Olin College of Engineering
ENGR2410 – Signals and Systems

Assignment 8

Problem 1 *(2 points)*

A. Verify that $\cos(1.1\pi n)$ aliases to $\cos(0.9\pi n)$ by creating a plot of $\cos(1.1\pi t)$ and $\cos(0.9\pi t)$ for $-7 \le t \le 7$ and then plotting $\cos(1.1\pi n)$ for $n \in \{-7, -6, ...7\}$ on the same axes. Explain clearly this plot.

B. Find the transform of $\cos(\Omega_0 n)$.

C. Sketch the transforms of $\cos(0.9\pi n)$ and $\cos(1.1\pi n)$ from -2π to 2π. Explain clearly.

Problem 2 *(3 points)* The transfer function of the first-order difference equation

$$y[n] - ay[n-1] = x[n] \qquad \text{is} \qquad H(\Omega) = \frac{1}{1 - ae^{-j\Omega}}.$$

A. Plot the magnitude and phase of $H(\Omega)$ from -3π to 3π when $a = 0.9$.

B. Since $e^x \approx 1 + x$ when $x \ll 1$,

$$H_{approx}(\Omega) = \frac{\frac{1}{a}}{\frac{1-a}{a} + j\Omega} \approx H(\Omega) = \frac{1}{1 - ae^{-j\Omega}}$$

when $\Omega \approx 2\pi n$. Make a Bode plot of both $H(\Omega)$ and $H_{approx}(\Omega)$ when $a = 0.9$ and $10^{-3} < \Omega < 2\pi$. What kind of filter is this?

C. Redo the part A when $a = -0.9$. What kind of filter is this? Explain clearly.

D. Find the transfer function for the difference equation below and plot it as in part A. What kind of filter is this? Explain clearly.

$$y[n] + 0.9y[n-2] = x[n]$$

E. Find the transfer function for the difference equation below and plot it as in part A. This is called a *comb filter.*

$$y[n] = x[n] - 0.9x[n-5]$$

F. Find and sketch the impulse response for the comb filter of part E. *This type of filter is a* finite impulse response *(FIR) filter. The first order difference equation of part A is an* infinite impulse response *(IIR) filter.*

Problem 3 *(5 points)*

A. Find and sketch the transfer function $H_c(j\omega)$ such that

$$y_c(t) = x_c\left(t - \frac{1}{3f_S}\right)$$

in the system shown below, assuming $x_c(t)$ is bandlimited by f_{max} such that the sampling frequency $f_S > 2f_{max}$.

$$\begin{tikzcd} x_c(t) \arrow[r] & \boxed{\text{C/D}} \arrow[r, "x_d[n]=x_c\left(\frac{n}{f_S}\right)"] & \boxed{H_d(\Omega)} \arrow[r, "y_d[n]"] & \boxed{\text{D/C}} \arrow[r, "y_c(t)=y_d[n],\ \text{if } t=\frac{n}{f_S}"] & y_c(t) \\ & f_S \arrow[u] & & f_S \arrow[u] & \end{tikzcd}$$

B. Find and sketch $H_d(\Omega)$.

C. Find the naive expression for $y_d[n]$ in terms of $x_d[n]$ by transforming $H_d(\Omega)$. Note that while your result is technically true, it cannot be applied literally! The next two parts give us the actual answer.

D. Assume $x_c(t) = \text{sinc}(\pi f_s t)$. Verify that $x_d[n] = \delta[n]$. Combine both plots in the same set of axes.

E. Find $y_c(t)$ and $y_d[n]$. Explain why $y_d[n] = h_d[n]$. Combine both plots in the same set of axes.