4 dimension de-Sitter space, second paper

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4. Motion equation

As usually action for free particle:

$$S = -mc \int_a^b ds \qquad ds = \sqrt{dx_\mu dx^\mu} \tag{1}$$

There $u_\mu = \frac{dx_\mu}{ds}$ and according with principle of least action:

$$\delta S = - - mc \int_a^b \frac{dx_\mu \delta dx^\mu}{ds} = -mc \int_a^b u_\mu d\delta x^\mu = 0$$

After integration by parts:

$$\delta S = -mcu_\mu \delta x^\mu \Big|_a^b + mc \int_a^b \delta x^\mu \frac{du_\mu}{dx} ds \rightarrow \frac{du_\mu}{ds} = 0 \tag{2}$$

Calculate $\omega_\mu = \frac{du_\mu}{ds}$, using ds from last paper:

$$x^\mu = \big(t \ , \ \ \theta \ , \ \ \varphi\big)$$

After reparameterize $R = 1$

$$u^\mu = \left(\frac{1}{\sqrt{-1+\dot\theta^2+\sin^2\theta\dot\varphi^2}} \ , \ \ \frac{\dot\theta}{\sqrt{-1+\dot\theta^2+\sin^2\theta\dot\varphi^2}} \ , \ \ \frac{\dot\varphi}{\sqrt{-1+\dot\theta^2+\sin^2\theta\dot\varphi^2}}\right) \tag{3}$$

With $\quad \gamma = (-1 + \dot\theta^2 + \sin^2\theta\dot\varphi^2)^{-\frac{1}{2}}$
And $\gamma' = \frac{\gamma}{ds} = \gamma^3(\sin\theta\cos\theta\dot\theta\dot\varphi^2 - \dot\theta\ddot\theta - \sin^2\theta\dot\varphi\ddot\varphi)$
in the end we have:

$$\omega^\mu = \big(\gamma' \ , \ \ \gamma'\dot\theta + \gamma^2\ddot\theta \ , \ \ \gamma'\dot\varphi + \gamma^2\ddot\varphi\big) = 0 \tag{4}$$

5. Motion equation for massless particle

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