Olin College of Engineering
ENGR2410 – Signals and Systems

Quiz 1

Name:____________________________________

You have 90 uninterrupted minutes to complete this quiz. You may use one page of notes in addition to the overview and tables document in the website. Use alternate methods to confirm or check your answers. Awareness of errors and/or the right answer is worth quite a bit of partial credit! Good luck!

Problem 1 Find the Fourier series coefficients c_n for the functions shown below such that

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi}{T}nt}$$

as well as the average power $< p >= \frac{1}{T}\int_{t_0}^{t_0+T} |v(t)|^2 dt$.

A. $v(t) = V + \frac{V}{2}\cos\left(\frac{2\pi}{T}t\right)$

B. $v(t) = V \sin\left(\frac{2\pi}{T}t\right) + \frac{V}{2}\cos\left(\frac{2\pi}{T}t\right)$

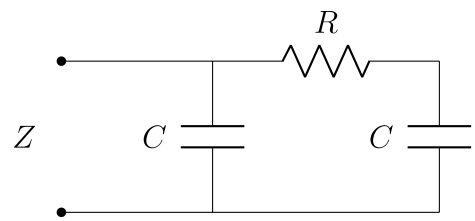
C. $v(t) = V \sin\left(\frac{2\pi}{T}t\right) + \frac{V}{2}\cos\left(2\frac{2\pi}{T}t\right)$

Problem 2

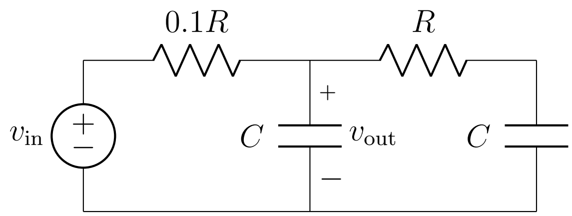
A. Show that the impedance Z in the circuit below is

$$Z = \frac{1}{Cs} || \left(R + \frac{1}{Cs} \right) = \frac{1}{Cs} \cdot \frac{s + \frac{1}{RC}}{s + \frac{2}{RC}}$$

where $||$ is the parallel operator, $R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2}$.

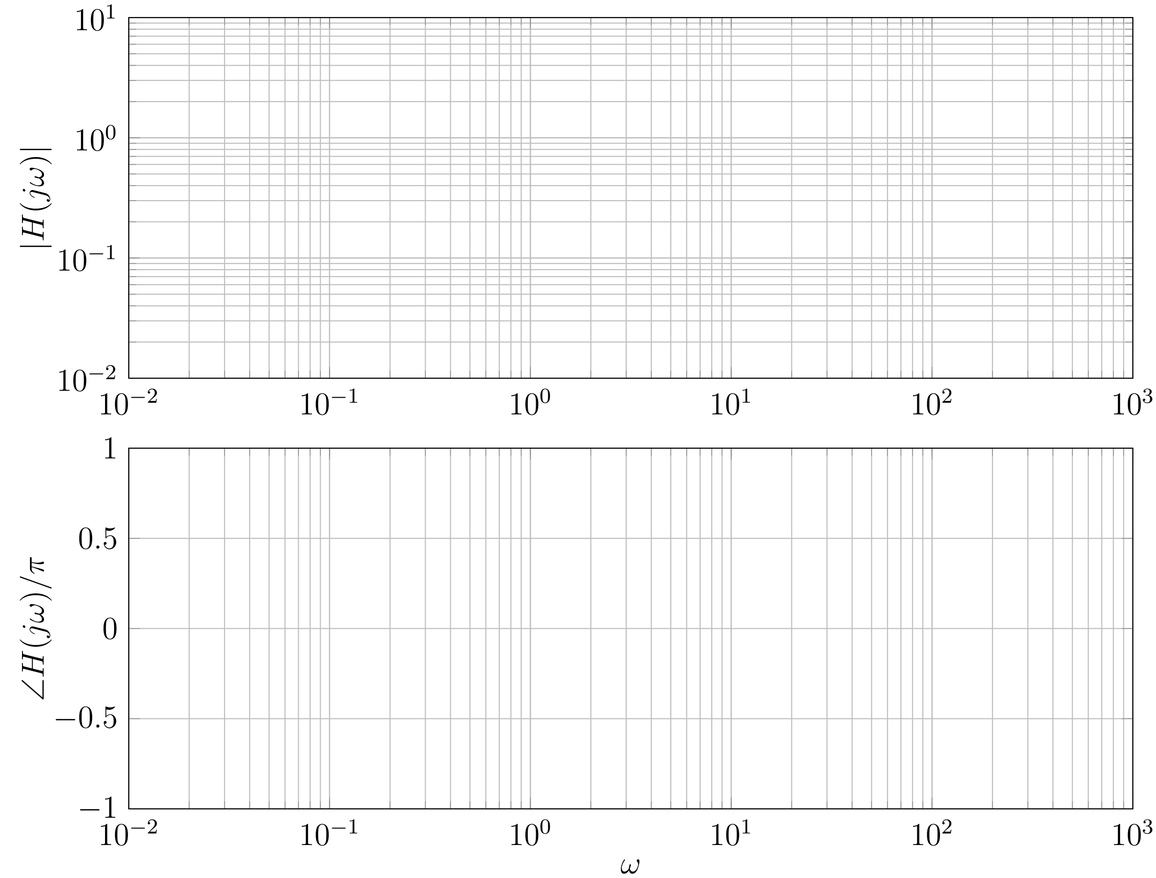


B. Find the transfer function V_{out}/V_{in} in the circuit below.



C. Find the natural frequencies of the system. Is the system underdamped ($\omega_0 \gg \alpha$), or overdamped? Make reasonable approximations (less than 10% error) when appropriate. *Hint: You may make the approximation* $\sqrt{26} \approx 5$.

D. Sketch the Bode plot using asymptotic approximations (i.e., let $\omega \to \infty$ and $\omega \to 0$). Again, make reasonable approximations. *Hint:* $\left|\frac{1+10j}{-9+12j}\right| = 0.67, \angle\left(\frac{1+10j}{-9+12j}\right) = -\pi/4.2$



E. Find $v_{out}(t)$ if $v_{in}(t)$ is a 1 Volt step at $t = 0$ (i.e., $v_{in}(t) = 1 \text{ V} \cdot u(t)$).

(i) Find the particular, or steady-state, solution when all transients have died.

(ii) Write the general solution, which includes the particular and the homogeneous solution before specifying the initial conditions.

(iii) The capacitor is initially discharged ($v_{out}(0) = 0$). The initial current through the capacitor is $i_C(0) = 10\ \mathrm{V}/R$, which means that the initial slope at $t = 0$ is $\dot{v}_{out}(0) = i_C(0)/C = 10\ \mathrm{V}/(RC)$. Use these initial conditions to find an algebraic expression for $v_{out}(t)$.

(iv) Sketch $v_{out}(t)$.