Olin College of Engineering
ENGR2410 – Signals and Systems

Assignment 4

Problem 1: *(2 points)* Assume that n is an integer. Show that if the complex coefficients $c_{-n} = c_n^*$, then

$$\sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi}{T}nt} = c_0 + \sum_{n=1}^{\infty} 2\mathrm{Re}\{c_n\}\cos\left(\tfrac{2\pi}{T}nt\right) + \sum_{n=1}^{\infty}(-2)\mathrm{Im}\{c_n\}\sin\left(\tfrac{2\pi}{T}nt\right)$$

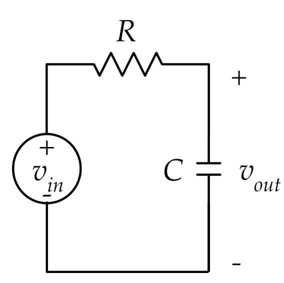
Problem 2: *(4 points)* In lecture we found out that if we try to represent the function

$$v(t) = \begin{cases} V & -T_1 + nT < t < T_1 + nT, n \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}$$

as $v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi}{T}nt}$ then $c_n = 2V\frac{T_1}{T}\mathrm{sinc}\left(\frac{2\pi}{T}nT_1\right)$.

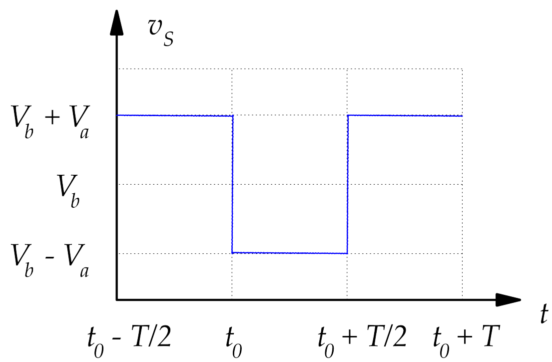
A. Plot the original $v(t)$ and the coefficients c_n when $T_1 = T/4$, $T_1 = T/8$, and $T_1 = T/16$. Use the *stem* command in Matlab. *Careful: the definition of sinc in Matlab is different than the one we used in class.*

B. Assume $v_{in}(t) = v(t)$ and $T_1 = T/8$. Plot the response of the circuit shown below using Fourier decomposition with complex exponentials. Plot both the input and the output using 2, 7, and 100 harmonics. Note how similar the responses are even though you are using a very crude approximation.



Problem 3: *(4 points)* This problem will introduce the concept of *average power* in a signal, and will explore the relationships across the frequency and time domains.

A. The voltage signal shown below is a square wave of amplitude V_a and average value V_b.



The *average power* of a signal is defined as

$$< P >= \frac{1}{T}\int_{t_0}^{t_0+T} |v_S|^2 dt$$

where $|v_S|^2 = v_S v_S^*$ is the square magnitude of the signal and the asterisk denotes complex conjugation. Show that the average power for v_S is $V_a^2 + V_b^2$.

B. Show that the average power of a complex exponential $c_n e^{j\omega t}$ is its squared magnitude, that is, $< P >= |c_n|^2$.

C. Find the Fourier decomposition of v_S assuming $t_0 = T/4$ (*note that this choice is irrelevant for this problem; think about why*) and using complex exponentials such that

$$v_S(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi}{T}nt}$$

D. Show that the average power of v_S can be separated into a sum of a term that depends on V_a and a term that depends on V_b. When computing the average power, you might find useful to know that

$$\sum_{n\in[1,3,5...\infty)} \frac{1}{n^2} = \frac{\pi^2}{8}$$

E. The average power of v_S that depends on V_a is called the AC, or alternating current power, while the power that depends on V_b is called the DC, or direct current power. For the square wave, show that the fundamental (frequency 1/T) contains $8/\pi^2 \approx 80\%$ of the total AC power.