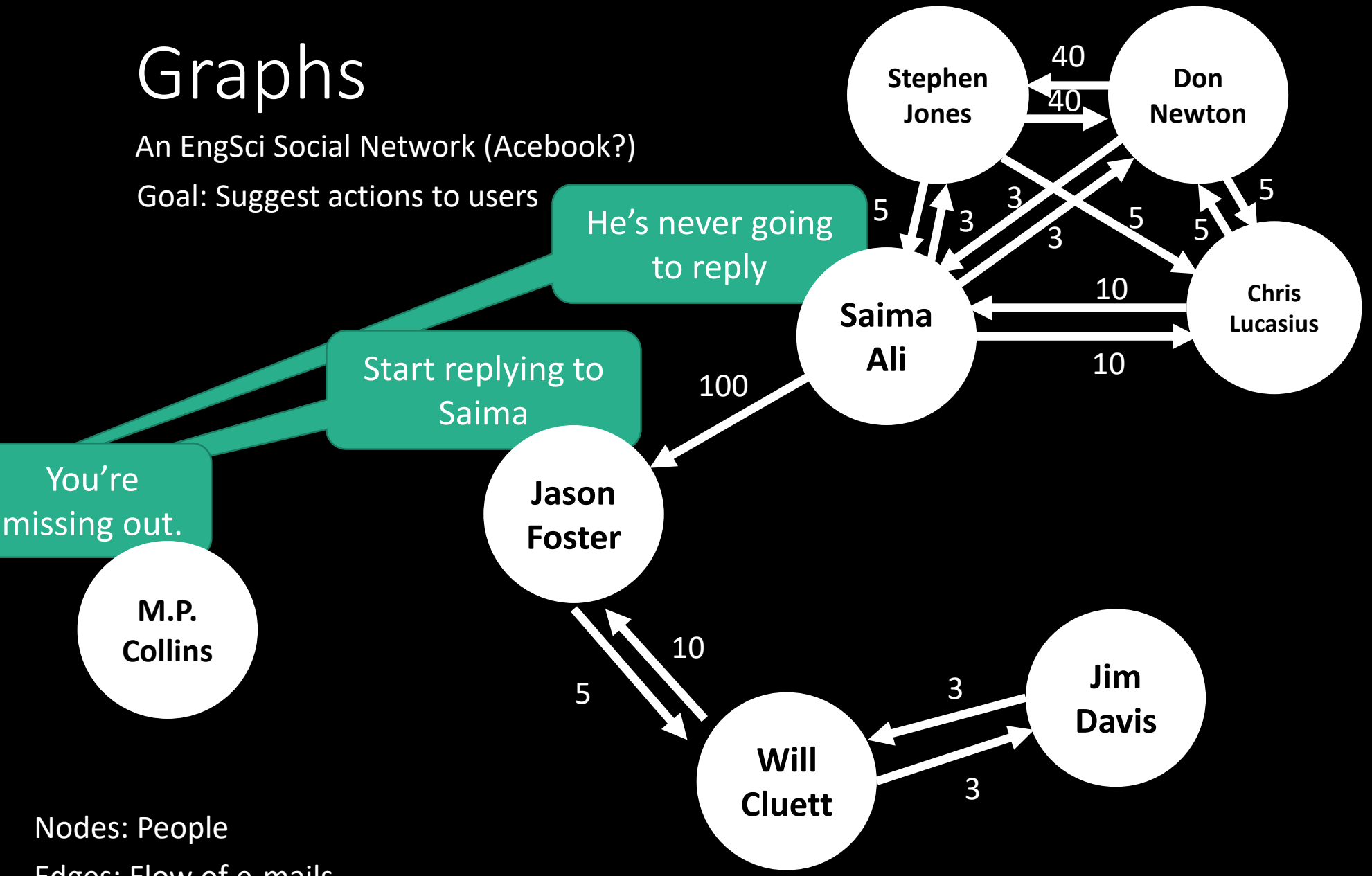

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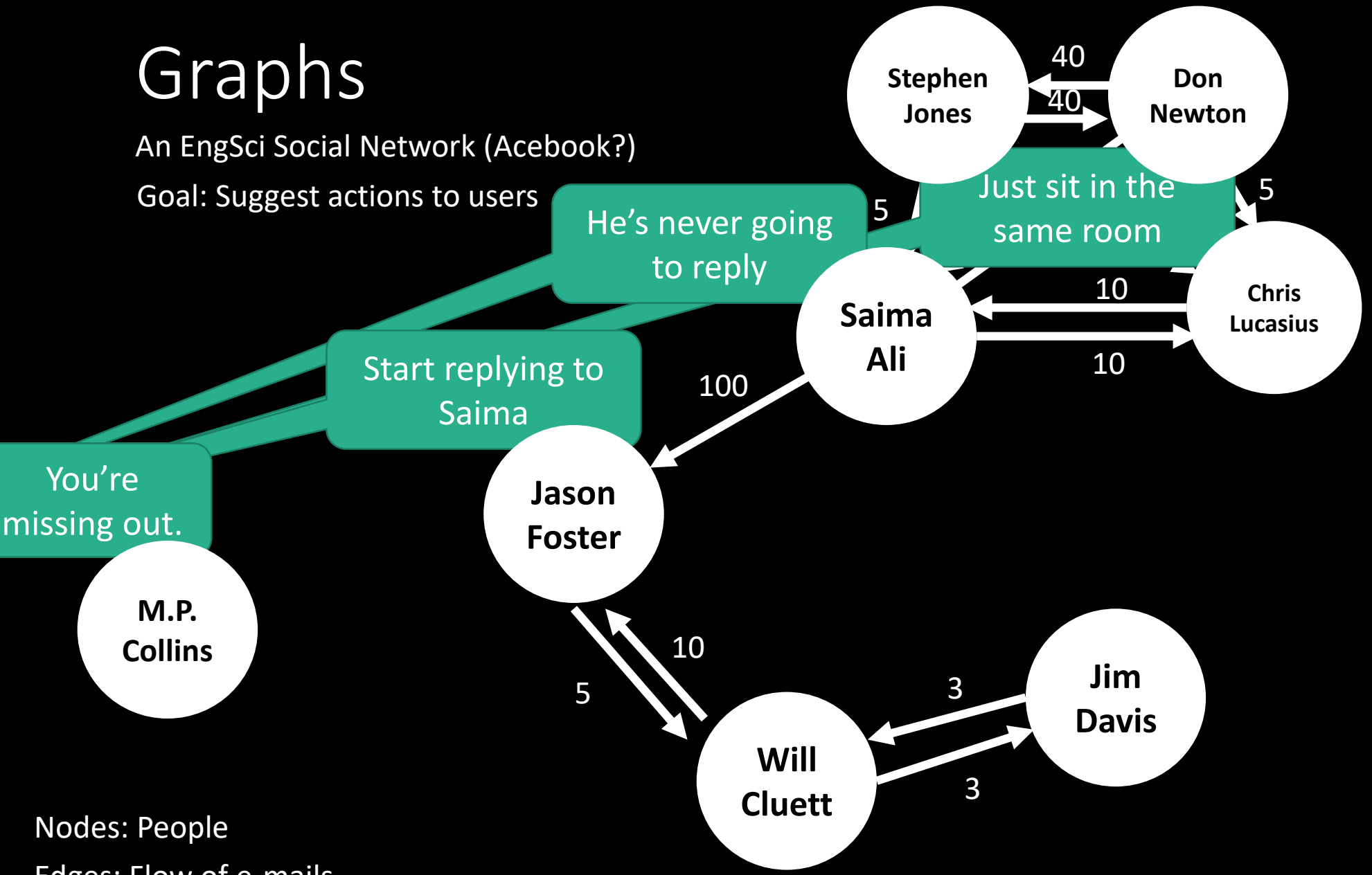

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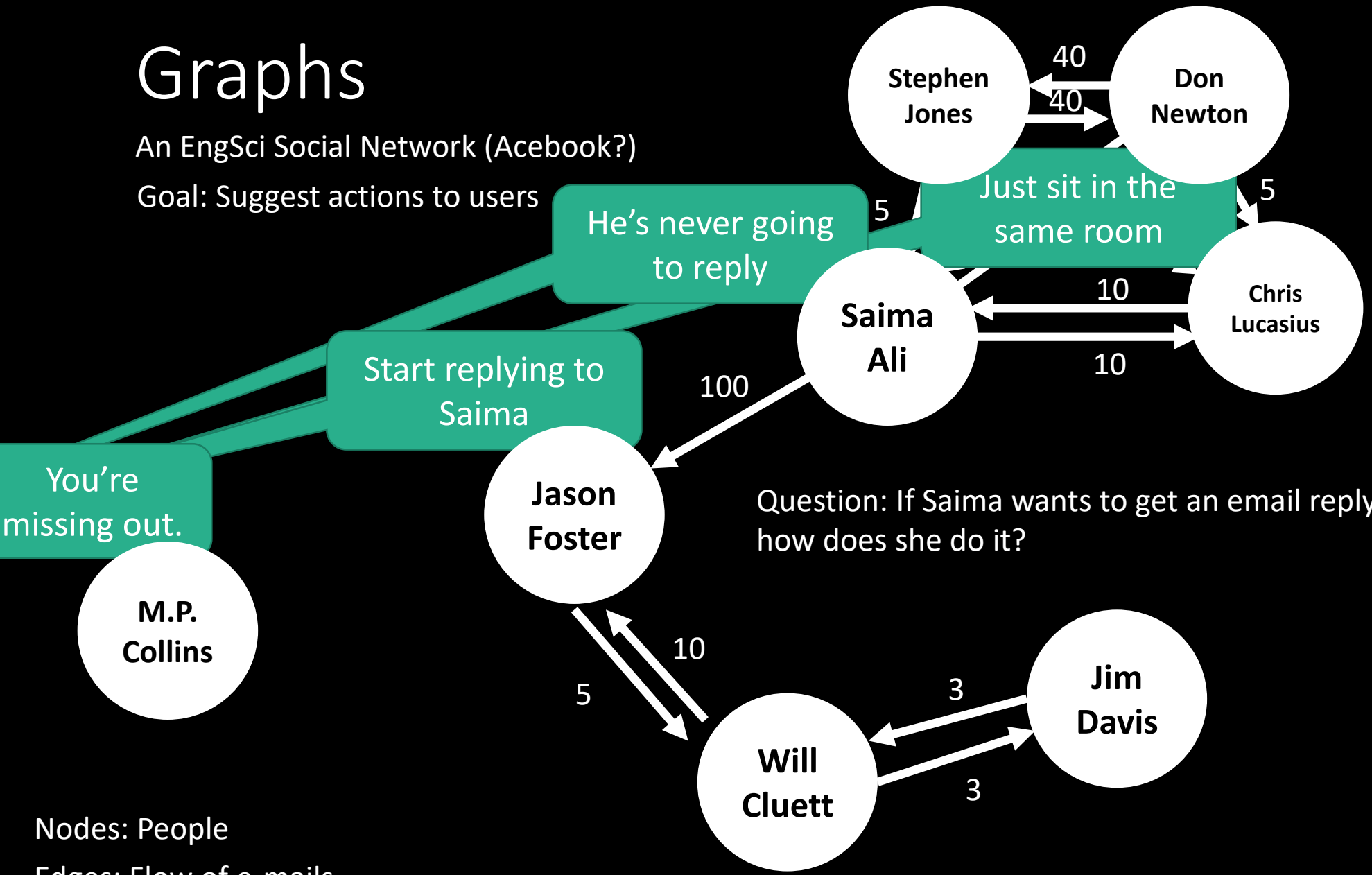

Question: If Saima wants to get an email reply from Prof Foster, how does she do it?

Nodes: People

Edges: Flow of e-mails

Graphs

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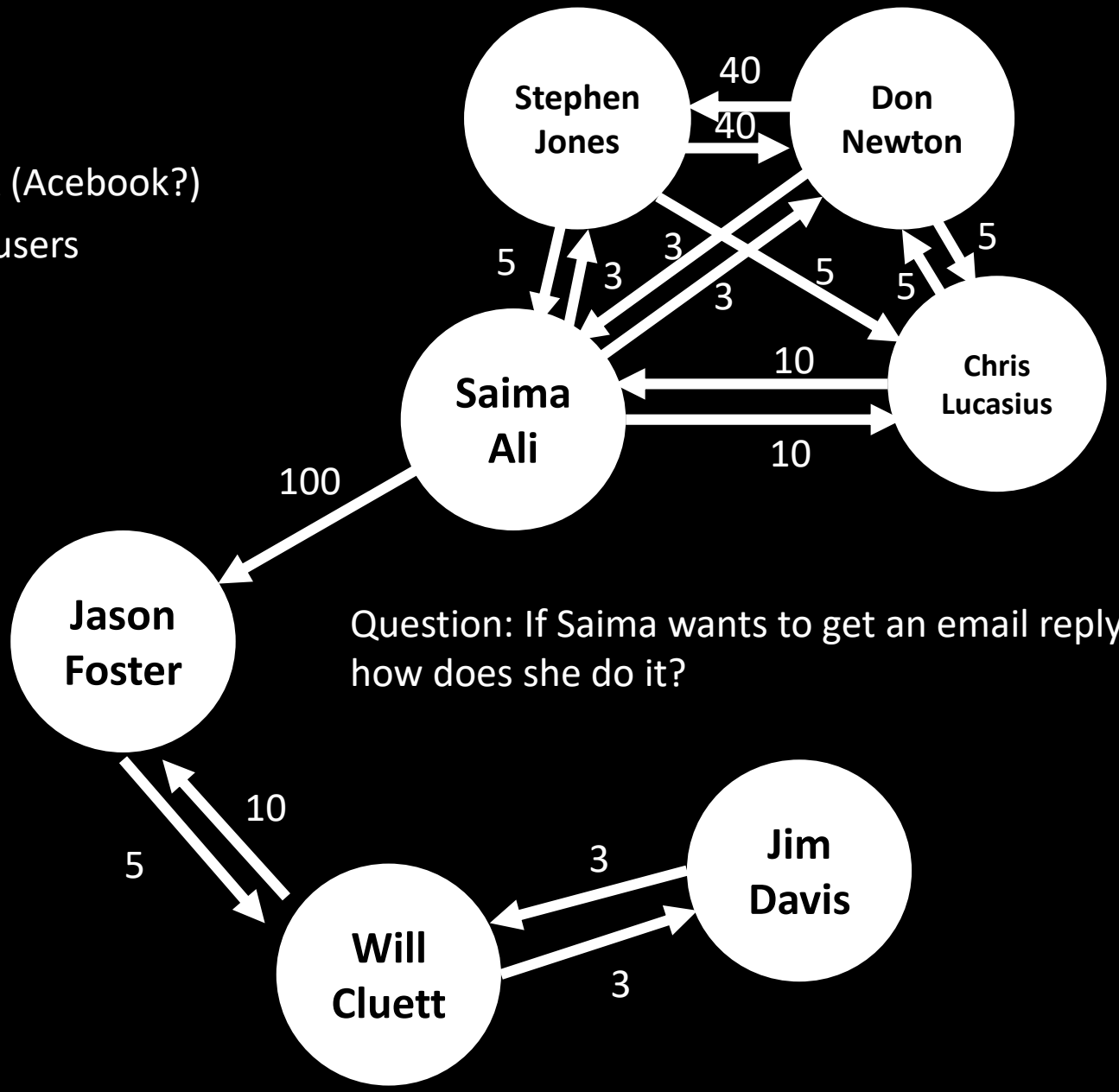

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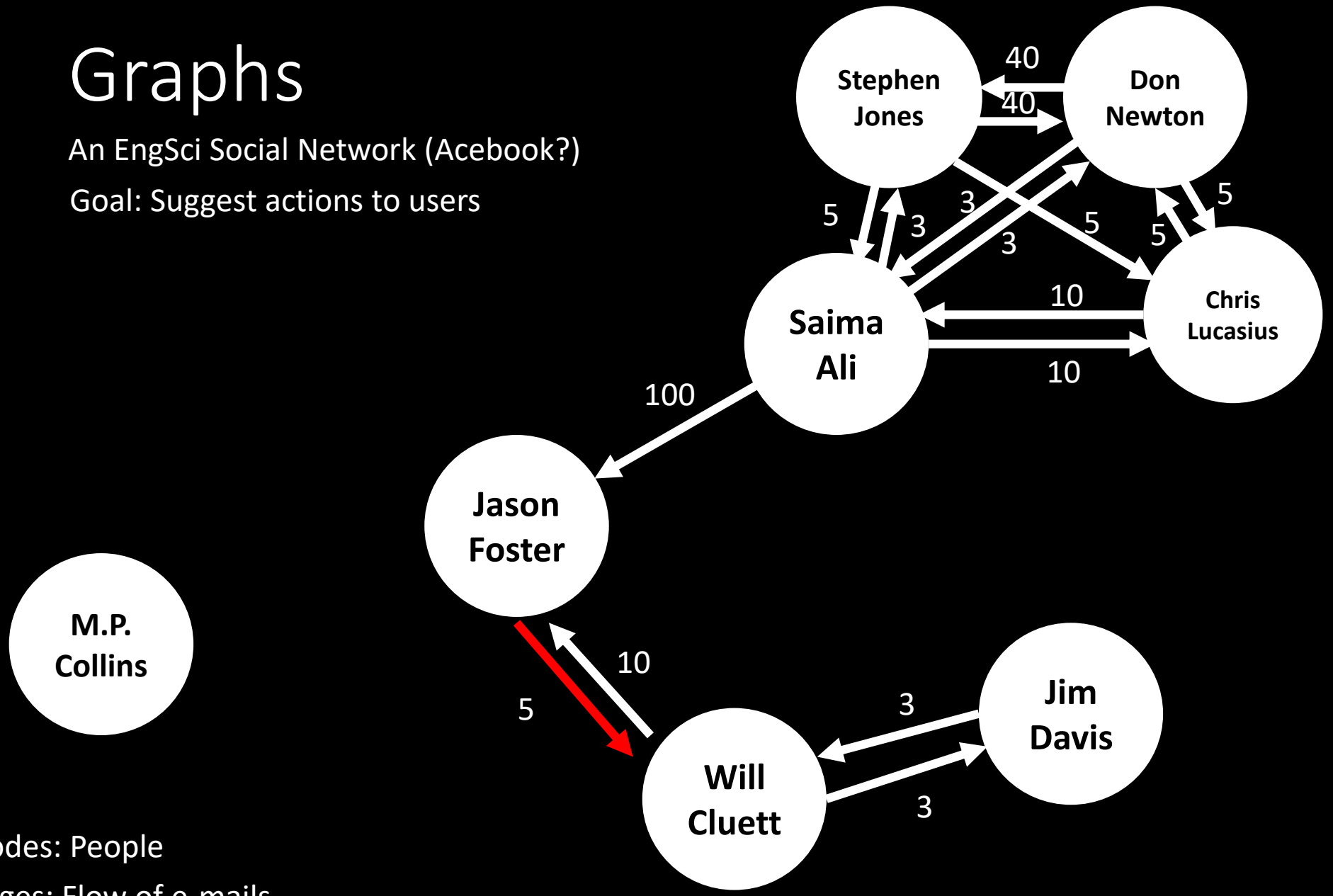

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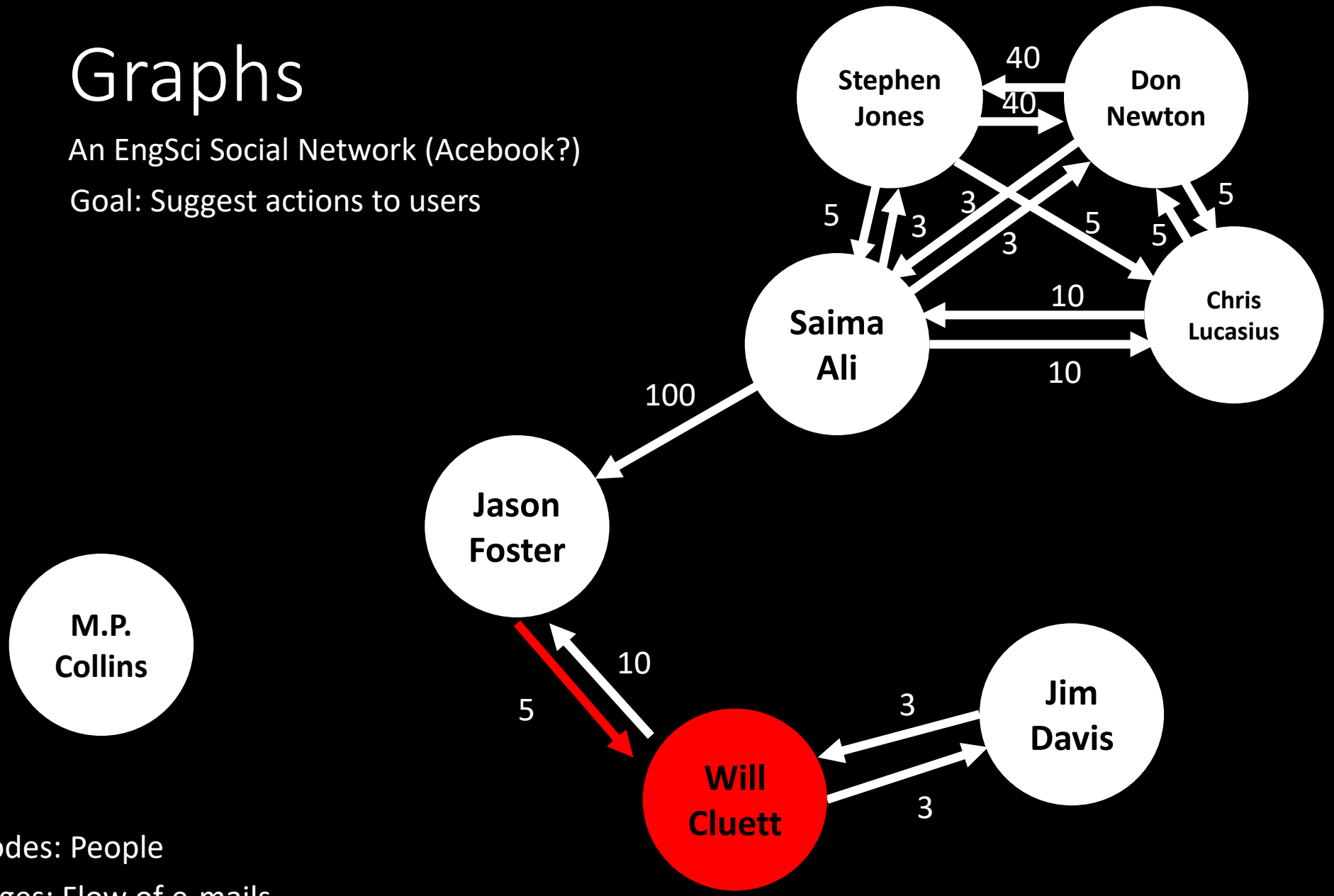

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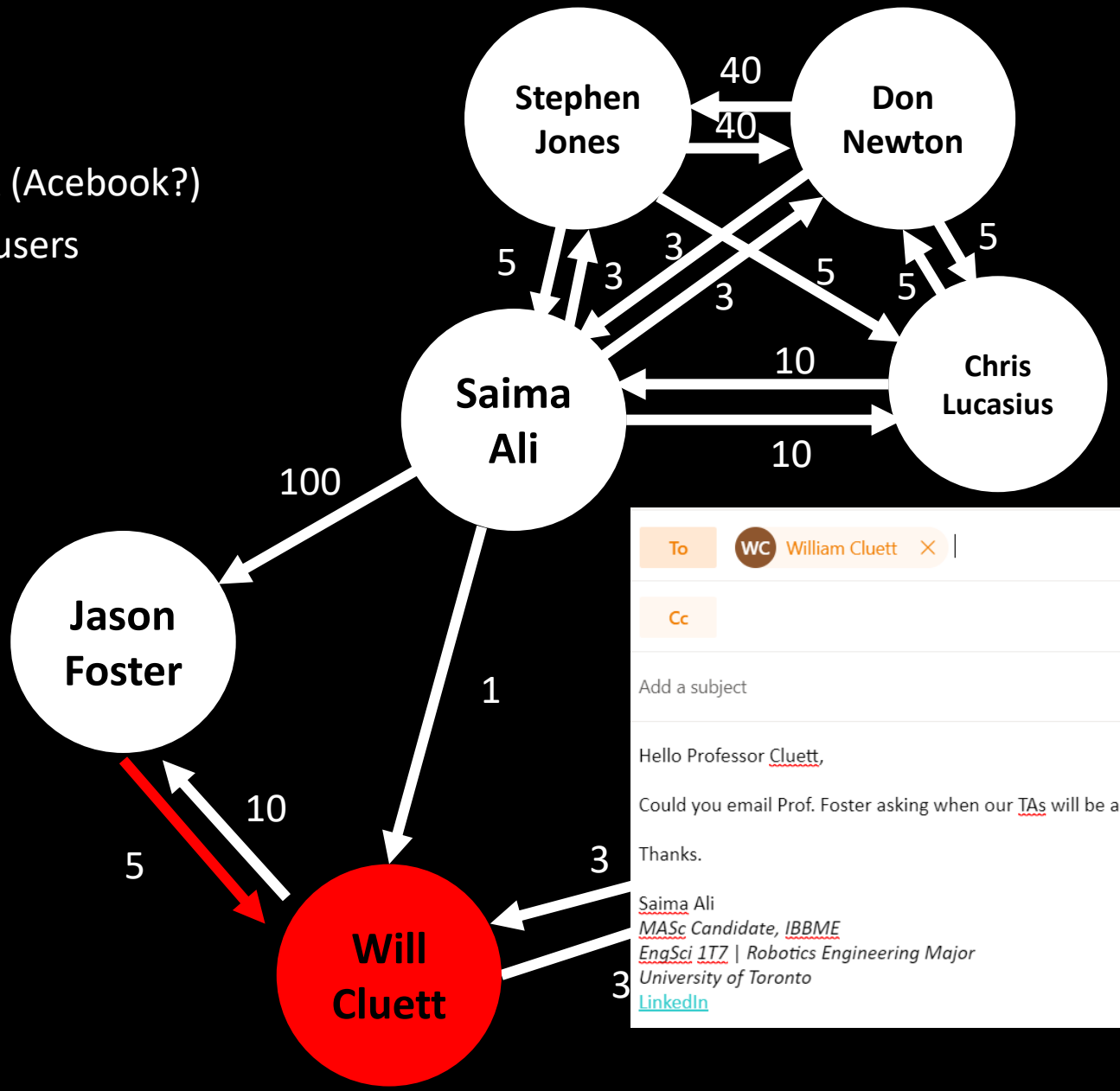

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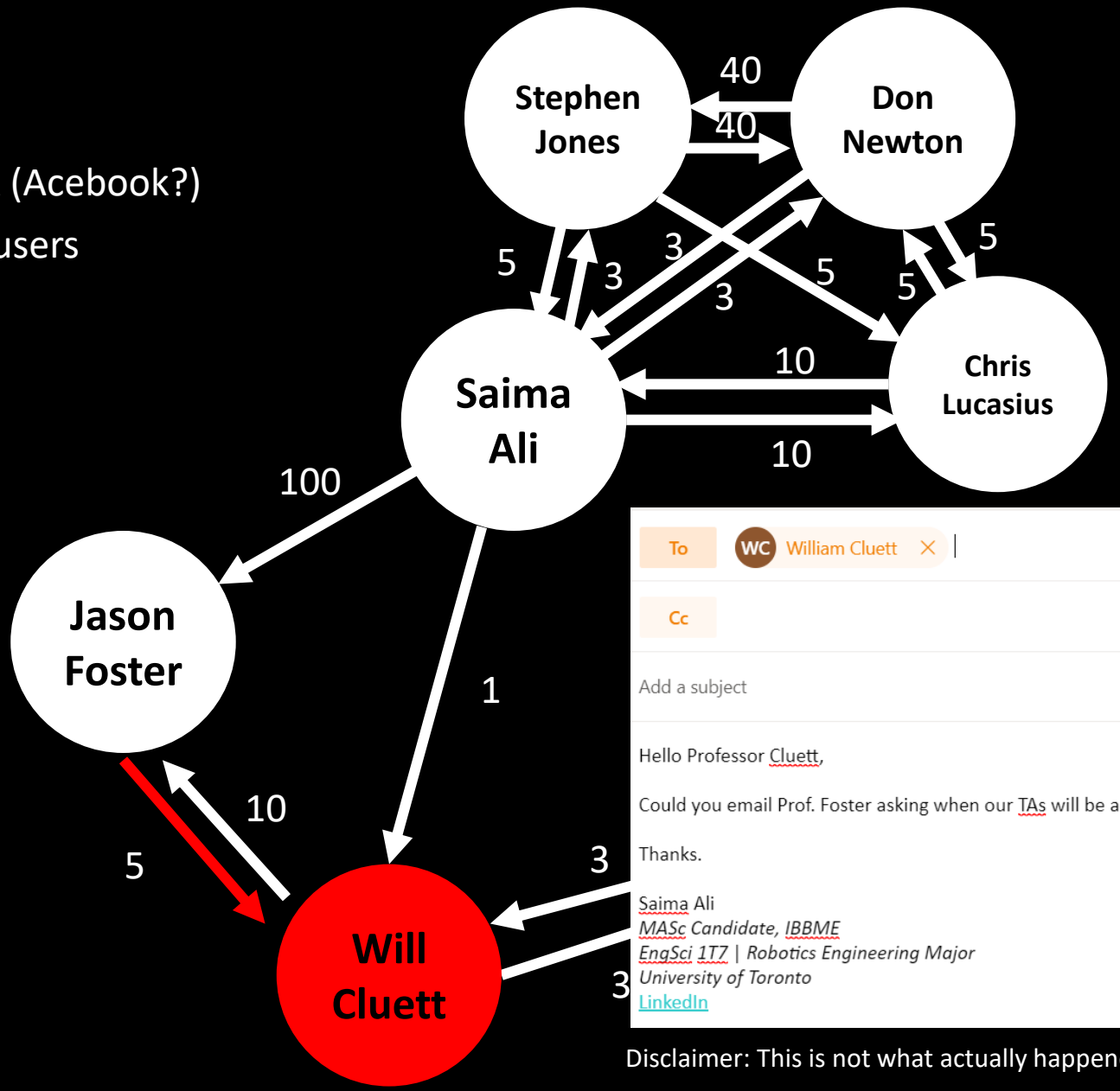

To WC William Cluett

Cc

Add a subject

Hello Professor Cluett,

Could you email Prof. Foster asking when our TAs will be added to Discourse?

Thanks.

Saima Ali
MASc Candidate, IBBME
EngSci 1T7 | Robotics Engineering Major
University of Toronto
LinkedIn

Disclaimer: This is not what actually happened

Nodes: People

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Graphs

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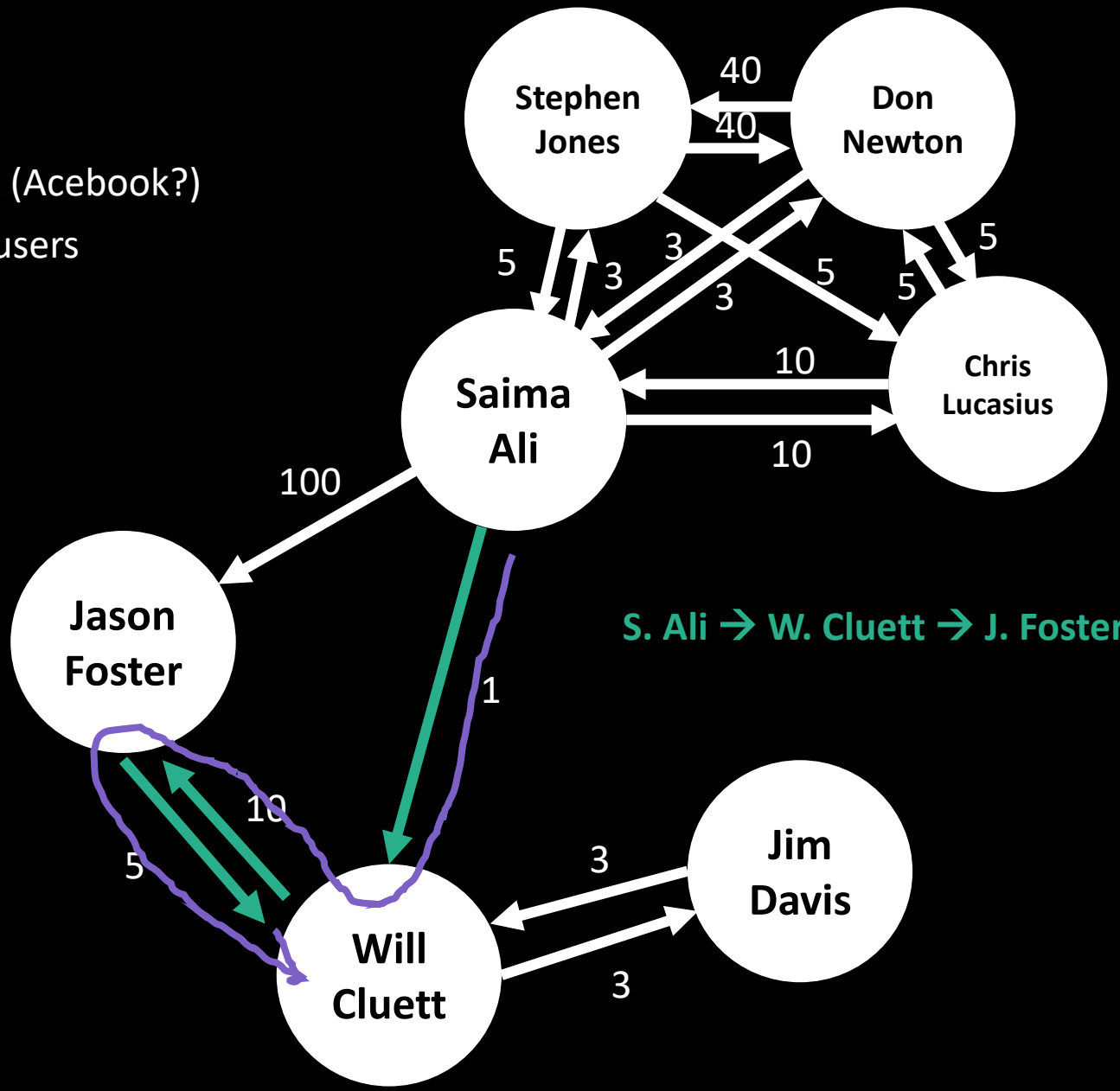

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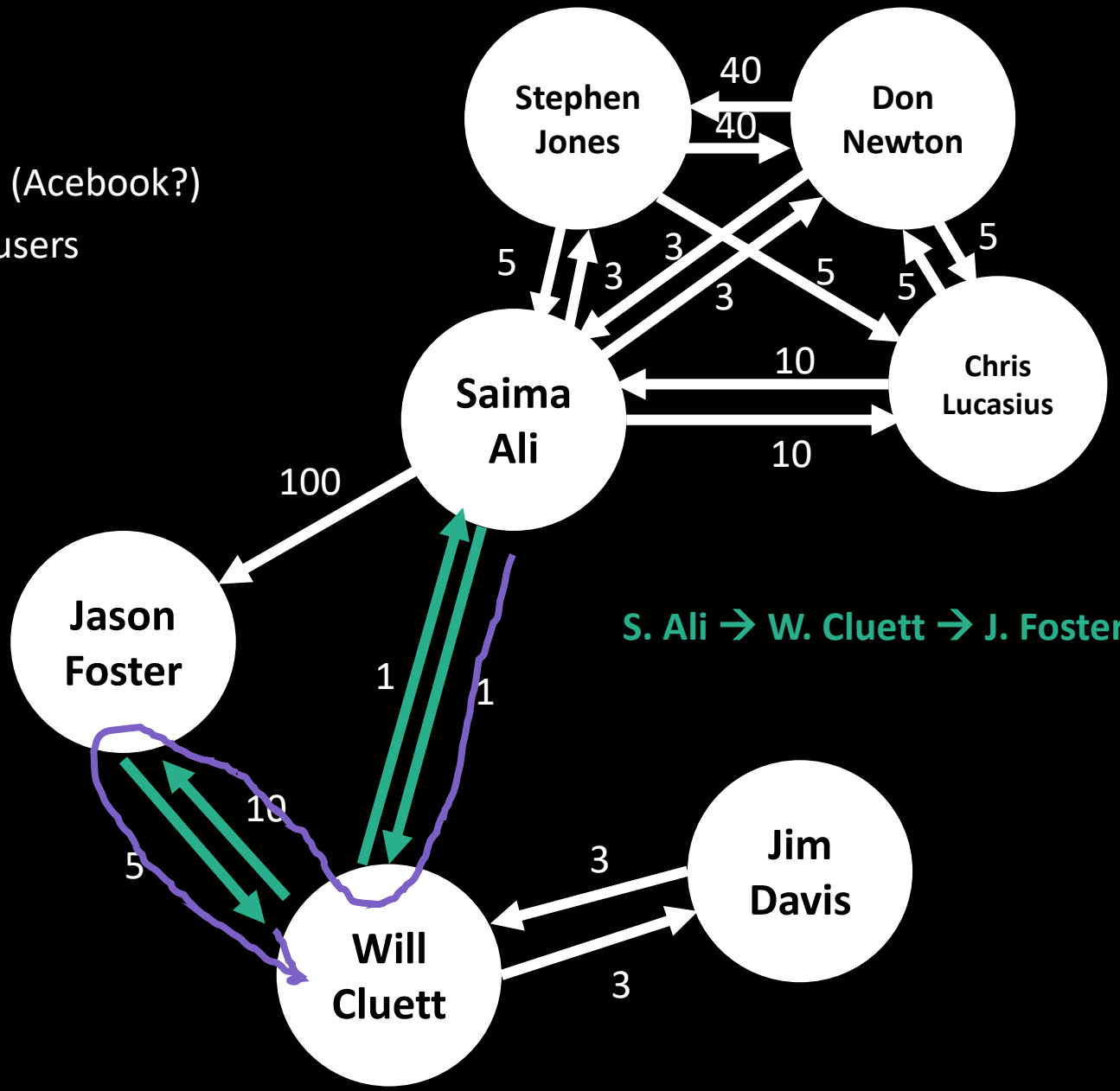

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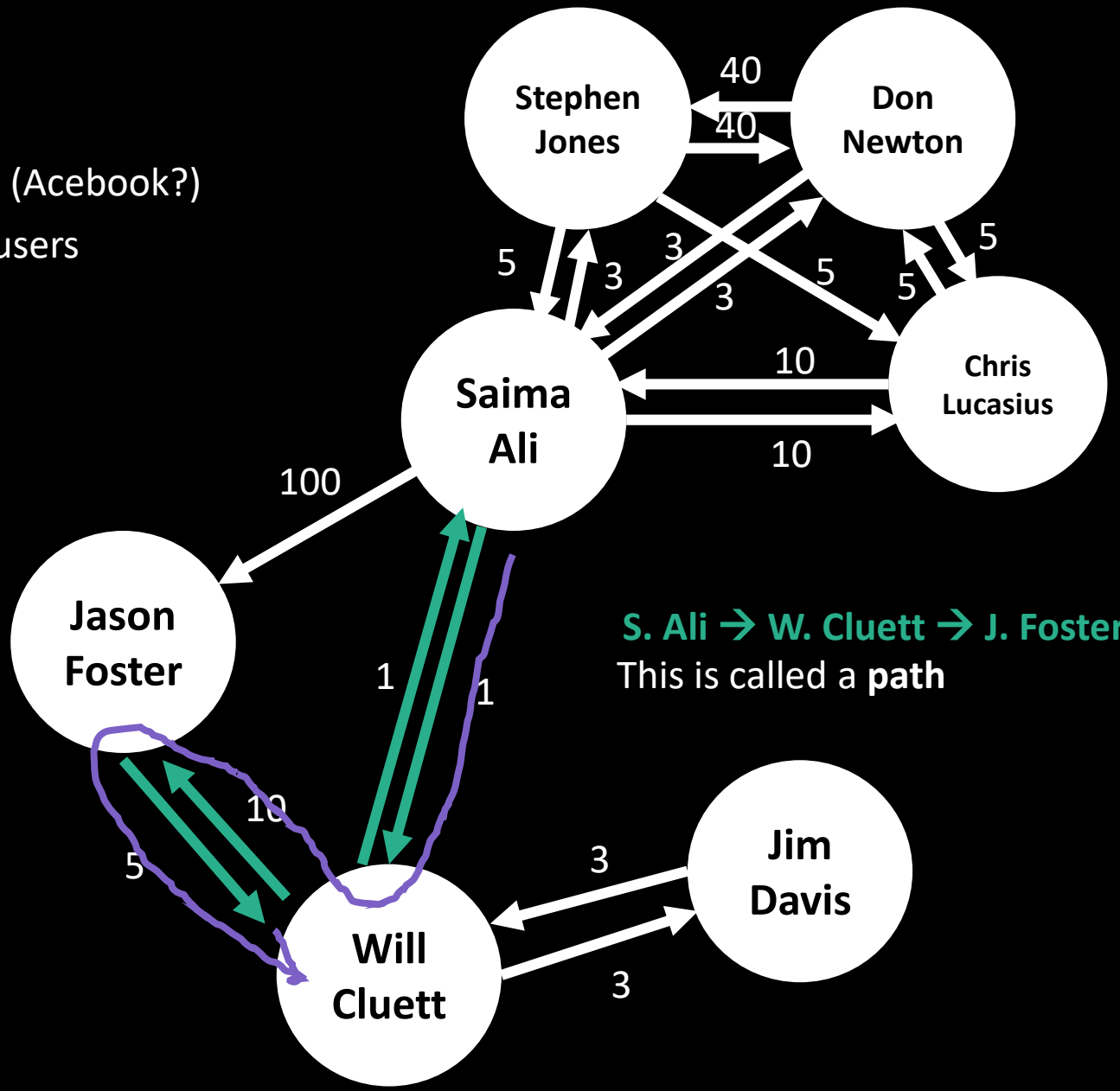

S. Ali → W. Cluett → J. Foster→ S. Ali

This is called a **path**

Nodes: People

Edges: Flow of e-mails

Recall: Sets

- Sets are collections of **unique entities**
- Our treatment of graphs will heavily rely on set notation

$\forall$ $\subset$

$\in$ $\cap$

$\exists$

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For all items x in the set of real numbers (for every real number)

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For all items x in the set of real numbers (for every real number)

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$$\subset$$

$$\cap$$

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There exists an item y in the set of real numbers

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For all items x in the set of real numbers (for every real number)
There exists an item y in the set of real numbers such that…

$$\mathbb{Z} \subset \mathbb{R}$$

The set of integers is a **subset** of the set of real numbers.

$$\{1, 2, 3\} \subseteq \{1, 2, 3\}$$

$$\cap$$

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$$\{1, 2, 3\} \subseteq \{1, 2, 3\}$$

$$\mathbb{Z} \subseteq \mathbb{Z} \cap \mathbb{R}$$

The set of integers is a **subset or equal to** the **intersection** between the set of integers and the set of real numbers.

Graphs

- Collection of **nodes** and **edges**

$$G(V, E)$$

Graphs

- Nodes: Denoted by a set V
- Edges: Denoted by a set E
  - Impose a rule that $\forall e \in E$, e connects exactly 2 nodes $u, v \in V$

Graphs

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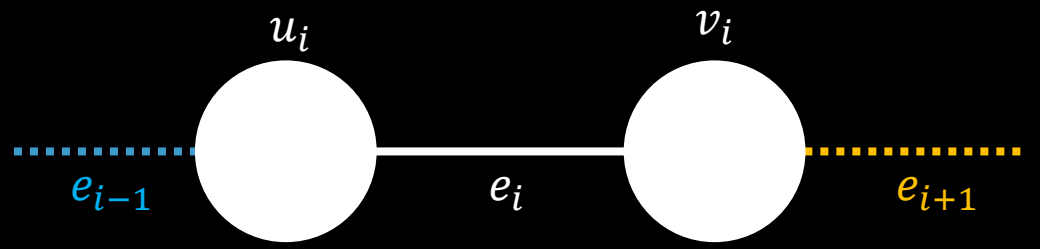
- Path: A set of unique edges $\{e_1, e_2, \dots, e_n\} \subseteq E$ for which the following is satisfied:
  - $e_i, i = 1 \dots n$ connects two nodes denoted $v_i, u_i \in V$
  - Then, $\forall i = \{1 \dots n-1\}$, $\exists w \in \{u_{i+1}, v_{i+1}\}$ such that $w \in \{v_i, u_i\}$
    - I.e. $\exists w \in \{u_{i+1}, v_{i+1}\}$ such that $w \in \{u_{i+1}, v_{i+1}\} \cap \{v_i, u_i\}$
    - The intersection must exist

Graphs



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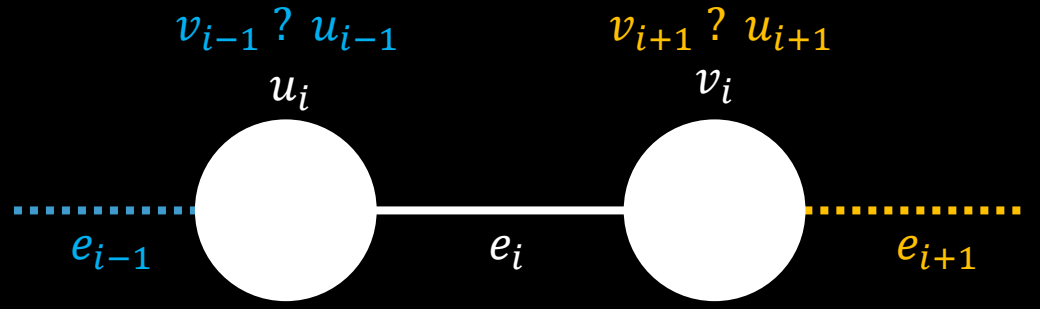
Graphs



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  - Ther
    $\dots, v_{i+1}\} \cap \{v_i, u_i\}$

Just make sure your path consists of lines (edges) which are always connected by circles (nodes/vertices)

Graphs

- Neighbors:

  Let $u, v \in V$ be 2 nodes on a graph, $G(V, E)$.

  u and v are neighbors $\Leftrightarrow \exists e \in E$ connecting u, v

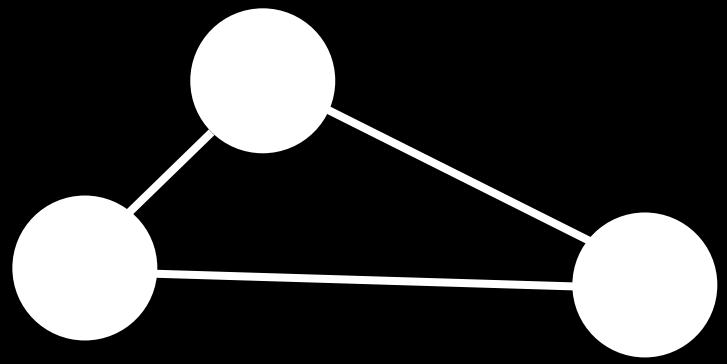
Graphs

- Neighbors:

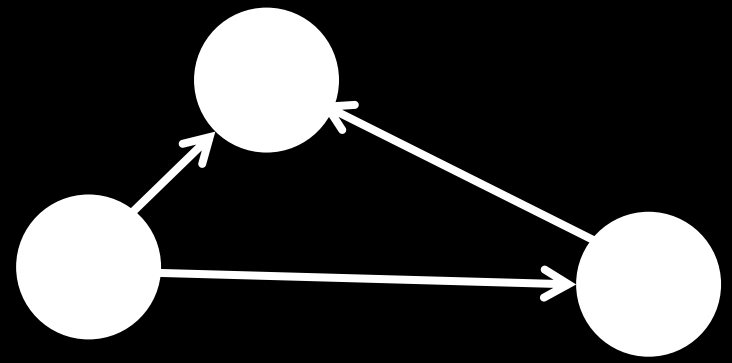
  Let $u, v \in V$ be 2 nodes on a graph, $G(V, E)$.

  u and v are neighbors $\Leftrightarrow \exists e \in E$ connecting u, v
- Just adjacent nodes (connected by an edge)

Graphs

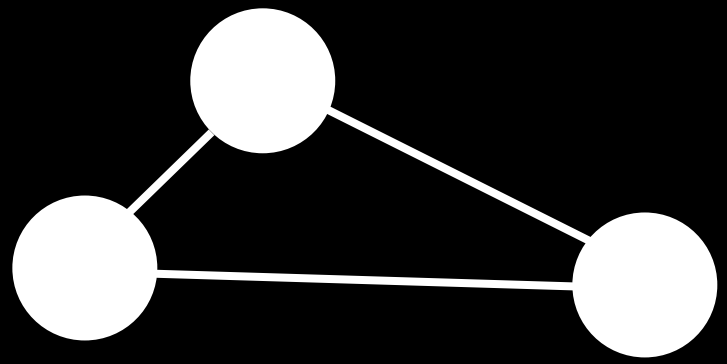


Undirected

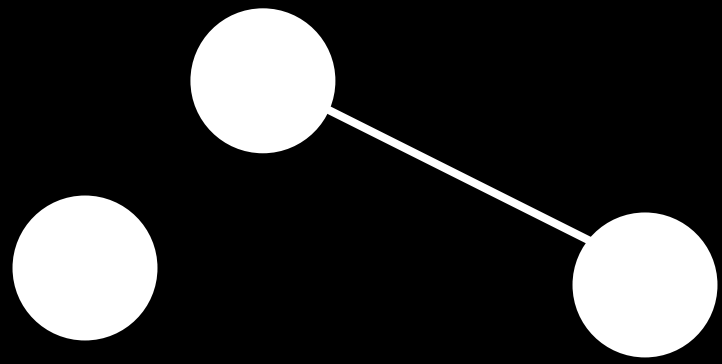


Directed

Graphs

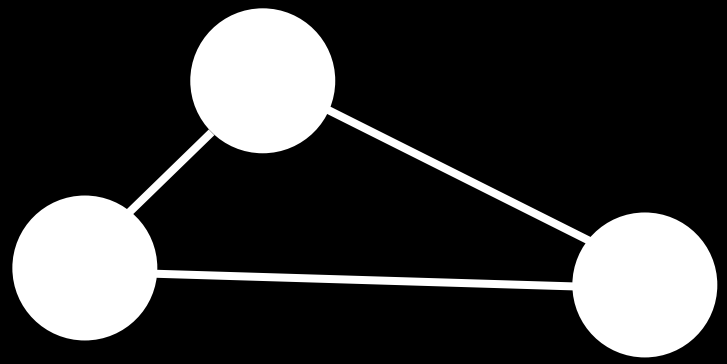


Connected



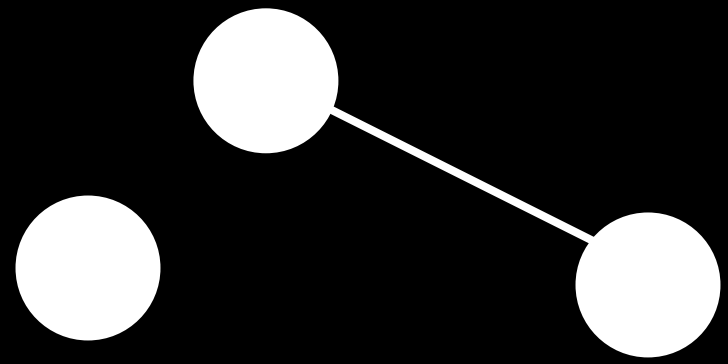
Disconnected

Graphs



Connected

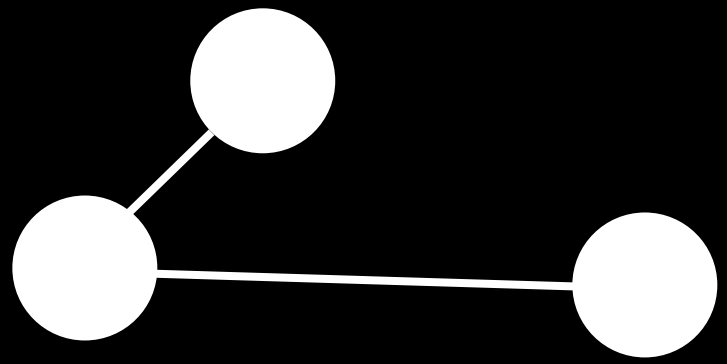
From any starting node, you can reach all other nodes



Disconnected

Graphs

Connected?



Graphs

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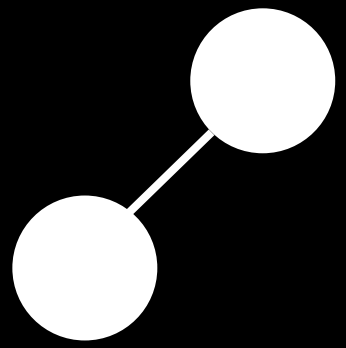

Graphs

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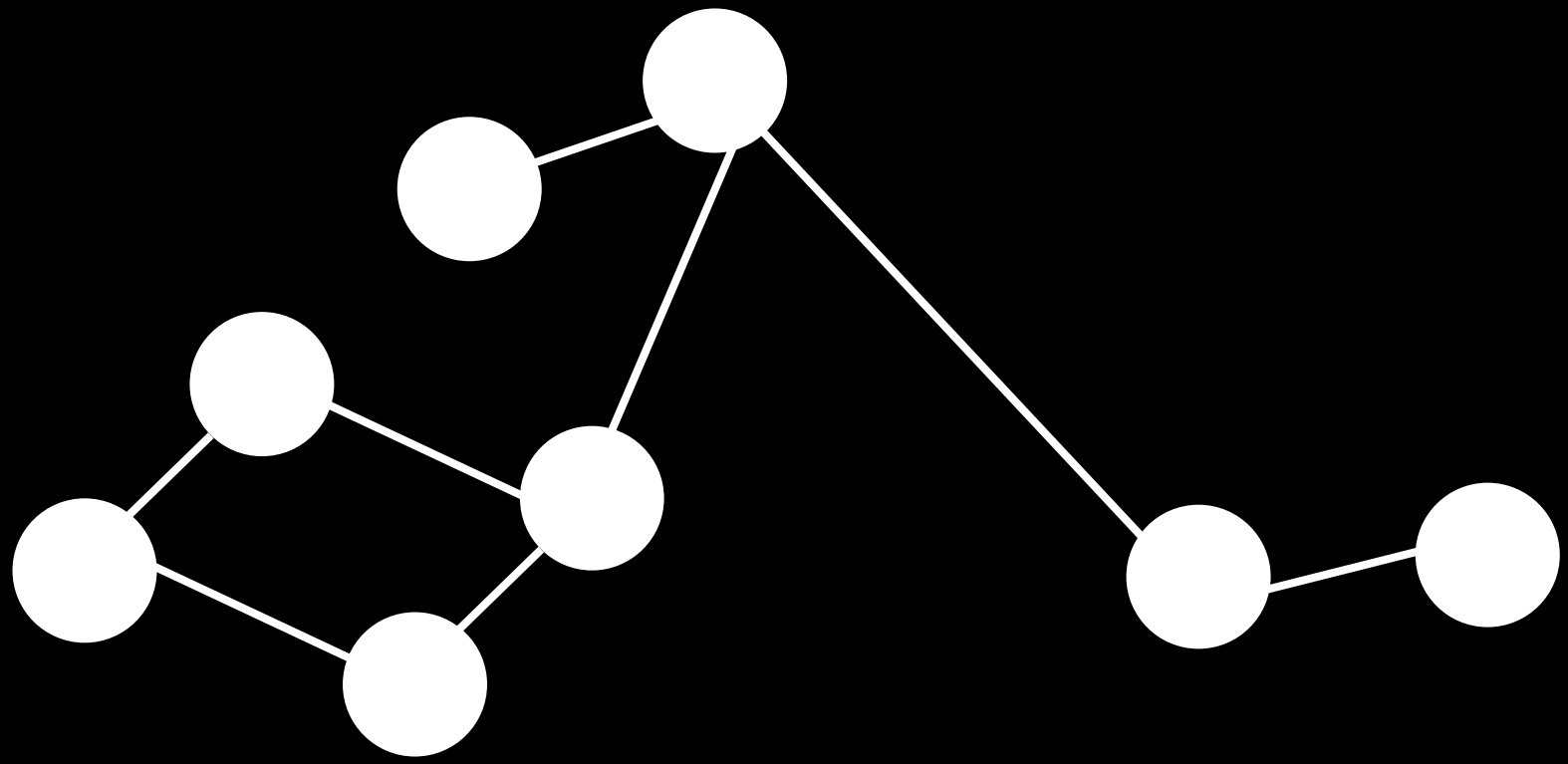

Graphs

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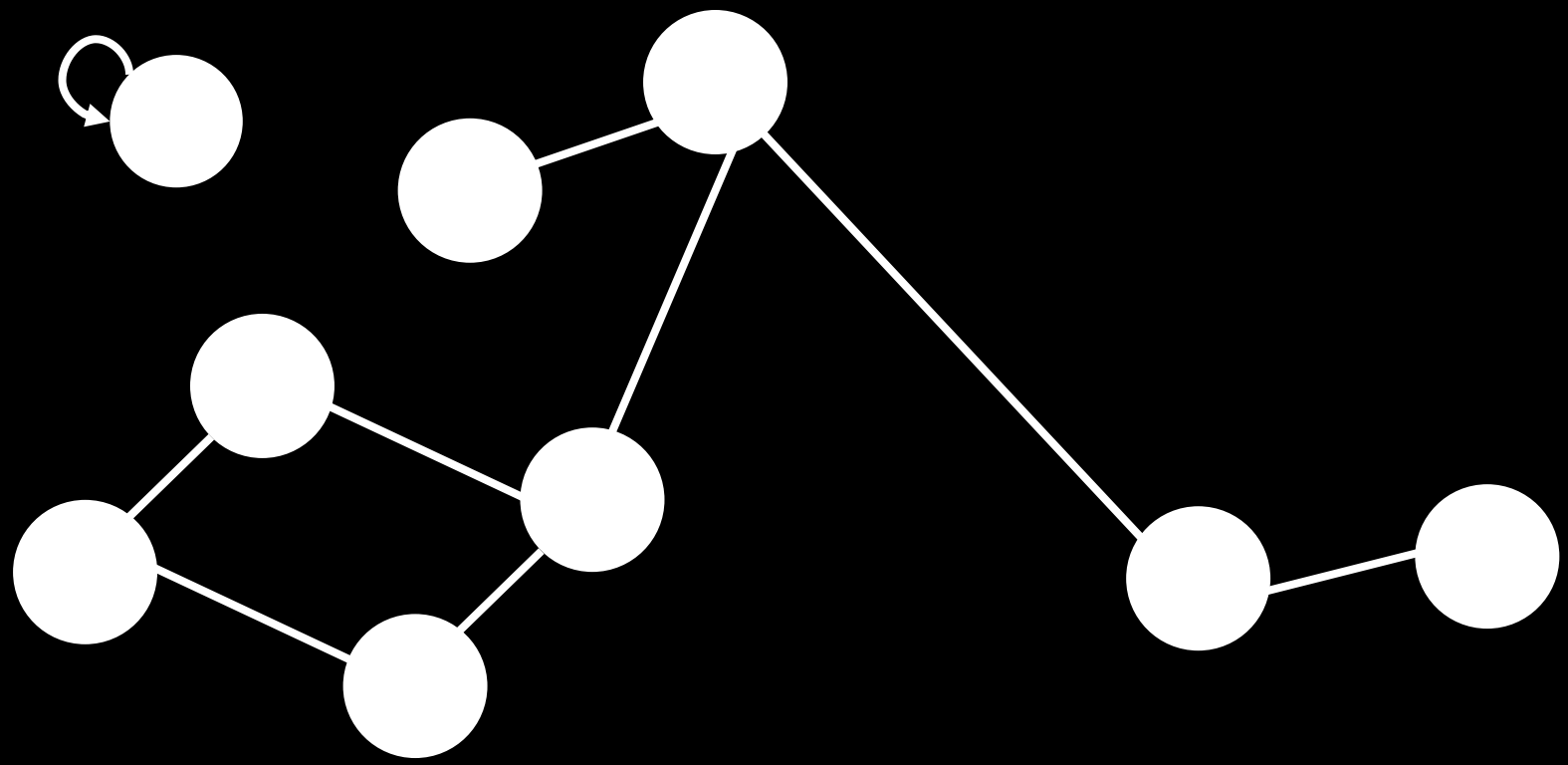
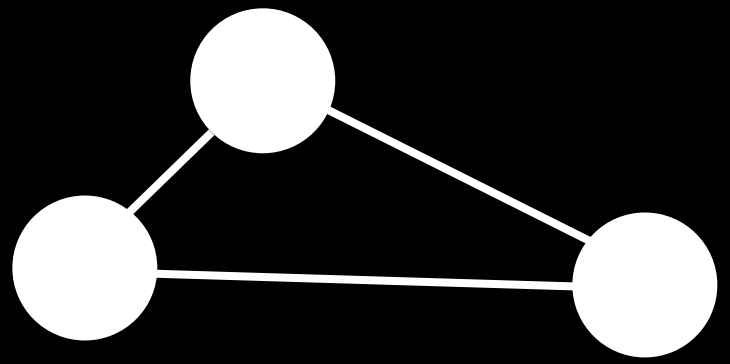
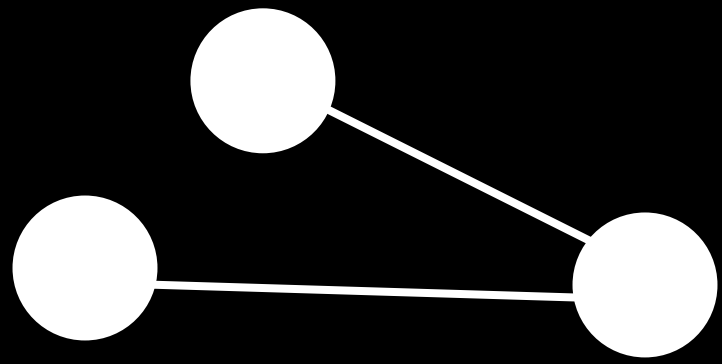


Graphs



Cyclic

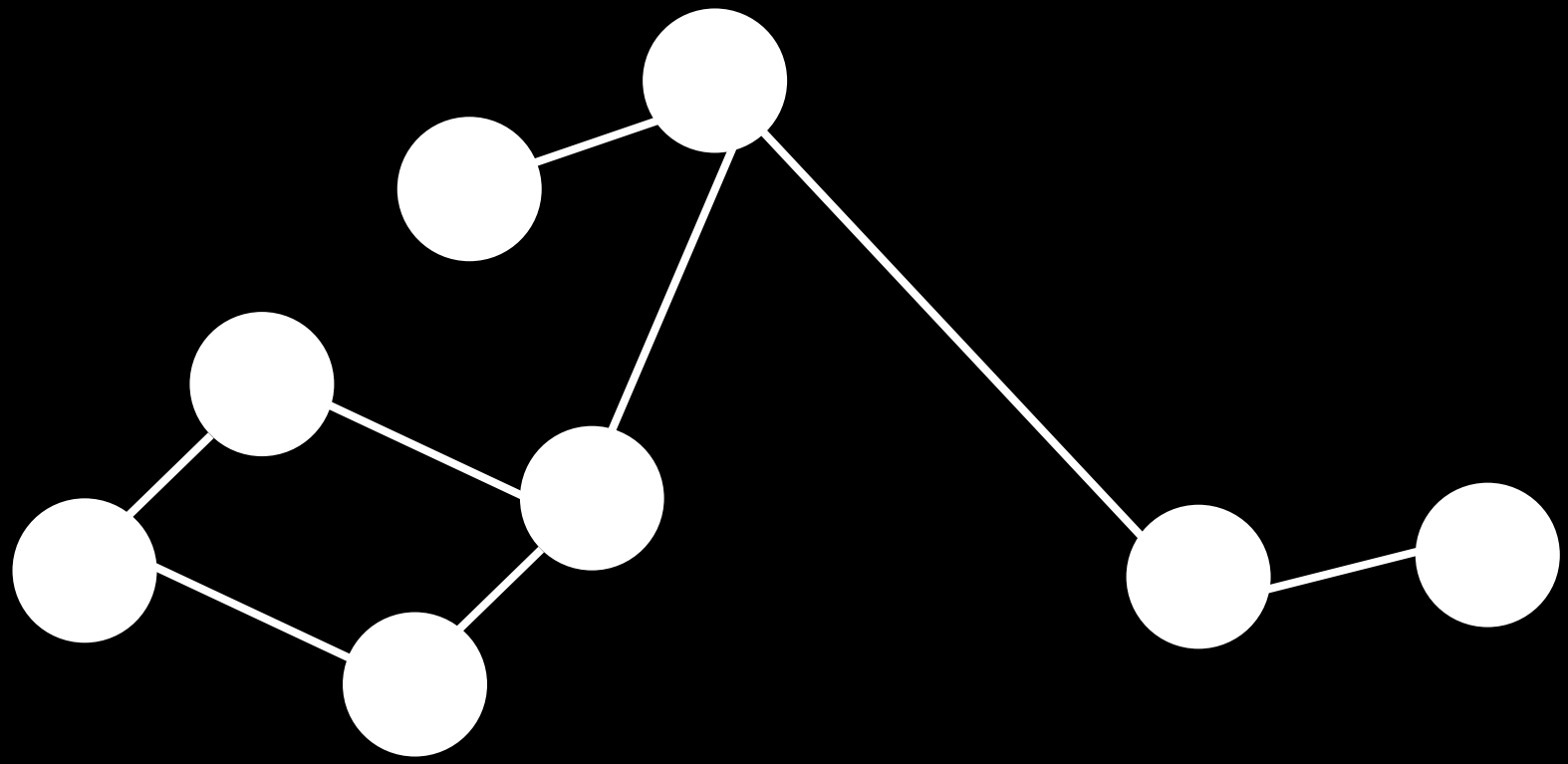
There exists a starting node for which there is a path to itself



Acyclic

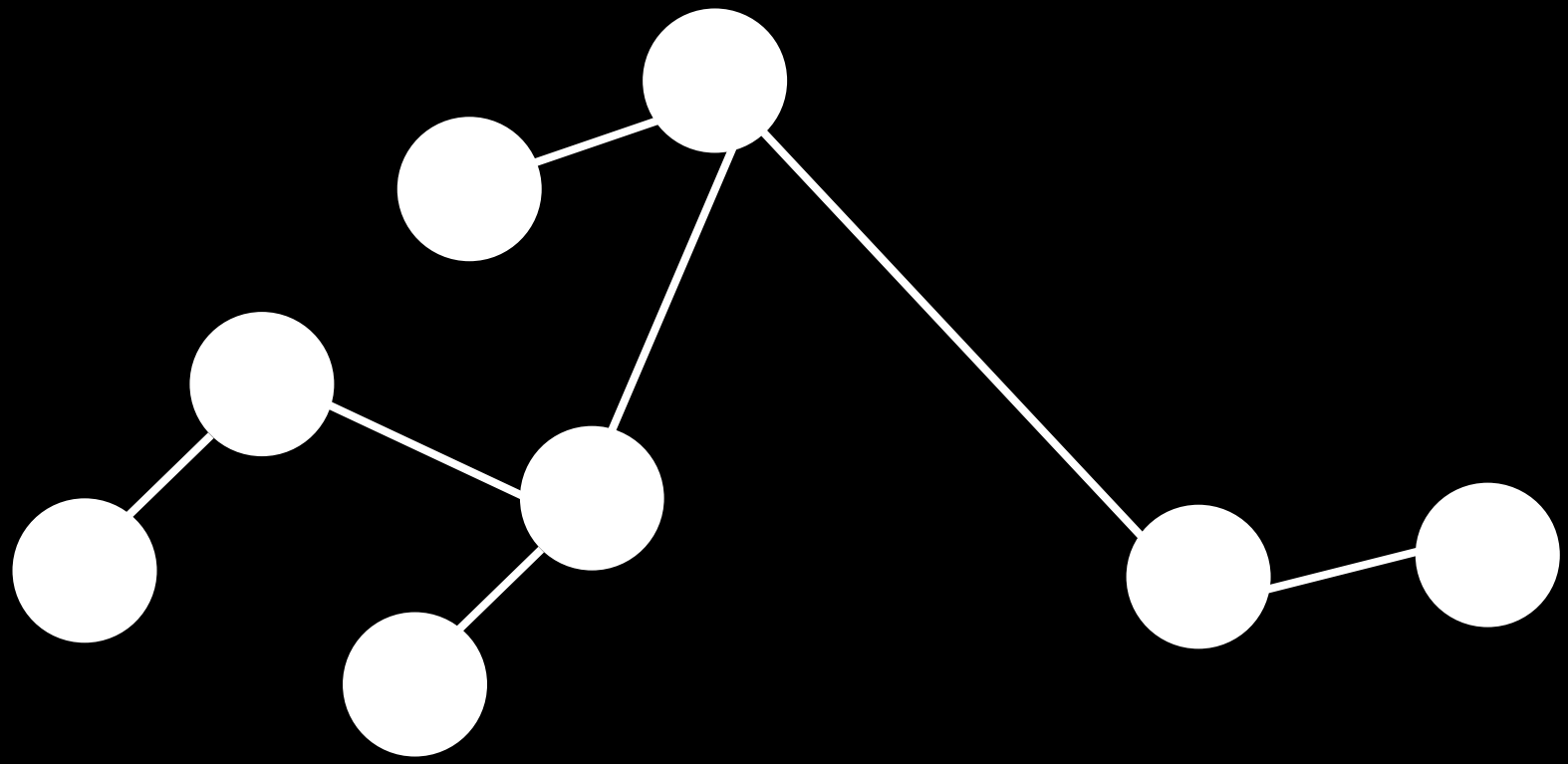
Graphs

Cyclic?



Graphs

Cyclic?



Graphs

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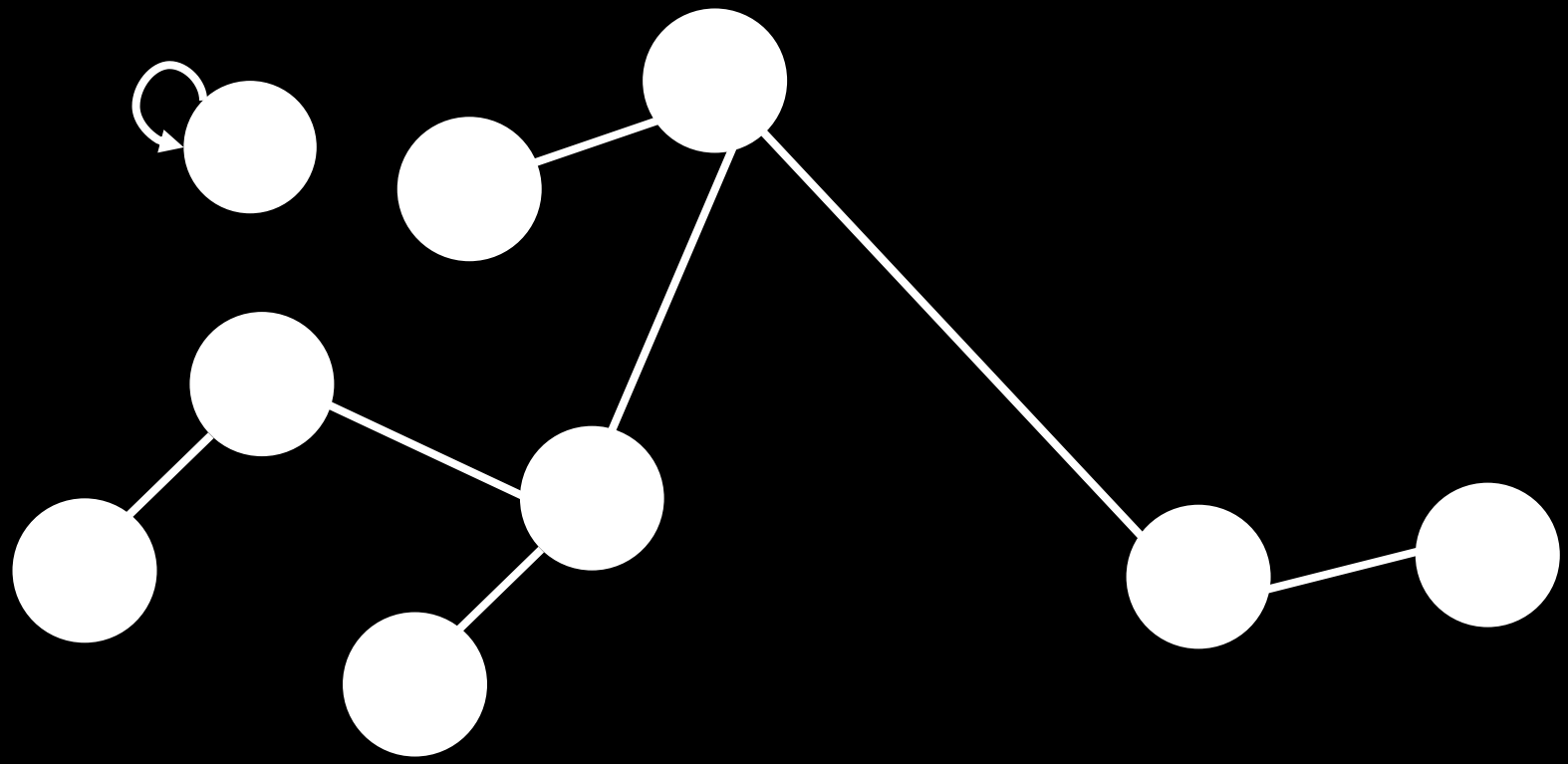

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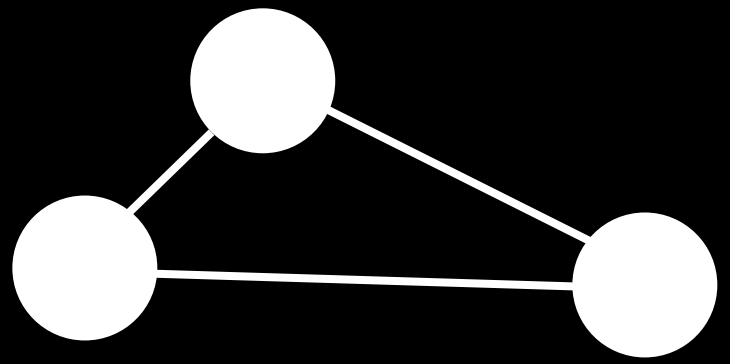
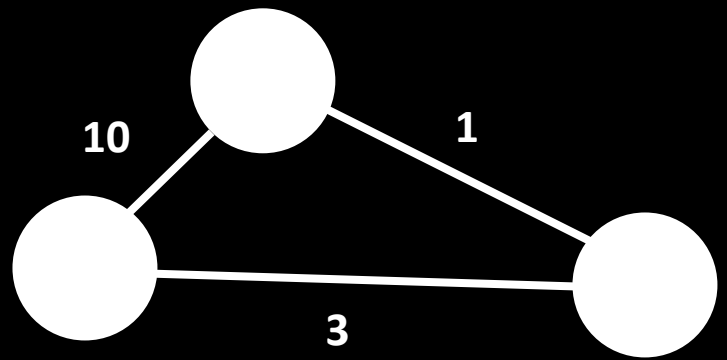
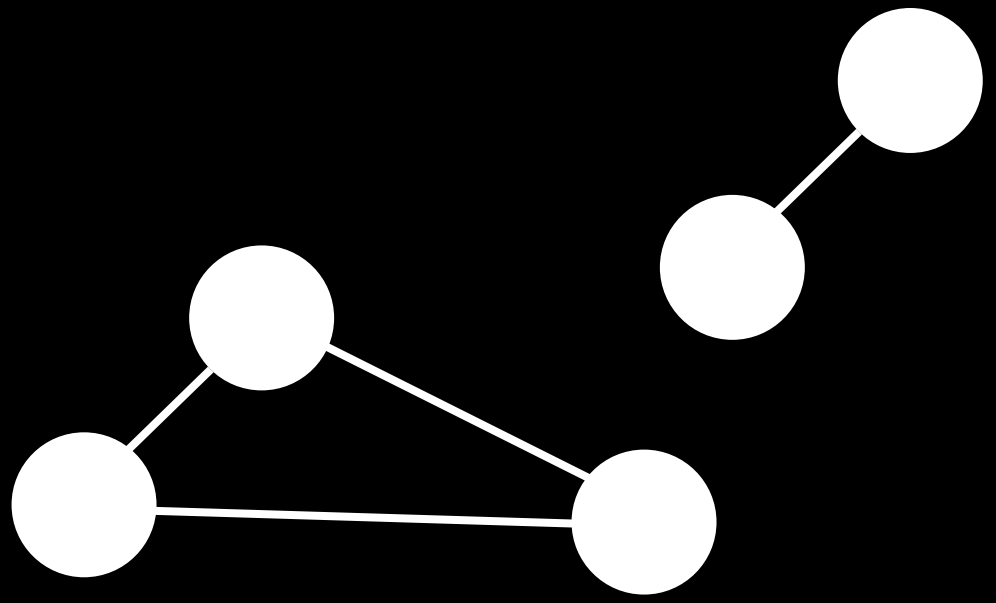


Unweighted



Weighted

Today



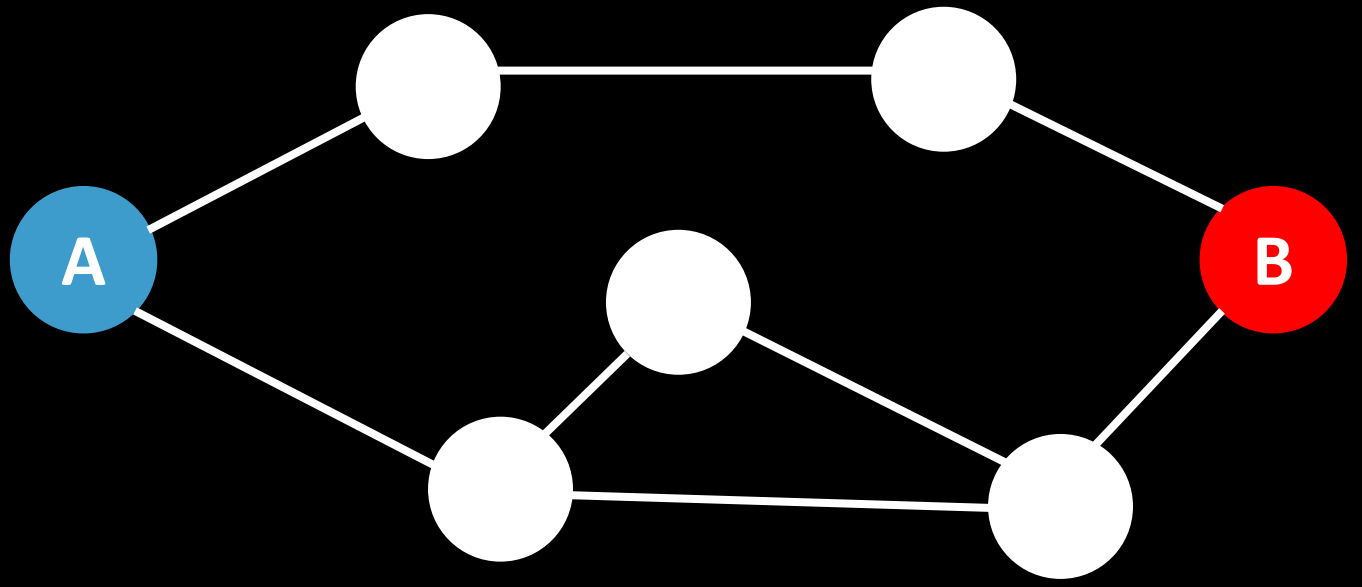
Undirected + Unweighted

Could be Cyclic

Could be Disconnected

Graphs

- Consider an arbitrary undirected, unweighted graph
- Pick 2 arbitrary vertices on the graph: **A** and **B**



Graphs

- Q: For an arbitrary graph, are we guaranteed that there is a **path** between nodes **A** and **B**?

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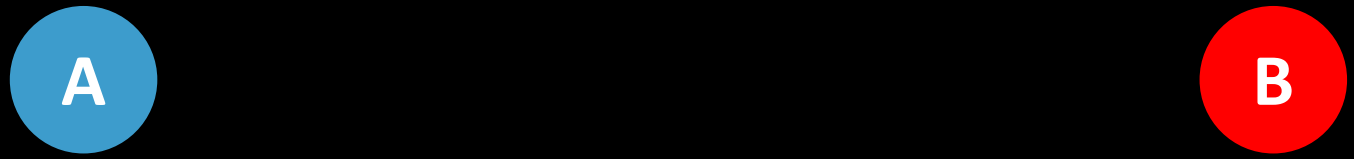

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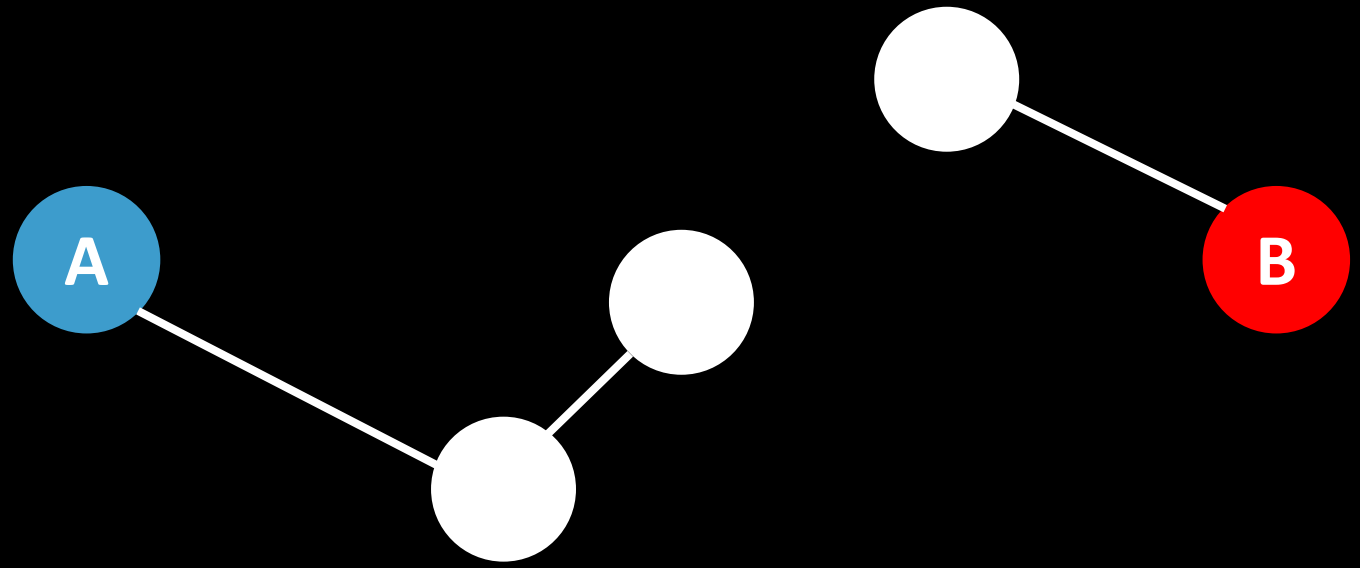

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- Q: If a path exists between **A** and **B**, is it unique?

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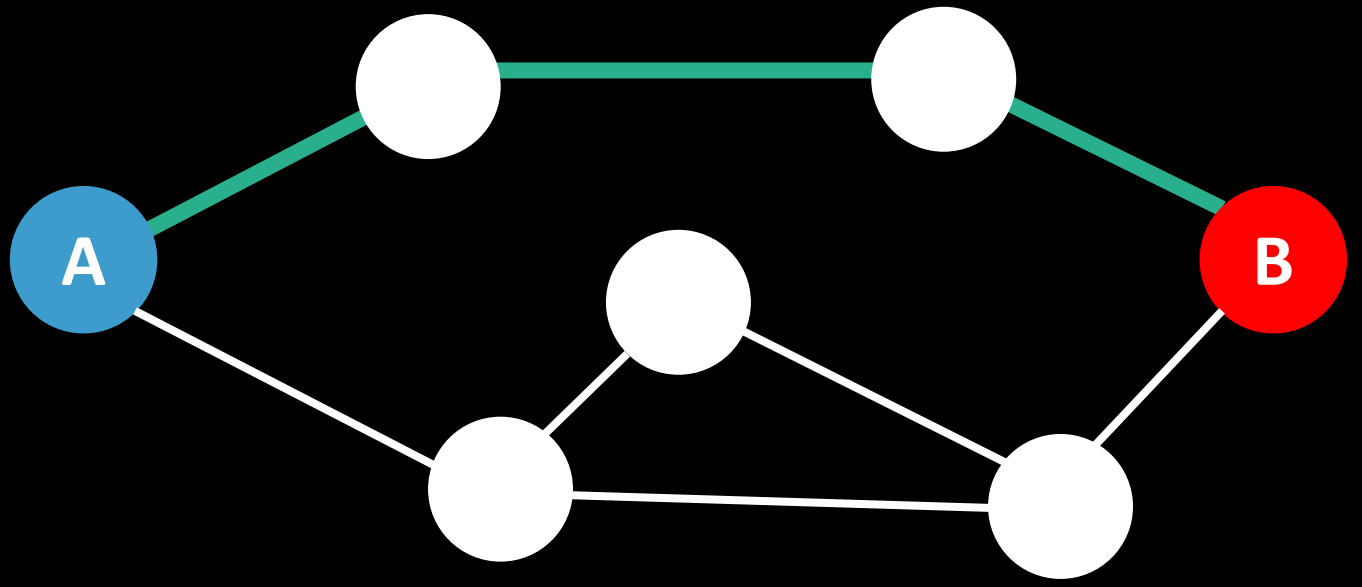

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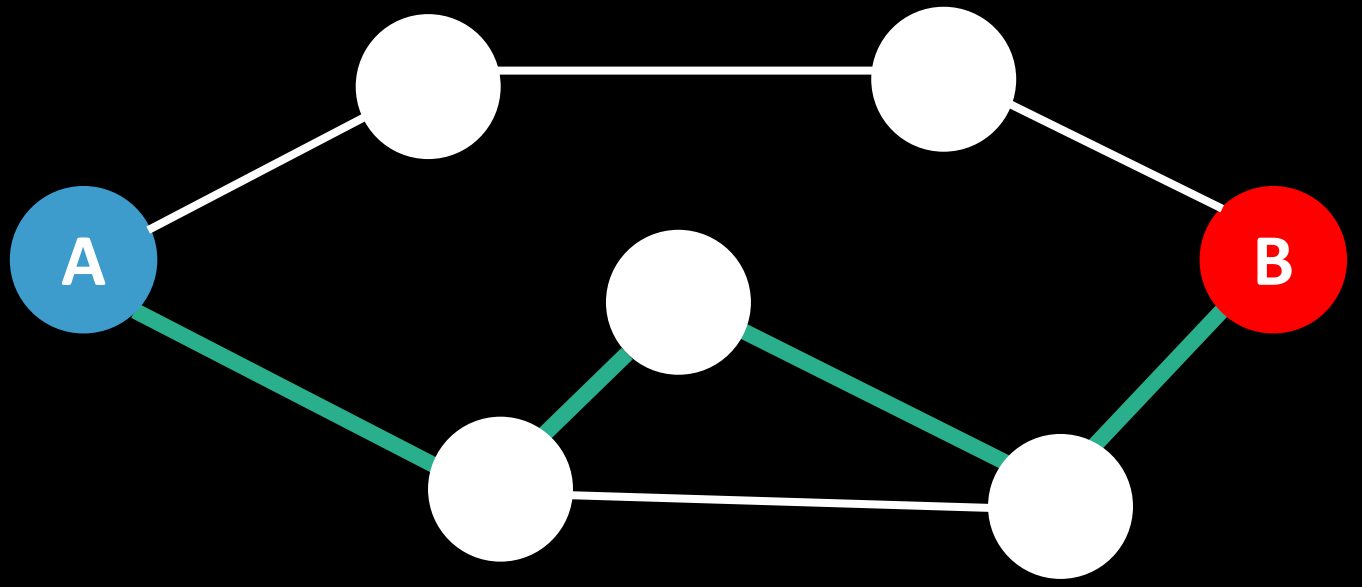

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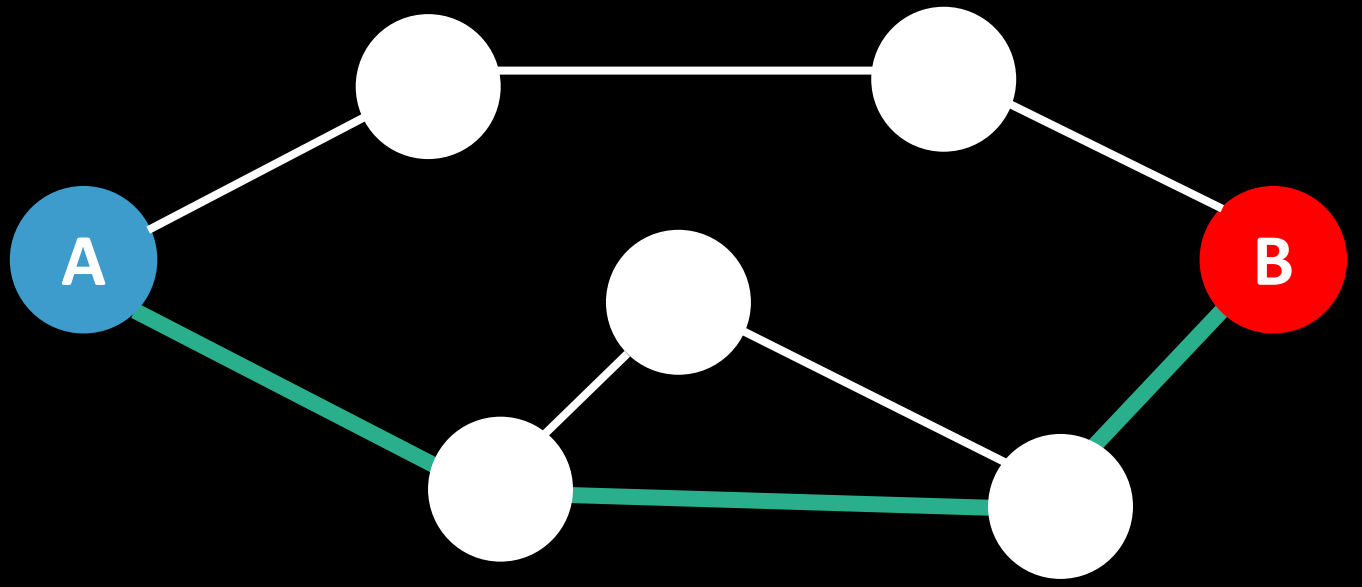

Graphs

Interested in finding…

- Whether a path exists from A → B
- A path from A → B with **minimal path length**

Definitions

For an undirected, unweighted graph:

- Path length: for a path from node A to node B, the **path length** is the number of edges the path is comprised of.
  - If the path is denoted by $\mathrm{P} = \{e_1, e_2, \ldots, e_n\}$, its length is $|P| = n$
- Level: The minimum number of edges between a node and the *starting node*
  - i.e. the length of the shortest path between a node and the starting node

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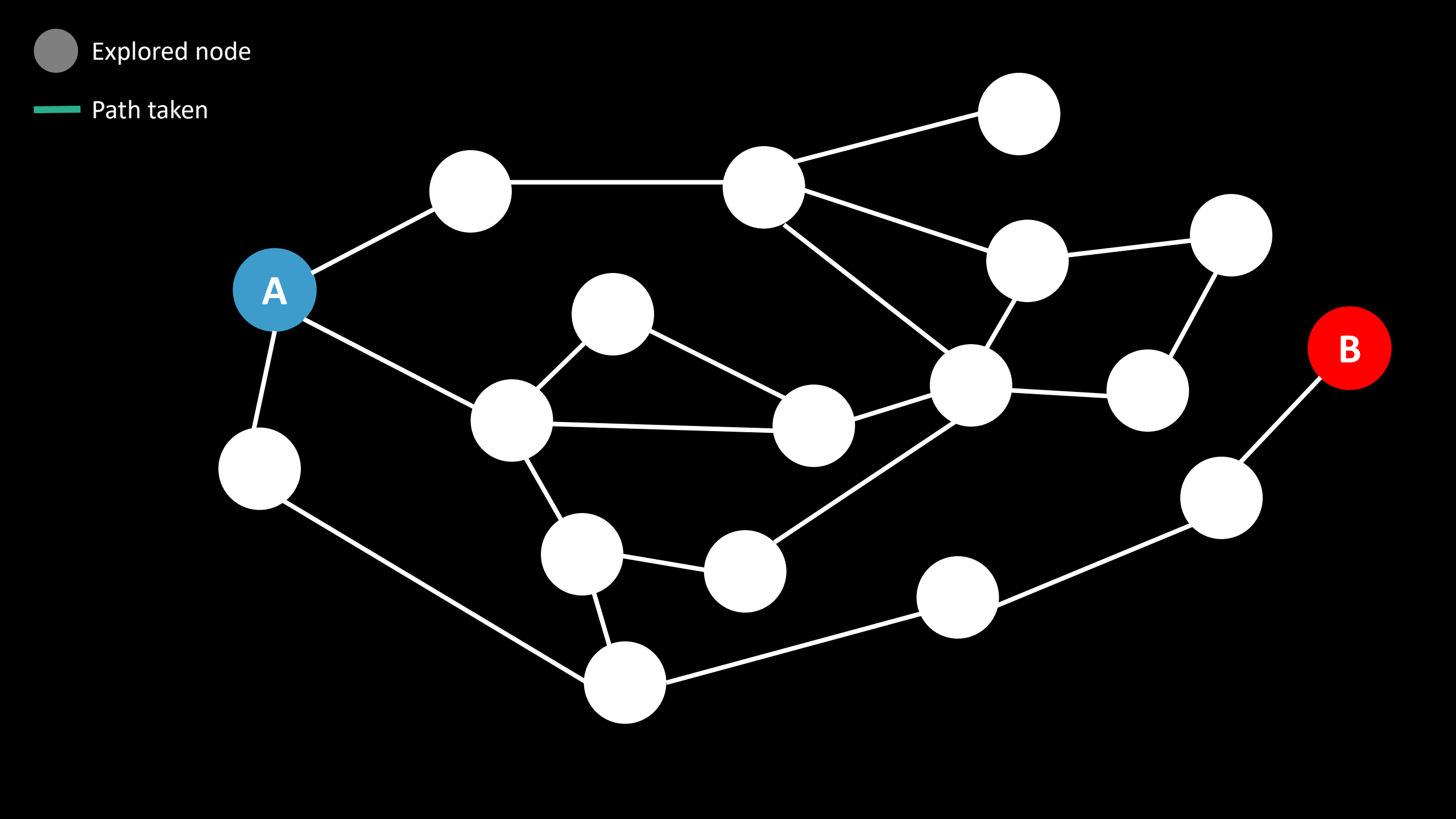
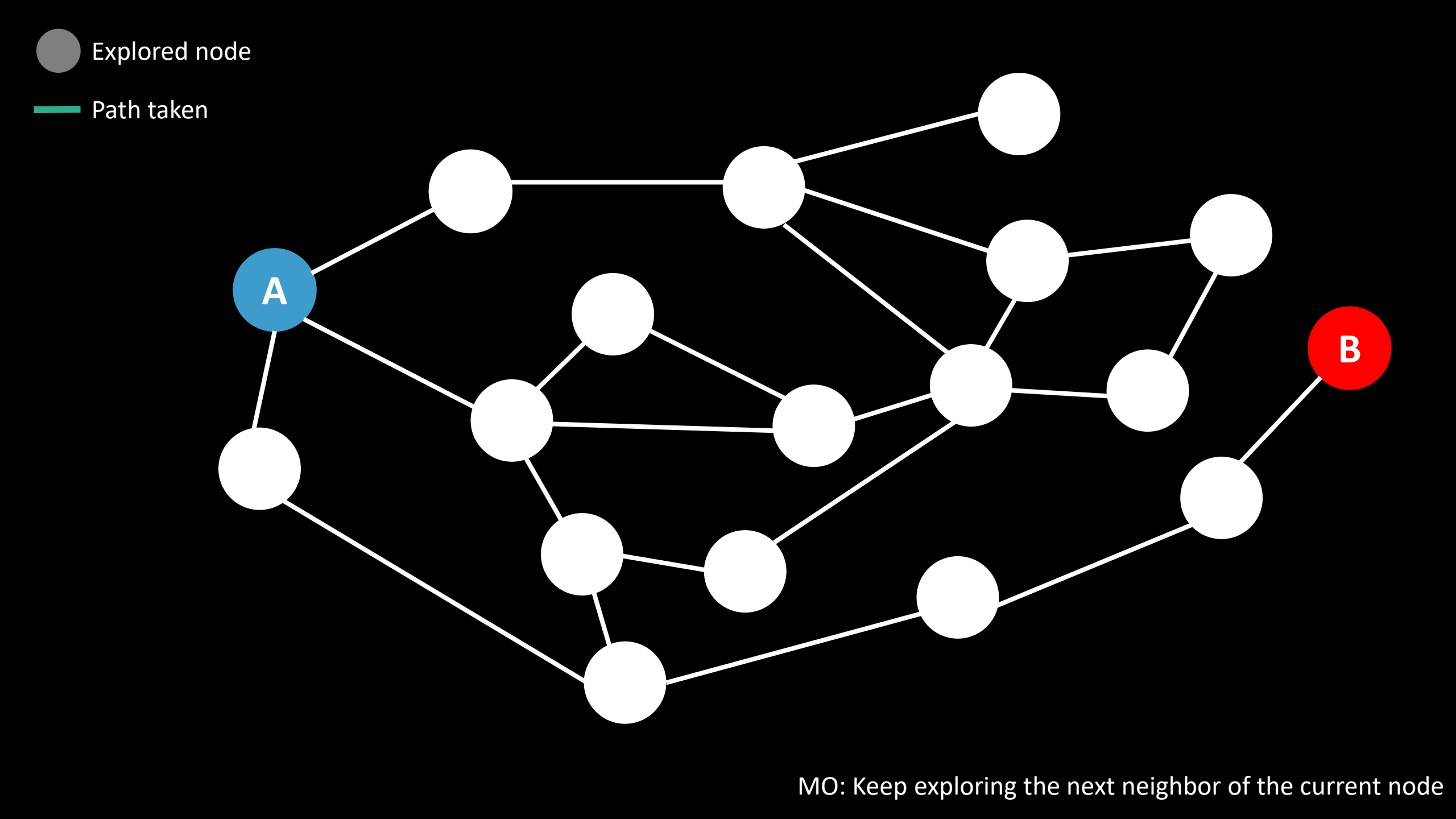
  - i.e. the length of the shortest path between a node and the starting node
  - Q: What is the starting node?
  - A: You choose, based on the context

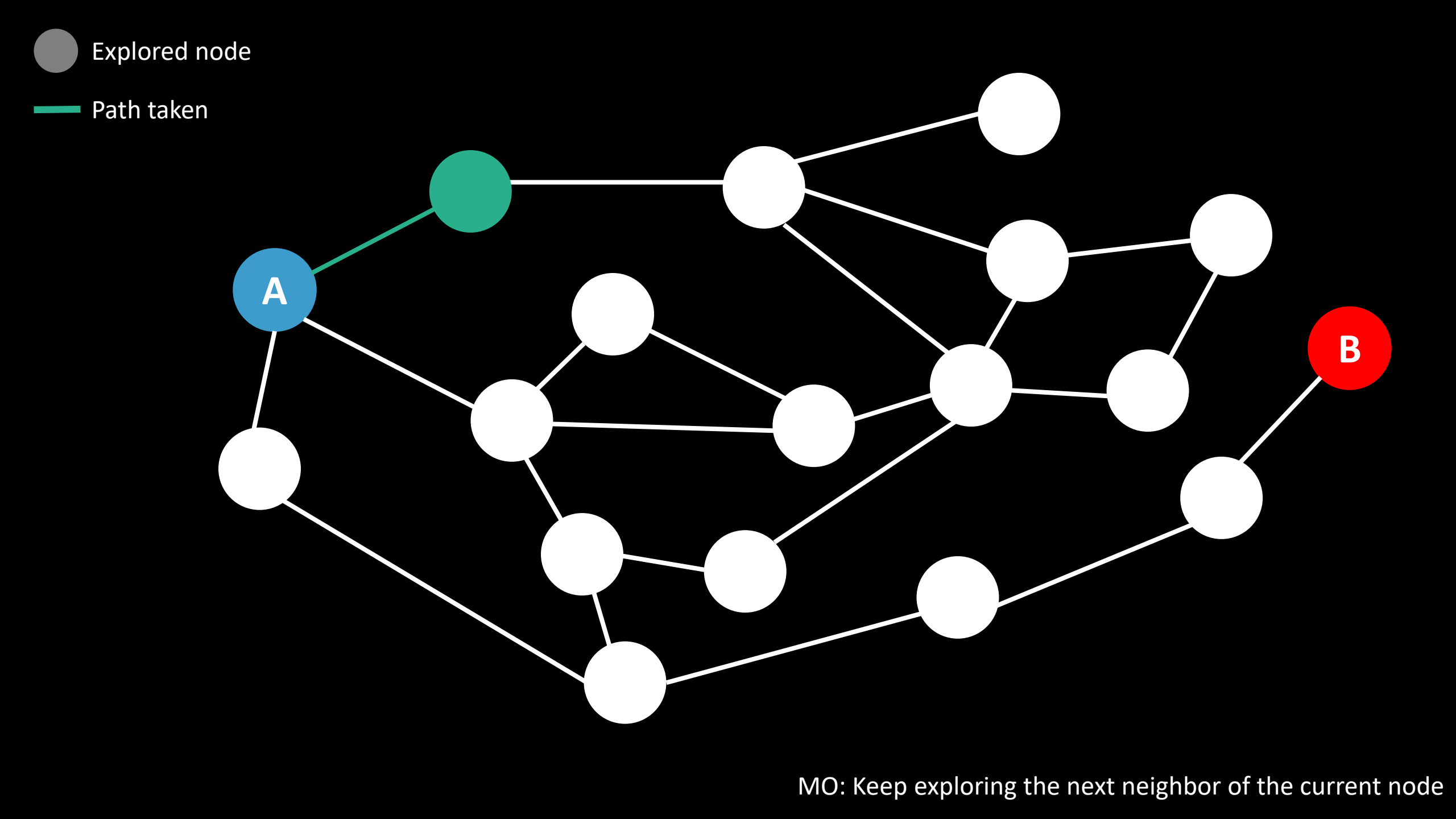
Path-finding

- Does a path exist from A→B?
- Keep track of:
  - Path taken
  - Nodes explored


Explored node
Path taken
A
B



MO: Keep exploring the next neighbor of the current node



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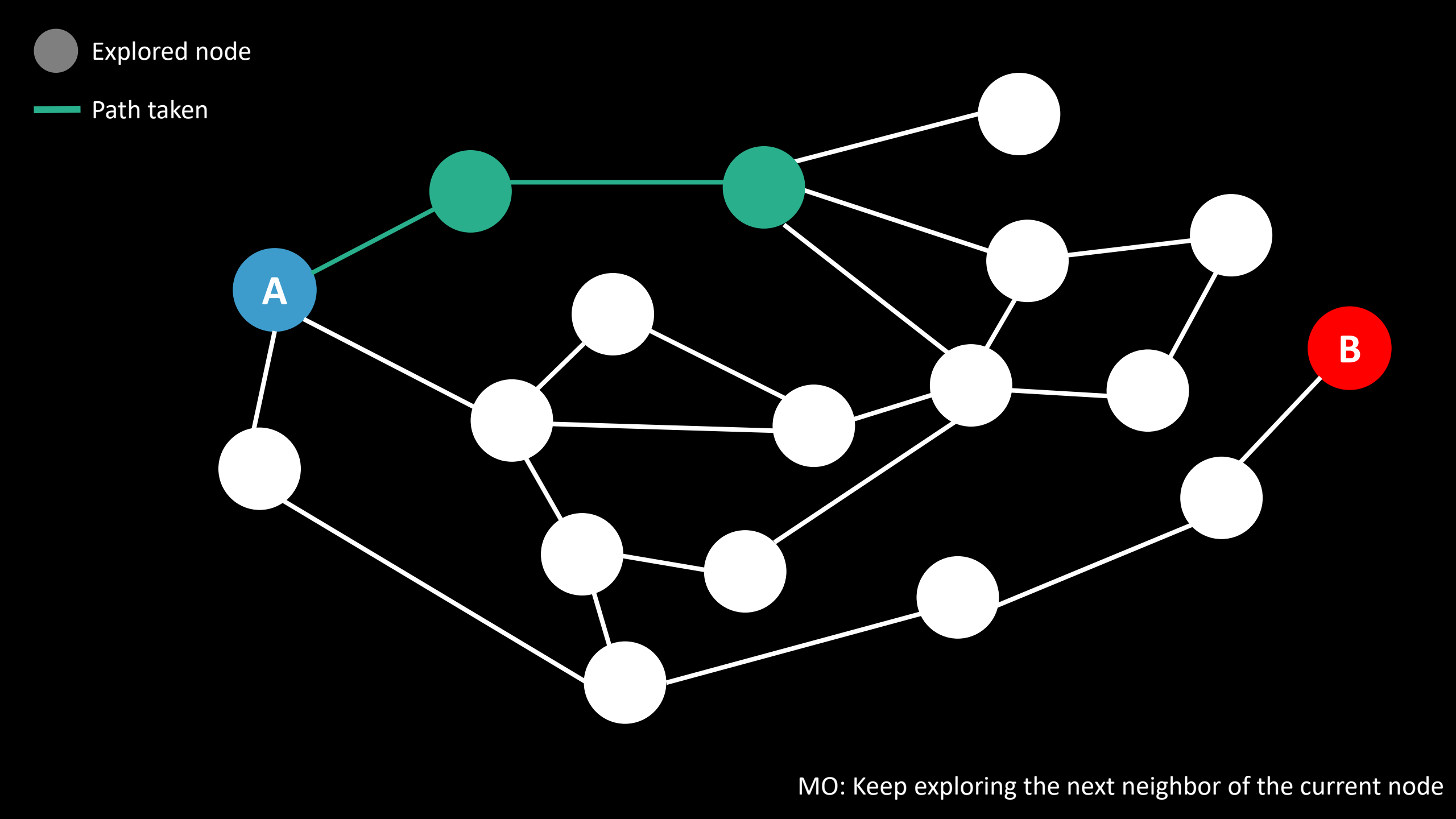

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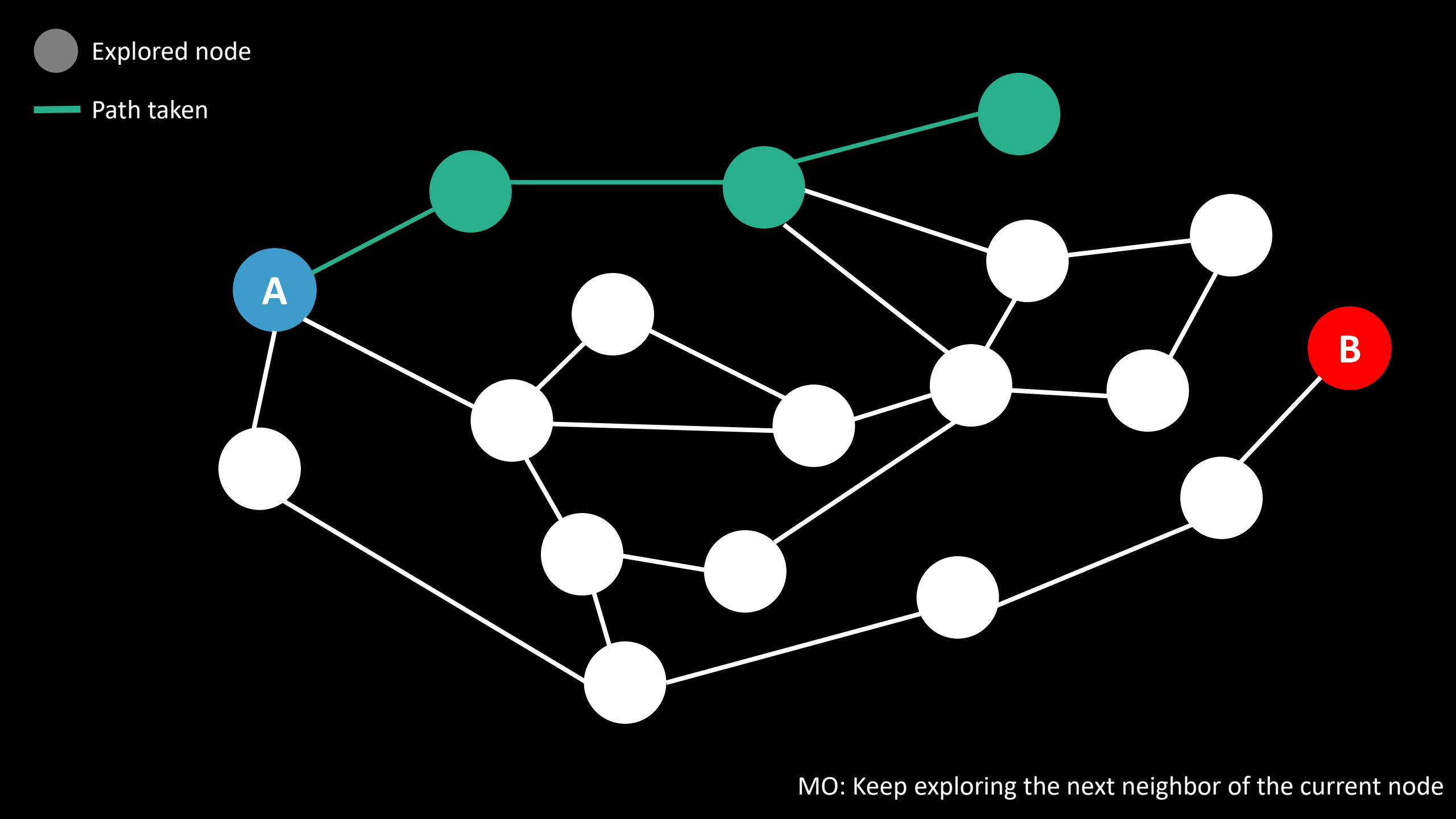

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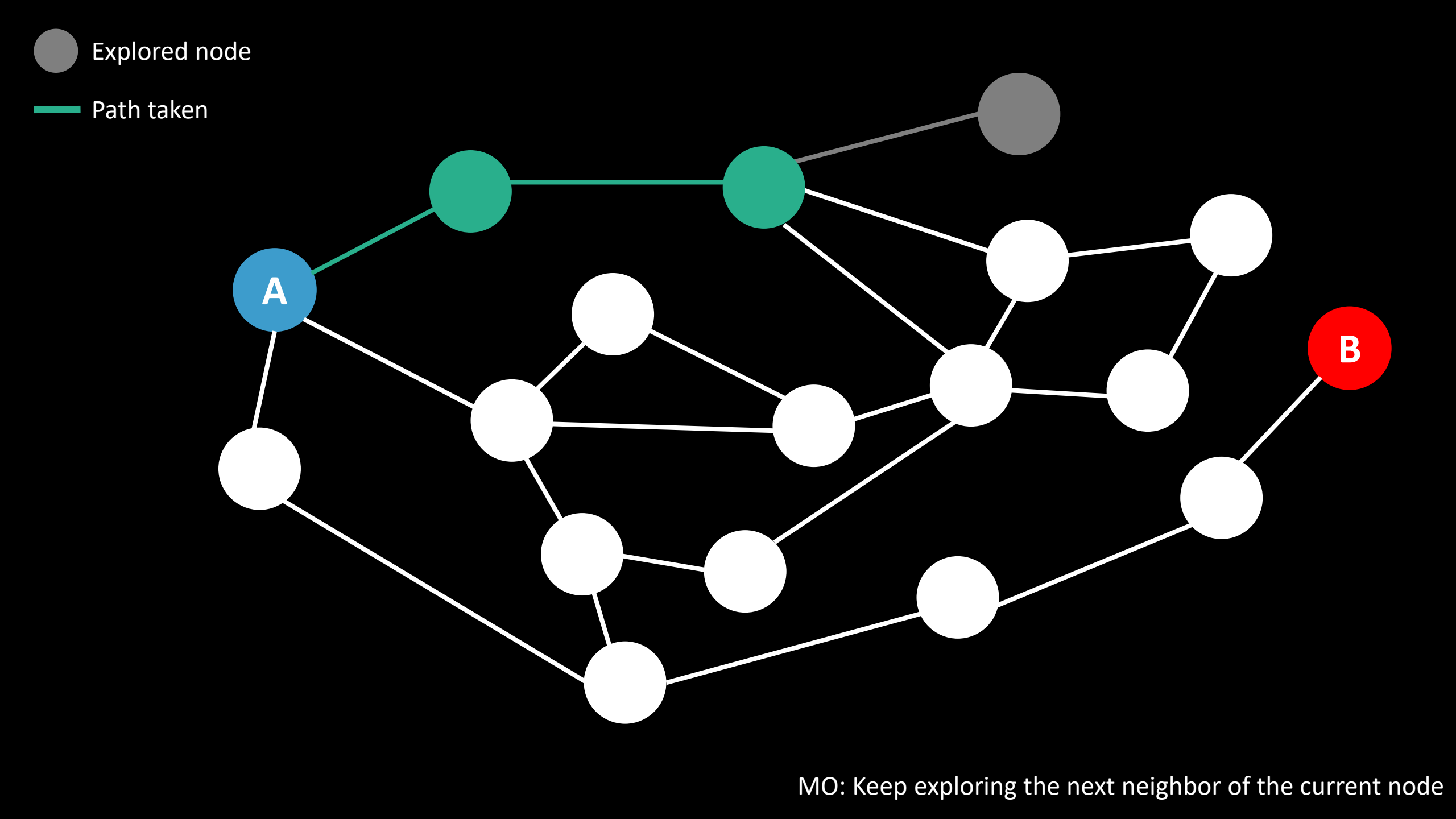

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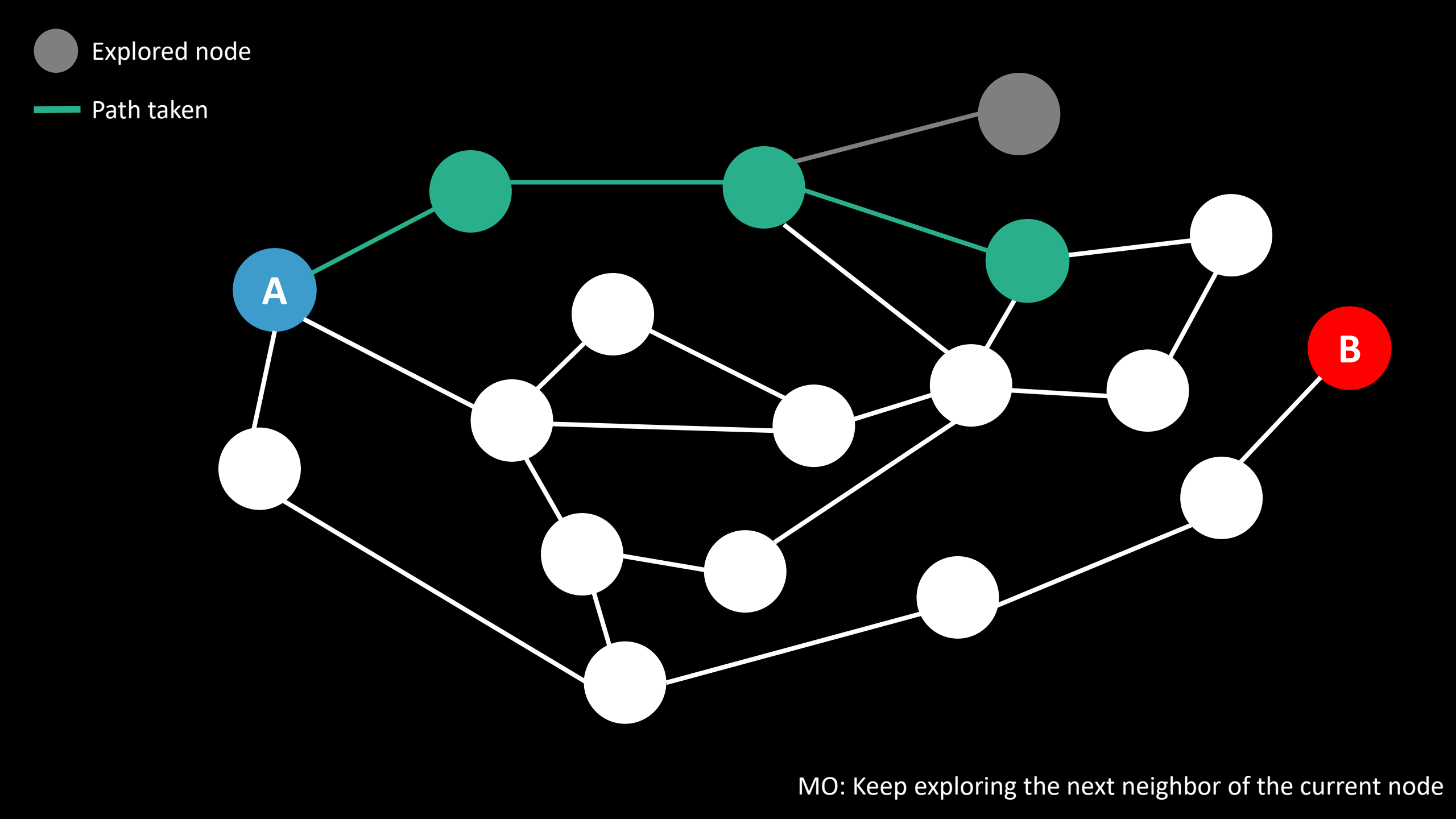

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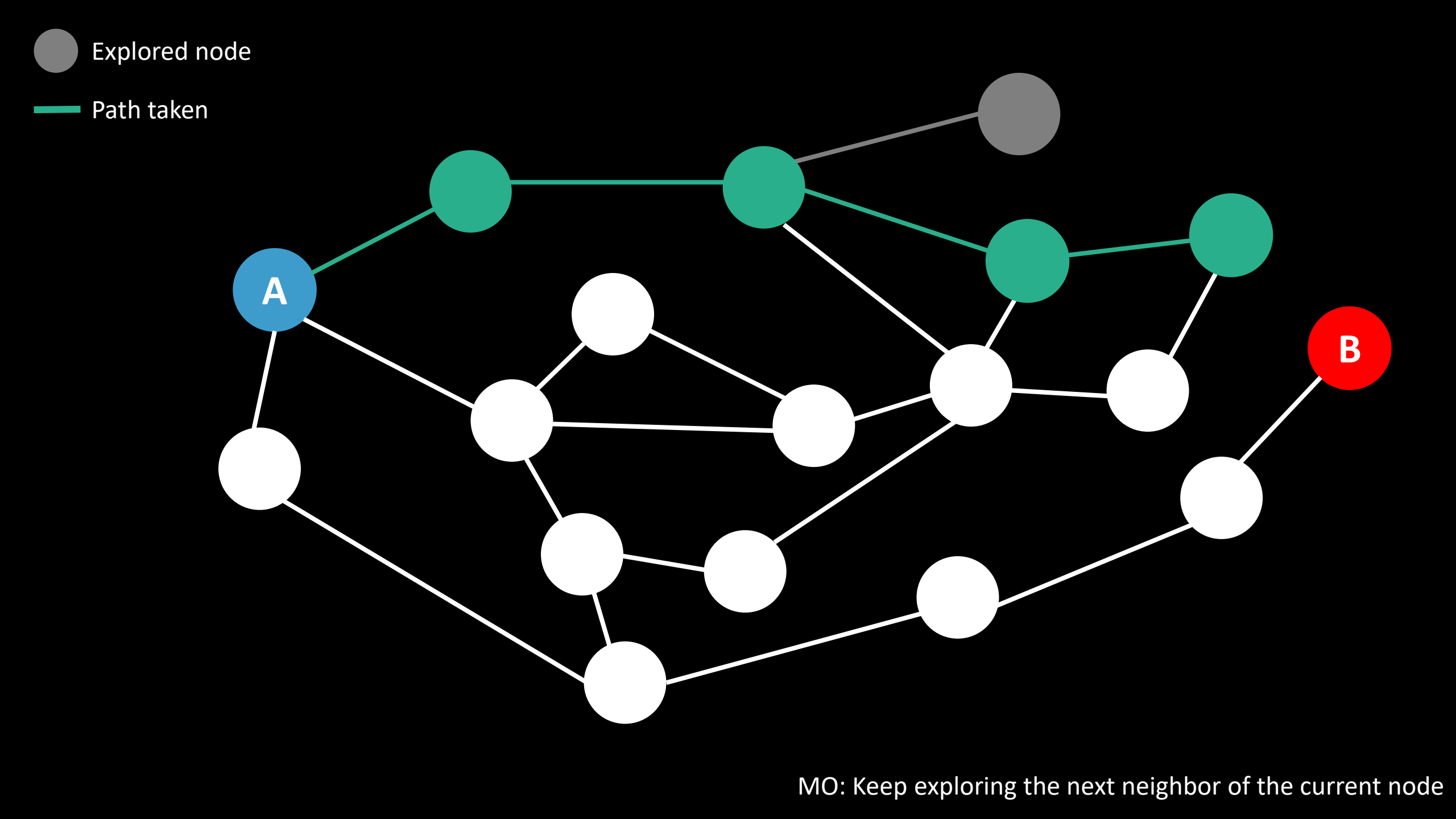

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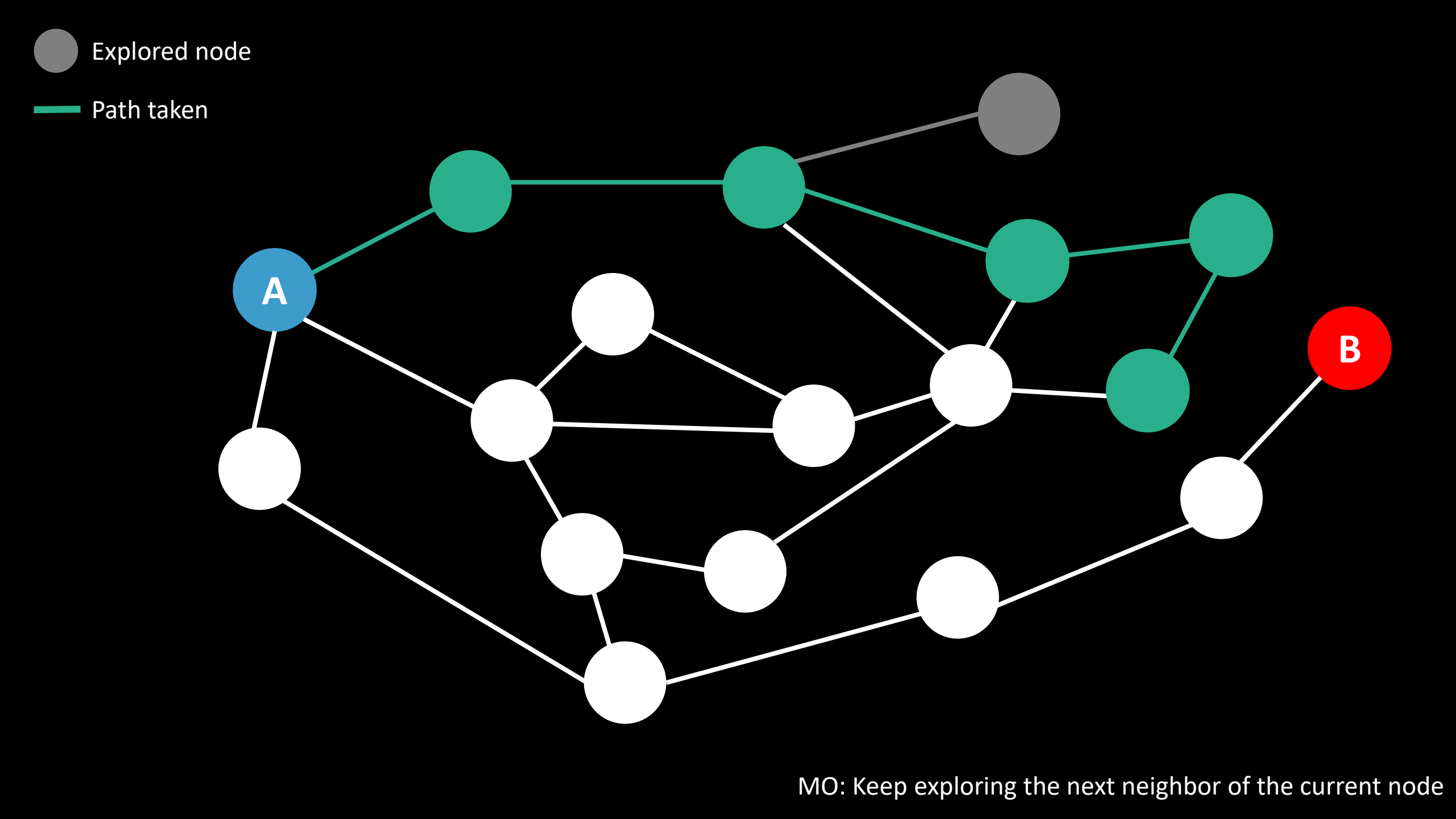

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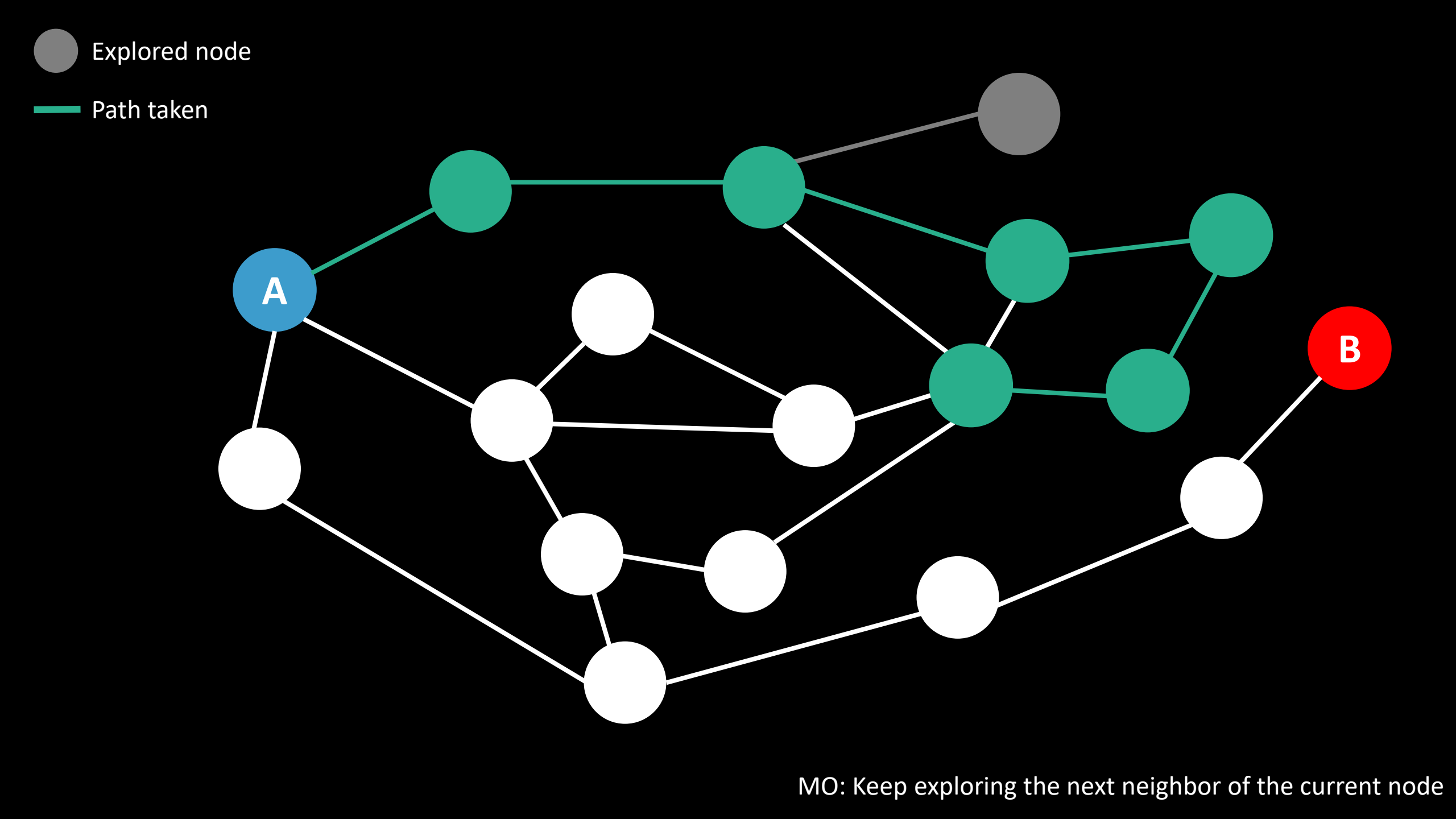

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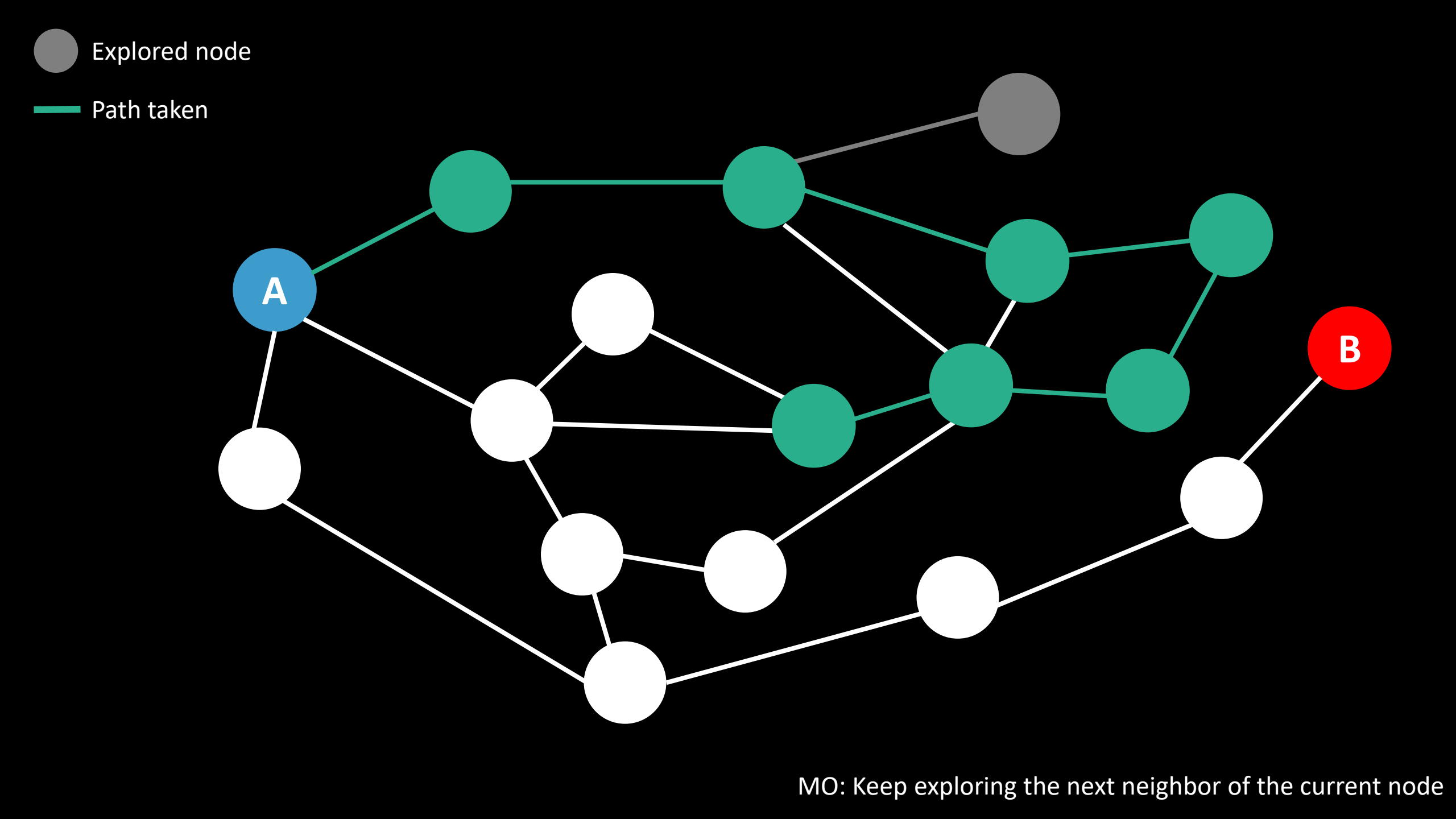

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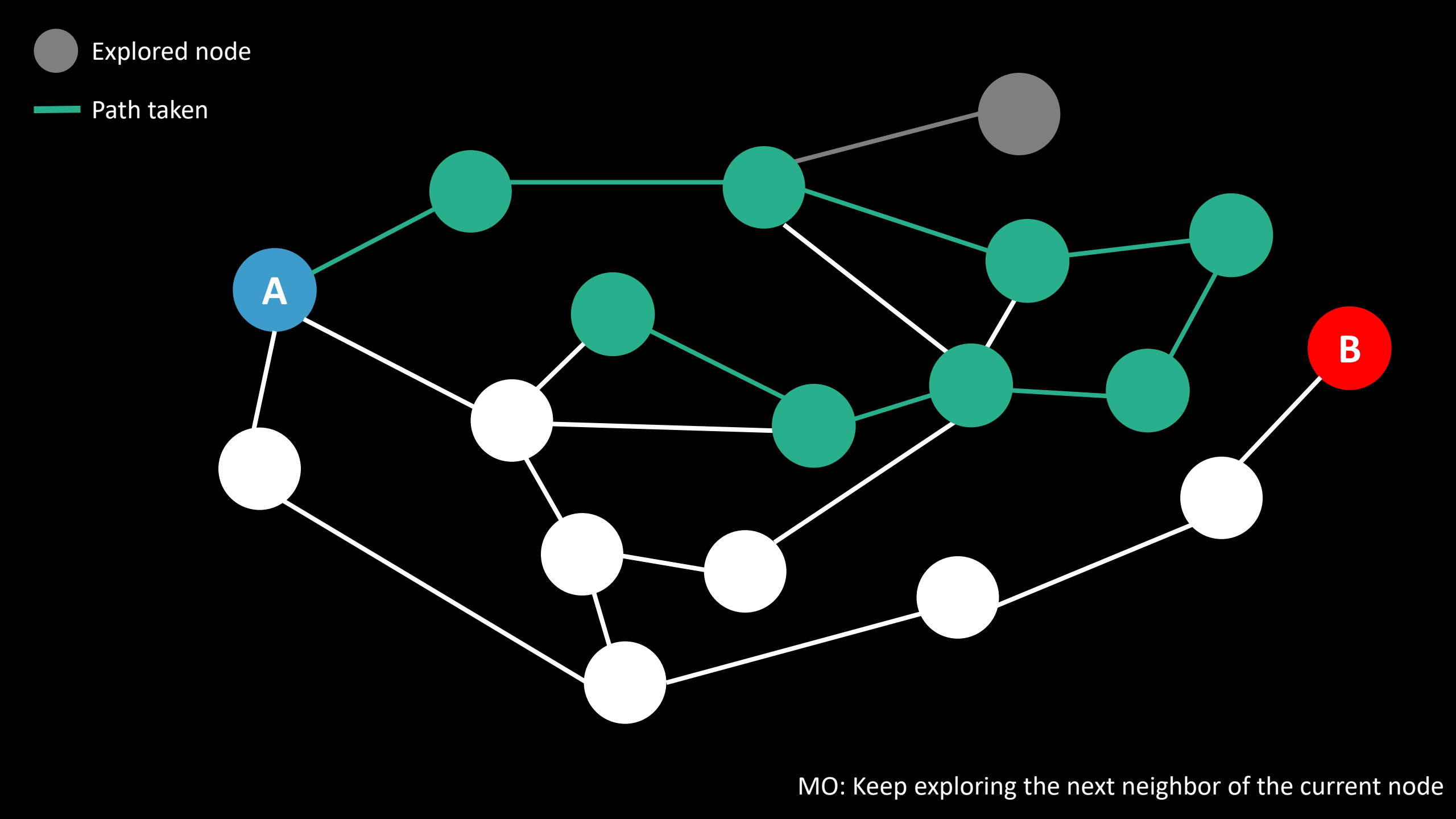

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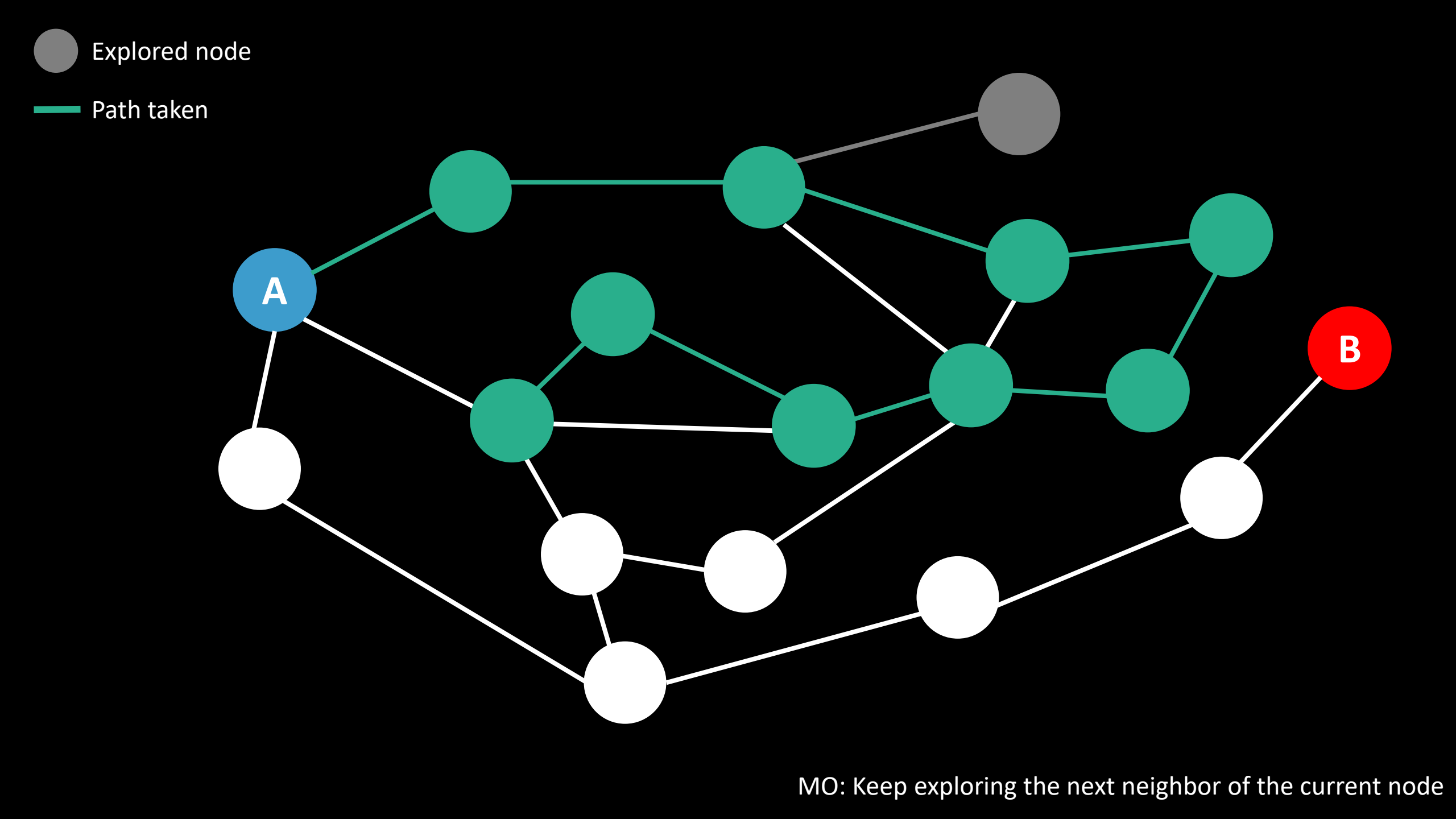

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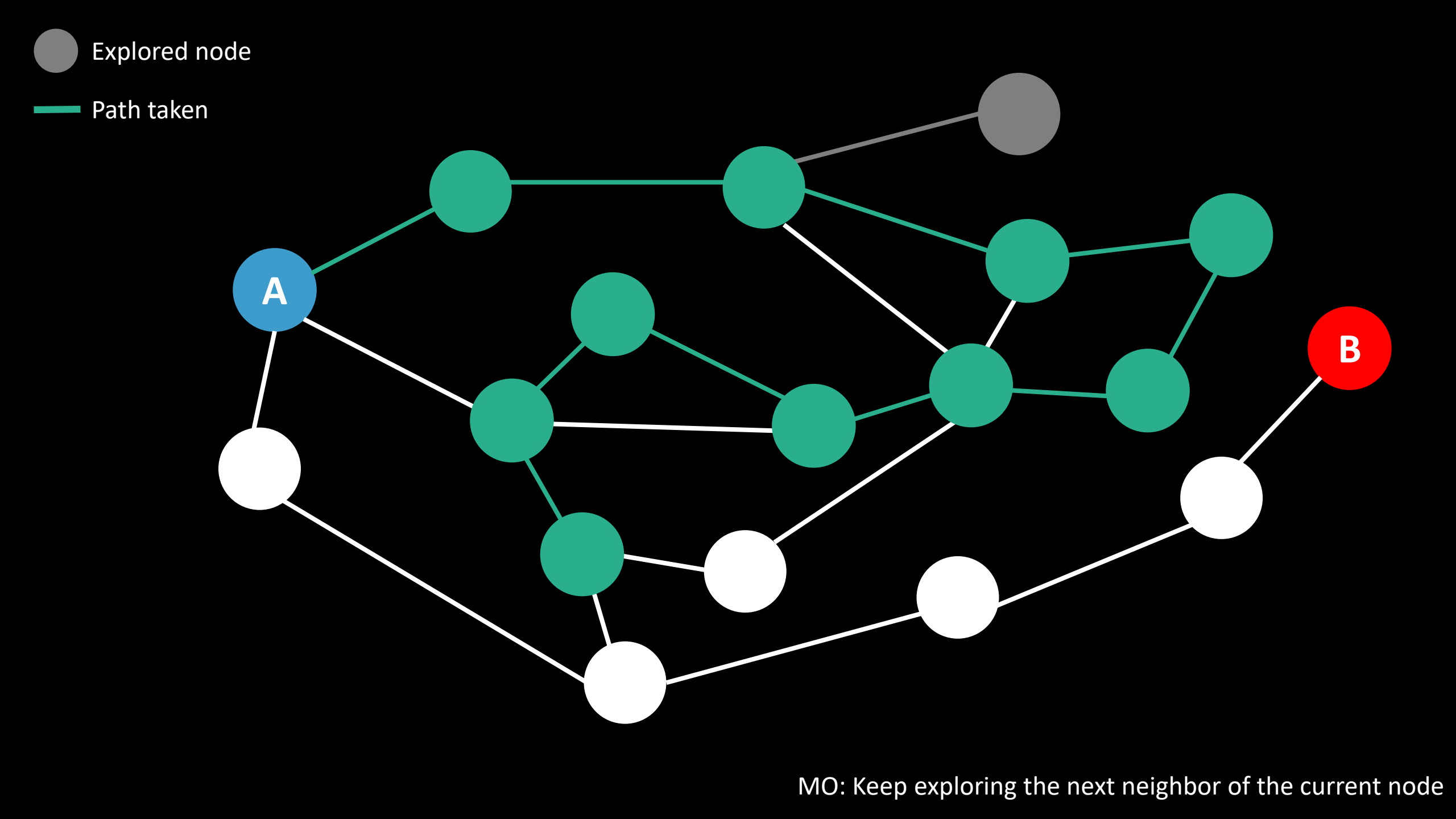

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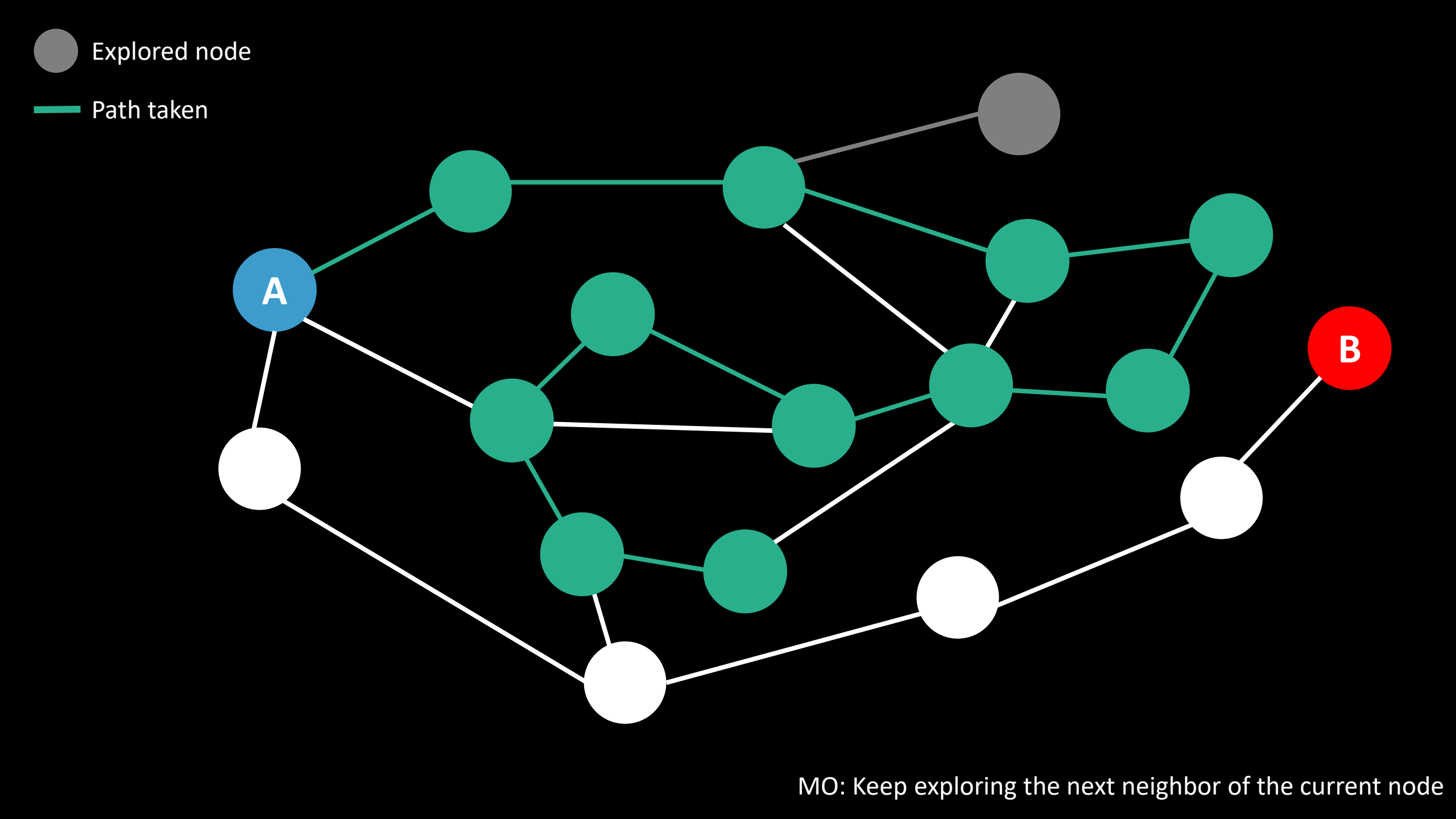

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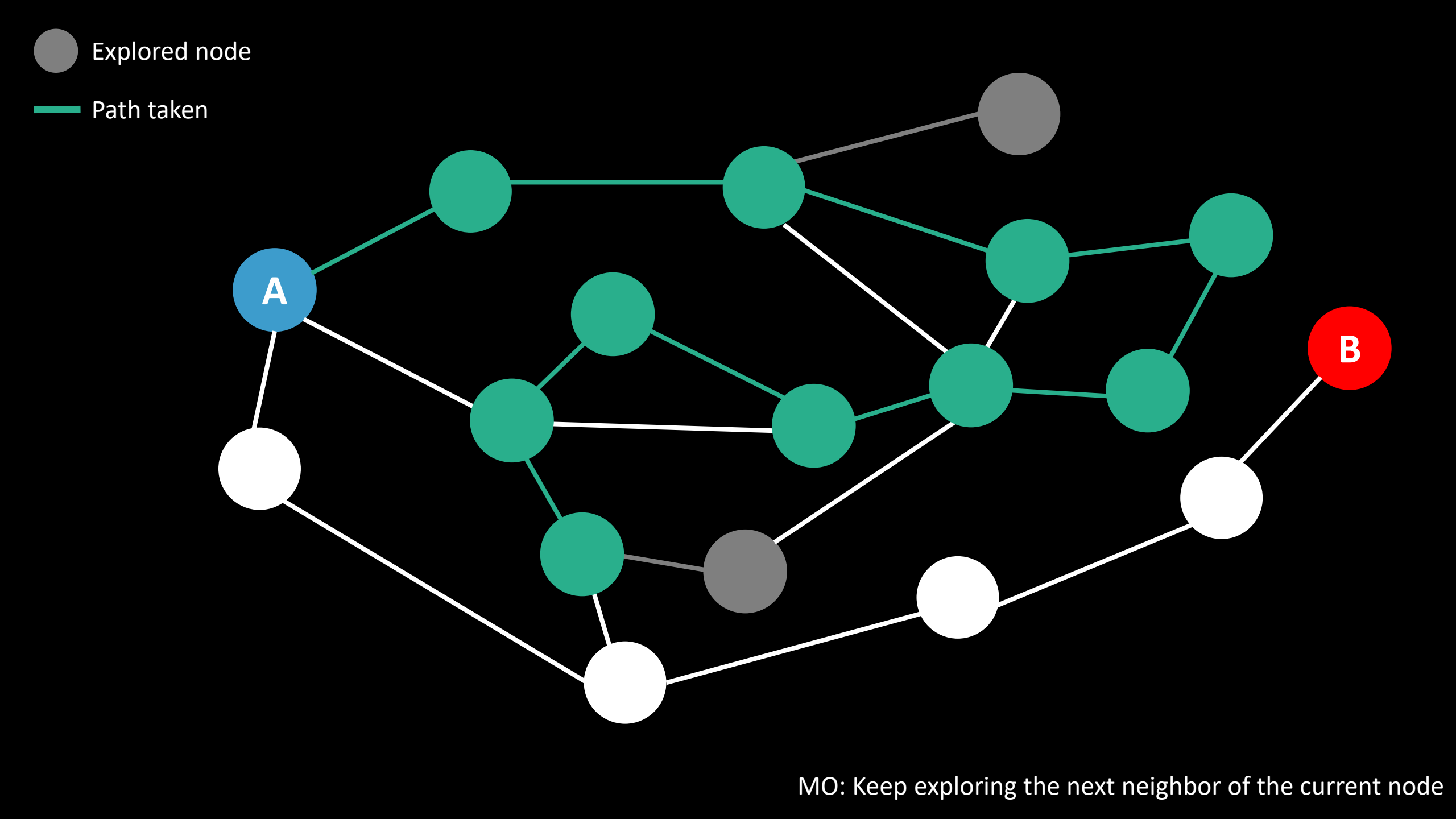

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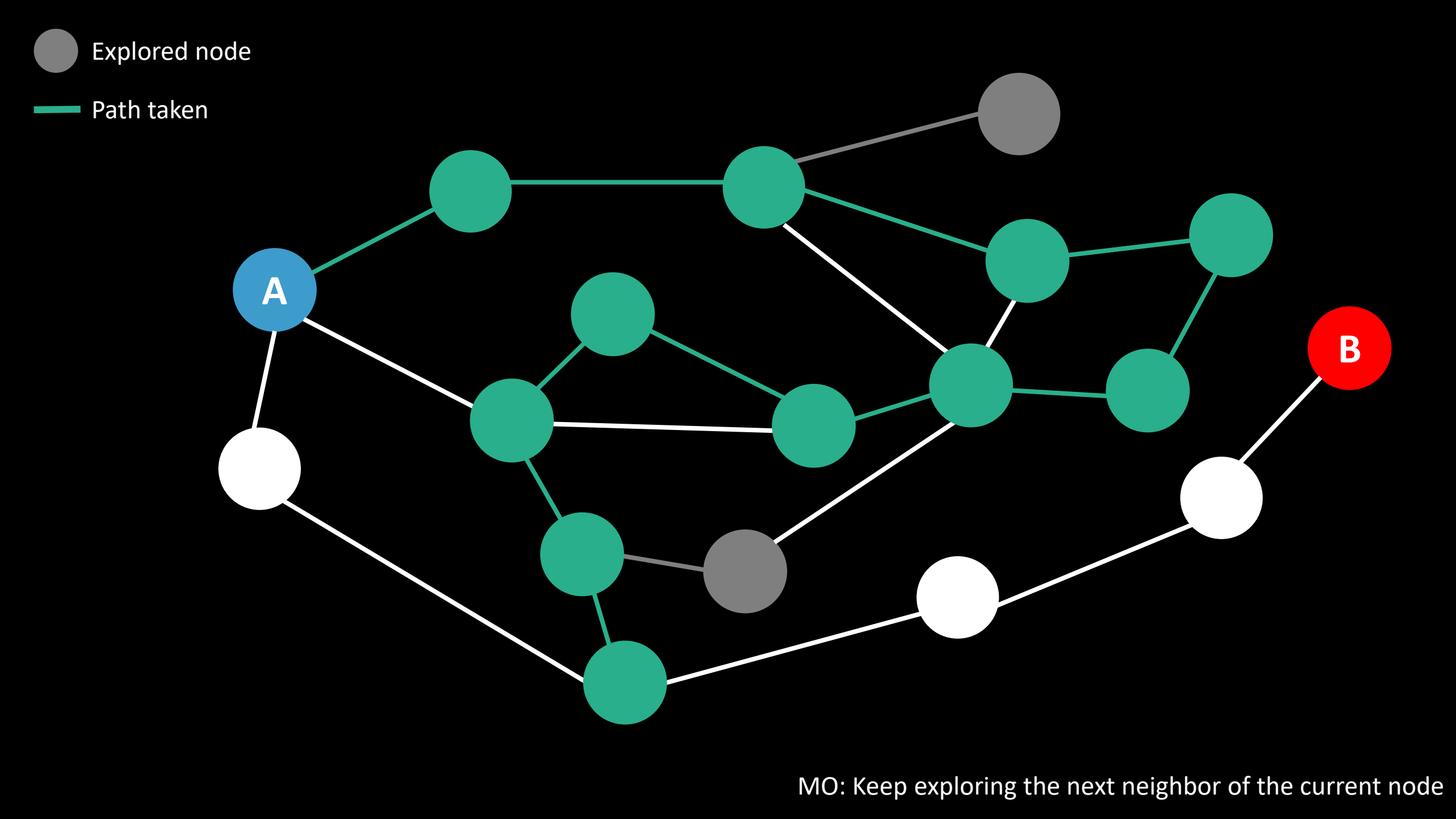

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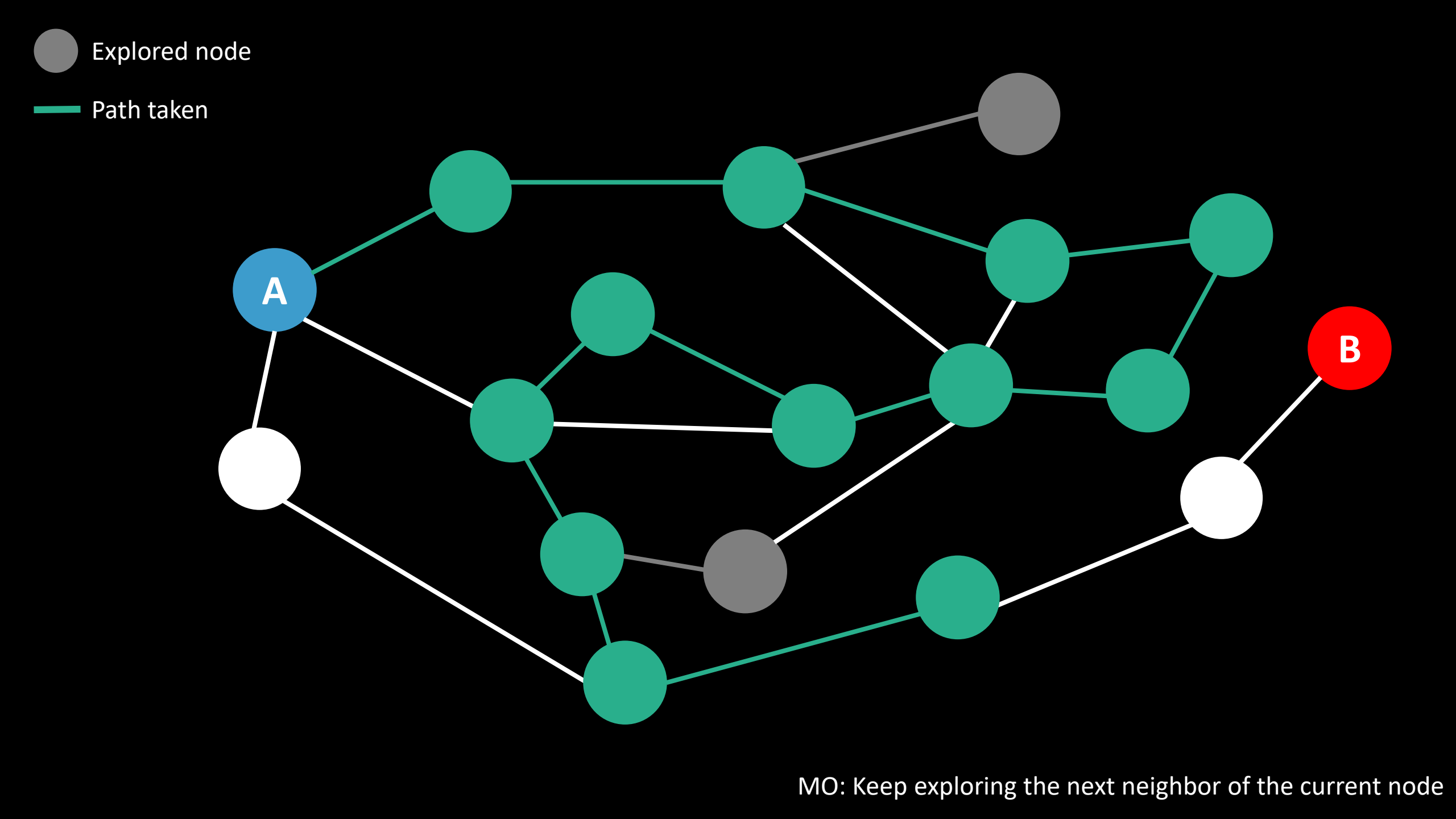

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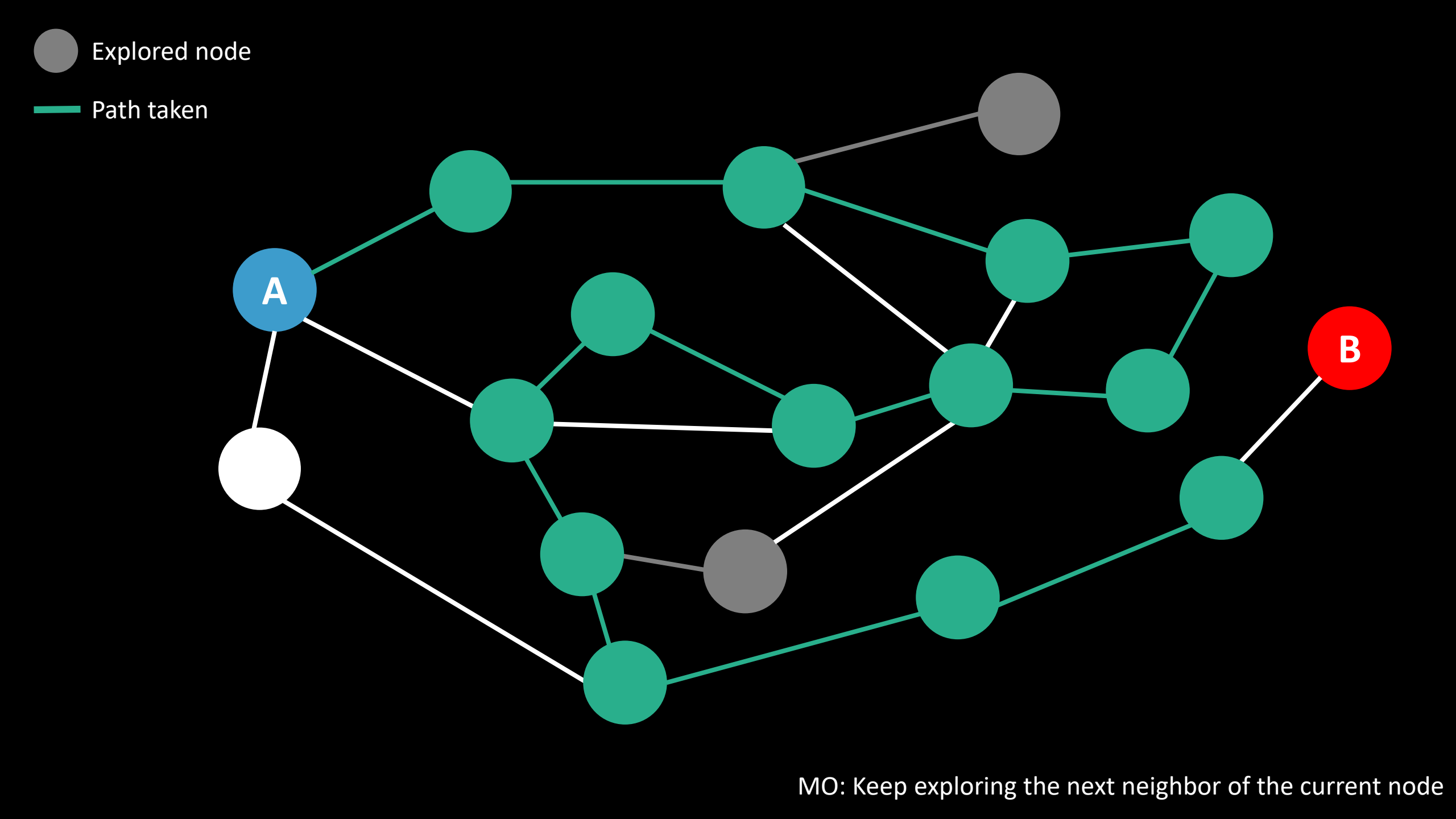

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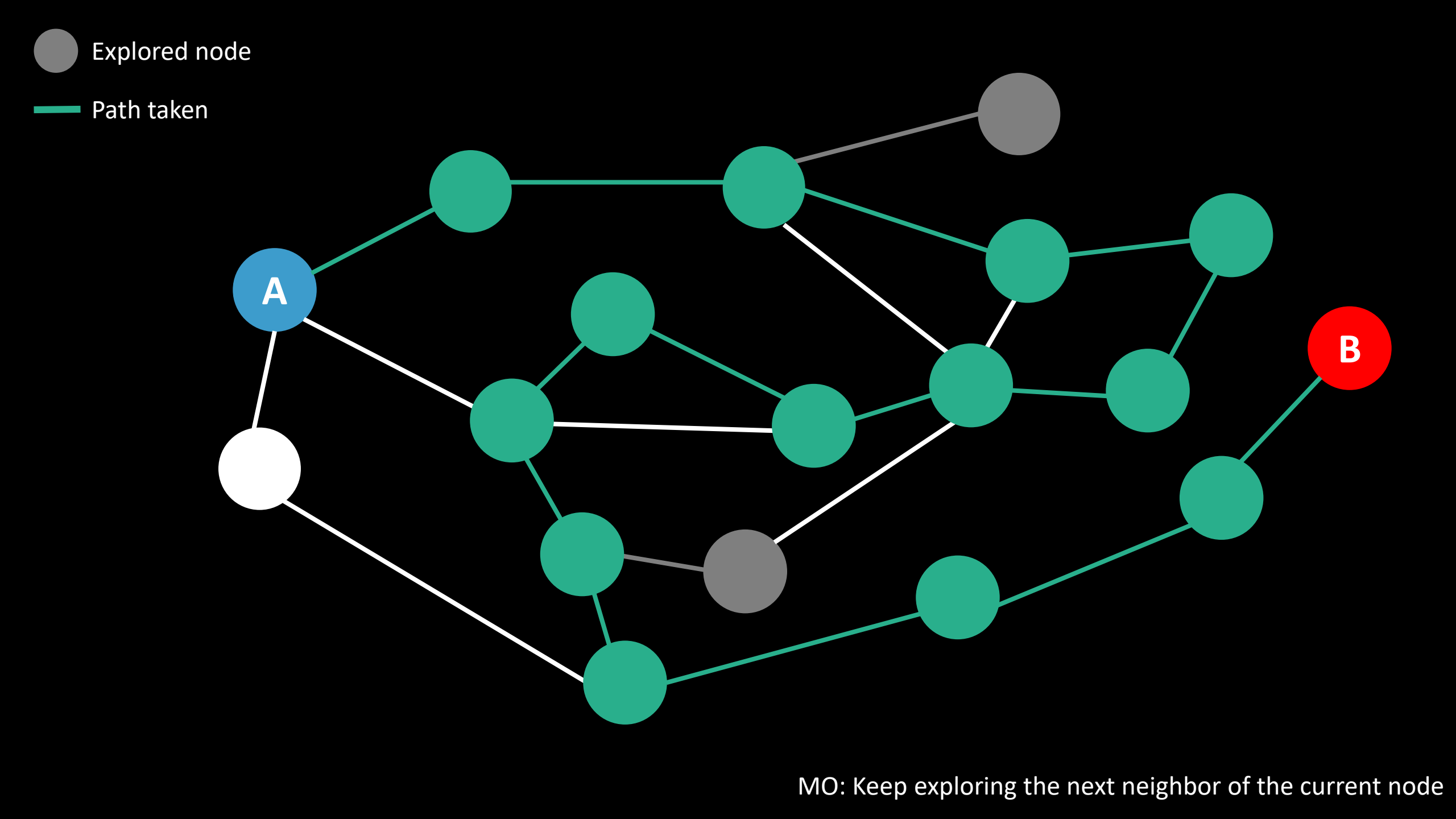

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Path-finding

- Does a path exist from A→B?
- Keep track of:
  - Path taken
  - Nodes explored: do not re-explore previously explored nodes

Path-finding

- Does a path exist from A→B?
- Keep track of:
  - Path taken
  - Nodes explored: do not re-explore previously explored nodes
- Identified that a path exists through Depth-First Search (DFS)

Depth-First Search

Algorithm

1. **Expand** starting node
2. Add all neighboring nodes to the **search frontier** (push onto stack)
3. **Pop** a node off the stack
4. **Expand** the node
5. Repeat 2-4 until : the ending node is found OR there are no nodes left on the stack

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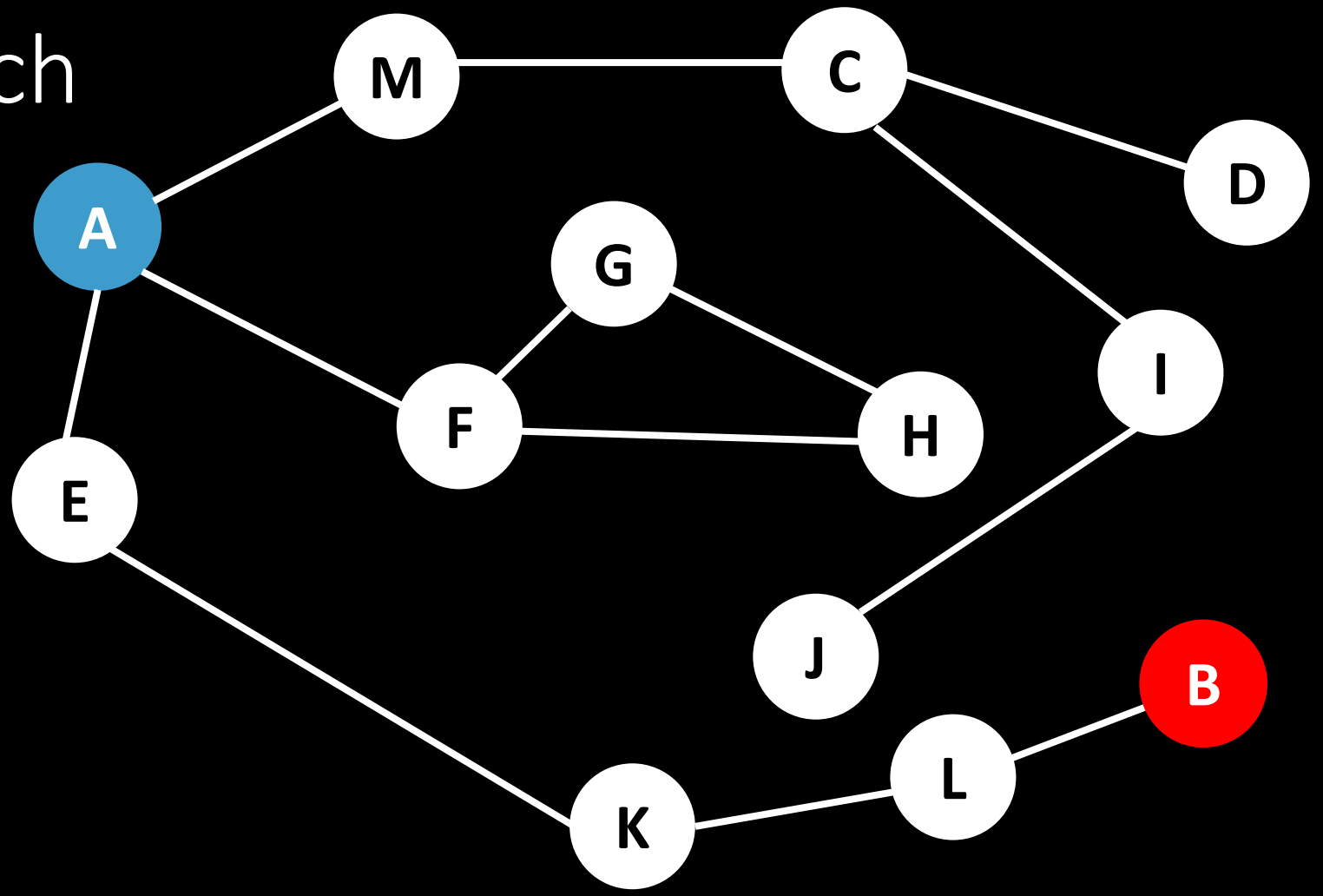

Search Frontier

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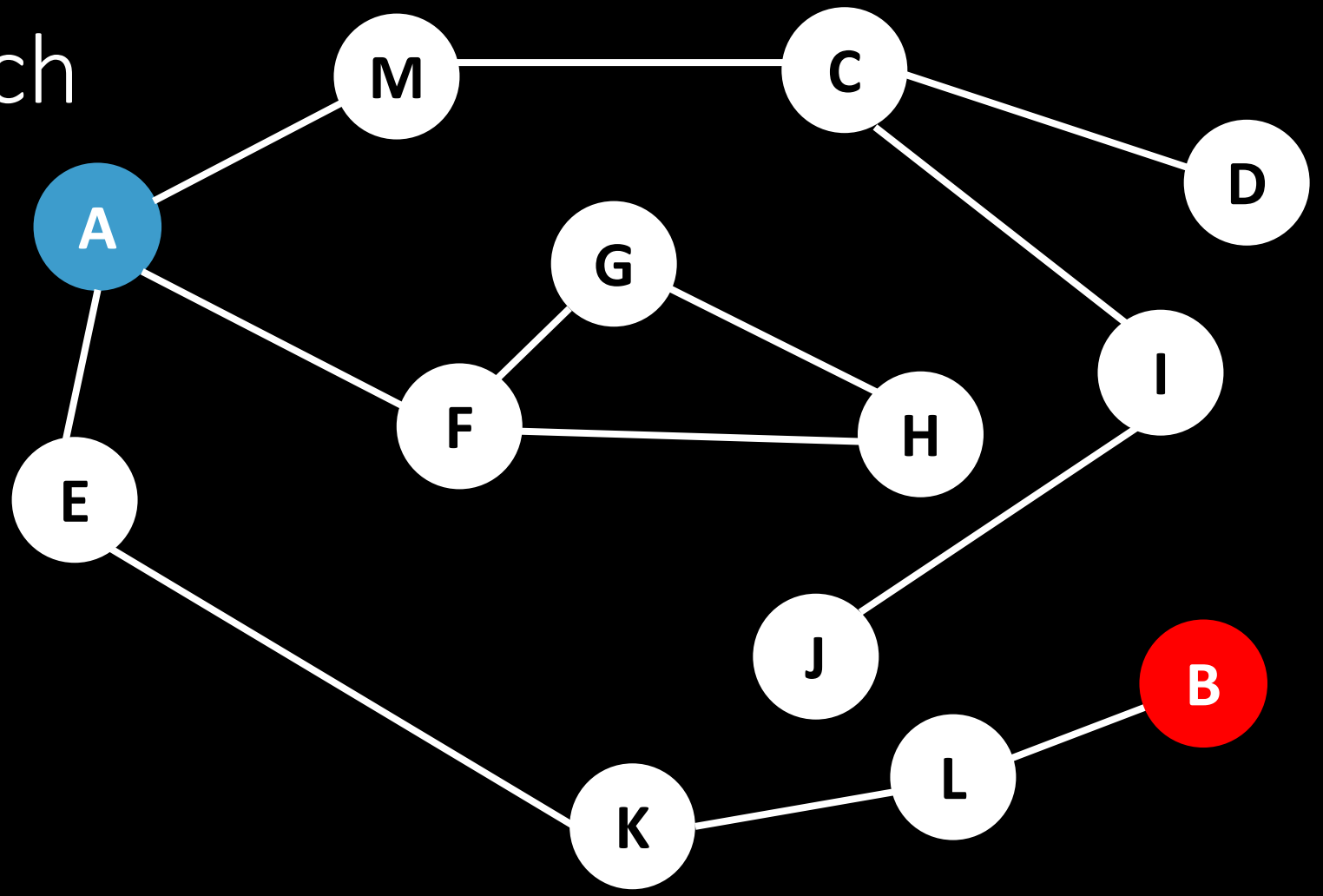

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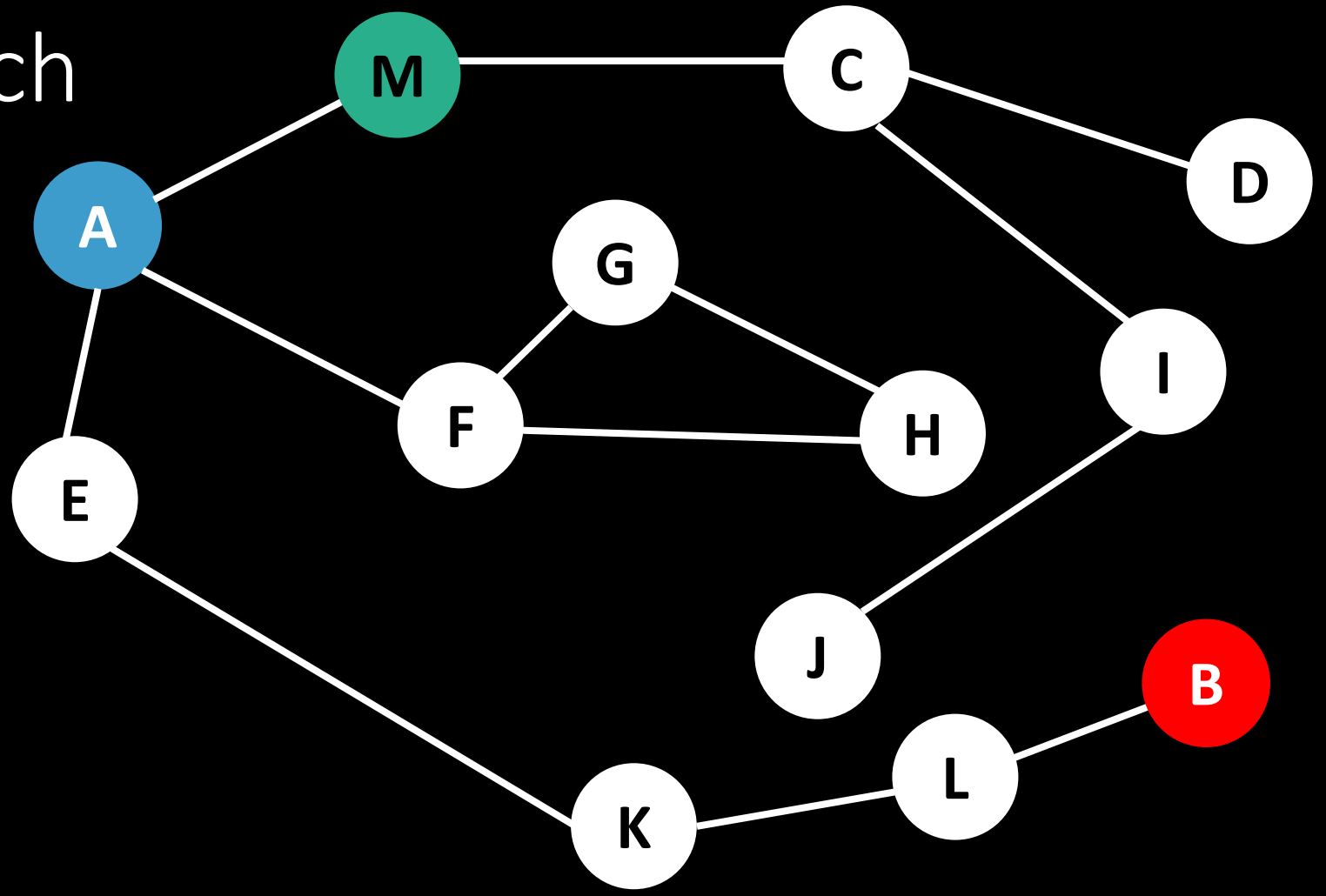

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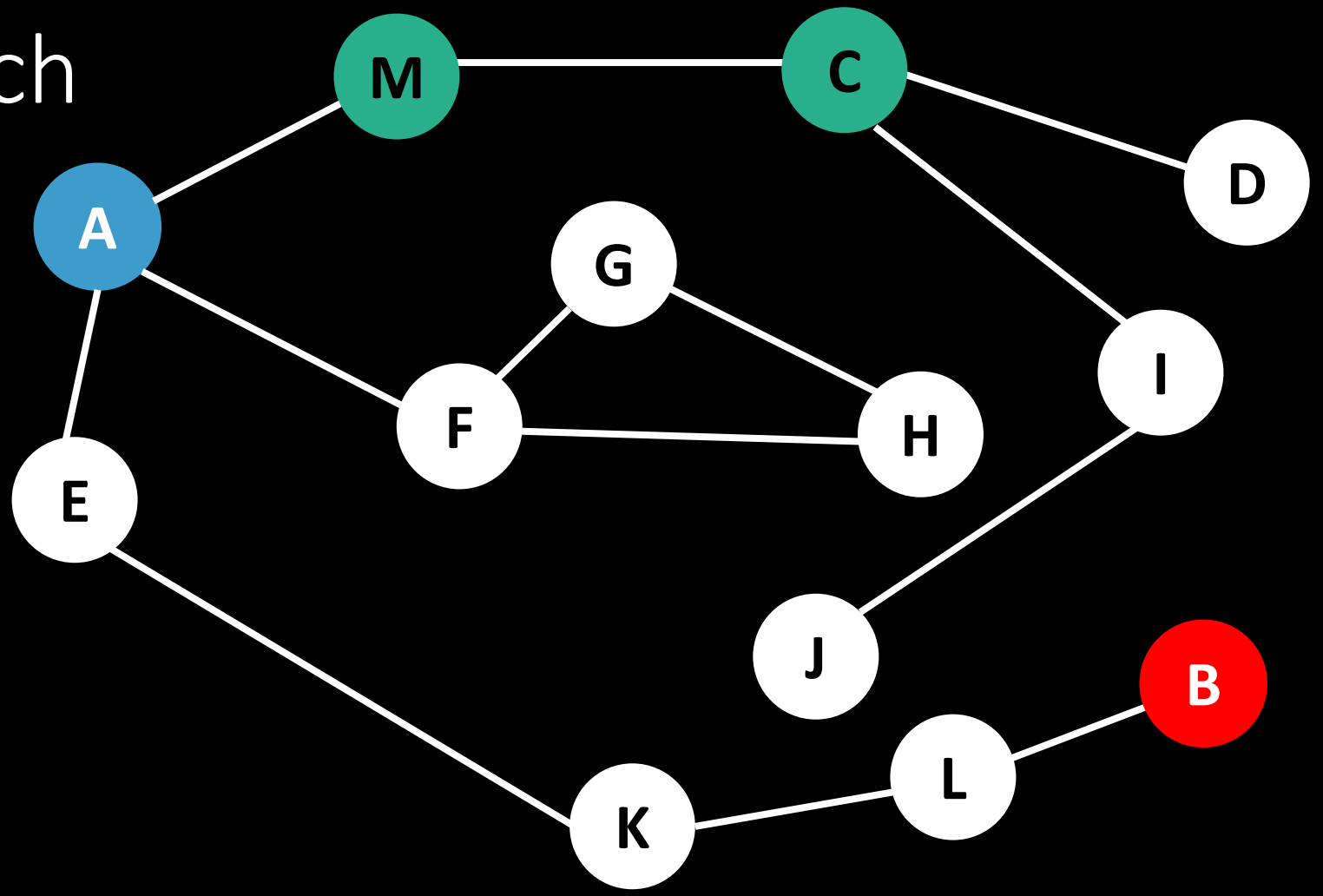

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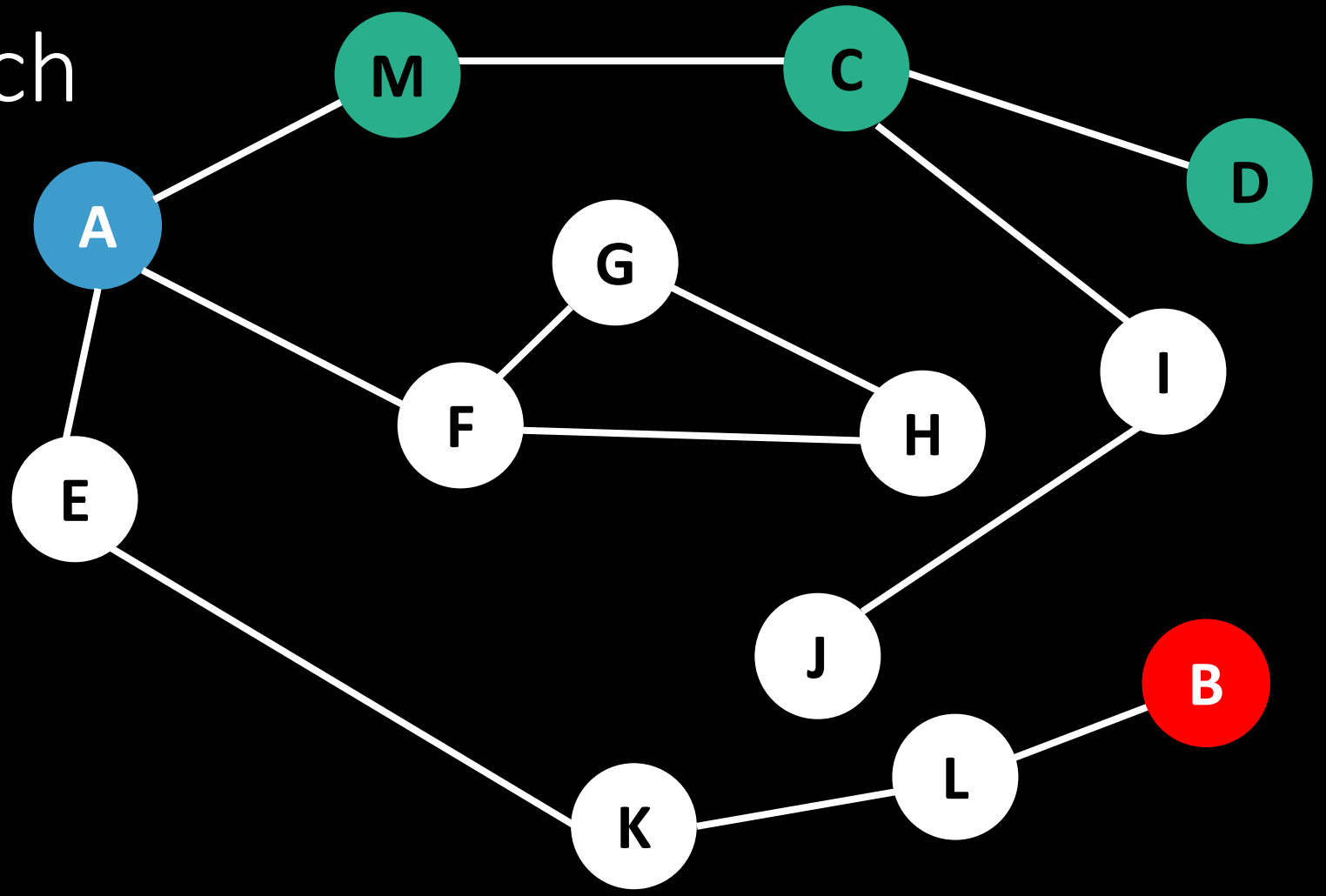

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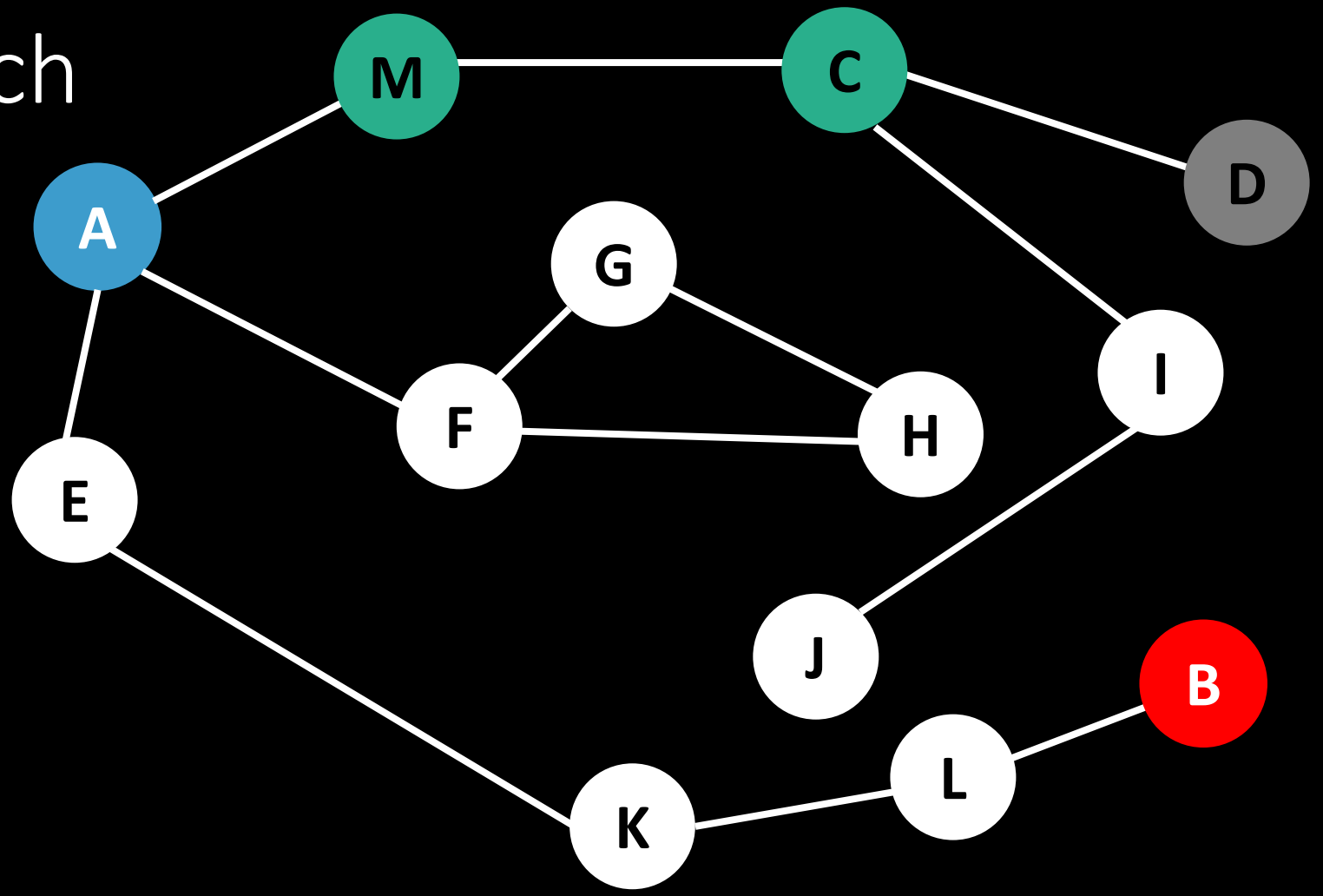

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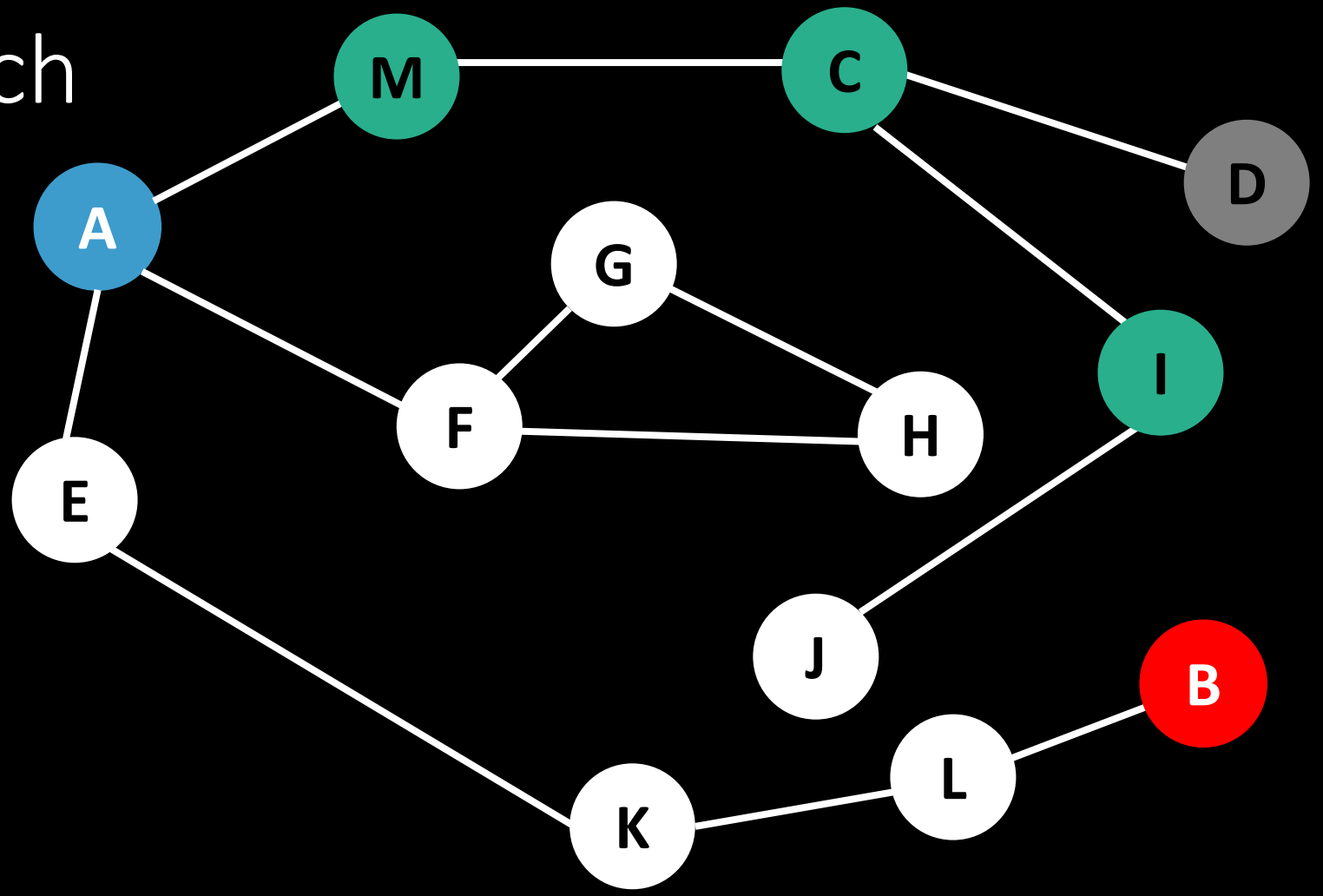

Search Frontier

A	M	C	D	I	J	F	E

Depth-First Search

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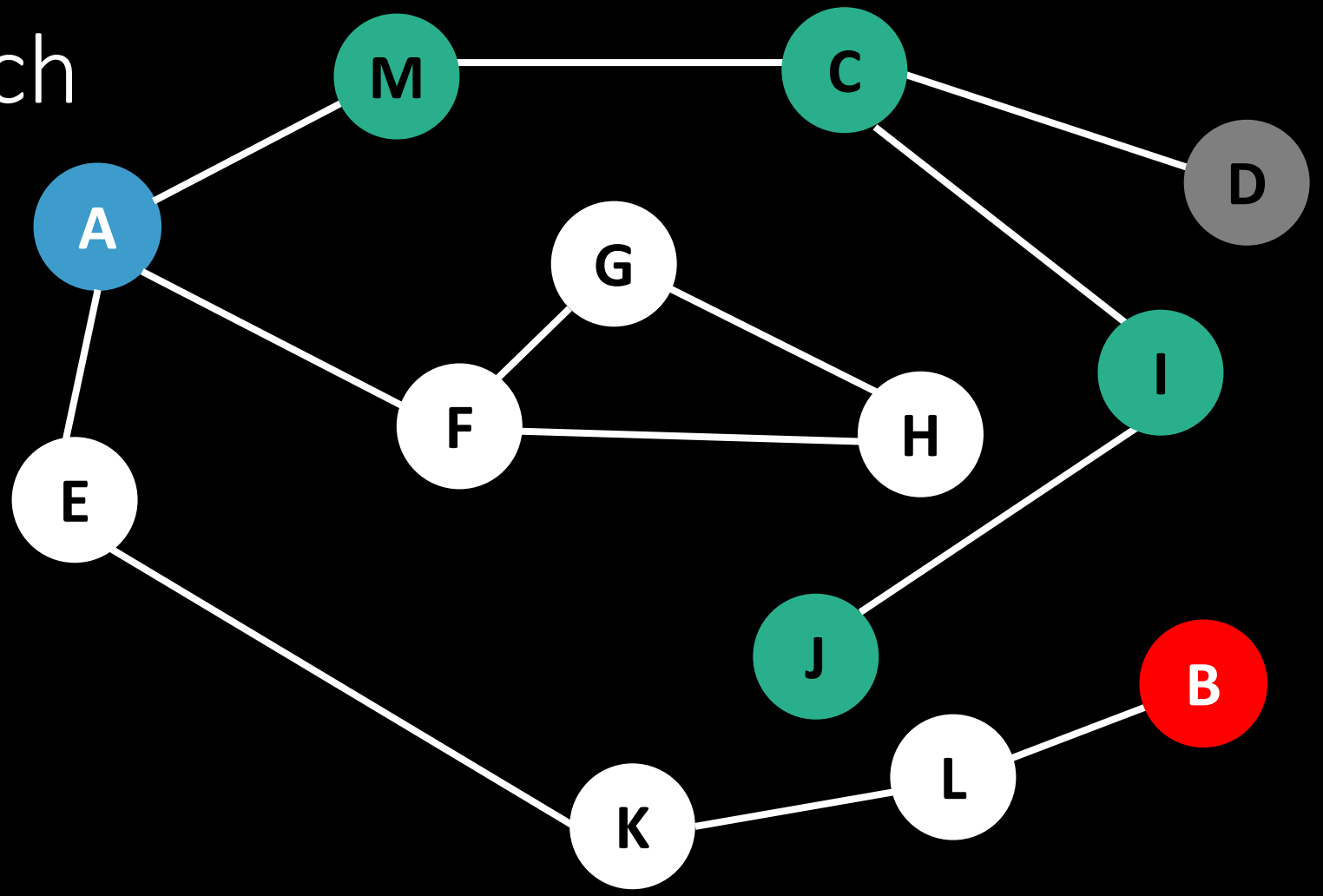

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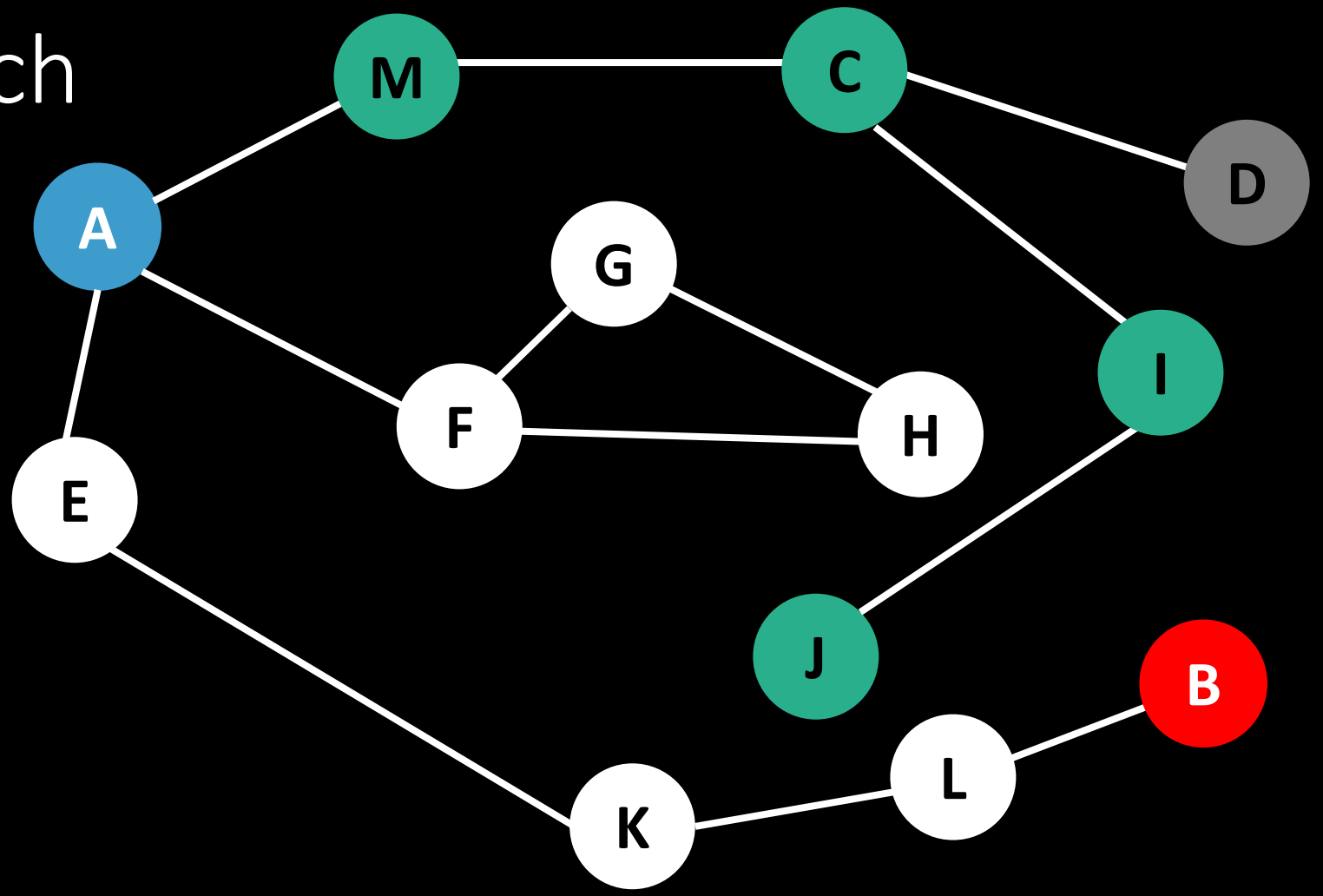

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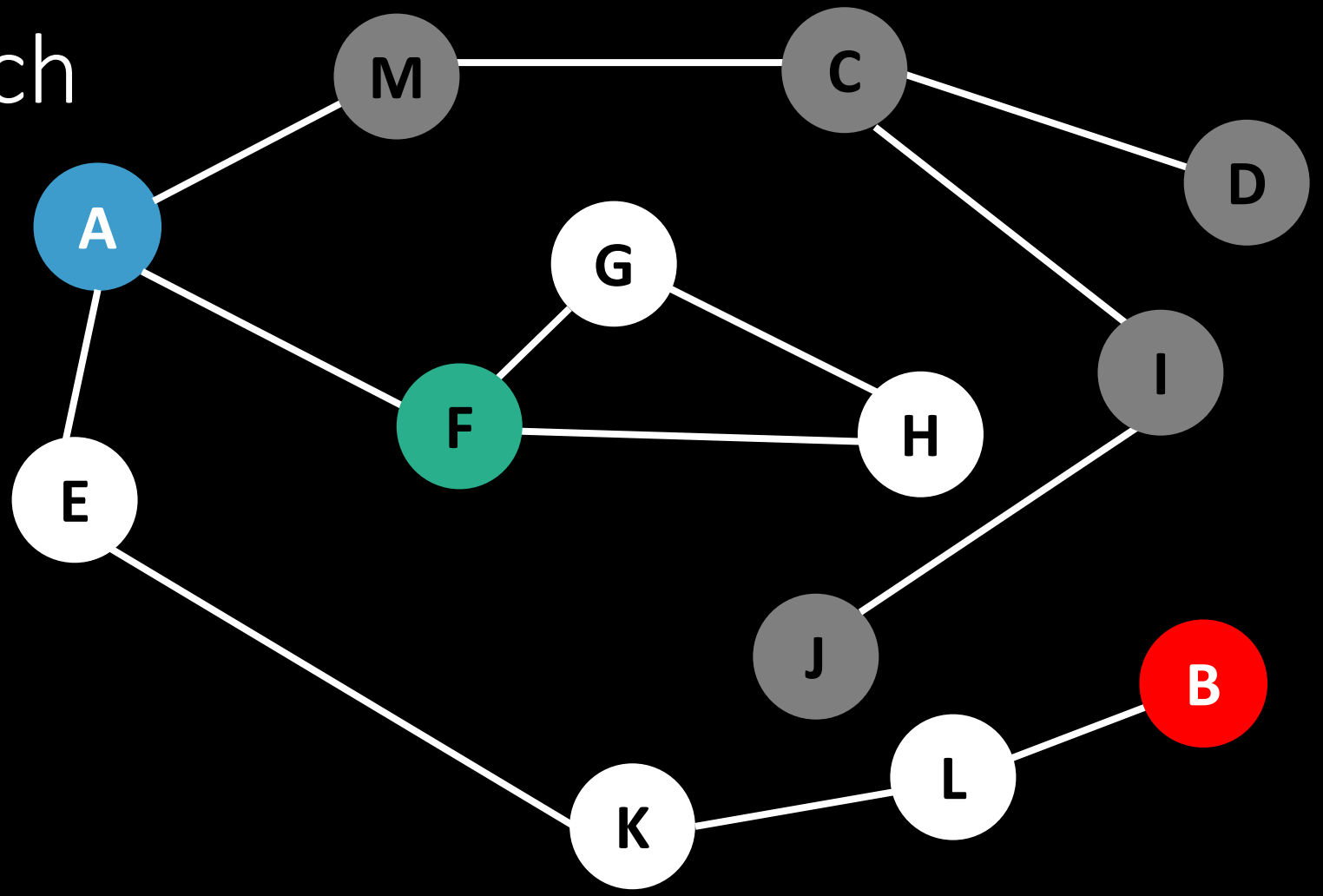

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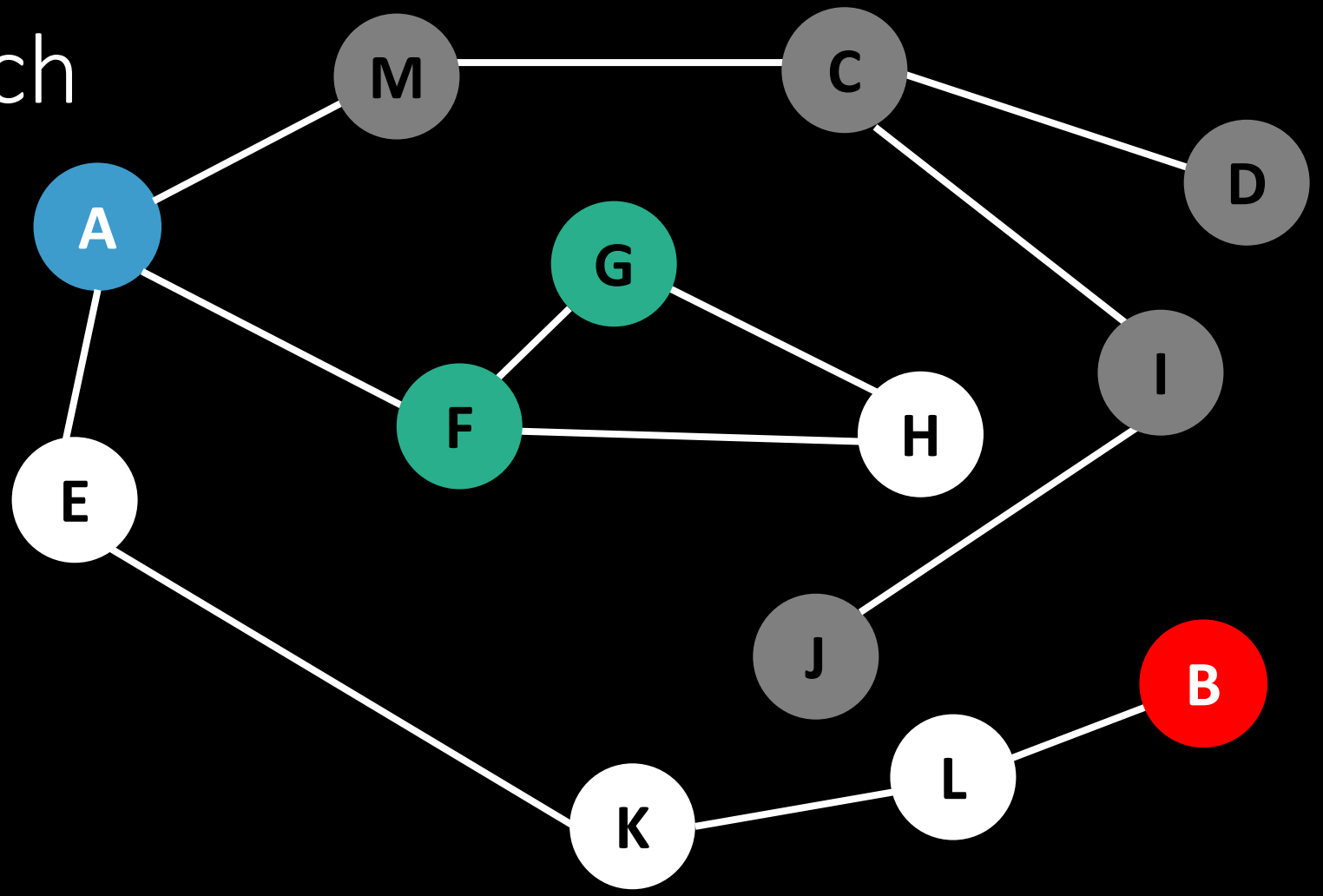

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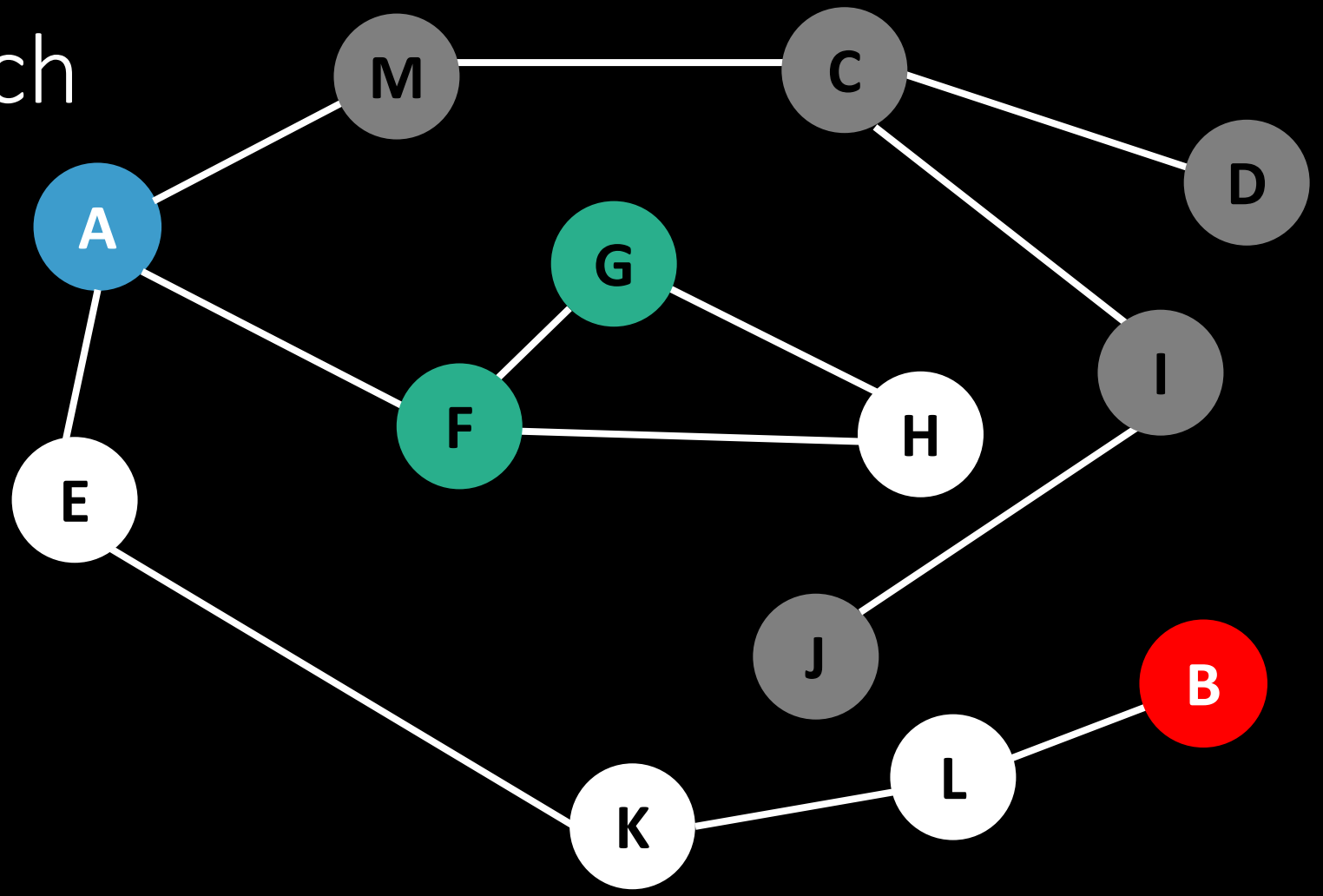

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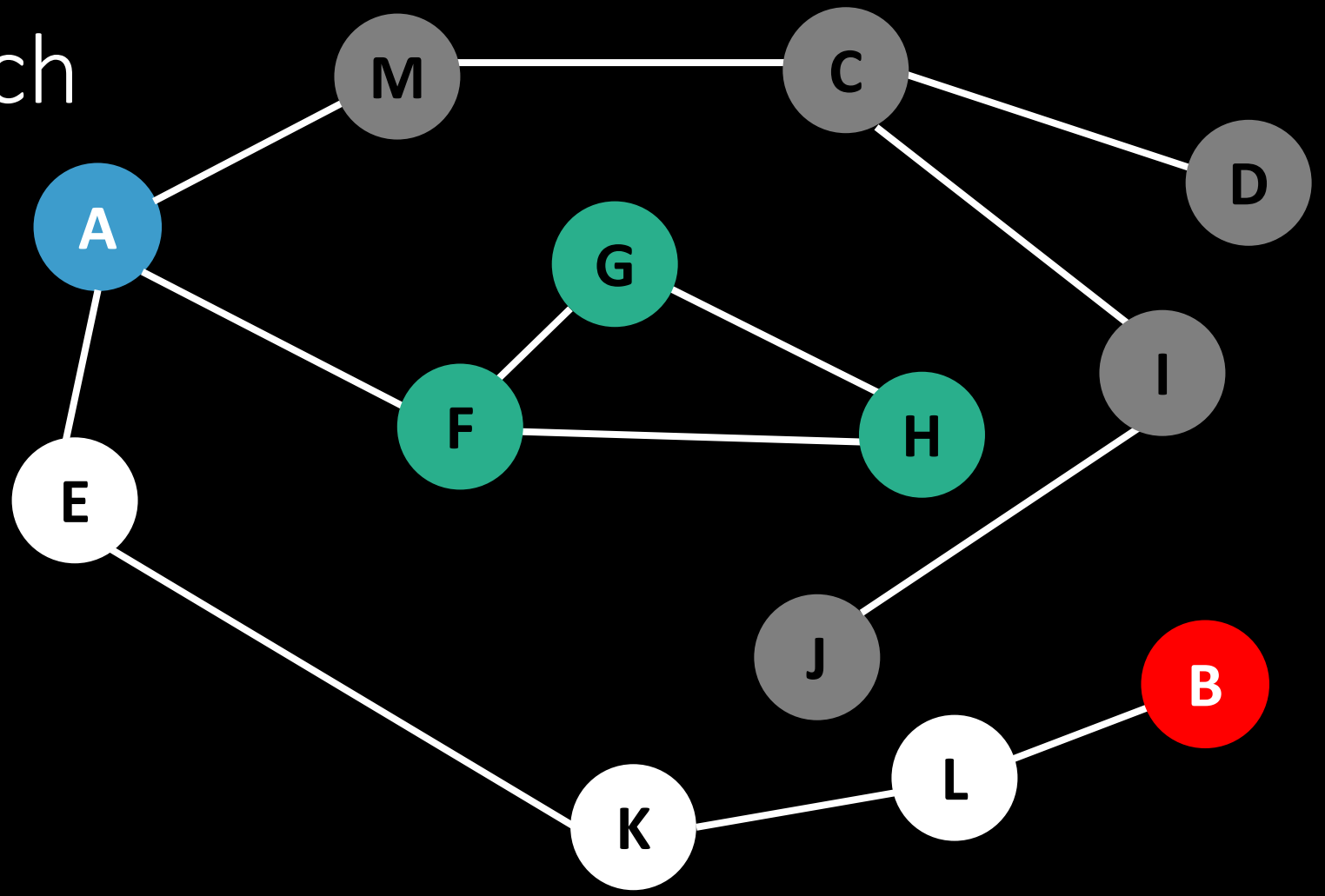

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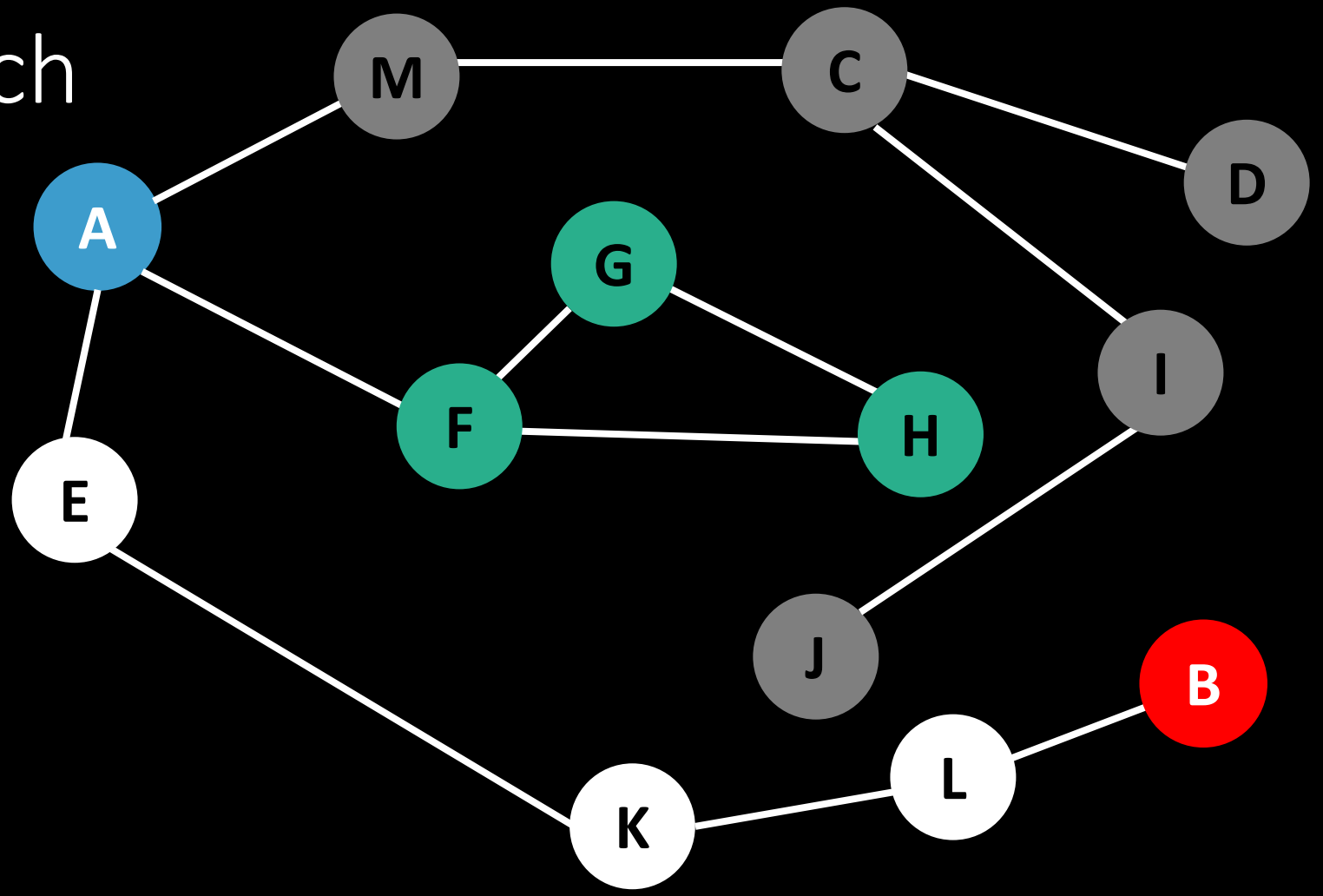

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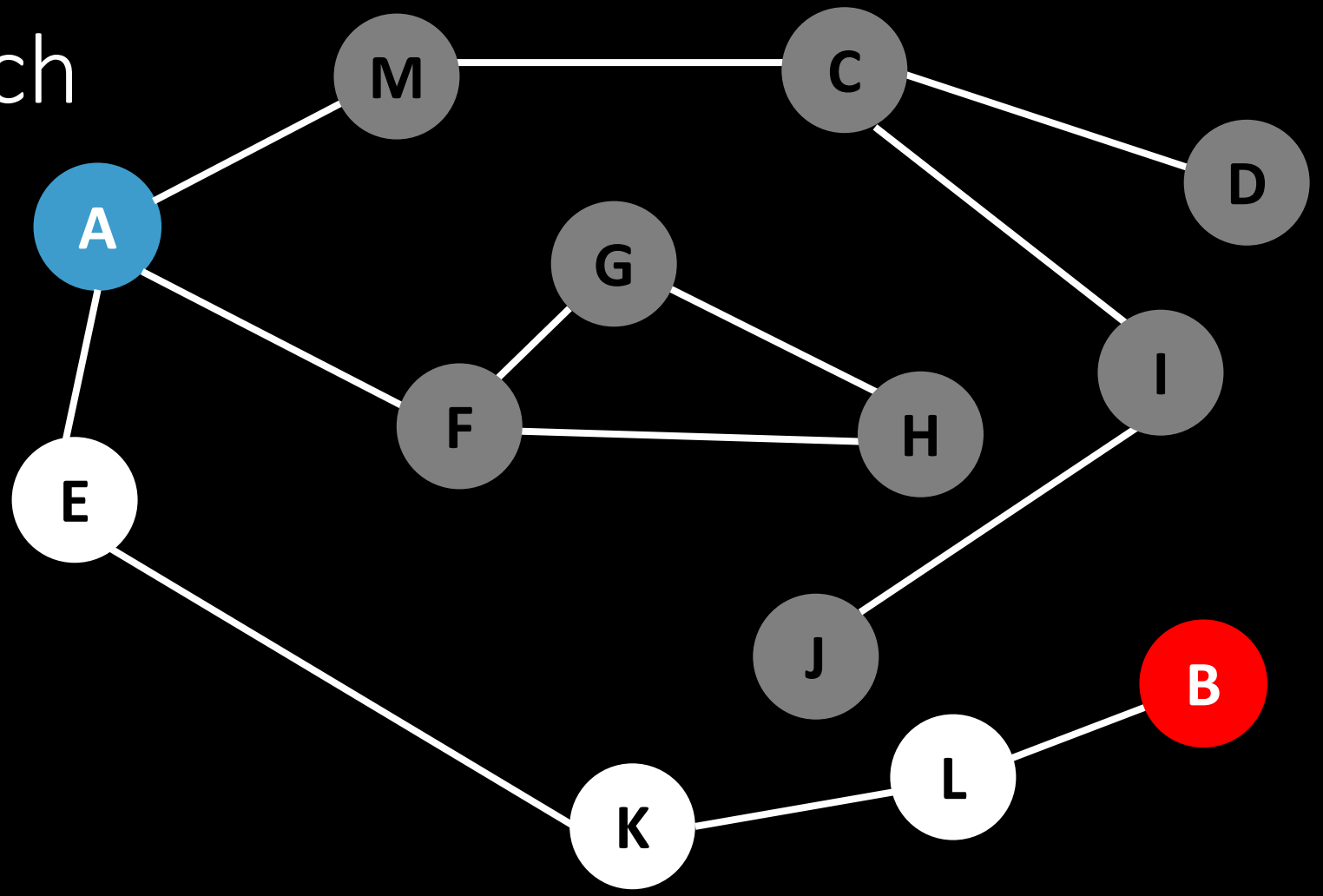

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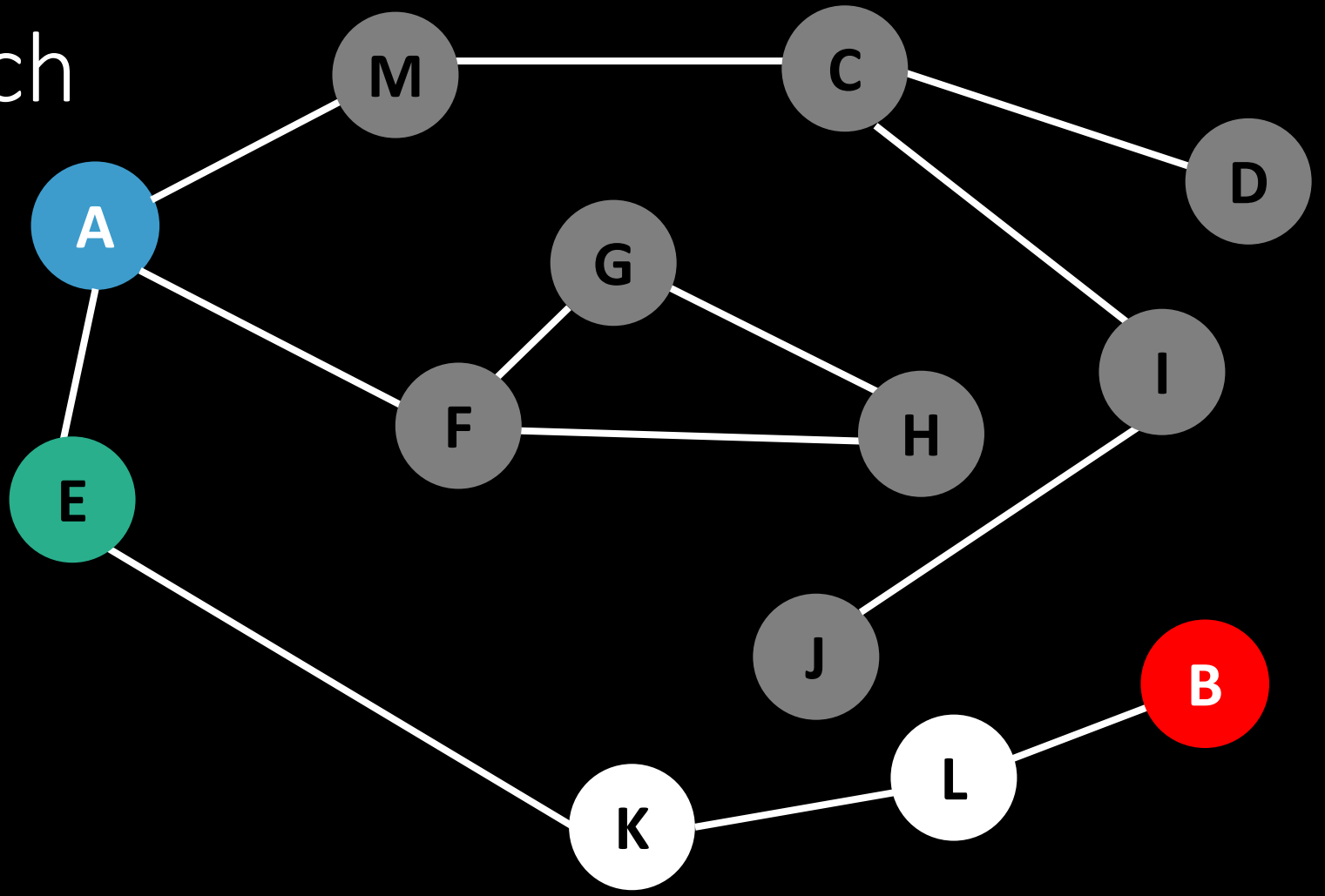

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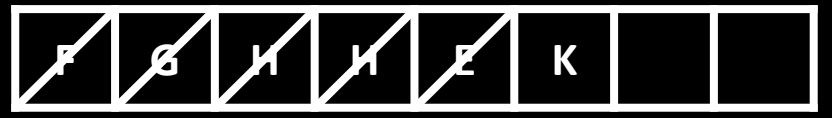

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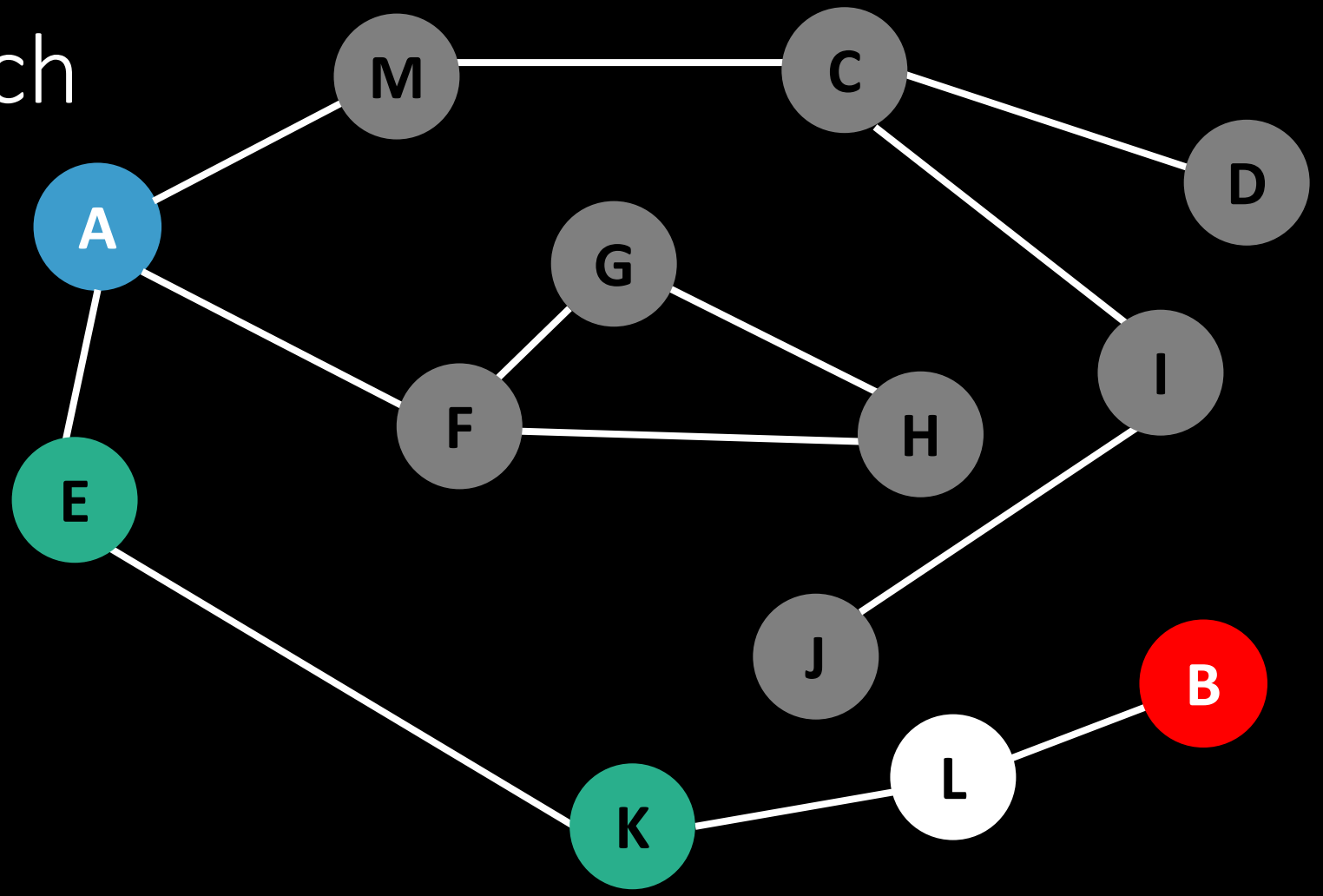

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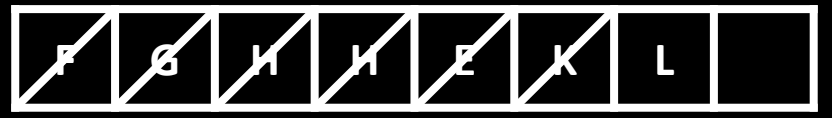

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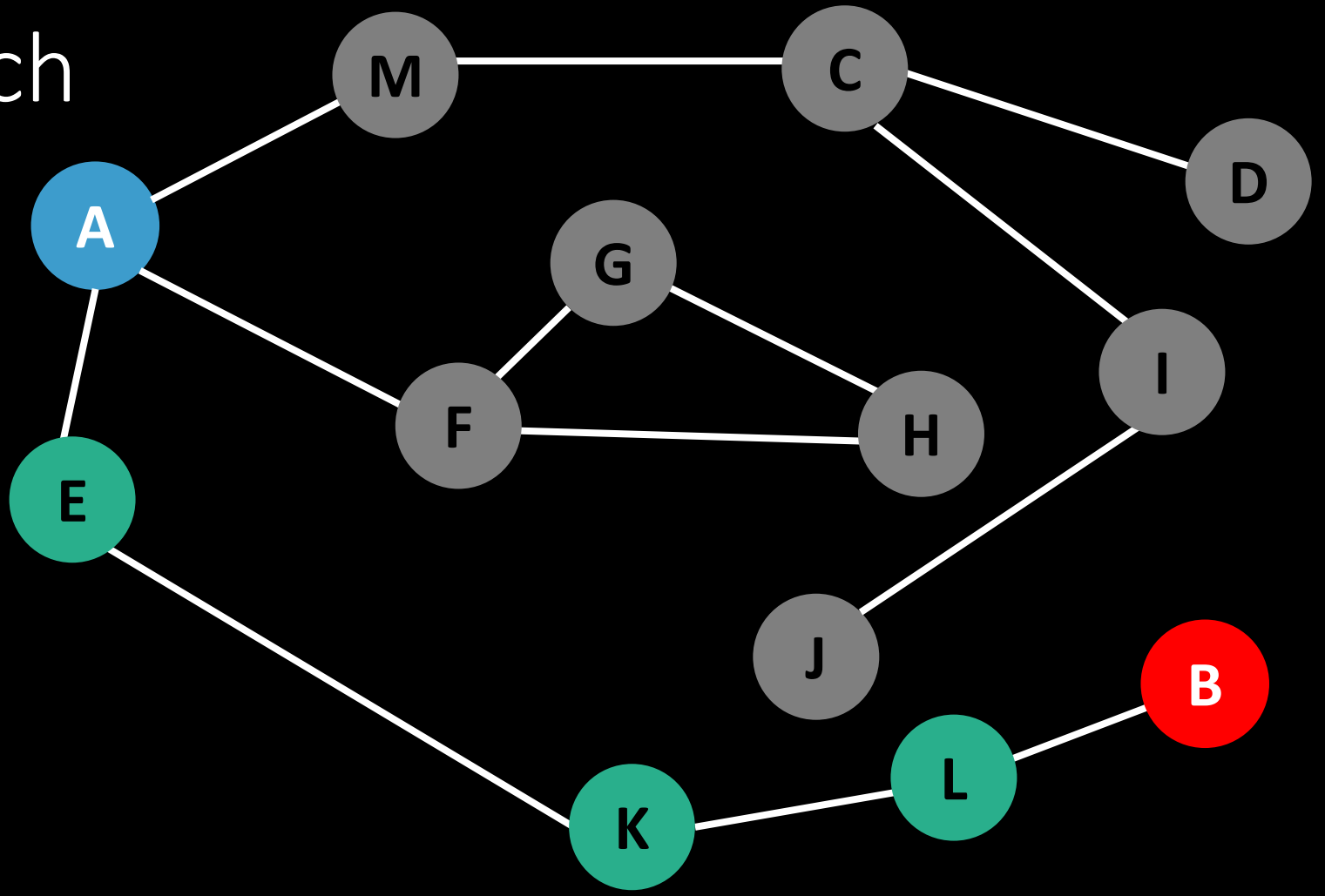

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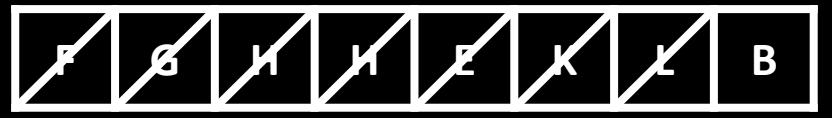

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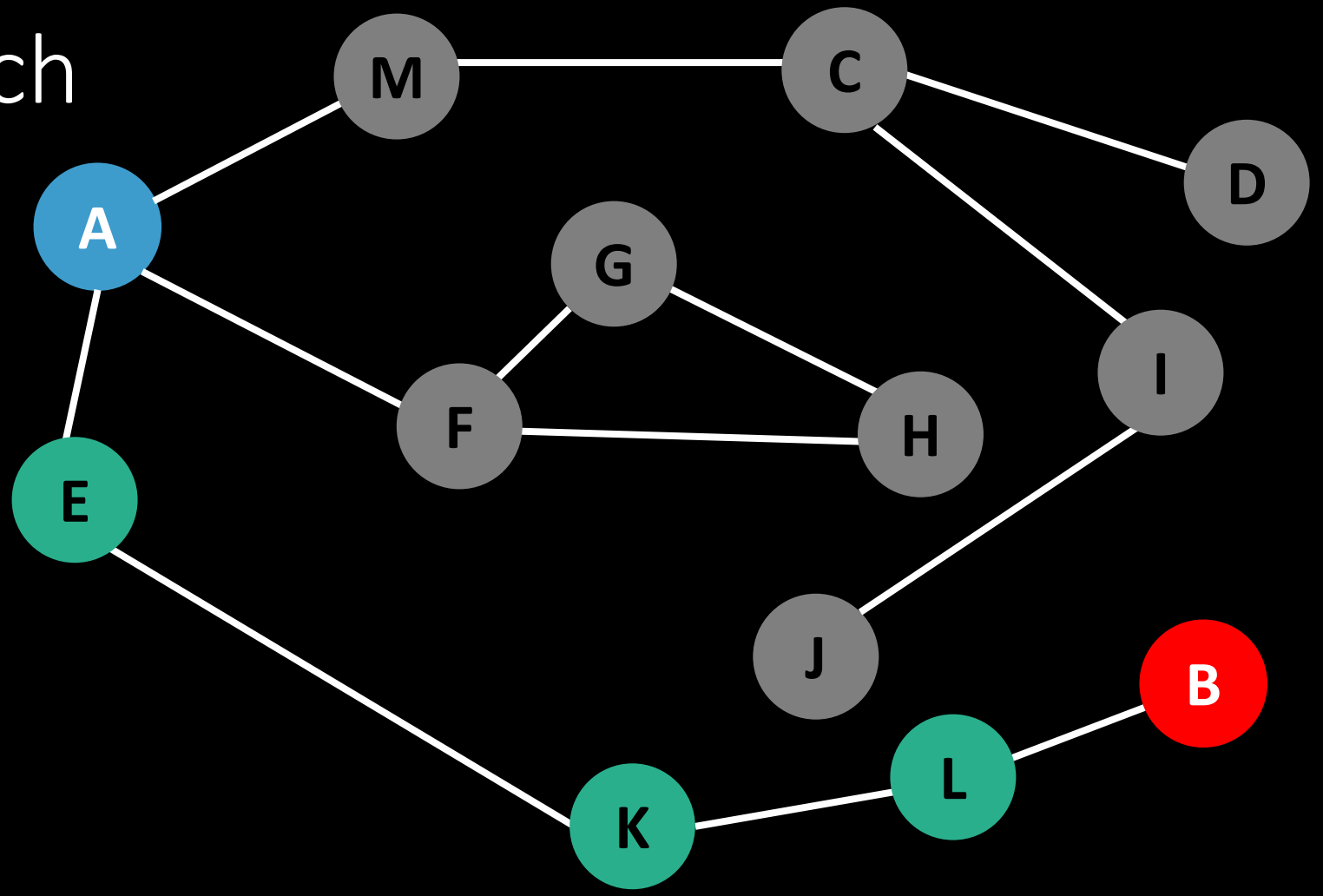

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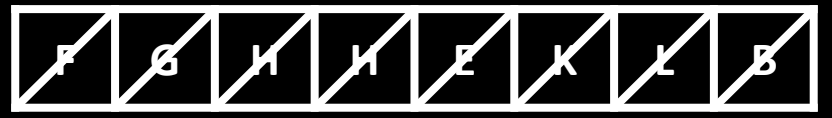


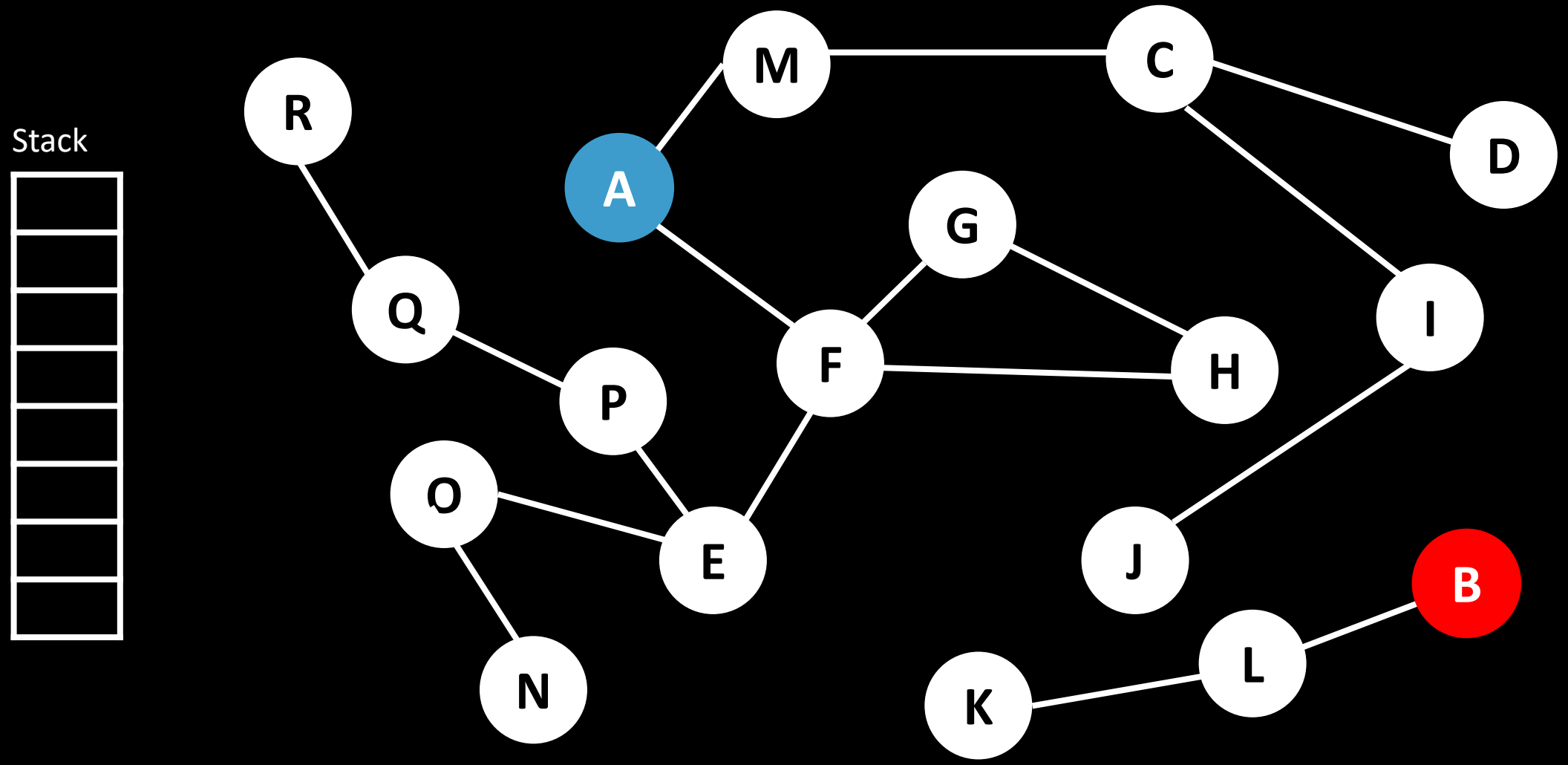
Depth-First Search

- Expands depth-first
- Does **not** give us a shortest path between two nodes
- Finds a path if one exists

Depth-First Search

- Expands depth-first
- Does **not** give us a shortest path between two nodes
- Finds a path if one exists
- Can be used for maze solving
  - We just want a path, any path, who cares about the length?
  - https://www.youtube.com/watch?v=Kv8SRv4HH1g
- There are other cool applications of DFS which we may get to later in the course

Another DFS Example



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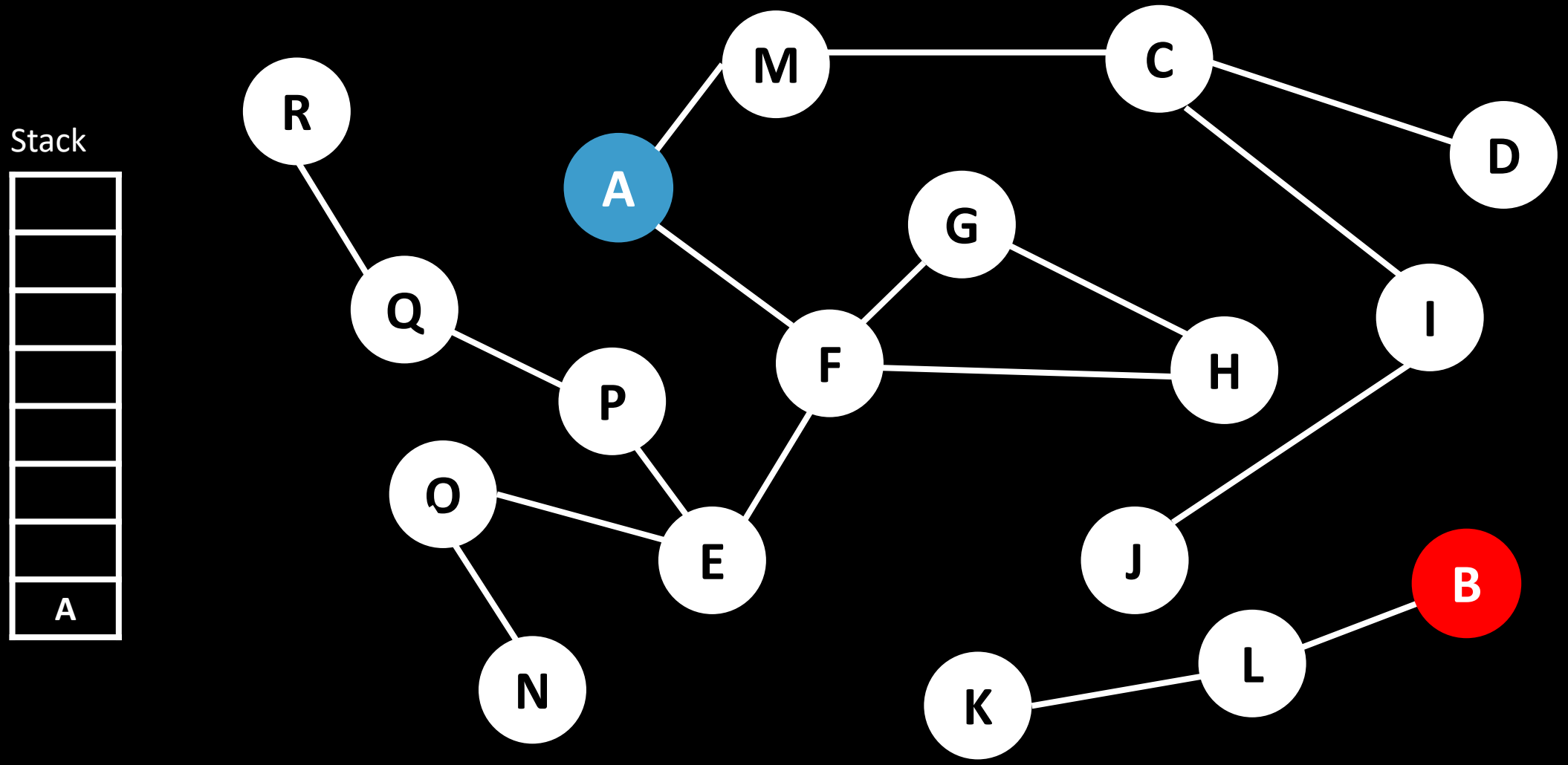

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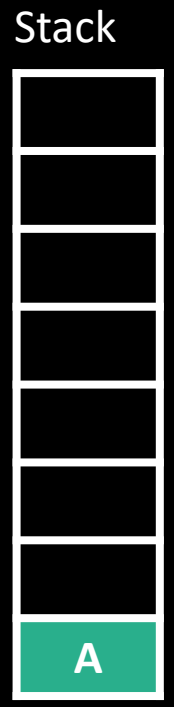

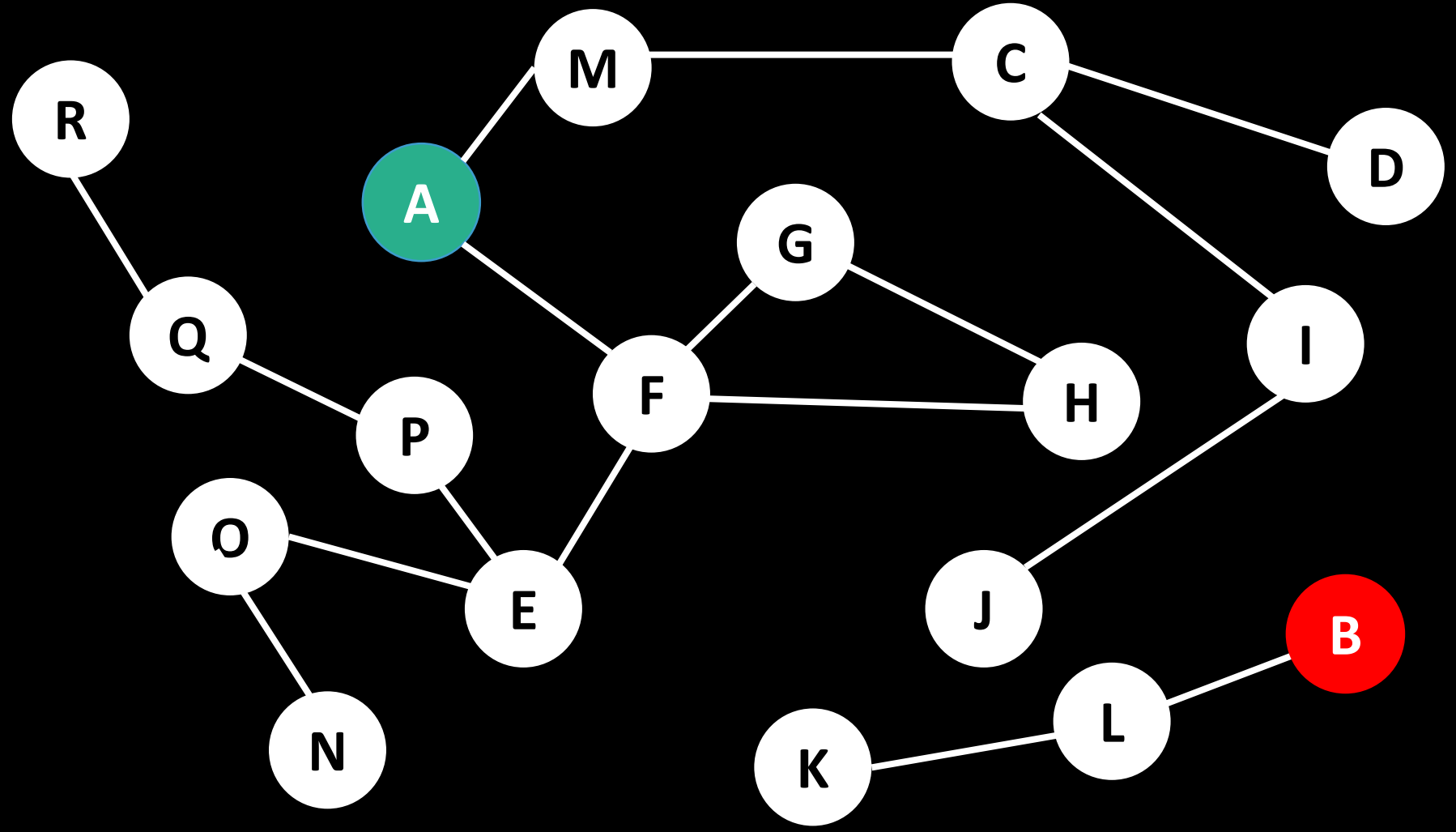

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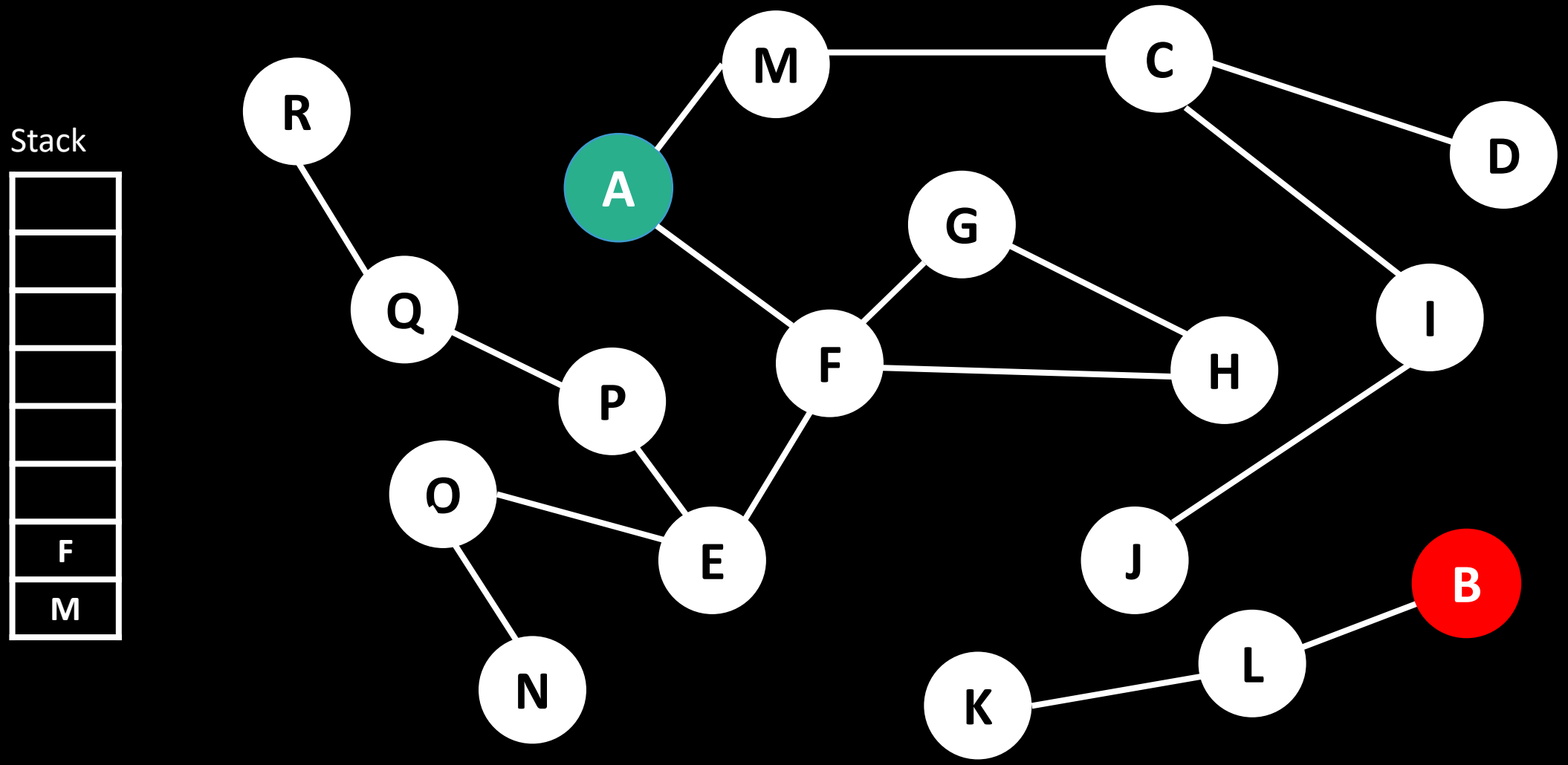

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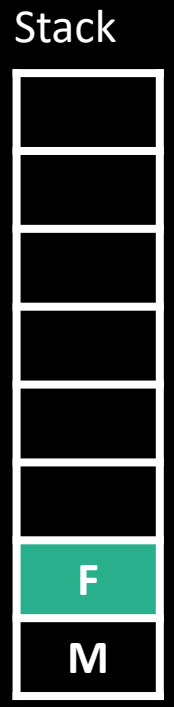

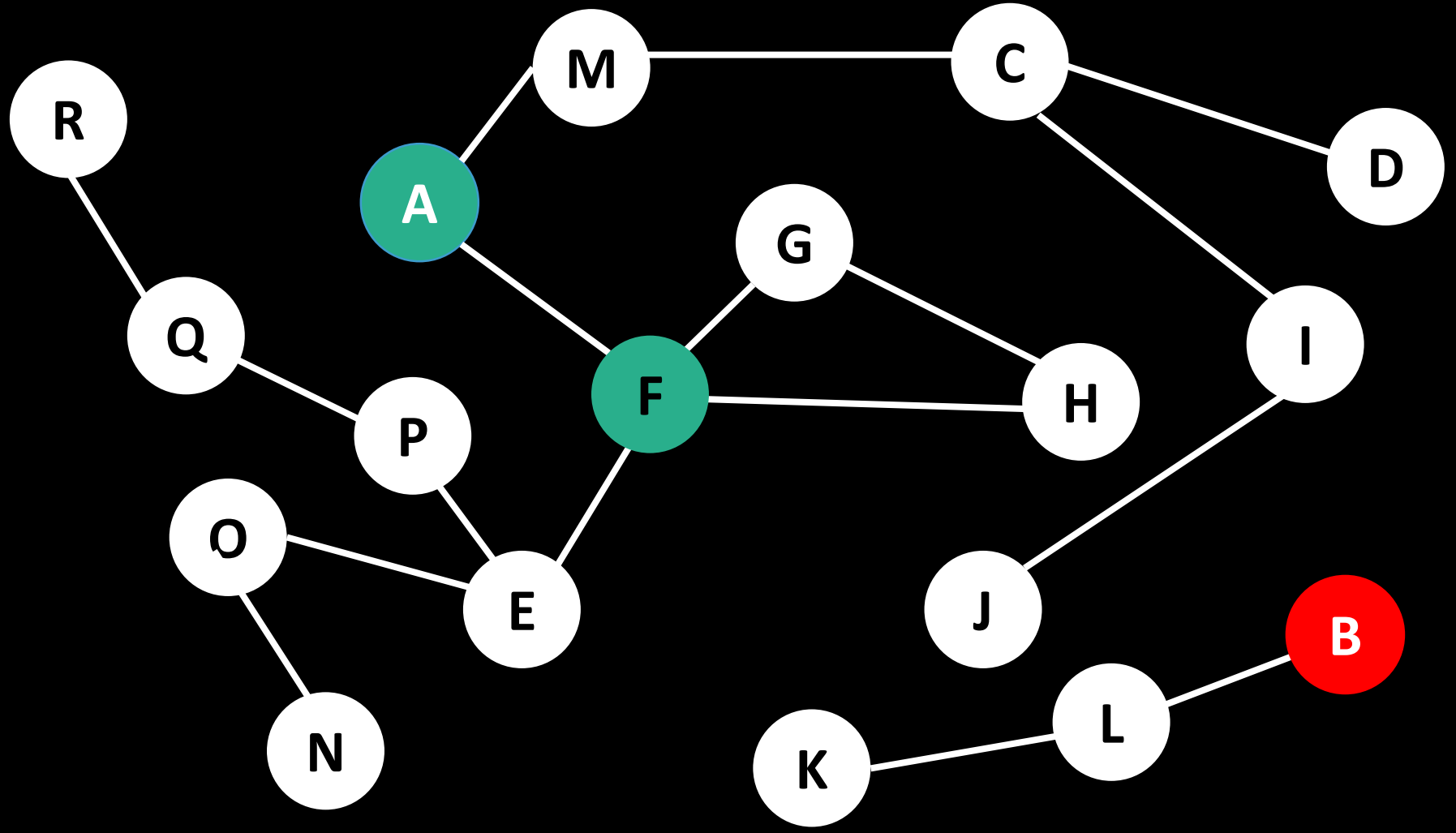

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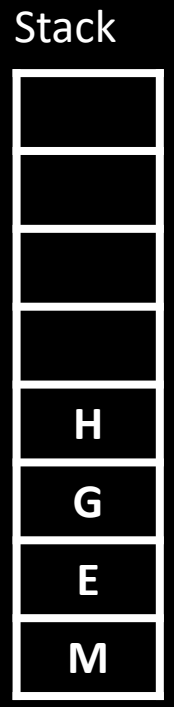

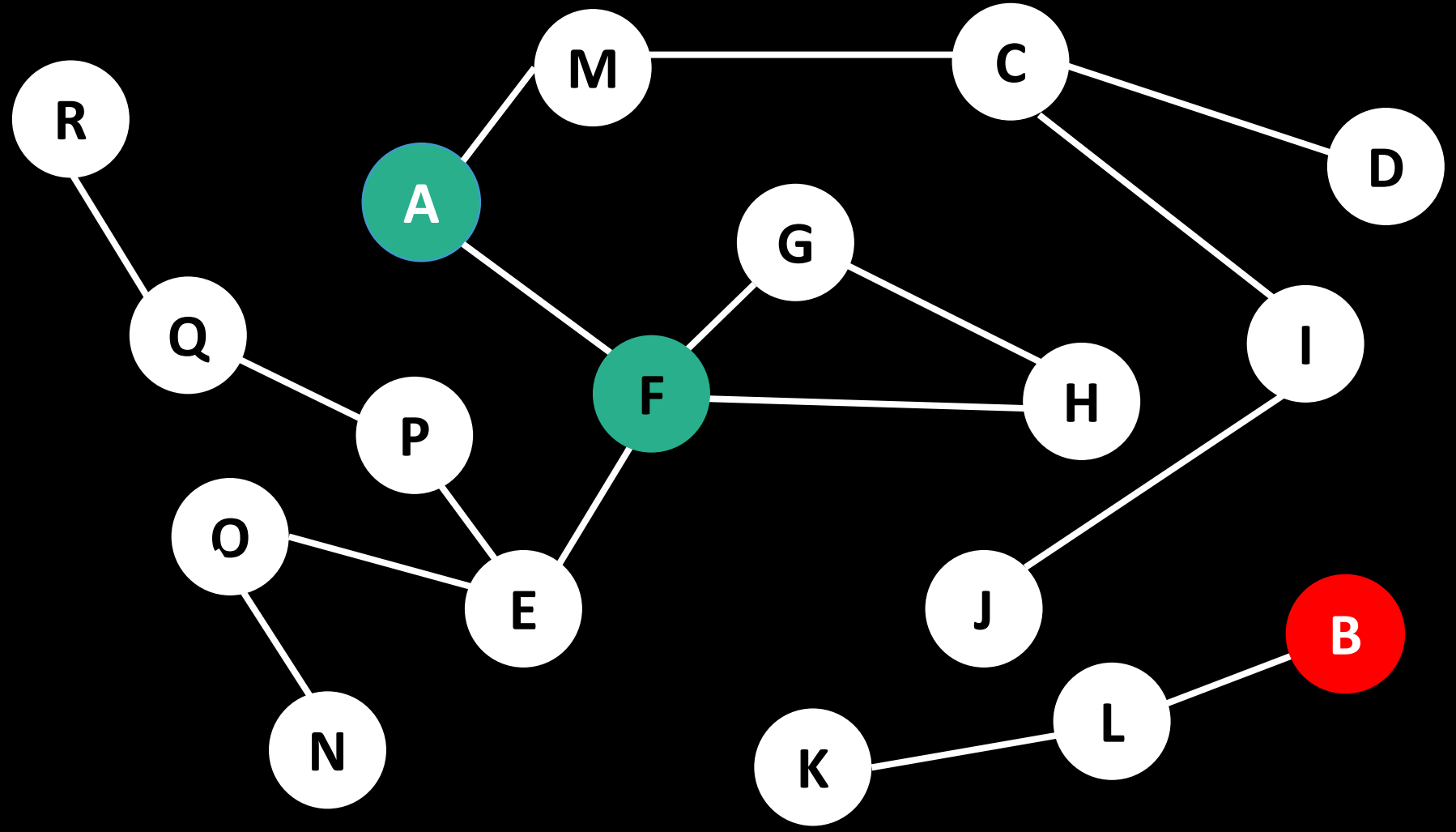

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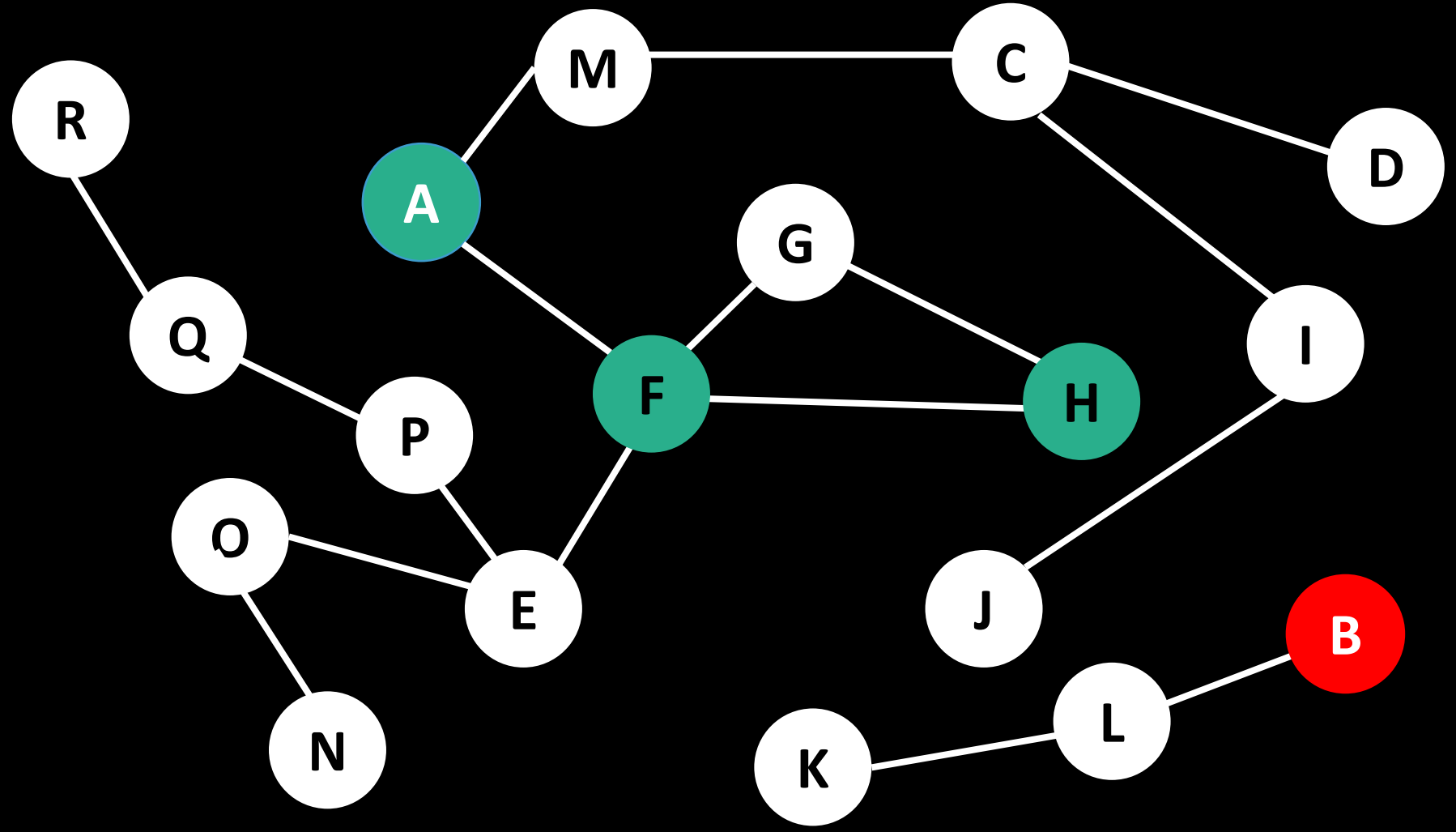

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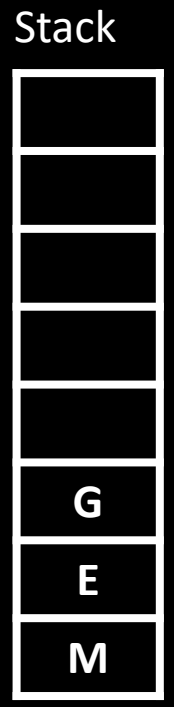

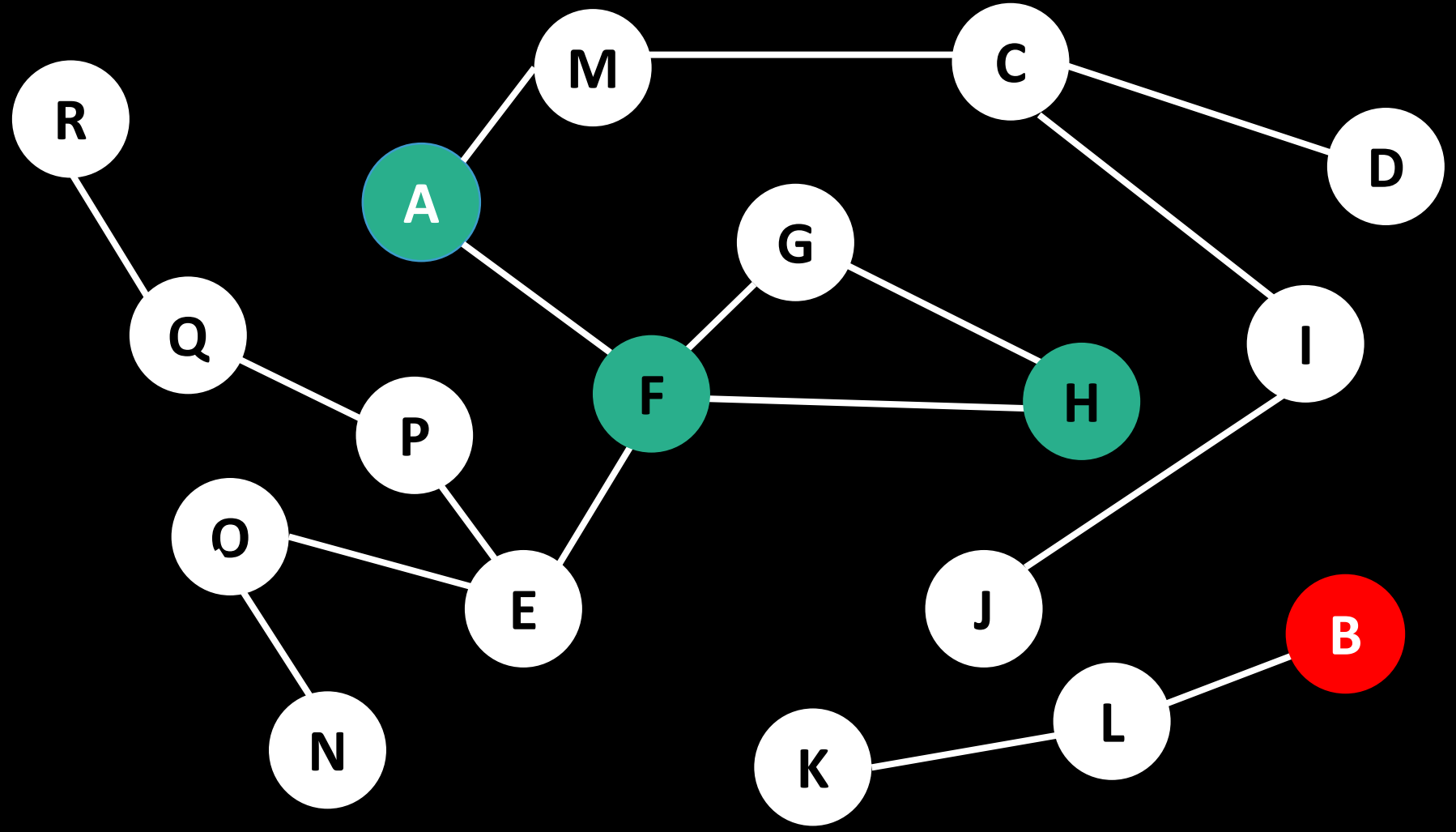

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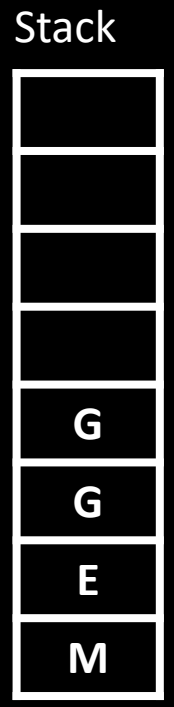

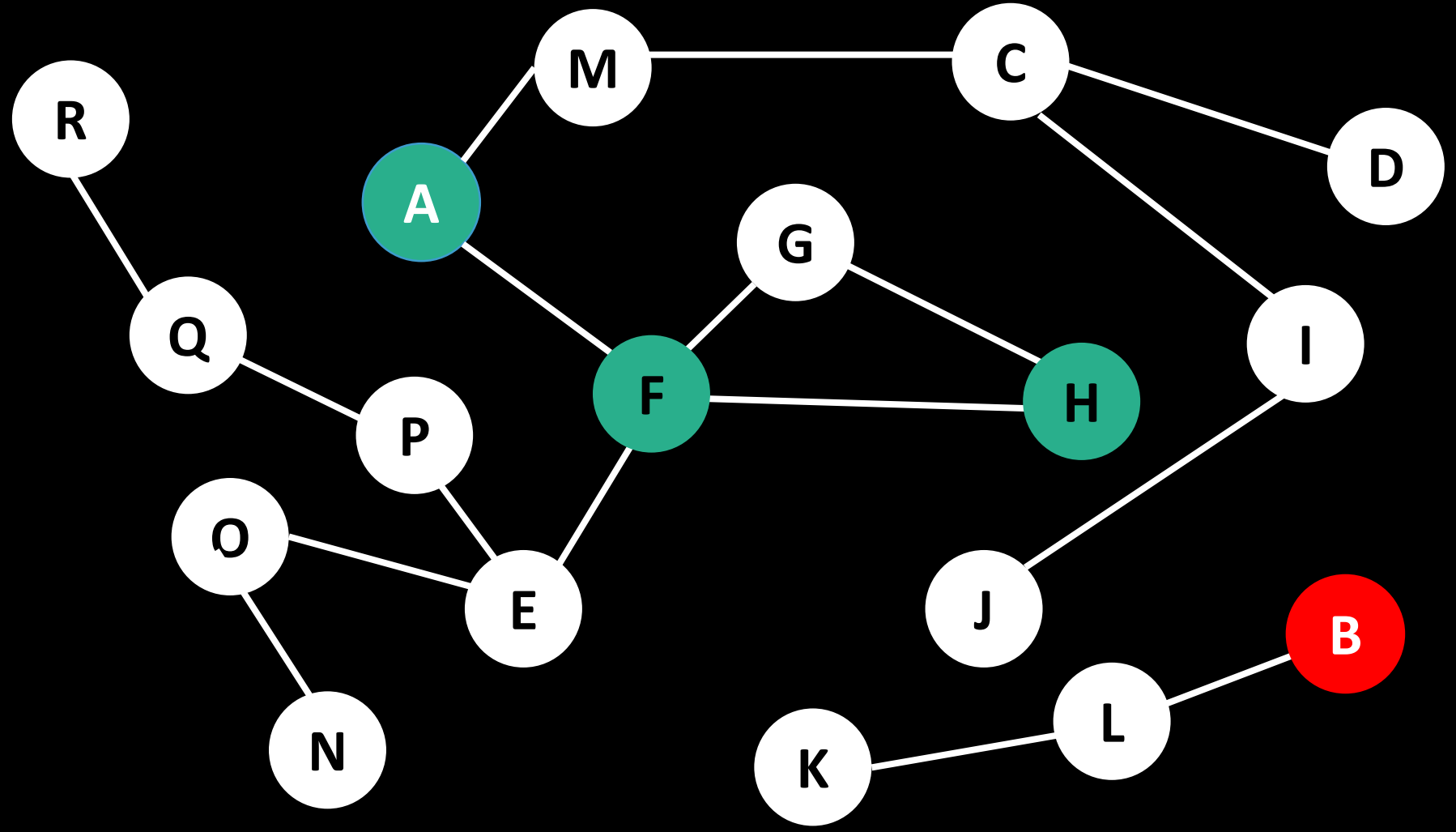

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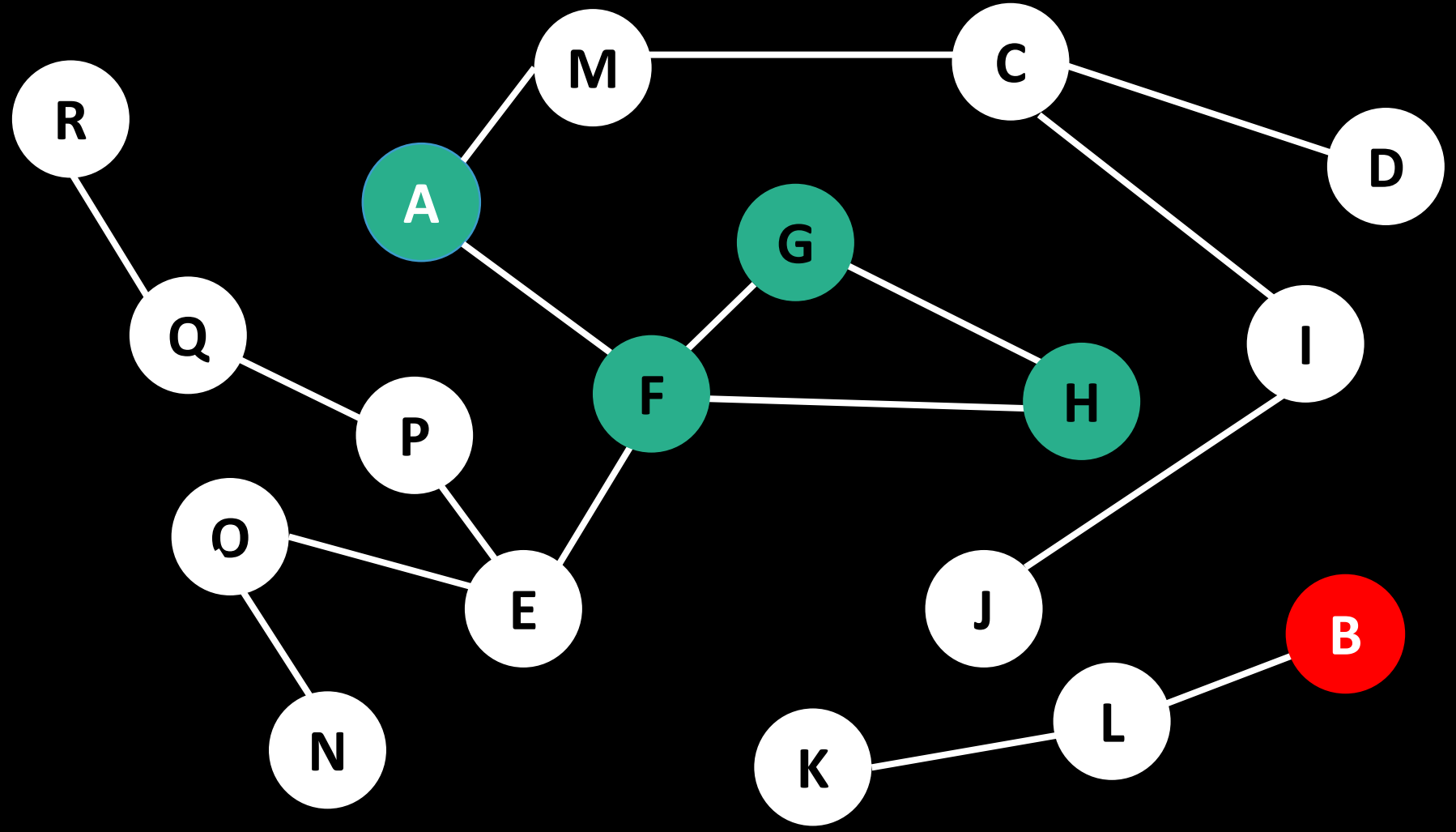

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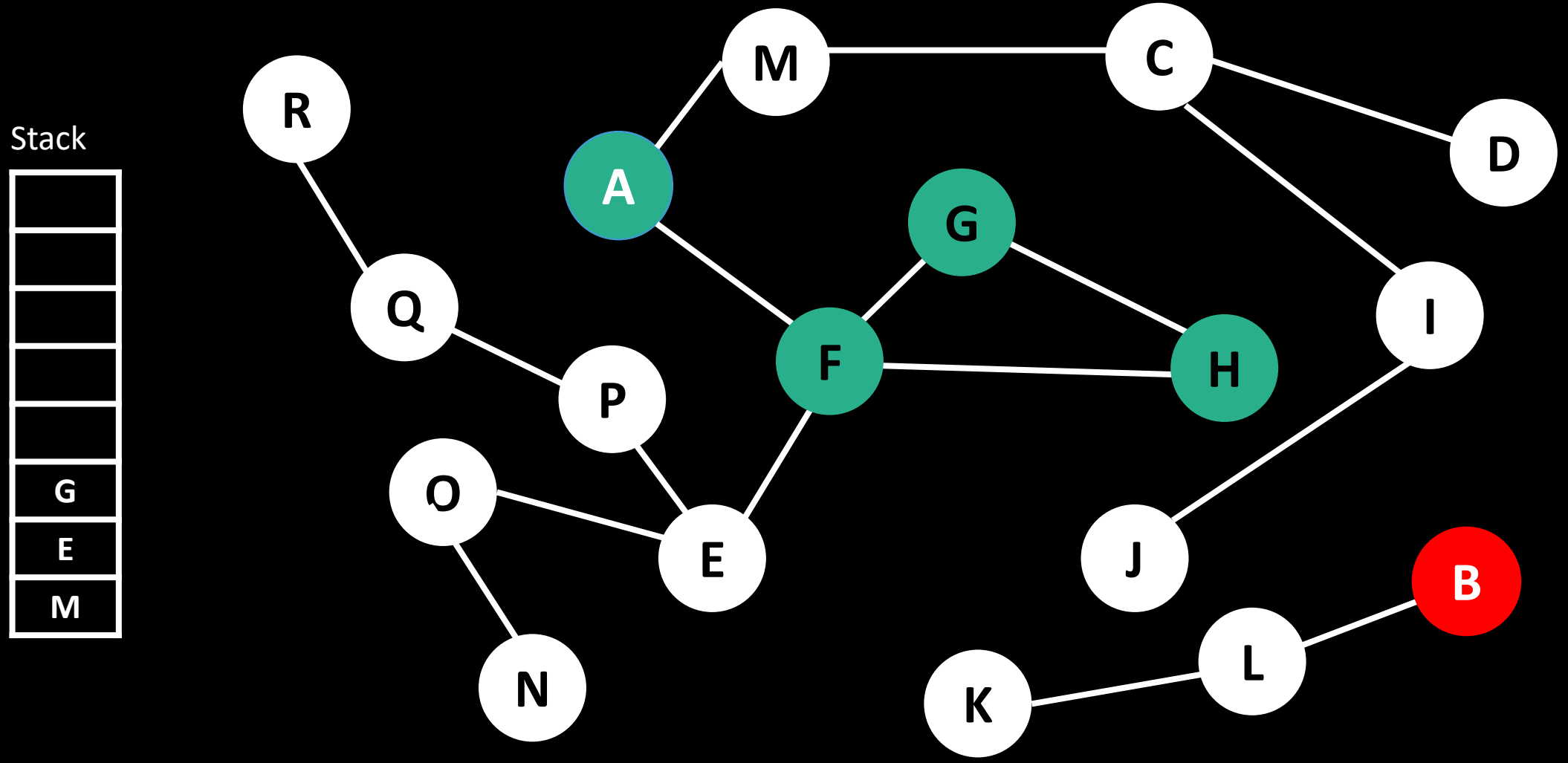

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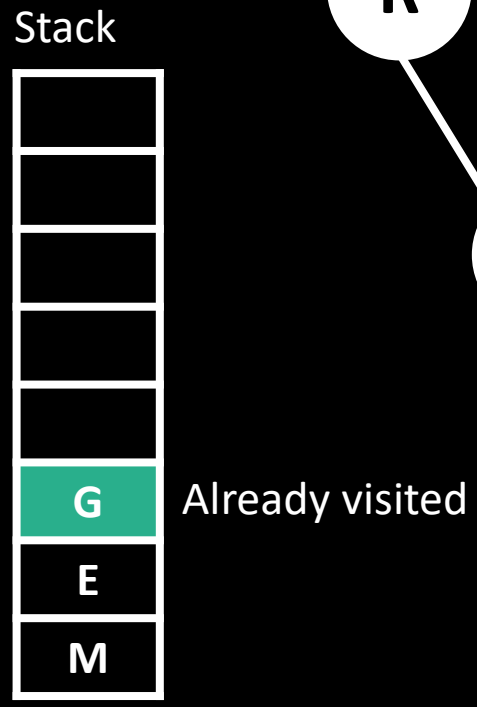

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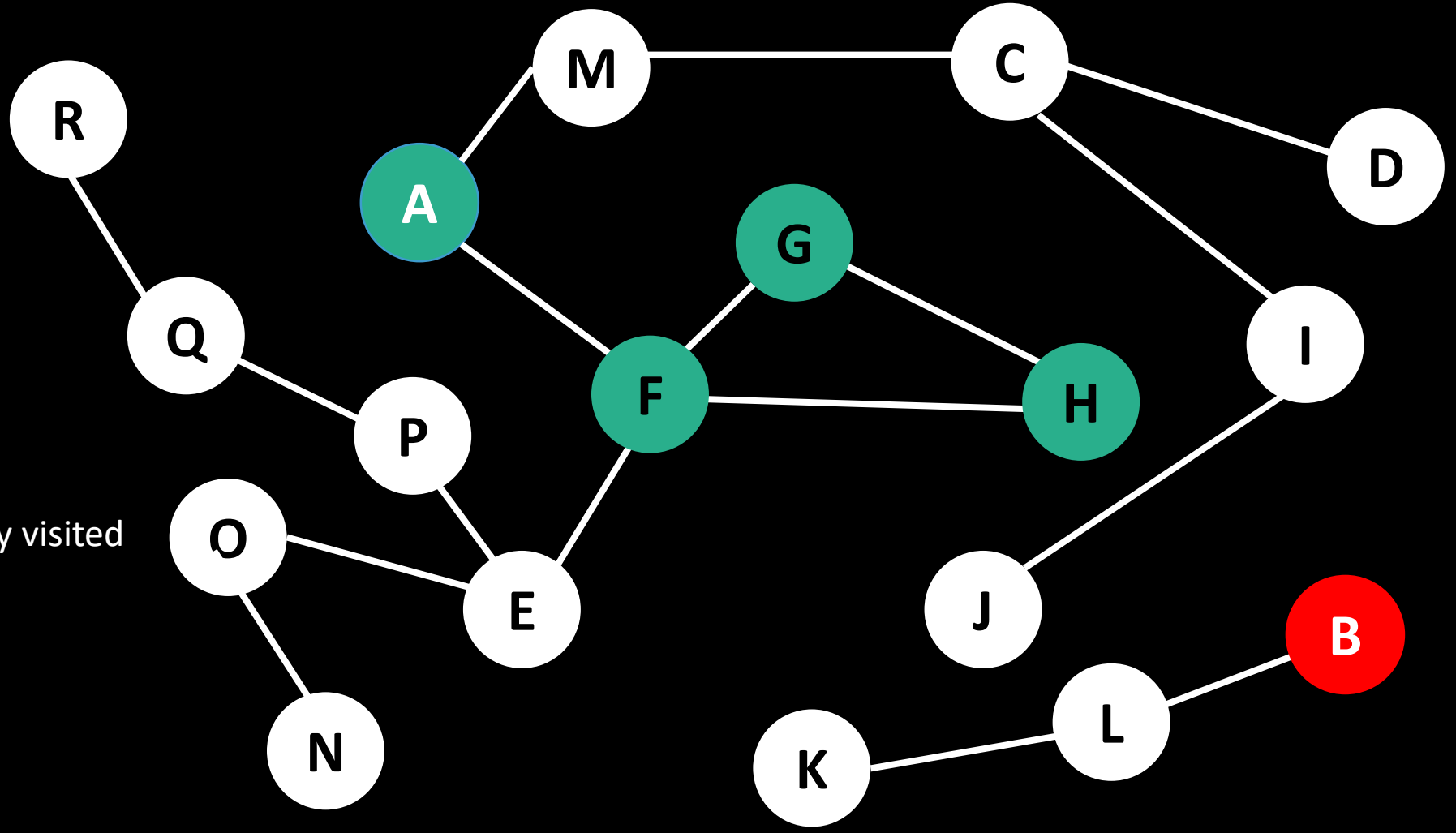

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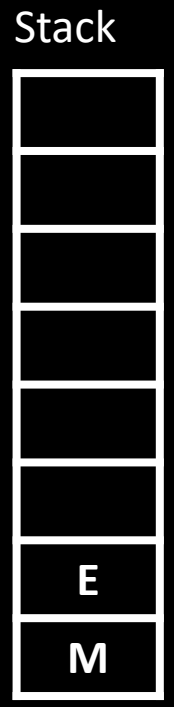

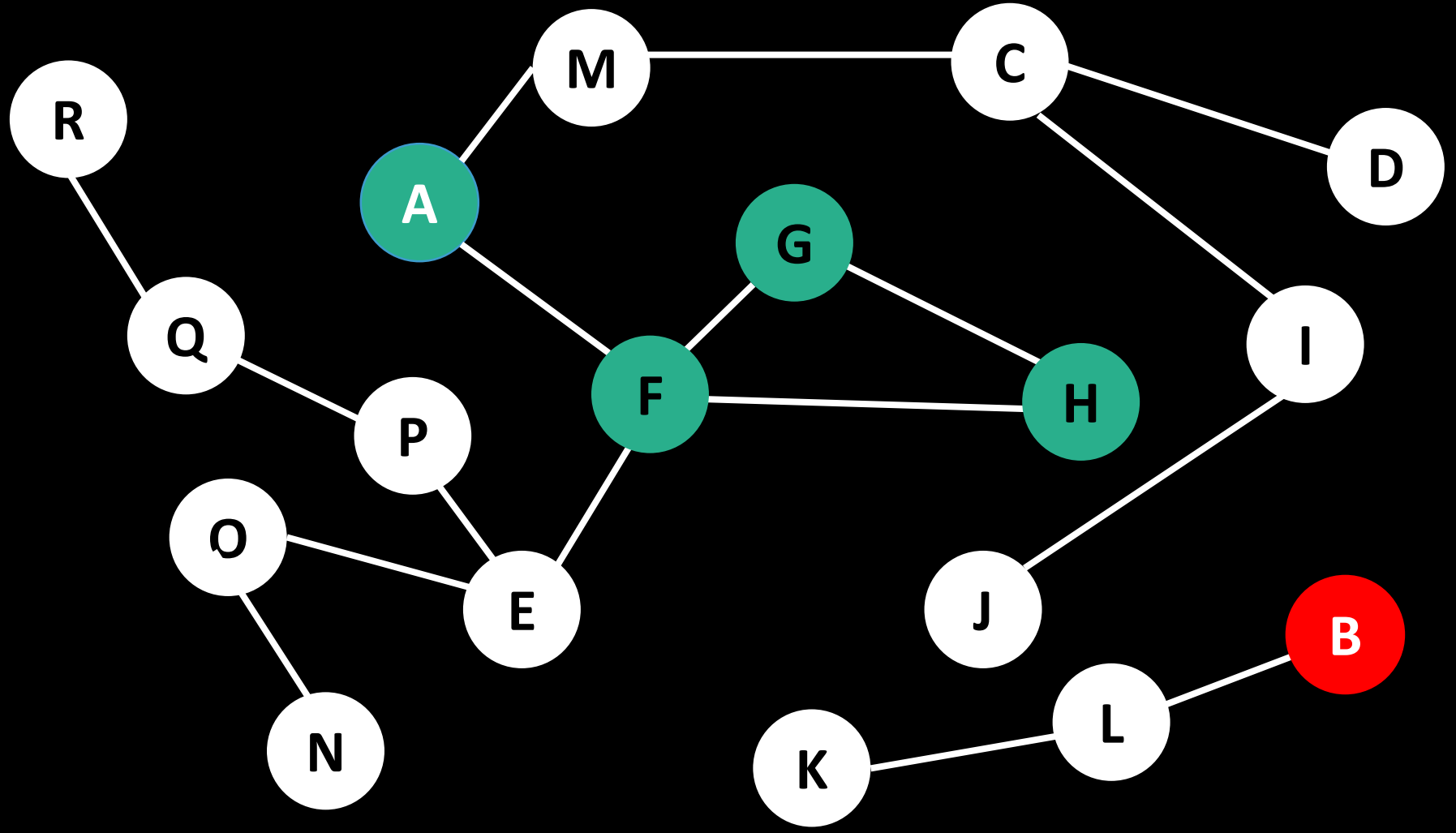

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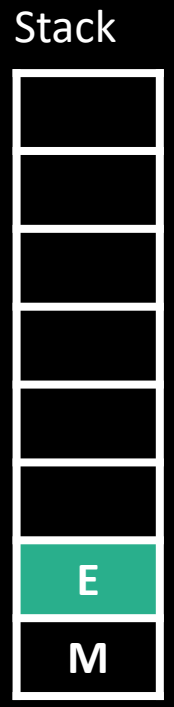

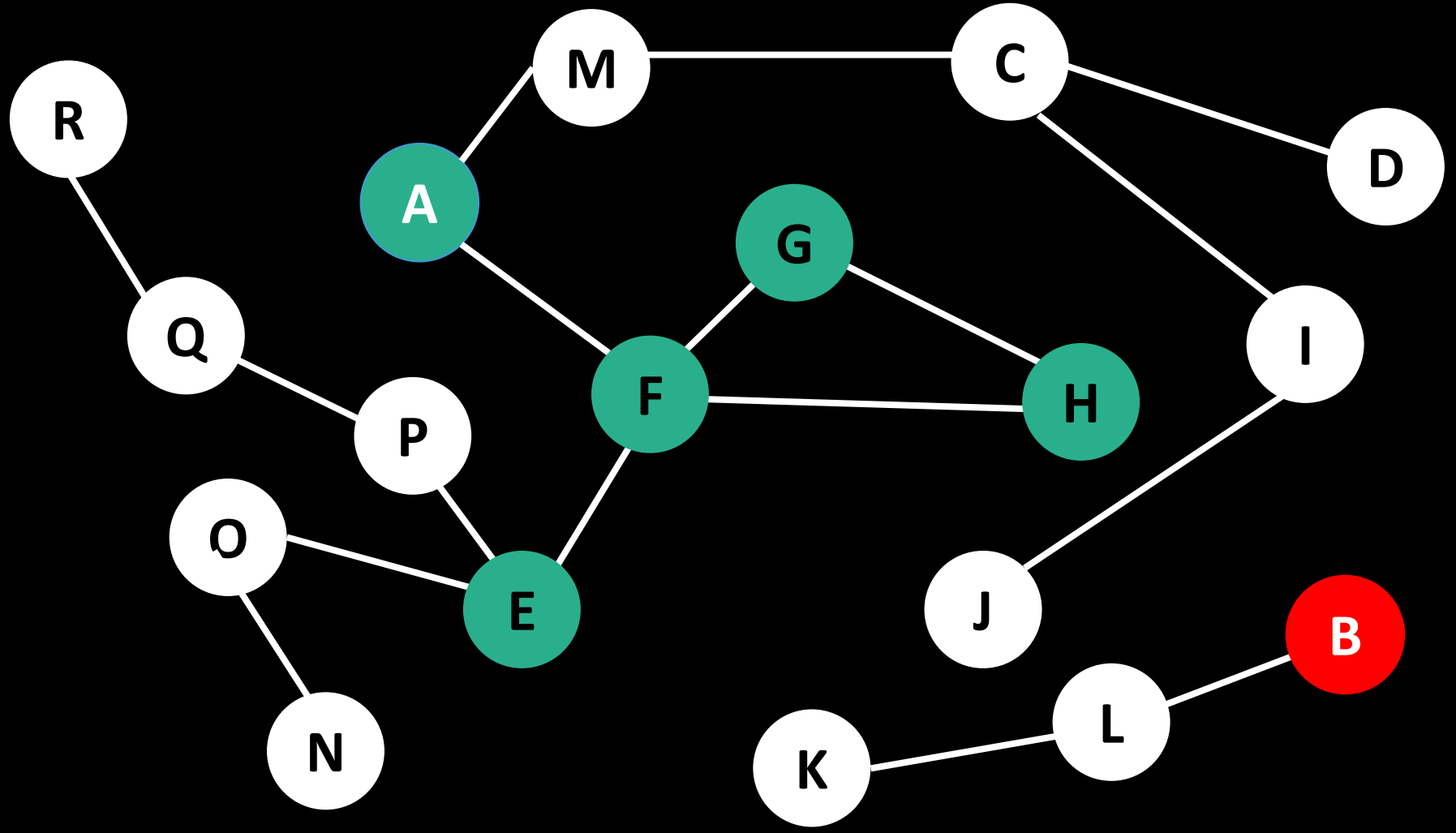

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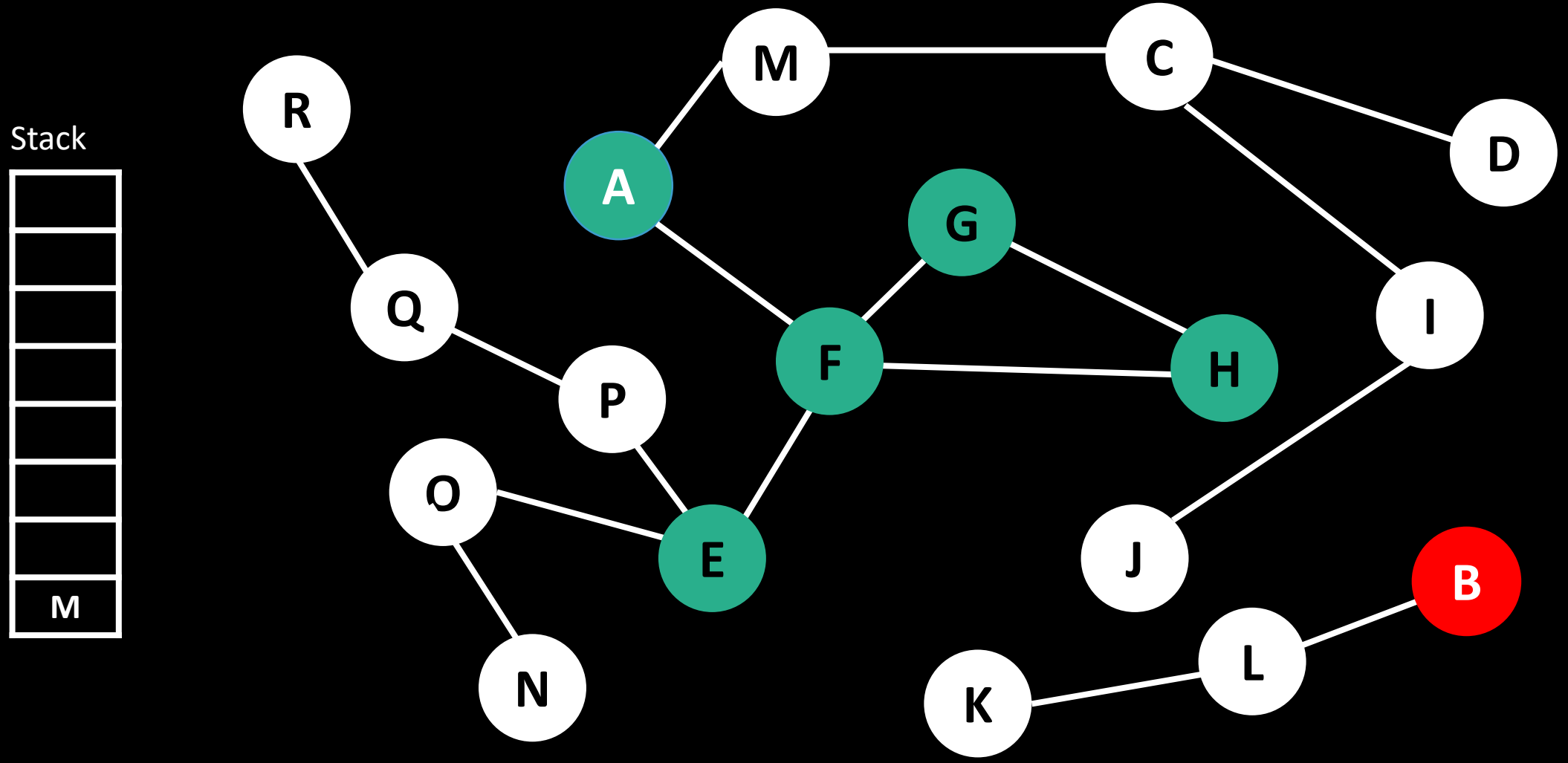

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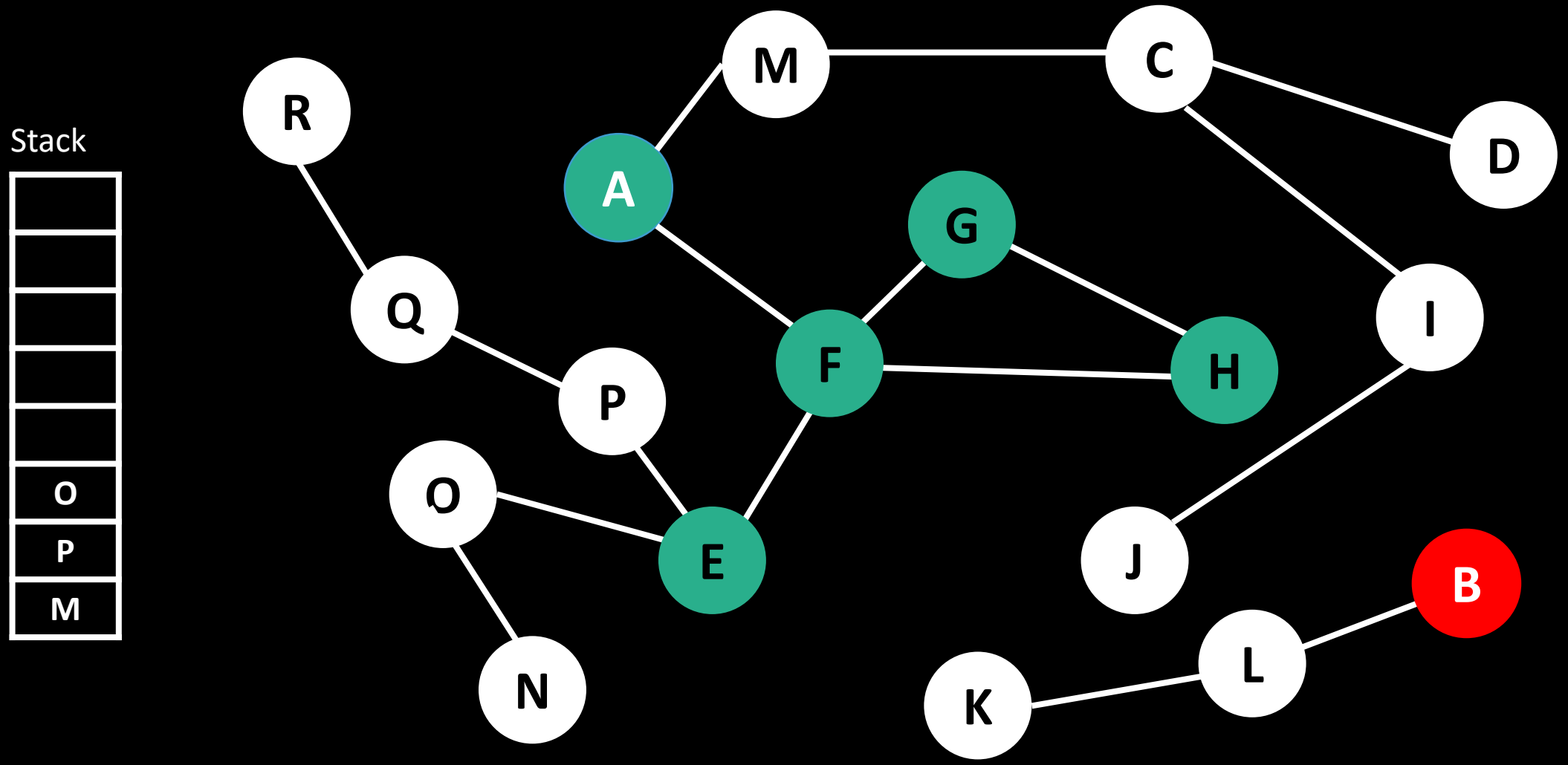

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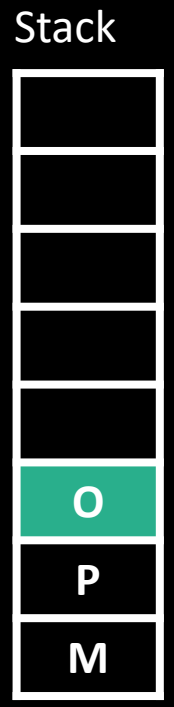

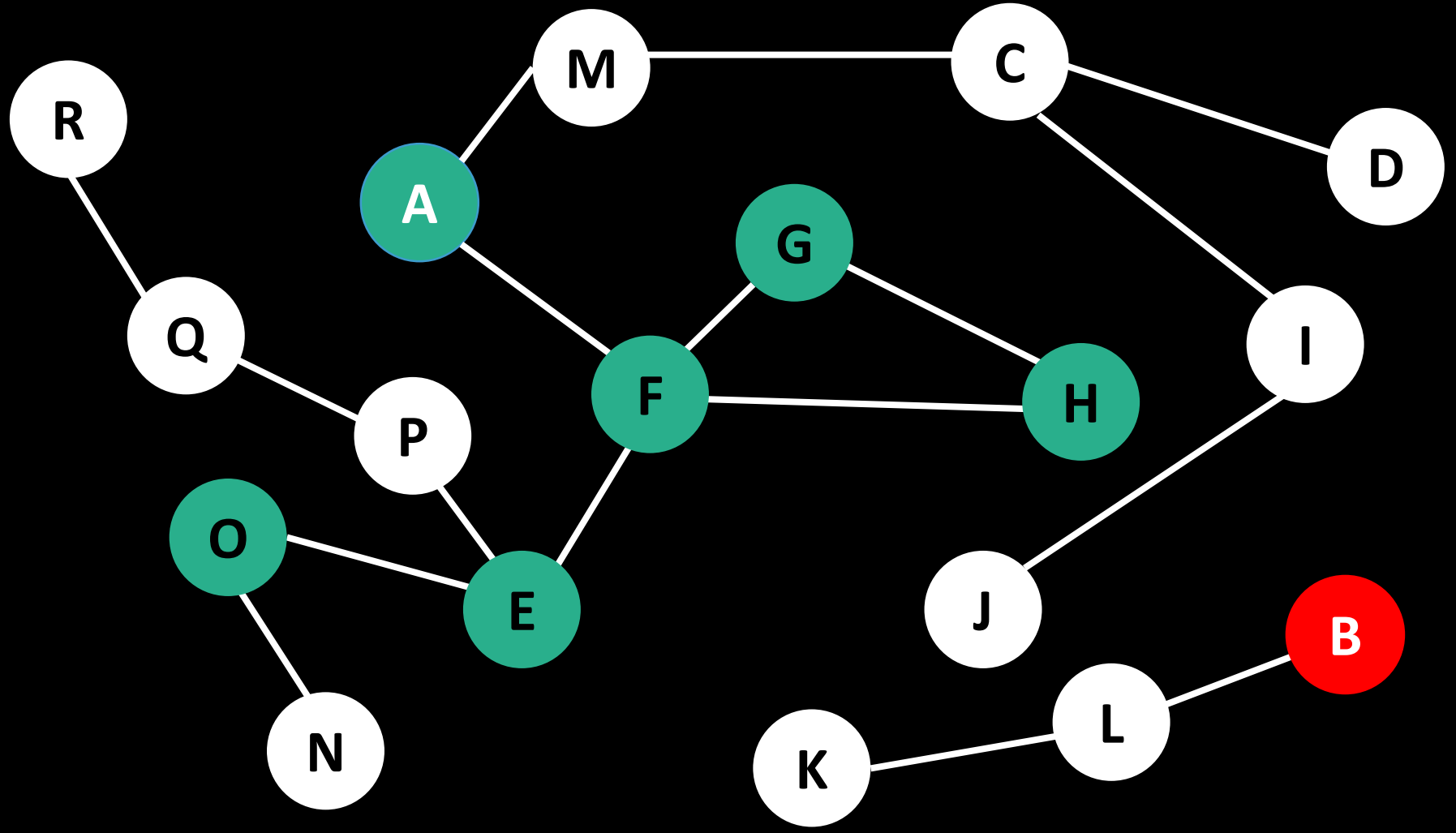

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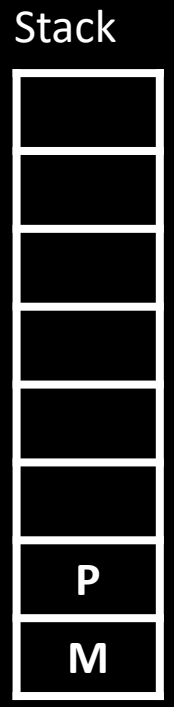

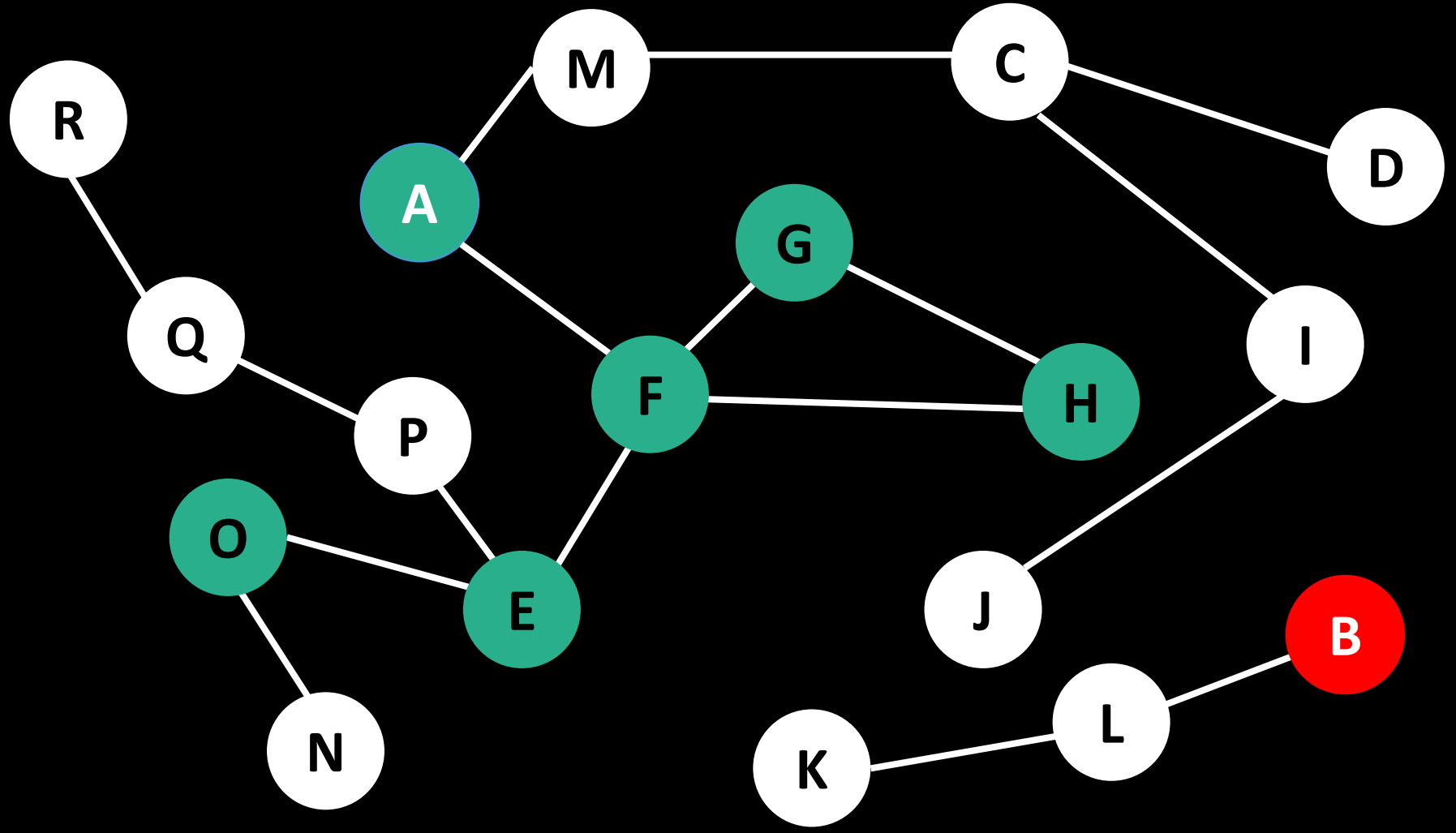

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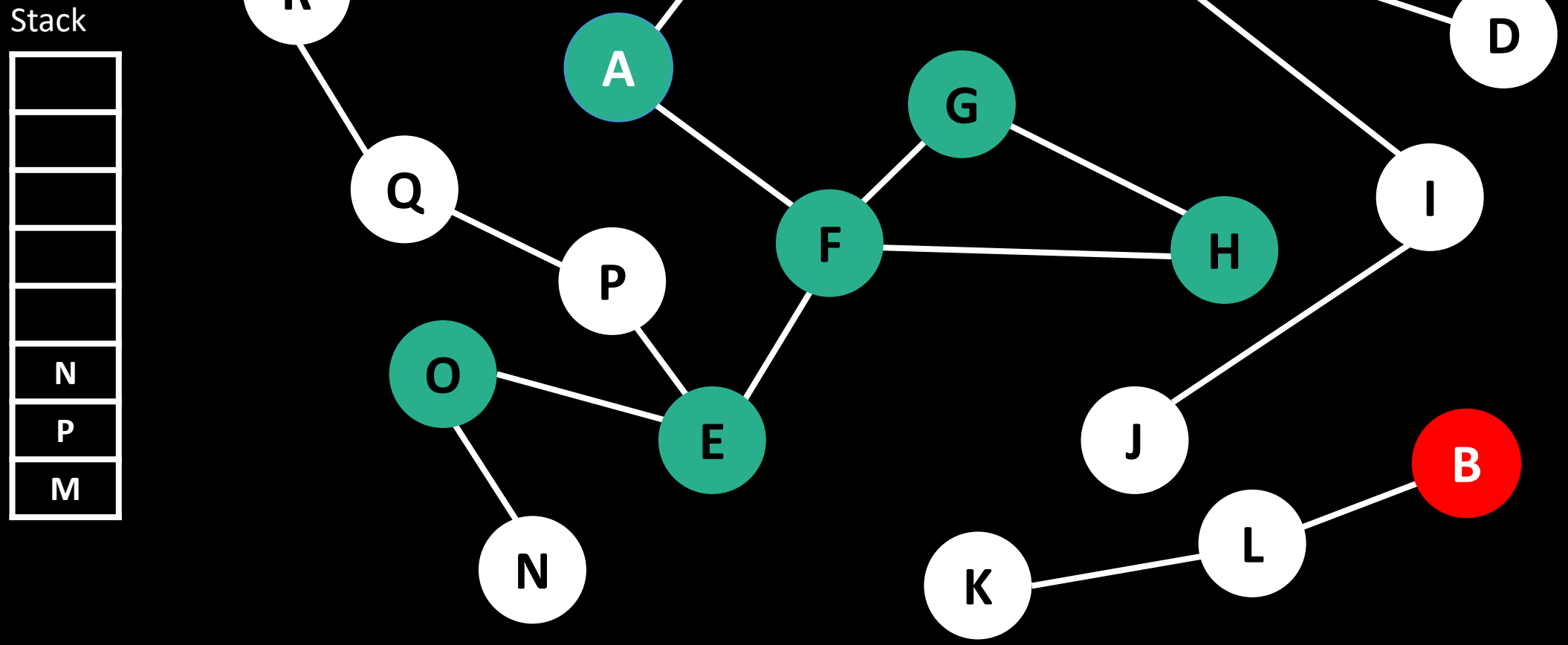

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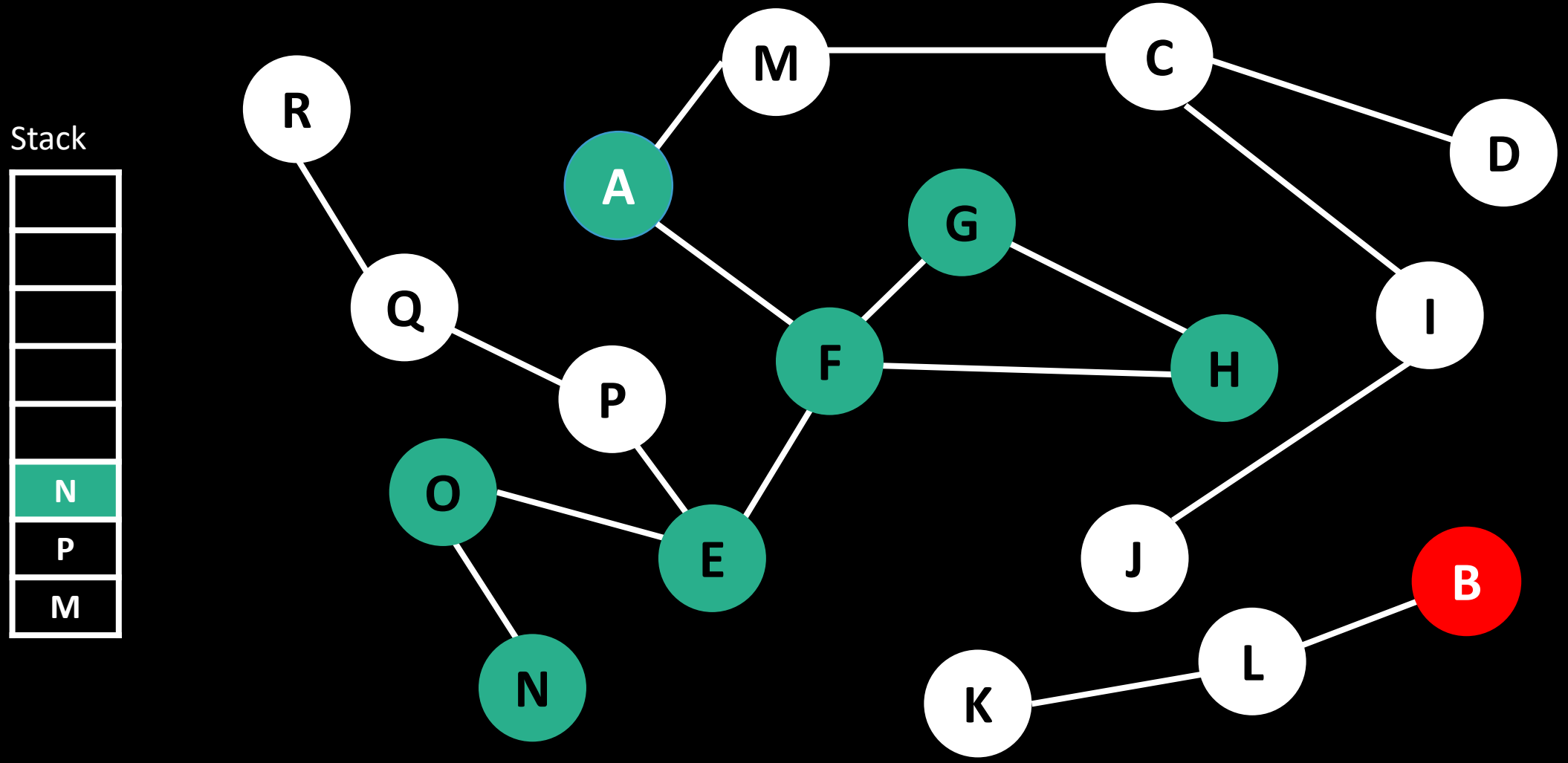

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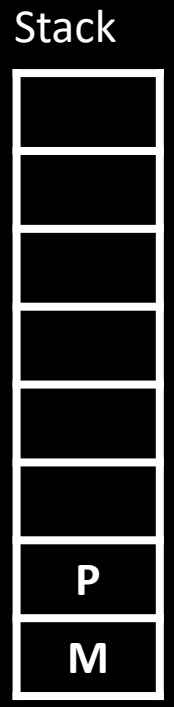

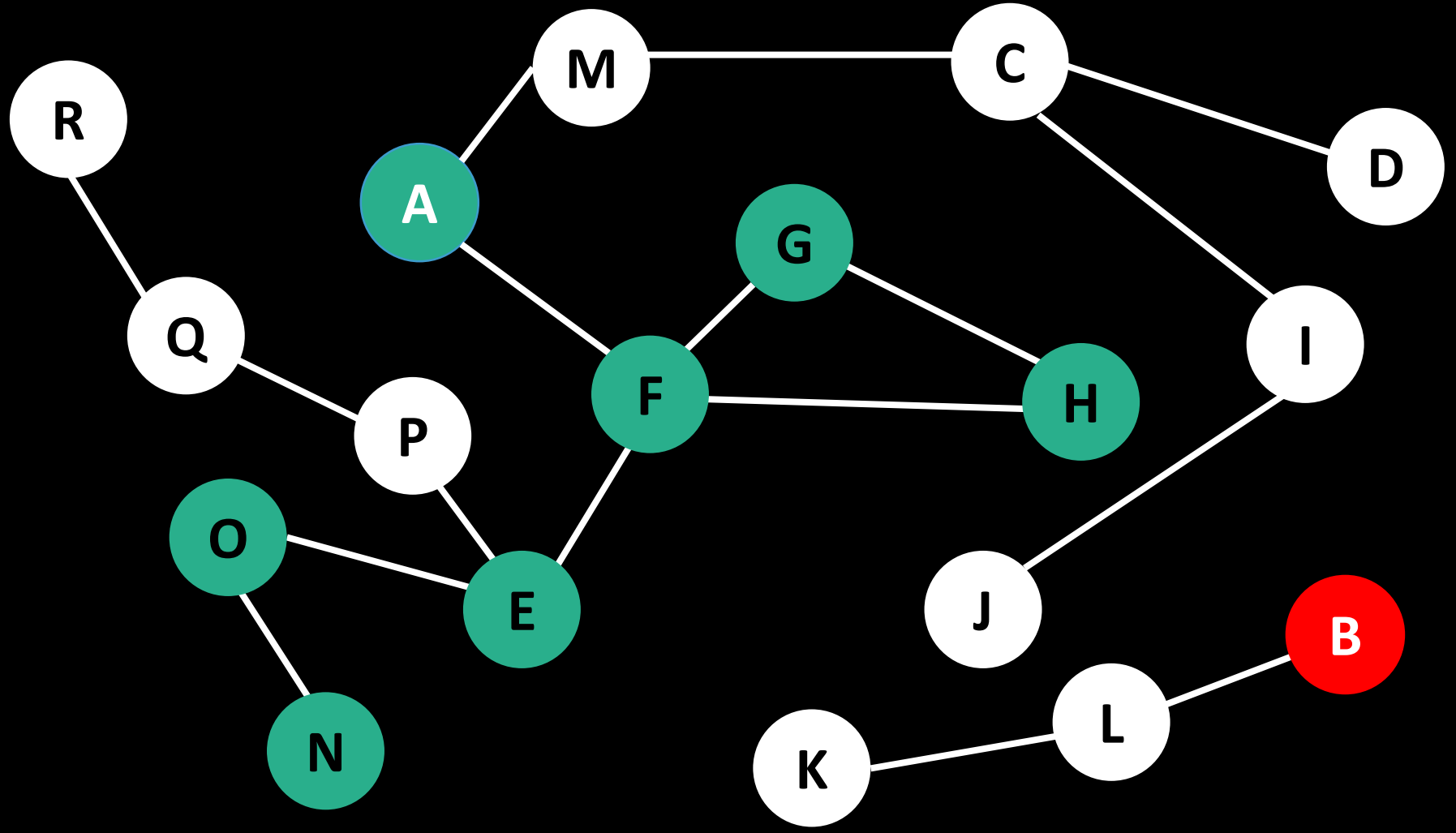

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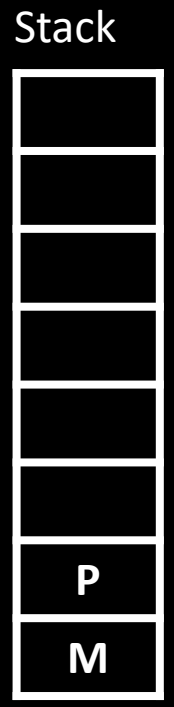
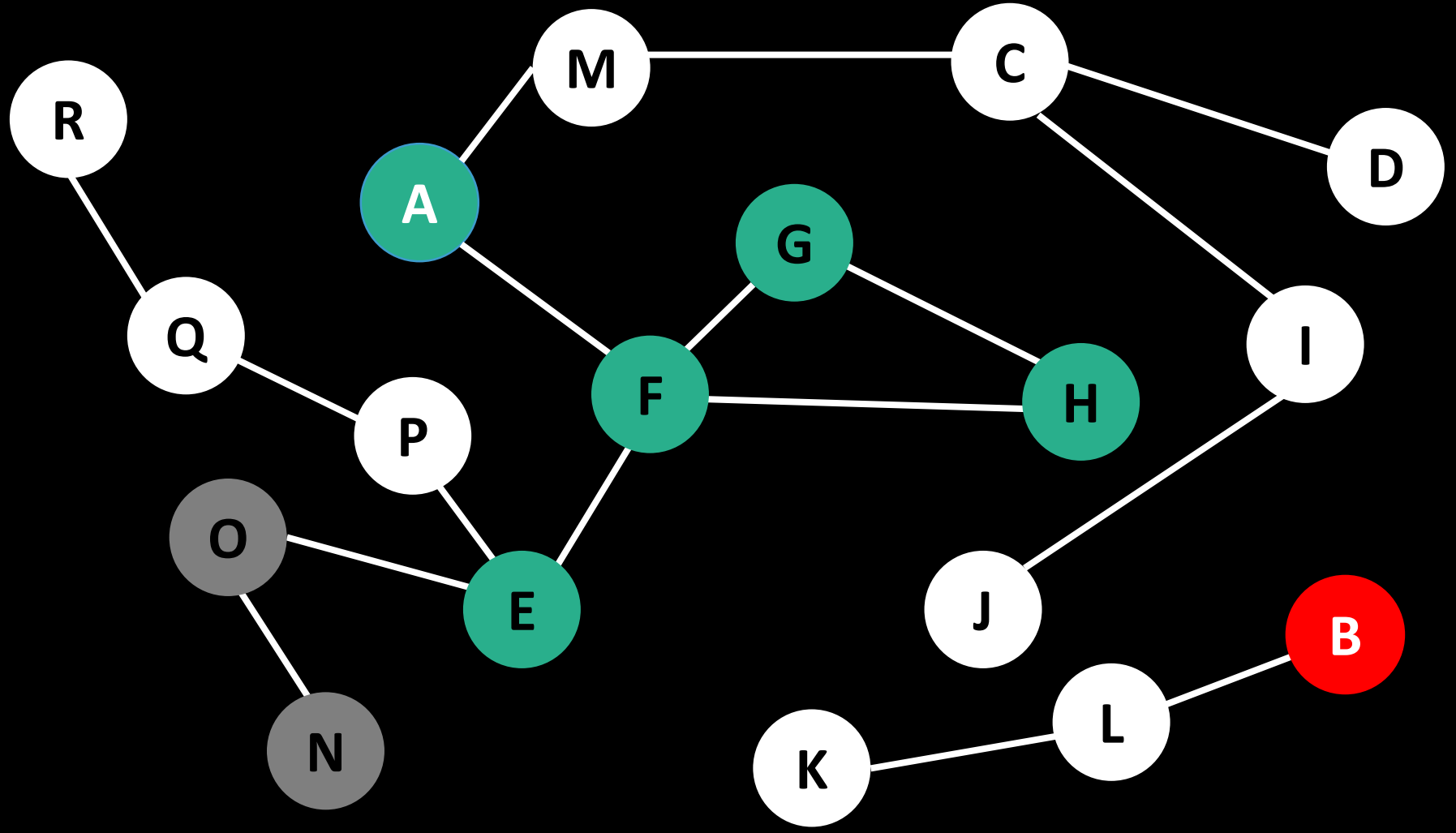
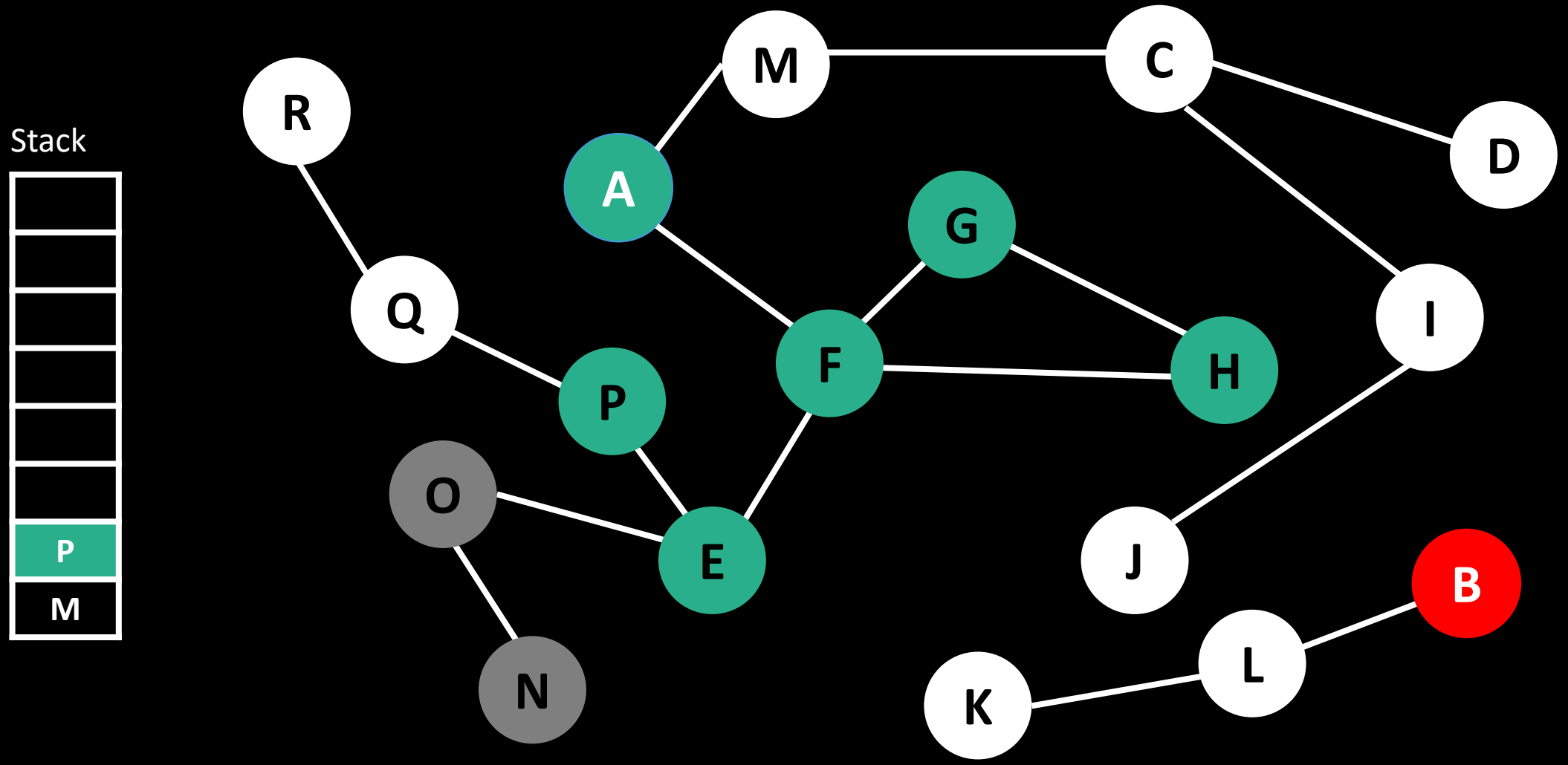
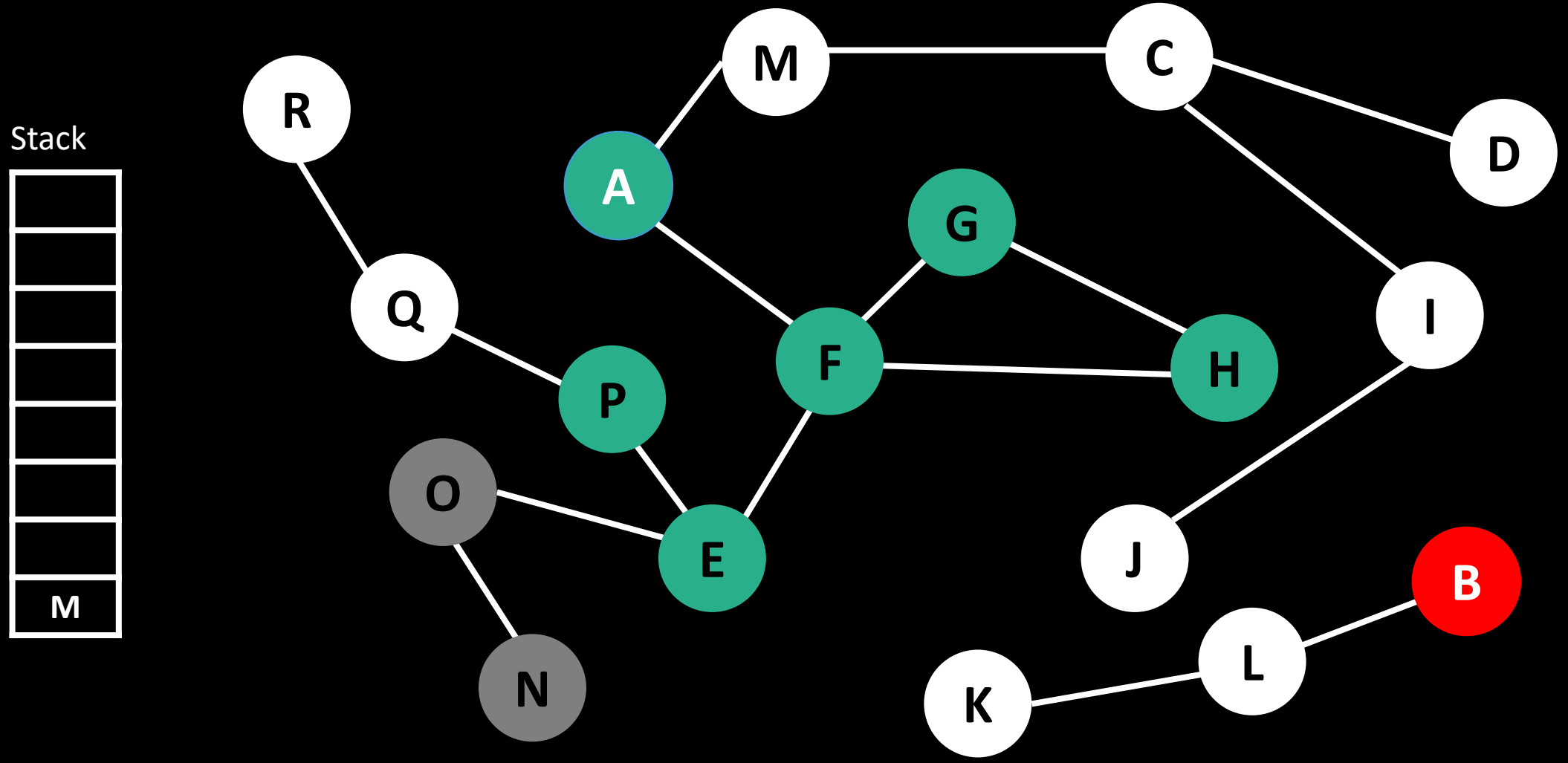




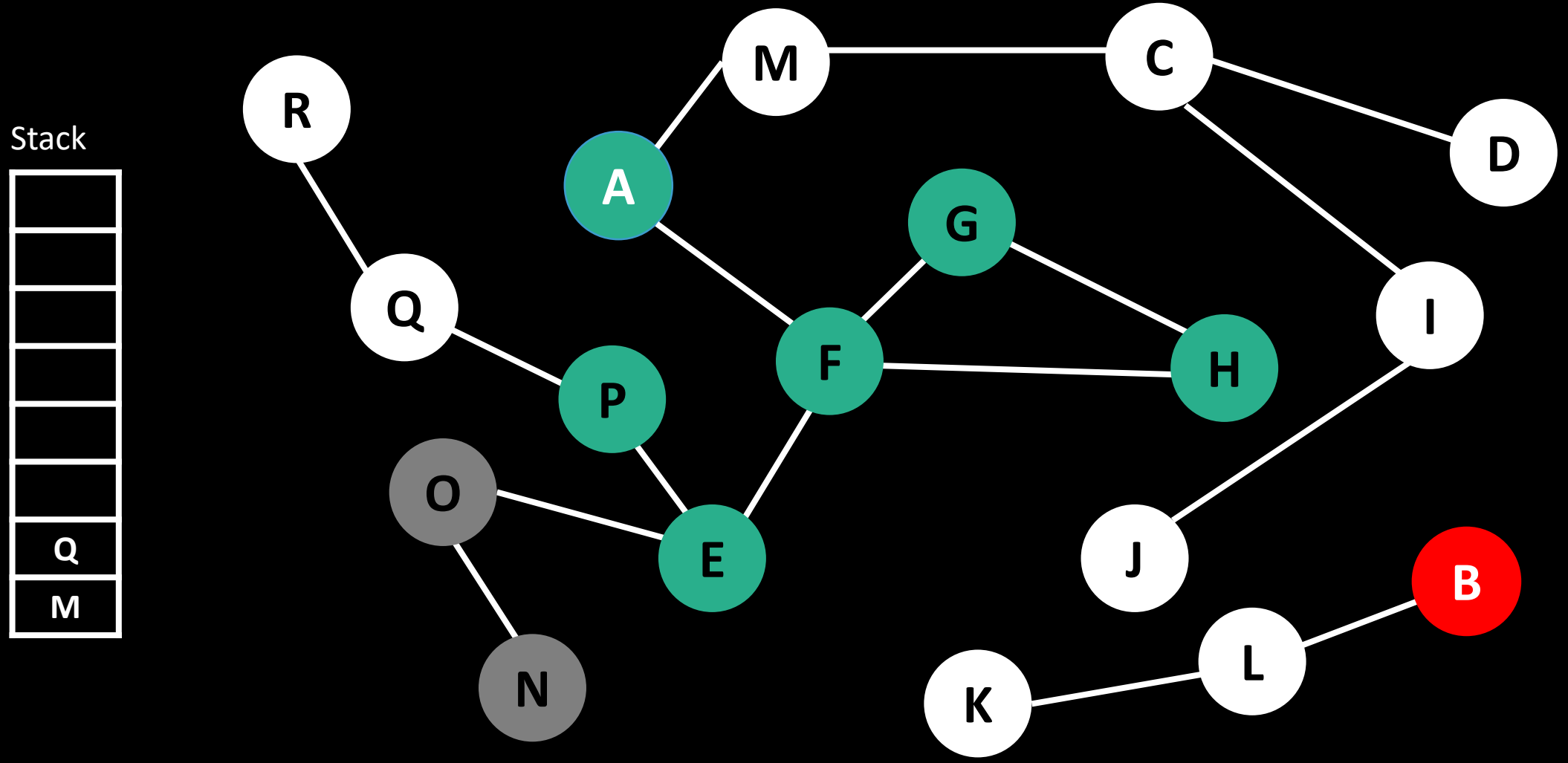
Another DFS Example



Another DFS Example



Another DFS Example



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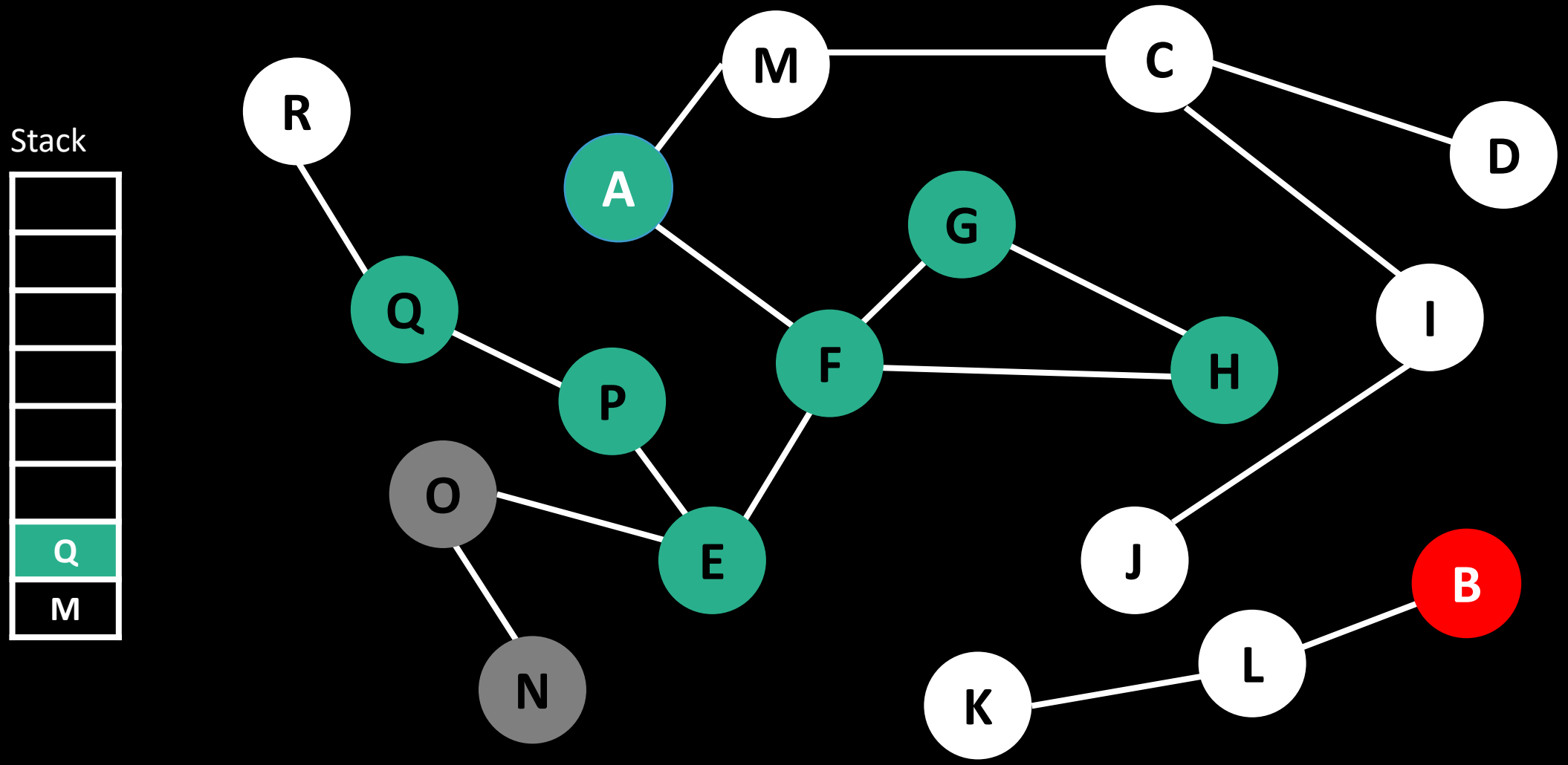

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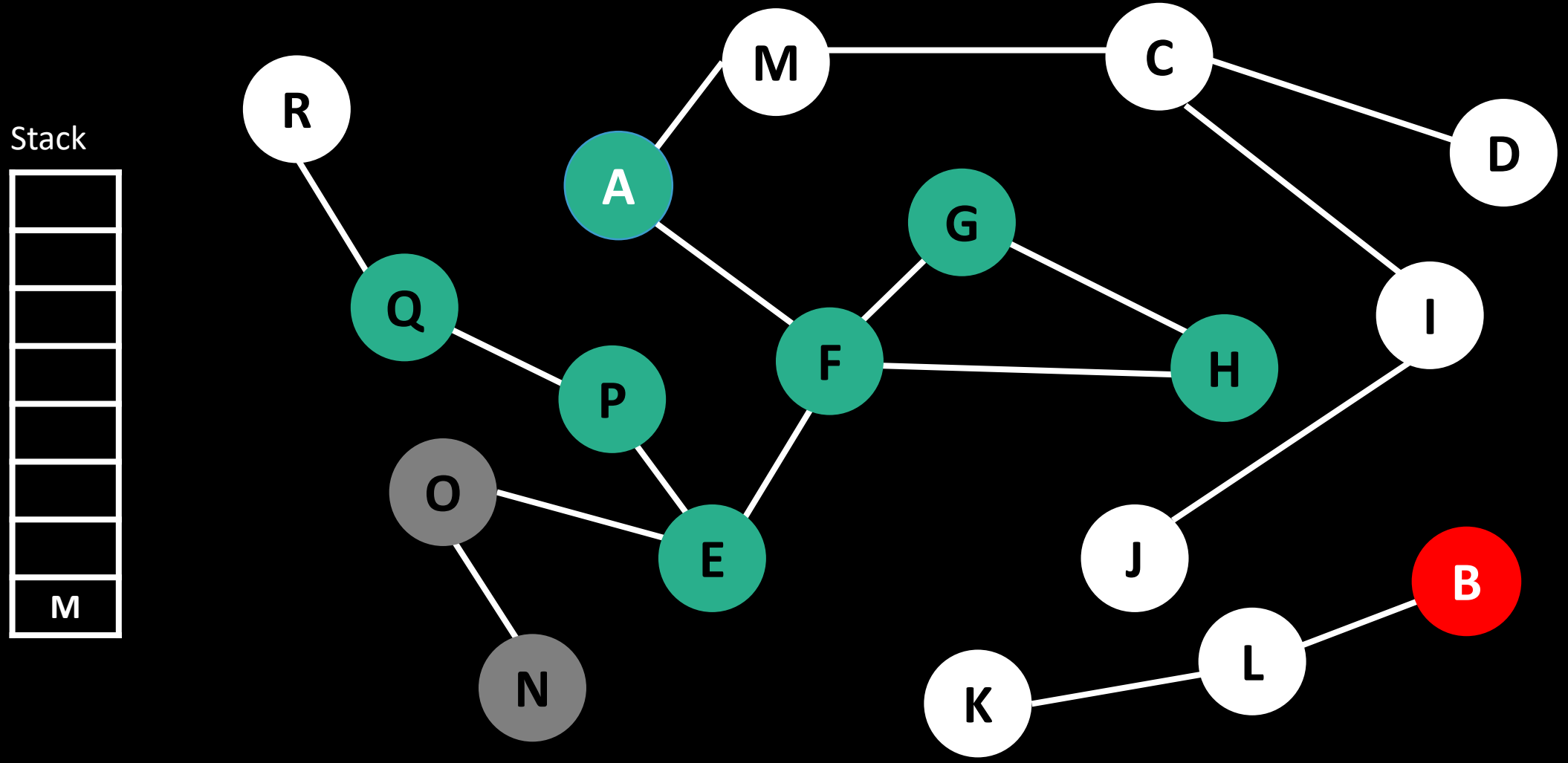

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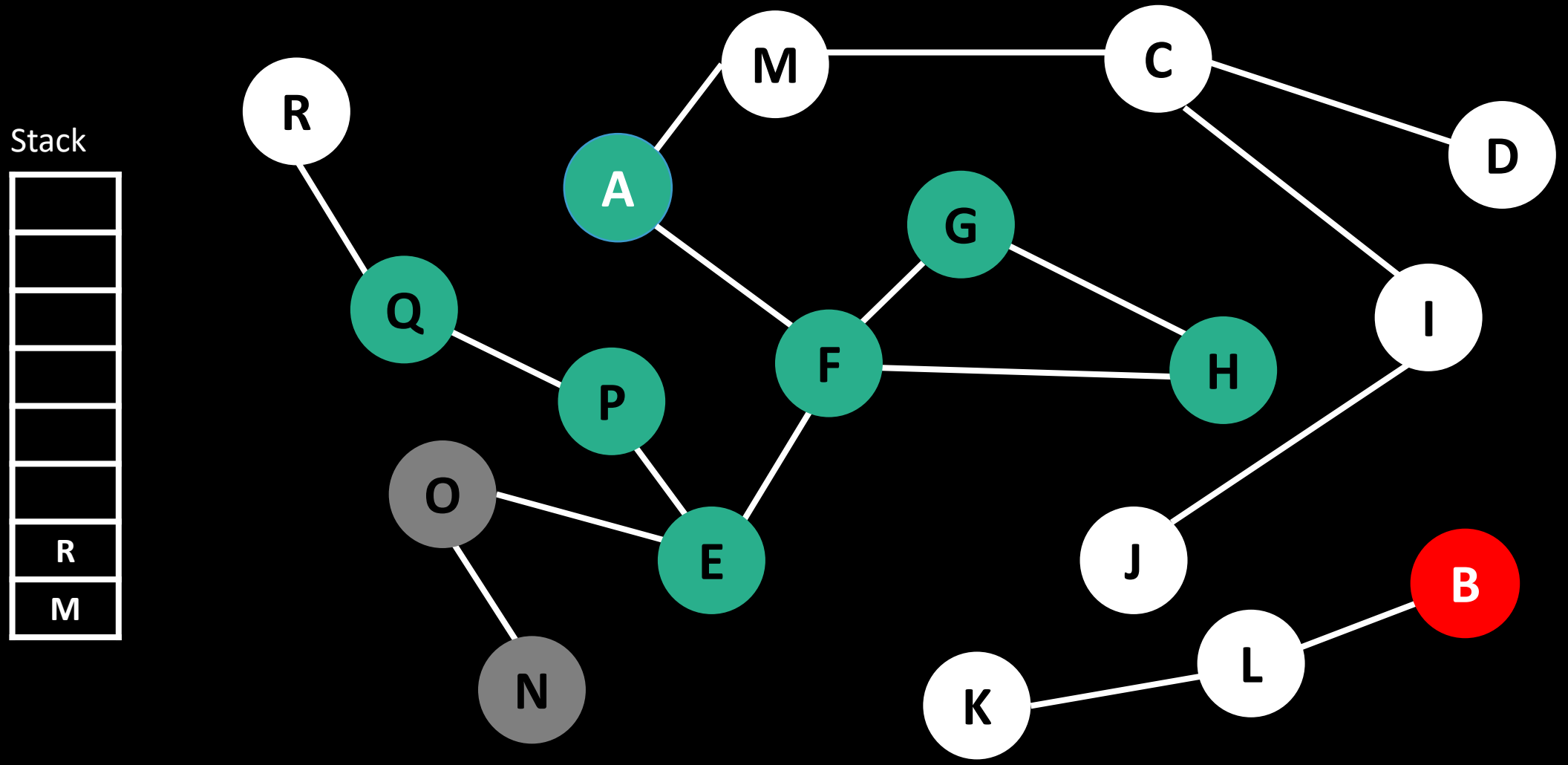

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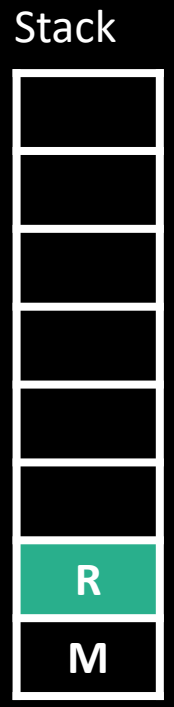

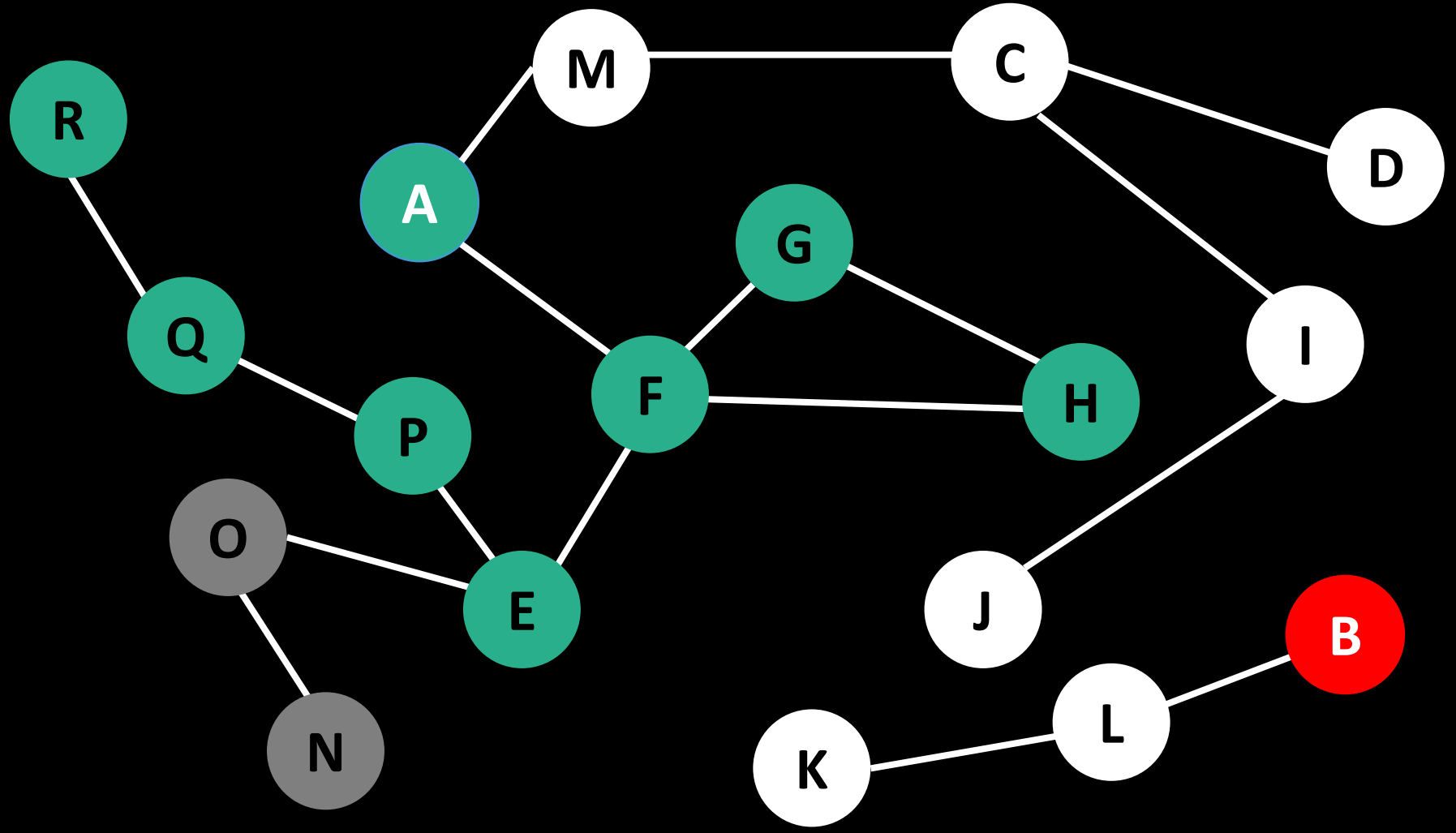

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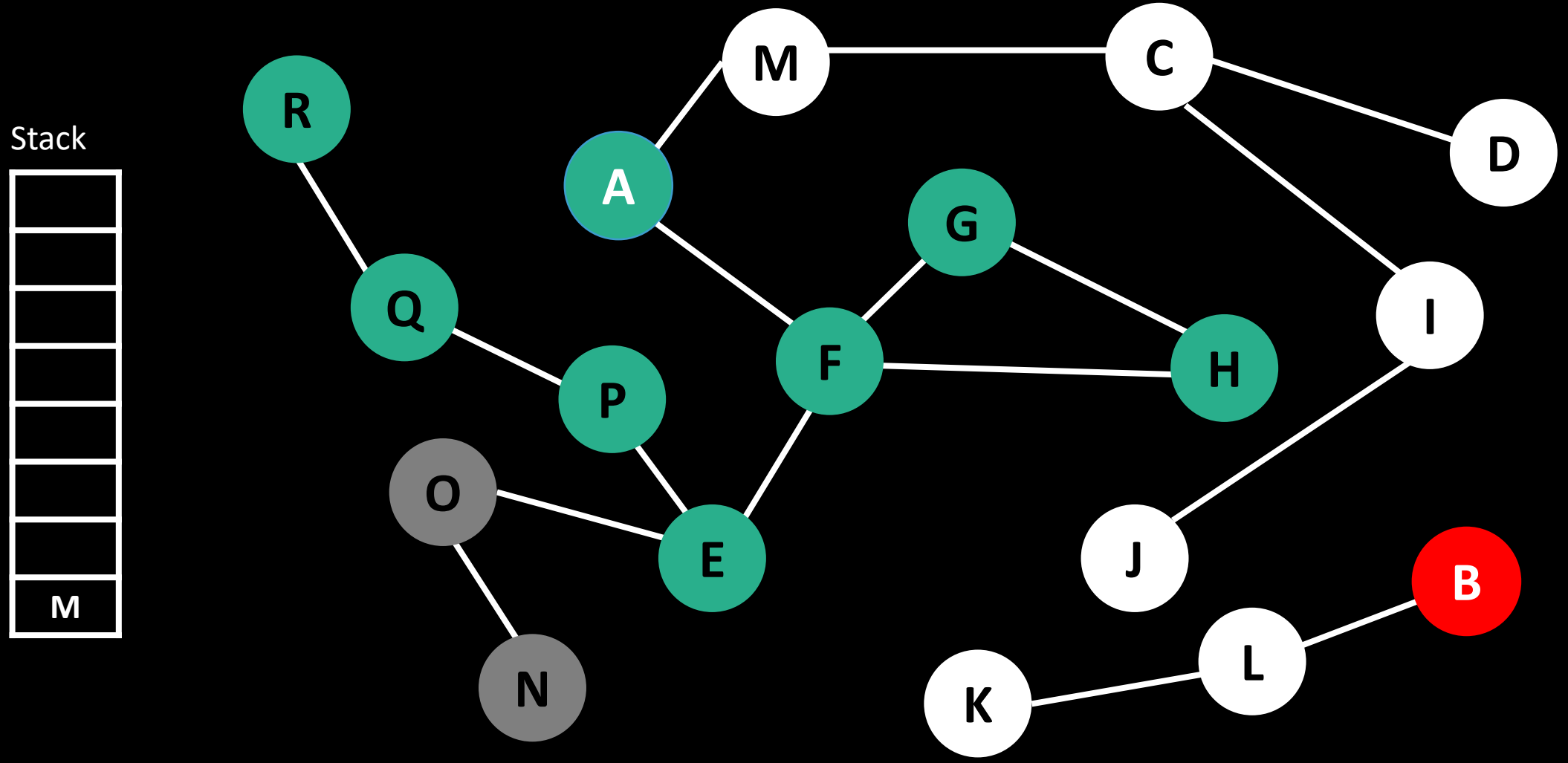

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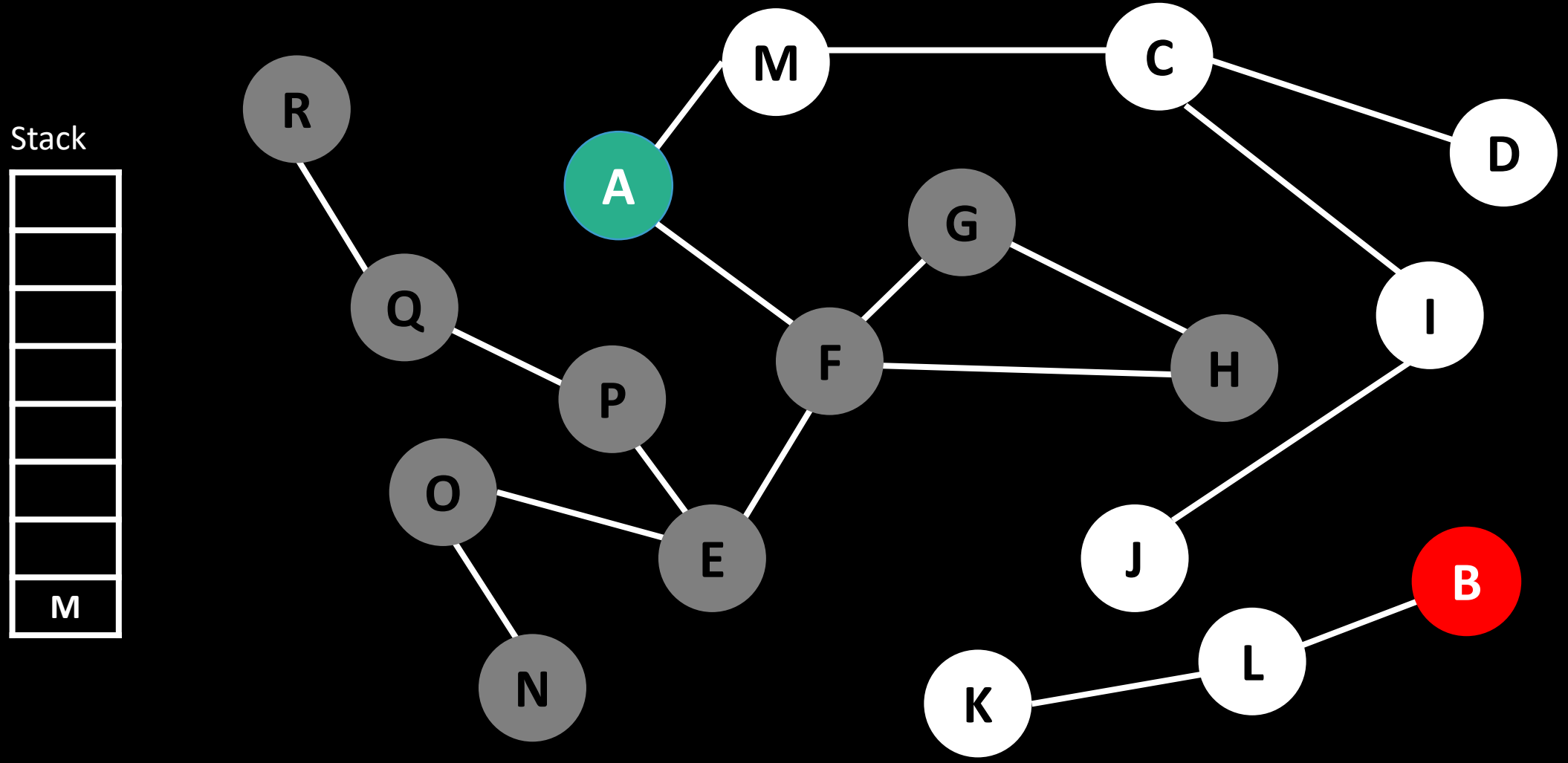

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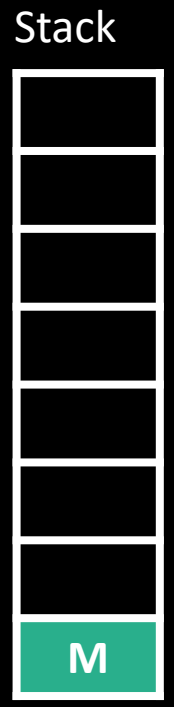

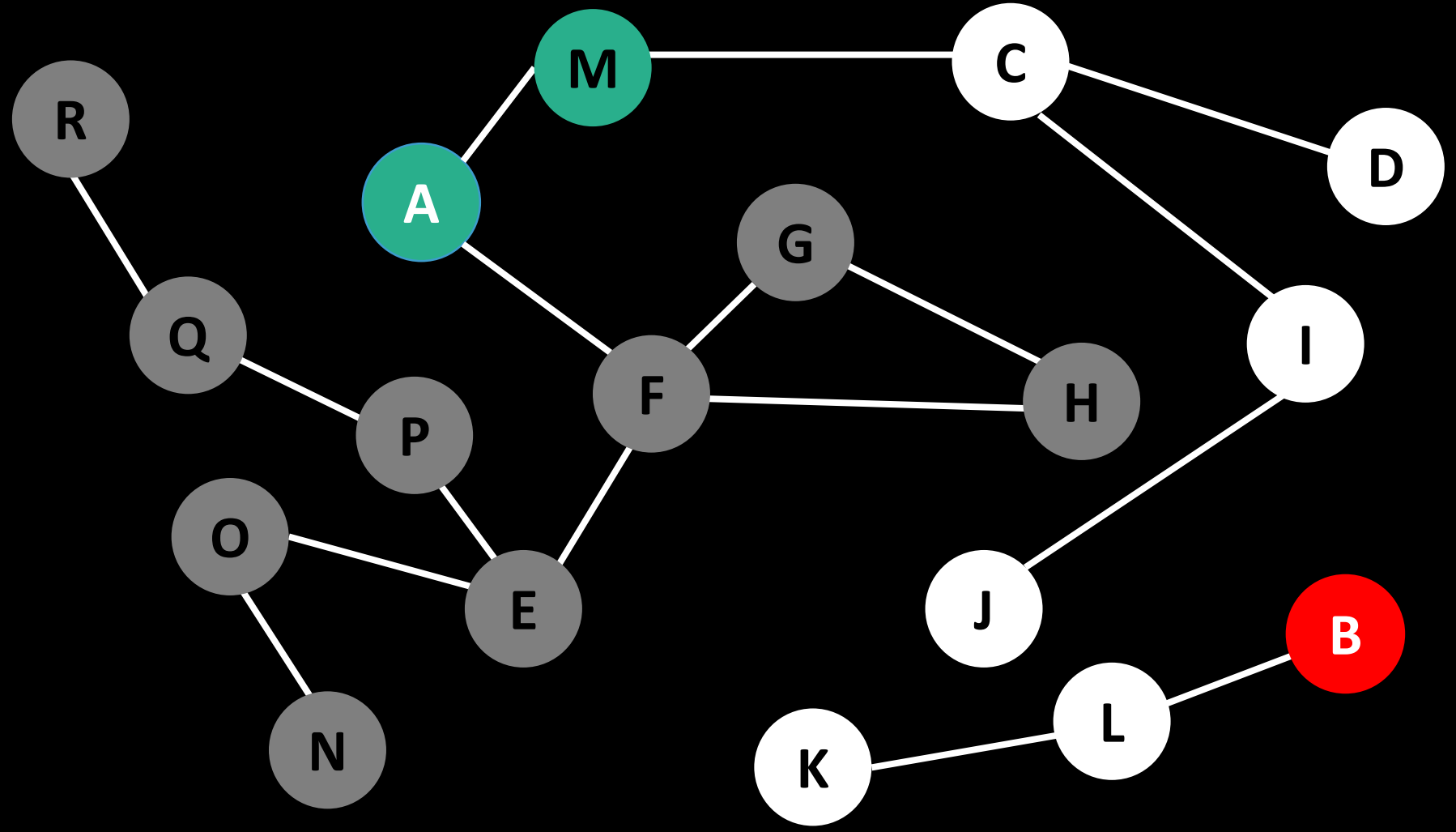

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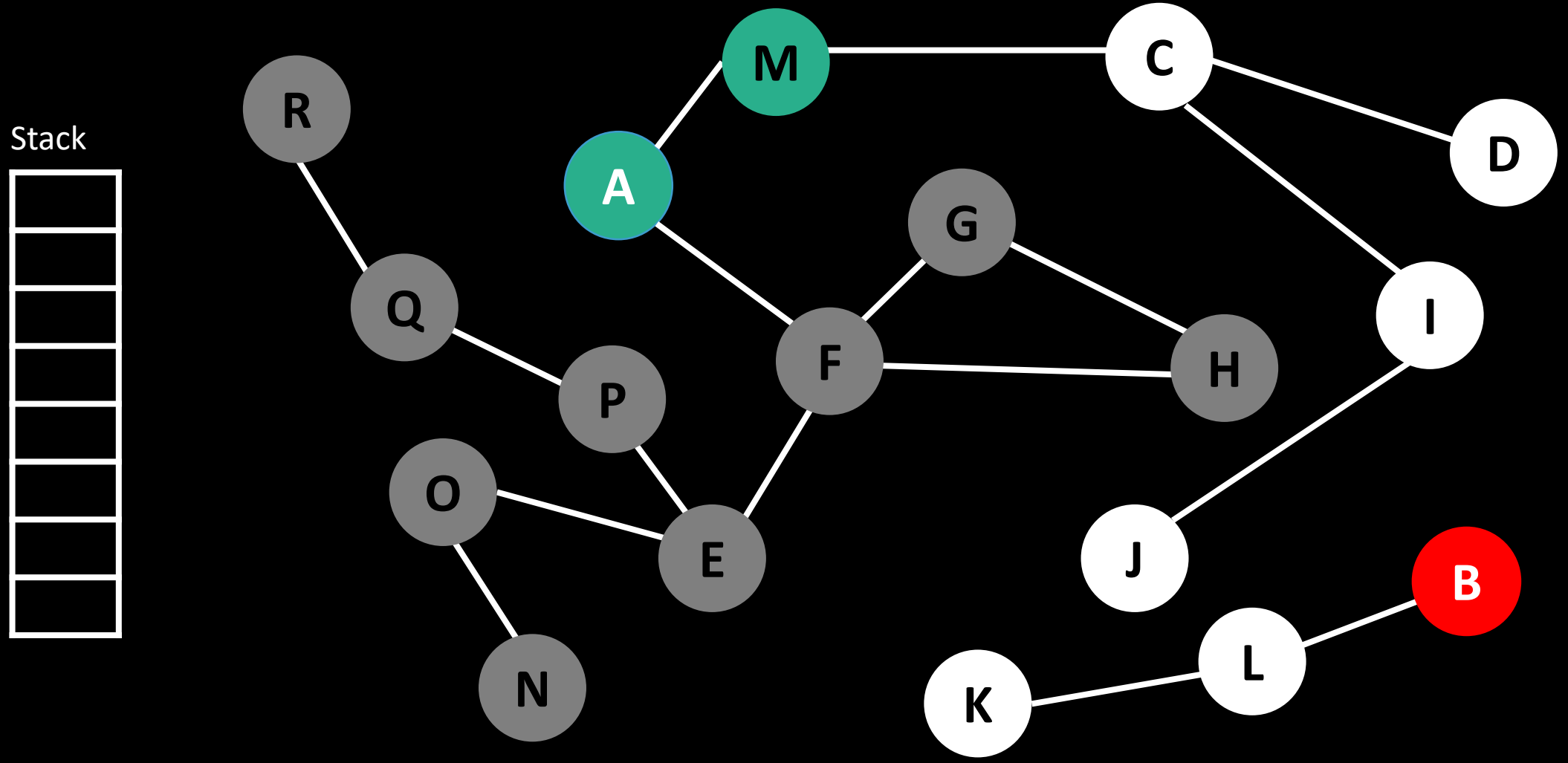
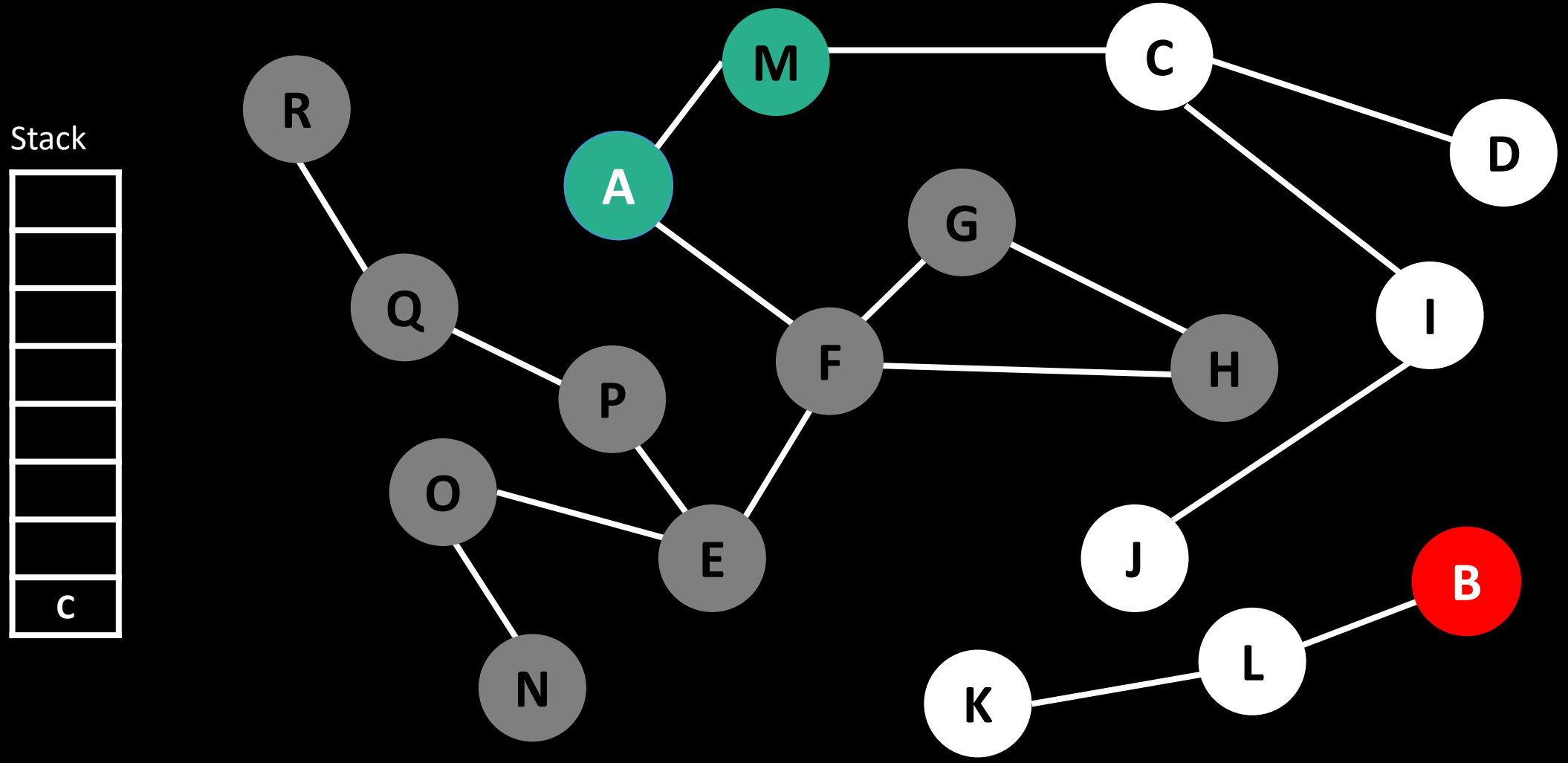


Another DFS Example



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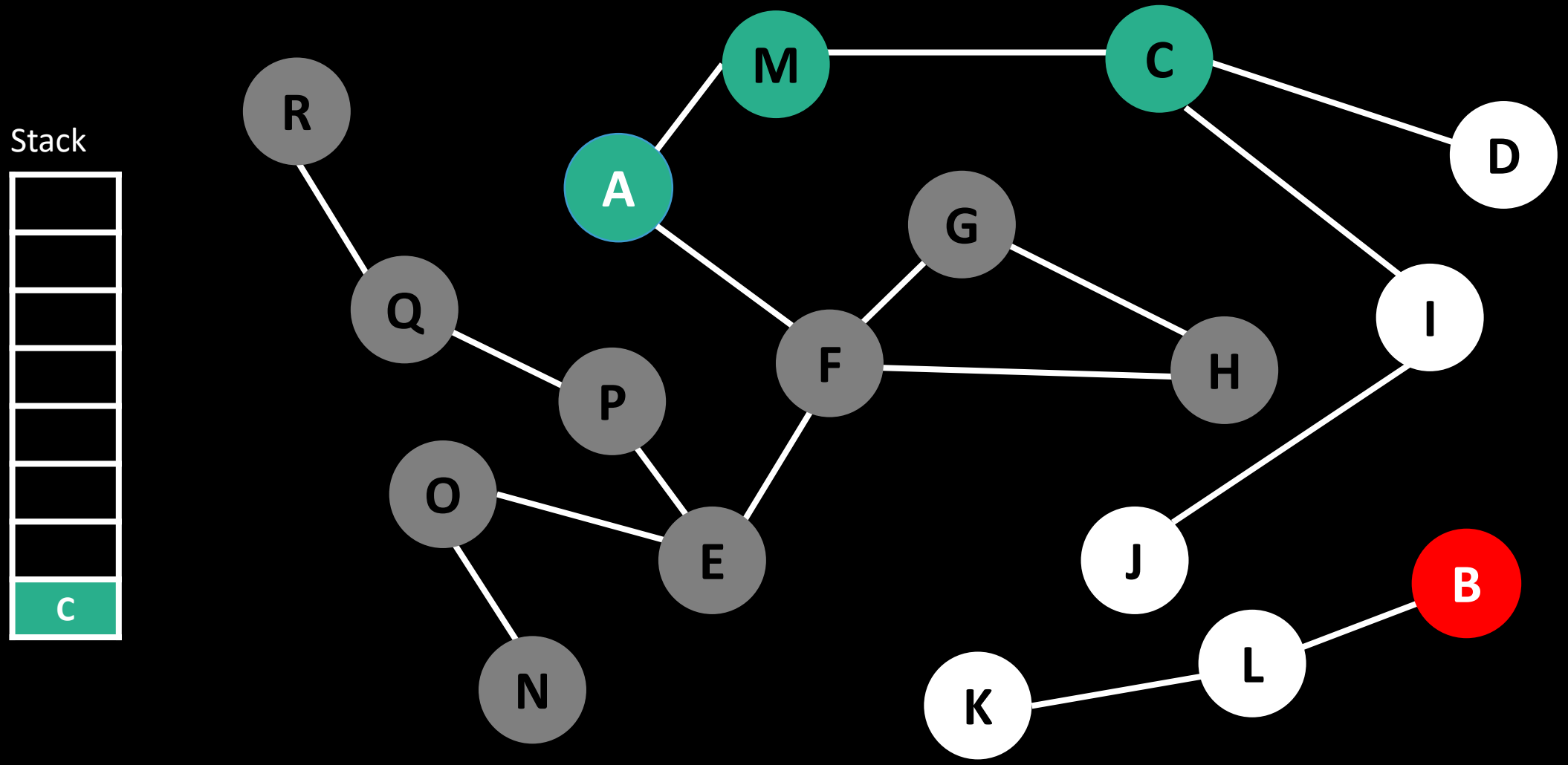

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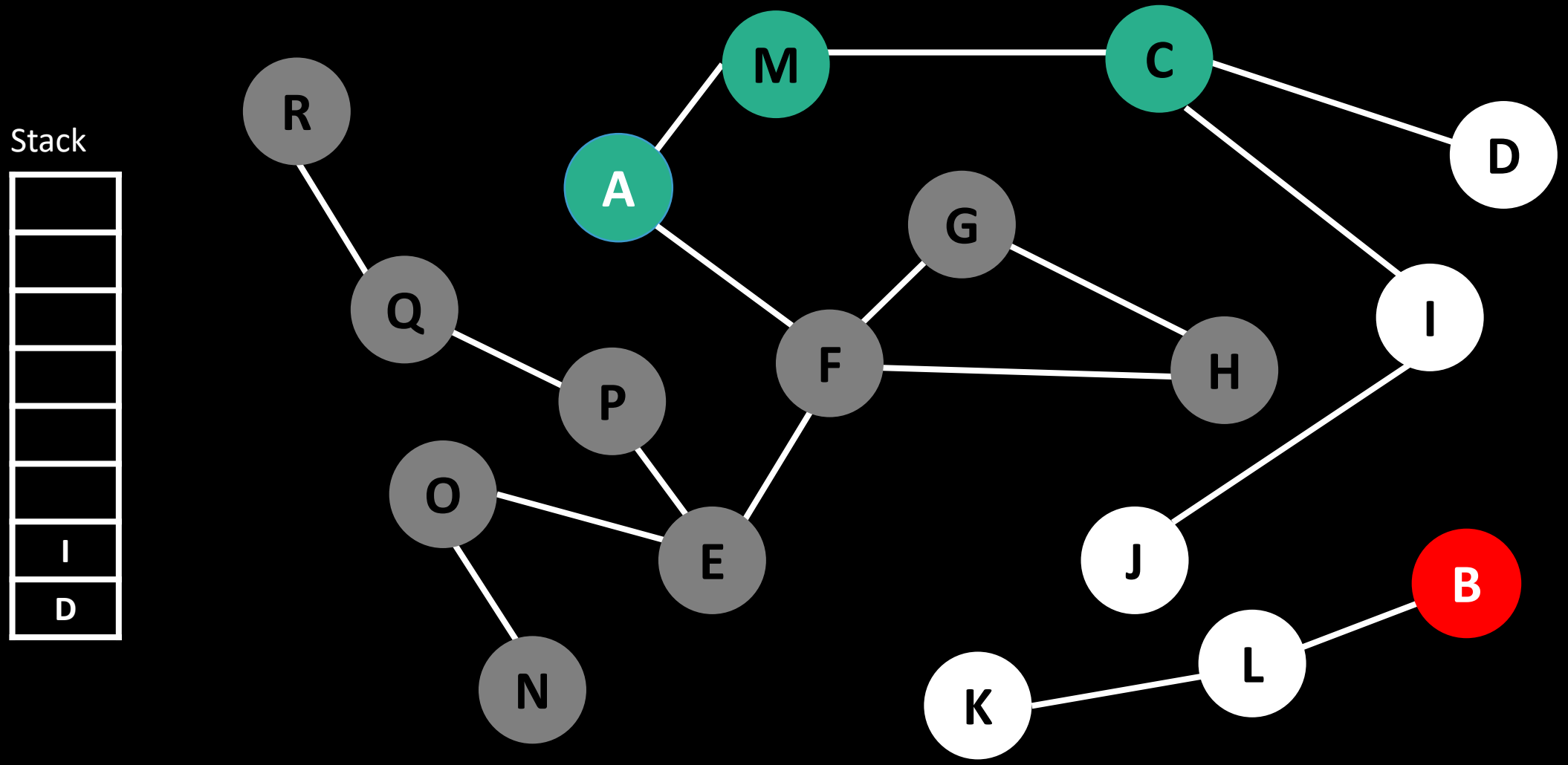

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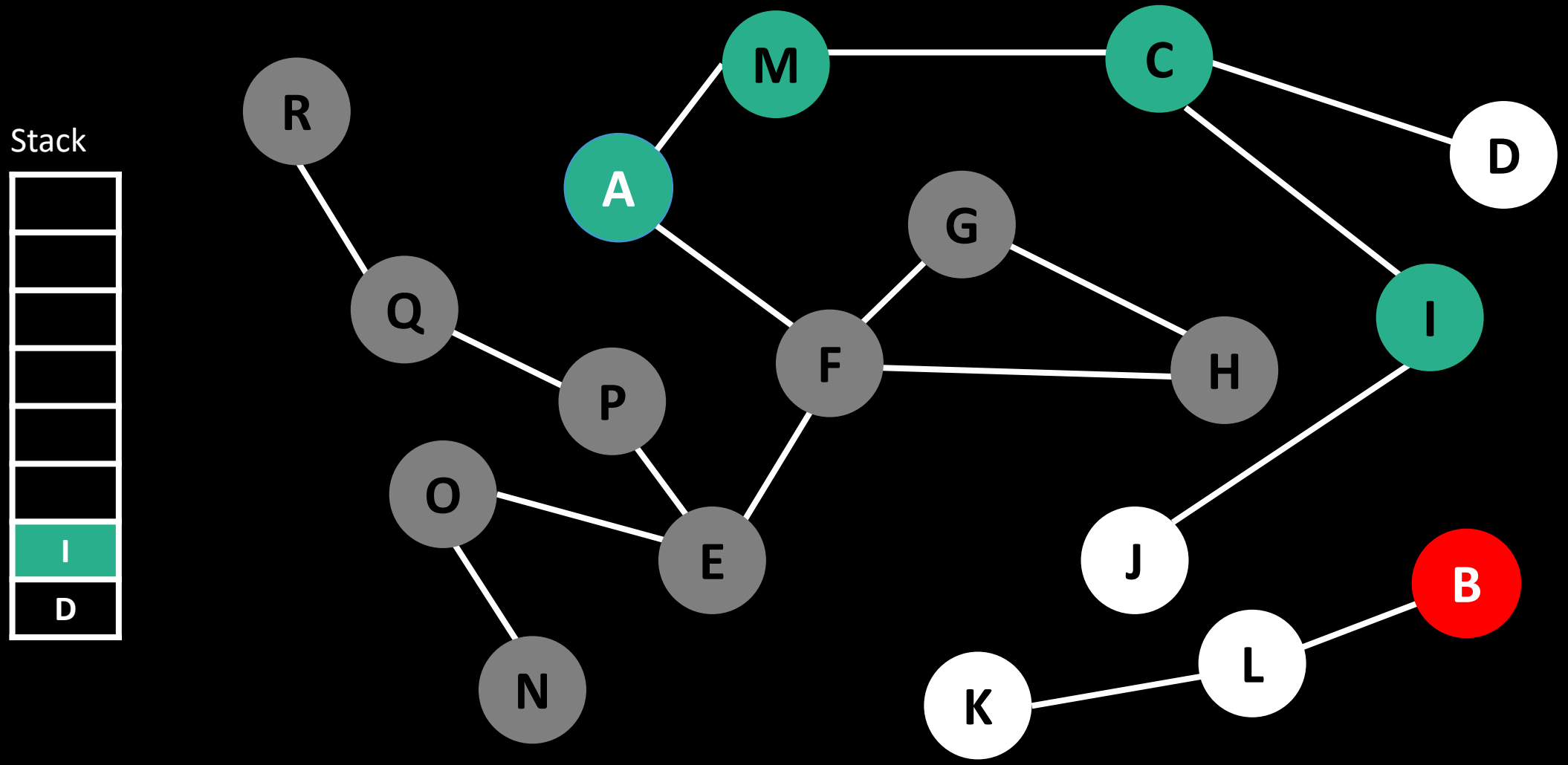

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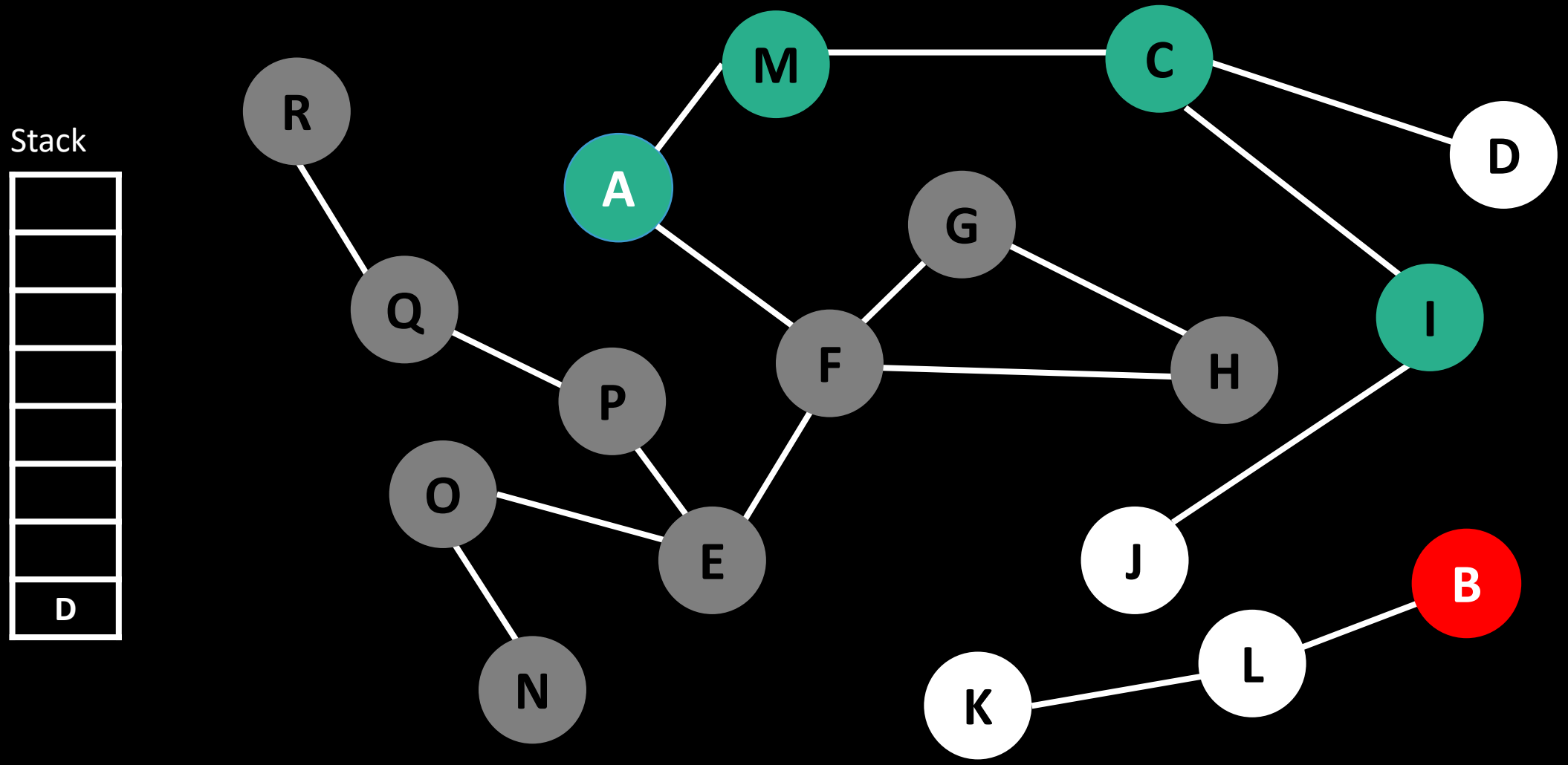
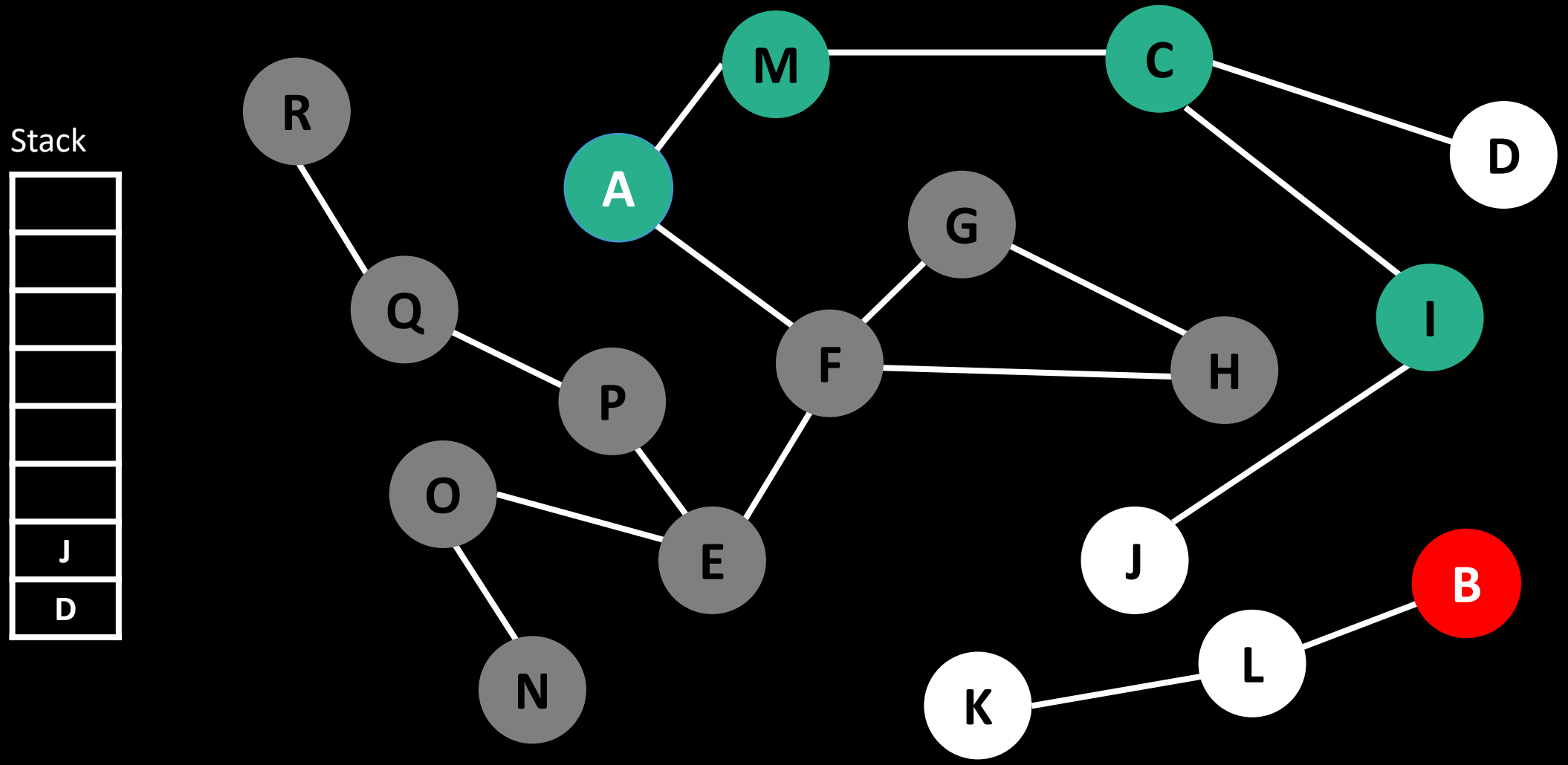


Another DFS Example



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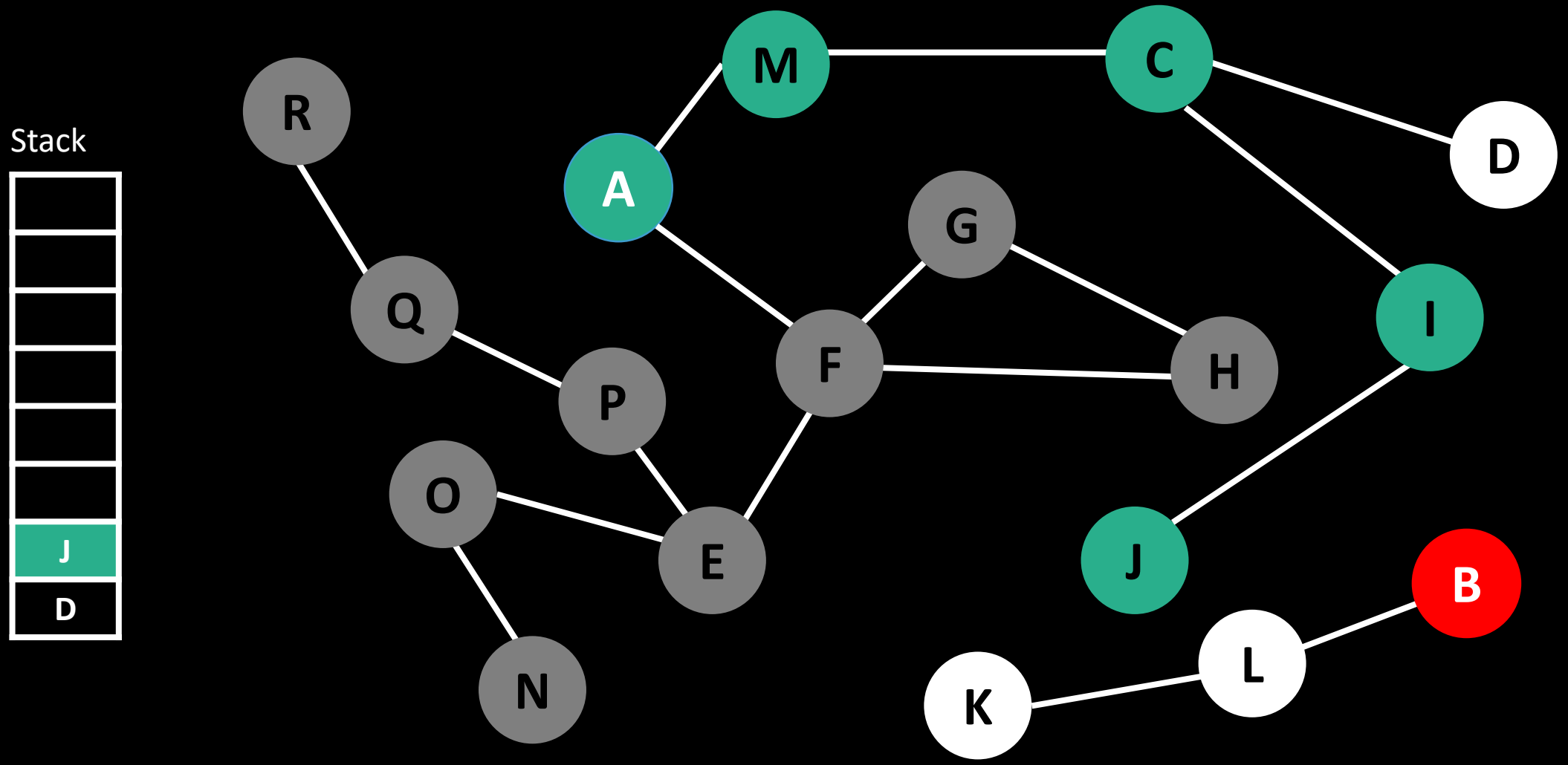

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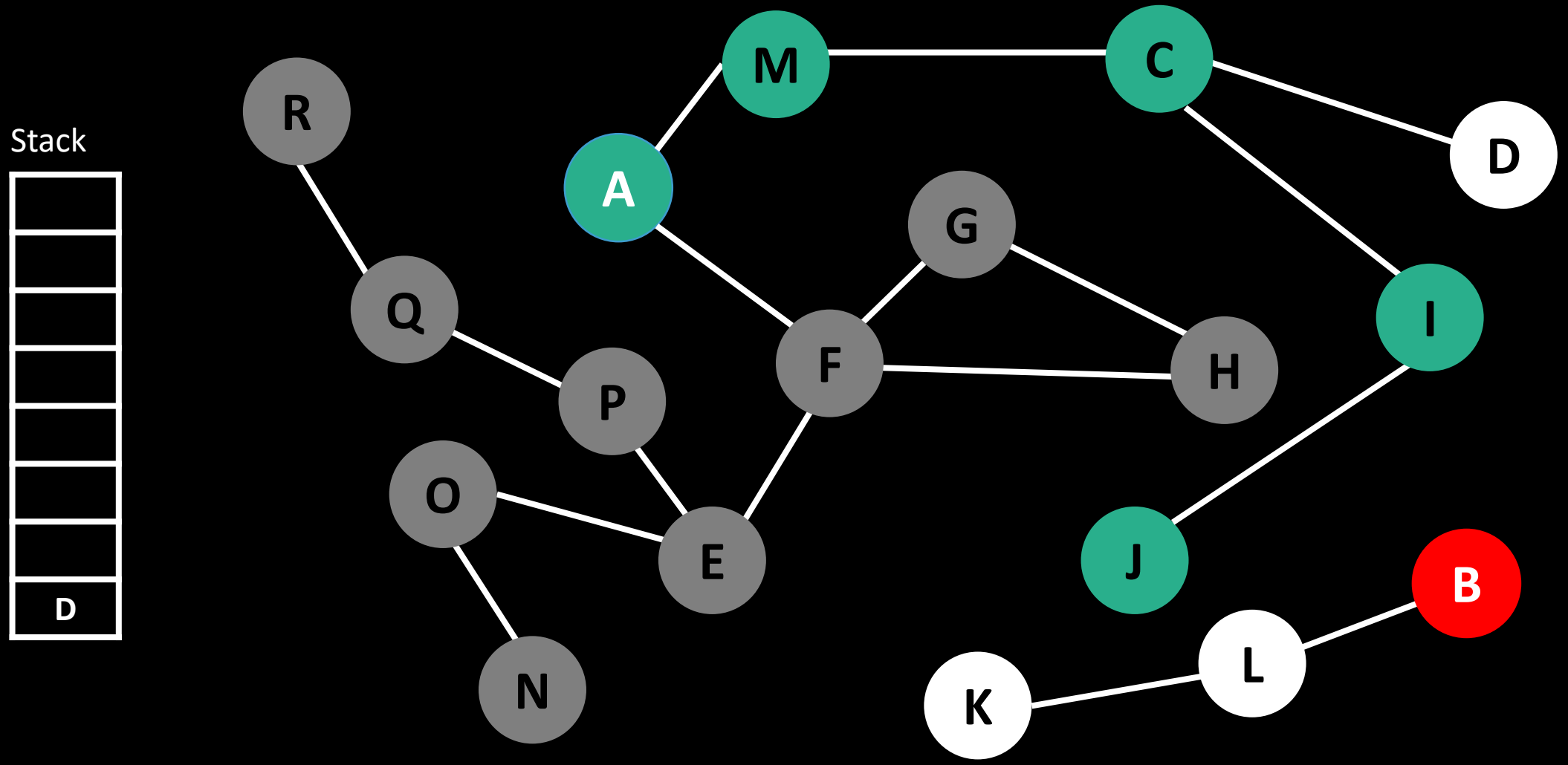
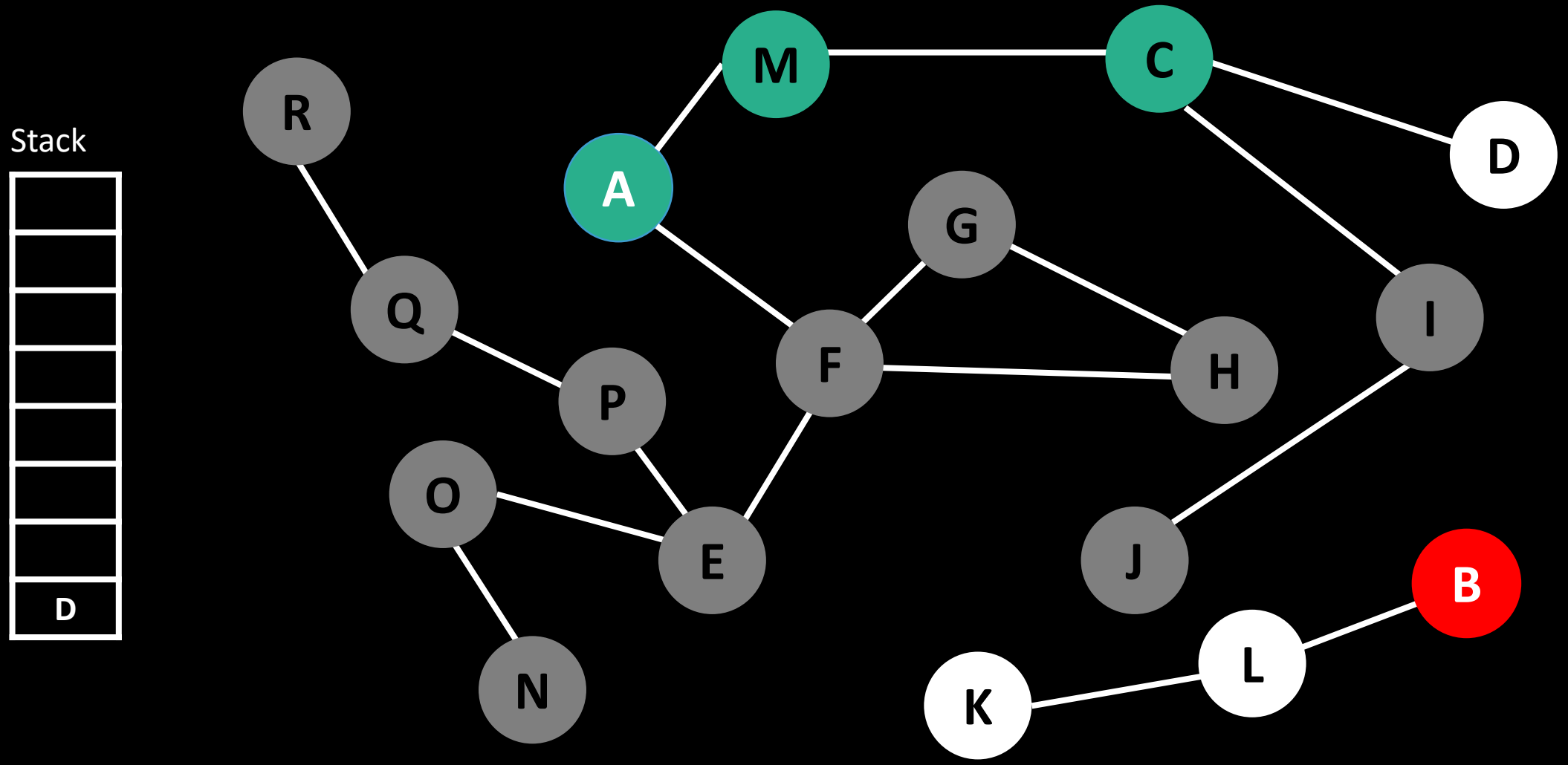


Another DFS Example



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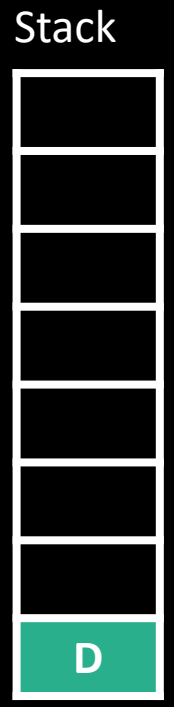

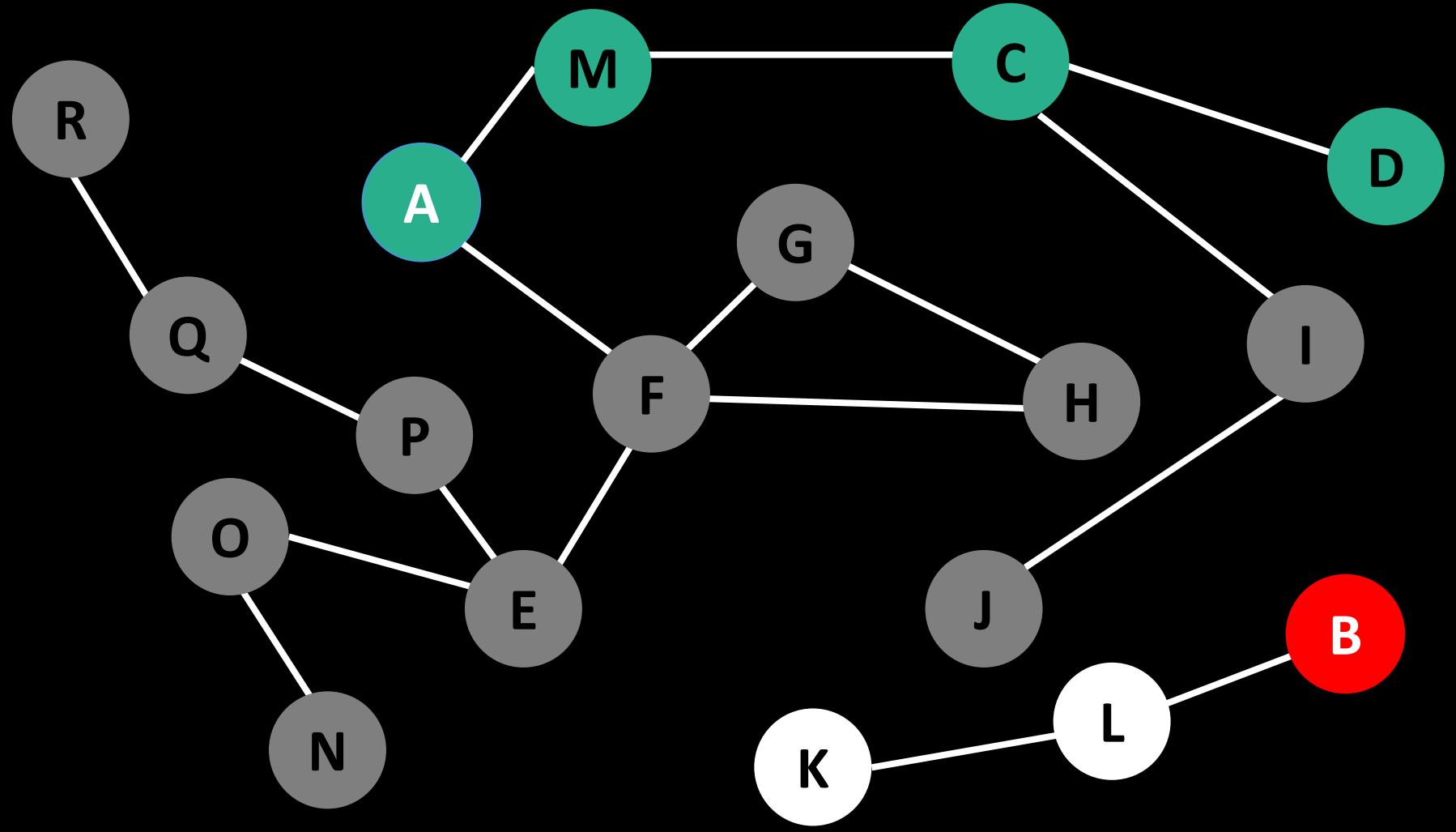

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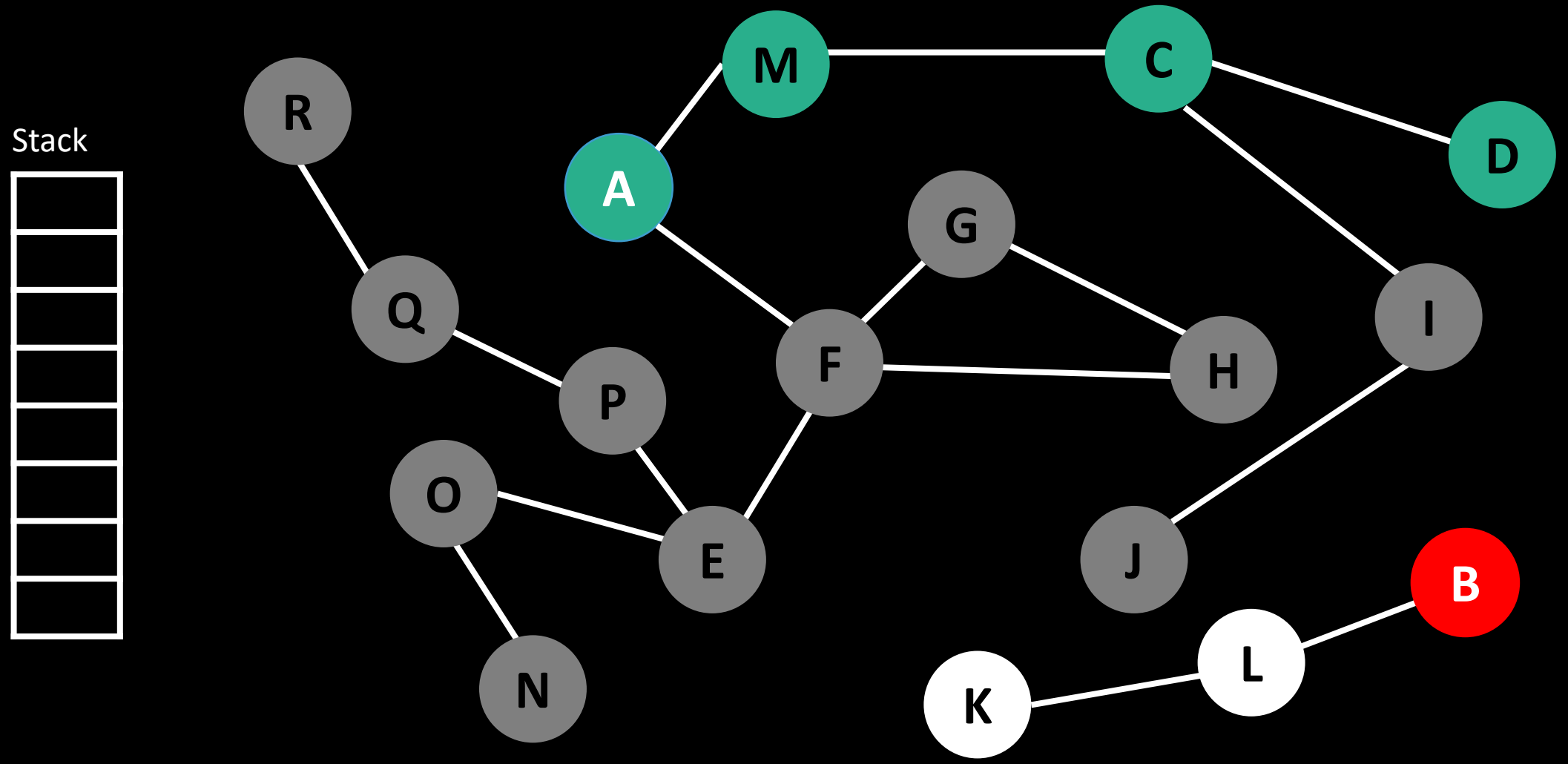
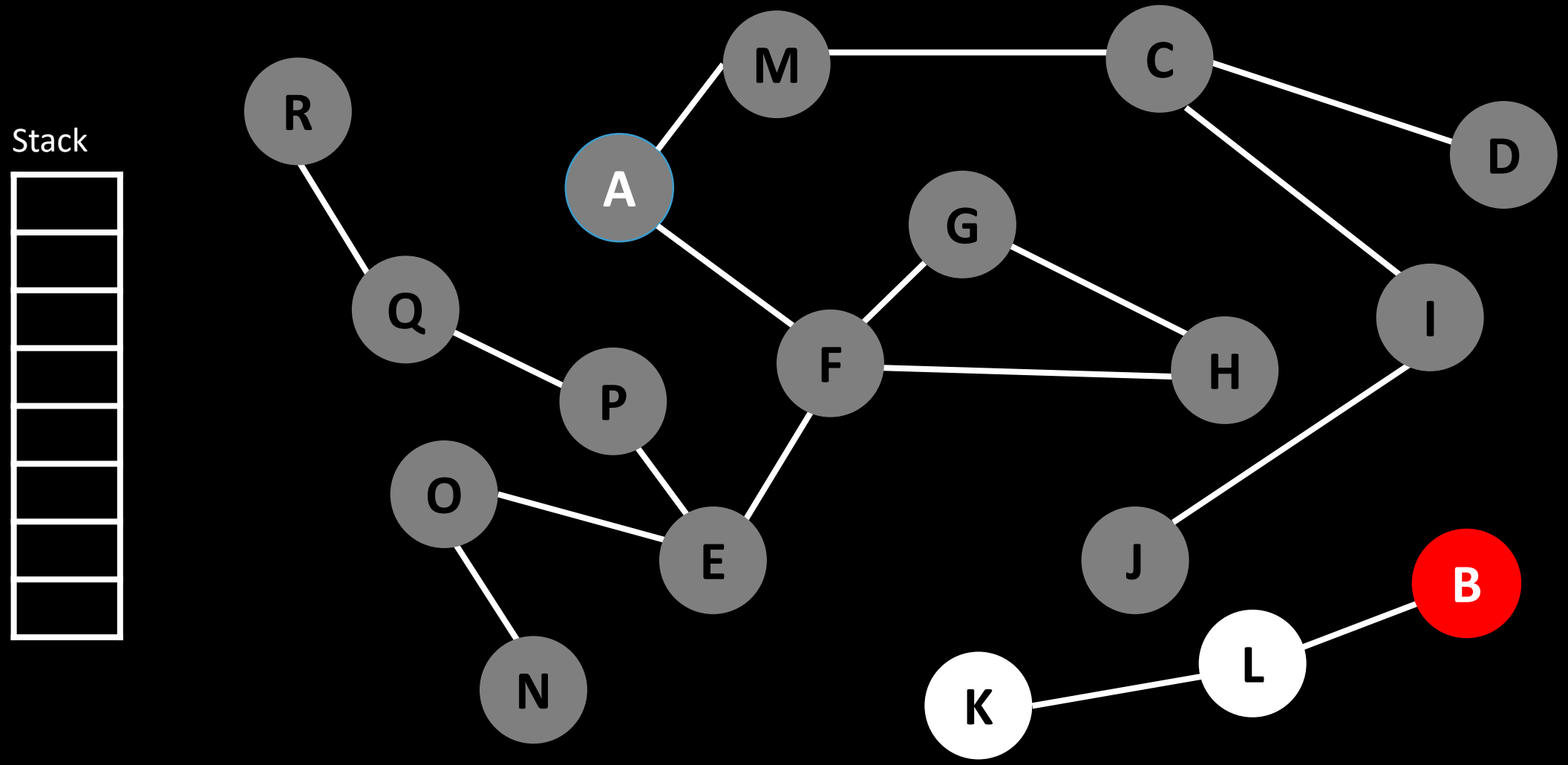
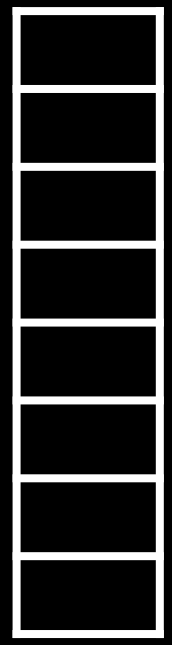


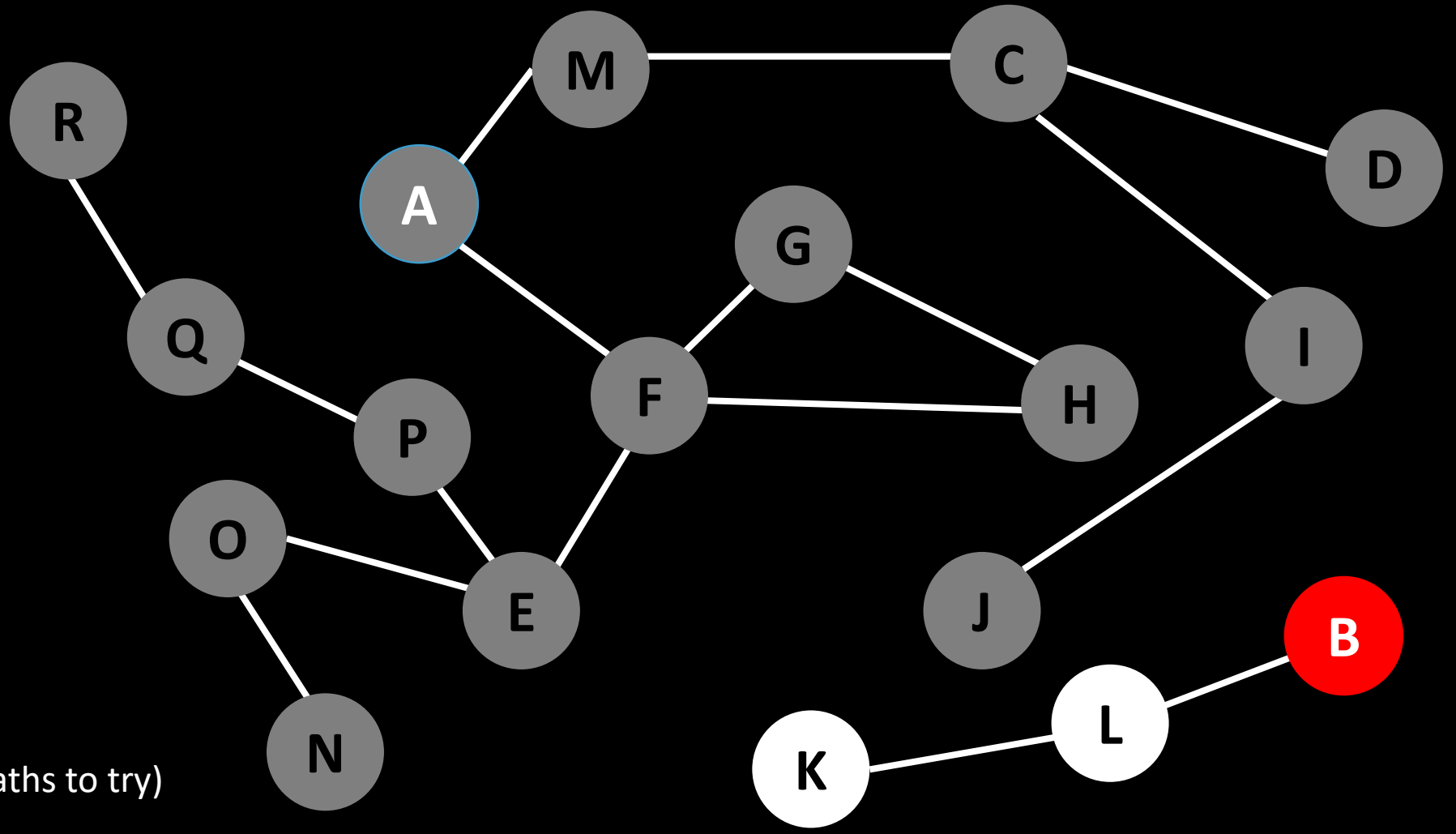
Another DFS Example



Another DFS Example

Stack





Stack is empty (i.e. no more paths to try)
We did not find B
Thus, no path exists from A → B

Depth-First Search PseudoCode

```
stack = []
stack.push(starting_node)

while (stack not empty) and (target not found):
        node = stack.pop()
        if node is visited:
                skip
        if node = target:
                target = found
        for all unvisited neighbors of node:
                stack.push(neighbor)
        node.visited = True
```

Depth-First Search PseudoCode

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while (stack not empty) and (target not found):
        node = stack.pop()
        if node is visited:
                skip
        if node = target:
                target = found
        for all unvisited neighbors of node:
                neighbor.prev = node
                stack.push(neighbor)
        node.visited = True
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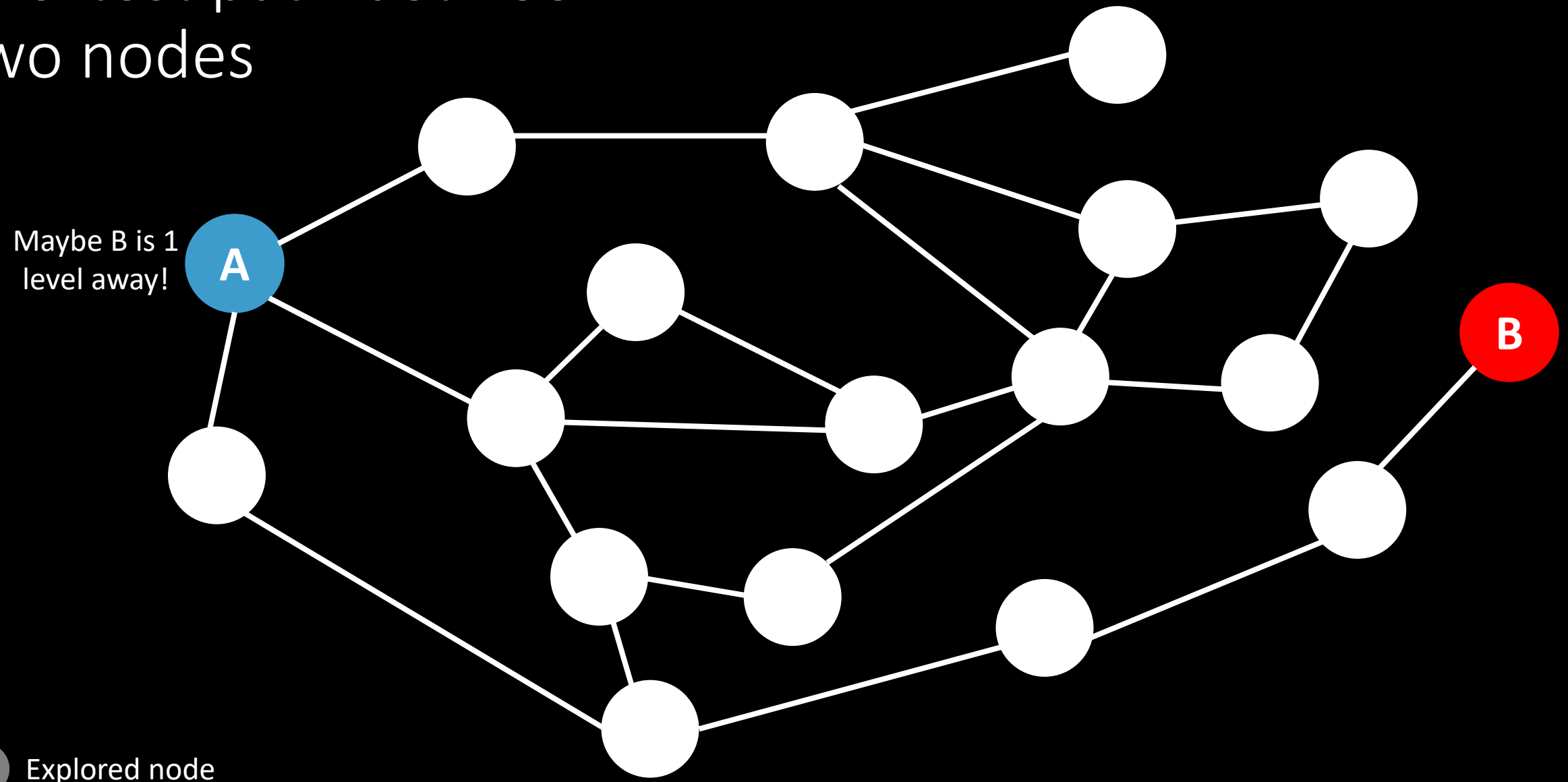
                target = found
        for all unvisited neighbors of node:
                neighbor.prev = node
                stack.push(neighbor)
        node.visited = True
```

Retrieving the path, if one is found

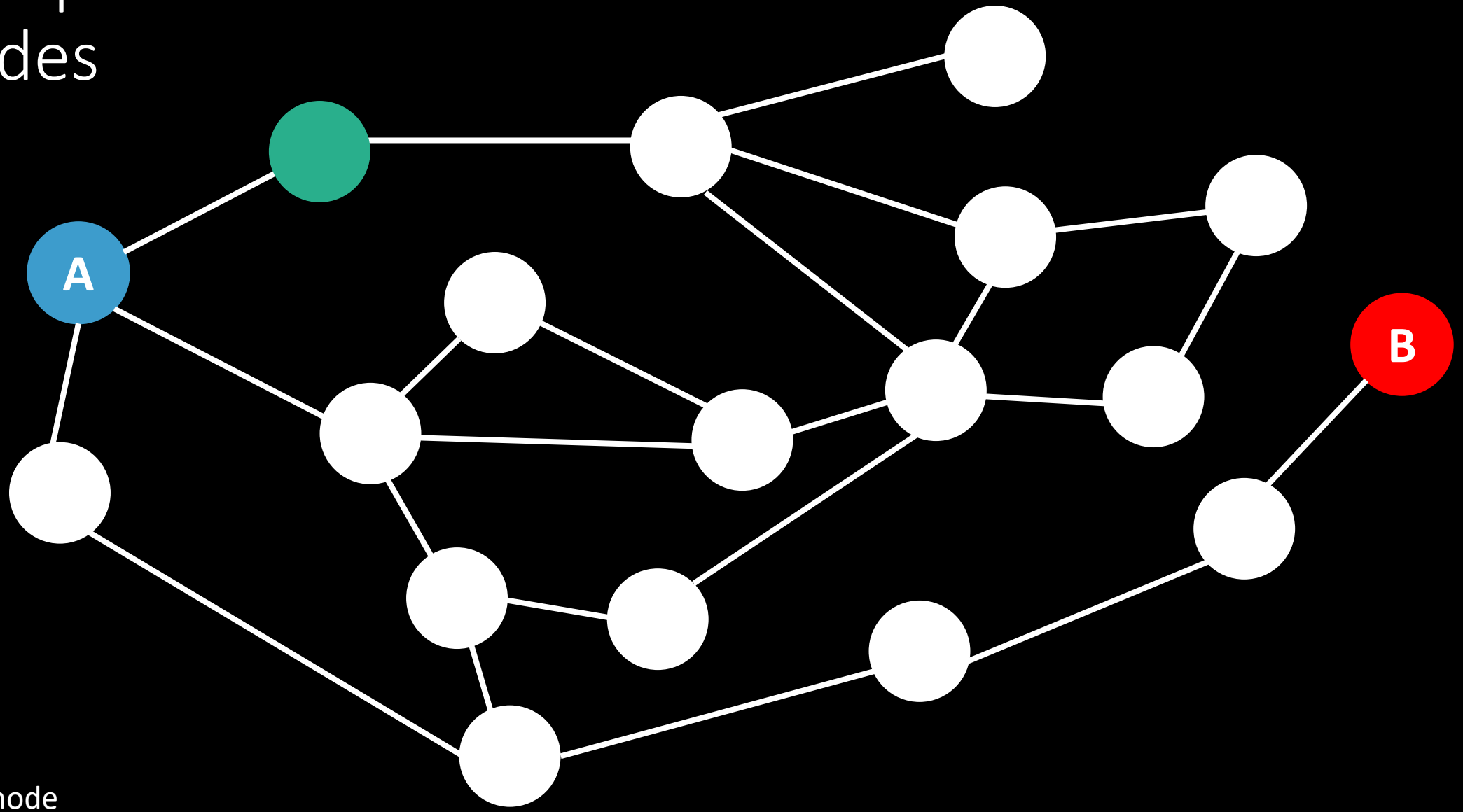
```
node = target
path = [node]
while node is not starting_node:
        path.push(node.prev)
        node = node.prev
```

Shortest path between two nodes



MO: Finish exploring current level before deepening the search

Shortest path between two nodes



Explored node

Path taken

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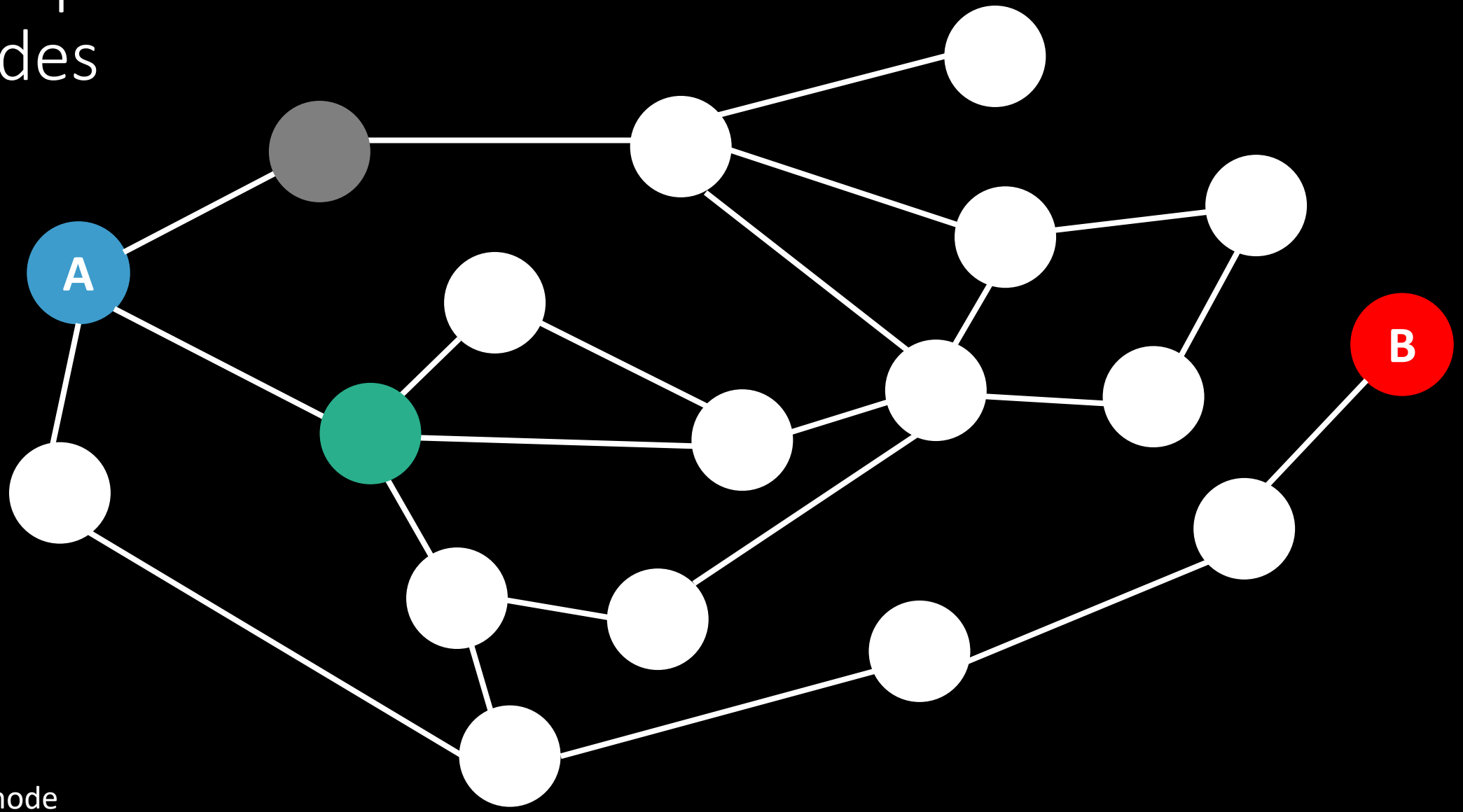
Shortest path between two nodes



Explored node

Path taken

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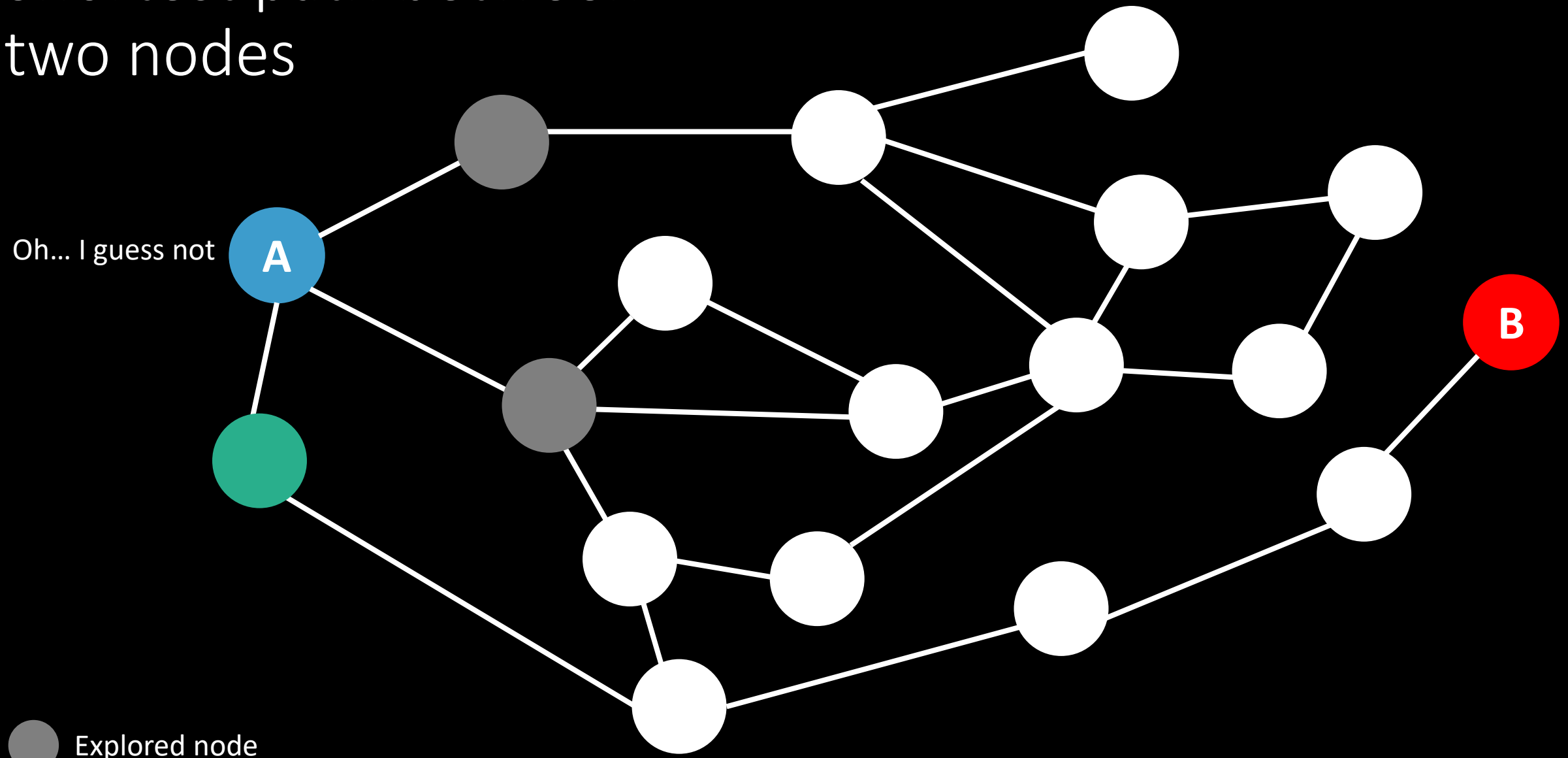

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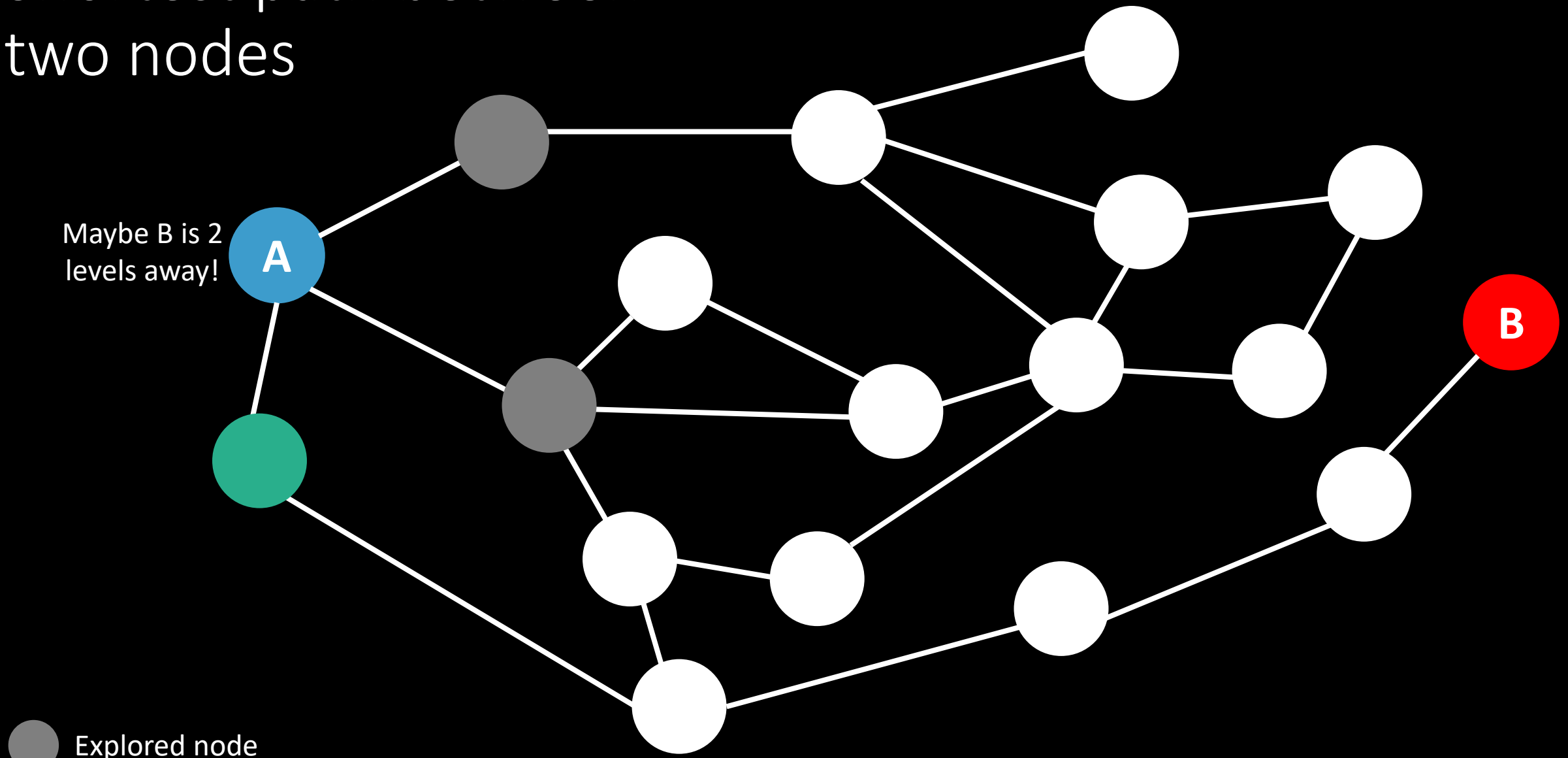

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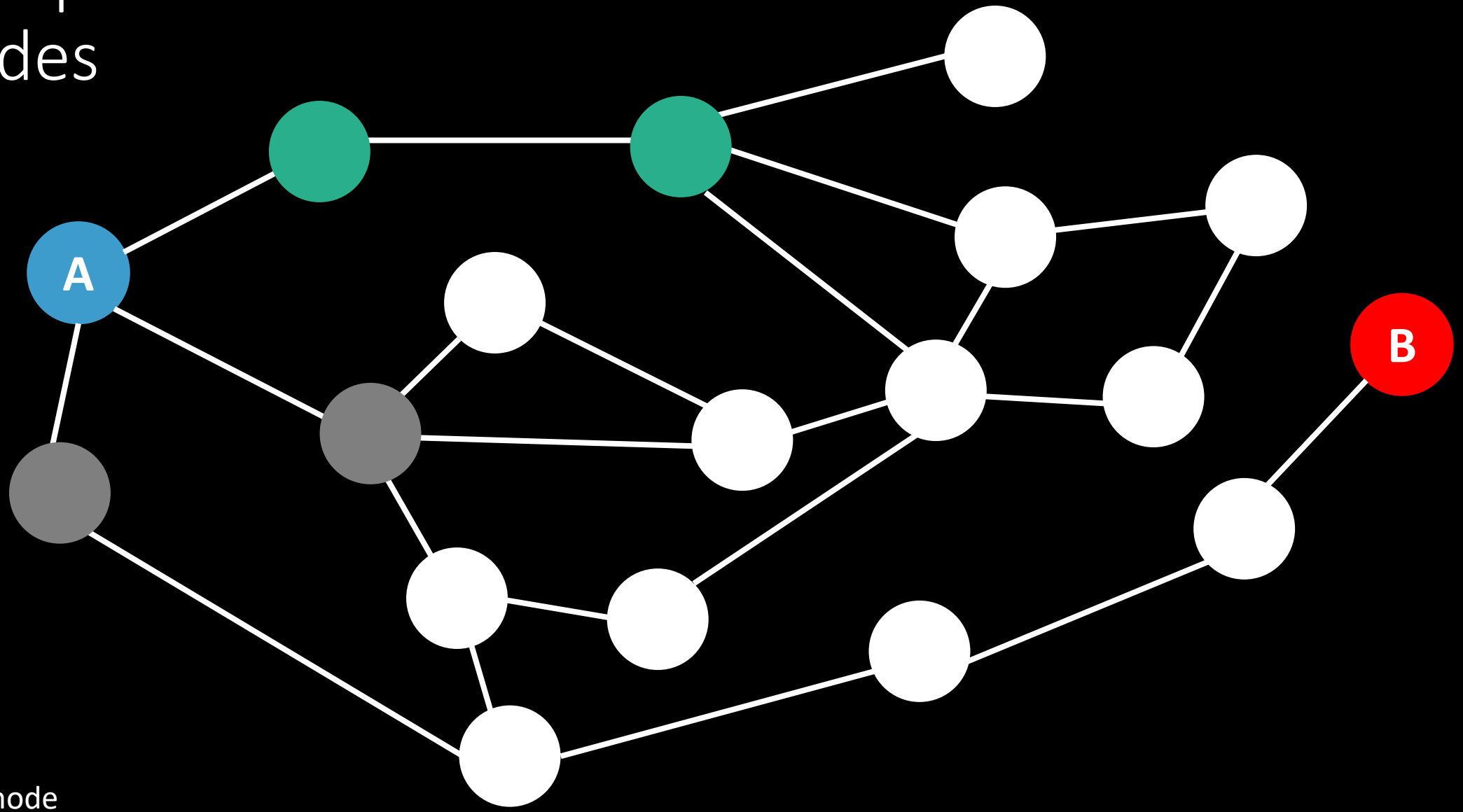

Explored node

Path taken

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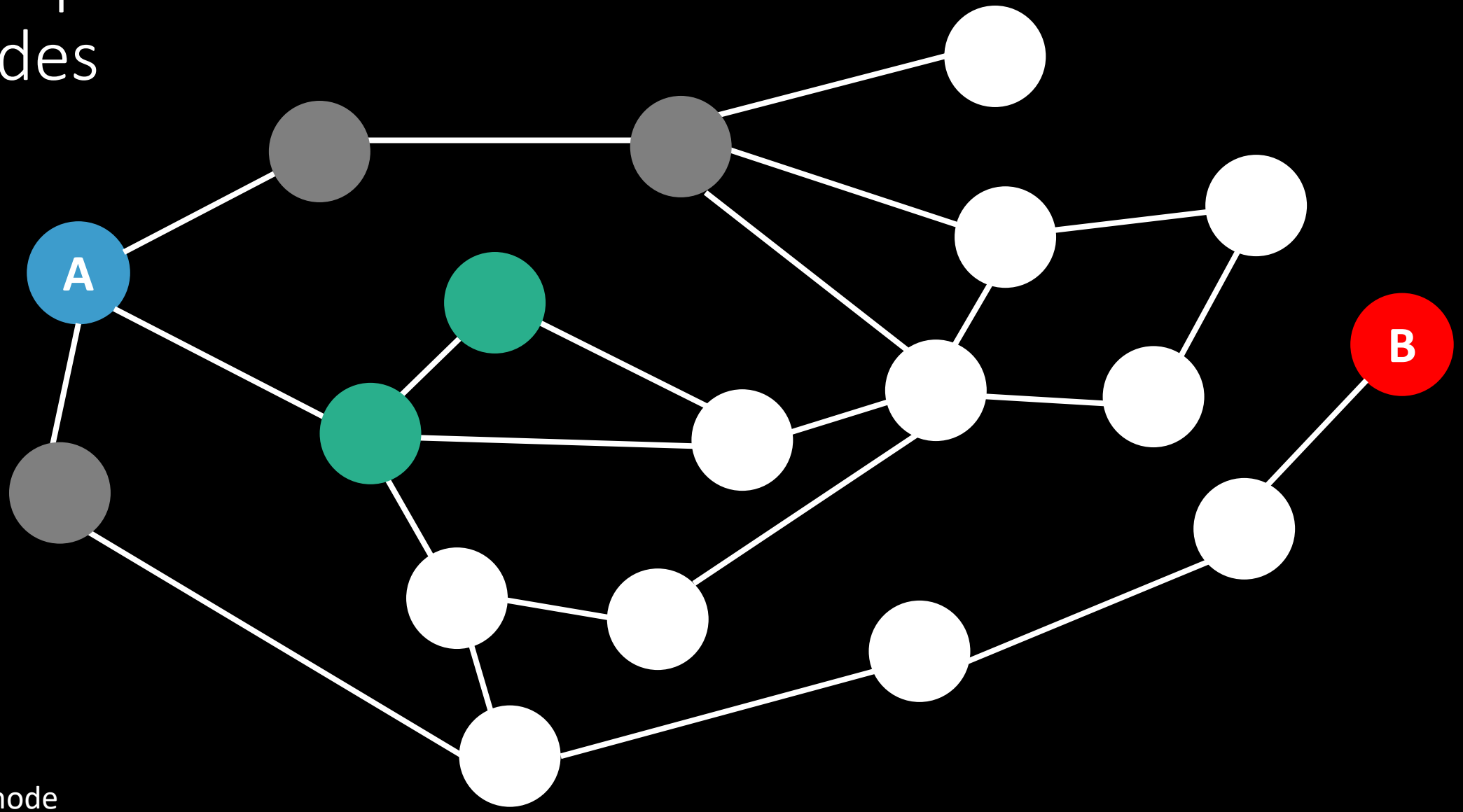

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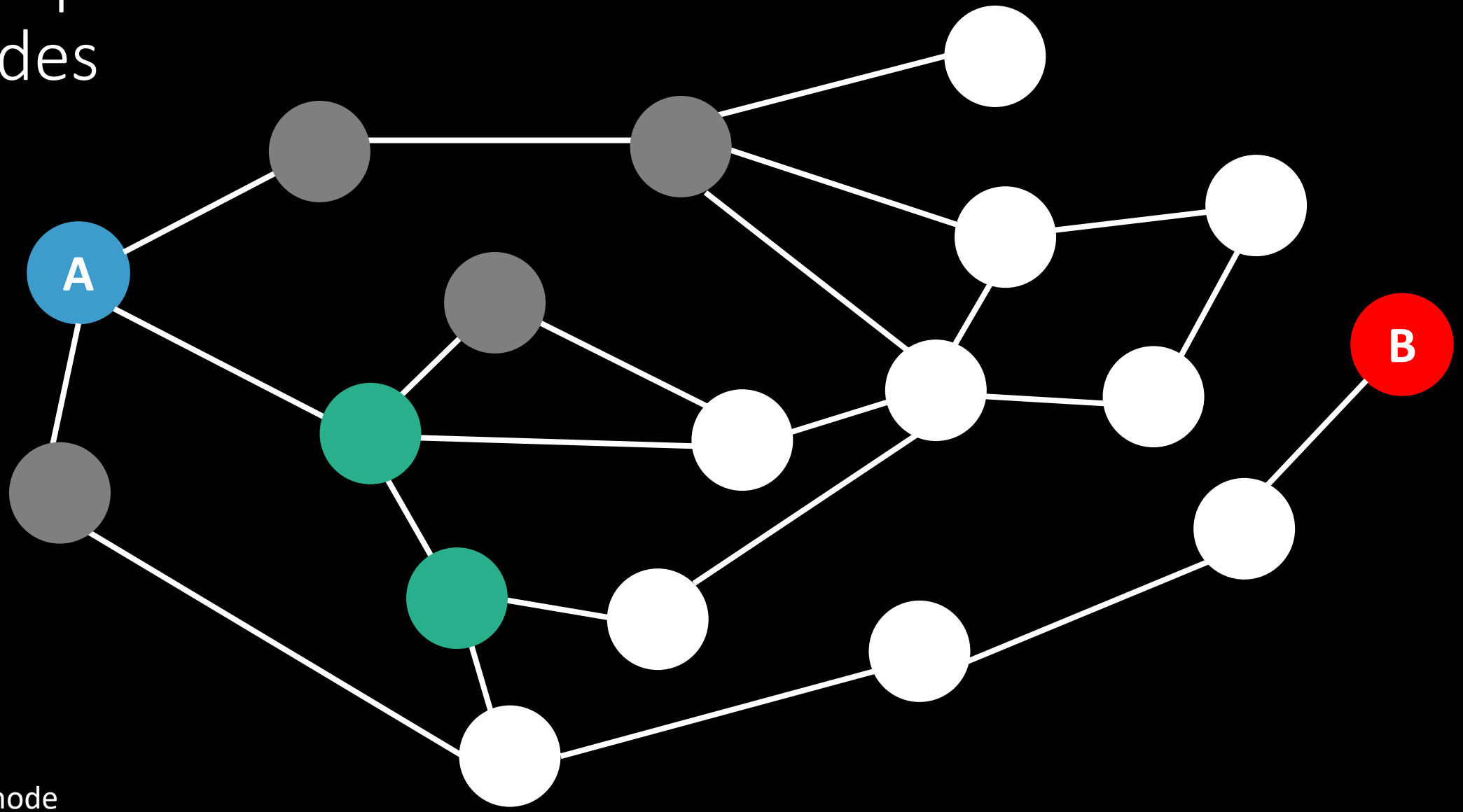

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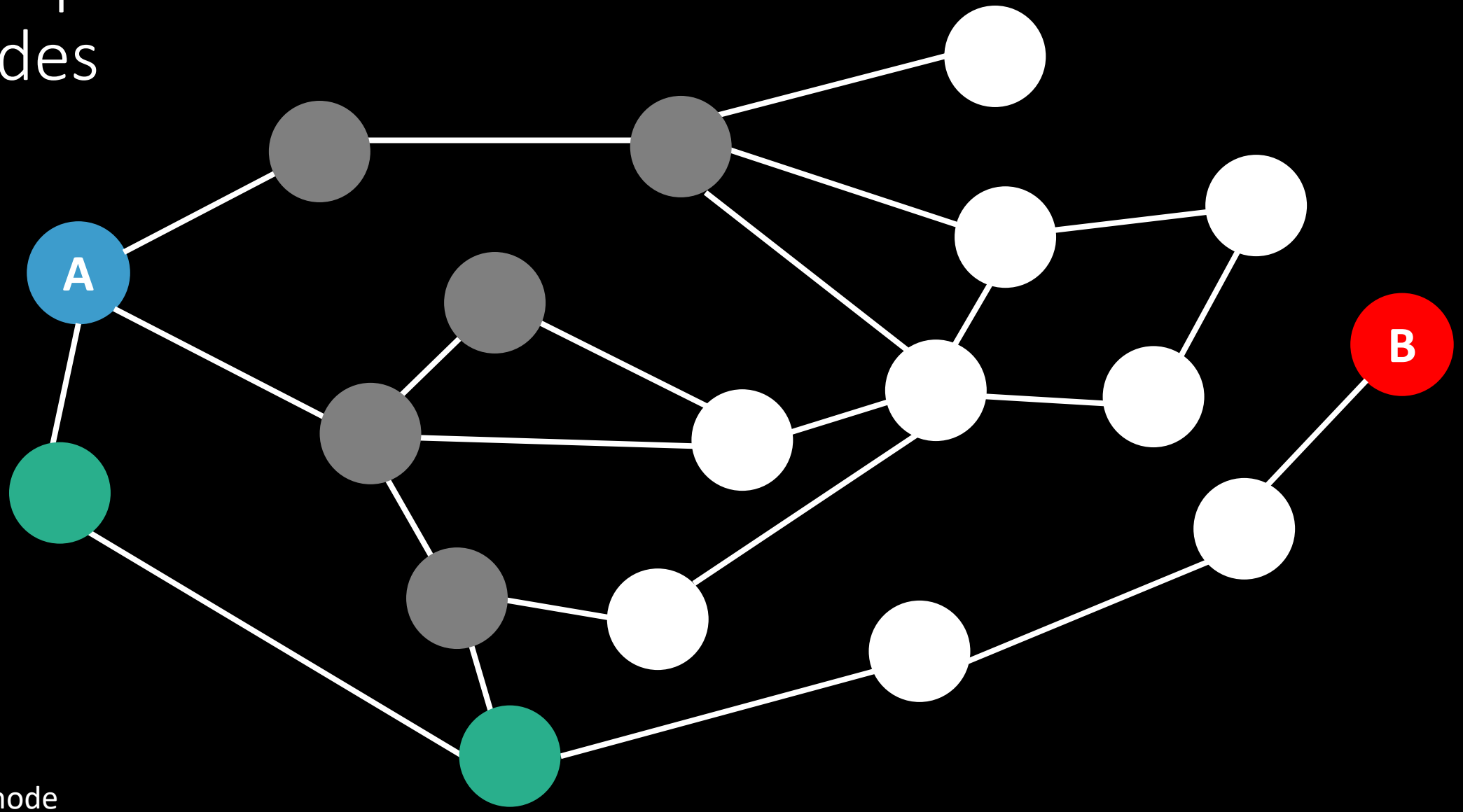

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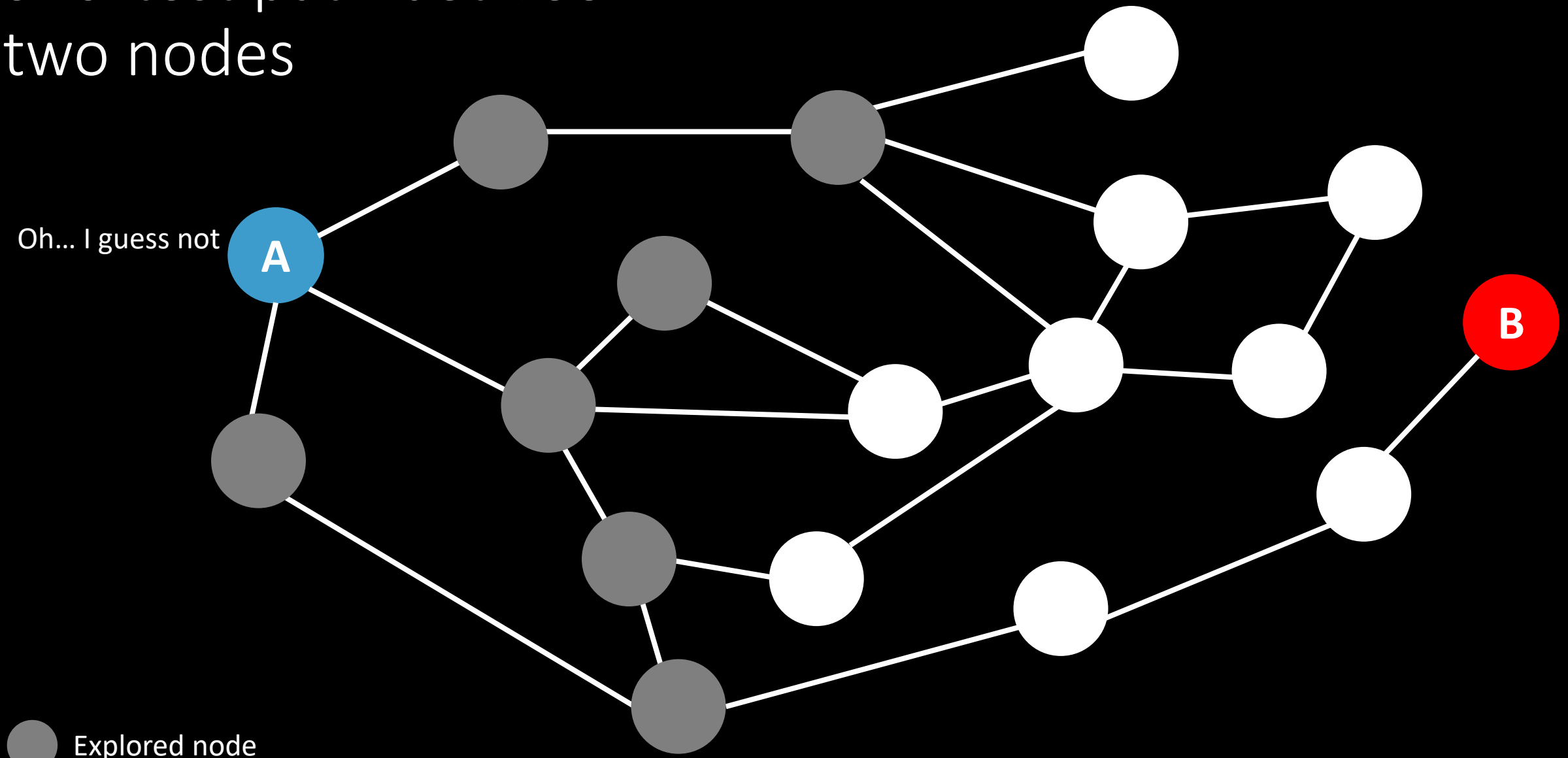

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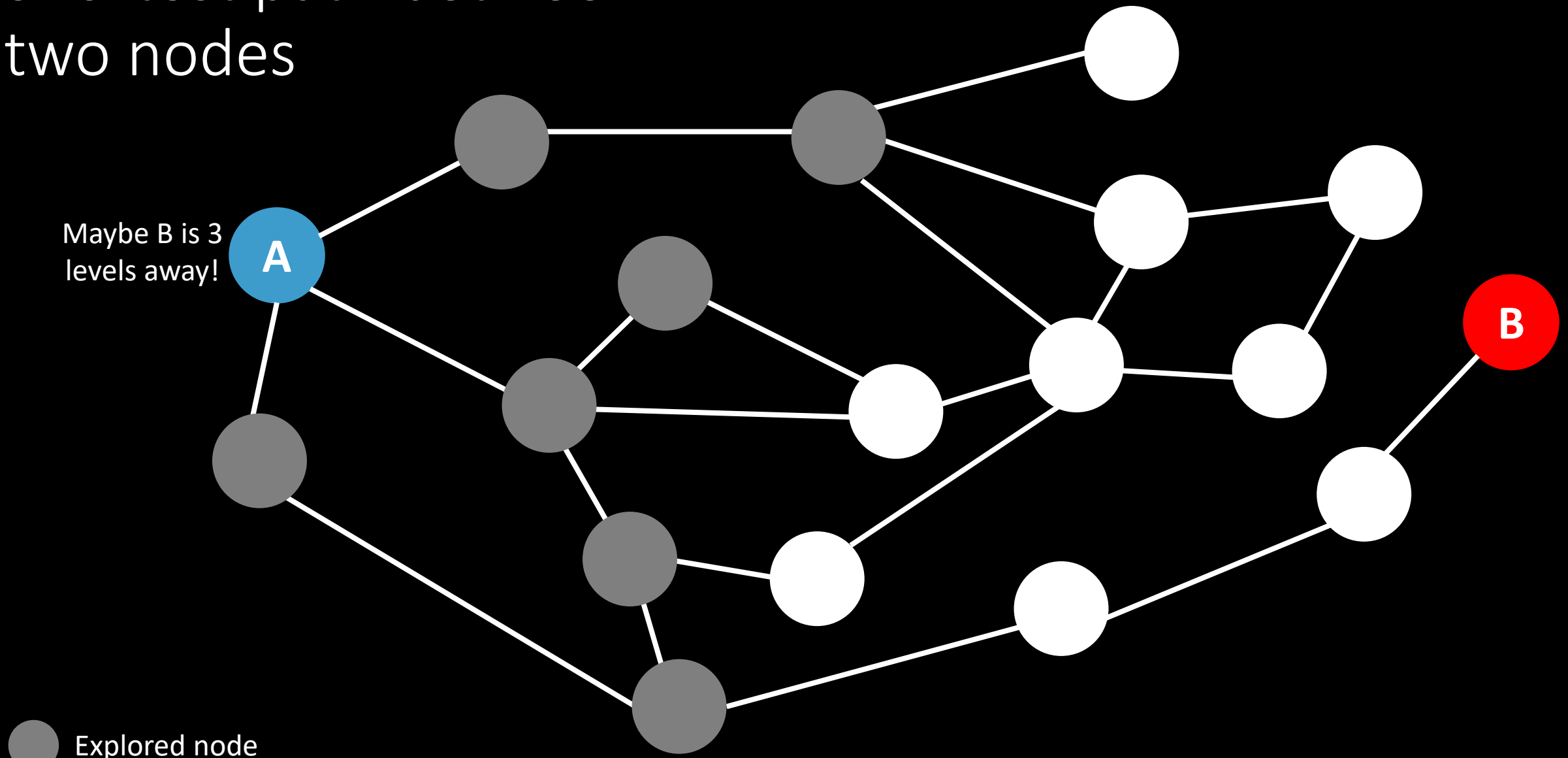

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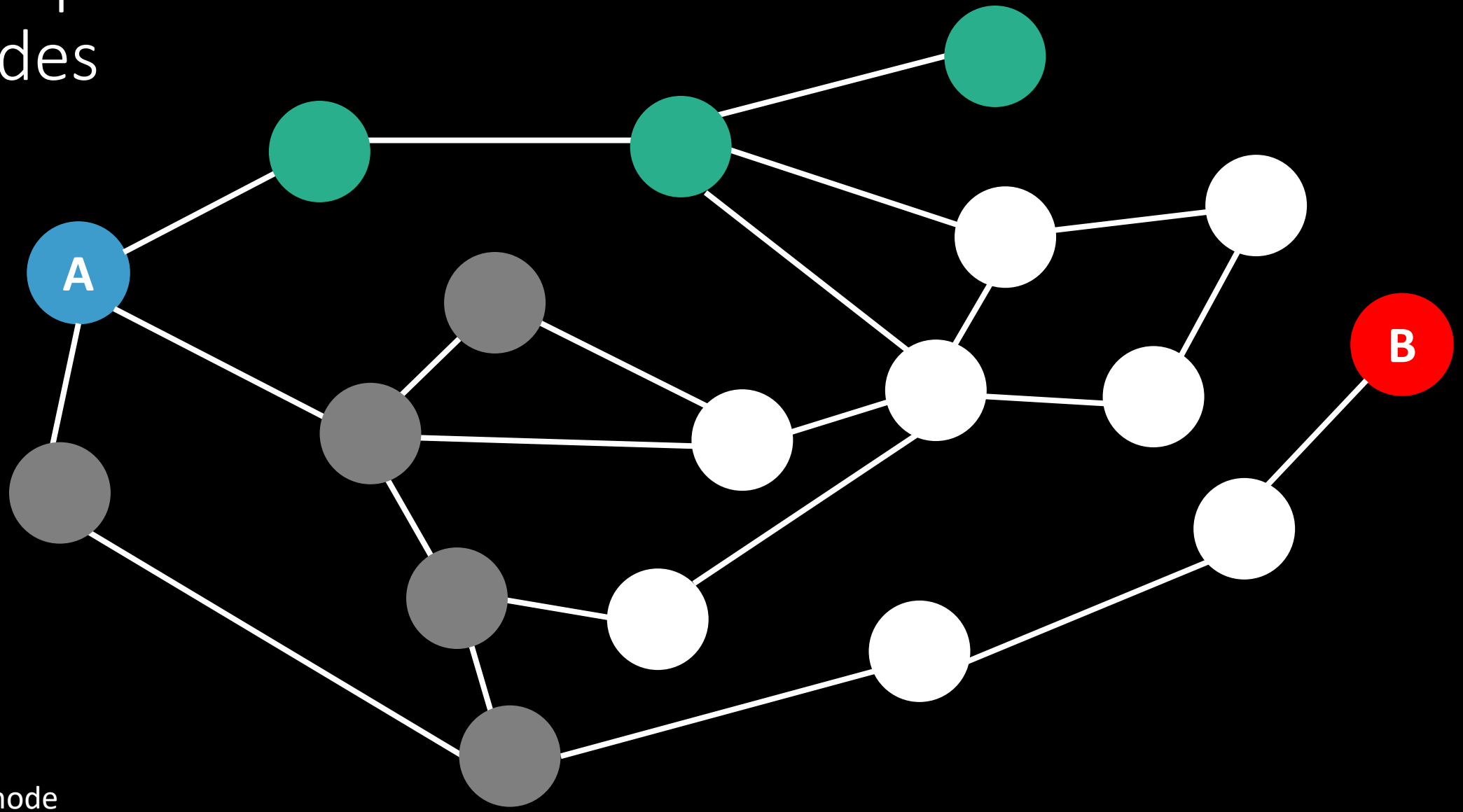

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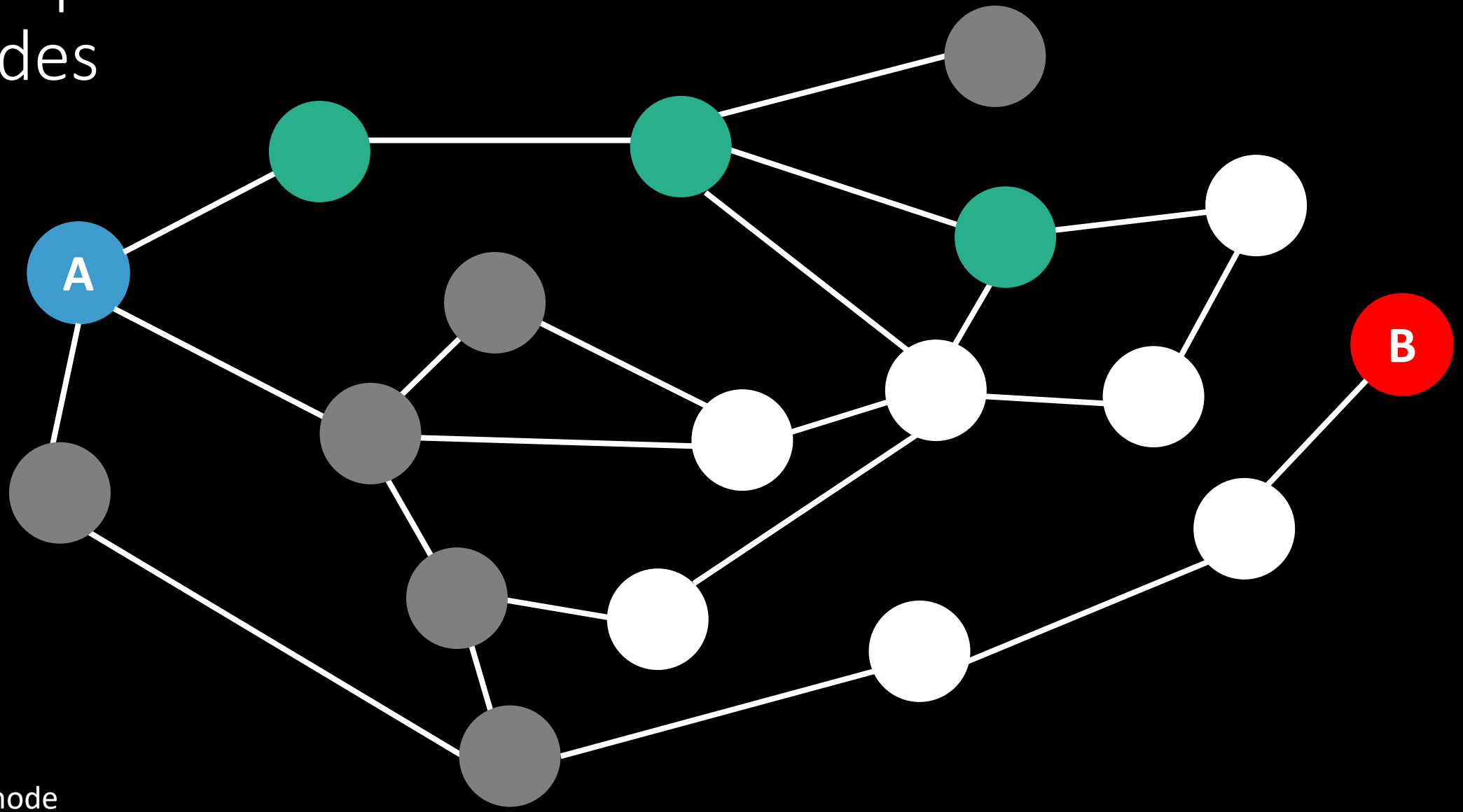

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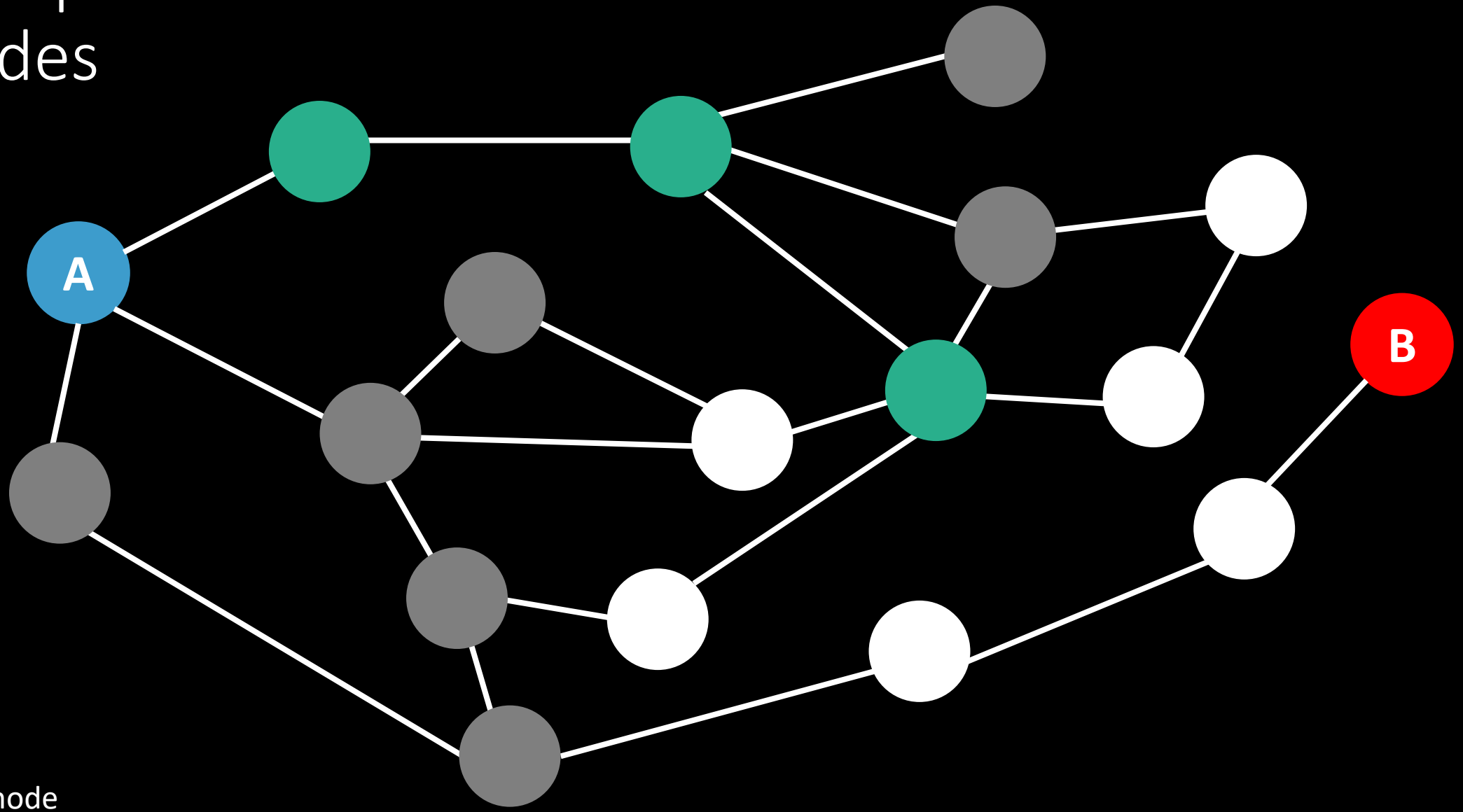

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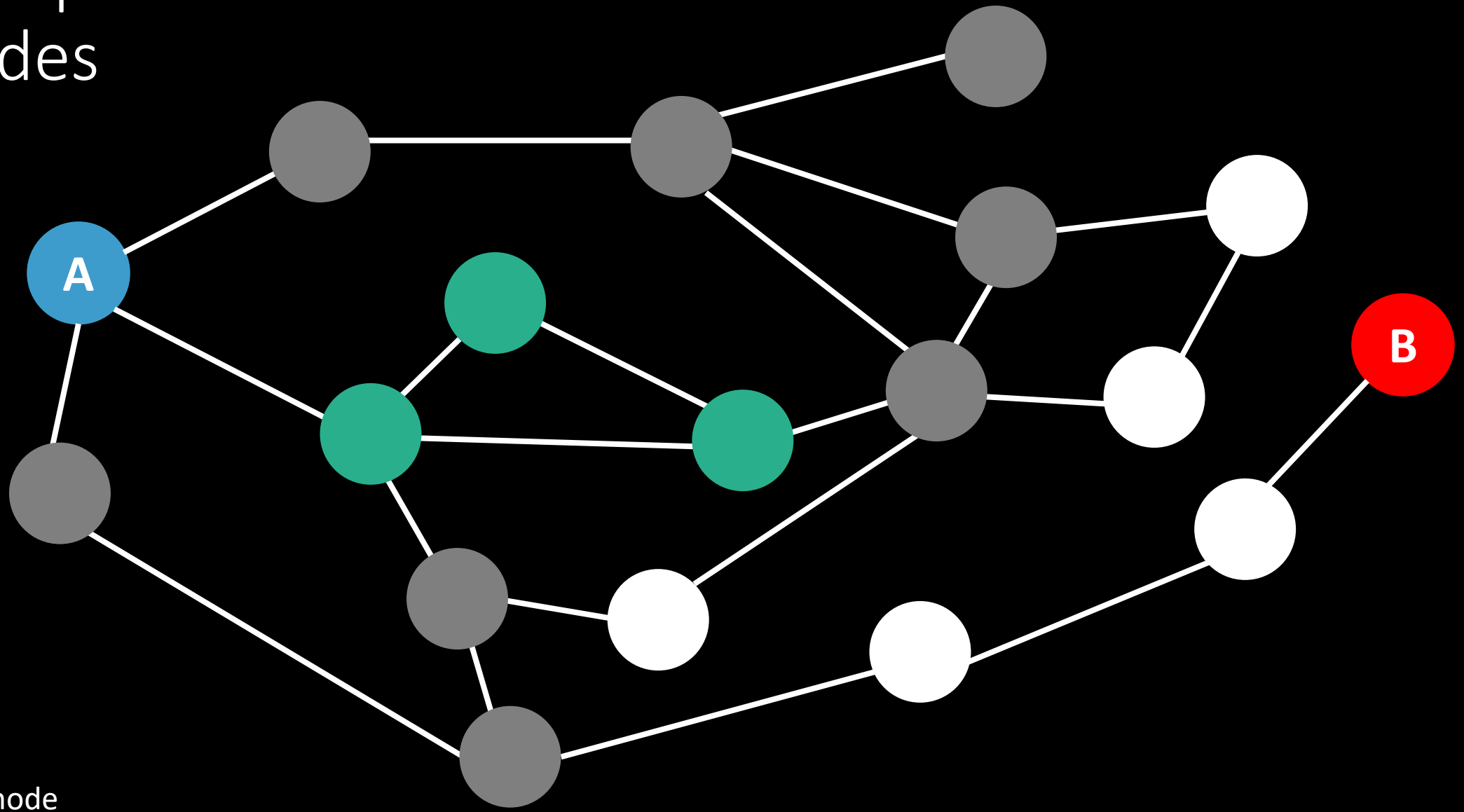

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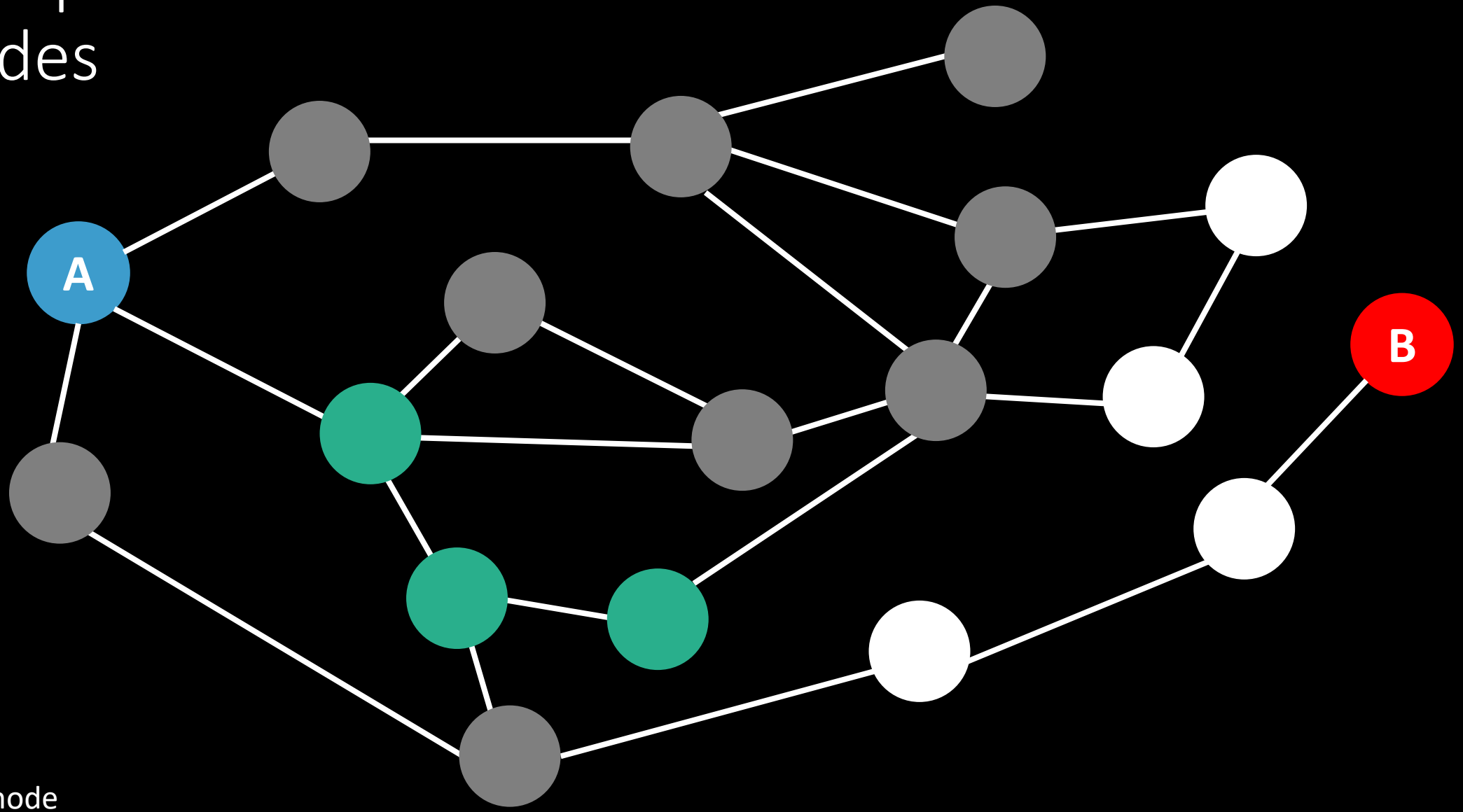

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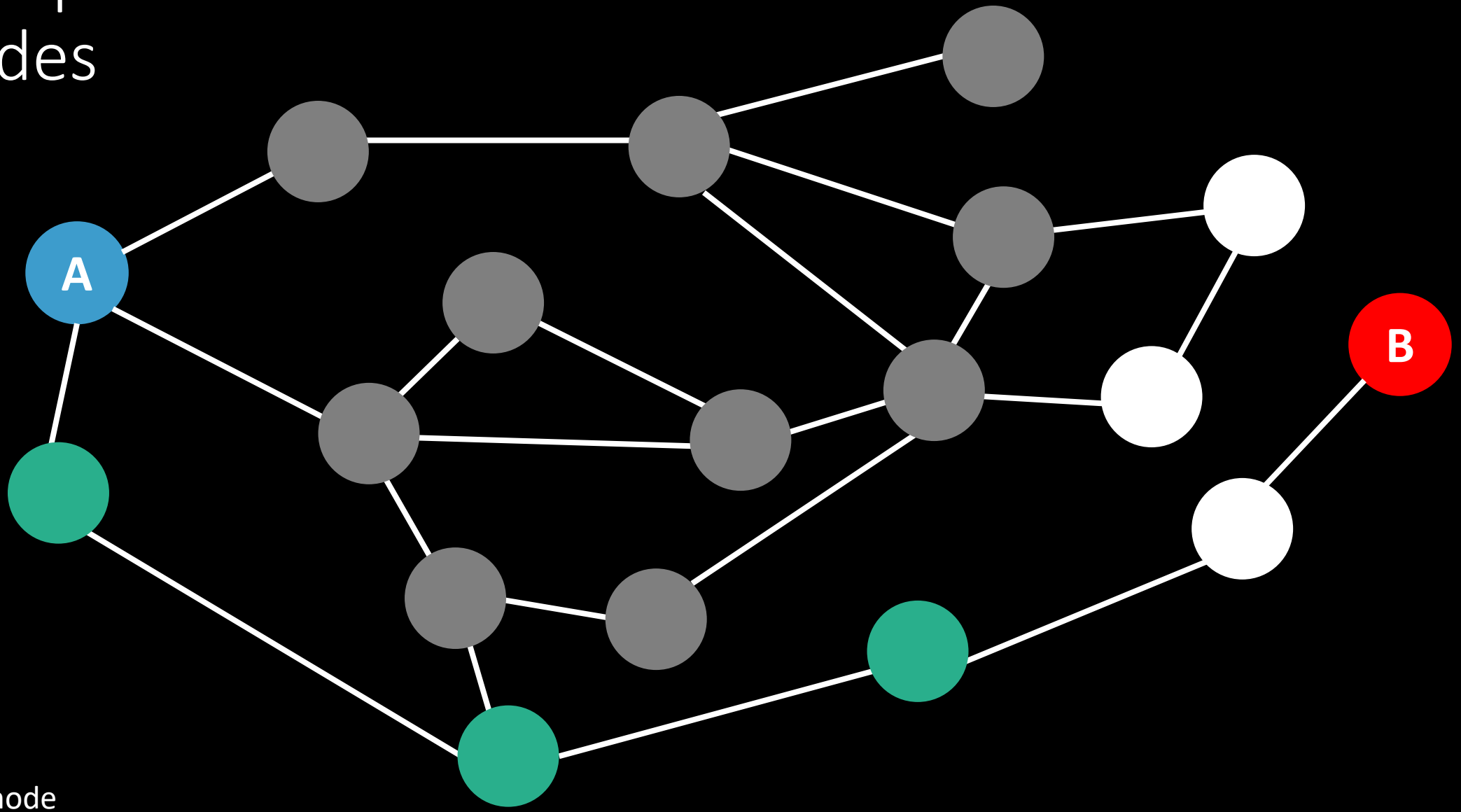

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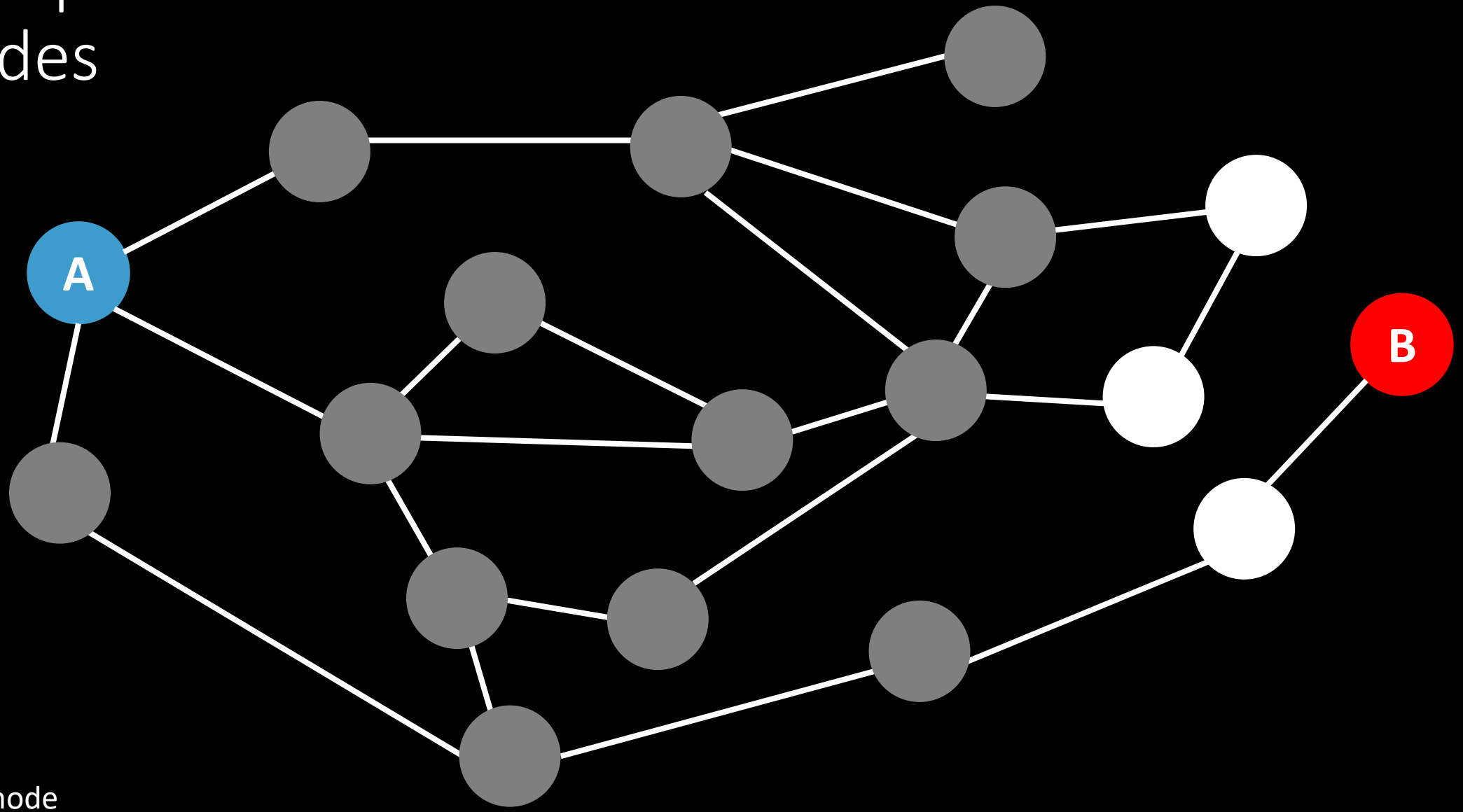

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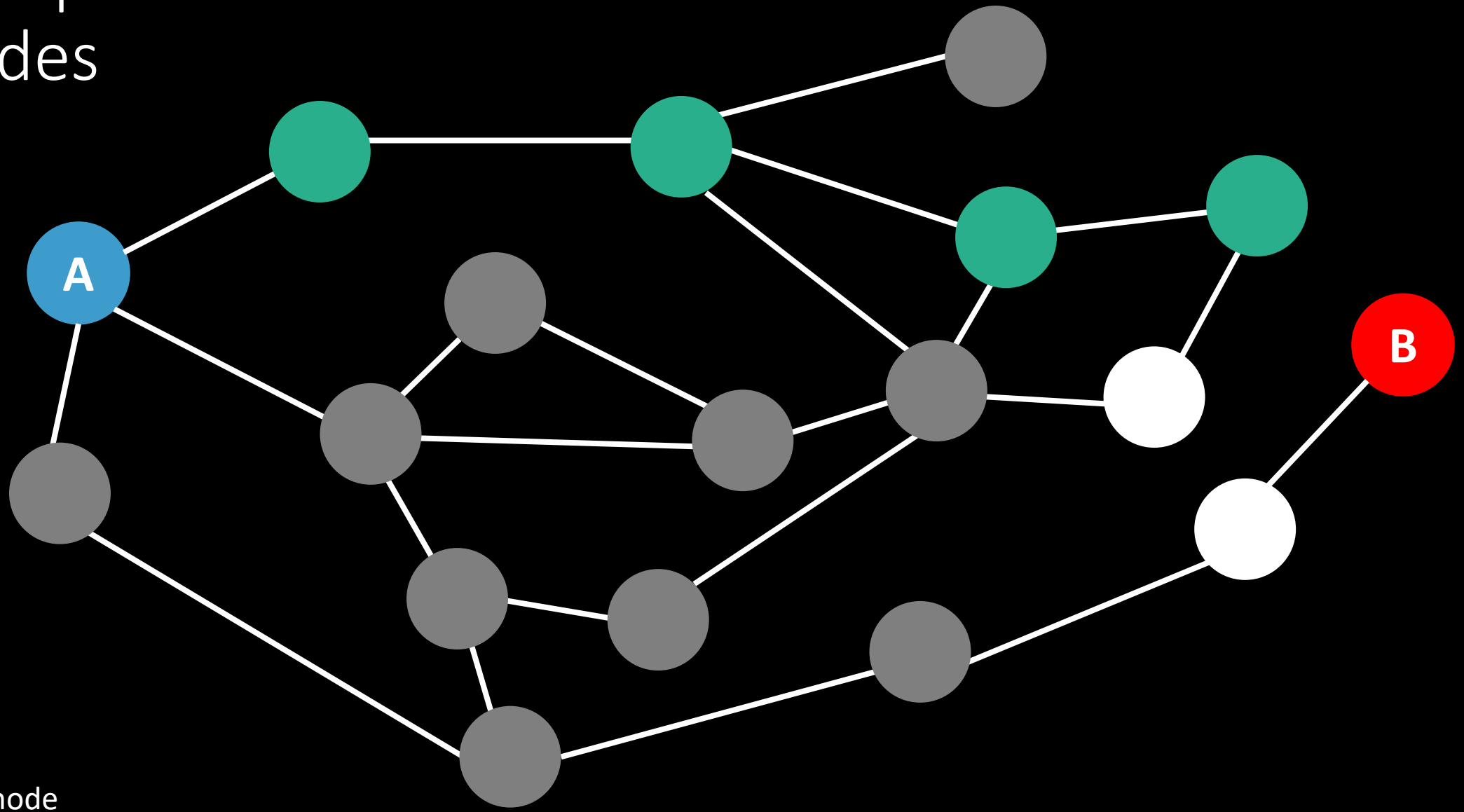

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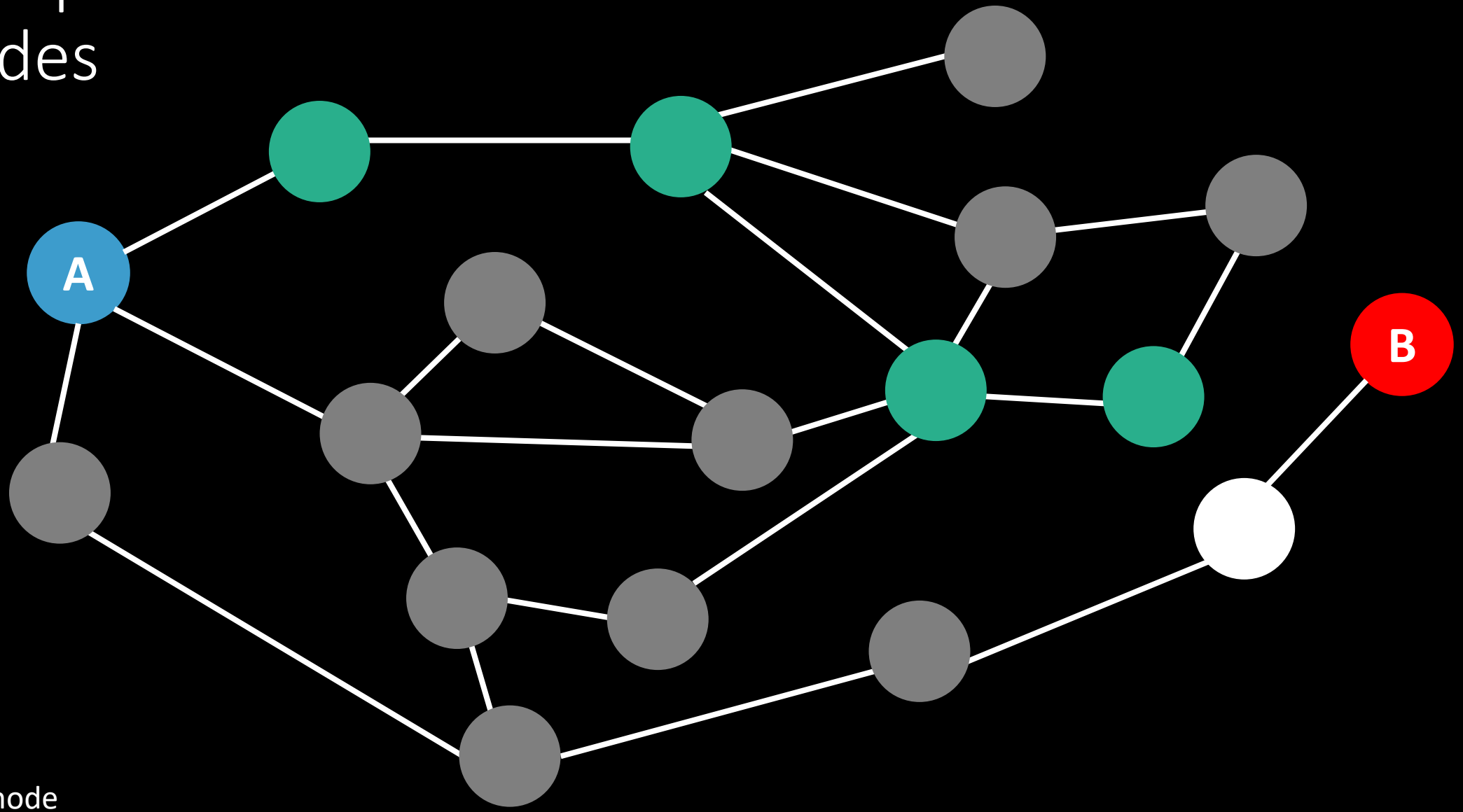

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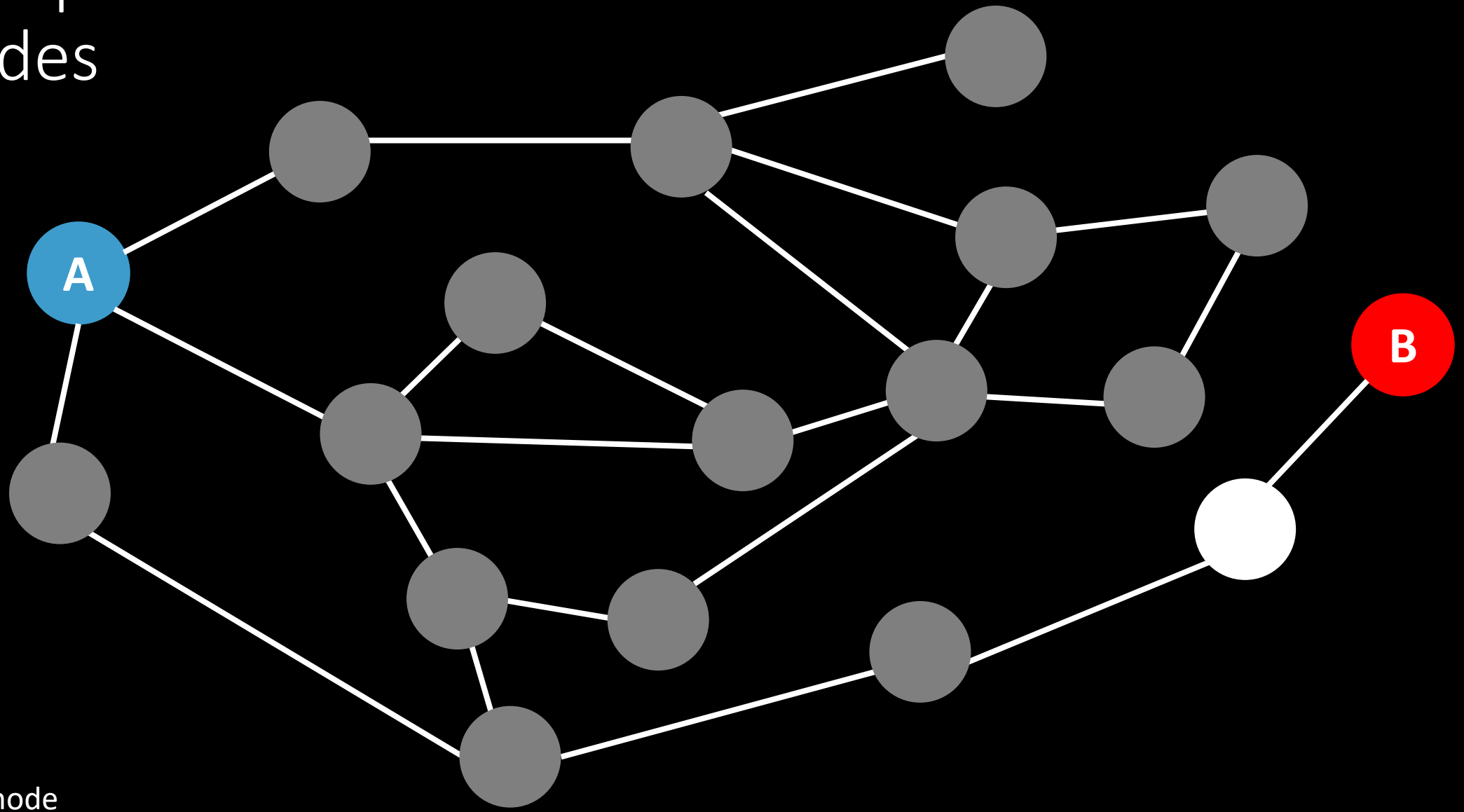

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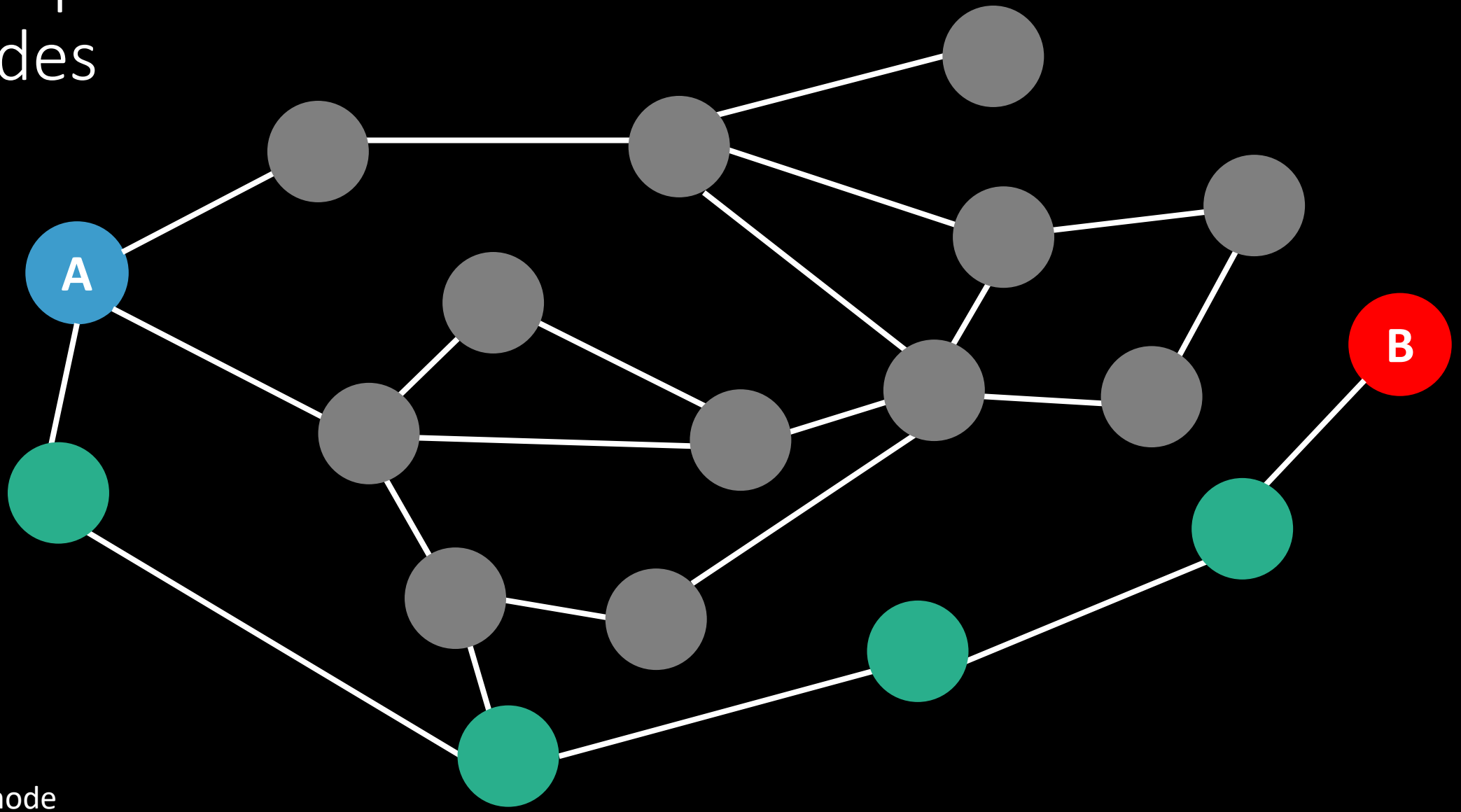

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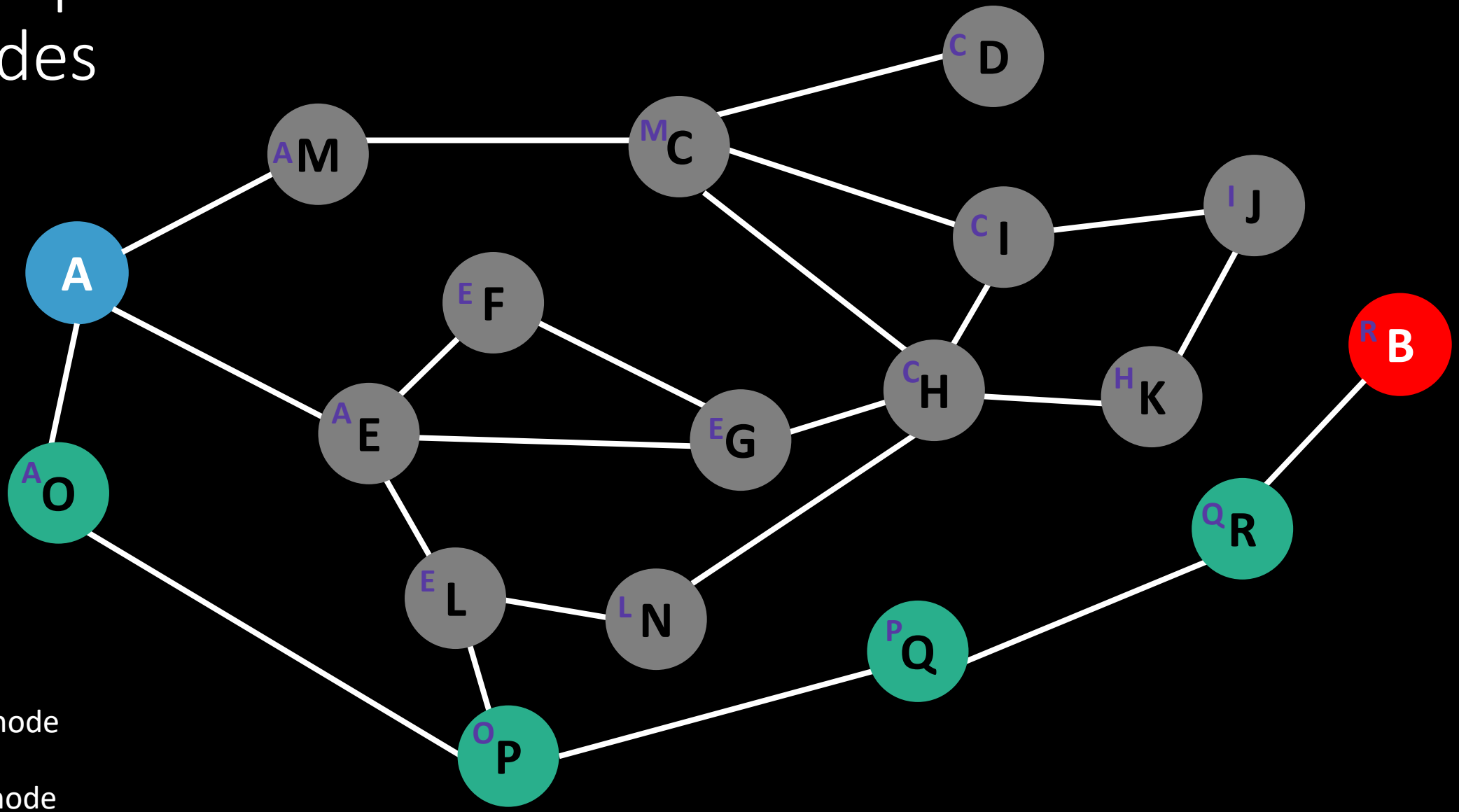
MO: Finish exploring current level before deepening the search

Shortest path between two nodes



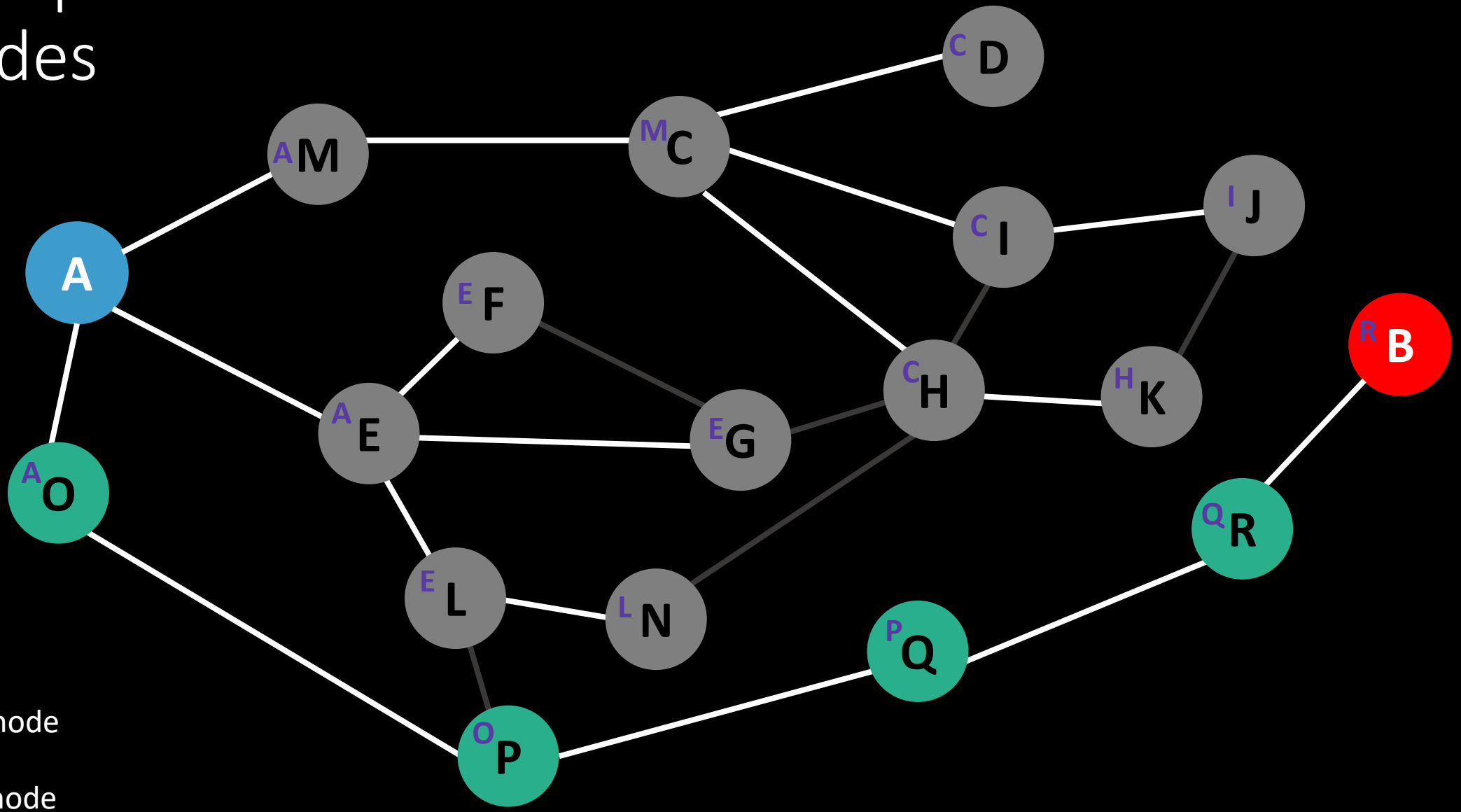
* Previous node

Explored node

Path taken

MO: Finish exploring current level before deepening the search

Shortest path between two nodes



* Previous node

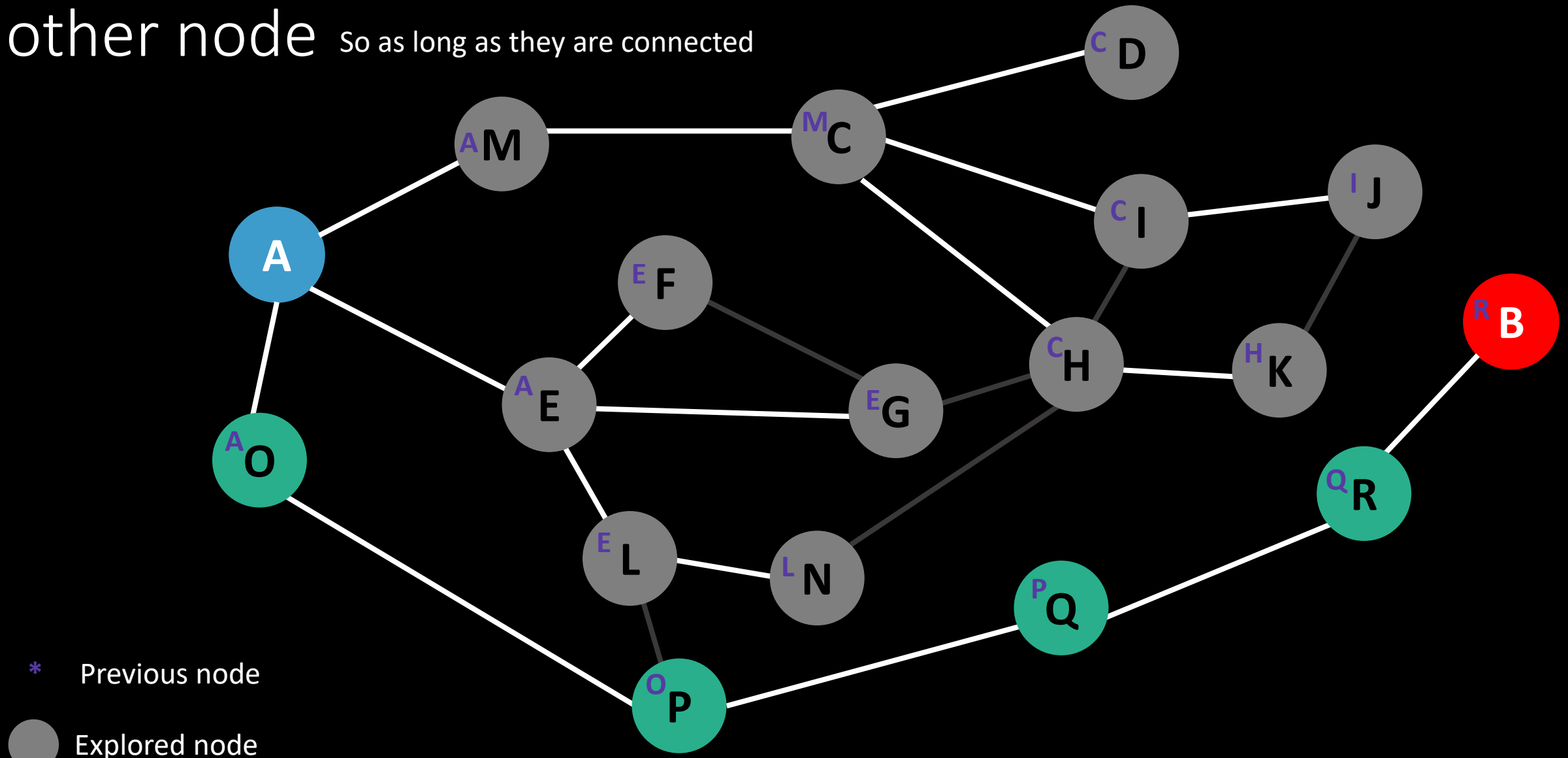
Explored node

Path taken

MO: Finish exploring current level before deepening the search

Shortest path between the starting node and **any** other node

So as long as they are connected



MO: Finish exploring current level before deepening the search

Breadth-First Search

Algorithm

1. **Expand** starting node
2. Add all neighboring nodes to the **search frontier** (enqueue)
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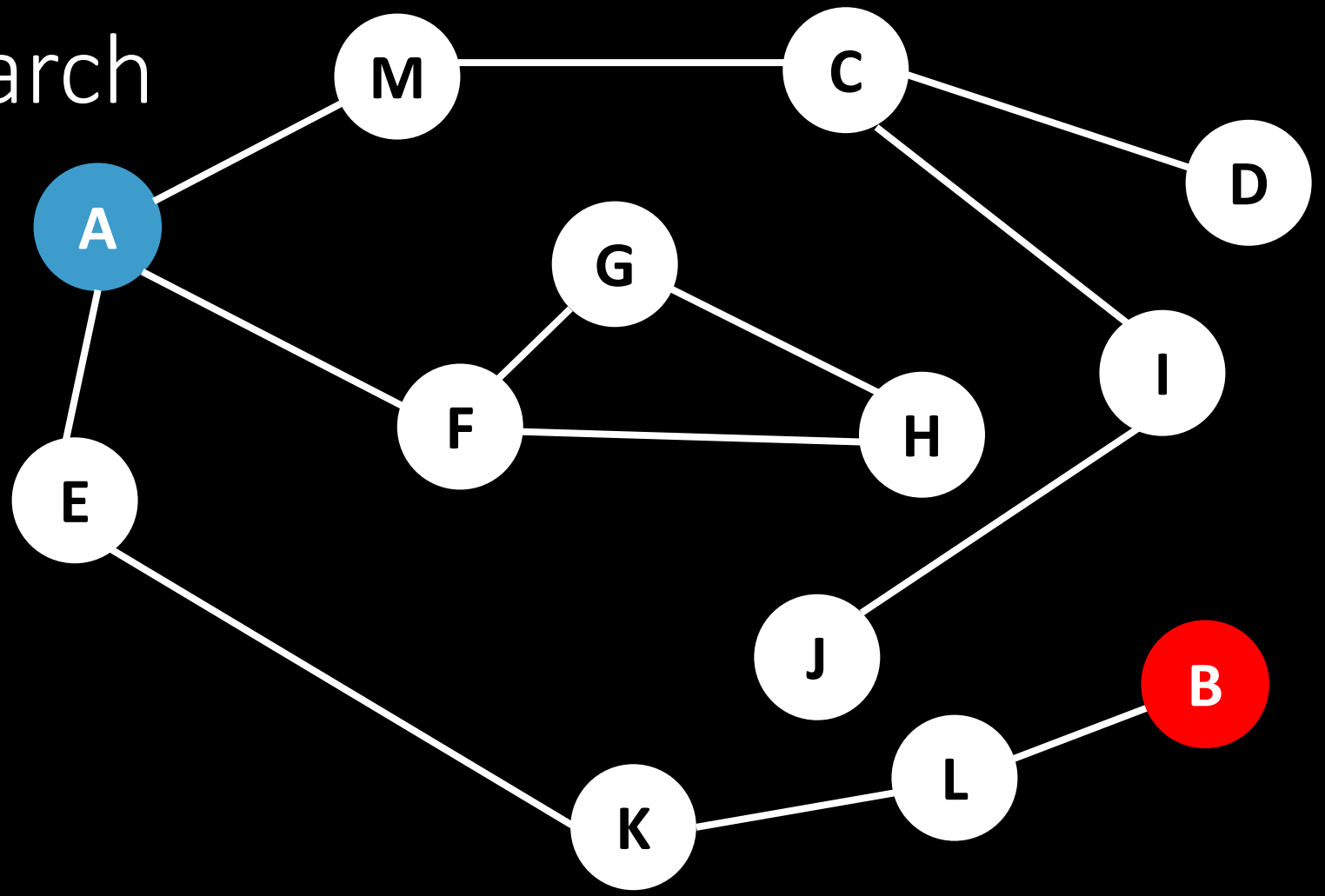

Search Frontier

A							

Breadth-First Search

Algorithm

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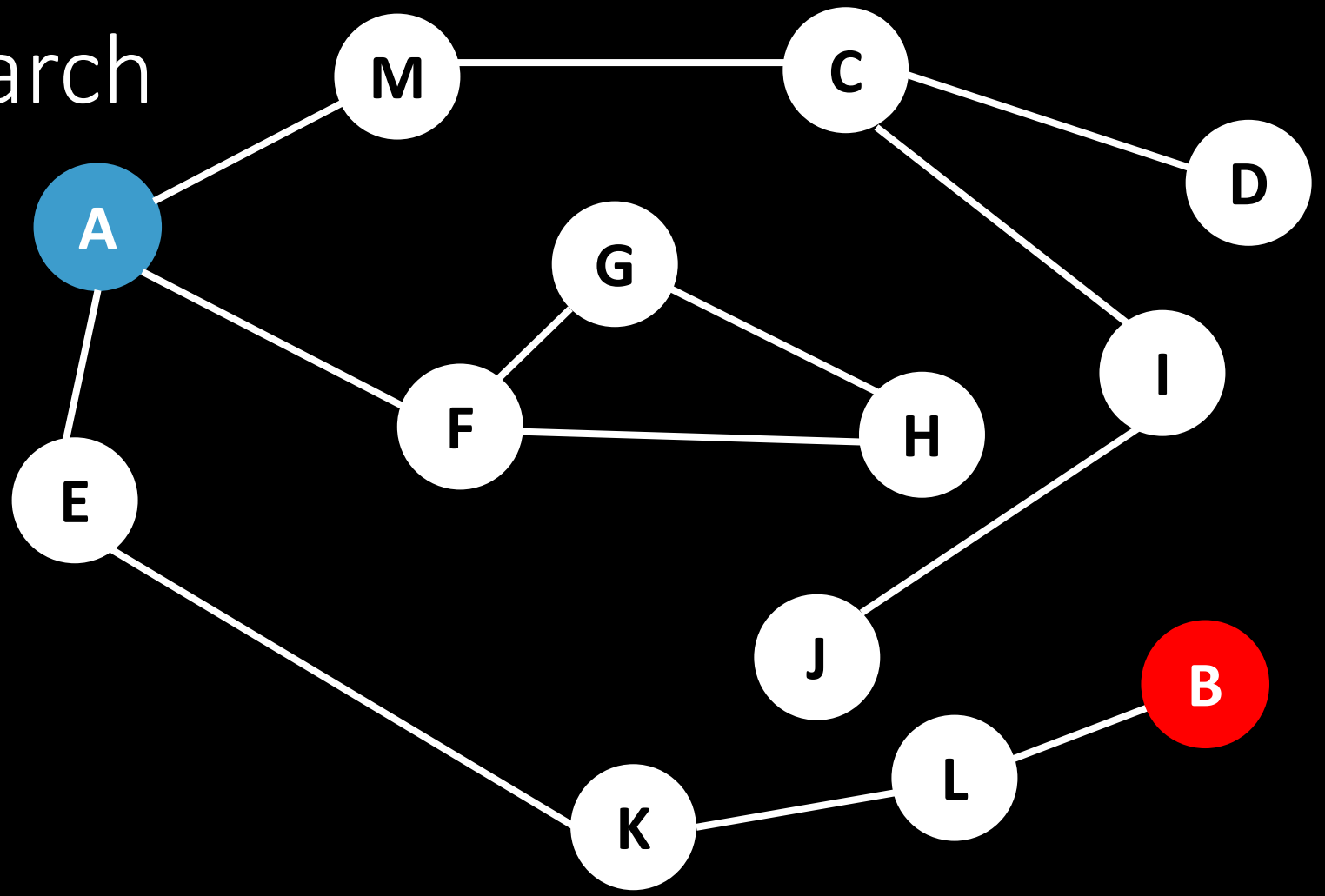

Search Frontier

~~A~~	M	F	E				

Breadth-First Search

Algorithm

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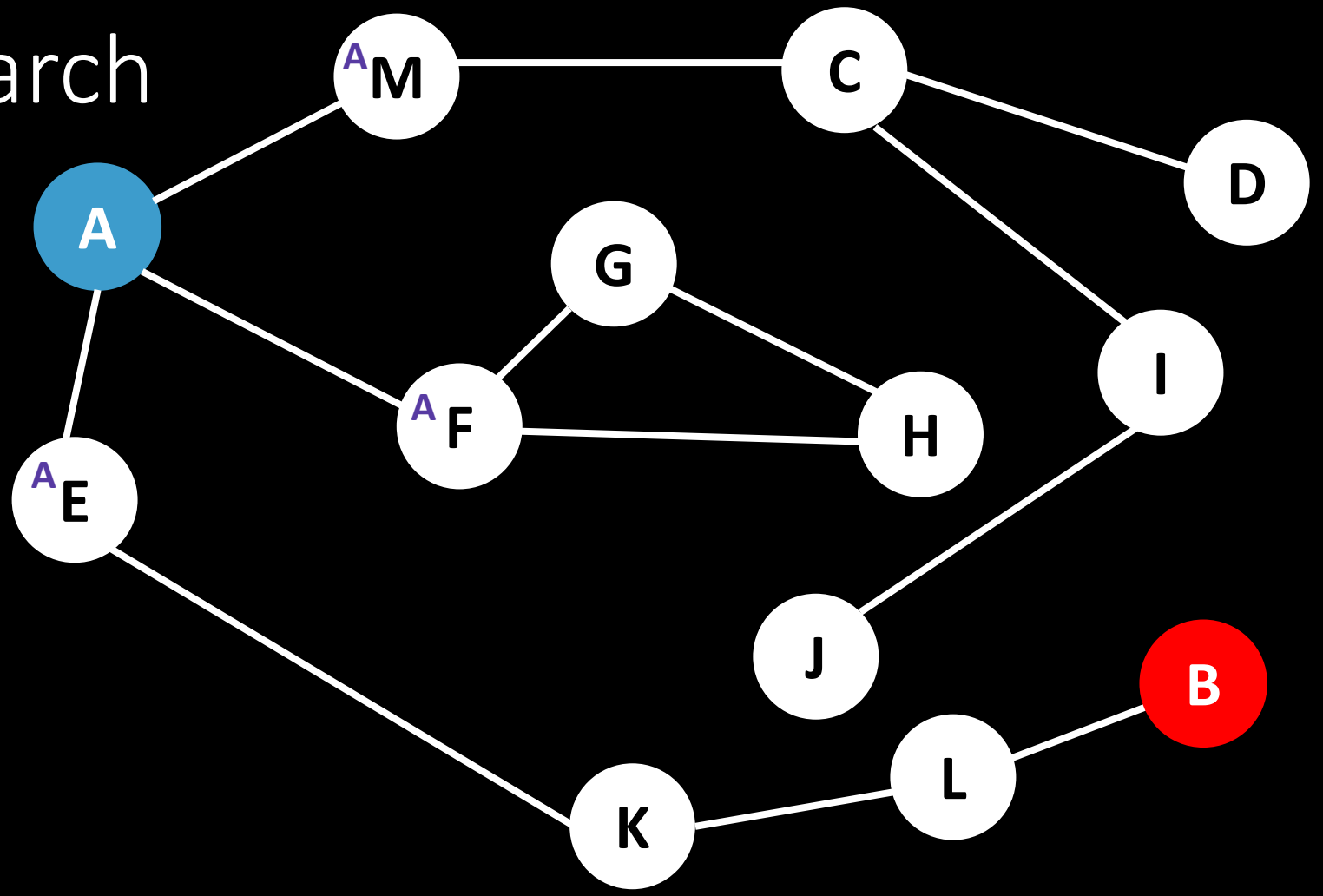

Search Frontier

~~A~~	M	F	E				

Breadth-First Search

Algorithm

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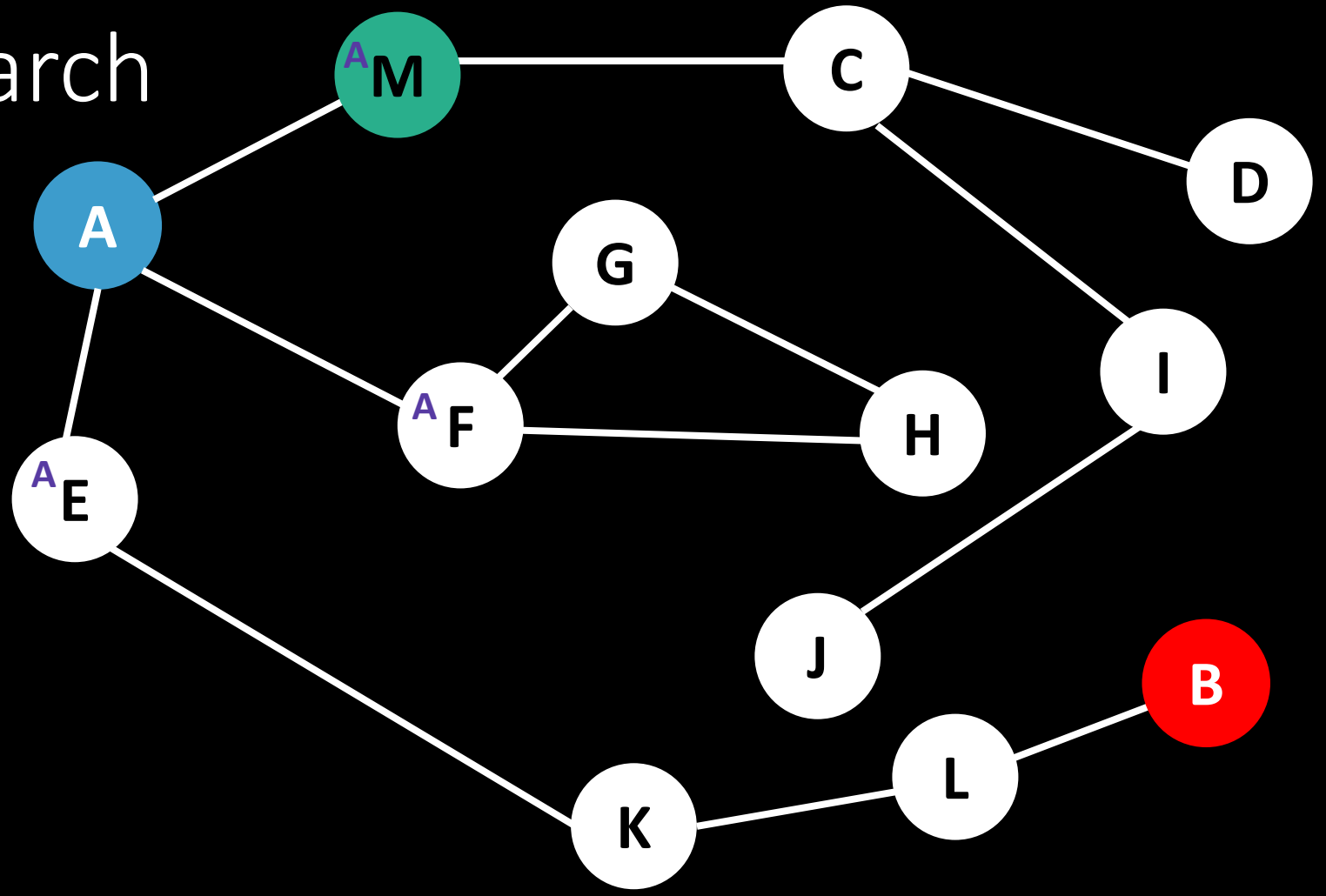

Search Frontier

A	M	F	E	C			

Breadth-First Search

Algorithm

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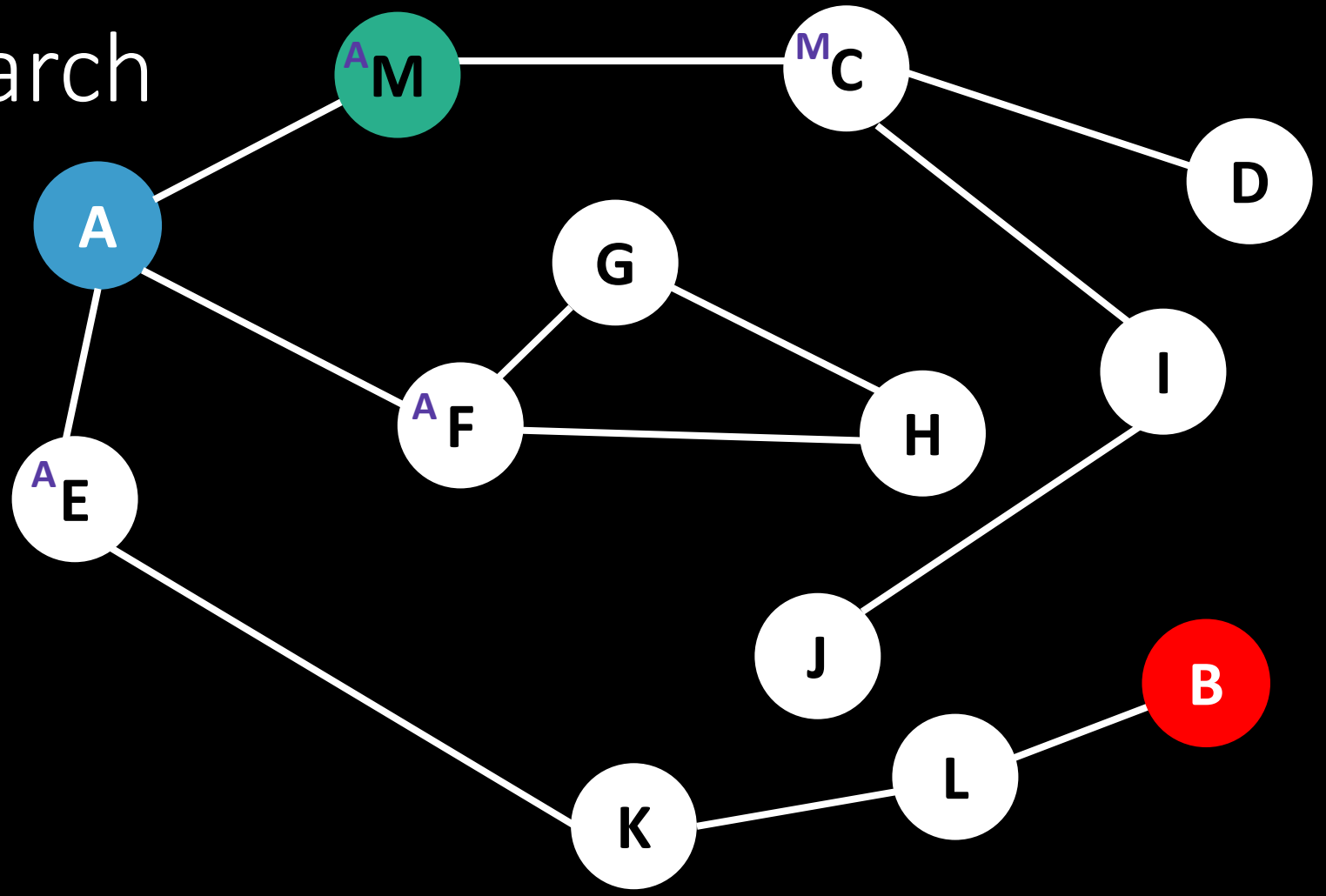

Search Frontier

A	M	F	E	C			

Breadth-First Search

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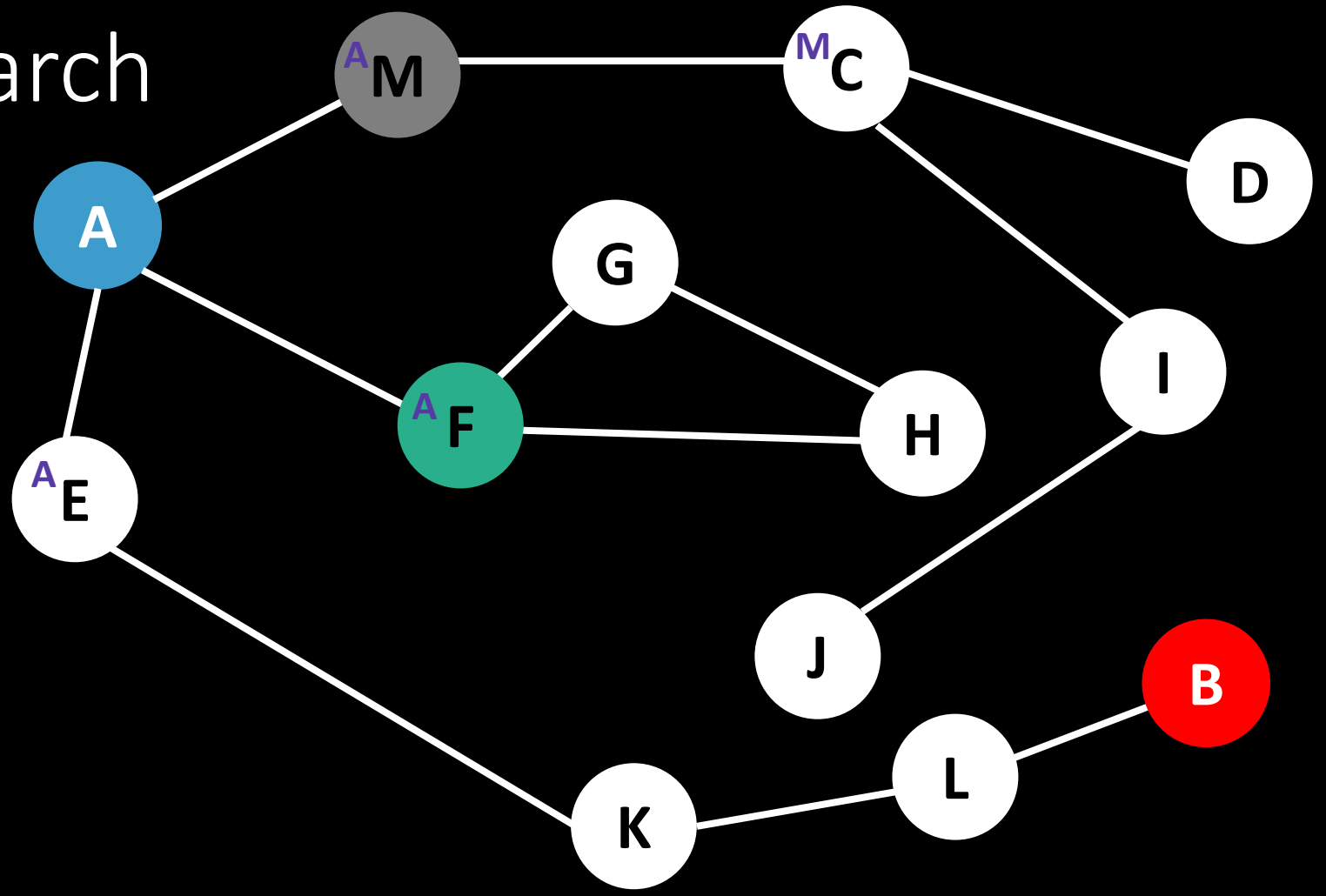

Search Frontier

A	M	F	E	C	G	H	

Breadth-First Search

Algorithm

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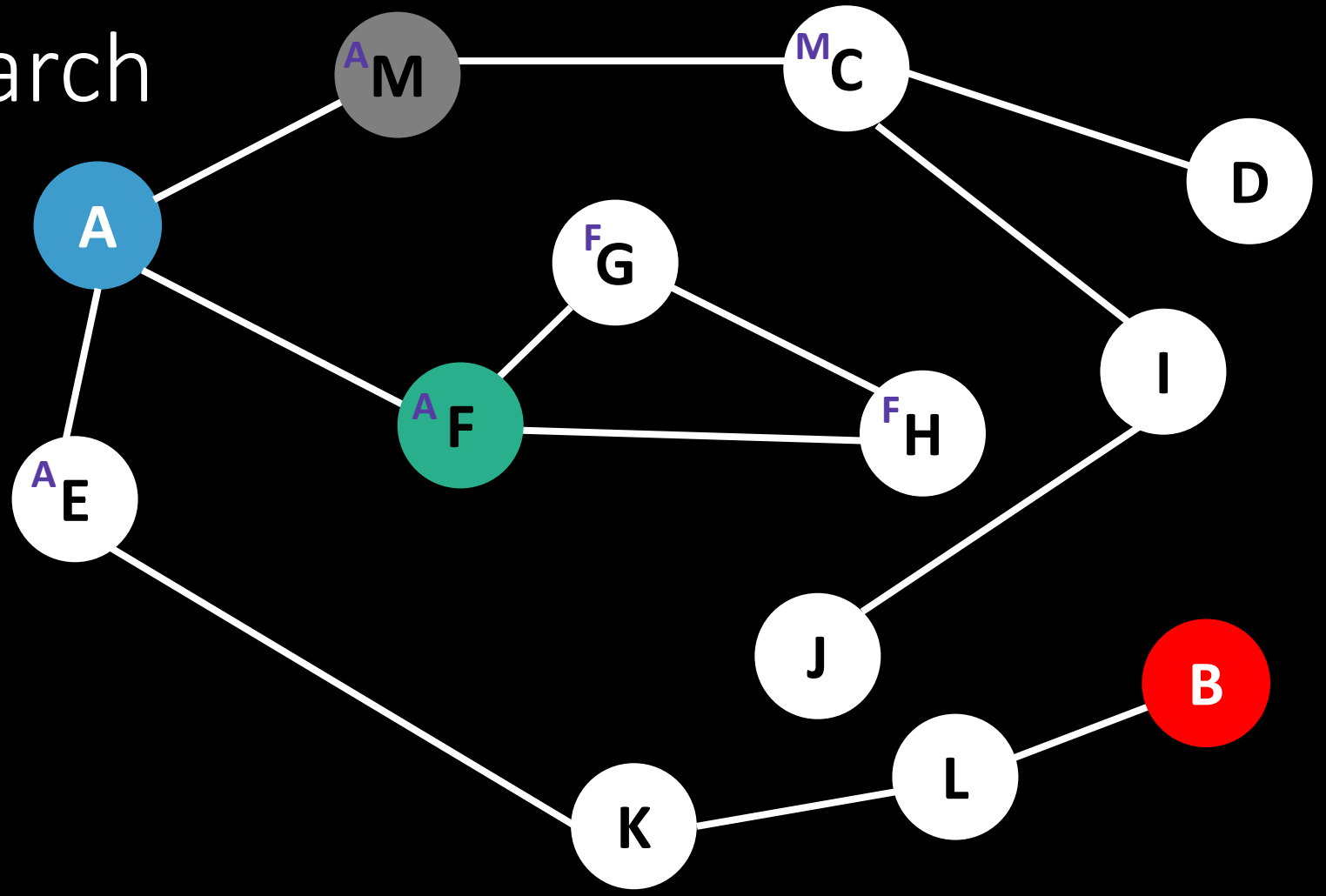

Search Frontier

A	M	F	E	C	G	H	

Breadth-First Search

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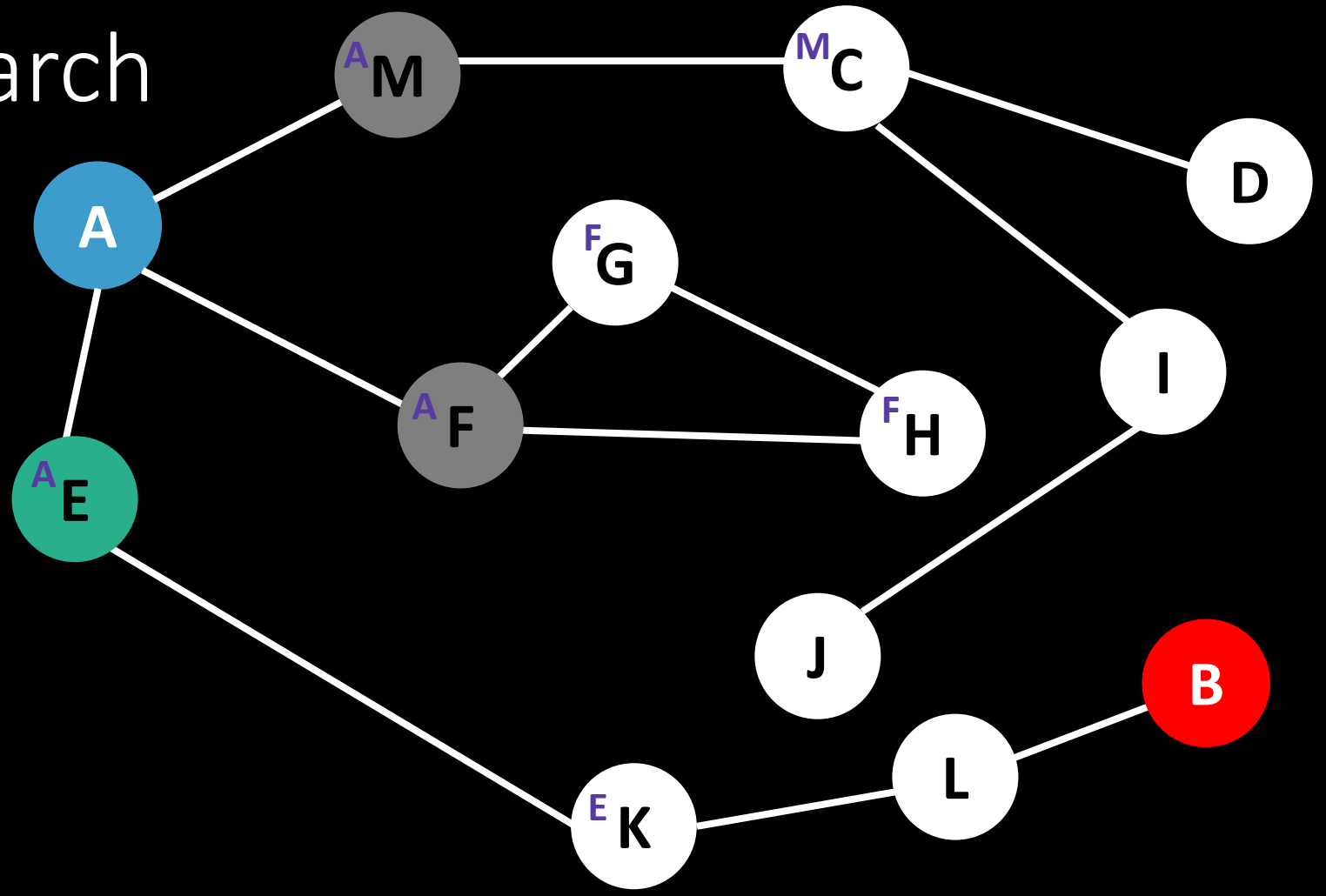

Search Frontier

~~A~~	~~M~~	~~F~~	~~E~~	C	G	H	K

Breadth-First Search

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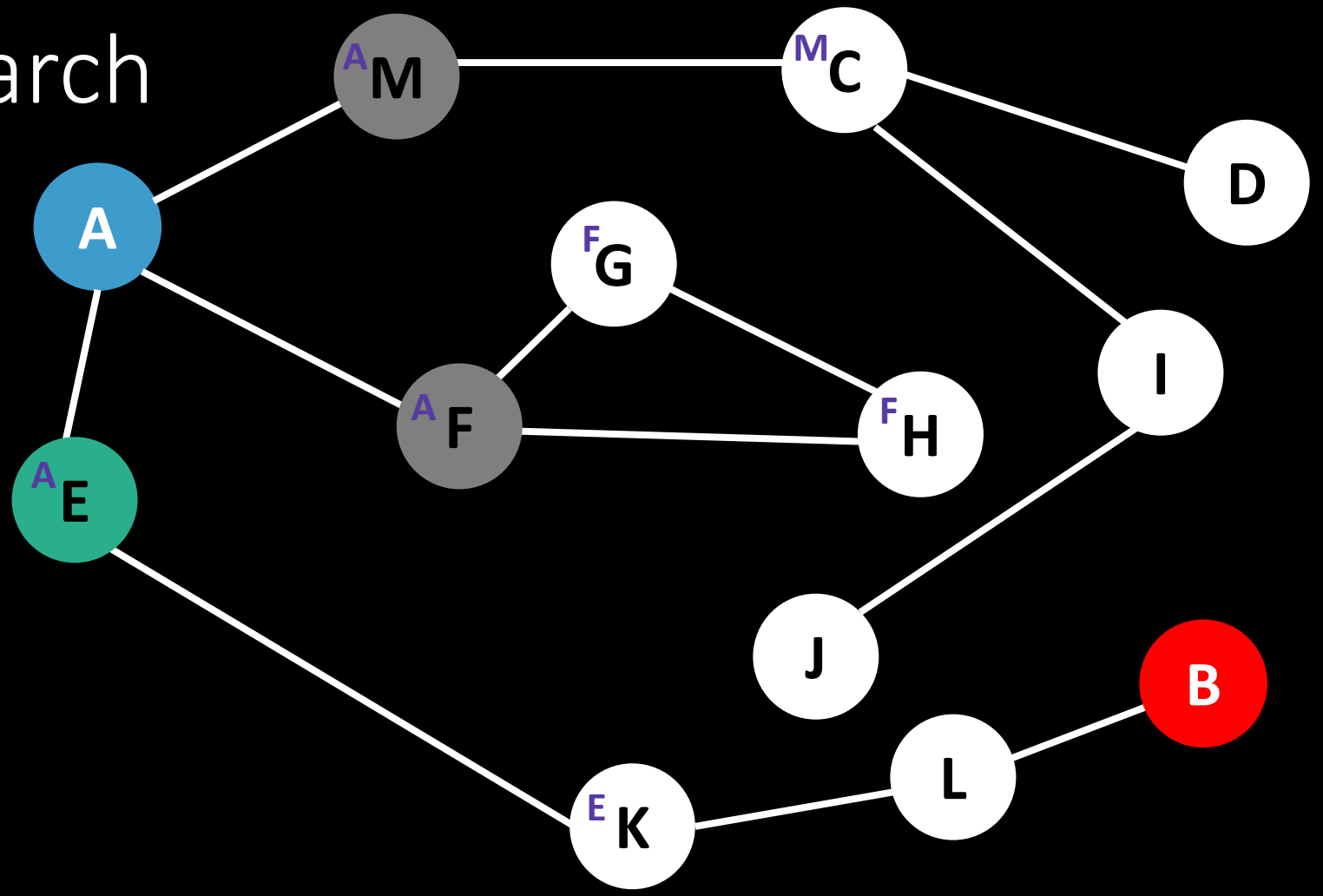

Search Frontier

C	G	H	K				

Breadth-First Search

Algorithm

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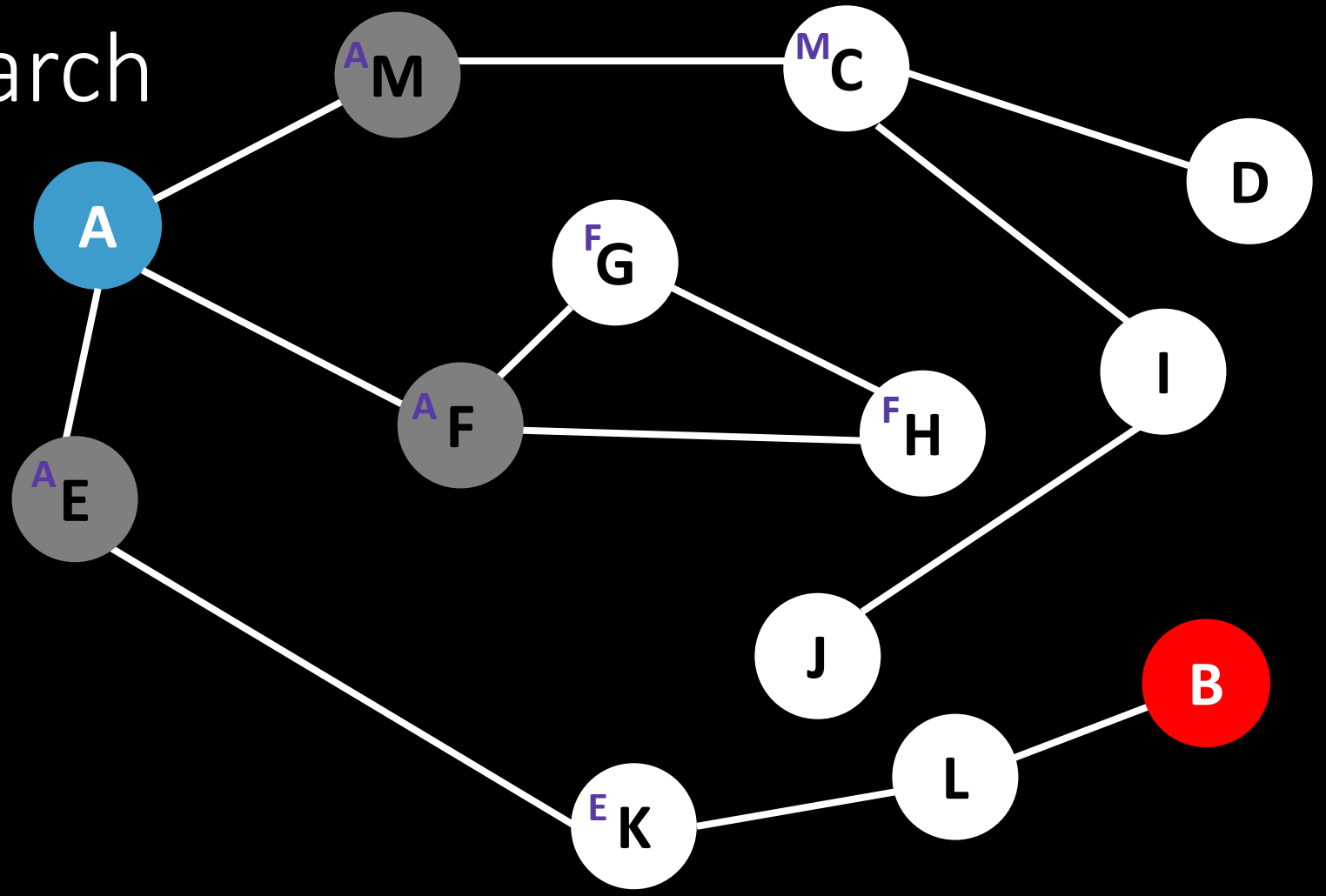

Search Frontier

C	G	H	K				

Breadth-First Search

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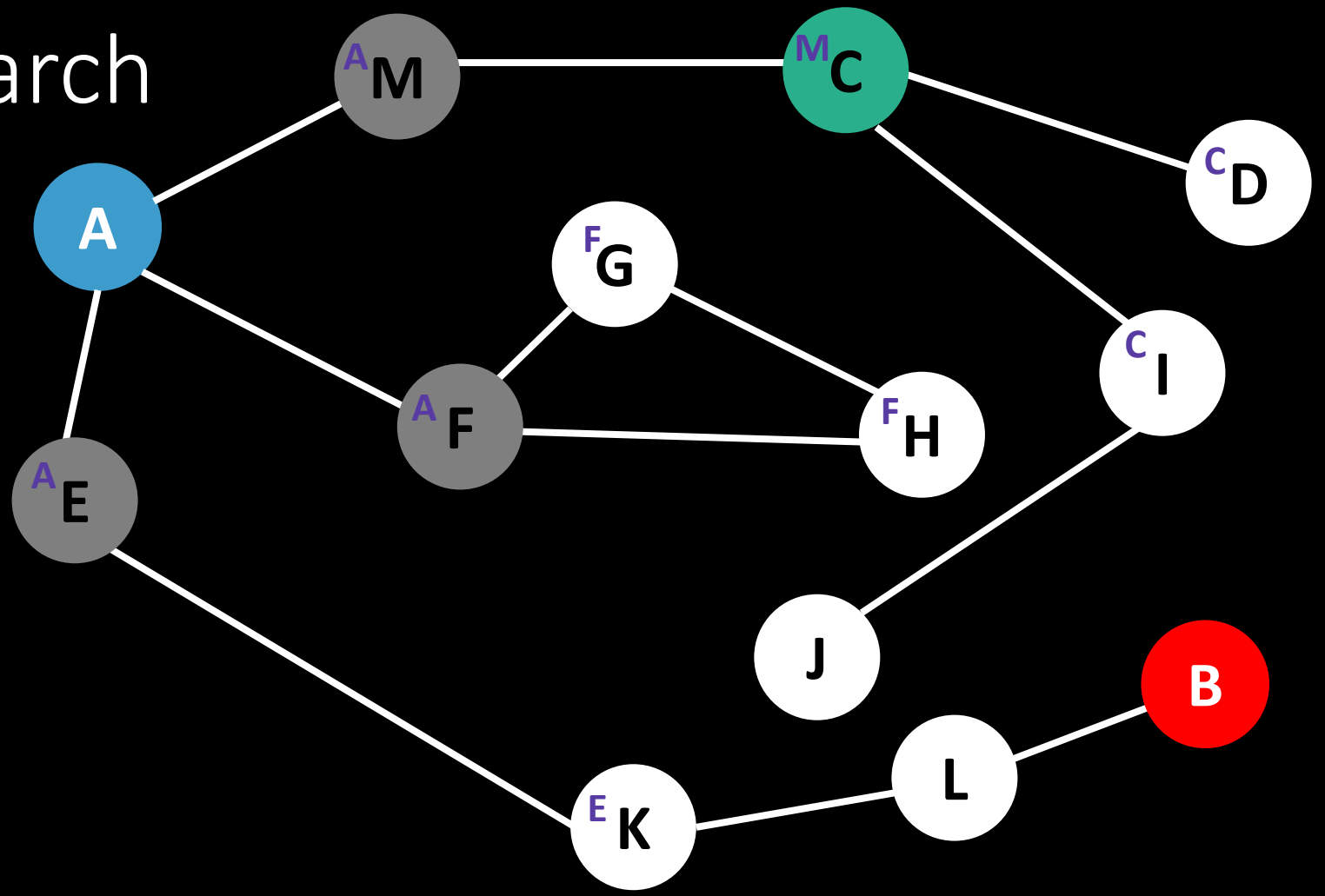

Search Frontier

C	G	H	K	D			

Breadth-First Search

Algorithm

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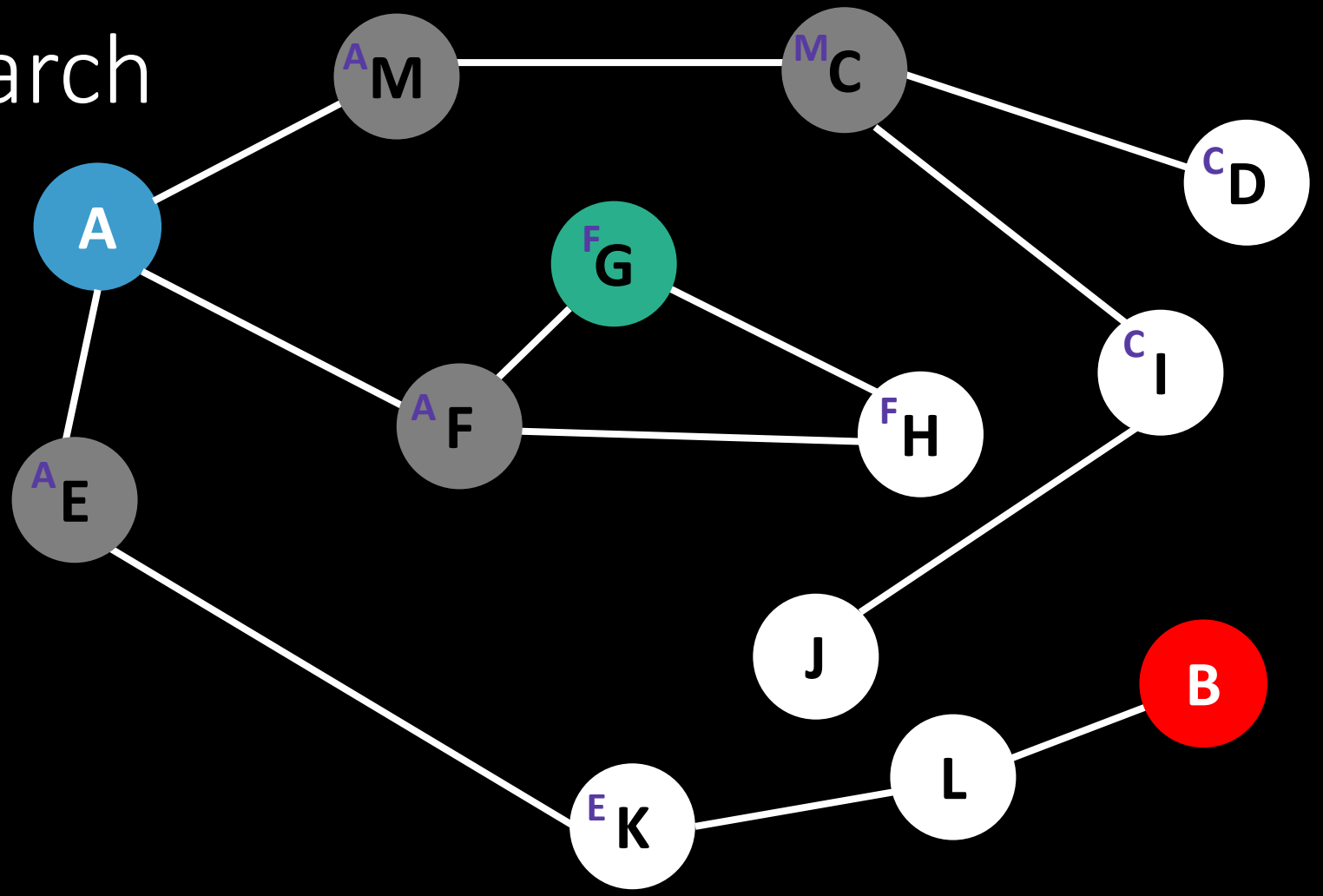

Search Frontier

C	G	H	K	D	I	H	

Breadth-First Search

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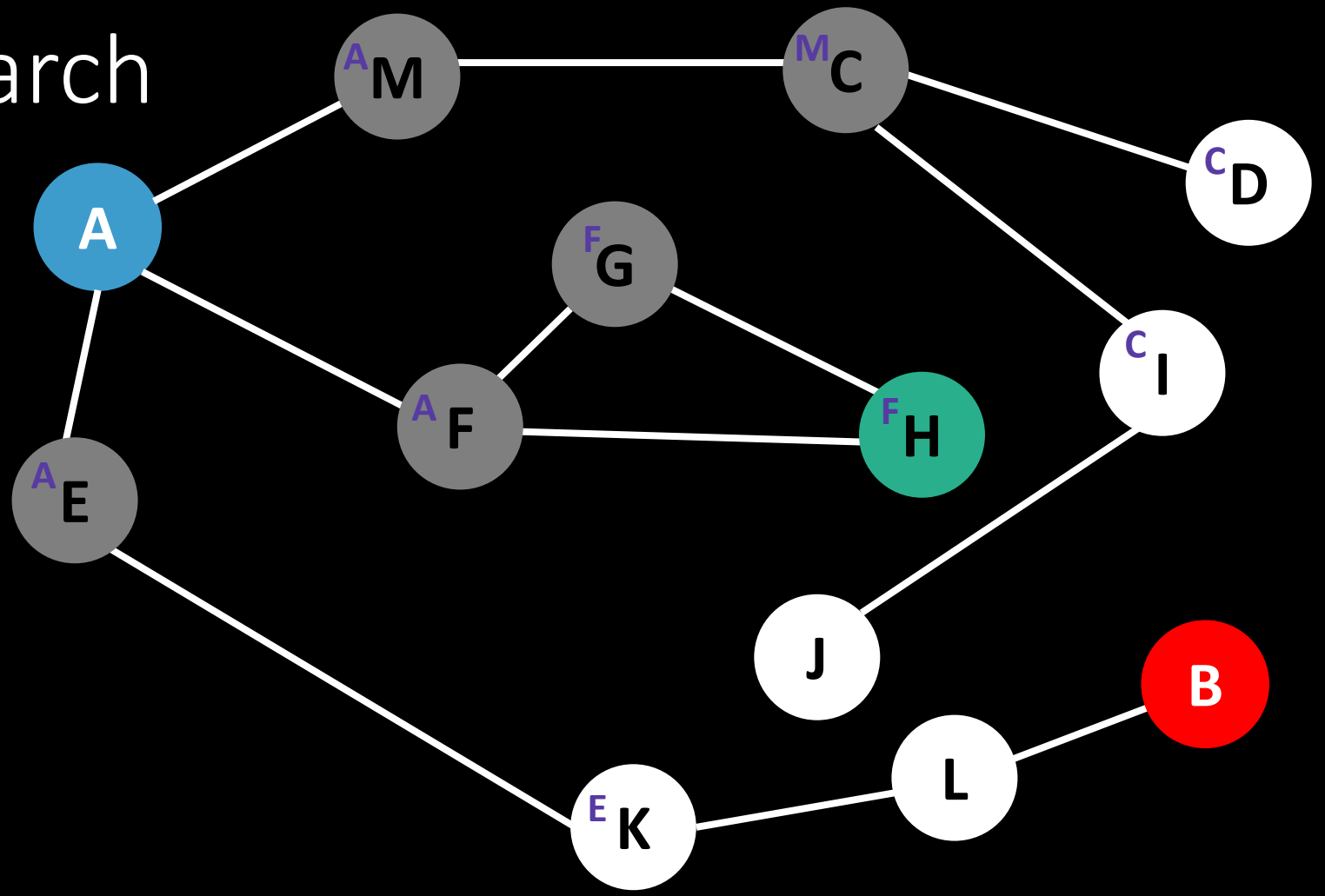

Search Frontier

C	G	M	K	D	I	H	

Breadth-First Search

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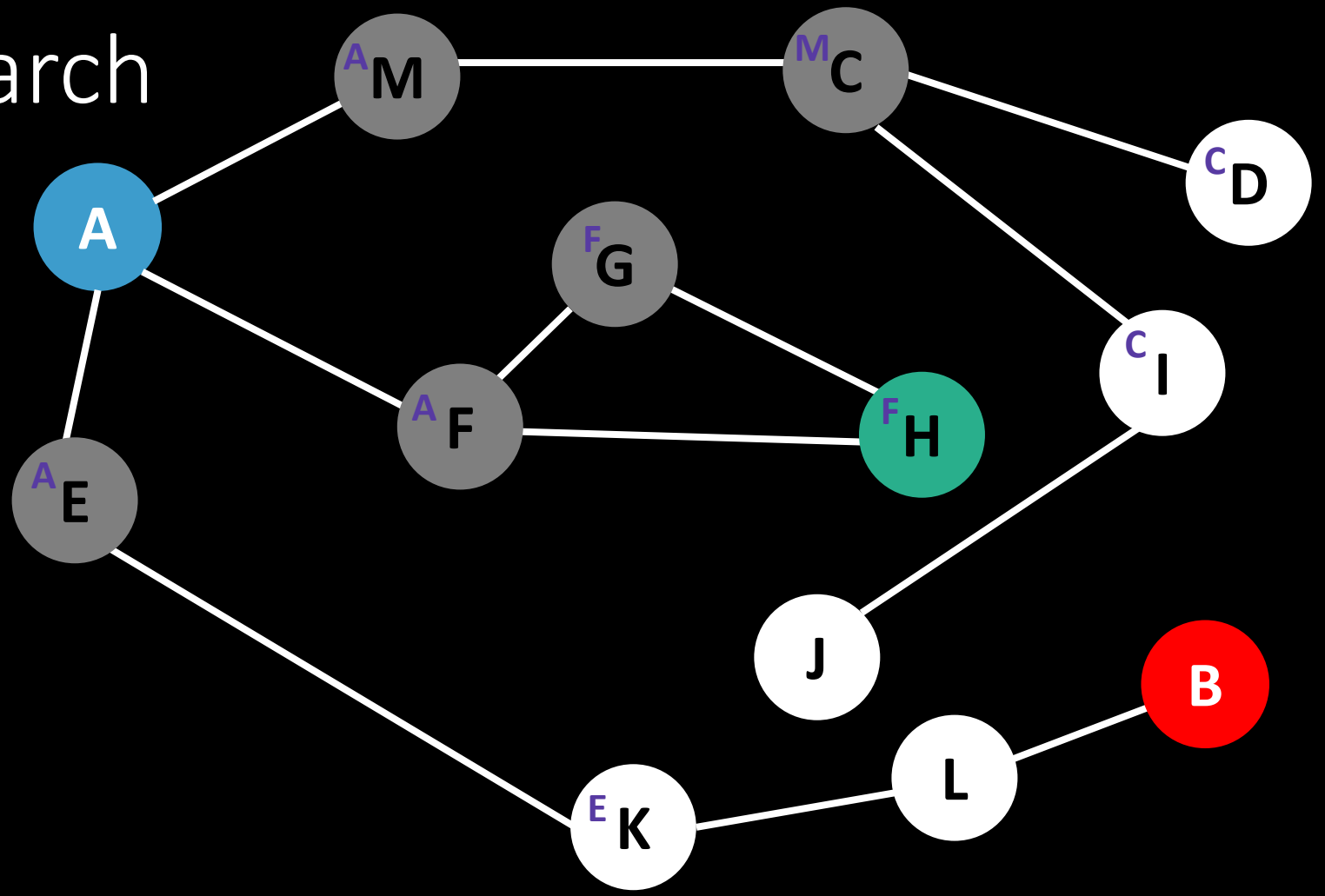

Search Frontier

C	G	M	K	D	I	H	

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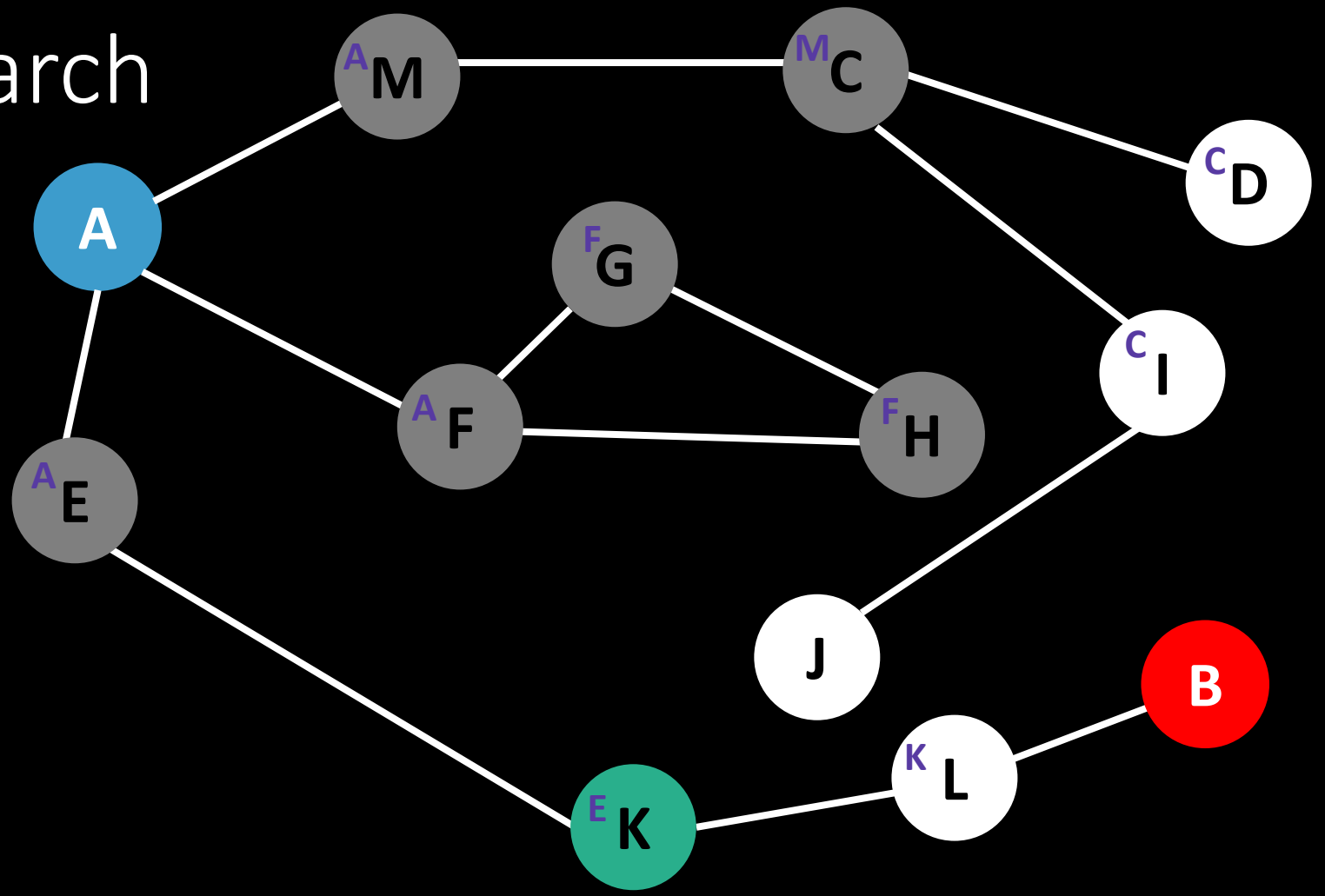

Search Frontier

~~C~~	~~G~~	~~H~~	~~K~~	D	I	H	L

Breadth-First Search

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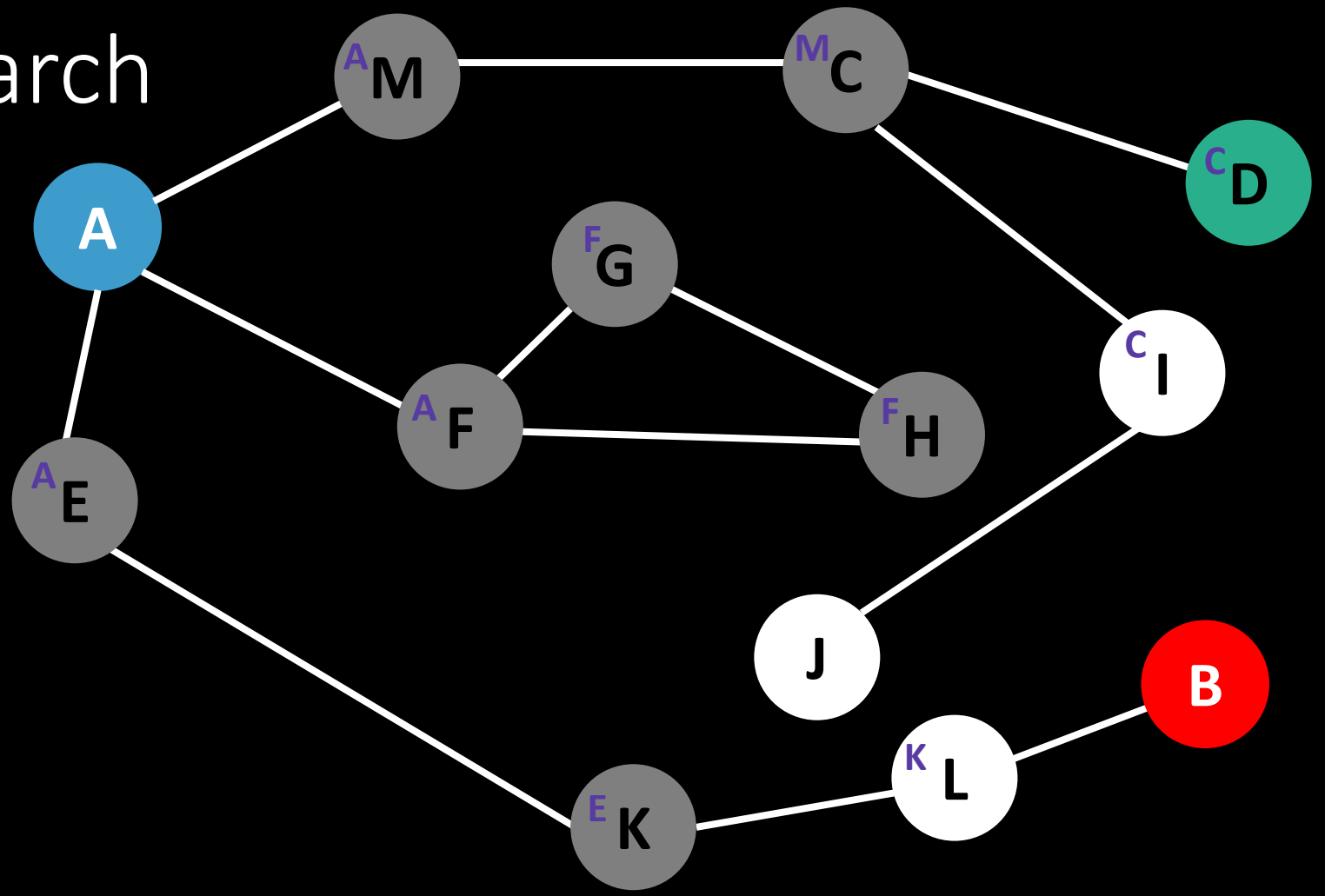

Search Frontier

D	I	H	L				

Breadth-First Search

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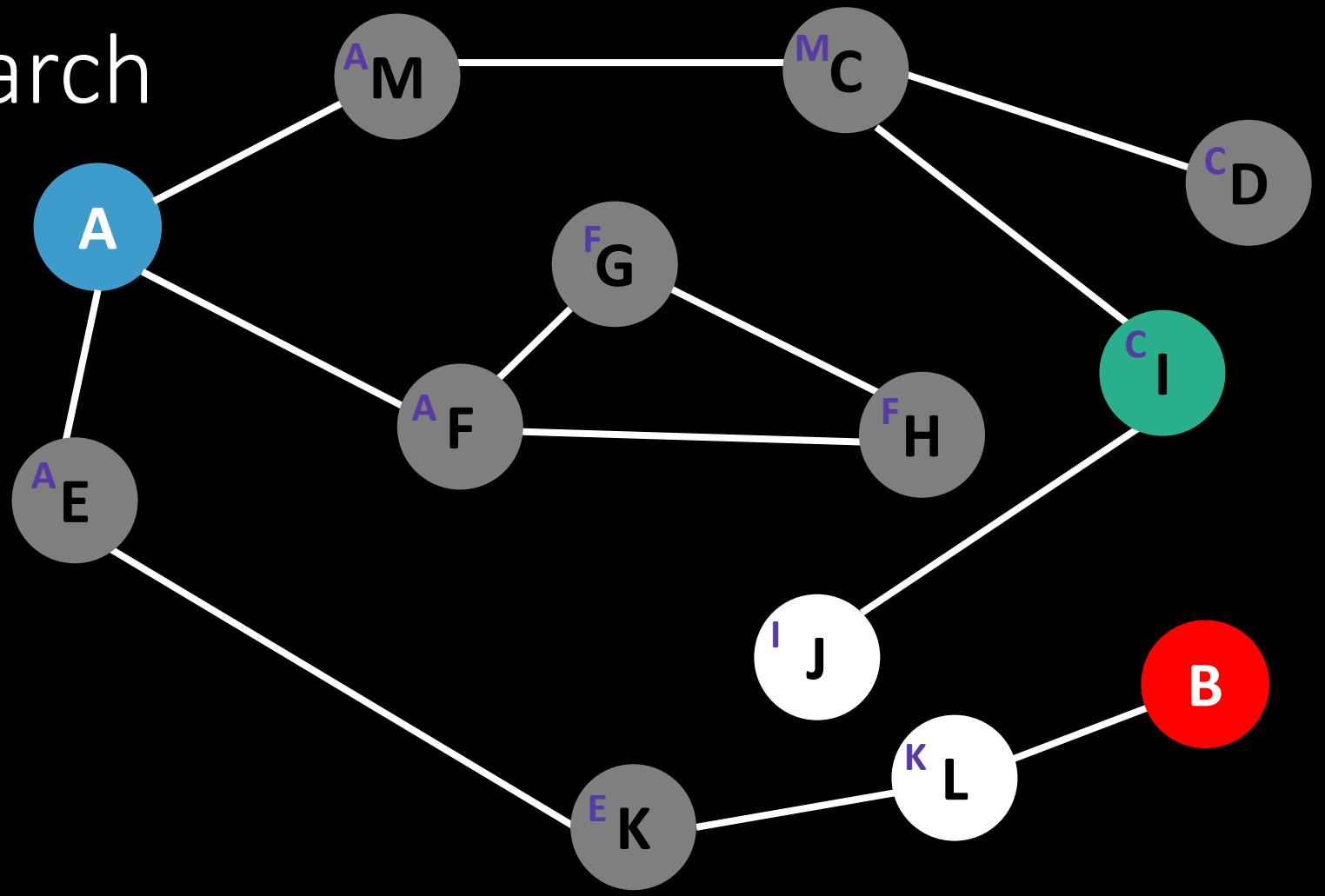

Search Frontier

~~D~~	~~I~~	H	L	J			

Breadth-First Search

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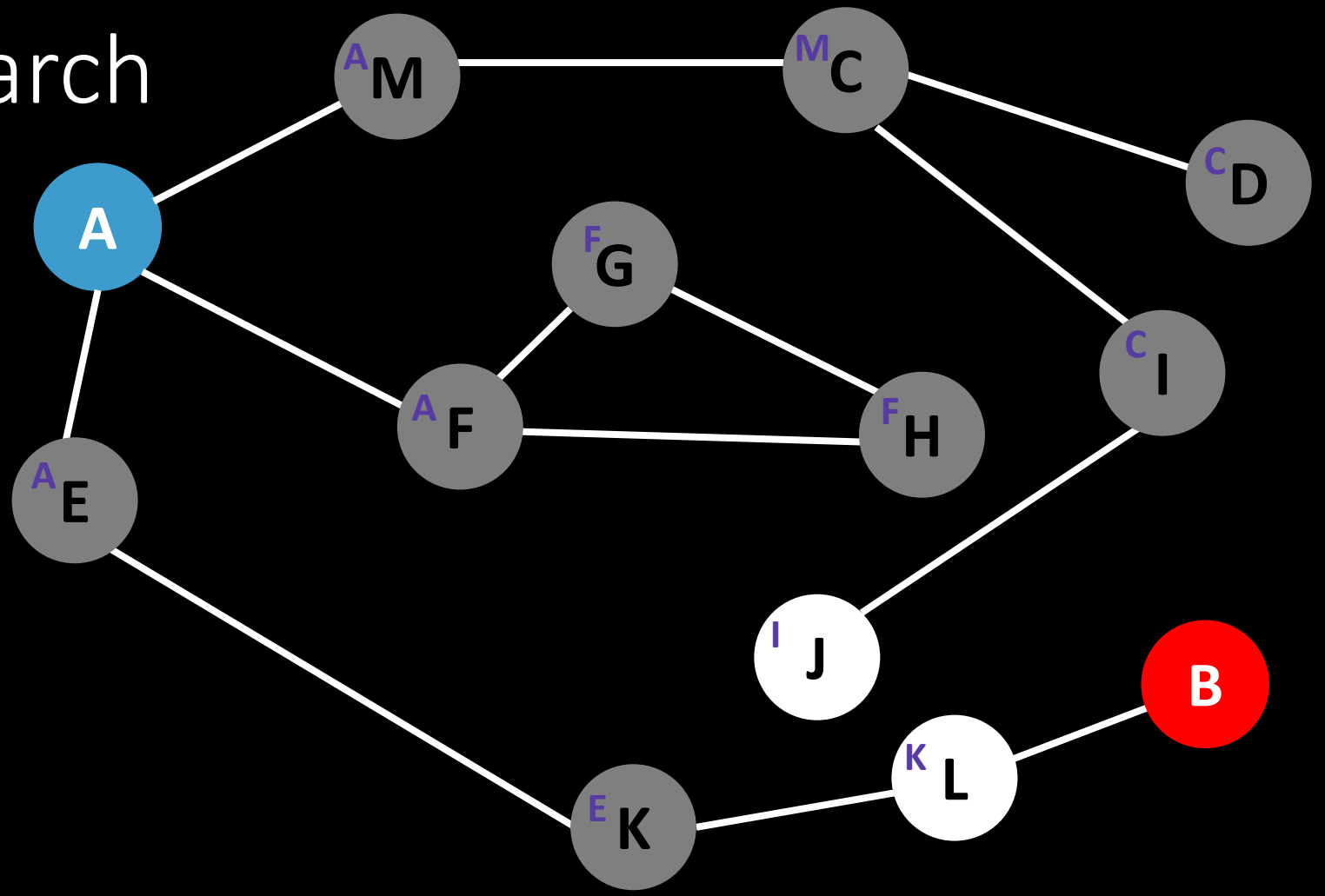

Already explored, do not re-expand

Search Frontier

D	I	H	L	J			

Breadth-First Search

Algorithm

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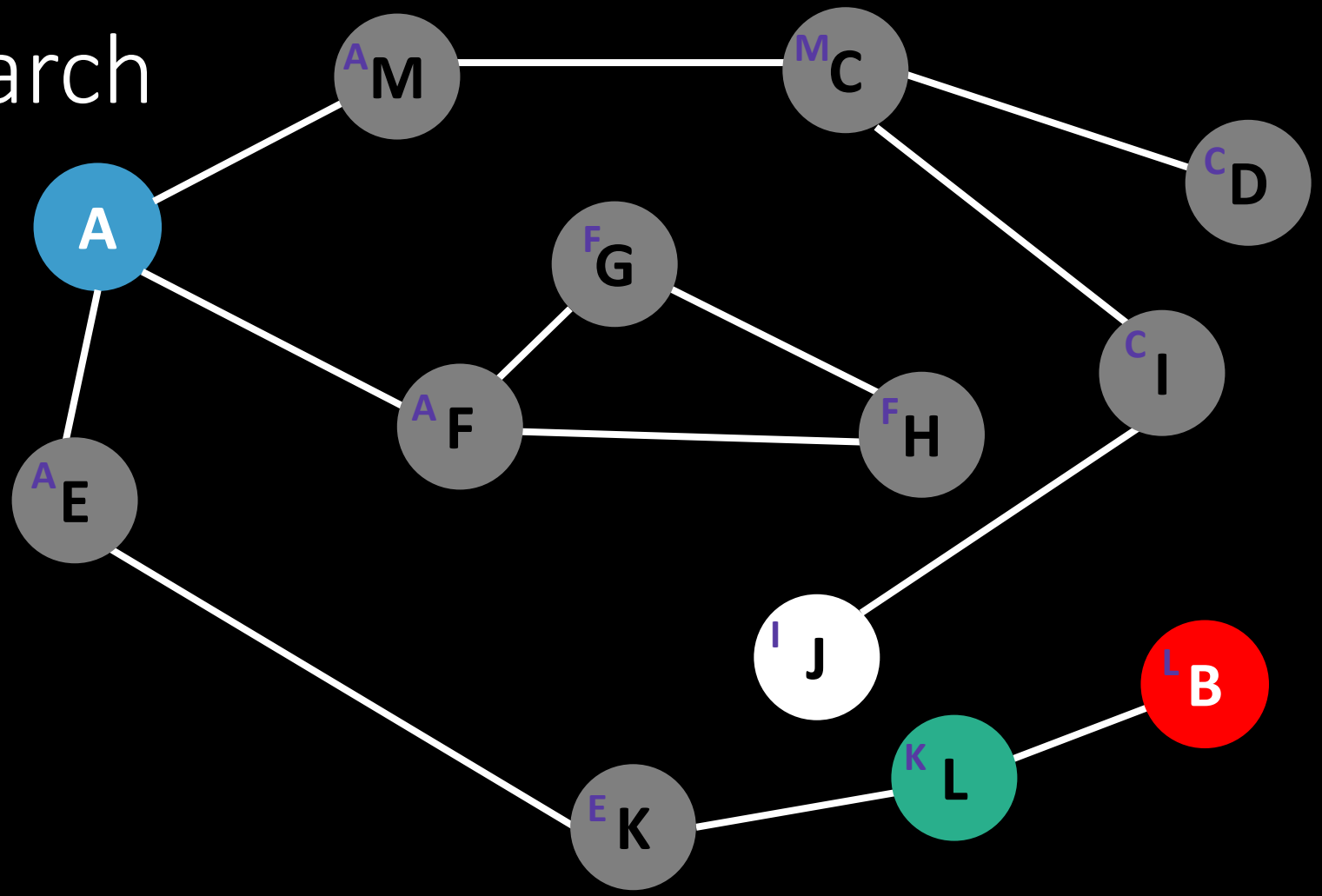

Search Frontier

D	I	H	L	J	B		

Breadth-First Search

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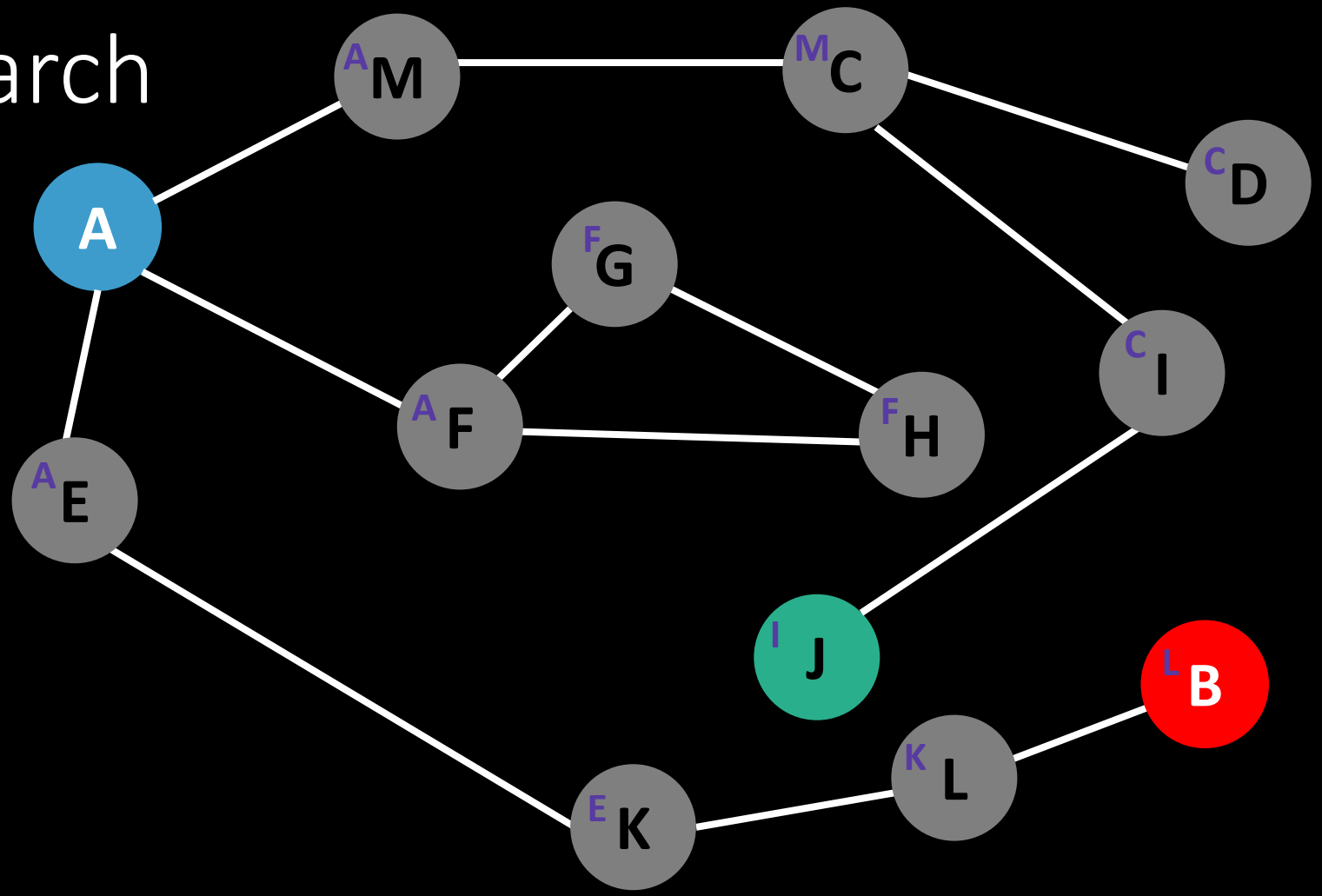
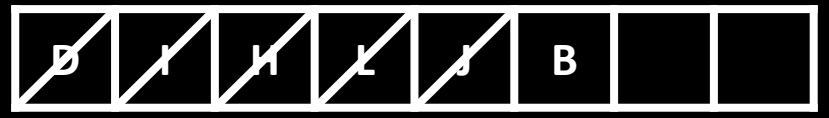


Search Frontier



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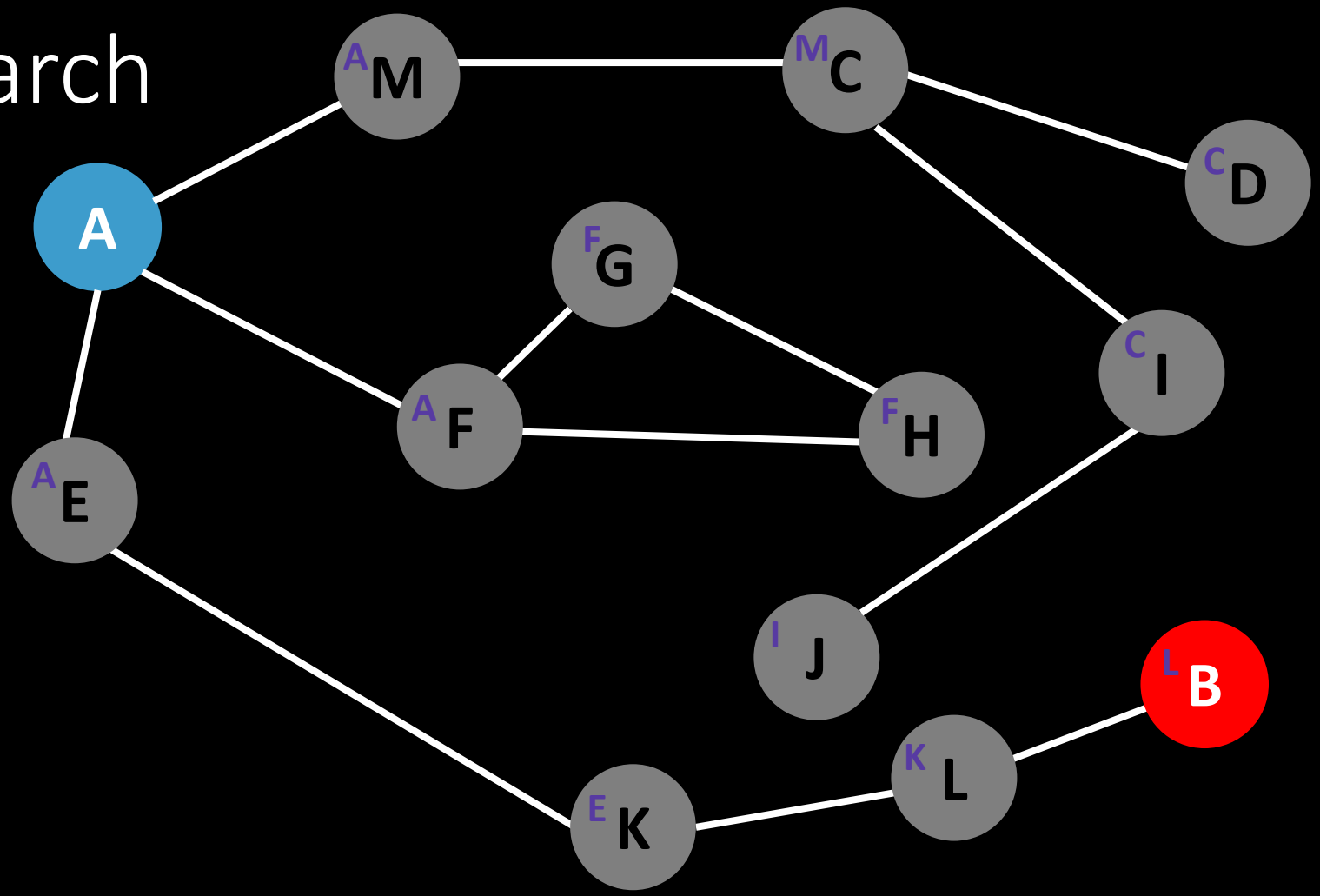
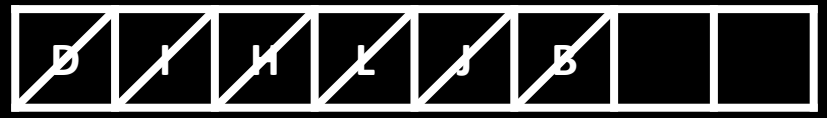


Search Frontier



Breadth-First Search

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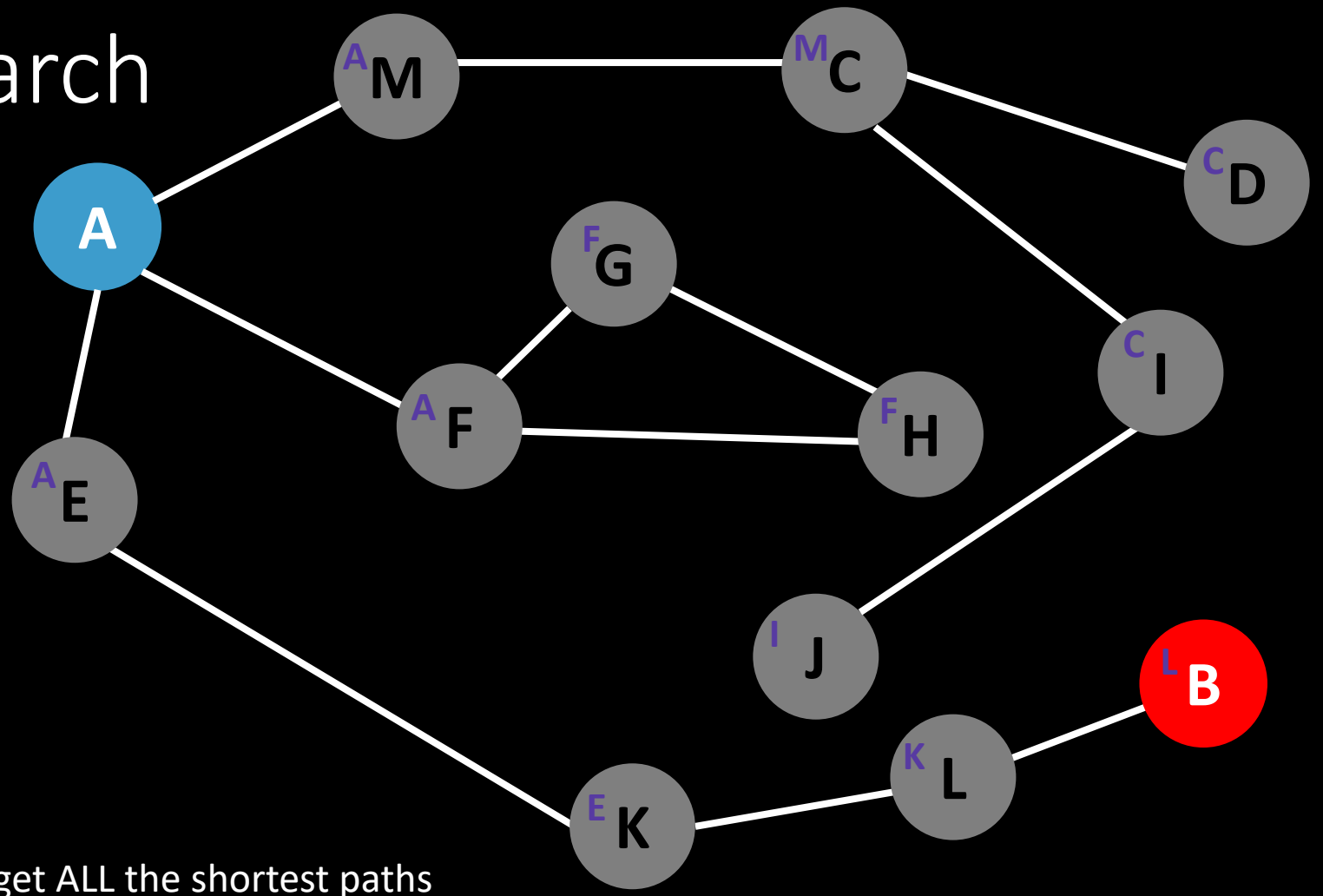
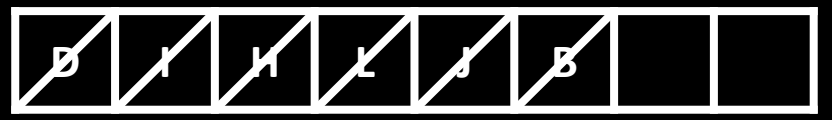
4. **Expand** the node
5. Repeat 2-4 until : the ending node is found OR there **are no nodes left in the queue**

} To get ALL the shortest paths from the starting node



Search Frontier

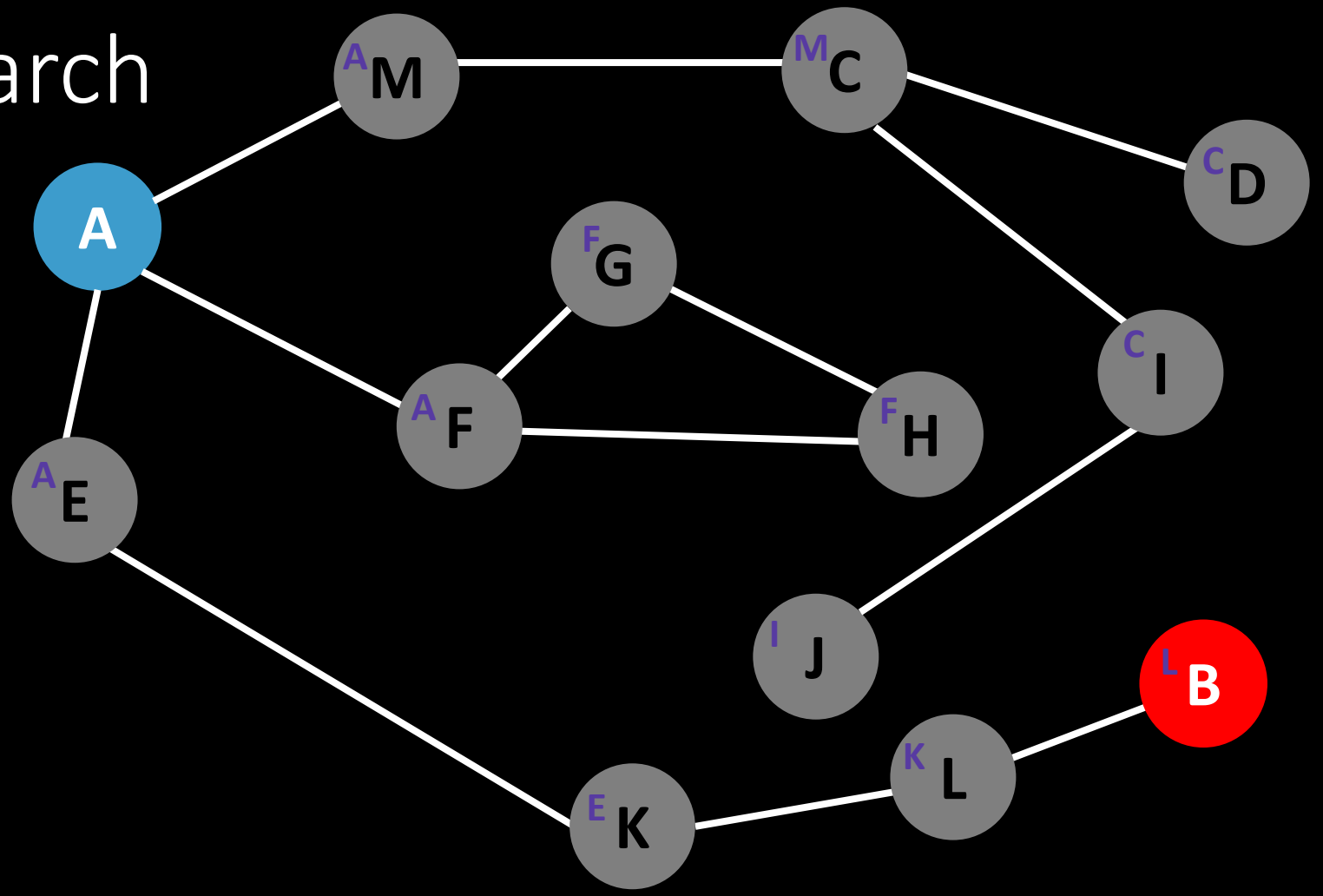


Breadth-First Search

- How to retrieve the shortest path between nodes A and B once the algorithm terminates?

Breadth-First Search

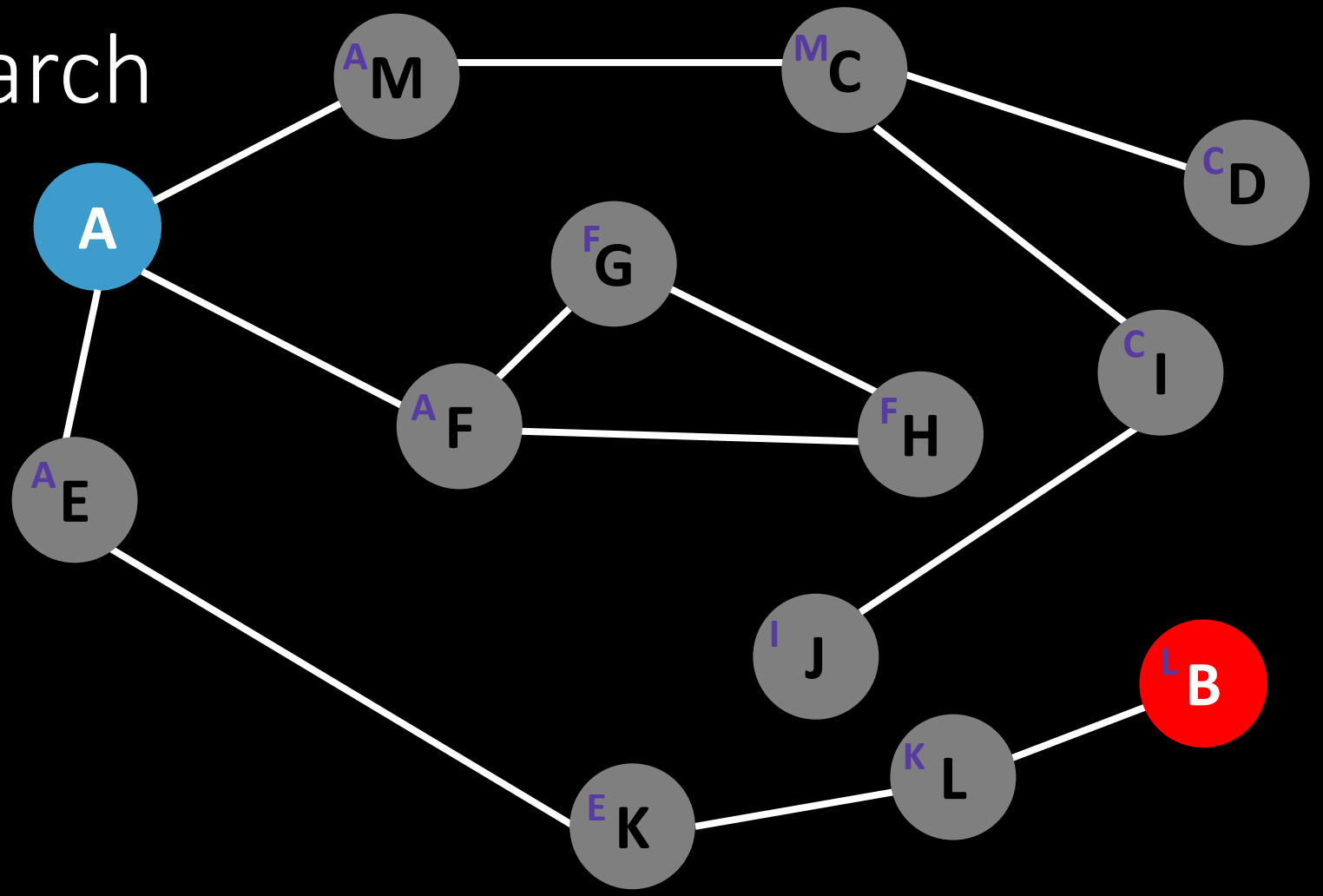
- How to retrieve the shortest path between nodes A and B once the algorithm terminates?



Breadth-First Search

Q: How to retrieve the shortest path between nodes A and B once the algorithm terminates?

A: Trace back using the indicator of the previous node



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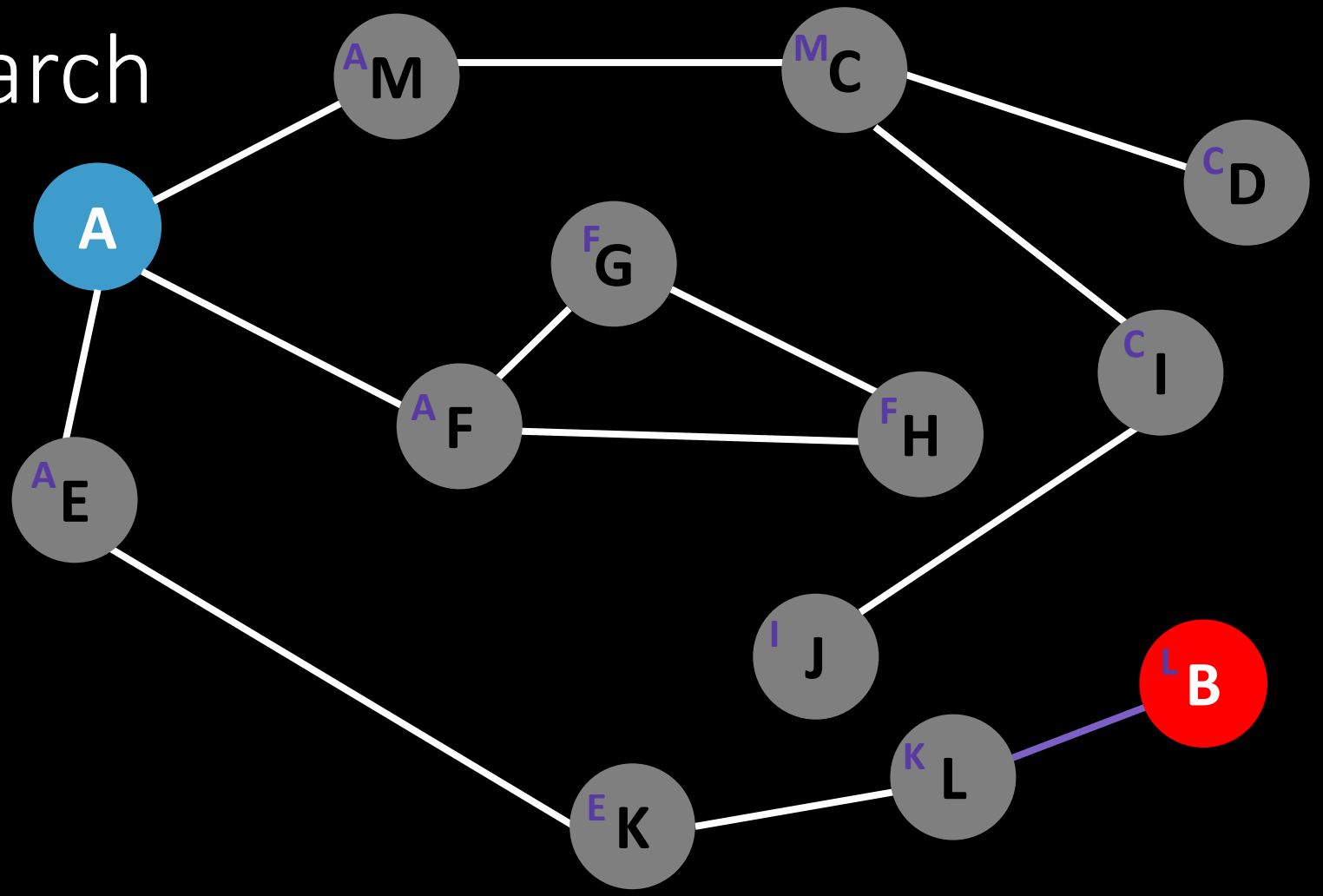

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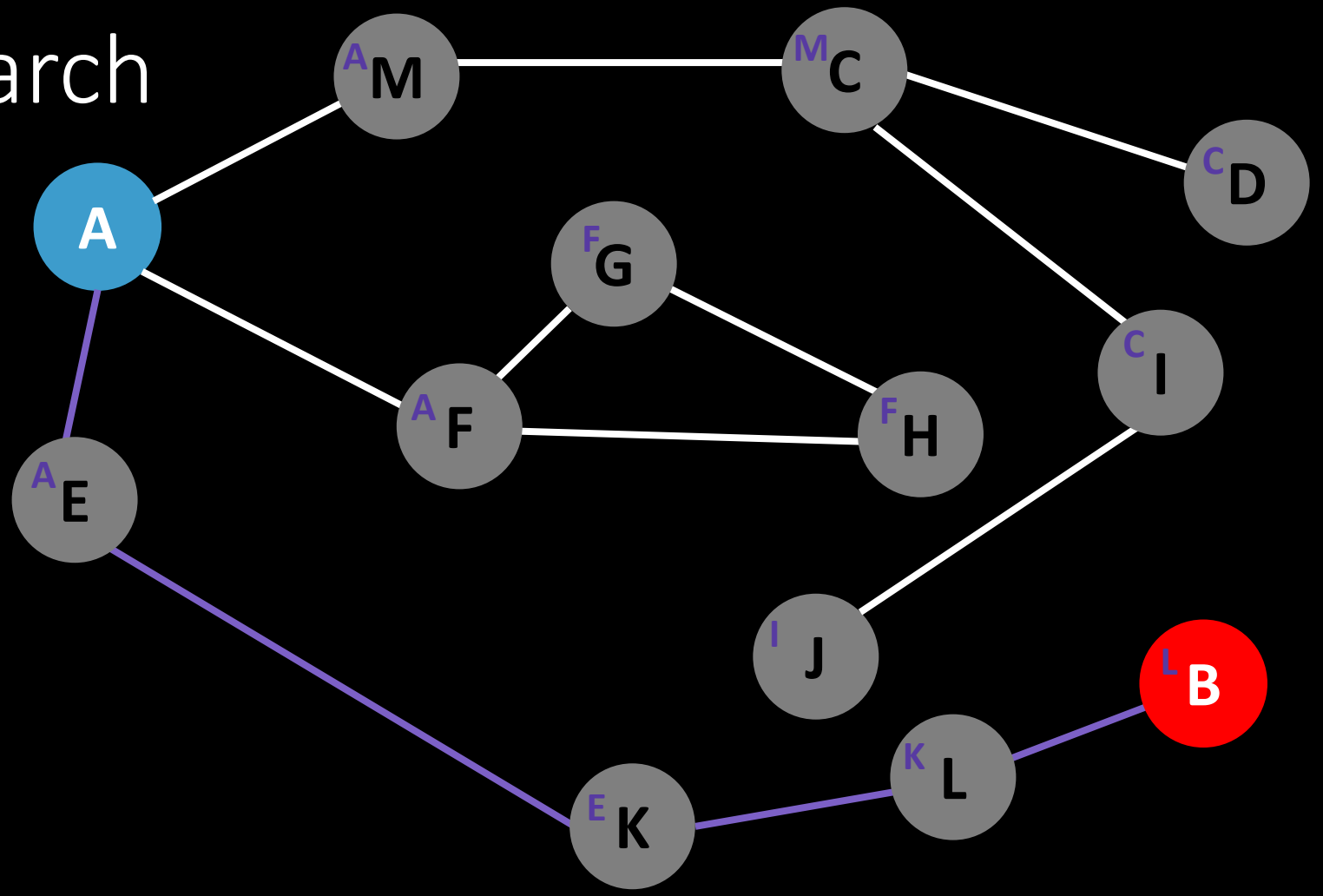
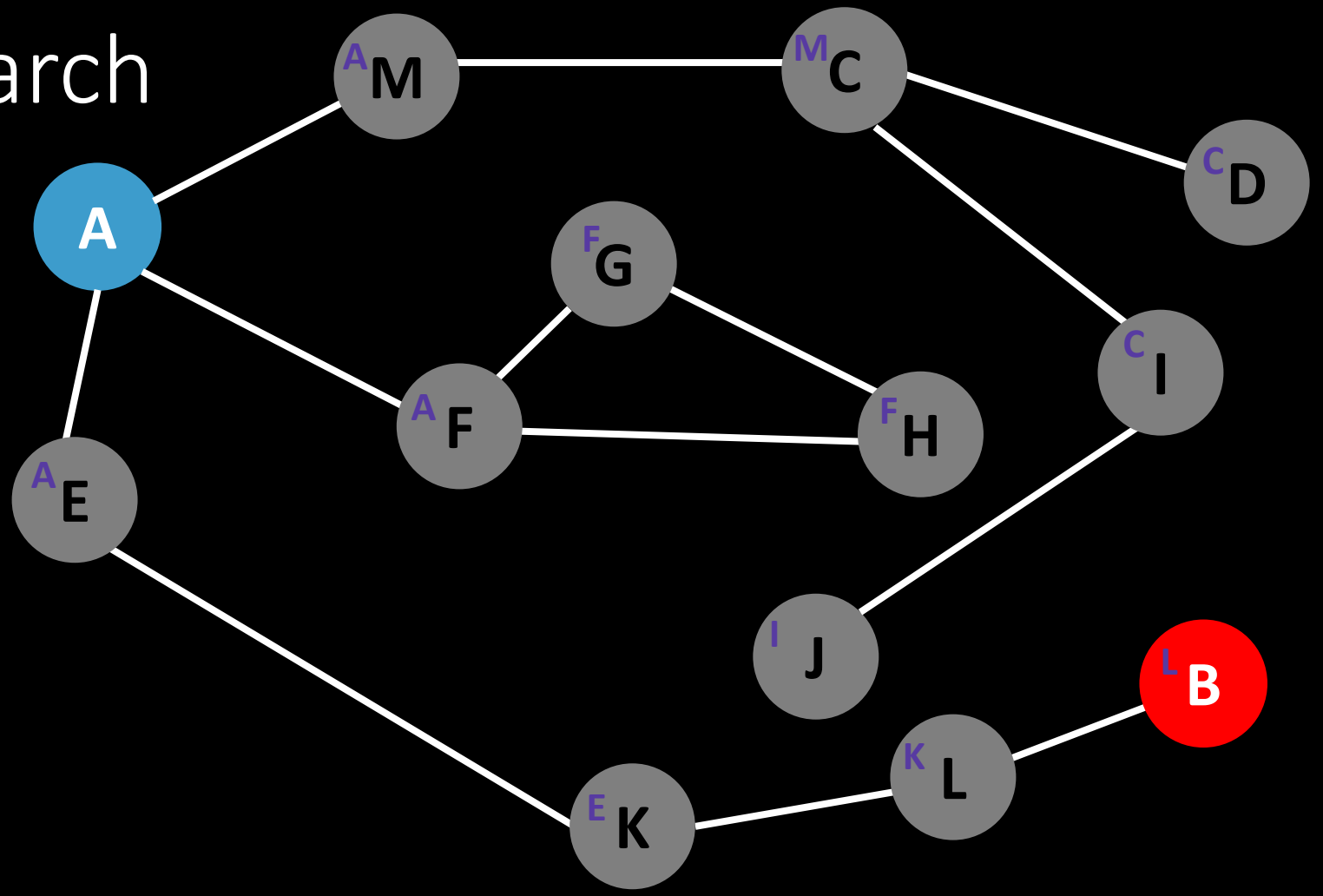


Breadth-First Search

Q: How to retrieve the shortest path between nodes A and H once the algorithm terminates?

A: Trace back using the indicator of the previous node



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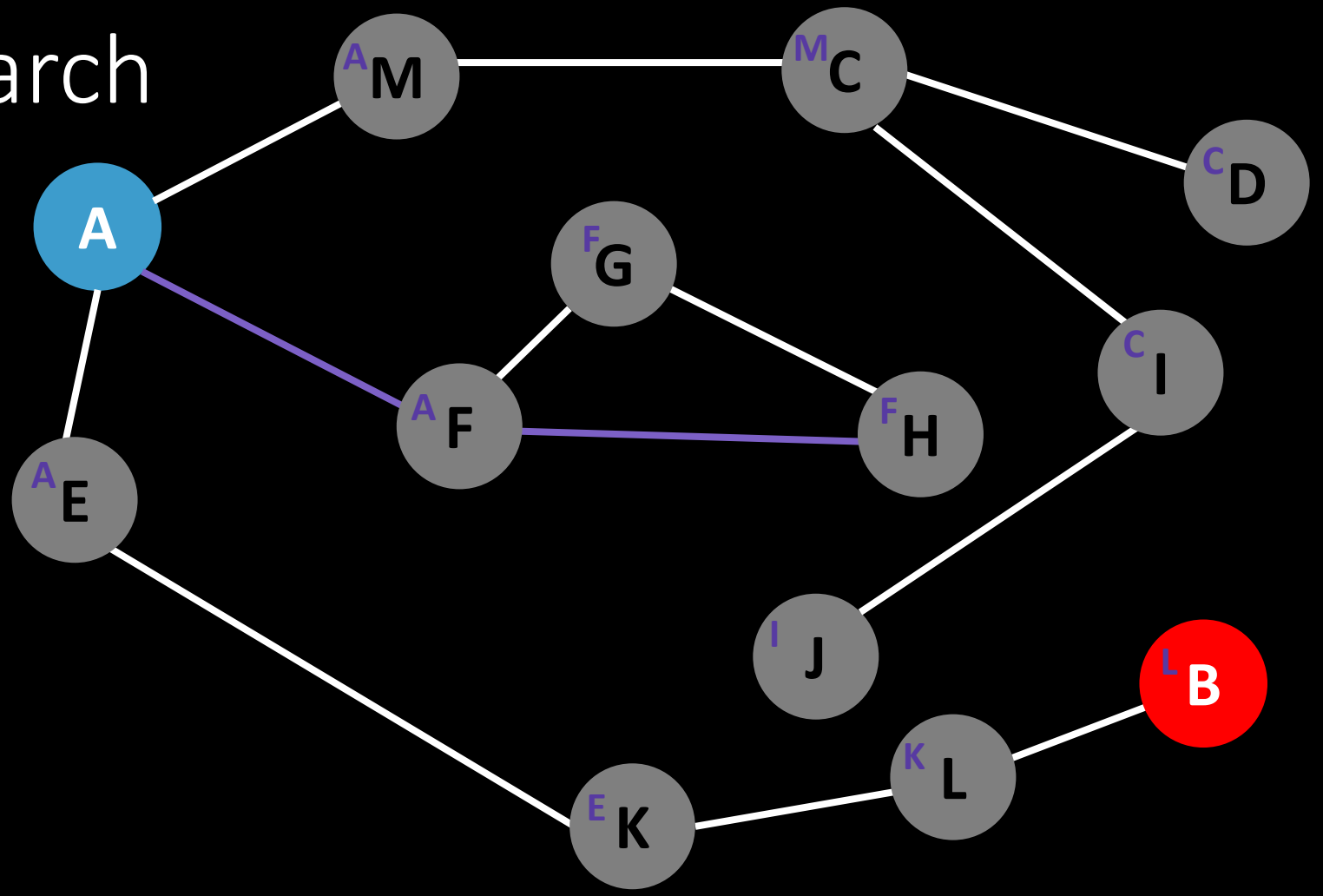
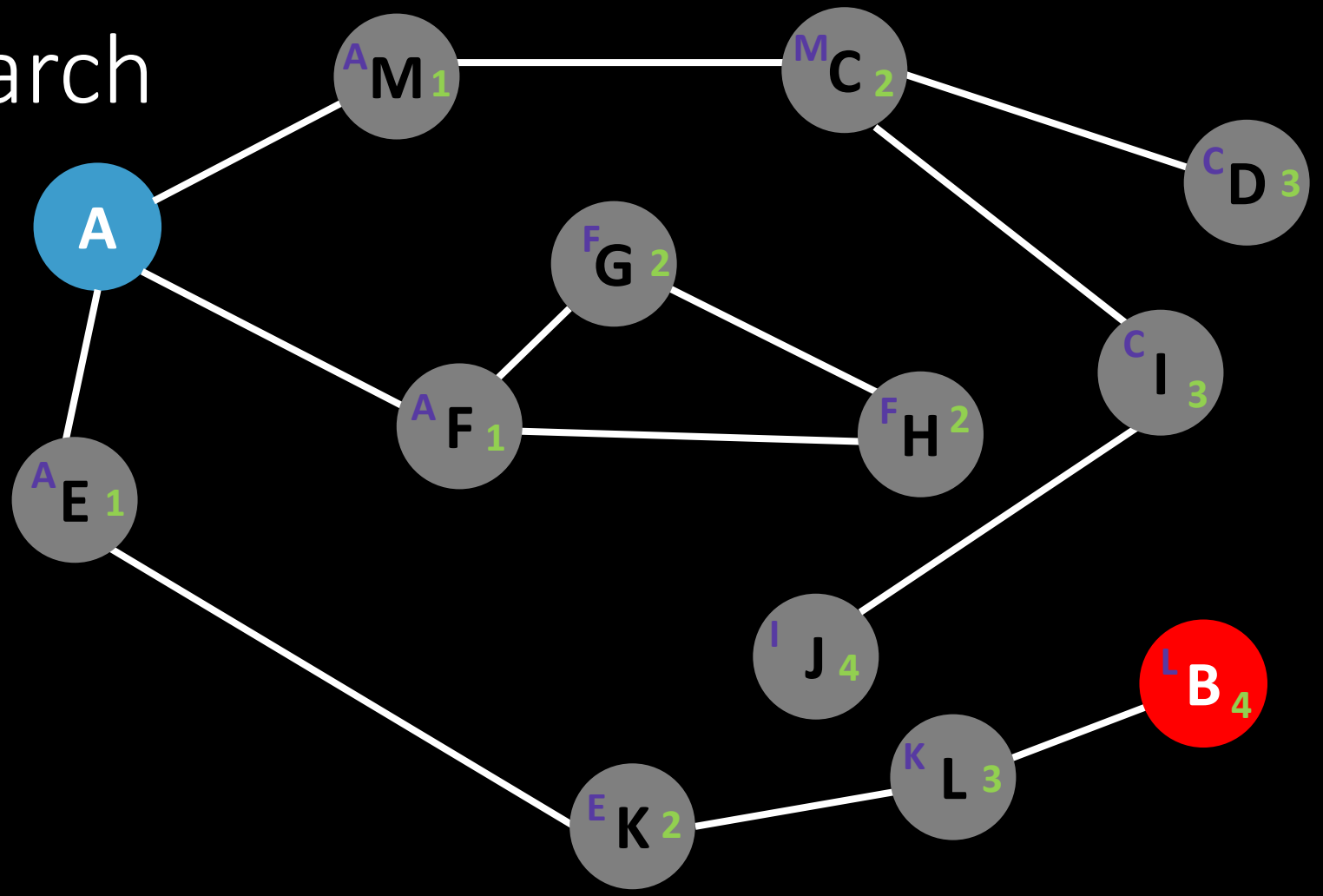


Breadth-First Search

- Sketch of proof that Breadth-First Search yields the shortest path

Breadth-First Search

- May also want to keep track of the node **level** uncovered by the BFS expansion



Depth-First Search PseudoCode

```
stack = []
stack.push(starting_node)

while (stack not empty) and (target not found):
        node = stack.pop()
        if node is visited:
                skip
        if node = target:
                target = found
        for all unvisited neighbors of node:
                neighbor.prev = node
                stack.push(neighbor)
        node.visited = True
```

Retrieving the path, if one is found

```
node = target
path = [node]
while node is not starting_node:
        path.push(node.prev)
        node = node.prev
```

Breadth-First Search PseudoCode

```
queue = []
queue.enqueue(starting_node)

while (queue not empty) and (target not found):
        node = queue.dequeue()
        if node is visited:
                skip
        if node = target:
                target = found
        for all unvisited neighbors of node:
                neighbor.prev = node
                queue.enqueue(neighbor)
        node.visited = True
```

Retrieving the path, if one is found

```
node = target
path = [node]
while node is not starting_node:
        path.push(node.prev)
        node = node.prev
```

Breadth-First Search PseudoCode

```
queue = []
queue.enqueue(starting_node)
level = 0
while (queue not empty) and (target not found):
        node = queue.dequeue()
        node.level = level++
        if node is visited:
                skip
        if node = target:
                target = found
        for all unvisited neighbors of node:
                neighbor.prev = node
                queue.enqueue(neighbor)
        node.visited = True
```

Retrieving the path, if one is found

```
node = target
path = [node]
while node is not starting_node:
        path.push(node.prev)
        node = node.prev
```

Breadth-First Search

- Consider when the graph edges do not have unit weight
- Can we still use BFS to find the shortest path?

Breadth-First Search

- Consider when the graph edges do not have unit weight
- Can we still use BFS to find the shortest path?
- Next lecture: finding the shortest path in weighted graphs
- Next week: how to implement these algorithms

Visualization Resource

- https://visualgo.net/en/dfsbfs
- https://workshape.github.io/visual-graph-algorithms/