Olin College of Engineering
ENGR2410 – Signals and Systems

Concept Overview

1. Complex exponentials are eigenfunctions of LTI systems.

$$e^{j\omega t} \longrightarrow \boxed{\text{LTI}} \longrightarrow H(j\omega)e^{j\omega t}$$

2. Most functions can be expressed as an infinitely dense sum (i.e., an integral) of exponentials. If the sum is discrete, the function must be periodic.

$$x(t) = \int_{\omega=-\infty}^{\omega=\infty} \underbrace{X(j\omega)}_{\text{frequency content}} e^{j\omega t}\frac{d\omega}{2\pi}$$

3. The frequency content of the output is the frequency content of the input multiplied by the transfer function. A *Bode plot* is a logarithmic plot of the transfer function.

$$y(t) = \int_{\omega=-\infty}^{\omega=\infty} \underbrace{H(j\omega)X(j\omega)}_{Y(j\omega)} e^{j\omega t}\frac{d\omega}{2\pi} \iff \begin{tikzcd} x(t) \arrow[r] \arrow[d, "\mathscr{F}"'] & \boxed{\text{LTI}} \arrow[r] & y(t) \\ X(j\omega) \arrow[r] & \boxed{H(j\omega)} \arrow[r] & Y(j\omega)= X(j\omega)H(j\omega) \arrow[u, "\mathscr{F}^{-1}"'] \end{tikzcd}$$

4. The frequency content of an impulse is constant over all frequencies. As a result, the frequency content of the impulse response is given by the transfer function.

$$\begin{tikzcd} \delta(t) \arrow[r] \arrow[d, "\mathscr{F}"'] & \boxed{h(t)} \arrow[r] & h(t) \\ 1 \arrow[r] & \boxed{H(j\omega)} \arrow[r] & H(j\omega) \arrow[u, "\mathscr{F}^{-1}"'] \end{tikzcd}$$

5. Convolution in time is equivalent to multiplication in frequency. Thus, for a system in the time domain,
   the output is the input convolved with the impulse response.
   But, the *transform of the* output is the *transform of the* input ~~convolved~~ **multiplied** with the *transform of the* impulse response.

$$\begin{tikzcd} x(t) \arrow[r] \arrow[d, "\mathscr{F}"'] & \boxed{h(t)} \arrow[r] & y(t) = x(t) * h(t) = \int_{\tau=-\infty}^{\tau=\infty} x(\tau)h(t-\tau)d\tau \\ X(j\omega) \arrow[r] & \boxed{H(j\omega)} \arrow[r] & Y(j\omega)= X(j\omega)H(j\omega) \arrow[u, "\mathscr{F}^{-1}"'] \end{tikzcd}$$

6. Exponentials are more generalized eigenfunctions of LTI systems (than complex exponentials).

$$e^{st} \longrightarrow \boxed{h(t)} \longrightarrow H(s)e^{st}$$

7. The eigenvalue of the eigenfunction e^{st} is the Laplace transform of $h(t)$ since

$$H(s)e^{st} = \underbrace{e^{st}}_{\text{input}} * \underbrace{h(t)}_{\text{system}} = \int_{\tau=-\infty}^{\tau=\infty} h(\tau)e^{s(t-\tau)}d\tau = e^{st}\underbrace{\int_{\tau=-\infty}^{\tau=\infty} h(\tau)e^{-s\tau}d\tau}_{H(s)}$$

8. A *pole-zero diagram* is a plot of the complex plane showing the values of s where the transfer function is zero (*zeroes*) or infinity (*poles*). A system is stable iff all the poles are in the left half-plane ($\Re\{s\} > 0$).