Olin College of Engineering
Linearity 1, 2017

Studio 10 Problems

(1) Pharmacokinetic and Thermal Models
The system of differential equations

$$\begin{cases} \dot{x} = -x \\ \dot{y} = x - \frac{1}{2}y \end{cases}$$

with initial condition $x(0) = 1$, $y(0) = 0$ is a simple model for concentration of alcohol in the stomach (x) and blood (y).

Use the eigenvectors and eigenvalues of the matrix $\begin{bmatrix} -1 & 0 \\ 1 & -\frac{1}{2} \end{bmatrix}$ to find $x(t)$ and $y(t)$. To do this you will have to find the eigenvalues and eigenvectors and in addition write the initial condition as a linear combination of the eigenvalues and eigenvectors.

Could the use of linear algebra have improved your ModSim project #2? If so, how (as specifically as possible)

(2) Second order as First order
The equation

$$\ddot{x} + 2Q\dot{x} + x = 0$$

can be rewritten as the equation

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = A\begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

for some matrix A.

(a) What is the matrix A?

(b) What are the eigenvalues and eigenvectors of the matrix A?

(c) Claim: The eigenvalues of A are the same as the roots of the characteristic polynomial of $\ddot{x} + 2Q\dot{x} + x$. *Why?*

(d) Claim: The eigenvectors of A are of the form $\begin{bmatrix} 1 \\ \lambda \end{bmatrix}$. *Why?*.

(3) Particular and Homogeneous

(a) Solve $A\mathbf{x} = \mathbf{b}$ with $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$.

(b) Solve $\mathbf{x}_{n+1} = A\mathbf{x}_n + \mathbf{b}_n$ with $A = \begin{bmatrix} \frac{1}{3} & \frac{3}{4} \\ \frac{2}{3} & \frac{1}{4} \end{bmatrix}$, $\mathbf{b}_n = \left(\frac{1}{3}\right)^n$ and $\mathbf{x}_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

(c) Solve $\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{b}(t)$ with $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{b}(t) = \begin{bmatrix} 0 \\ e^{-t} \end{bmatrix}$.

(d) Solve $x_{n+1} = x_n + 2x_{n-1}$ with $x_0 = 1$ and $x_1 = 1$.

(e) Solve $\ddot{x} + 3\dot{x} + 4x = e^{-t}$ with $x(0) = 1$ and $\dot{x}(0) = -1$.

* In what way is the solution method for all of these problems similar?

(4) Concept Map

Review the key ideas that you have learned this semester and how they are connected to each other. If you are so moved, create a study guide.

(5) Quiz Question

Choose two concepts from this class that are related to each other. Write a quiz question whose solution uses these concepts.