

Olin College of Engineering
ENGR2410 – Signals and Systems

Assignment 8

Problem 1 (2 points)

- A. Verify that $\cos(1.1\pi n)$ aliases to $\cos(0.9\pi n)$ by creating a plot of $\cos(1.1\pi t)$ and $\cos(0.9\pi t)$ for $-7 \leq t \leq 7$ and then plotting $\cos(1.1\pi n)$ for $n \in \{-7, -6, \dots, 7\}$ on the same axes. Explain clearly this plot.
- B. Find the transform of $\cos(\Omega_0 n)$.
- C. Sketch the transforms of $\cos(0.9\pi n)$ and $\cos(1.1\pi n)$ from -2π to 2π . Explain clearly.

Problem 2 (3 points) The transfer function of the first-order difference equation

$$y[n] - ay[n-1] = x[n] \quad \text{is} \quad H(\Omega) = \frac{1}{1 - ae^{-j\Omega}}.$$

- A. Plot the magnitude and phase of $H(\Omega)$ from -3π to 3π when $a = 0.9$.
- B. Since $e^x \approx 1 + x$ when $x \ll 1$,

$$H_{approx}(\Omega) = \frac{\frac{1}{a}}{\frac{1-a}{a} + j\Omega} \approx H(\Omega) = \frac{1}{1 - ae^{-j\Omega}}$$

when $\Omega \approx 2\pi n$. Make a Bode plot of both $H(\Omega)$ and $H_{approx}(\Omega)$ when $a = 0.9$ and $10^{-3} < \Omega < 2\pi$. What kind of filter is this?

- C. Redo the part A when $a = -0.9$. What kind of filter is this? Explain clearly.
- D. Find the transfer function for the difference equation below and plot it as in part A. What kind of filter is this? Explain clearly.

$$y[n] + 0.9y[n-2] = x[n]$$

- E. Find the transfer function for the difference equation below and plot it as in part A. This is called a *comb filter*.

$$y[n] = x[n] - 0.9x[n-5]$$

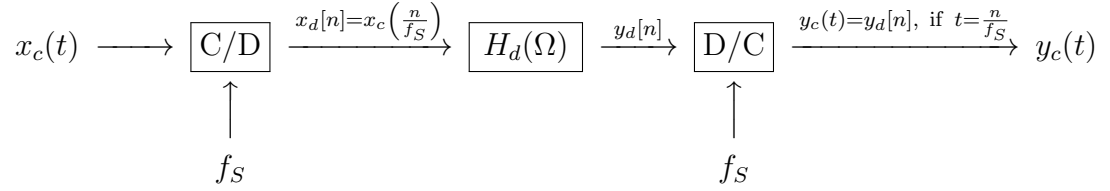
- F. Find and sketch the impulse response for the comb filter of part E. *This type of filter is a finite impulse response (FIR) filter. The first order difference equation of part A is an infinite impulse response (IIR) filter.*

Problem 3 (5 points)

- A. Find and sketch the transfer function $H_c(j\omega)$ such that

$$y_c(t) = x_c\left(t - \frac{1}{3f_s}\right)$$

in the system shown below, assuming $x_c(t)$ is bandlimited by f_{max} such that the sampling frequency $f_s > 2f_{max}$.



- B. Find and sketch $H_d(\Omega)$.
- C. Find the naive expression for $y_d[n]$ in terms of $x_d[n]$ by transforming $H_d(\Omega)$. Note that while your result is technically true, it cannot be applied literally! The next two parts give us the actual answer.
- D. Assume $x_c(t) = \text{sinc}(\pi f_s t)$. Verify that $x_d[n] = \delta[n]$. Combine both plots in the same set of axes.
- E. Find $y_c(t)$ and $y_d[n]$. Explain why $y_d[n] = h_d[n]$. Combine both plots in the same set of axes.