Olin College of Engineering
ENGR2410 – Signals and Systems

Assignment 6

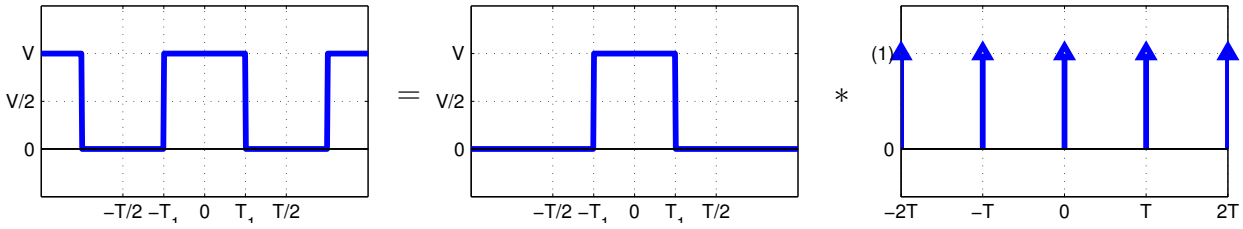
Problem 1 *(10 points)*

A. Find an expression for $\mathscr{F}\{v(t)\}$, where

$$v(t) = \begin{cases} V & -T_1 + nT < t < T_1 + nT, n \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}.$$

Hint: In lecture 6, we showed that convolution with a shifted impulse creates a copy of the original signal shifted by that amount. Use this to express $v(t)$ as the convolution of a pulse and an impulse train, as shown below.

$$v(t) = \left\{ \begin{cases} V & -T_1 < t < T_1 \\ 0 & \text{otherwise} \end{cases} \right\} * \sum_{k=-\infty}^{+\infty} \delta(t - kT)$$



B. Graph $\mathscr{F}\{v(t)\}$ for the case $T_1 = T/4$ and $V = V_S$.

C. Show that the Fourier transform you found is equivalent to the coefficients for the even square wave with period T and pulse width $2T_1$,

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi}{T}nt}, \qquad c_n = 2V\frac{T_1}{T}\text{sinc}\left(2\pi n\frac{T_1}{T}\right)$$

D. *Optional:* In Quiz 5, you found that the frequency content of a period T function only exists in the harmonics of $2\pi/T$ (multiples of $\frac{2\pi}{T}$) and must be zero elsewhere. Can you generalize the transform of a periodic function $x(t) = x(t+T)$? For consistency of notation, define $x_T(t)$ as a single period of $x(t)$ and $X_T(j\omega) = \mathscr{F}\{x_T(t)\}$.

E. *Optional:* Generalize the equivalence to series

$$c_n = \frac{1}{T} \cdot X_T\left(j\frac{2\pi}{T}n\right).$$