4 dimension de-Sitter space, first paper

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1. Metric

$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & R^2 & \\ & & & R^2 \sin^2\theta \end{pmatrix} \qquad g^{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & \frac{1}{R^2} & \\ & & & \frac{1}{R^2 \sin^2\theta} \end{pmatrix}$$

It's easy to notice, that $\quad g_{\mu\gamma}g^{\gamma\mu} = \delta^{\nu}_{\mu}$ and

$$ds^2 = -dt^2 + dR^2 + R^2 d\theta^2 + R^2 \sin^2\theta d\varphi^2 \tag{1}$$

$$\sqrt{-det(g_{\mu\nu})} = \sqrt{-(-R^2\sin^2\theta)} = R^2\sin\theta = J_{R,\varphi,\theta} \tag{2}$$

2. Parallel Trasport

We are going to find how change a vector after parallel Transport on dx^j

$$dA_k = \Gamma^{l}_{jk} A_l dx_j \tag{3}$$

With Christoffel's symbols:

$$\Gamma^i_{kl} = \frac{1}{2} g^{im} (\partial_i g_{mk} + \partial_k g_{mi} - \partial_m g_{ki}) \qquad (4)$$

There are only three non-zero components in Christoffel's tensor in our metric:

$$\Gamma^2_{21} = \cot\theta \quad \Gamma^1_{22} = -\sin\theta\cos\theta \quad \Gamma^2_{12}\cot\theta$$

And for transport along a closed loop:

$$\triangle A_k = \oint \Gamma^l_{jk} A_l dx_j$$

And we can calculate that the angle which vector turns ,after transport along a closed loop on constant-radius sphere, equal the solid angle of the loop.

3. Differentiation

Usually we write : $A_i = \frac{\partial x'^k}{\partial x^i} A'_k$ and $dA_i = \frac{\partial x'^k}{\partial x^i} dA'_k$ for base substitution. But in this case, we have:

$$dA_i = \frac{\partial x'^k}{\partial x^i} dA'_k + A'_k d\frac{\partial x'^k}{\partial x^i} = \frac{\partial x'^k}{\partial x^i} dA'_k + A'_k \frac{\partial^2 x'^k}{\partial x^i \partial x^l} dx'^l; \frac{\partial^2 x'^k}{\partial x^i \partial x^l} \neq 0$$

So differentiation is difference of two infinitely close vectors, and we must do parallel trasport on this infinitely close distance dx^i. According with second paragraph get: $\delta A^i = \Gamma^i_{ki} A^k dx^i$, to sum up:

$$DA^i = (dA^i - \delta A^i)dx^l = (\frac{\partial A^i}{\partial x^l} - \Gamma^i_{ki} A^k dx^i)dx^l = A^i_{;l} dx^l \qquad (5)$$

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