Olin College of Engineering
ENGR2410 – Signals and Systems

Assignment 9

Problem 1 *(2 points)* Show Parseval's Theorem

$$\int_{t=-\infty}^{t=\infty} [x(t)]^2 dt = \int_{\omega=-\infty}^{\omega=\infty} |X(j\omega)|^2 \tfrac{d\omega}{2\pi}$$

where

$$|X(j\omega)|^2 = X(j\omega)X^*(j\omega)$$

$x(t)$ is real, and $X^*(j\omega)$ is the complex conjugate of $X(j\omega)$.
Hint:

$$X^*(j\omega) = \int_{t=-\infty}^{t=\infty} x(t)e^{j\omega t} dt$$

Problem 2 *(4 points)* A non-ideal filter or communication channel $H(j\omega)$ with finite *transition band* Δf between the *passband* (where $H(j\omega) \neq 0$) and *stopband* (where $H(j\omega) = 0$) can be modeled as $H(j\omega) = H_0(j\omega) * H_\Delta(j\omega)$ where

$$H_0(j\omega) = \begin{cases} 1 & -2\pi f_0 < \omega < 2\pi f_0 \\ 0 & \text{otherwise} \end{cases} \qquad H_\Delta(j\omega) = \begin{cases} \frac{1}{2\pi\Delta f} & -2\pi\Delta f/2 < \omega < 2\pi\Delta f/2 \\ 0 & \text{otherwise} \end{cases}$$

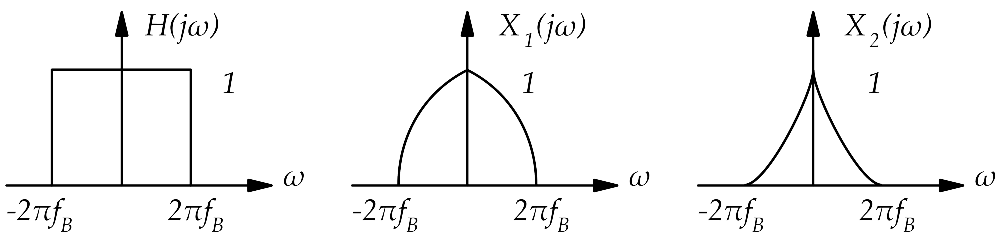
A. Find an expression for $h_0(t)$, the impulse response of the ideal channel.

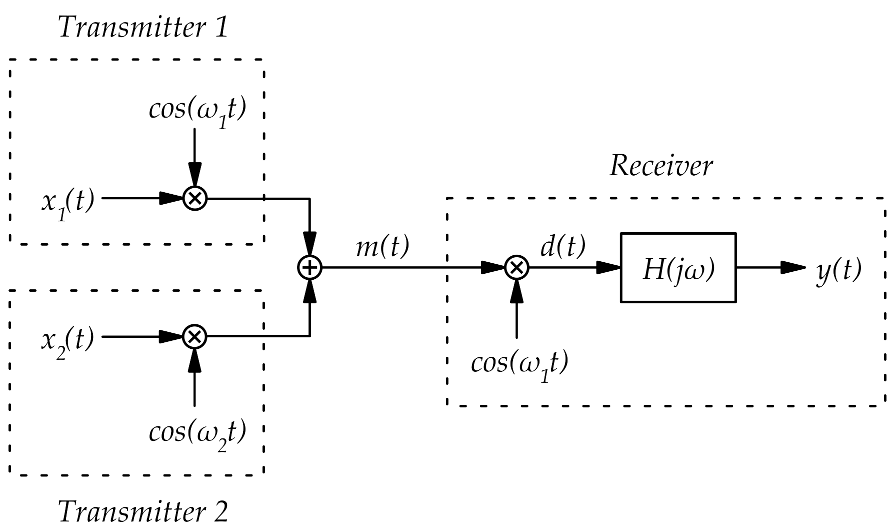
B. Find an expression for $H(j\omega)$ and sketch it.

C. Find an expression for the impulse response $h(t)$ of the non-ideal channel.

D. Plot $h(t)$ and $h_0(t)$ on the same axes when $f_0 = 10$ kHz and $\Delta f = 2$ kHz and sketch the associated $H(j\omega)$. Repeat the plot for the same values when $\Delta f = 5$ kHz. Compare the plots. For what values of Δf is the effect of the non-ideal transition band noticeable? Is the effect what you would expect? Explain.

Problem 3 *(4 points)* The system shown below represents a basic communication system where two messages $x_1(t)$ and $x_2(t)$ share a common communication channel. Signals $x_1(t)$ and $x_2(t)$ are bandlimited to f_B and have a frequency content as shown below. The receiver has an ideal low-pass filter $H(j\omega)$ with a cutoff frequency of f_B as shown below.





A. What would happen if $\omega_1 = \omega_2 = 0$? Find $y(t)$ in terms of $x_1(t)$ and/or $x_2(t)$, and show its frequency content.

B. Find constraints on ω_1 and ω_2 such that there is no frequency interference (aliasing). Show the frequency content of $m(t)$ and $d(t)$ under these constraints. *Note: There may be multiple solutions; just find one that works.*

C. Show the frequency content and find an algebraic expression for $y(t)$ in terms of $x_1(t)$ and/or $x_2(t)$ assuming the constraints of part B.