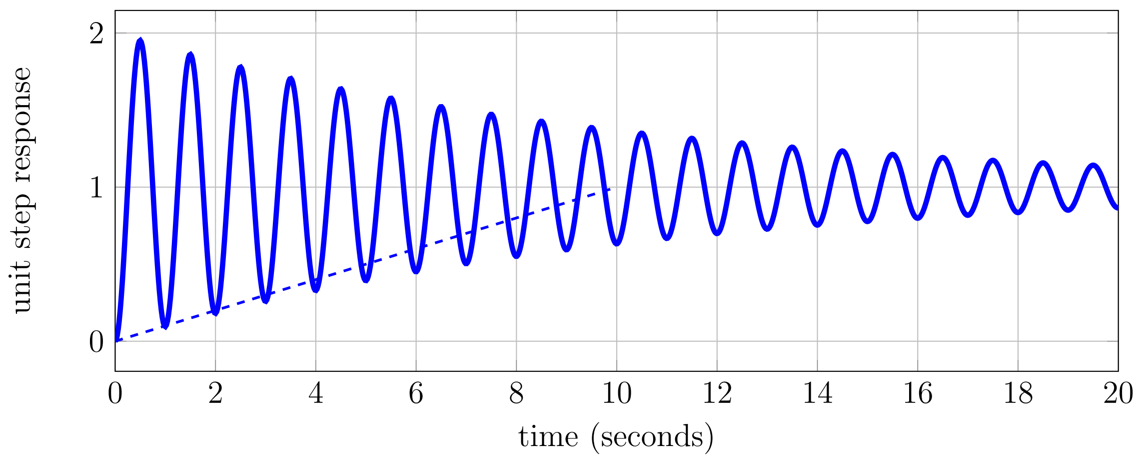
Olin College of Engineering
ENGR2410 – Signals and Systems

Quiz 2

Name:____________________________________

You have 90 uninterrupted minutes to complete this quiz. You may use one page of notes in addition to the overview and tables document in the website. Use alternate methods to confirm or check your answers. Awareness of errors and/or the right answer is worth quite a bit of partial credit! Good luck!

Problem 1 A system has no zeros and the unit step response shown below. You may make reasonable approximations. *Hint: The dashed line is tangent to the envelope of the oscillations at* $t = 0$.



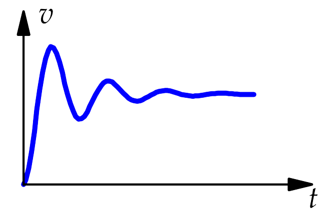
A. Draw the pole zero diagram of the system, labeling the location of the poles clearly.

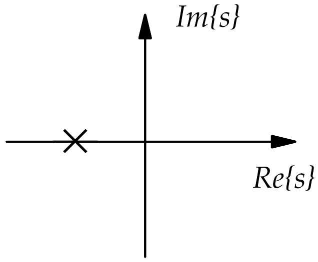
B. Compute the quality factor Q of this system.

C. Write the transfer function of the system.

Problem 2 For each part, indicate whether the system described at the beginning may be equivalent to the system representations listed afterwards. You may choose different values to match each representation. Note that any number of the representations below may be equivalent, including all or none. Provide a specific example for representations that are equivalent or clear justifications for representations that are not. *An answer without justification will not receive credit.*

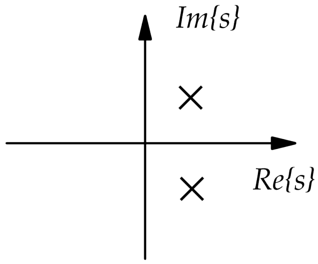
A. A system is described by the step response





Circle your answer:

equivalent **not equivalent**



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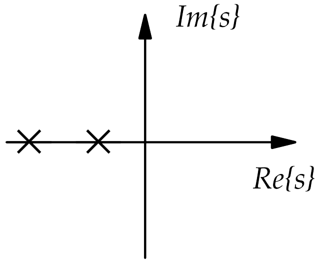

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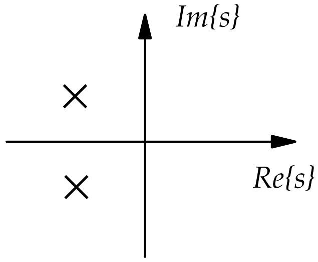

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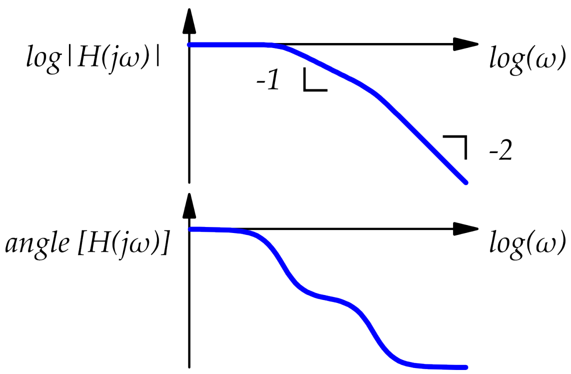

Circle your answer:

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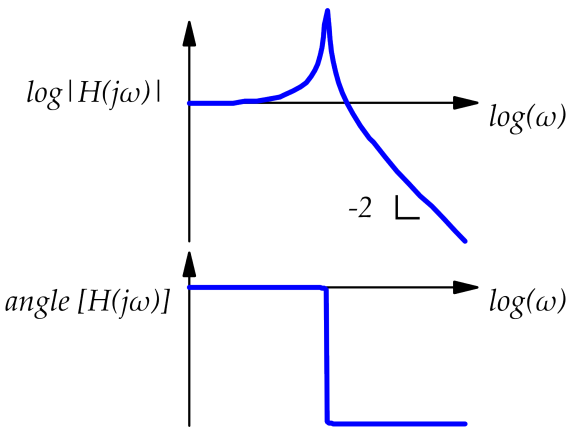
B. A system is described by the transfer function

$$\frac{K}{s^2 + 2\alpha s + \omega_n^2}$$

where K, α and ω_n are **real** and **positive**.



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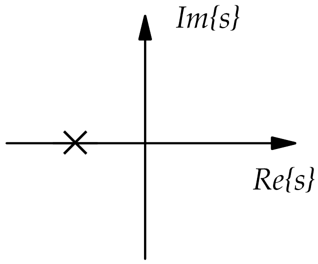

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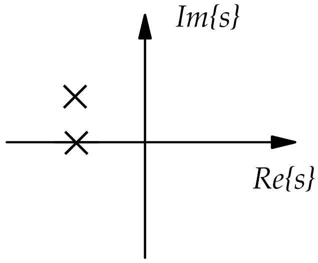

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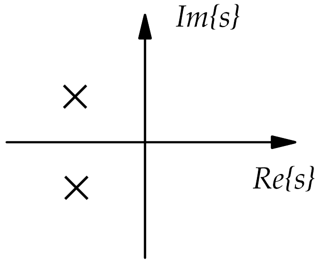

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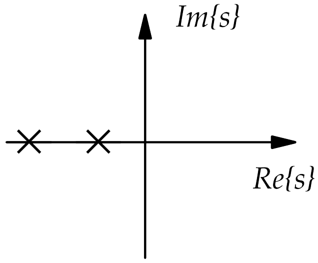

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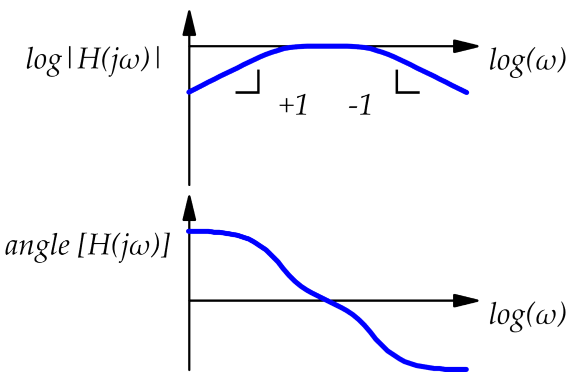
C. A system is described by the transfer function

$$\frac{s}{(s+a)(s+b)}$$

where a and b are **real** and **positive**, and $a \neq b$.



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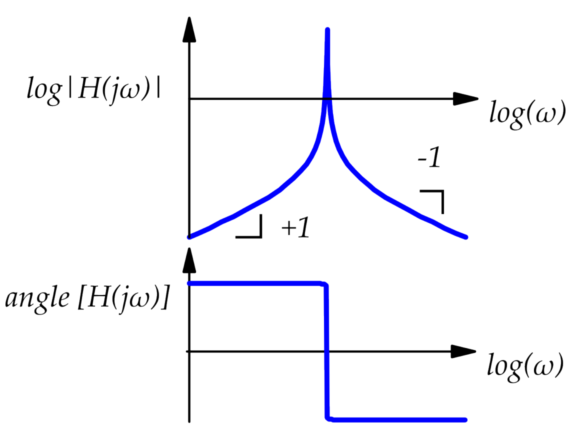

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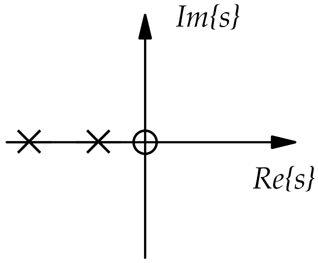

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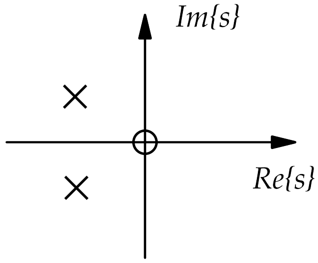

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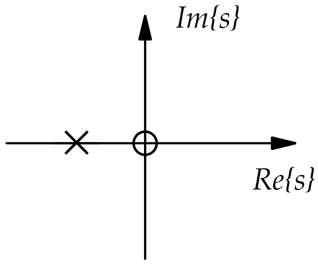

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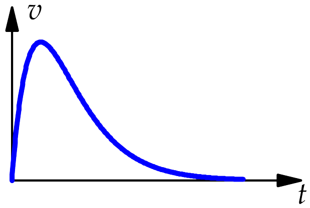
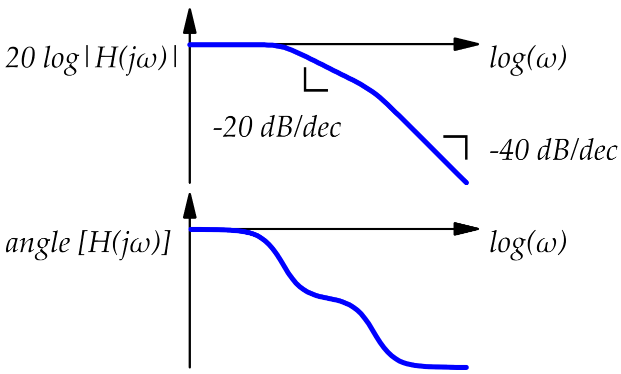
D. A system is described by the impulse response





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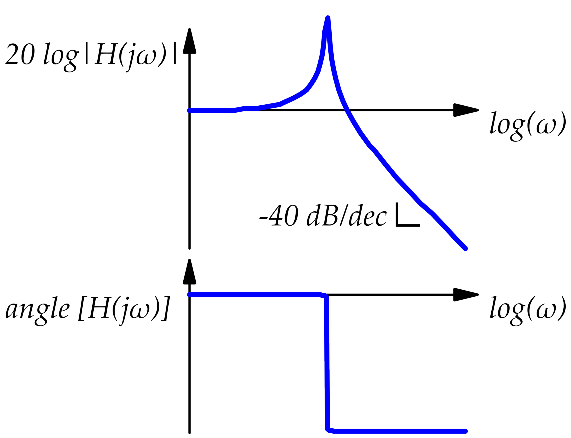

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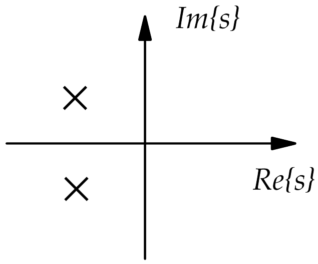

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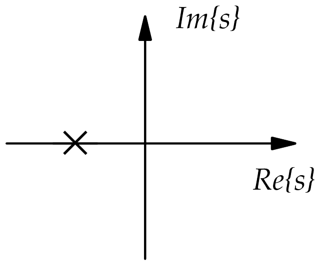

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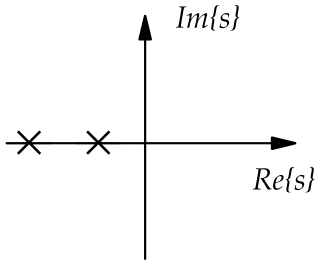

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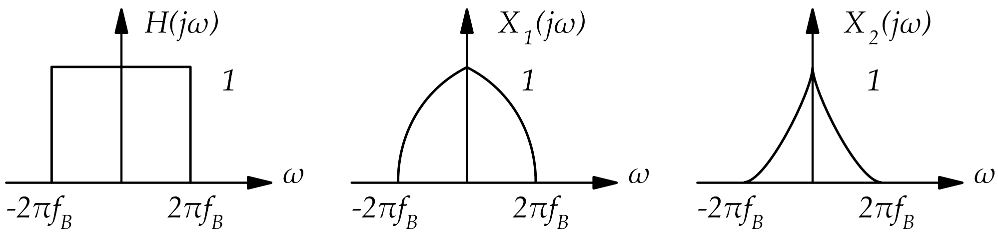
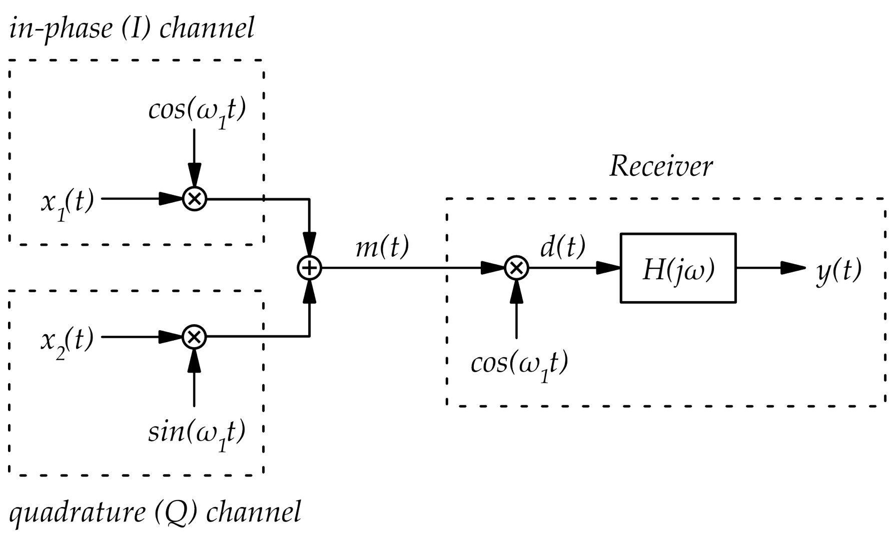
Problem 3 The system shown below introduces the concept of *quadrature*, where we send multiple signals that share not only a common channel, but also the same frequency band. Signals $x_1(t)$ and $x_2(t)$ are bandlimited to f_B and have a frequency content as shown below. The receiver has an ideal low-pass filter $H(j\omega)$ with a cutoff frequency of f_B as shown below.





A. Show that the frequency content of $M(j\omega)$ is

$$M(j\omega) = \frac{1}{2}X_1(\omega - \omega_1) + \frac{1}{2}X_1(\omega + \omega_1) - j\frac{1}{2}X_2(\omega - \omega_1) + j\frac{1}{2}X_2(\omega + \omega_1)$$

The result can be either the expression above or neatly labeled sketches of both the real and imaginary parts of $M(j\omega)$. *Hint: You can use either equations or sketches to find the solution, but using both may help you avoid algebra mistakes.*

B. Find an expression for $y(t)$. Justify your answer clearly.

C. Find an expression for $y(t)$ if $m(t)$ is multiplied by $\sin(\omega_1 t)$ in the receiver instead of multiplied by $\cos(\omega_1 t)$. *Hint: You can use your intuition (and should!) to guess the answer, but a clear justification will get more points.*