# Olin College of Engineering
# ENGR2410 – Signals and Systems

## Assignment 8

**Problem 1** In this problem, you will analyze several discrete time filters. Recall that the transfer function of the first-order difference equation

$$y[n] - ay[n-1] = x[n] \qquad \text{is} \qquad H(\Omega) = \frac{1}{1 - ae^{-j\Omega}}.$$

A. Plot the magnitude and phase of $H(\Omega)$ from $-3\pi$ to $3\pi$ when $a = 0.9$.

B. Since $e^x \approx 1 + x$ when $x \ll 1$, we can examine the behavior of the filter when $\Omega \approx 2\pi n$.

$$H(\Omega) \approx \frac{1}{1 - a(1 - j\Omega)} = \frac{\frac{1}{a}}{\frac{1-a}{a} + j\Omega} = H_{approx}(\Omega)$$

Make a Bode plot of both $H(\Omega)$ and $H_{approx}(\Omega)$ when $a = 0.9$ and $10^{-3} < \Omega < 2\pi$. What kind of filter is this?

C. Redo the part A when $a = -0.9$. What kind of filter is this? Explain clearly.

D. Find the transfer function for the difference equation below and plot it as in part A. What kind of filter is this? Explain clearly.

$$y[n] + 0.9y[n-2] = x[n]$$

E. Find the transfer function for the difference equation below and plot it as in part A. This is called a *comb filter*.

$$y[n] = x[n] - 0.9x[n-5]$$

F. Find and sketch the impulse response for the comb filter of part E. *This type of filter is a* finite impulse response *(FIR) filter. The first order difference equation of part A is an* infinite impulse response *(IIR) filter.*

**Problem 2** In this problem, you will find the impulse response of an analog delay filter implemented using a digital filter when the delay is smaller than the sampling frequency of the digital filter.

A. Find and sketch the transfer function $H_c(j\omega)$ such that

$$y_c(t) = x_c\left(t - \frac{1}{3f_S}\right)$$

in the system shown below, assuming $x_c(t)$ is bandlimited by $f_{max}$ such that the sampling frequency $f_S > 2f_{max}$.

$$x_c(t) \longrightarrow \underset{\substack{\uparrow \\ f_S}}{\boxed{\text{C/D}}} \xrightarrow{x_d[n]=x_c\left(\frac{n}{f_S}\right)} \boxed{H_d(\Omega)} \xrightarrow{y_d[n]} \underset{\substack{\uparrow \\ f_S}}{\boxed{\text{D/C}}} \xrightarrow{y_c(t)=y_d[n],\text{ if } t=\frac{n}{f_S}} y_c(t)$$

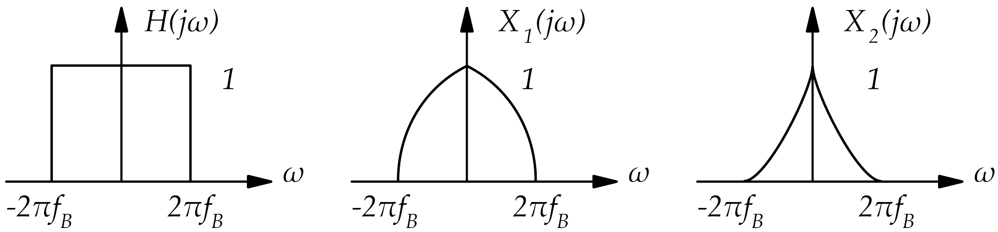
B. Find and sketch $H_d(\Omega)$.

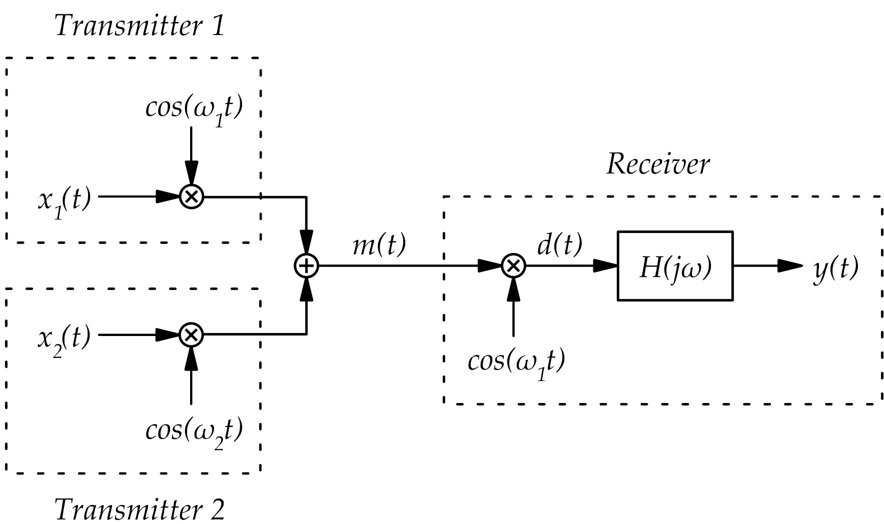
C. Find the naive expression for $y_d[n]$ in terms of $x_d[n]$ by transforming $H_d(\Omega)$. Note that while your result is technically true, it cannot be applied literally! The next two parts give us the actual answer.

D. Assume $x_c(t) = \text{sinc}(\pi f_s t)$. Verify that $x_d[n] = \delta[n]$. Combine both plots in the same set of axes.

E. Find $y_c(t)$ and $y_d[n]$. Explain why $y_d[n] = h_d[n]$. Combine both plots in the same set of axes.

**Problem 3** The system shown below represents a basic communication system where two messages $x_1(t)$ and $x_2(t)$ share a common communication channel. Signals $x_1(t)$ and $x_2(t)$ are bandlimited to $f_B$ and have a frequency content as shown below. The receiver has an ideal low-pass filter $H(j\omega)$ with a cutoff frequency of $f_B$ as shown below.




A. What would happen if $\omega_1 = \omega_2 = 0$? Find $y(t)$ in terms of $x_1(t)$ and/or $x_2(t)$, and show its frequency content.

B. Find constraints on $\omega_1$ and $\omega_2$ such that there is no frequency interference (aliasing). Show the frequency content of $m(t)$ and $d(t)$ under these constraints. *Note: There may be multiple solutions; just find one that works.*

C. Show the frequency content and find an algebraic expression for $y(t)$ in terms of $x_1(t)$ and/or $x_2(t)$ assuming the constraints of part B.

**Course feedback**

Feel free to send any additional feedback directly to us.

Name (optional):

A. End time: How long did the assignment take you?

B. Are the lectures understandable and engaging?

C. Was the assignment effective in helping you learn the material?

D. Are you getting enough support from the teaching team?

E. Are the connections between lecture and assignment clear?

F. Are the objectives of the course clear? Do you feel you are making progress towards those objectives?

G. Anything else?