TD4 : Full State Feedback

Jan, 17, 2022 - It Chivorn

Problem 1.1 Consider the regulator system shown in Figure 1. The plant is given by

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$

where,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The system uses the state feedback control $\mathbf{u} = -K\mathbf{x}$. Let us choose the desired closed-loop poles at

$$s = -2 \pm j4, \; s = -10$$

(We make such a choice because we know from experience that such a set of closed-loop poles will result in a reasonable or acceptable transient response.) Determine the state feedback gain matrix K.

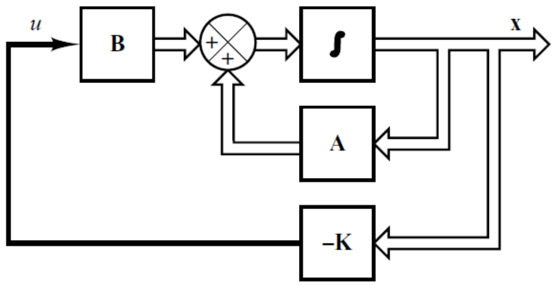

Figure 1: Regulator system.

Problem 1.2 The system is given

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathbf{u}(t)$$
$$\mathbf{y}(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(t)$$

Desired closed-loop poles are $s = -5, \; s = -6$.

1. Determine stability and controllability of a system.
2. Express the gain controller K where the closed loop control $\mathbf{u} = r - K\mathbf{x}$. Calculate steady state error of the closed loop system.
3. Add the gain k_o into the controller where, $\mathbf{u} = k_o r - K\mathbf{x}$. Choose the gain k_o that made zero steady state error.

Problem 1.3 Consider Type 1 Servo System when the Plant Has An Integrator as following in Figure 2.

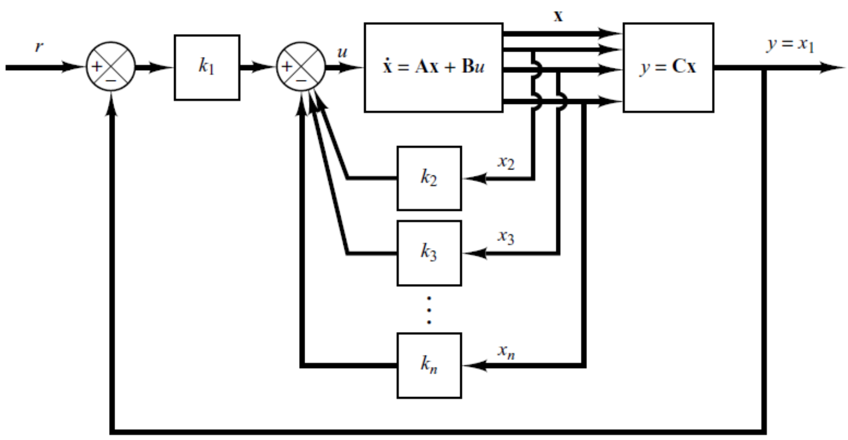

Figure 2: Servo system when the plant has an integrator.

Assume that the plant is defined by

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$
$$\mathbf{y} = C\mathbf{x} = x_1$$

Design a type 1 servo system when the plant transfer function has an integrator. Assume that the plant transfer function is given by

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1}{s(s+1)(s+2)}$$

The desired closed-loop poles are $s = -2 \pm j2\sqrt{3}$ and $s = -10$. Assume that the system configuration is the same as that shown in Figure 2 and the reference input r is a step function.

Problem 1.4 If the plant has no integrator (type 0 plant), the basic principle of the design of a type 1 servo system is to insert an integrator in the feedforward path between the error comparator and the plant, as shown in Figure 3.

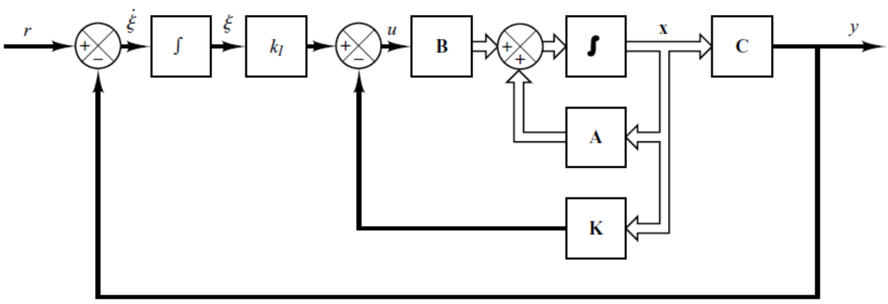

Figure 3: Servo system when the plant has no integrator.

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$
$$\mathbf{y} = C\mathbf{x}$$
$$\dot{\xi} = \mathbf{r} - \mathbf{y}$$
$$\mathbf{u} = -K\mathbf{x} + k_I\xi$$

Consider the system has no integrator represented in state space as following

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.5 & 0 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}, \; C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

1. Determine statbility and controllability of the system.
2. Express closed loop state space model.
3. Design the controller gain K.

End