Olin College of Engineering
ENGR2410 – Signals and Systems

Quiz 10

Instructions

A. Collaboration is not allowed on quizzes.

B. Students may only use a page of notes and the tables from the website during the quizzes.

C. Time is limited to one continuous hour.

D. Quizzes are due at the beginning of lecture on Thursday.

E. Late or missed quizzes will be given a score of zero. Any excuses must come directly from the Office of Student Life.

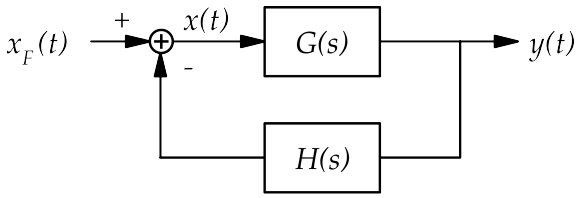
F. The two lowest quiz scores will be eliminated to allow for unforeseeable circumstances.

G. In case of doubt, students are expected to base their behavior on the values expressed in the Honor Code.

Name:

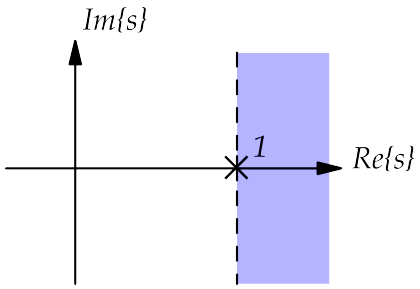
Start time:

Problem 1 *(4 points)* In the system shown below, $G(s) = \frac{1}{s-1}$ and $H(s) = K$, where K is a real constant. Assume the system and all inputs are causal.



A. Draw a pole-zero map of $G(s)$ indicating its region of convergence (ROC).

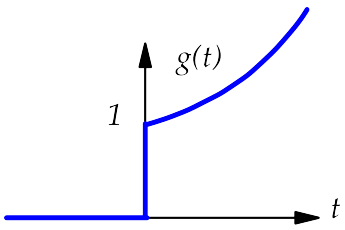
Solution:
$G(s)$ has a single pole at $s = 1$. Since it is causal, the ROC must extend to $+\infty$.



B. Find an algebraic expression and sketch the impulse response of $G(s)$.

Solution:

$$g(t) = \mathscr{L}^{-1}\{G(s)\} = e^t u(t)$$



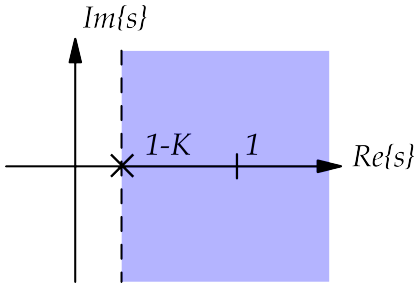
C. Find the overall transfer function $G_F(s) = Y(s)/X_F(s)$.

Solution:

$$G_F(s) = \frac{\frac{1}{s-1}}{1 + \frac{K}{s-1}} = \frac{1}{s - 1 + K}$$

D. Draw a pole-zero map of $G_F(s)$ indicating its region of convergence (ROC).

Solution:
$G_F(s)$ has a single pole at $s = 1 - K$. Since both $G(s)$ and $H(s)$ are causal, the ROC of $G_F(s)$ must extend to $+\infty$.



E. Find a requirement on K that makes the system stable.

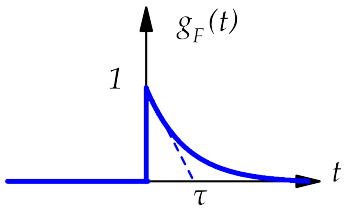
Solution:
For any system to be stable, its region of convergence must include the $j\omega$ axis.

$$1 - K < 0 \Rightarrow K > 1$$

F. Find an algebraic expression and sketch the impulse response of $G_F(s)$ assuming a value of K that makes the system stable.

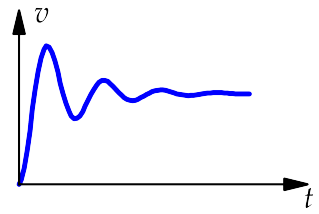
Solution:

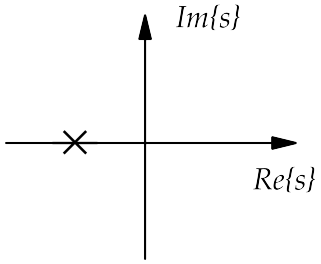
$$g_F(t) = \mathscr{L}^{-1}\{G_F(s)\} = e^{-(K-1)t}u(t)$$



Problem 2 *(6 points)* For each part, indicate whether the system described at the beginning may be equivalent to the system representations listed afterwards. You may choose different values to match each representation. Note that any number of the representations below may be equivalent, including all or none. Provide a specific example for representations that are equivalent or clear justifications for representations that are not. *An answer without justification will not receive credit.*

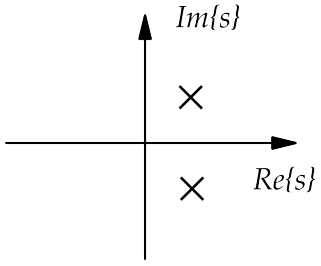
A. A system is described by the step response





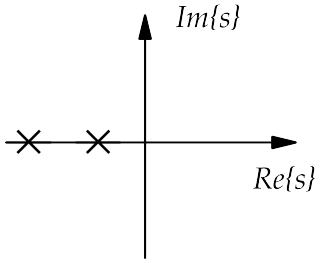
Solution:
equivalent **not equivalent**

Oscillations imply 2nd order system. Single pole implies 1st order system.



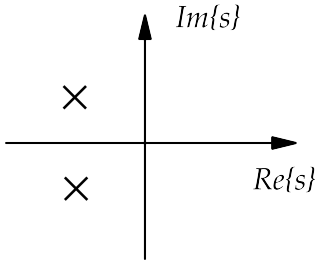
Solution:
equivalent **not equivalent**

Decaying oscillations imply stable system. Poles in RHP imply unstable system.



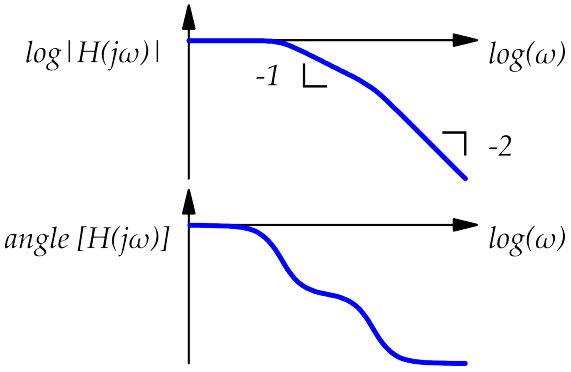
Solution:
equivalent **not equivalent**

Oscillations imply complex conjugate poles.



Solution:
equivalent **not equivalent**

2nd order underdamped low pass: $\frac{1}{s^2+0.01s+1}$



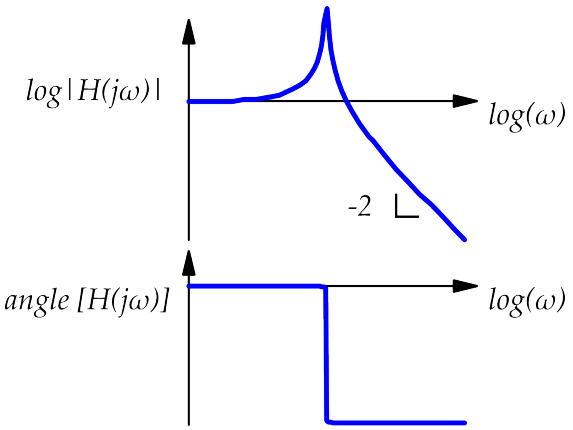
Solution:
equivalent **not equivalent**

Distinct transitions imply real poles. Oscillations imply complex conjugate poles.

B. A system is described by the transfer function

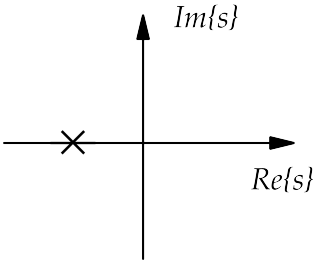
$$\frac{K}{s^2 + 2\alpha s + \omega_n^2}$$

where K, α and ω_n are **real** and **positive**.



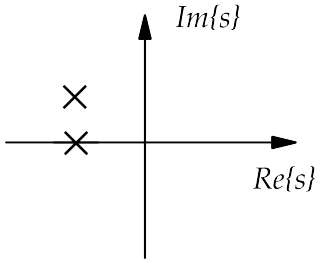
Solution:
equivalent **not equivalent**

Underdamped 2nd order low pass, $\alpha \ll \omega_0$: $\frac{1}{s^2+0.01s+1}$



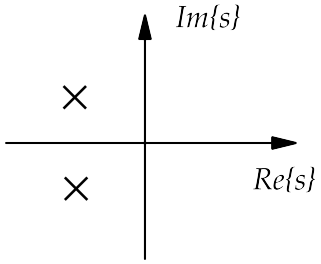
Solution:
equivalent **not equivalent**

2nd order denominator implies two poles.



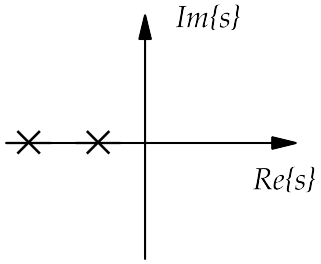
Solution:
equivalent **not equivalent**

Real system implies complex conjugate poles.



Solution:
equivalent **not equivalent**

Underdamped 2nd order low pass, $\alpha \ll \omega_0$: $\frac{1}{(s+0.1+j)(s+0.1-j)}$



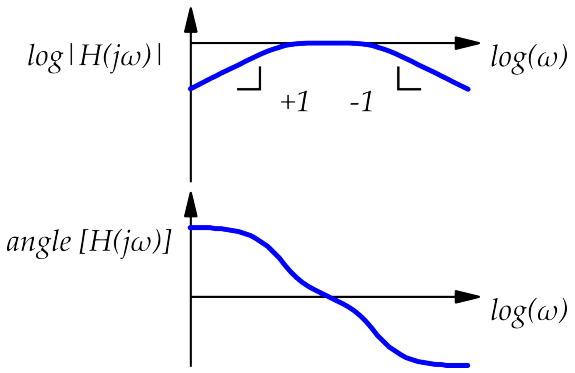
Solution:
equivalent **not equivalent**

Overdamped 2nd order low pass, $\alpha \gg \omega_0$: $\frac{1}{(s+1)(s+100)}$

C. A system is described by the transfer function

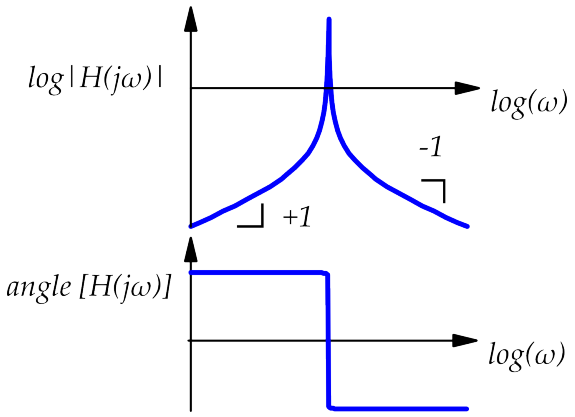
$$\frac{s}{(s+a)(s+b)}$$

where a and b are **real** and **positive**, and $a \neq b$.



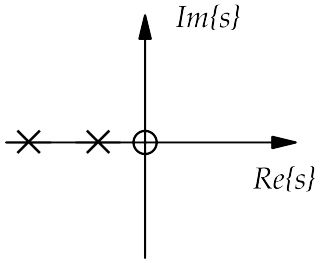
Solution:
equivalent **not equivalent**

Overdamped 2nd order band pass, $\alpha \gg \omega_0$: $\frac{s}{(s+1)(s+100)}$



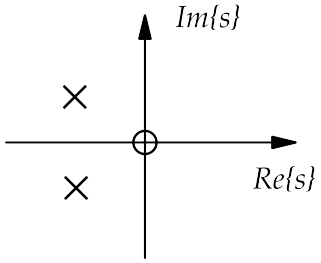
Solution:
equivalent **not equivalent**

Underdamped peak implies complex conjugate poles.



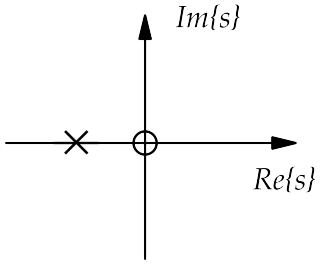
Solution:
equivalent **not equivalent**

Overdamped 2nd order band pass, $\alpha \gg \omega_0$: $\frac{s}{(s+1)(s+100)}$



Solution:
equivalent **not equivalent**

Poles in map are complex conjugate. System poles are real.



Solution:
equivalent **not equivalent**

System has two poles. Map has single pole.

Course feedback
Feel free to send any additional feedback directly to us.

Name (optional):

A. End time: How long did the quiz take you?

B. Was the quiz a fair measure of your understanding?

C. Was the assignment effective preparation for the quiz?

D. Is the Monday session effective?

E. Are the connections between lecture, assignment and quiz clear?

F. Are the objectives of the course clear? Do you feel you are making progress towards those objectives?

G. Anything else?

Assignment grades

Date:

Assignment number:

Group member 1:

Grade:

Group member 2:

Grade:

Group member 3:

Grade: