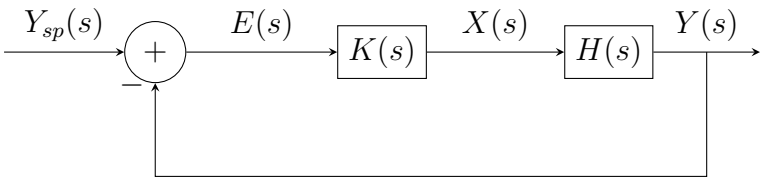
Olin College of Engineering
ENGR2410 – Signals and Systems

Assignment 10

Problem 1 *(2 points)* Use the Laplace transform to verify that the step response of the system $\dot{y} + y = x$ is $y(t) = (1 - e^{-t})u(t)$. Be sure to indicate the regions of convergence of any functions in the s-plane. You will have to refresh your partial fraction expansions.

Problem 2 *(2 points)* In this problem, you will explore the properties of integral control, as compared to proportional control. Recall that with proportional control, the DC gain depends on the amount of feedback, K_p. This is known as *offsett error*. Integral control eliminates this error at the cost of introducing oscillations and possibly making the overall system unstable.

A. Find the DC gain of the system $Y(s)/Y_{sp}(s)$ below if you use an integral controller $K(s) = K_I/s$ for any $H(s)$ and verify that it is independent of the value of K_I.



B. Assume $H(s) = \frac{1/\tau}{s+1/\tau}$. Find $Y(s)/Y_{sp}(s)$. Find the pole(s) of the system assuming $K_I \gg 1/\tau$. Compare the pole-zero diagram and step response of $H(s)$ and $Y(s)/Y_{sp}(s)$ in this case.

Problem 3 *(2 points)* Use Matlab to analyze the behavior of the systems listed. Feel free to use the code shown below.

```
s=tf('s');h=(s^2+1)/(s^2+3*s+1)
subplot 311;bode(h)
subplot 312;pzmap(h)
subplot 313;step(h)
```

For each system, note the relationship between all three plots: order of the system, number of poles and zeros, real or complex poles, oscillations and so forth. Hand in a couple of sentences for each system describing its behavior and any notable characteristics concisely.

A. $\dfrac{s}{s+1}$

B. $\dfrac{s}{s^2+100s+1}$

C. $\dfrac{s}{s^2+s+1}$

D. $\dfrac{s}{s^2+0.1s+1}$

E. $\dfrac{s^2-0.01s+1}{s^2+0.01s+1}$

F. $\dfrac{s^2+0.1s+1}{s^2+0.11s+1}$

Problem 4 *(4 points)* You are asked to stabilize the system

$$H(s) = \frac{1}{s^2-0.01s+1}$$

Do the algebra by hand in this problem. Matlab will introduce numerical errors that will give you the wrong answer!

A. Plot the step response and pole-zero map of this system using Matlab.

B. Use the pole-zero map to show the effect of using proportional control on this system. Show the step response of at least two feedback gains to illustrate. Can you stabilize the system?

C. Repeat part B using integral control.

D. Repeat part B using differential (or derivative) control.