

1. $ac + bc'd + abd$
 $= ac + bc'd + abd(c + c')$ // multiply by 1
 $= ac + bc'd + abdc + abdc'$ // expand
 $= ac + abdc + bc'd + abdc'$ // associative property
 $= ac(1+bd) + bc'd(1+a)$ // factor out ac and bc'd
 $= ac + bc'd$
2. $(a + c)(b + c' + d)(a + b + d) = (a + c)(b + c' + d)$
 $=> (a+c)(b+c'+d)(a+b+d)$
 $=> (a+c)((b+c')(a+b) + d)$ // factor out d because if d is true then both exp are true
 $=> (a+c)(ab + bb \ bc' + ac' + d)$ // multiply out the right side
 $=> (a+c)(b(a+1 + c') + ac' + d)$ // factored out b
 $=> (a+c)(b + c'(a) + d)$ // resolved to 1 since $x \vee 1 = 1$
 $=> (ab + ac' + ad + bc + cd)$ // multiplied everything
 $=> a(b+c'+d) + cb + cc' + cd$ // added 0 or cc'
 $=> a(b+c'+d) + c(b + c' + d)$ // factored out
 $=> (a+c)(b+c'+d)$ // here you go'
3. $f(a,b,c,d) = ac + bc'd + abd$
 $f(a,b,c,d) = cf(a,b,1,d) + c'f(a,b,0,d)$ // shannon expansion
 $cf(a,b,1,d) = c(ac + abd)$
 $cf(a,b,0,d) = c'(bd + abd)$
 $ac + abcd + bc'd + abcd$
 $ac(1+bd + bd) + bc'd$
 $= ac + bc'd$
4. $f(a,b,c,d) = (a + c)(b + c' + d)(a + b + d)$
 $f(a,b,c,d) = cf(a,b,1,d) + c'f(a,b,0,d)$ // shannon expansion
 $f(a,b,1,d) = (b+d)(a+b+d)$
 $f(a,b,0,d) = (a)(a+b+d)$
 $f(a,b,c,d) = c(b+d)(a+b+d) + c'(a)(a+b+d)$
 $= (cb + cd)(a+b+d) + c'a(a+b+d)$
 $= (abc + bc + bcd) + (acd + bcd + cd) + (c'a + abc' + ac'd)$
// remove terms
 $(abc + abc' = ab(c + c') = ab)$
 $(bc + bcd => bc(1+d) = bc$
 $acd + ac'd => ad(c+c')$
leftover terms becomes result
 $= (ab) + bc + ad + ac' + cd + cc' => (a+c)(b+c' + d)$

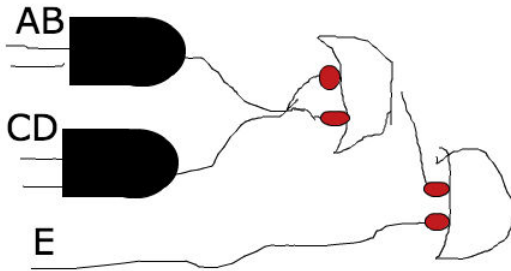
5. Bubble pushing

$$f(A,B,C,D,E) = \neg(AB+CD) + \neg E$$

STEPS : Turn the AND gate connecting AB and CD into an OR gate, reversing the bubbles.

Afterwards turn the AND gate connecting $\neg(AB+CD)$ and E into an OR gate, reversing the bubbles.

There are five literals, four operators, 4 gates, 9 nets, and 12 pins



6. Truth table below. The functions below are using the IDs as input.
b.

$$d0 = F(A,B,C,D,E) = \sum (\min(3,4,5,6,7, 13,14,15,16,23,24,25,26,27))$$

$$d1 = \sum (\min(2,3,5,6,8,11,12,15,17,18,21,22,24,27,28,31))$$

$$b1 = \sum (\min(0,8,9,10,16,17,18,19,20,24,25,26,27,28,29,30))$$

c.

$$d0 = F(A,B,C,D,E) = \prod (\min(0,1,2,7,8,9,10,11,12,17,18, 19,20,21,22,27,28,29,30,31))$$

$$d1 = F(A,B,C,D,E) = \prod (\min(0,3,4,7,9,10,13,14,16,19,20,23,25,26,29,30))$$

$$b1 = \prod (\min(1,2,3,4,5,6,7,11,12,13,14,15,21,22,23,31))$$

ID	x0	x1	y0	y1	b0	d0	d1	b1	
0	0	0	0	0	0	0	0	0	0-0-0
1	0	0	0	0	1	0	1	1	0-0-1
2	0	0	0	1	0	0	1	1	0-1-0
3	0	0	0	1	1	1	0	1	"0-1-1"
4	0	0	1	0	0	1	0	1	"0-2-0"
5	0	0	1	0	1	1	1	1	"0-2-1"
6	0	0	1	1	0	1	1	1	"0-3-0"
7	0	0	1	1	1	0	0	1	"0-3-1"
8	0	1	0	0	0	0	1	0	"1-0-0"
9	0	1	0	0	1	0	0	0	"1-0-1"
10	0	1	0	1	0	0	0	0	"1-1-0"
11	0	1	0	1	1	0	1	1	"1-1-1"
12	0	1	1	0	0	0	1	1	"1-2-0"
13	0	1	1	0	1	1	0	1	"1-2-1"
14	0	1	1	1	0	1	0	1	"1-3-0"
15	0	1	1	1	1	1	1	1	"1-3-1"
16	1	0	0	0	0	1	0	0	"2-0-0"
17	1	0	0	0	1	0	1	0	"2-0-1"
18	1	0	0	1	0	0	1	0	"2-1-0"
19	1	0	0	1	1	0	0	0	"2-1-1"
20	1	0	1	0	0	0	0	0	"2-2-0"
21	1	0	1	0	1	0	1	1	"2-2-1"
22	1	0	1	1	0	0	1	1	"2-3-0"
23	1	0	1	1	1	1	0	1	"2-3-1"
24	1	1	0	0	0	1	1	0	"3-0-0"
25	1	1	0	0	1	1	0	0	"3-0-1"
26	1	1	0	1	0	1	0	0	"3-1-0"
27	1	1	0	1	1	0	1	0	"3-1-1"
28	1	1	1	0	0	0	1	0	"3-2-0"
29	1	1	1	0	1	0	0	0	"3-2-1"
30	1	1	1	1	0	0	0	0	"3-3-0"
31	1	1	1	1	1	0	1	1	"3-3-1"

