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A Two-Phase Algorithm to Solve a 3-Dimensional Pallet Loading Problem

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Abstract

The pallet loading problem (PLP) is an NP-hard problem and is generally tackled by attempting to maximize the number of boxes that can be loaded orthogonally on a pallet. Several methods have been proposed in the literature that focus on maximizing the volume utilization and stability of the pallet. In this paper, a two-phase algorithm is proposed to solve a 3-dimensional PLP while considering humidity, and storage time. In the first phase, the boxes should fit completely within the pallet without any overhang, i.e. the edges of the boxes should be parallel to the edges of the pallet, where the boxes can be rotated by 90°. In the second phase, the number of horizontal layers per pallet is calculated based on three parameters, i.e. maximum allowable height, maximum allowable weight, and dynamic compressive strength of boxes at the bottom layer. The maximum allowable height and maximum allowable weight for the pallet are known. The dynamic compressive strength represents the compressive strength of a box under real conditions. It is a function of the static compressive strength (strength under lab conditions), humidity, storage time etc. Here, static compressive strength of a pallet is calculated using the modified McKee formula. The horizontal layers are loaded using the same pattern to avoid overlapping. Results show the efficiency of the two-phase algorithm in maximizing the number of boxes per pallet (i.e. pallet utilization) and stability.

Keywords: *pallet loading problem; heuristics; dynamic compressive strength; pallet utilization; overhang; overlap; pallet strength*

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1. Introduction

In a PLP, boxes with same or varying dimensions and weights are arranged on a rectangular pallet with known dimensions and weight to improve performance measures such as number of boxes per pallet (or pallet utilization) and stability. We denote the dimensions of boxes as $(a, b, h \text{ and } w)$, where a is the length of the box, b the width of the box, h the height of the box, and w is the weight of the box, respectively. We can also define the dimensions of the pallet as $(L, W, H_{max} \text{ and } M_{max})$, L is the pallet length, W the pallet width, H_{max} the maximum allowable pallet load height, and M_{max} the maximum allowable weight on the pallet$(L \geq a,\ W \geq b)$. Generally, PLP is divided into two categories based on the size of the loaded boxes [1]. The first category is the uniform PLP, where the boxes are identical (homogeneous boxes). The second category of the PLP is the mixed PLP, where the sizes of boxes are non-identical, i.e. mixed boxes of different sizes $(a_i,\ b_i,\ h_i, w_i)\ \forall\ i = 1, \ldots, q$, with q being the total number of different box sizes [2]. In the latter, the boxes can be classified as weakly heterogeneous or strongly heterogeneous based on the sizes of the boxes in the system [3], [4]. The boxes must be loaded onto the pallet completely without any overlap and overhang. In this regard, several techniques have been used in engineering literature to load the boxes onto a pallet. The first technique is Layer-by-Layer, wherein the height of boxes should be the same, so all boxes in the layer should be in contact with the same surface [5], [6, p.]. Another technique divides the pallet layout into blocks, where the boxes can be loaded in multiple orientations within the block [5], [7]–[11].

Regardless of category, several performance measures are taken into account when solving a PLP, such as pallet utilization, stability of the pallet, and strength of the pallet, considering constraints like humidity, storage time, etc. Generally, improving the pallet utilization will increase stability of the pallet as well. Recent studies focus on maximizing pallet utilization and the stability. This study arose from a research initiative at DHL Supply Chain. DHL Supply Chain is researching the best methods to load cases on the pallet considering humidity and storage time. Change in humidity affects the stability, the strength of the pallet and box mechanical strength over time [12]. As the pallet stability depends on the number of horizontal layers per pallet, and the layout pattern of boxes per layer, this represents a 3-dimensional manufacturer's PLP to determine the maximum number of boxes per pallet. In this study, a two-phase algorithm is proposed to solve the 3-dimensional PLP with identical boxes, considering storage time and humidity without any pallet overlap and overhang. In this first phase, the number of the identical boxes per horizontal layer and corresponding box loading layout or pattern on the pallet is determined using five heuristics, i.e. one-block heuristic, three-block heuristic, five-block heuristic, hollow block heuristic and G5-heuristic [9]–[11] . In the second phase, for each of the five heuristics, the number of horizontal layers per pallet is calculated based on H_{max} M_{max}, and dynamic compressive strength of the pallet. Consequently, the number of boxes per pallet is determined for each of the five heuristics based on the number of boxes per horizontal layer and the number of horizontal layers. Finally, the box loading layout which results in the maximum number of boxes per pallet is chosen.

The remaining parts of this paper are as follows: Section 2 provides a review of relevant literature. Section 3 describes the proposed methodology in detail. Finally, Section 4 illustrates the results and conclusion.

2. Literature Review

Most research studies divide PLP into two types based on the size of the boxes [1]. The first type is the manufacturer's pallet loading problem (MPLP), where similar products are assigned into identical boxes. The second type is the distributor's pallet loading problem (DPLP), where dissimilar products are loaded into the heterogeneous boxes with different sizes based on the customer demand. Many researchers have studied the MPLP [13], [14] and DPLP [15], [16] to maximize the number of boxes per pallet and to improve pallet volume utilization and pallet stability. Researchers have used a broad set of methods including exact methods (mathematical models), meta-heuristics (genetic algorithm, tabu search, etc), and heuristics to solve the MPLP. For example, a two-phase algorithm to solve the 3-dimesional MPLP is described in [6]. In the first phase, an integer linear programming model was utilized to maximize the number of boxes that can be loaded on the pallet [6]. In the second phase, a second mathematical model was developed based on the results of the first phase to create the box loading layout or pattern on the pallet [6]. A tabu search algorithm was used in [13] to solve the MPLP. Additionally, several heuristics have been proposed in literature to solve the MPLP, such as the one-block heuristic, two-block heuristic, three-block heuristic, diagonal block heuristic, hollow block heuristic, G4-heuristic, and G5-heuristic [9]–[11].

This paper focuses on solving a real life case study of MPLP with additional constraints including humidity and storage time. There are limited studies of the MPLP considering humidity, and storage time. Malasri *et al.* explained the effect of temperature and humidity on the strength of softwood pallets [17]. Their study were based on a compressive test performed by using a compressive test device to define the performance of the pallets. The results have shown an inverse relationship between compressive strength and temperature. Fadiji *et al.* illustrated the negative impact of relative humidity and temperature on compressive strength, i.e. the compressive strength of paperboard package decreases with increasing humidity and drop in temperature [18]. The authors used the Lansmont compression tester-squeezer in their experiment [18]. Furthermore, finite element modeling was also used to analyse results obtained. Bandyopadhyay *et al*. [19] illustrated that the mechanical properties such as the modulus of elasticity, yield stress, and tensile stress decrease as the humidity increases. They developed a mathematical model to define the transfer of moisture through paperboard. The mathematical model was developed based on conduction and convection rules. In this paper, the box layout on the pallet and the number of boxes per horizontal layer are determined in the first phase using the best among the five heuristics, i.e. one-block heuristic, three-block heuristic, five-block heuristic, hollow block heuristic, and G5-heuristic [9]–[11], designed to avoid overlap. Then, the number of horizontal layers per pallet is calculated in the second phase considering the storage time, and humidity constrained by H_{max}, M_{max} and the dynamic strength. Finally, the maximum number of boxes per pallet is determined based on results from the two phases.

3. Methodology Used:

In this paper, the number of boxes loaded per horizontal layer and the number of horizontal layers on the pallet are calculated using a two-phase algorithm to maximize the number of boxes per pallet. This study ensures that the boxes fit completely on the pallet without any overlap and overhang. If pallet area divided by box area is < 101 then Martins and Dell show that either one-block heuristic, three-block heuristic, five-block heuristic [9], [11], hollow-block heuristic or the G5-heuristic [10] finds the maximum number of identical boxes per layer. In the second phase, the number of horizontal layers per pallet is calculated based on the H_{max} , M_{max}, and dynamic compressive strength parameters considering humidity and storage time. Finally, pallet volume utilization post algorithm run shows the efficacy of the proposed algorithm.

3.1 Phase One: Box Arrangement by Layer

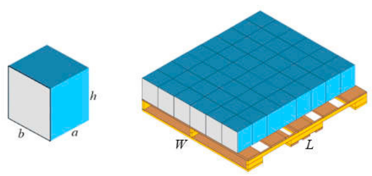
In this section, one-block heuristic, three-block heuristic, five-block heuristic, hollow-block heuristic, and G5-heuristic used to arrange identical boxes on the layer with known and fixed dimensions. Any one of the heuristics might yield the maximum number of boxes per layer in a pallet, depending on the box dimensions. Table 1 shows an example to examine the proposed algorithm.

Table 1. Box and Pallet Dimensions.

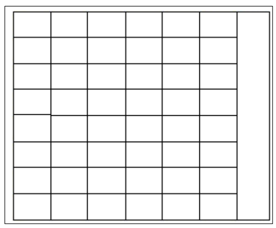
Box length $a\ (in)$	Box width $b\ (in)$	Box height $h\ (in)$	Box weight $w\ (lb)$	Pallet length $L\ (in)$	Pallet width $W\ (in)$	Pallet height $H\ (in)$	Pallet weight $M\ (lb)$	$H_{max}(in)$	$M_{max}\ (lb)$
5	7	9	3	48	40	5	50	50	5000

This paper used *feasible* partition, *efficient* partition, and *perfect* partition to fit the boxes completely on the pallet without overlap and overhang. Assuming that (m, n) denotes an ordered pair of non-negative integers satisfying $m * a + n * b \leq S$ for a pallet dimension S, *(L or W)*, then the ordered pair (m, n) is called a *feasible* partition of S. If m and n also satisfy $0 \leq S - m * a - n * b < a$, then (m, n) is called an *efficient* partition of S. If m and n satisfy $m * a + n * b = S$, then (m, n) is called a *perfect* partition of S. The set of all *feasible* partitions of pallet dimension S is denoted as *F(S, a, b)*. The set of all *efficient* partitions of pallet dimension S is denoted as *PE(S, a, b)*, and the set of all *perfect* partitions of pallet dimension S is denoted as *P(S, a, b)* [20], [21].

The one-block heuristic is used to load the boxes on a horizontal layer without overhang. The boxes can be loaded in either H-box or V-box orientation based on the ratio between the dimensions of the pallet and the dimensions of the boxes. For example, identical boxes with known dimensions and weight $(a, b, h\ and\ w)$ are loaded on the pallet with dimensions $(L, W, H_{max}\ and\ M_{max})$, where h is dimension perpendicular to the surface of the pallet as shown in Figure 1. If $\left\lfloor \frac{L}{a} \right\rfloor * \left\lfloor \frac{W}{b} \right\rfloor > \left\lfloor \frac{W}{a} \right\rfloor * \left\lfloor \frac{L}{b} \right\rfloor$, where the length of the boxes is parallel to the length of the pallet, then the horizontal layout pattern for boxes in this layer is called H-box pattern, else, if the length of box is parallel to the width of the pallet, the box vertical layout pattern is called V-boxes pattern. All boxes in the layer should be loaded on the pallet in uniform orientation, which means all boxes must have either an H-box or V-box layout (Figure 1). There are three possibilities to load the boxes in the layer, where either $a, b\ or\ h$ is perpendicular to the surface of pallet, respectively.



Arrangement of Boxes per Horizontal Layer using the One-Block Heuristic



Top view for the horizontal layer using the One-Block Heuristic

Figure 1. One-Block Heuristic.

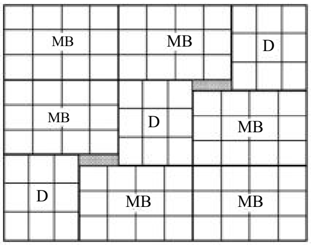
The layout of the pallet can also be divided into three or five blocks. Boxes in each block should be arranged uniformly in either an H-box or V-box orientation. Then the number of boxes per layer can be determined by the three-block heuristic or the five-block heuristic [9], [22].

Three-Block Heuristic Five-Block Heuristic

Figure 2. The Three-block Heuristic and Five-block Heuristic [22].

The hollow block heuristic can also be utilized to load identical boxes in a layer. First the heuristic divides the layer layout into diagonal and main blocks, as given in Figure 2. Diagonal blocks (D) and main blocks (MB) are created with boxes loaded in different directions, where these blocks should cover the entire length and width of pallet [10].

Figure 3. Hollow Block [10].



The G5-heurstic is an important heuristic in the PLP literature that maximizes the number of boxes per layer. It relies on dividing a layer on a pallet into five blocks (four blocks on the corner, and one in the centre). The dimensions of the central block is based on the dimensions of the four corner blocks. Each block is loaded independently, and may be loaded using a different heuristic. Therefore, within the 5 blocks, to maximize the number of boxes and minimize the waste area on a block, the number of boxes loaded each block can be determined by any one among one-block heuristic, three-block heuristic, hollow block heuristic or five-block heuristic. Then the sum of maximum number of boxes generated for each block is equal to the total number of boxes per layer [10].

The maximum number of boxes per layer is determined using all of the five aforementioned heuristics. Then the optimal solution, which gives the maximum number of cases per layer, among the five heuristics is chosen for three scenarios. The three scenarios are with *h, b* and *a* as perpendicular dimension to the pallet (shown in Table 2). The maximum number of boxes per layer for the example in Table 1 was obtained using the five-block heuristic.

Table 2. The Maximum Number of Boxes per Pallet.

Solution #	Dimension Perpendicular to Base	Boxes/Layer
1C	h	54
2C	b	40
3C	a	29

3.2 Phase Two: Computing the Number of Layers per Pallet

After getting the maximum number of identical boxes per layer, in the next phase, the number of the horizontal layers per pallet are calculated considering three parameters, (a) total height of the horizontal layers should not exceed the H_{max}, (b) the total weight on the pallet should not be more than M_{max}, and (c) the load on the boxes in the bottom layer of pallet should be less than its dynamic compressive strength considering humidity and storage time.

3.2.1 Number of Horizontal Layers based on the H_{max}

This section calculates maximum number of horizontal layers per pallet by constraining based on only H_{max} of the load on the pallet. The H_{max} is either based on storage on pallet within warehouse, i.e. racks, floor etc. or based on transportation constraints, i.e. trailer internal height, number of layers loaded etc. Equation 1 calculates the maximum number of horizontal layers per pallet. This paper assumes that all the layers will have the same pattern where overlap between layers is not allowed, therefore, the all the horizontal layers have the same height.

$$The\ Number\ of\ Horizontal\ Layers_{Hmax} = \left\lfloor \frac{H_{max}}{Boxe'sDimension\ Perpendicular\ to\ Base} \right\rfloor \tag{1}$$

Table 3 shows the number of boxes, layers and the load weight per pallet considering the H_{max} for the example in Table 1.

Table 3. The Number of Layers based on the H_{max}.

Solution #	Dimension Perpendicular to Base	Boxes/ Layer	Layers/Pallet (Based on H_{max})	Boxes/ Pallet	Load Weight from Boxes
1C	h	54	5	270	810
2C	b	40	7	280	840
3C	a	29	10	290	870

3.2.2 Number of layers based on M_{max}

Next, the number of horizontal layers per pallet is identified based on the M_{max}. Considering the pallet's maximum weight capacity to define the number of the horizontal layers per pallet is important for several reasons. First, if there is no height limit, the number of horizontal layers will be determined based on the maximum weight capacity per pallet. Second, the total weight of the boxes per pallet when loading on the shelves or the trailers should not be more than the maximum weight capacity of the pallet. Third, if the multiple pallets could be stacked on top of each other, the maximum weight of the bottom layer should be taken into consideration. Equation 2 defines the number of horizontal layers per pallet based on the pallet's maximum weight capacity or allowable weight:

$$The\ Number\ of\ Layers_{M_{max}} = \left\lfloor \frac{M_{max}}{Number\ of\ Boxes\ per\ Layer * w} \right\rfloor \tag{2}$$

The total weight of the boxes in each of three scenarios shown does not exceed the M_{max} for a pallet. When the pallets do not have limitations for the maximum height, the number of layers per pallet should be determined based on the pallet's maximum weight, constrained by the dynamic strength of the boxes in the bottom layer. Table 4 presents the results for the example in Table 1 based on the M_{max}.

Table 4. The Number of Layers per Pallet based on the M_{max}.

Solution #	Dimension Perpendicular to Base	Boxes/ Layer	Weight/Layer	Layers/ Pallet (Based on M_{max})
1C	h	54	162	30
2C	b	40	120	41
3C	a	29	87	57

3.2.3 Number of Layers Considering the Dynamic Compressive Strength of Boxes

Compressive strength is the capacity of a material or structure to withstand loads without any deterioration. For a pallet, compressive strength is the maximum weight that the boxes in the bottom layer can support without any damage. In this section, pallet dynamic compressive strength is calculated through modifying the McKee formula by adding considerations for humidity, storage time, and the pallet shape coefficients. McKee formula is usually used to find the static compressive strength, which is a theoretical value under ideal laboratory conditions, with the temperature and humidity controlled within 73˚± 2˚F and 50% ± 2% RH respectively [22]. On the other hand, under real conditions, dynamic compressive strength is the actual compressive strength of a box. Equations 3, 4, and, 5 show the static compressive strength, dynamic compressive strength, and actual load on boxes in the bottom layer. ECT is the Carton Edge Crush Test, which measures the ability of a corrugated board to sustain a top-to-bottom load [24]. CAL is calliper of the corrugated board, PER is box perimeter from top view, S_S is the static compressive strength, F_o is the orientation coefficient form strength, F_T is the storage time coefficient, F_H is the humidity coefficient, and F_G is the pallet shape coefficient (Table 5). Additionally, w_{ij} is the weight of box j at horizontal layer i, n_i is the number of boxes per horizontal layer i, m is the number of horizontal layers, and k is the number of boxes per horizontal layer.

$$Static\ Compression\ Strength\ (S_S) = 5.874 * ECT * CAL^{0.508} * PER^{0.492} * F_o \tag{3}$$

$$\text{Dynamic } Compression \text{ Strength } (S_D) = S_S * F_T * F_H * F_G \tag{4}$$

$$Actual\ load\ on\ boxes\ in\ the\ bottom\ layer = \frac{\sum_{i=2}^{m}\sum_{j=1}^{k} w_{ij}}{\sum_{i=2}^{m} n_i} \tag{5}$$

Table 5. Dynamic Strength Factors [23].

Storage Time (F_T)		Relative Humidity (F_H)		Pallet Surface Gapped (F_G)		Perpendicular Dimension to Base (F_o)	
0	1	0~0.45	1.1	Yes	0.92	The Second Shortest Dimension of Box	1
1~3	0.7	0.45~0.55	1	No	1	Shortest Dimension of Box	0.9
4~10	0.65	0.55~0.65	0.9			Longest Dimension of Box	0.8
11~30	0.6	0.65~0.75	0.8				
31~90	0.55	0.75~0.85	0.7				
91~120	0.5	0.85~1	0.5				
121~300	0.45						

Table 6 shows the maximum number of the horizontal layers that can be loaded on the pallet considering the dynamic compressive strength, where $ECT = 35.7$ and $CAL = 0.159$. The results shown in Tables 3, 4 and 6 show that maximum number of horizontal layers are obtained by using S_D.

Table 6. The Maximum Number of Layers based on the Dynamic Strength.

Solution #	Dimension Perpendicular to Base	Static Strength	Dynamic Strength (S_D)	Layer/ Pallet (Based on S_D)
1C	h	314.82	188.26	62
2C	b	382.08	228.48	76
3C	a	453.36	271.11	90

3.2.4 Maximum Number of Boxes per pallet considering all Three Parameters

The results obtained in sections 3.2.1, 3.2.2 and 3.2.3 could be different due to different constraints on number of layers. Equation 6 calculates the maximum number of horizontal layers per pallet including all the constraints, i.e. H_{max}, M_{max} and compressive strength of boxes to avoid any pallet damage:

$$Number\ of\ layers\ per\ pallet = min\left(\frac{H_{max}}{\text{Dimension Perpendicular to Base}}, \frac{M_{max}}{w * Number\ of\ Layer\ per\ layer}, \frac{S_D}{w}\right) \quad (6)$$

Table 7 lists the maximum number of boxes and horizontal layers that can be loaded on the pallet using a proposed two-phase algorithm for the example in Table 1. The number of boxes and the number of layers per pallet, calculated using Equation 6, are determined based on the H_{max}. The table shows that the maximum number of boxes per pallet is 290 when the box length is perpendicular to the base. It is worth noticing that, the maximum number of boxes per pallet depends on the number of boxes per layer and number of horizontal layers per pallet simultaneously. For instance, when the box length in perpendicular to the base, the number of boxes per layer is 29 with 10 horizontal layers giving the total number of boxes per pallet equal to 290. In contrast, with the box height perpendicular to the base, the maximum number of boxes per layer is 54, but this layout can only have maximum 5 horizontal layers, thereby making the total number of boxes per pallet equal to 270.

Table 7. The Number of Boxes per Pallet using the Proposed Algorithm.

Solution #	Dimension Perpendicular to Base	Number of Layers/ Pallet	Number of Boxes/Layer	Number of Boxes/Pallet
1C	h	5	54	270
2C	b	7	40	280
3C	a	10	29	290

3.3 Pallet Load Pallet Volume Utilization

The parameter pallet load pallet utilization or simply called pallet utilization is calculated using Equation 7 to evaluate the efficacy of the two-phase algorithm:

$$Pallet\ Volume\ Utilization\ (PVU) = \frac{Number\ of\ Boxes\ per\ Pallet * (a * b * h)}{(L * W * H_{max})} * 100\% \quad (7)$$

The pallet volume utilization for the example in Table 1 is 95.16%. This means that approximately 4.84% of the total load volume on the pallet is empty space.

4. Results and Conclusion

This section shows the results from 15 real-life datasets from the DHL Supply Chain using the two-phase algorithm. The analysis here compares total number of boxes per pallet using the manual pallet loading method or one block heuristic and the proposed algorithm. Table 8 and Table 9 list the results obtained, for the 15 datasets, using manual method and the proposed algorithm, respectively. The maximum number of boxes per pallet, is calculated using Equations 8 and 9, to test the performance of both methods.

$$Max\ Number\ of\ Boxes\ \ by\ H_{max} \approx \left[\frac{(L * W * H_{max})}{(a * b * h)}\right] \quad (8)$$

$$Max\ Number\ of\ Boxes\ \ by\ M_{max} \approx \left[\frac{M_{max}}{w}\right] \quad (9)$$

Table 8. Number of Layers and Boxes Loaded on the Pallet considering the H_{max}, M_{max} and SD using Manual Approach.

S.No.	Box Length a (in)	Box Width b (in)	Box Height h (in)	Box Weight w (lb)	Pallet Length L (in)	Pallet Width $W(in)$	$H_{max}(in)$	$M_{max}(lb)$	Vertical Dim	Number of Horizontal Layers	Number of Boxes (Manual)	Max Number of Boxes by H_{max}	Max Number of Boxes by M_{max}	PVU %
1	9.375	4.812	5.375	1.344	46.9	38.3	43.0	419.3	h	8	288	319	312	90.3%
2	9.625	7.250	6.750	1.500	45.8	38.6	40.5	225.0	h	6	144	153	150	94.1%
3	14.750	7.500	10.130	15.630	45.0	37.0	40.5	937.8	h	4	48	61	60	78.7%
4	15.630	11.560	10.690	31.685	46.9	38.8	42.8	1267.4	h	4	36	41	40	87.8%
5	9.250	8.625	7.750	3.805	46.2	35.1	51.5	456.6	a	5	100	136	120	73.5%
6	6.125	4.750	6.250	1.945	47.6	39.4	30.5	497.9	b	6	252	315	256	80.0%
7	23.250	13.310	10.500	48.460	48.0	40.0	42.0	1163.0	h	4	24	25	24	96.0%
8	23.500	14.440	7.000	13.005	48.0	40.0	42.0	390.2	h	6	24	34	30	70.6%
9	23.250	13.310	10.500	44.236	48.0	40.0	42.0	1061.7	h	4	24	25	24	96.0%
10	18.560	10.810	7.000	24.480	48.0	40.0	42.0	1175.0	h	6	48	58	48	82.8%
11	10.940	8.375	10.500	18.768	48.0	40.0	42.0	1351.3	h	4	64	84	72	76.2%
12	15.750	12.750	8.500	12.061	48.0	40.0	93.5	1459.4	h	11	99	106	121	93.4%
13	14.750	9.875	3.375	7.231	48.0	40.0	45.2	1041.3	h	12	144	177	144	81.4%
14	6.500	3.438	6.813	0.316	48.0	40.0	40.9	155.5	h	6	468	516	492	90.7%
15	18.000	12.000	3.000	16.900	38.0	30.0	28.8	676.0	b	2	40	51	40	78.4%

Table 9. Number of Layers and Boxes Loaded on the Pallet considering the H_{max}, M_{max} and SD using the Proposed Algorithm

S.No.	Box Length a (in)	Box Width b (in)	Box Height h (in)	Box Weight w (lb)	Pallet Length L (in)	Pallet Width $W(in)$	$H_{max}(in)$	$M_{max}(lb)$	Vertical Dim	Number of Horizontal Layers	Number of Boxes (Proposed Algorithm)	Max Number of Boxes by H_{max}	Max Number of Boxes by M_{max}	PVU%
1	9.375	4.812	5.375	1.344	46.9	38.3	43.0	419.3	*h*	8	312	319	312	97.8%
2	9.625	7.250	6.750	1.500	45.8	38.6	40.5	225.0	*h*	6	144	153	150	94.1%
3	14.750	7.500	10.130	15.630	45.0	37.0	40.5	937.8	*h*	4	60	61	60	98.4%
4	15.630	11.560	10.690	31.685	46.9	38.8	42.8	1267.4	*h*	4	40	41	40	97.6%
5	9.250	8.625	7.750	3.805	46.2	35.1	51.5	456.6	*h*	6	114	136	120	83.8%
6	6.125	4.750	6.250	1.945	47.6	39.4	30.5	497.9	*b*	6	252	315	256	80.0%
7	23.250	13.310	10.500	48.460	48.0	40.0	42.0	1163.0	*h*	4	24	25	24	96.0%
8	23.500	14.440	7.000	13.005	48.0	40.0	42.0	390.2	*h*	6	30	34	30	88.2%
9	23.250	13.310	10.500	44.236	48.0	40.0	42.0	1061.7	*h*	4	24	25	24	96.0%
10	18.560	10.810	7.000	24.480	48.0	40.0	42.0	1175.0	*h*	6	48	58	48	82.8%
11	10.940	8.375	10.500	18.768	48.0	40.0	42.0	1351.3	*h*	4	72	84	72	85.7%
12	15.750	12.750	8.500	12.061	48.0	40.0	93.5	1459.4	*h*	11	99	106	121	93.4%
13	14.750	9.875	3.375	7.231	48.0	40.0	45.2	1041.3	*a, b, h*	3, 4, 12	144	177	144	81.4%
14	6.500	3.438	6.813	0.316	48.0	40.0	40.9	155.5	*h*	6	492	516	492	95.3%
15	18.000	12.000	3.000	16.900	38.0	30.0	28.8	676.0	*b, h*	2	40	51	40	78.4%

For example, the number of boxes per pallet for dataset 1 using the manual calculation is 288 with pallet volume utilization of 90.3%, while the number of boxes per pallet increased to 312 using the proposed two-phase algorithm with a pallet utilization of 97.8%. As shown, the maximum boxes per pallet for dataset 1 was constrained by M_{max}. On the other hand, dataset 12 is constrained by H_{max}, where $\lfloor H_{max}/h \rfloor = 11\ layers/pallet$), with 106 boxes per pallet which is less than the maximum number of boxes per pallet based on M_{max}. For dataset 15, maximum number of boxes are found, constrained by M_{max}, when box width or height is perpendicular to base. Additionally, the pallet utilization for dataset 14 is 90.7% and 95.3% by using the manual and proposed algorithm respectively. On careful observation, it can be seen that by using mixed mode, H-box and V-box orientation simultaneously improves the pallet utilization.

Table 10 presents the % improvement with the proposed algorithm over the manual method. The results illustrate that the maximum improvement is 25.0%; it is observed in dataset 3. In this regard, the proposed algorithm improved PVU% results anywhere from 5.1% to 25% based on the obtained results. The average improvement over 15 datasets is 6.7%.

Table 10. % Improvement of the PVU.

S.No.	PVU%		%
	Proposed Algorithm	Manual Method	Improvement
1	97.8%	90.3%	8.3%
2	94.1%	94.1%	0.0%
3	98.4%	78.7%	25.0%
4	97.6%	87.8%	11.2%
5	83.8%	73.5%	14.0%
6	80.0%	80.0%	0.0%
7	96.0%	96.0%	0.0%
8	88.2%	70.6%	24.9%
9	96.0%	96.0%	0.0%
10	82.8%	82.8%	0.0%
11	85.7%	76.2%	12.5%
12	93.4%	93.4%	0.0%
13	81.4%	81.4%	0.0%
14	95.3%	90.7%	5.1%
15	78.4%	78.4%	0.0%
	Min		5.1%
	Max		25.0%
	Average		6.7%

This paper tackled the 3-dimensional MPLP to maximize pallet volume utilization. The research can be extended in future to include maximizing utilization while considering overlap and overhang, model for mixed box sizes etc.

References

[1] T. J. Hodgson, "A combined Approach to the Pallet Loading Problem.," IIE Transactions, vol. 14, no. 3, pp. 175-82, Sep. 1982.

[2] A. P. Davies and E. E. Bischoff, "Weight distribution considerations in container loading," *European Journal of Operational Research*, vol. 114, no. 3, pp. 509–527, May. 1999.

[3] J. F. Gonçalves and M. G. C. Resende, "A parallel multi-population biased random-key genetic algorithm for a container loading problem," *Computers & Operations Research*, vol. 39, no. 2, pp. 179–190, Feb. 2012.

[4] H. C. W. Lau, T. M. Chan, W. T. Tsui, G. T. S. Ho, and K. L. Choy, "An AI approach for optimizing multi-pallet loading operations," *Expert Systems with Applications*, vol. 36, no. 3, pp. 4296–4312, Apr. 2009.

[5] F.-H. F. Liu and C.-J. Hsiao, "A three-dimensional pallet loading method for single-size boxes. Journal of the Operational Research Society, vol. 48, no. 7, pp. 726-735, Jul. 1997.

[6] K. C. Wu and C. J. Ting, "A Two-phase algorithm for the manufacturer's pallet loading problem," in *2007 IEEE International Conference on Industrial Engineering and Engineering Management*, Singapore, pp. 1574–1578, Dec. 2007.

[7] E. E. Bischoff, F. Janetz, and M. S. W. Ratcliff, "Loading pallets with non-identical items," *European Journal of Operational Research*, vol. 84, no. 3, pp. 681–692, Aug. 1995.

[8] Harold J. Steudel, "Generating Pallet Loading Patterns: A Special Case of the Two-Dimensional Cutting Stock Problem," *Management Science*, no. 10, pp. 997-1004, Oct. 1979.

[9] G. H. A. Martins and R. F. Dell, "The minimum size instance of a Pallet Loading Problem equivalence class," *European Journal of Operational Research*, vol. 179, no. 1, pp. 17–26, May 2007.

[10] G. H. A. Martins and R. F. Dell, "Solving the pallet loading problem," *European Journal of Operational Research*, vol. 184, no. 2, pp. 429–440, Jan. 2008.

[11] J. Nelissen, "New Approaches to Pallet Loading Problem.pdf." 1993.

[12] T. M. Berry, A. Ambaw, T. Defraeye, C. Coetzee, and U. L. Opara, "Moisture adsorption in palletised corrugated fibreboard cartons under shipping conditions: A CFD modelling approach," *Food and Bioproducts Processing*, vol. 114, pp. 43–59, Mar. 2019.

[13] E. Silva, J. F. Oliveira, and G. Wäscher, "The pallet loading problem: a review of solution methods and computational experiments," *International Transactions in Operational Research*, vol. 23, no. 1–2, p. 147, 20160101.

[14] R. Morabito and R. Farago, "A Tight Lagrangean Relaxation Bound for the Manufacturer's Pallet Loading Problem," vol. 2, no.1, pp. 57-76, Jul. 2002.

[15] M. Schuster, R. Bormann, D. Steidl, S. Reynolds-Haertle, and M. Stilman, "Stable stacking for the distributor's pallet packing problem," in *2010 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Taipei, pp. 3646–3651, Oct. 2010.

[16] J. Terno, G. Scheithauer, U. Sommerweiû, and J. Riehme, "An efficient approach for the multi-pallet loading problem," *European Journal of Operational Research*, p. 10, 1997.

[17] S. Malasri *et al.*, "Effect of Temperature on Static and Impact Properties of New Softwood Pallets," *International Journal of Advanced Packaging Technology*, vol. 1, no. 1, pp. 30–39, Nov. 2013.

[18] T. Fadiji, C. Coetzee, and U. L. Opara, "Compression strength of ventilated corrugated paperboard packages: Numerical modelling, experimental validation and effects of vent geometric design," *Biosystems Engineering*, vol. 151, pp. 231–247, Nov. 2016.

[19] A. Bandyopadhyay, B. V. Ramarao, and S. Ramaswamy, "Transient moisture diffusion through paperboard materials," *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, vol. 206, no. 1–3, pp. 455–467, Jul. 2002.

[20] E. E. Bischoff and W. B. Dowsland, "An Application of the Micro to Product Design and Distribution," *al Research Society*, vol. 33, no. 3, p. 11, 1982.

[21] K. A. Dowsland, "The Three-Dimensional Pallet Chart: An Analysis of the Factors Affecting the Set of Feasible Layouts for a Class of Two-Dimensional Packing Problems," vol. 35, no. 10, pp. 895-905, Oct. 1984.

[22] B. Frank, "Corrugated Box Compression-A Literature Survey: Corrugated Box Compression-A Literature Survey," *Packaging Technology and Science*, vol. 27, no. 2, pp. 105–128, Feb. 2014.

[23] Cape Pack user guide. http://docs.esko.com/docs/en-us/cape/2.13/userguide/cp213 retail user guide-master.pdf

[24] Correll JD, inventor; CORRELL CONCEPTS LP, assignee. Crush-resisting paperboard clamshell carton. United States patent US 6,223,979. 2001 May 1.