Olin College of Engineering
ENGR2410 – Signals and Systems

Quiz 8

Instructions

A. Collaboration is not allowed on quizzes.

B. Students may only use a page of notes and the tables from the website during the quizzes.

C. Time is limited to one continuous hour.

D. Quizzes are due at the beginning of lecture on Thursday.

E. Late or missed quizzes will be given a score of zero. Any excuses must come directly from the Office of Student Life.

F. The two lowest quiz scores will be eliminated to allow for unforeseeable circumstances.

G. In case of doubt, students are expected to base their behavior on the values expressed in the Honor Code.

Name:

Start time:

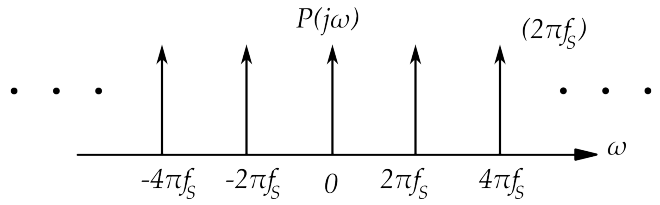
Problem 1 *(5 points)* If the frequency content of a signal is concentrated in a narrow band, we can sample at much lower frequencies.

A. Sketch $P(j\omega)$, the transform of $p(t)$, where

$$p(t) = \sum_n \delta(t - n/f_S)$$

Solution:

$$P(j\omega) = \sum_n 2\pi f_S \delta(\omega - 2\pi n f_S)$$



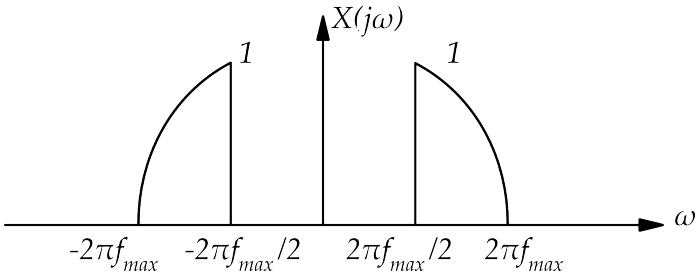
B. If $x_S(t) = x(t)p(t)$, find an expression for $X_S(j\omega)$ in terms of $X(j\omega)$.

Solution:

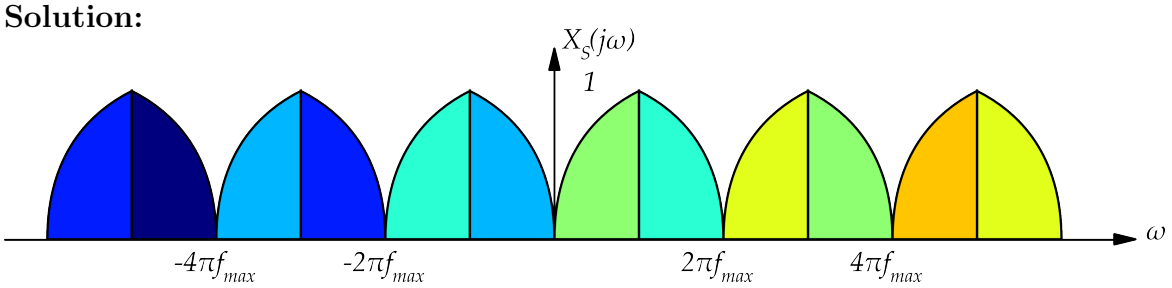
$$X_S(j\omega) = \frac{1}{\cancel{2\pi}} X(j\omega) * \sum_n \cancel{2\pi} f_S \delta(\omega - 2\pi n f_S)$$

$$X_S(j\omega) = f_S X(\omega - 2\pi n f_S)$$

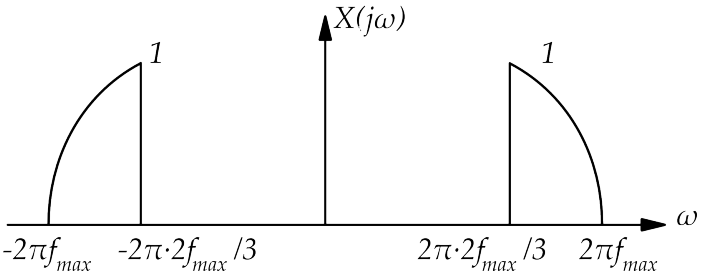
C. Assume that $X(j\omega)$, the frequency content of $x(t)$, is bounded between f_{max} and $f_{max}/2$ as shown below. Show that you can avoid aliasing by sampling at f_{max}. Sketch clearly and carefully $X_S(j\omega)$, the frequency content of $x_S(t)$.



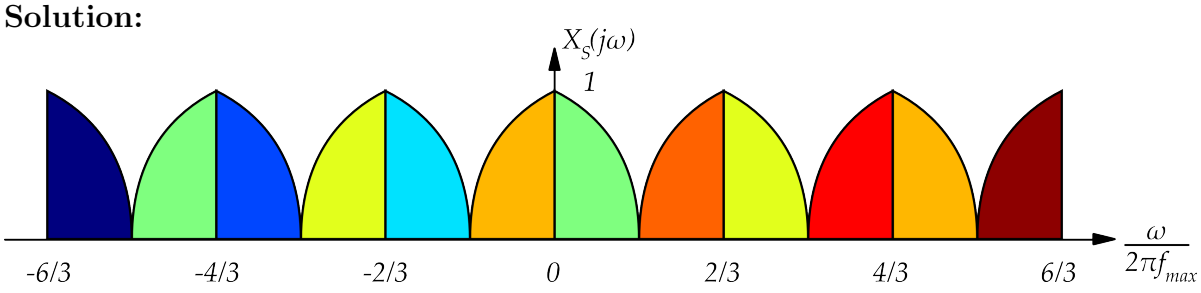
Solution:



D. Assume that $X(j\omega)$, the frequency content of $x(t)$, is bounded between f_{max} and $2f_{max}/3$ as shown below. Show that you can avoid aliasing by sampling at $2f_{max}/3$. Sketch clearly and carefully $X_S(j\omega)$, the frequency content of $x_S(t)$.



Solution:



Problem 2 *(5 points)* The system

$$y[n] = \frac{1}{3}x[n-1] + \frac{1}{3}x[n] + \frac{1}{3}x[n+1]$$

is a moving average.

A. Find the transfer function $H(\Omega) = Y(\Omega)/X(\Omega)$ for this system. *Hint: it can be simplified into two terms.*

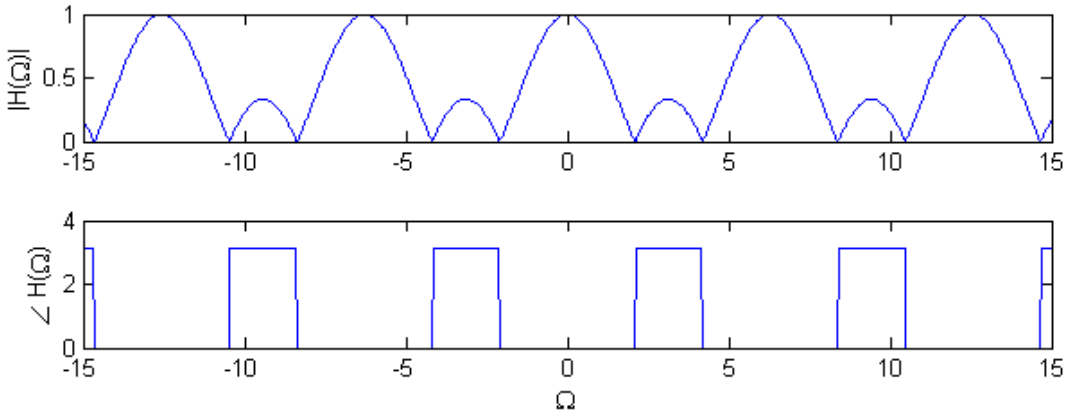
Solution:

$$Y(\Omega) = \frac{1}{3}X(\Omega)e^{-j\Omega} + \frac{1}{3}X(\Omega) + \frac{1}{3}X(\Omega)e^{j\Omega}$$

$$H(\Omega) = \frac{X(\Omega)}{Y(\Omega)} = \frac{1}{3} + \frac{1}{3}\left(e^{-j\Omega} + e^{j\Omega}\right) = \frac{1}{3} + \frac{2}{3}\cos(\Omega)$$

B. Sketch the magnitude and phase of $H(\Omega)$.

Solution:



C. Find the frequency Ω_0 where the magnitude of $H(\Omega)$ goes to zero.

Solution:

$$H(\Omega) = 0 \Leftrightarrow \cos(\Omega_0) = -\frac{1}{2}$$

$$\Omega_0 = 2\pi/3 + 2k\pi$$

and

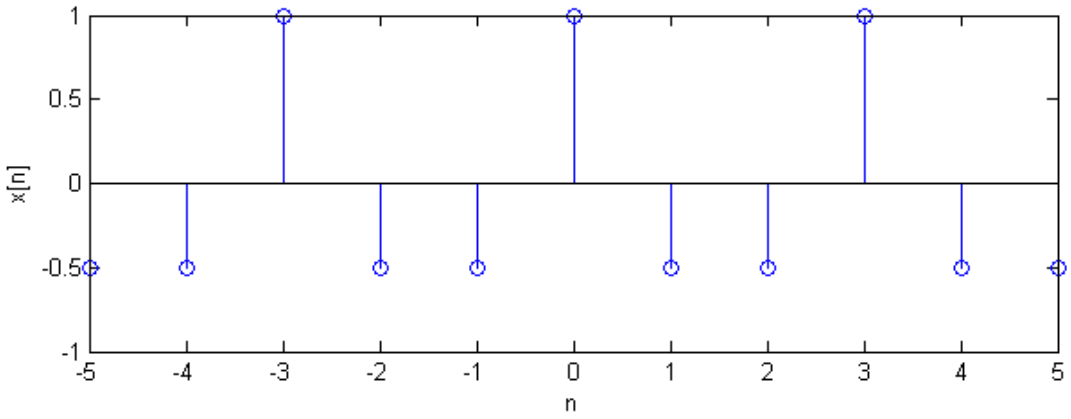
$$\Omega_0 = -2\pi/3 + 2k\pi$$

for all integers k.

D. What is the output if the input is $x[n] = \cos(\Omega_0 n)$. Sketch this input. Does it make sense that the output is zero?

Solution:
If $x[n] = \cos(\Omega_0 n) = \cos\left(\frac{2\pi}{3}n\right)$. Then

$$y[n] = \frac{1}{3}\cos\left[\frac{2\pi}{3}(n-1)\right] + \frac{1}{3}\cos\left(\frac{2\pi}{3}n\right) + \frac{1}{3}\cos\left[\frac{2\pi}{3}(n+1)\right]$$



By trigger identity:

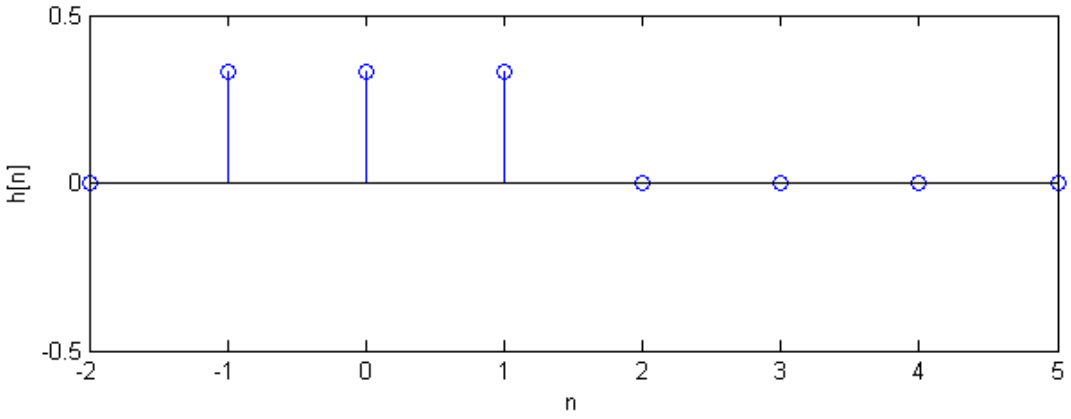
$$y[n] = \frac{2}{3}\cos\left(\frac{2\pi}{3}n\right)\cos\left(\frac{2\pi}{3}\right) + \frac{1}{3}\cos\left(\frac{2\pi}{3}n\right)$$

$$y[n] = -\frac{1}{3}\cos\left(\frac{2\pi}{3}n\right) + \frac{1}{3}\cos\left(\frac{2\pi}{3}n\right) = 0$$

E. Find and expression and sketch $h[n]$, the impulse response of the system. *Hint: You can find it relatively easily in either domain.*

Solution:

$$h[n] = \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n+1]$$



Course feedback
Feel free to send any additional feedback directly to us.

Name (optional):

A. End time: How long did the quiz take you?

B. Was the quiz a fair measure of your understanding?

C. Was the assignment effective preparation for the quiz?

D. Is the Monday session effective?

E. Are the connections between lecture, assignment and quiz clear?

F. Are the objectives of the course clear? Do you feel you are making progress towards those objectives?

G. Anything else?

Assignment grades

Date:

Assignment number:

Group member 1:

Grade:

Group member 2:

Grade:

Group member 3:

Grade: