AL-NAHARIN UNIVERSITYCOLLEGE OF ENGINEERING DEPARTMENT OF ELECTRONIC AND COMMUNICATIONS

الاسم :علي حسين عبد الامير

الموظوع:Matrix

التاريخ:28/4/2024

Introduction

Matrix is a rectangular array of numbers, symbols, points, or characters each belonging to a specific row and column. A matrix is identified by its order which is given in the form of rows × and columns. The numbers, symbols, points, or characters present inside a matrix are called the elements of a matrix. The location of each element is given by the row and column it belongs to.

Matrices are important for students of class 12 and also have great importance in engineering mathematics as well. In this introductory article on matrices, we will learn about the types of matrices, the transpose of matrices, the rank of matrices, the adjoint and inverse of matrices, the determinants of matrices, and many more in detail.

What are Matrices?

Matrices are rectangular arrays of numbers, symbols, or characters where all of these elements are arranged in each row and column. An array is a collection of items arranged at different locations.

Let's assume points are arranged in space each belonging to a specific location then an array of points is formed. This array of points is called a matrix. The items contained in a matrix are called Elements of the Matrix. Each matrix has a finite number of rows and columns and each element belong to these rows and columns only. The number of rows and columns present in a matrix determines the order of the matrix. Let's say a matrix has 3 rows and 2 columns then the order of the matrix is given as 3×2.

Matrices Definition

as [P]m×n where P is the matrix, m is the number of rows and n is the number of columns. Matrices in maths are useful in solving numerous problems of linear equations and many more.

Order of Matrix

Order of the Matrix tells about the number of rows and columns present in a matrix. Order of a matrix is represented as the number of rows times the number of columns. Let's say if a matrix has 4 rows and 5 columns then the order of the matrix will be 4×5. Always remember that the first number in the order signifies the number of rows present in the matrix and the second number signifies the number of columns in the matrix.

Matrices Examples

Examples of matrices are mentioned below:

$$\text{Example: } \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}, \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 6 \\ 4 & -2 & 5 \end{bmatrix}_{3\times 3}$$

Operation on Matrices

Matrices undergo various mathematical operations such as addition, subtraction, scalar multiplication, and multiplication. These operations are performed between the elements of two matrices to give an equivalent matrix that contains the elements which are obtained as a result of the operation between elements of two matrices. Let's learn the operation of matrices.

Addition of Matrices

In addition of matrices, the elements of two matrices are added to yield a matrix that contains elements obtained as the sum of two matrices. The addition of matrices is performed between two matrices of the same order. Let's learn through an example.

Example: Find the sum of $\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$ **and** $\begin{bmatrix} 2 & 3 \\ 6 & 7 \end{bmatrix}$

Solution:

Here, we have $A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$ *and* $B = \begin{bmatrix} 2 & 3 \\ 6 & 7 \end{bmatrix}$

$$A + B = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 6 & 7 \end{bmatrix}$$

$$\Rightarrow A + B = \begin{bmatrix} 1+2 & 2+3 \\ 4+6 & 5+7 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 10 & 12 \end{bmatrix}$$

Subtraction of Matrices

Subtraction of Matrices is the difference between the elements of two matrices of the same order to give an equivalent matrix of the same order whose elements are equal to the difference of elements of two matrices. The subtraction of two matrices can be represented in terms of the addition of two matrices. Let's say we have to subtract matrix B from matrix A then we can write A – B. We can also rewrite it as A + (-B). Let's solve an example

Example: Subtract $\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$ from $\begin{bmatrix} 2 & 3 \\ 6 & 7 \end{bmatrix}$.

Answer:

Let us assume $A = \begin{bmatrix} 2 & 3 \\ 6 & 7 \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$

$$A - B = \begin{bmatrix} 2 & 3 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$$

$$\Rightarrow A - B = \begin{bmatrix} 2-1 & 3-2 \\ 6-4 & 7-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

Scalar Multiplication of Matrices

Scalar Multiplication of matrices refers to the multiplication of each term of a matrix with a scalar term. If a scalar let’s ‘k’ is multiplied by a matrix then the equivalent matrix will contain elements equal to the product of the scalar and the element of the original matrix. Let’s see an example:

Example: Multiply 3 with $\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$.

Solution:

$$3[A] = \begin{bmatrix} 3 \times 1 & 3 \times 2 \\ 3 \times 4 & 3 \times 5 \end{bmatrix}$$

$$\Rightarrow 3[A] = \begin{bmatrix} 3 & 6 \\ 12 & 15 \end{bmatrix}$$

Multiplication of Matrices

In the multiplication of matrices, two matrices are multiplied to yield a single equivalent matrix. The multiplication is performed in the manner that the elements of the row of the first matrix multiply with the elements of the columns of the second matrix and the product of elements are added to yield a single element of the equivalent matrix. If a matrix $[A]_{i\times j}$ is multiplied with matrix $[B]_{j\times k}$ then the product is given as $[AB]_{i\times k}$. Let's see an example.

Example: Find the product of $\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$ **and** $\begin{bmatrix} 2 & 3 \\ 6 & 7 \end{bmatrix}$

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 3 \\ 6 & 7 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 6 & 7 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 1 \times 2 + 2 \times 6 & 1 \times 3 + 2 \times 7 \\ 4 \times 2 + 5 \times 6 & 4 \times 3 + 5 \times 7 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 18 & 17 \\ 38 & 47 \end{bmatrix}$$

Properties of Matrix Addition and Multiplication

Properties followed by Multiplication and Addition of Matrices is listed below:

- A + B = B + A (Commutative)
- (A + B) + C = A + (B + C) (Associative)
- AB ≠ BA (Not Commutative)
- (AB) C = A (BC) (Associative)
- A (B+C) = AB + AC (Distributive)

Transpose of Matrix

Transpose of Matrix is basically the rearrangement of row elements in column and column elements in a row to yield an equivalent matrix. A matrix in which the elements of the row of the original matrix are arranged in columns or vice versa is called Transpose Matrix. The transpose matrix is represented as A^T. if $A = [a_{ij}]_{m\times n}$, then $A^T = [b_{ij}]_{n\times m}$ where $b_{ij} = a_{ji}$.

Let's see an example:

Types of Matrices

Based on the number of rows and columns present and the special characteristics shown, matrices are classified into various types.

- **Row Matrix:** A Matrix in which there is only one row and no column is called Row Matrix.
- **Column Matrix:** A Matrix in which there is only one column and now row is called a Column Matrix.
- **Horizontal Matrix:** A Matrix in which the number of rows is less than the number of columns is called a Horizontal Matrix.
- **Vertical Matrix:** A Matrix in which the number of columns is less than the number of rows is called a Vertical Matrix.
- **Rectangular Matrix:** A Matrix in which the number of rows and columns are unequal is called a Rectangular Matrix.
- **Square Matrix:** A matrix in which the number of rows and columns are the same is called a Square Matrix.
- **Diagonal Matrix:** A square matrix in which the non-diagonal elements are zero is called a Diagonal Matrix.
- **Zero or Null Matrix:** A matrix whose all elements are zero is called a Zero Matrix. A zero matrix is also called as Null Matrix.
- **Unit or Identity Matrix:** A diagonal matrix whose all diagonal elements are 1 is called a Unit Matrix. A unit matrix is also called an Identity matrix. An identity matrix is represented by I.

- **Symmetric matrix:** A square matrix is said to be symmetric if the transpose of the original matrix is equal to its original matrix. i.e. $(A^T) = A$.
- **Skew-symmetric Matrix:** A skew-symmetric (or antisymmetric or antimetric[1]) matrix is a square matrix whose transpose equals its negative i.e. $(A^T) = -A$.
- **Orthogonal Matrix:** A matrix is said to be orthogonal if $AA^T = A^TA = I$
- **Idempotent Matrix:** A matrix is said to be idempotent if $A^2 = A$
- **Involutory Matrix:** A matrix is said to be Involutory if $A^2 = I$.
- **Upper Triangular Matrix:** A square matrix in which all the elements below the diagonal are zero is known as the upper triangular matrix
- **Lower Triangular Matrix:** A square matrix in which all the elements above the diagonal are zero is known as the lower triangular matrix
- **Singular Matrix:** A square matrix is said to be a singular matrix if its determinant is zero i.e. $|A|=0$
- **Nonsingular Matrix:** A square matrix is said to be a non-singular matrix if its determinant is non-zero.

References:

https://www.geeksforgeeks.org/matrices/