

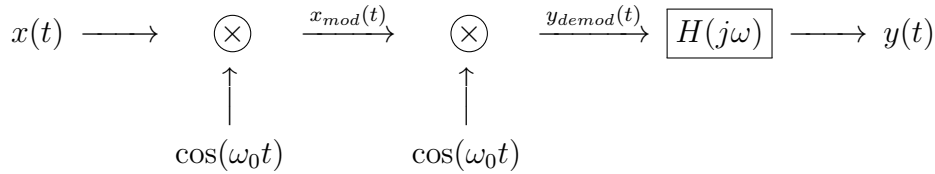
Olin College of Engineering

ENGR2410 – Signals and Systems

Reference 8b

Modulation

Modulation allows us to move the information in a signal to another *frequency band*. Demodulation is the process of recuperating the original signal. This is the basis of communication engineering. The most basic process is shown below, where a signal $x(t)$ is modulated by multiplying it by a sinusoid, and demodulated by multiplying it again with the same sinusoid.



Assume $x(t) \xleftrightarrow{\mathcal{F}} X(j\omega)$, and $X(j\omega)$ is bandlimited to $2\pi f_{max}$, where $2\pi f_{max} \ll \omega_0$. In the time domain,

$$x_{mod}(t) = x(t) \cos(\omega_0 t)$$

$$y_{demod} = x(t) \cos^2(\omega_0 t) = x(t) \left[\frac{1}{2} + \frac{1}{2} \cos(2\omega_0 t) \right]$$

In the frequency domain,

$$X_{mod}(j\omega) = \frac{1}{2\pi} X(j\omega) * [\pi\delta(\omega + \omega_0) + \pi\delta(\omega - \omega_0)] = \frac{1}{2} X(\omega + \omega_0) + \frac{1}{2} X(\omega - \omega_0)$$

$$Y_{demod}(j\omega) = \frac{1}{2\pi} \left[\frac{1}{2} X(\omega + \omega_0) + \frac{1}{2} X(\omega - \omega_0) \right] * [\pi\delta(\omega + \omega_0) + \pi\delta(\omega - \omega_0)]$$

$$Y_{demod}(j\omega) = \frac{1}{4} X(\omega + 2\omega_0) + \frac{1}{2} X(\omega) + \frac{1}{4} X(\omega - 2\omega_0)$$

If $H(j\omega)$ is an ideal low pass filter with cutoff frequency at $2\pi f_{max}$, $Y(j\omega) = \frac{1}{2} X(j\omega)$ and $y(t) = \frac{1}{2} x(t)$.