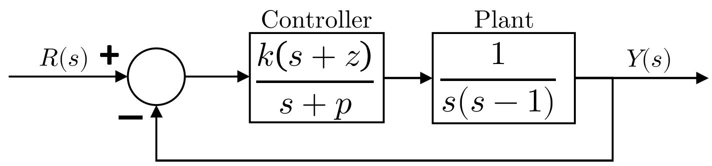
Department of Electrical and Energy Engineering
Institute of Technology of Cambodia

I4-GEE-EA-MCS : Modern Control Systems

TD5

Ex1. Consider the following block diagram. The system open-loop unstable (without feedback).



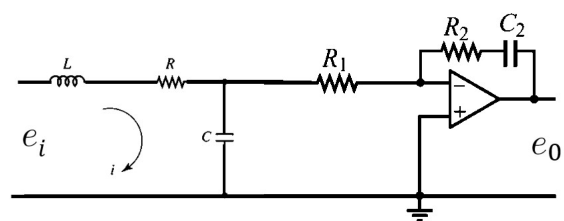
a. Express the closed-loop transfer function.
b. Choose the closed-loop poles ($s = -1, s = -2 - j, s = -2 + j$), then solve k, z and p values.
c. Calculate steady state error for unit step input.

Ex2. Consider a system described be the following differential equations

$$\dddot{q} + 9\ddot{q} + 7\dot{q} + q = 3\dddot{u} + 2\ddot{u} + 9\dot{u}$$

Express the system in state-space model using Controllable Canonical Form.

Ex3. Consider the electrical circuit LRC.

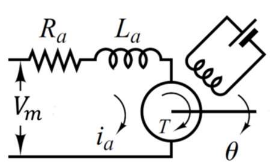


$$R_1 = 1, R_2 = 2, R = 10, \qquad L = 2, \qquad C = 3, C_2 = 5$$

Express the transfer function

$$H(s) = \frac{E_0(s)}{E_i(s)}$$

Ex4. A PMDC motor is a Permanent Magnet DC Motor with equivalent circuit as



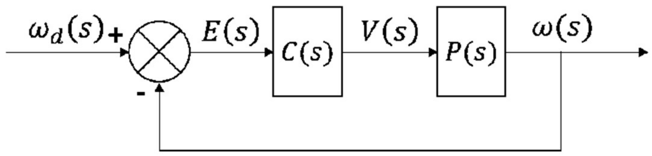
$$v_m = Ri + L\frac{di}{dt} + K\dot{\theta}$$
$$J\ddot{\theta} = Ki - b\theta$$

v_m is an input voltage, and $\dot{\theta}$ is an output measuring.

1. Obtain the stat-space model of the systems.
   Where, R=10, J=5, L=4, b=0.9, and K=1
2. Calculate the steady-state $\dot{\theta}_{ss}$ using state-space model, when step input is given.
   (Hint; $\dot{x} = 0$)

Ex5. Determine controller gain K_p and K_i in closed-loop system by the following condition;

- Steady state error of Ramp input $e_{ss}(ramp) = 1\%$
- Damping ratio $\xi = 0.8$



$$P(s) = \frac{1}{s+1}, \qquad C(s) = K_p + \frac{K_i}{s}$$

$$\text{Hint; } T(s) = \frac{N(s)}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Ex6. Consider the inverted-pendulum system shown in Figure 1. Since in this system the mass is concentrated at the top of the rod, the center of gravity is the center of the pendulum ball. For this case, the moment of inertia of the pendulum about its center of gravity is small, and we assume $I = 0$. Then the mathematical linear model for this system becomes as follows:

$$ml^2\ddot{\theta} + ml\ddot{x} = mgl\theta$$
$$(M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta} = u$$

1. Express that the system differential equation as following
$$Ml\ddot{\theta} = b\dot{x} + (m+M)g\theta - u$$
$$M\ddot{x} = -b\dot{x} - mg\theta + u$$
2. Write the state space model of system.
If u is the control input and x and θ are the output.
3. Is the system stable? Assume $= 0$.

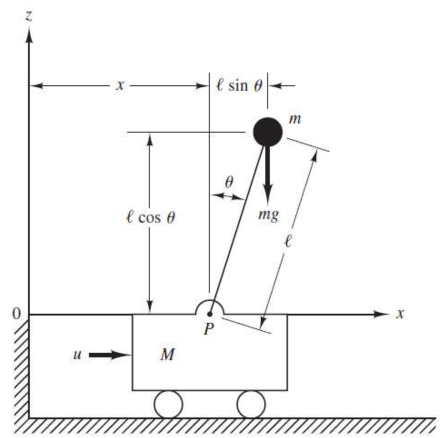


Figure 1. Inverted Pendulum system
$M = 1, m = 0.2, l = 0.2,$
$b = 10^{-3}, g = 10$

Ex7. Consider Two interacting tank system shown in Figure 2 in series with outlet flowrate being function of the square root of tank height can be modeled by

$$A_1\dot{h}_1 = F - R_1\sqrt{h_1 - h_2}$$
$$A_2\dot{h}_2 = R_1\sqrt{h_1 - h_2} - R_2\sqrt{h_2}$$

Where, F is the control input, $y = h_2$ is the output, A_1, A_2, R_1 and R_2 are constants. $A = 1, A_2 = 2, R_1 = R_2 = 2$

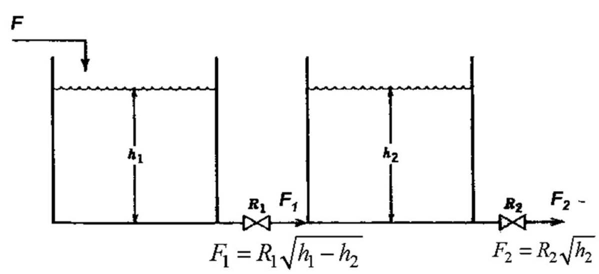
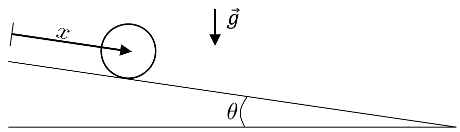


Figure 2. Two interacting Tank system

1. Choose state variable and write down the state equation.
$$\dot{\mathrm{x}} = \mathrm{f}(\mathrm{x}, \mathrm{u})$$
2. Linearize the system around $h_{1s} = 5, h_{2s} = 1$ under control constant input $F_s = 4$.

$$\text{Hint,} \qquad \frac{d}{dt}\left(\sqrt{x(t) + a}\right) = \frac{\frac{d}{dt}(x(t))}{2\sqrt{x(t) + a}}$$

Ex8. Consider the simple ball and beam system as following

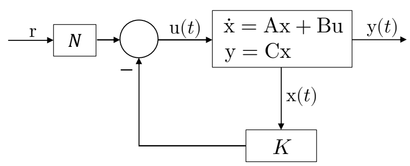


$$\ddot{x}(t) = G\theta(t)$$
$$y(t) = x(t)$$

where G is constant, θ is angle input, and y is position output.

1. Transfer function
   a. Express the transfer function of the system.
   $$H(s) = \frac{Y(s)}{U(s)}$$
   b. Find $y(t)$ for step input $u(t) = a$. (using inverse Laplace transform).
2. State Space model
   a. Choose the state variable and represents in state space model.
   b. Determine controllability of the system.
   c. The system uses the state feedback control $\mathrm{u} = rN - K\mathrm{x}$. Let us choose the desired closed-loop poles at $s = -10$ and $s = -4$.

   Determine the state feedback gain matrix K.



   d. Express the closed-loop state space model.
   $$\dot{\mathrm{x}}_n = A_n\mathrm{x}_n + B_n\mathrm{u}_n$$
   $$\mathrm{y}_n = C_n\mathrm{x}_n$$
   e. Calculate steady state error with step input $\mathrm{u}_n = a$.

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