Olin College of Engineering
ENGR2410 – Signals and Systems

Reference 8b

Modulation

Modulation allows us to move the information in a signal to another *frequency band*. Demodulation is the process of recuperating the original signal. This the basis of communication engineering. The most basic process is shown below, where a signal $x(t)$ is modulated by multiplying it by a sinusoid, and demodulated by multiplying it again with the same sinusoid.

$$
\begin{tikzcd}
x(t) \arrow[r] & \otimes \arrow[r, "x_{mod}(t)"] & \otimes \arrow[r, "y_{demod}(t)"] & \boxed{H(j\omega)} \arrow[r] & y(t) \\
& \cos(\omega_0 t) \arrow[u] & \cos(\omega_0 t) \arrow[u] & &
\end{tikzcd}
$$

Assume $x(t) \overset{\mathscr{F}}{\Longleftrightarrow} X(j\omega)$, and $X(j\omega)$ is bandlimited to $2\pi f_{max}$, where $2\pi f_{max} \ll \omega_0$. In the time domain,

$$x_{mod}(t) = x(t)\cos(\omega_0 t)$$

$$y_{demod} = x(t)\cos^2(\omega_0 t) = x(t)\left[\frac{1}{2} + \frac{1}{2}\cos(2\omega_0 t)\right]$$

In the frequency domain,

$$X_{mod}(j\omega) = \frac{1}{2\pi}X(j\omega) * [\pi\delta(\omega+\omega_0) + \pi\delta(\omega-\omega 0)] = \frac{1}{2}X(\omega+\omega_0) + \frac{1}{2}X(\omega-\omega_0)$$

$$Y_{demod}(j\omega) = \frac{1}{2\pi}\left[\frac{1}{2}X(\omega+\omega_0) + \frac{1}{2}X(\omega-\omega_0)\right] * [\pi\delta(\omega+\omega_0) + \pi\delta(\omega-\omega_0)]$$

$$Y_{demod}(j\omega) = \frac{1}{4}X(\omega+2\omega_0) + \frac{1}{2}X(\omega) + \frac{1}{4}X(\omega-2\omega_0)$$

If $H(j\omega)$ is an ideal low pass filter with cutoff frequency at $2\pi f_{max}$, $Y(j\omega) = \frac{1}{2}X(j\omega)$ and $y(t) = \frac{1}{2}x(t)$.