Olin College of Engineering
ENGR2410 – Signals and Systems

Quiz 9

Instructions

A. Collaboration is not allowed on quizzes.

B. Students may only use a page of notes and the tables from the website during the quizzes.

C. Time is limited to one continuous hour.

D. Quizzes are due at the beginning of lecture on Thursday.

E. Late or missed quizzes will be given a score of zero. Any excuses must come directly from the Office of Student Life.

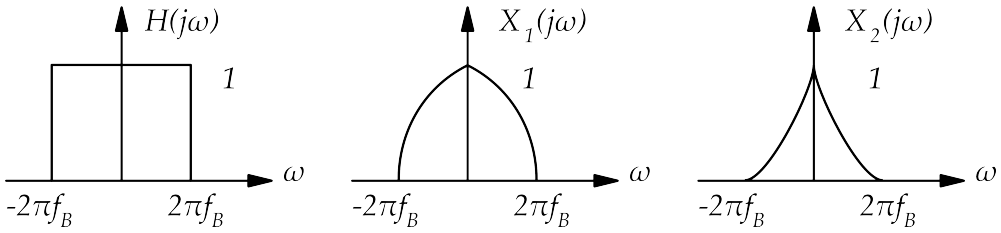
F. The two lowest quiz scores will be eliminated to allow for unforeseeable circumstances.

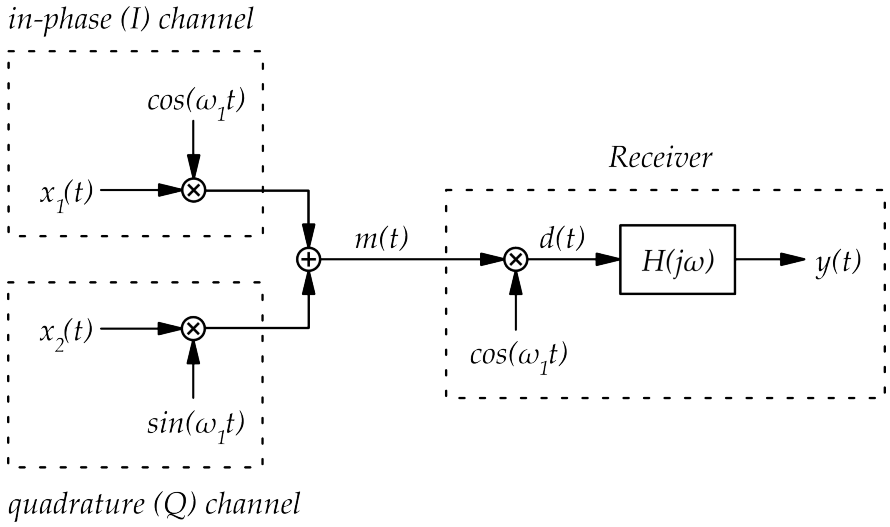
G. In case of doubt, students are expected to base their behavior on the values expressed in the Honor Code.

Name:

Start time:

Problem 1 *(10 points)* The system shown below introduces the concept of *quadrature*, where we send multiple signals that share not only a common channel, but also the same frequency band. Signals $x_1(t)$ and $x_2(t)$ are bandlimited to f_B and have a frequency content as shown below. The receiver has an ideal low-pass filter $H(j\omega)$ with a cutoff frequency of f_B as shown below.





A. Show that the frequency content of $M(j\omega)$ is

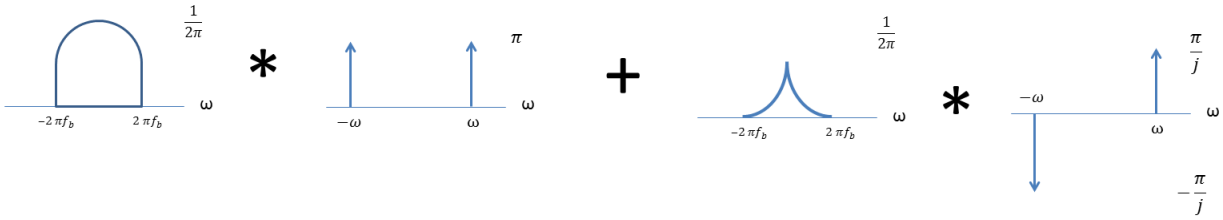
$$M(j\omega) = \frac{1}{2}X_1(\omega - \omega_1) + \frac{1}{2}X_1(\omega + \omega_1) - j\frac{1}{2}X_2(\omega - \omega_1) + j\frac{1}{2}X_2(\omega + \omega_1)$$

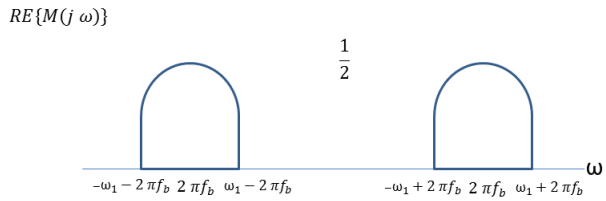
The result can be either the expression above or neatly labeled sketches of both the real and imaginary parts of $M(j\omega)$. *Hint: You can use either equations or sketches to find the solution, but using both may help you avoid algebra mistakes.*

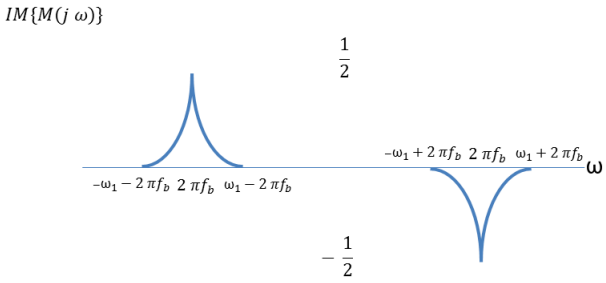
Solution:

$$m(t) = x_1 \cos(\omega_1 t) + x_2 \sin(\omega_1 t)$$

$$M(j\omega) = \frac{1}{2\pi}X_1(j\omega) * [\pi\delta(\omega-\omega_1)+\pi\delta(\omega+\omega_1)] + \frac{1}{2\pi}X_2(j\omega) * [\frac{\pi}{j}\delta(\omega-\omega_1)-\frac{\pi}{j}\delta(\omega+\omega_1)]$$



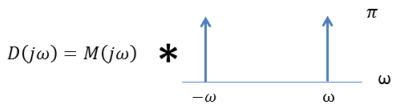


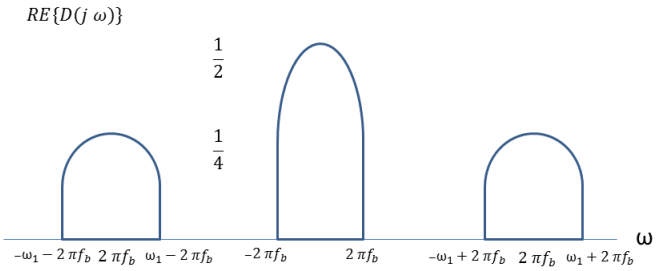


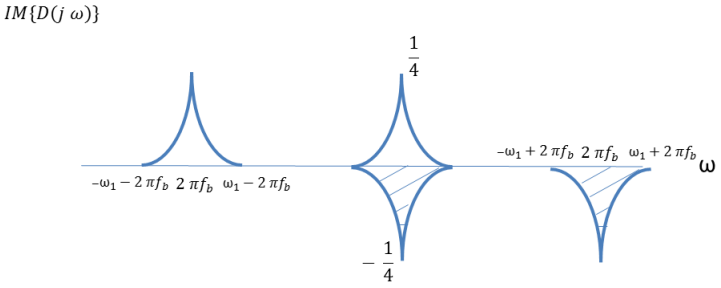
B. Find an expression for $y(t)$. Justify your answer clearly.

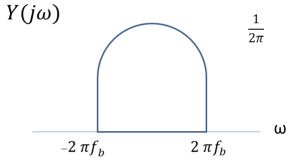
Solution:

$$D(j\omega) = \frac{1}{2\pi} M(j\omega) * [\pi\delta(\omega - \omega_1) + \pi\delta(\omega + \omega_1)] = \frac{1}{2}[M(\omega - \omega_1) + M(\omega + \omega_1)]$$









$Y(j\omega)$ has no imaginary part and the high-frequency component is filtered by $H(j\omega)$. Therefore, $Y(j\omega) = \frac{1}{2}X_1(j\omega)$ and $y(t) = \frac{1}{2}x_1(t)$.

C. *Bonus:* Find an expression for $y(t)$ if $m(t)$ is multiplied by $\sin(\omega_1 t)$ in the receiver instead of multiplied by $\cos(\omega_1 t)$. *Hint: You can use your intuition (and should!) to guess the answer, but a clear justification will get more points.*

Solution:

$$D(j\omega) = \frac{1}{2\pi} M(j\omega) * [-\frac{j\pi\delta}{2}\delta(\omega - \omega_1) + \frac{j\pi\delta}{2}\delta(\omega + \omega_1)]$$

$$\begin{aligned} D(j\omega) = &-\frac{j}{4}X_1(\omega - 2\omega_1) + \frac{j}{4}X_1(\omega) - \frac{j}{4}X_1(\omega) + \frac{j}{4}X_1(\omega + 2\omega_1) \\ &-\frac{1}{4}X_2(\omega - 2\omega_1) + \frac{1}{4}X_2(\omega) + \frac{1}{4}X_2(\omega) - \frac{1}{4}X_2(\omega + 2\omega_1) \end{aligned}$$

$$Y(j\omega) = \frac{j}{4}X_1(\omega) - \frac{j}{4}X_1(\omega) + \frac{1}{4}X_2(\omega) + \frac{1}{4}X_2(\omega) = \frac{1}{2}X_2(\omega) \Rightarrow y(t) = \frac{1}{2}x_2(t)$$

Course feedback
Feel free to send any additional feedback directly to us.

Name (optional):

A. End time: How long did the quiz take you?

B. Was the quiz a fair measure of your understanding?

C. Was the assignment effective preparation for the quiz?

D. Is the Monday session effective?

E. Are the connections between lecture, assignment and quiz clear?

F. Are the objectives of the course clear? Do you feel you are making progress towards those objectives?

G. Anything else?

Assignment grades

Date:

Assignment number:

Group member 1:

Grade:

Group member 2:

Grade:

Group member 3:

Grade: