Olin College of Engineering
ENGR2410 – Signals and Systems

Quiz 7

Instructions

A. Collaboration is not allowed on quizzes.

B. Students may only use a page of notes and the tables from the website during the quizzes.

C. Time is limited to one continuous hour.

D. Quizzes are due at the beginning of lecture on Thursday.

E. Late or missed quizzes will be given a score of zero. Any excuses must come directly from the Office of Student Life.

F. The two lowest quiz scores will be eliminated to allow for unforeseeable circumstances.

G. In case of doubt, students are expected to base their behavior on the values expressed in the Honor Code.

Name:

Start time:

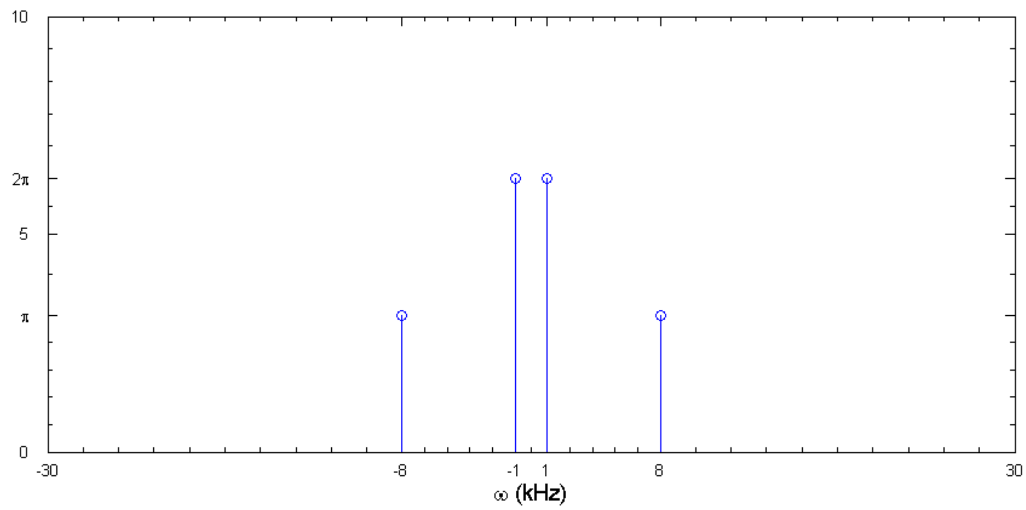
Problem 1 *(6 points)* For consistency throughout this problem, sketch the all Fourier transforms from -30 kHz to 30 kHz.

A. Sketch the Fourier transform of $x(t) = 2\cos(2\pi \cdot 1 \text{ kHz} \cdot t) + \cos(2\pi \cdot 8 \text{ kHz} \cdot t)$.

Solution:
The Fourier Transform of $\cos(\omega_0 t)$ is

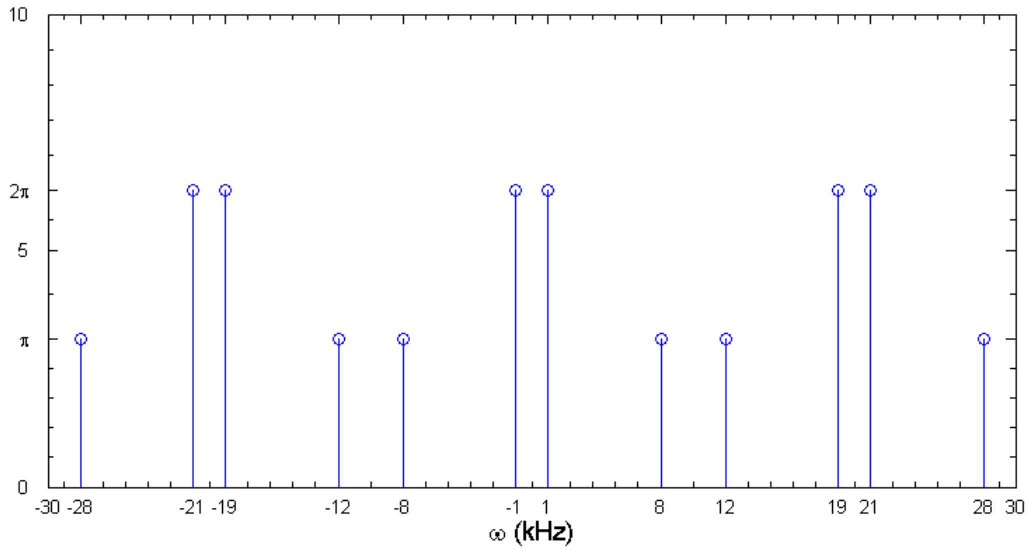
$$\mathscr{F}\{\cos(\omega_0 t)\} = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0),$$

using superposition we get



B. $x(t)$ is sampled at 20 kHz. Sketch the Fourier transform of the resulting function, $x_{S1}(t)$.

Solution:
Samping at 20 kHz, alters the Fourier transform to be periodic with a period of 20 kHz (i.e. the solution repeats every 20 kHz) and we get



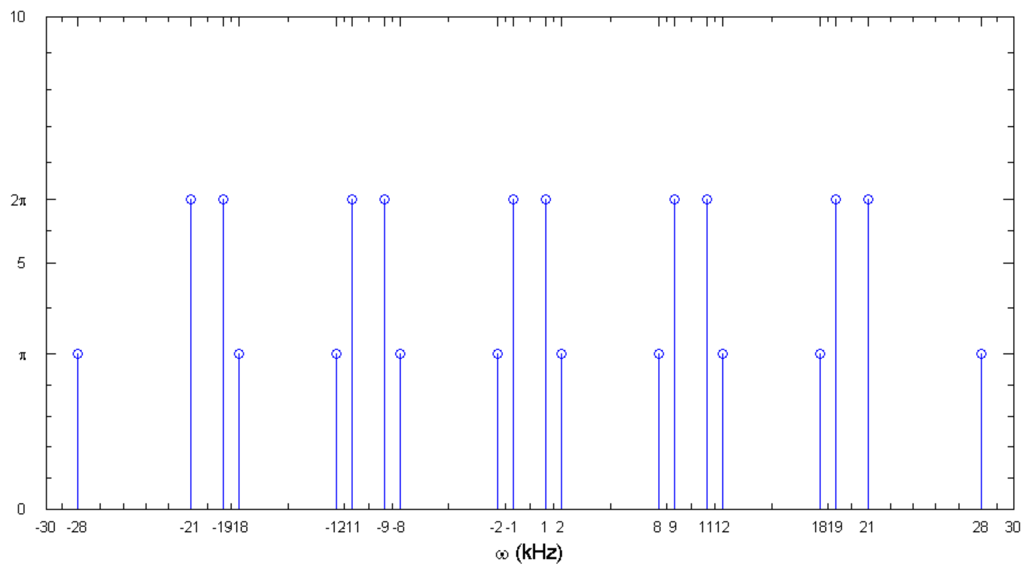
C. $x_{S1}(t)$ is passed through an ideal low-pass filter from -10 kHz to 10 kHz. Write an expression for the filtered output $y_1(t)$.

Solution:
Filtered at 10 kHz, all of the data above 10 kHz vanishes and we get the original Fourier transform back, which corresponds to the original function.

$$y_1(t) = 2\cos(2\pi \cdot 1 \text{ kHz} \cdot t) + \cos(2\pi \cdot 8 \text{ kHz} \cdot t)$$

D. $x(t)$ is sampled at 10 kHz. Sketch the Fourier transform of the resulting function, $x_{S2}(t)$.

Solution:
Sampling at 10 kHz alters the Fourier transform to be periodic with a period of 10 kHz (i.e. the solution repeats every 10 kHz) and we get



Notice the overlapping solutions. This will lead to aliasing.

E. $x_{S2}(t)$ is passed through an ideal low-pass filter from -10 kHz to 10 kHz. Write an expression for the filtered output $y_2(t)$.

Solution:
The overlapping solutions introduce two additional cosines at 2 kHz and 9 kHz yielding:

$$y_2(t) = 2\cos(2\pi \cdot 1 \text{ kHz} \cdot t) + \cos(2\pi \cdot 2 \text{ kHz} \cdot t) + \cos(2\pi \cdot 8 \text{ kHz} \cdot t) + 2\cos(2\pi \cdot 9 \text{ kHz} \cdot t)$$

A fascinating debate is whether the human brain samples what we see, and if so, what is the sampling frequency. Check out all the sampling language in this letter. However, newer evidence like this article suggests that sampling may not be enough to explain our perception.

Problem 2 *(4 points)* Find an algebraic expression for the inverse Fourier transform of $X(j\omega)$. *This filter is known as a* raised-cosine filter *and is very important in digital communications. For example, check out this article. You just found the impulse response of the filter.*

$$X(j\omega) = \begin{cases} \frac{1}{2}\left[1+\cos\left(\frac{\omega}{2f_S}\right)\right] & -2\pi f_S \leq w \leq 2\pi f_S \\ 0 & \text{otherwise} \end{cases}$$

Solution:
The original equation can be decomposed into the product

$$X(j\omega) = \frac{1}{2}\left[1+\cos\left(\frac{\omega}{2f_S}\right)\right] \times \begin{cases} 1 & -2\pi f_S \leq \omega \leq 2\pi f_S \\ 0 & \text{otherwise} \end{cases}$$

The inverse Fourier transform is then

$$\begin{aligned} x(t) =& \frac{1}{2}\mathscr{F}^{-1}\left\{1+\cos\left(\frac{\omega}{2f_S}\right)\right\} * \mathscr{F}^{-1}\left\{\begin{cases} 1 & -2\pi f_S \leq \omega \leq 2\pi f_S \\ 0 & \text{otherwise} \end{cases}\right\} \\ =& \frac{1}{2}\left[\delta\left(t\right)+\frac{1}{2}\delta\left(t+\frac{1}{2f_s}\right)+\frac{1}{2}\delta\left(t+\frac{1}{2f_s}\right)\right] * 2f_s\text{sinc}(2\pi f_s t) \\ =& f_s\text{sinc}(2\pi f_s t)+\frac{1}{2}f_s\text{sinc}(2\pi f_s t-\pi)+\frac{1}{2}f_s\text{sinc}(2\pi f_s t+\pi) \end{aligned}$$

Course feedback
Feel free to send any additional feedback directly to us.

Name (optional):

A. End time: How long did the quiz take you?

B. Was the quiz a fair measure of your understanding?

C. Was the assignment effective preparation for the quiz?

D. Is the Monday session effective?

E. Are the connections between lecture, assignment and quiz clear?

F. Are the objectives of the course clear? Do you feel you are making progress towards those objectives?

G. Anything else?

Assignment grades

Date:

Assignment number:

Group member 1:

Grade:

Group member 2:

Grade:

Group member 3:

Grade: