TD 1 : (Review) Feedback Control Systems

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Laplace Transform and Transfer Function

The **Laplace transformation** for a function of time, $f(t)$, is

$$F(s) = \int_0^{\infty} f(t)e^{-st}dt = \mathscr{L}\{f(t)\}$$

The **inverse Laplace transform** is written as

$$f(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} f(t)e^{-st}dt = \mathscr{L}^{-1}\{F(s)\}$$

The **transfer function** model enables us to determine the output response $y(t)$ to any change in an input $x(t)$ and it provides a basis for determining important system response characteristics without solving the complete differential equation. As defined, the transfer function is a rational function in the complex variable $s = \sigma \pm j\omega$, that is

$$H(s) = \frac{Y(s)}{X(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + ... + b_1 s + b_0}{a_m s^m + a_{m-1}s^{m-1} + ... + a_1 s + a_0}$$

It is often convenient to factor the polynomials in the numerator and denominator, and to write the transfer function in terms of those factors:

$$H(s) = \frac{N(s)}{D(s)} = K\frac{(s-z_1)(s-z_2)...(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)...(s-p_{m-1})(s-p_m)}$$

Where the numerator and denominator polynomials, $N(s)$ and $D(s)$.

Final Value Theorem of Laplace Transform : $f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$
Initial Value Theorem of Laplace Transform : $f(0^+) = \lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$

Problem 1.1 Sketch of the block diagram ,then find the transfer function of the following function below

$$u(t) = K_p e(t) + K_i\int_0^t e(t)dt + K_d\frac{de(t)}{dt}$$

Problem 1.2 Figure 1 shows a RLC circuit. Express the system transfer function $H(s)$ using Laplace transform, where $H(s) = \frac{V_o(s)}{V_{in}(s)}$.

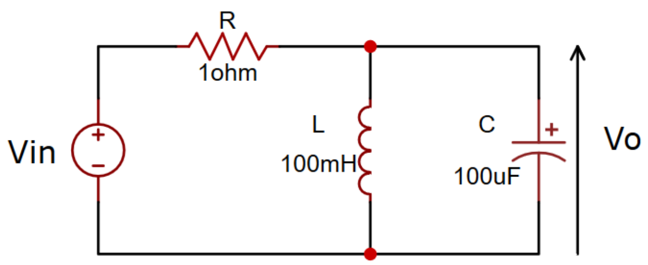

Figure 1: RLC circuit in Problem 1.2

Problem 1.3 A linear system is described by the differential equation:

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = -6\frac{du}{dt} - 8u$$

1. Sketch the block diagram of the differential equation.
2. Find the transfer function of $H(s) = \frac{Y(s)}{U(s)}$
3. Find system poles and zeros and Sketch the pole-zero diagram
4. The unit step input $u(t)$ and find the output $y(t)$