

## THE VEHICLE SCHEDULING PROBLEM: A SURVEY

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## SUMMARY

*The vehicle scheduling problem is concerned with routing a fleet of vehicles each with a capacity constraint and which are based at a central depot, to visit a set of delivery points. The optimality criterion is most frequently taken as the total distance travelled which is to be minimized. This paper discusses this problem, surveys the literature on it and presents some new ideas on heuristic solution procedures.*

## 1. INTRODUCTION

A recent article in *Management Today*, the journal of the British Institute of Management, suggested to distribution managers that if "you have not looked at your distribution system for 4 years, you are probably paying 200% more than you need". In this day and age of severe economic conditions, the physical distribution function in many companies is under considerable pressure. Recent increases in oil prices and in wage levels have upset the balance of costs, making it necessary to maintain a close check on such problems as depot numbers and location, vehicle scheduling, fleet size and mix and replacement policies, this paper examines one of these activities in detail: the vehicle scheduling problem. The literature on it is surveyed and some new ideas on heuristic solution procedures are presented.

The *vehicle scheduling problem* (VSP) is concerned with the scheduling of a number of vehicles which must visit a number of locations in order to pick up or deliver some commodity or perform some service. The terms *vehicle* and *location* have been used in order to provide a definite scenario, but many applications of the VSP model have nothing to do with actual vehicles visiting locations. The problem is sometimes referred to in the literature as the truck dispatching, transportation routing, vehicle routing, or the delivery problem.

The version of the VSP examined here is concerned with a given number of vehicles all based at a central depot. There is a set of locations scattered around the depot, each with a known demand for some service that the vehicles can provide. The vehicles have the same capacity to provide this service, which may, for example, be based on a weight or volume restriction relating to the dimensions of a commodity that is to be delivered or picked up. Each vehicle must be assigned a tour beginning at the depot, visiting a number of locations in a prescribed sequence ending at the depot with the guarantee that the total service requirement of the tour does not exceed vehicle capacity. The objective is to assign at least one tour to each vehicle so that each location is visited by exactly one vehicle and some optimality criterion is minimized. The optimality criterion is typically total distance travelled, total time taken or total cost involved in servicing all locations, but sometimes one of the above subject to using the least number of vehicles and occasionally evening out the work load of the vehicles is also important.

There are a number of variations on the basic problem that have been described. It is sometimes assumed that each vehicle not only has a capacity but also a time or distance constraint on the length of its tour. Further, these two characteristics may vary from vehicle to vehicle. Clarke and Wright [15] provided a heuristic procedure for the problem of variable capacity, and Golden [32] has formulated and examined the problem where time constraints are also present. Vehicles may also have several compartments with similar or different capacities [16]. Sometimes it is necessary to have each arc as well as each node in the transportation network visited by a vehicle. Orloff [57] has called this scenario the General Routing Problem (GRP). Orloff and Caprera [58] have developed a heuristic procedure for the GRP which apparently solves relatively large problems. If it is impossible to schedule all customers within a given day, customers may be given a priority and all customers with the highest priority must be scheduled. Pierce [59] discusses the implications of the existence of deadlines for service but also of times before which a visit is not permissible. Of course these two together result in a time window during which a visit has to be made. It may be necessary to have inter-location travel times (as well as distances) and location service times available to cater for these time windows. Russell [64] has presented a heuristic which accommodates time windows. Wren and Holliday [83], and Gillett and Johnson [31] discuss the problem with multiple depots. Christofides et al. [13] have developed and compared a number of methods for the case where a number of distinct services (or commodities) are to be performed. Each vehicle has a certain capacity for each service.

All the above variations arise from the wide diversity of possible applications for the VSP. Reported applications to real-world problems include: school bus scheduling [1,5,18,25,51,56,58,65], public bus scheduling [66,68], beer delivery [16], milk collection [26], printing press scheduling [35], cement delivery [69], garbage collection [4,49,74], newspaper distribution [33,63] medical specimen collection [50], retail goods delivery [23,42], fuel oil delivery [28,43], bulk mail conveyance [61], meal delivery [8], mass transit crew scheduling [3], postal truck scheduling [67].

Most of the above problem areas can be described as 'tactical' in the sense that they examine problems with a short term viewpoint. There are several problems in which the VSP can be employed in a 'strategic' sense. Examples of these include the design of fixed areas for delivery [10], vehicle fleet size and mix [20], development of delivery costs for use in depot location studies [76], and determining the level of service to offer to customers [77].

## 2. VSP FORMULATIONS

In this section, we formulate the VSP as an integer programming problem. Let  $n$  denote the number of locations,  $p$  the number of feasible tours,  $l_j$  the length of tour  $j$ ,  $\delta_{ij} = 1$  if the  $i$ th location is on tour  $j$  and 0 otherwise, and  $Y_j = 1$  if tour  $j$  is chosen and 0 otherwise. Then VSP has been formulated by Balinski and Quandt [2] as

$$\begin{aligned} \text{Minimize } & \sum_{j=1}^p l_j Y_j \\ \text{subject to } & \sum_{j=1}^p \delta_{ij} Y_j = 1, \quad i = 1, \dots, n, \text{ and} \\ & Y_j = 0 \text{ or } 1, \quad j = 1, \dots, p \end{aligned}$$

This formulation is unfortunately not very useful as there is likely to be an enormous number of feasible tours or variables  $Y_j$ ,  $j = 1, \dots, p$ . However, Balinski and Quandt did manage to reduce this number by the use of "dominated tours" - tours which could never be part of an optimal solution. Using a Gomory cutting plane method, they found approximate solutions to problems with up to 270 locations and 15 feasible tours. However, any realistic application is likely to contain considerably more tours; so we turn to more useful formulations.

A formulation similar to that given above has been developed by Foster and Ryan [24]. These authors relax the solution space by enumerating only routes with special characteristics (termed 'petal' routes). The solution approach

used is also different from that of Balinski and Quandt in that they relax the integrality requirement and define the set of feasible tours to allow for any other constraints that may be necessary in a given context. For a solution to the resulting LP to be interpreted as a schedule, one must ensure that the variables have values of only 0 or 1. This can be done using standard branch and bound techniques. However, Foster and Ryan used alternative methods. Using the revised simplex method and a column generating technique, they have been able to find approximate solutions to problems with up to 100 locations in reasonable computing time. This method has been applied by Crawford and Sinclair [16].

An alternative formulation of the problem may be described as follows. Let  $m$  denote the number of vehicles,  $C$  the vehicle capacity,  $D_i$  the demand at location  $i$ ,  $d_{ij}$  the cost of travel from location  $i$  to location  $j$ , and  $x_{ijk} = 1$  if vehicle  $k$  travels directly from location  $i$  to location  $j$  and 0 otherwise. Location 1 represents the central depot. Then the VSP can be expressed as:

$$\text{Minimize} \quad \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^m d_{ij} x_{ijk} \quad (1)$$

$$\text{subject to:} \quad \sum_{i=1}^n \sum_{k=1}^m x_{ijk} = 1, \quad j = 2, \dots, n \quad (2)$$

$$\sum_{i=1}^n x_{ipk} - \sum_{j=1}^n x_{pjk} = 0, \quad k = 1, \dots, m, \\ p = 1, \dots, n \quad (3)$$

$$\sum_{i=1}^n D_i \left\{ \sum_{j=1}^n x_{ijk} \right\} \leq C, \quad k = 1, \dots, m \quad (4)$$

$$\sum_{i=2}^n x_{i1k} \leq 1, \quad k = 1, \dots, m \quad (5)$$

$$q_i - q_j + n \sum_{k=1}^m x_{ijk} \leq n-1, \quad \text{for some} \\ \text{real numbers } q_i, \quad i = 2, \dots, n, \\ j = 2, \dots, n, \text{ and } i \neq j \quad (6)$$

$$x_{ijk} = 0 \text{ or } 1, \quad i = 1, \dots, n \\ j = 1, \dots, n, \quad k = 1, \dots, m \quad (7)$$

These expressions can be explained as follows:

(1) the total delivery cost is to be minimized; (2) Exactly one vehicle must visit each location (other than the depot);

(3) Each tour must comprise a continuous sequence of arcs; (4) Vehicle capacity cannot be violated; (5) No more than  $m$  vehicles can be used (if demand is such that a vehicle must be used for more than one tour, each successive tour is considered to involve a new vehicle); (6) These are  $n^2 - 3n + 2$  subtour-breaking constraints as explained in Golden et al [33].

Other IP formulations are given in Golden [32], Fisher and Jaikumar [21], and Christofides et al. [14]. However, even the formulation given above will have an enormous number of variables and constraints for a modestly-sized VSP. Thus its value lies not in its practicality as a way of solving the VSP directly, but more in its ability to yield insights which may be useful in the development of heuristics. As an example of this process, Fisher and Jaikumar [22] formulate a more general version of the VSP which includes vehicle capacity, a time span during which each location is to be visited, and a time constraint on each tour. They discovered that two well-known combinatorial optimization problems are embedded in their formulation, namely the generalized assignment problem and the travelling salesman problem. By using techniques already available for these two problems, they were able to develop a heuristic based on Benders decomposition which has impressive computational results.

Before surveying the VSP literature, we should mention the existence of previous books and surveys on the subject. The book Distribution Management, published in 1971 by Eilon et al. [20] contains a chapter on the VSP and, although its results are now somewhat dated, it has developed into something of a classic text on distribution. Two other books have recently appeared which are also devoted to distribution: An Analytical Approach to Physical Distribution Management by Wills [82] and Operational Distribution Research by Mercer et al. [53]. The first is more formal, the second is concerned mainly with case studies. Many O.R. texts contain sections devoted to the travelling salesman problem and its extensions to vehicle scheduling. Expository papers due to Bodin [6], Christofides [11], Golden [32], Mole [55], Pierce [59] and Turner et al. [73] have appeared over the last decade. Although all are excellent pieces of work, each is somewhat limited in scope or rather out of date. The intention of the present paper is to present an up-to-date broad survey.

### 3. EARLY WORK ON THE VSP - THE SAVINGS APPROACH

Two of the earliest references on this problem are due to Garvin et al. [28] who formulated the problem as a mixed I.P. in the hope that its special structure could be

exploited, and Dantzig and Ramser [17] who solved an appropriate L.P. in the hope of obtaining near-optimal solutions. An alternative approach was developed by Clarke and Wright [15] in 1964 to include variable capacity and heralded the advent of what has become known as the "savings" concept. Their heuristic which is still one of the most widely used today [36] begins by creating a separate tour for each location:  $\langle l \rightarrow i \rightarrow l \rangle, i = 2, 3, \dots, n$ . The possibility of combining various tours is then explored by calculating a list of potential savings. As an example, suppose the tour to location  $i$  and the tour to location  $j$  were combined, with  $i$  visited first, i.e.,  $\langle l \rightarrow i \rightarrow j \rightarrow l \rangle$  and this tour was feasible. Then the saving in cost,  $s_{ij}$ , is the difference between costs of the two original tours,  $2d_{li} + 2d_{lj}$ , (assuming  $d_{ij} = d_{ji}$  for all  $i$  and  $j$ ) and the cost of the new tour,  $d_{li} + d_{ij} + d_{jl}$ , i.e.

$$s_{ij} = d_{li} + d_{lj} - d_{ij}$$

The list of  $s_{ij}$ 's is scanned in order of nonincreasing size and tours are combined whenever feasible. There are two versions of this heuristic, the "multiple", in which many routes are developed in parallel, and the "sequential", in which each route is completed before the next is started. This method was also developed independently by Webb [78].

This heuristic as it stands has a number of faults. Firstly, it is myopic in the sense that it does not look ahead to discover the consequence of taking advantage of a particular saving. Secondly, its decisions are irreversible. Once an arc is accepted as part of a tour it is never discarded, and consequently one cannot predict the final number of routes. There have been a number of attempts to overcome these deficiencies. Tillman and Cochran [70] developed a procedure that was more long-sighted in that rather than choosing the largest saving, they selected the best and second-best savings at each iteration. However, Foulds et al. [26] state that this modification requires an inordinate amount of computational time. Knowles [43] embedded the savings scheme within a tree search in an attempt to discover which savings should be accepted. A similar approach was also adopted by Holmes and Parker [40]. Tillman and Hering [73] have employed more elaborate "look ahead" schemes. Gaskell [29] and Yellow [84] have replaced  $s_{ij}$  by  $s_{ij} - \theta d_{ij}$  where  $\theta$  is a real variable. By varying  $\theta$ , one can place differing amounts of emphasis on the  $i$ - $j$  travel cost (for some experimentation on the value of  $\theta$ , see [50] and [33]). Gaskell also proposed the measure  $s_{ij} = (\bar{d} + |d_{li} - d_{lj}| - d_{ij})$  where  $\bar{d}$  is the average  $d_{li}$ . However, the results do not suggest that the ideas are substantially better than that of Clarke and Wright.

Using an approach that is completely different from the savings scheme, Tyagi [75] presented a "nearest neighbour" heuristic. That is, starting with any location,  $l$  (other than the depot), one progressively builds up a tour by always selecting as the next arc the one leading to the location closest to  $l$ , subject to feasibility. When this is no longer possible, the tour is terminated with a return to the depot. Certain rules of thumb are described in order to attempt to construct reasonable tours. Once all the tours have been identified, locations on them may be re-ordered using a travelling salesman problem (TSP) algorithm. The TSP will be described in the next section. Golden et al. [33] report that the Tyagi algorithm performs poorly.

#### 4. RECENT WORK ON THE VSP

As can be seen from the last section, much of the early work on the VSP evolved from the savings scheme of Clarke and Wright. However, many attempts to apply this work to real world problems in the early 1970's lead to disappointing results as documented by Jones [41], Menzies [52], and Wentworth [80] among others. This has caused later workers to explore ideas that are in some cases radically different.

We begin by describing exact solution procedures, i.e. which guarantee an optimal solution. Some of these methods are based on the travelling salesman problem, i.e., find a minimal cost tour for a travelling salesman who must visit a number of cities exactly once each, returning to the city from whence he started. Mathematically, one must

$$\text{Minimize} \quad \sum_{i=1}^n \sum_{j=1}^n d_{ij} x_{ij} \quad (8)$$

$$\text{subject to} \quad \sum_{i=1}^n x_{ij} = 1, \quad j = 1, \dots, n \quad (9)$$

$$\sum_{j=1}^n x_{ij} = 1, \quad i = 1, \dots, n \quad (10)$$

$$q_i - q_j + nx_{ij} \leq n - 1, \quad i = 1, \dots, n, \\ j = 1, \dots, n, \quad i \neq j \quad (11)$$

for some real numbers  $q_i$

$$x_{ij} = 0 \text{ or } 1, \quad i = 1, \dots, n, \quad j = 1, \dots, n \quad (12)$$

where  $x_{ij} = 1$  if the salesman proceeds directly from city  $i$  to city  $j$ , and 0 otherwise;  $d_{ij}$  = the distance from city  $i$  to city  $j$ , and  $n$  = the number of cities.

There are some obvious similarities between the formulations (1) - (7) and (8) - (12). Thus the application of TSP methods appeared quite promising at first. Indeed, close relationships between the TSP and VSP were established by Eilon et al. [20], Hayes [37], Turner et al. [73] and Christofides [11]. A VSP method based on Little's [47] branch and bound TSP algorithm has been described by Eilon et al., [20], Turner et al. [73] and Christofides and Eilon [9]. It is motivated by the consideration that if the individual tours of a feasible VSP solution,  $S$ , are combined to form one tour, this tour passes through the depot  $m$  times. If the depot is replaced by  $m$  artificial depots, all at the original depot location, then  $S$  represents a solution to an  $(n+m-1)$  - city TSP. (Recall that location 1 represents the original depot). Inter-depot travel is prohibited by defining the appropriate travel to be prohibitively high or some value  $\lambda$ . We have now set ourselves a TSP in which each "salesman" (vehicle) has a capacity - restriction which significantly reduces the number of feasible solutions. The smallest number of artificial depots to create can be found by comparing total demand with vehicle capacity. Little's method or some other TSP procedure can now be applied with the modification that potential tours are checked for capacity feasibility. The TSP approach has the additional attribute that by altering the value of  $\lambda$ , mentioned above, the VSP can be solved with alternative objectives [11].

Pierce [60] has developed a different approach based on set partitioning, but related to Balinski and Quandt's formulation.[2] First all possible feasible single-vehicle tours are identified. The minimal cost for the subset of locations in each tour can be calculated using any of the TSP algorithms available, such as that of Held and Karp [38] and [39]. Next a matrix  $A = \{a_{ij}\}$  is formed where  $a_{ij} = 1$  if location  $i$  is on tour  $j$ , and 0 otherwise. Now, of course, in any feasible solution each location has to be a member of exactly one tour. This corresponds to selecting a set,  $T$ , of columns (tours) of  $A$  with minimum total cost where each row (location) of  $A$  has a unit entry in exactly one of element of  $T$ . The cost of each column is the value of minimal TSP solution. This problem of column selection is equivalent to the set partitioning problem for which efficient solution procedures are available [e.g., 48]. Unfortunately the enormous number of columns that appear in even small VSP applications limits the practicality of this approach.



Christofides et al. [13] proposed a backtracking procedure which is designed to overcome this problem. Rather than generate all feasible tours a priori, it is possible to generate them only as required. A branching process is used together with both upper and lower bounds to guide the selection of tours. The lower bounds can be calculated using any TSP algorithm. However, it is still not clear that sufficient reductions in the number of possibilities can be made in order to guarantee minimal solution in reasonable computational time.

The VSP can also be formulated as a dynamic programming problem as has been done by Eilon et al. [20], and Turner et al. [73]. Once again the enormous number of possibilities places severe restrictions on the size of problem that can be effectively solved.

It may well be that the finding of an efficient (polynomial time) algorithm is an impossible task. This remark is based on the fact that the VSP is NP - hard [44], which makes it extremely likely that any algorithm will be of exponential time complexity. For a thorough discussion of the theory of NP-Completeness the reader is referred to the excellent work by Garey and Johnson [27]. Given this gloomy state of affairs we turn our attention to heuristic methods.

## 5. HEURISTIC APPROACHES TO THE VSP

Heuristics for the VSP can be classified into three classes:

- (1) Route first, (RF)
- (2) Cluster first, (CF)
- (3) Relaxed optimisation, (RO)

In the RF methods, tours are progressively built up initially. This is done by either accepting arcs successively as part of the initial solution or by inserting new locations one at a time into existing fledgling tours. The decision as to which arc is accepted, or which new location is inserted and where, is made on the basis of some evaluation system which indicates the potential worth of each possible choice. It is usual to restrict the possibilities to choices which maintain feasibility. When all locations have been assigned a tour, the initial solution may then be subjected to some improvement strategies. These amount to either exchanging groups of arcs, reordering locations on a given tour, or assigning a location to a different tour.

In the CF methods no attempt is made initially to select arcs for tours. Instead the set of locations is

partitioned into subsets - each subset ultimately comprising the locations for a single tour. This process is called *clustering* and is sometimes carried out by using information about the spatial layout of the locations, e.g., polar coordinates with the depot as origin, or by solving a generalised assignment problem. Once the locations have been clustered, each cluster is subjected to a TSP method in order to determine the best sequence of locations for each tour.

The RO methods can often be classed strictly as RF methods, but they have a distinguishing characteristic which makes a separate classification worthwhile, i.e., if they are operated for enough computational time (almost always a prohibitive amount for anything other than trivial problems) they guarantee optimality. What is done is to relax the requirement of grinding on relentlessly to the absolute minimal solution; hence the name of the class. This relaxation may come about in one of several ways. For instance, rather than generate all possible feasible tours at the outset and then systematically go about selecting the best subset, one can generate tours as needed and terminate before all possibilities have been examined.

The savings scheme and its various modifications outlined in the last section all belong to the RF class. There has been considerable progress made with methods of this type in the last few years, one of the highlights being the use of sophisticated computer science techniques by Golden et al. [33] to substantially reduce computation times. However, it does not seem to be very efficient to enumerate a large number of possible savings, because when a few early savings are made most of the rest become infeasible. Mole and Jameson [45] recognised this in developing a generalised savings scheme. It involves inserting locations into partial tours and ensuring that each partial tour does not intersect itself, a condition which obviously holds in any low-cost solution. Finally, a refinement subroutine is used to improve the final tours by reassigning a location to a different tour. Golden et al. divided the area containing all locations up into a series of identical rectangles and accept an arc as part of a tour only if it connects locations within the same or neighbouring rectangles. As mentioned earlier, they use special techniques to reduce running time. They also attempt to improve the final tours produced.

Robbins et al. [62] presented an RF method which built an initial solution via the basic savings scheme. This is then improved using a concept called *r-optimality*. Basically it involves replacing *r* arcs in the solution by another *r* arcs if total cost is lowered and feasibility is maintained. When it is impossible to find such an

improvement the routine is terminated. This can be done for progressively increasing values of  $r$ . The method was developed for the TSP by Lin [45], and improved by Lin and Kernighan [46]. It was applied to the VSP by Christofides and Eilon for  $r=3$ . Robbins et al. use only 2 optimality. The method of Lin and Kernighan has been successfully applied to the VSP by Russell [64]. A feasible starting tour, is, however required, and the results are starting point-dependent.

In 1979 Buxey [7] modified the savings approach by bringing in a probabilistic element. Rather than doggedly accepting an arc representing the next biggest savings on the list, he selects the next arc on the basis of a Monte Carlo simulation and assigns it a specific direction of travel. The method appears to yield improved results for certain well-known test problems in reasonable running time. Also in 1979, Christofides et al. presented an RF method which has two phases. In phase one, tours are progressively built up by inserting locations one at a time into existing tours by means of a scoring system. After each insertion  $r$ -optimal methods are used to see if a resequencing of locations on the tour which has just been expanded will lead to an improvement. The method is such that not all locations can necessarily be assigned routes. In phase two all unassigned locations are given tours if possible using a more sophisticated scoring system and  $r$  optimality; if not, phase one is repeated until all locations are assigned.

Finally we mention Doll [19] who has the simplest RF procedure of all. He simply establishes the number of schedules required per day, the number of vehicles, identifies any potentially embarrassing geographical barriers and creates tear drop - shaped routes on a scale map of the delivery area.

We turn now to CF methods. In 1972 Wren and Holliday [83] presented a method which uses information about the spatial layout of the locations. A location list is generated in order of angular coordinates from the depot. An axis is chosen which passes through the most sparsely populated area. Locations are considered one at a time starting from the axis, and are either added to existing tours or used to create new ones to minimize the mileage added and with the consideration that feasibility must be maintained. Improvement strategies that reassign locations to different tours and resequence locations on one tour are then employed. Finally, the axis is rotated through  $90^\circ$ ,  $180^\circ$  and  $270^\circ$  and the process is repeated at each position with the best solution being chosen.

In 1974 Gillett and Miller [30] and Gillett and Johnson [31] detailed a similar method. Once the position of the axis is chosen, they "sweep" with the axis in a clockwise or anticlockwise direction using the depot location as a pivot. Locations are assigned to a single tour as they are swept until a further assignment would violate feasibility; at this point a new tour is started. Different initial positions of the axis are selected and the reverse direction of sweep is also employed. After each  $360^\circ$  sweep is completed, locations are sequenced by a TSP method. Finally, the best solution is selected from all those generated.

Another imaginative CF method due to Fisher and Kaikumar [21],[22] has already been described in section 2. It has outperformed many of the methods mentioned earlier on standard test problems. However, there are difficulties in accommodating many practical constraints with this method. But this line of attack does show promise.

Finally we briefly mention two RO methods. The first reference to Foster and Ryan has been described in section 2. The second is a relaxation of the exact tree-search method of Christofides mentioned earlier in this section. The relaxation came about by not generating all possible tours and by terminating the tree search early.

We come now to a comparison of these methods. There exists a number of standard problems in Eilon et al. [20] (as well as a few from other sources) on which new heuristics have traditionally been tried. Many papers contain results of applying various methods to these problems and the following is merely a distillation of this information which has been guided by the authors' own experience. One must keep in mind that two factors are important: the running time of a method and the quality of the solution it produces. Mole reports that the methods of Russell, Wren and Holliday, Gillett and Miller, and Foster and Ryan all require broadly similar computational times and produce solution of similar quality as mentioned earlier, Buxey's results are limited but encouraging. Golden et al. applied their savings modification to a large scale newspaper distribution problem, and while it is uncertain that their solutions are of high quality, their method is very fast. Christofides et al. recently report that their RO tree search and RF two-phase methods appear to yield better quality solutions than those of the standard savings scheme, the sweep methods, or that of Mole and Jameson, Further, the two-phase method was the fastest. Christofides et al. also report four cases for which the sweep algorithm does not perform well in relation to the other methods tested. These cases involved locations clustered into groups.

They also suggest that the sweep algorithm would not be satisfactory in non-Euclidean problems. Finally, Fisher and Jaikumar [22] report that their generalised assignment method produced in 50% of the cases better quality solutions in less time than the Christofides et al. two-phase method.

We end this section with the observation that despite the early disappointments there are now many computer packages available commercially. Greenway [36] in 1975 and more recently an article in *Which Computer?* [81] give comprehensive details of available packages. It can be seen from the *Which Computer?* article that these packages can now cater for a wide variety of specific delivery problems. For example, the *Mover* package of Christofides [12] offer alternative objectives, as well as accommodating different types of vehicles with or without compartments, different shift times and rest periods, time windows and priorities for customers, loading and unloading times and vehicle or product mix restrictions. Notice also the modern tendency to offer a computerised road-network with different speeds for different types of road. Thus the routes developed by the procedures will at least be operational. The methods may not guarantee optimal routes, but they can usually be relied upon to produce cost improvements in even small distribution systems.

## 6. DIRECTIONS FOR FURTHER RESEARCH

The vehicle scheduling problem has exercised the minds of a large number of researchers over the last fifteen years and a great number of methods have been developed. A significantly large proportion of the authors have examined the "savings" method and proposed variations to overcome its shortcomings. The reason for the amount of work spent on the savings method may be related to the simplicity of the procedure. It is easy to understand and to write computer programs using the approach. However, surprisingly little work has been done on the analysis of the savings approach. The exceptions to this are Webb [79] and Golden [34], who both examine sequential algorithms. Webb's results are somewhat inconclusive, and Golden produced a worst case analysis showing a ratio of the sequential savings solution to the optimum solution is greater than  $(6 \log_2 n + 5)/21$ , i.e., it is possible to construct a VSP where the sequential savings method performs arbitrarily badly.

What empirical evidence exists is based on a relatively small number of problems, most of which are pseudo-real, i.e., those problems which are claimed to be

real are generally abstracted from real life problems rather than being the complete problem. There exists, therefore, much work to be done in this area both in the refining of existing heuristics as well as in the design of new methods. This process would undoubtedly be aided by the publication of real life problems on which to test new techniques, but also of problems for which the optimal solution is known.

A number of directions for future work look promising. It is likely that the best route to an exact procedure lies through TSP algorithms using appropriate measures to develop bounds (even tight bounds for the TSP can be very loose bounds for the VSP) [14], and possibly using configuration constraints, i.e., location groups which are infeasible, dominated or provably non-optimal are removed. The most promising heuristics for practical problems would appear to be the tree search heuristic of Christofides et al. and the method of Fisher and Jaikumar. This former heuristic can be programmed to develop a few alternative routes satisfying the characteristics of a particular transport system. A similar characteristic is available with the integer programming approach of Foster and Ryan, though this method is less discriminating in the selection of the initial routes. An area that deserves more examination is the use of seed points from which to generate routes [37,13,22]. The Fisher and Jaikumar paper contains the interesting suggestion that a seed point need not be a customer location, but some other convenient point. Finally, a simple measure which could be examined in more detail as an alternative to the savings is the extra mileage criterion,  $m(i,k,j) = d_{ik} + d_{kj} - d_{ij}$ . This measure leads essentially to radial routes rather than the circumferential routes of the savings method.

We have described the significant methods which have been developed to solve the vehicle scheduling problem. It is clear that we are a long way from being able to solve large scale practical problems exactly, although heuristic procedures exist and are widely used in industrial situations. However, much work remains to be done in this area, both in deriving new exact procedures, and in evaluating and developing new heuristics. This effort is fuelled by an appreciation of the impact that the distribution of goods has on us all.

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