Case Study II: Root-Finding

MTH 3150: Numerical Methods and Scientific Computing

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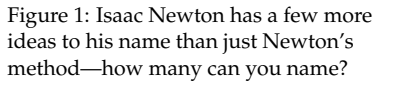

Figure 1: Isaac Newton has a few more ideas to his name than just Newton's method—how many can you name?

Overview

IN THIS CASE STUDY we examine iterative methods for finding roots of single nonlinear equations[1]. We want you to develop and implement algorithms for root-finding using the *bisection method*, *Newton's method*, *linear interpolation* (also known as secant method), and *inverse quadratic interpolation*, and determine the convergence properties of these methods by solving a given problem and comparing your performance to that of MATLAB's *fzero*.

[1] A root or zero of f is any value x which satisfies $f(x) = 0$.

A Floating Duck

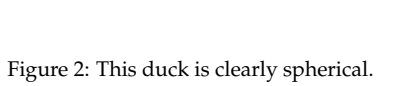

Figure 2: This duck is clearly spherical.

IMPORTANT PROBLEMS can be found in the most unlikely places. In the book *Physical Modeling in Matlab*, author Allen Downey poses the following problem (to be solved by root-finding)

> The density of a duck, ρ, is $0.3 g/cm^3$ (0.3 times the density of water). The volume of a sphere with radius r is $\frac{4}{3}\pi r^3$. If a sphere with radius r is submerged in water to a depth d, the volume of the sphere below the water line is
>
> $$\text{volume} = \frac{\pi}{3}(3rd^2 - d^3),\ d < 2r \tag{1}$$
>
> An object floats at the level where the weight of the displaced water equals the total weight of the object.[2] Assuming that a duck is a sphere with radius 10 cm, at what depth does a duck float?

[2] Whose principle is this, and which other forces does it ignore?

Objectives

THIS CASE STUDY has several goals: first, we want you to examine a variety of methods used for finding solutions to single nonlinear equations; we want you to implement *bisection method*, *Newton's method*, *linear interpolation*, and *inverse quadratic interpolation* in a general setting, and validate them on a problem with a known solution[3]; and third, we want you to write a report in which you put these various pieces together in a professionally-written document and in which you demonstrate your understanding of the methods and their context. In particular, we want you to

[3] In this case we mean a problem whose solution can be found by exactly—you might need to remind yourself how to solve polynomial equations.

- Develop a set of functions in MATLAB in order to solve the duck problem using *bisection method*, *Newton's method*, *linear interpolation*, and *inverse quadratic interpolation* and compare the performance of your algorithms in terms of rate of convergence and computation time to that of *fzero*.

Report

Prepare a brief typewritten report of roughly 4 pages in length in which you review the methods used, your implementation using code snippets as evidence, and your results. At a minimum your report should include:

- a section which briefly discusses the methods used, and places them in the broader context of methods for finding roots of nonlinear functions[4].
- a section which briefly discusses your implementation of the methods on the duck problem, using small pieces of code as evidence.
- a section which briefly discusses the results of your simulations, including (but not limited to)
  - a graph which compares the rate of convergence and computation time for each method when applied to the duck problem.
- a section which briefly summarizes your findings, including questions you have about this case study and any reflection you care to provide.

[4] This means you need to **read** about methods for root-finding from a variety of sources, i.e. not just Wikipedia.