Introduction

- We use 2 currencies as underlyings (use £ and \$ to make intuition easier)
- When a market maker launches a bank, it defines:
    - Borrowing rates
    - Lending rates
    - Market in FX swap points:

Time 0	Sell $\$S_0$ / buy £1	Buy $\$S_0$ / sell £1
Time T	Buy \$F / sell £1	Sell $\$F_0$ / buy £1

- Fundamental equations and these 3 primitive pairs are enough to define markets in collateralized lending rates
- In the following cases, we assume the collateral in escrow is the minimum required to meet the ***net*** obligations at time T

Definitions

x - call on £ denominated in \$ (bid side)
x' - put on £ denominated in \$ (bid side)
x^* - call on £ denominated in \$ (offer side)
$x^\wedge$ - put on £ denominated in \$ (offer side)

x^0 - call on \$ denominated in £ (bid side)
x'^0 - put on \$ denominated in £ (bid side)
x^{*0} - call on \$ denominated in £ (offer side)
$x^{\wedge 0}$ - put on \$ denominated in £ (offer side)

$r_\$$ - the rate at which the market maker lends \$
$r_£$ - the rate at which the market maker lends £

$r_{\$coll£}$ - the rate at which the market maker lends \$ collateralized in £
$r_{£coll\$}$ - the rate at which the market maker lends £ collateralized in \$

$r_{\$coll£}$ - the rate at which the market maker lends \$ collateralized in £
$r_{£coll\$}$ - the rate at which the market maker lends £ collateralized in \$

Lending rate equation 1 (\$-\$)

User 1 is a bank sponsor who funds in **\$**. "User 2" is actually multiple users in the market, so the impact for User 2 is not symmetrical to the impact for User 1.

At time 0:

User 1		**User 2**
Buys a put on £1, $K=S_0$ at x'		Sells a put on £1, $K=S_0$ at x' Posts $\$S_0$ collateral *(alternatively, buys a discounted underlying)*
Buys a discounted underlying on £1 (ie. S – call on £1, $K=S_0$) at $(S_0 - x^*)$		Sells a discounted underlying on £1 (ie. S – call on £1, $K=S_0$) at $(S_0 - x^*)$ Posts $\$S_0$ collateral *(alternatively, buys a put)*
User 1 spends $[S_0 - (x^* - x')]$ ie. "lends" $[S_0 - (x^* - x')]$		

At time T:

User 1 is repaid $\$S_0$

- User 1 receives S_0 from escrow
  (Fundamental equation 2: $P_K + (S - C_K) = K$)

Lending rate equation 1 $[S_0 - (x^* - x')] * (1 + r_\$) = S_0$ User 1 will only put on this package of trades when $r_\$$ exceeds it's hurdle rate (as a minimum, $x^* > x'$)

Lending rate equation 2a(i) ($-£)

User 1 is a bank sponsor who funds in **$**. “User 2” is actually multiple users in the market, so the impact for User 2 is not symmetrical to the impact for User 1.

At time 0:

User 1		**User 2**
Buys £1 / sells $\$S_0$ spot		Sells £1 / buys $\$S_0$ spot
Agrees to buy \$F / sell £1 at time T Posts £1 as collateral		Agrees to sell \$F / buy £1 at time T Posts \$F as collateral
User 1 spends $\$ S_0$ ie. “lends” $\$ S_0$		

At time T:

User 1 is repaid \$F

- User 1 receives \$F, pays £1 and receives £1 from escrow
- Net, User 1 receives \$F

Lending rate equation 2a(i) (with forward contract) $\mathbf{S_0 * (1 + r_{\$coll£}) = F}$ User 1 will offer the forward at F based on its hurdle rate for $r_{\$coll£}$, which in turn is dependent on its deposit funding rate and return on equity hurdle.

Lending rate equation 2a(ii) ($-£)

User 1 is a bank sponsor who funds in **$**. "User 2" is actually multiple users in the market, so the impact for User 2 is not symmetrical to the impact for User 1.

At time 0:

User 1		**User 2**
Buys a call on £1, K=S_0 at $x	⟷	Sells a call on £1, K=S_0 at $x Posts £1 collateral *(alternatively, buys a discount bond)*
Buys a discount bond on £1 at (S_0 - x^)	⟷	Sells a discount bond at (S_0 - x^) Posts $$S_0$ collateral *(alternatively, buys a call)*
Agrees to buy $F / sell £1 at time T Posts call option and discount bond as collateral	⟷	Agrees to sell $F / buys £1 at time T Posts $F as collateral Alternatively, could post: call on $1.3 + discount bond on $1.3 + ($F - $1.3) ➔ could be £ funded MM (equation 4)
↓		
User 1 spends [S_0 - (x^ - x)] ie. "lends" [S_0 - (x^ - x)]		

At time T:

User 1 is repaid $F

- User 1 receives £1 for the long call and long discount bond
- User 1 receives $F and pays £1
- Net, User 1 receives $F from escrow

Lending rate equation 2a (with forward contract)
[S_0 - ($x^ - $x)] * (1 + $r_{\$coll£}$) = F
User 1 will offer the forward at $F based on its hurdle rate for $r_{\$coll£}$, which in turn is dependent on its deposit funding rate and return on equity hurdle.
$r_{\$coll£} = r_£ + [(F – S_0) / S_0]$ ⟵ *TBD if this identity holds in crypto markets*

Lending rate equation 2c ($-£) (without forward contract)

User 1 is a bank sponsor who funds in **$**. "User 2" is actually multiple users in the market, so the impact for User 2 is not symmetrical to the impact for User 1.

At time 0:

User 1		**User 2**
Sells $1.3 / buys £1 spot	⟷	Buys $1.3 / sells £1 spot
Buys a call on $1.3, K=£1 at £$x^0$	⟷	Sells a call on $1.3, K=£1 at £$x^0$ Posts $1.3 as collateral *(alternatively, buys a discount bond)*
Buys a discount bond on $1.3 at (£1 – £$x^{\wedge 0}$)	⟷	Sells a discount bond on $1.3 at (£1 – £$x^{\wedge 0}$) Posts £1 as collateral *(alternatively, buys a call)*
↓		
User 1 spends $[1.3 - ($x^{\wedge 0}$ - x^0)/1.3] ie. "lends" $[1.3 - ($x^{\wedge 0}$ - x^0)/1.3]		

At time T:

User 1 is repaid $1.3

- User 1 receives $1.3 from escrow for the long call and long discount bond

Lending rate equation 2c (without forward contract) **$[1.3 - (x^{\wedge 0} - x^0)/1.3] * (1 + r_{\$coll£}) = 1.3$** User 1 will only put on this package of trades when $r_{\$coll£}$ exceeds its hurdle rate (as a minimum, $x^{\wedge 0} > x^0$)

Lending rate equation 3 (£-£)

User 1 is a bank sponsor who funds in **£**. "User 2" is actually multiple users in the market, so the impact for User 2 is not symmetrical to the impact for User 1.

At time 0:

User 1		**User 2**
Buys a put on \$1.3, K=£1 at £x'^{0}	⟷	Sells a put on \$1.3, K=£1 at £x'^{0} Posts £1 collateral *(alternatively, buys a discounted underlying)*
Buys a discounted underlying on \$1.3 (ie. \$1.3 – call on on \$1.3, K=£1) at (£1 - £x^{*0})	⟷	Sells a discounted underlying on \$1.3 (ie. \$1.3 – call on on \$1.3, K=£1) at (£1 - £x^{*0}) Posts £1 collateral *(alternatively, buys a put)*
↓		
User 1 spends £[1 - (x^{*0} - x'^{0})] ie. "lends" £[1 - (x^{*0} - x'^{0})]		

At time T:

User 1 is repaid £1

- User 1 receives £1 from escrow
  (Fundamental equation 2: $P_K + (S - C_K) = K$)

Lending rate equation 3 **$[1 - (x^{*0} - x'^{0})] * (1 + r_£) = 1$** User 1 will only put on this package of trades when $r_£$ exceeds it's hurdle rate (as a minimum, $x^{*0} > x'^{0}$)

Lending rate equation 4a(i) (£-\$) [non-standard forward market quotation (buy £F^0 / sell \$1.3)]

User 1 is a bank sponsor who funds in **£**. "User 2" is actually multiple users in the market, so the impact for User 2 is not symmetrical to the impact for User 1.

At time 0:

User 1		**User 2**
Buys \$1.3 / sells £1 spot		Sells £1 / buys \$$S_0$ spot
Agrees to buy £F^0 / sell \$1.3 at time T Posts \$1.3 as collateral		Agrees to sell \$F / buy £1 at time T Posts \$F as collateral
User 1 spends £1 ie. "lends" £1		

At time T:

User 1 is repaid \$F

- User 1 receives £F^0, pays \$1.3 and receives \$1.3 from escrow
- Net, User 1 receives £F^0

Lending rate equation 2a(i) (with forward contract) **$1 * (1 + r_{£coll\$}) = F^0$** User 1 will offer the forward at £F^0 based on its hurdle rate for $r_{£coll\$}$, which in turn is dependent on its deposit funding rate and return on equity hurdle.

Lending rate equation 4a(ii) (£-$) [non-standard forward market quotation (buy £F^0 / sell $1.3)]

User 1 is a bank sponsor who funds in **£**. "User 2" is actually multiple users in the market, so the impact for User 2 is not symmetrical to the impact for User 1.

At time 0:

User 1		**User 2**
Buys a call on $1.3, K=£1 at $$x^0$	⟷	Sells a call on $1.3, K=£1 at $x0 Posts $1.3 collateral *(alternatively, buys a discount bond)*
Buys a discount bond on $1.3 at (£1 - $x^{\wedge 0}$)	⟷	Sells a discount bond on $1.3 at (£1 - $x^{\wedge 0}$) Posts £1 collateral *(alternatively, buys a call)*
Agrees to buy £F^0 / sell $1.3 at time T Posts call option and discount bond as collateral	⟷	Agrees to sell £F^0 / buy $1.3 at time T Posts £$F^0$ as collateral Alternatively, could post: call on £1 + discount bond on £1 + (£F^0 - £1) ➔ could be $ funded MM (equation 2)
↓		
User 1 spends £[1 - ($x^{\wedge 0} – x^0$)] ie. "lends" £[1 - ($x^{\wedge 0} – x^0$)]		

At time T:

User 1 is repaid £F^0

- User 1 receives $1.3 for the long call and long discount bond
- User 1 receives £F^0 and pays $1.3
- Net, User 1 receives £F^0 from escrow

Lending rate equation 4a (non-standard forward market quotation (buy £F^0 / sell $1.3) **[1 - ($x^{\wedge 0} – x^0$)] * (1 + $r_{£coll\$}$) = F^0** User 1 will offer the forward at £F^0 based on its hurdle rate for $r_{£coll\$}$, which in turn is dependent on its deposit funding rate and return on equity hurdle. ***$r_{£coll\$} = r_\$ + [(F^0 – 1) / 1]$*** ⟵ *TBD if this identity holds in crypto markets*

<u>Lending rate equation 4b (£-\$) [standard forward market quoting convention (sell $\$F_0$ / buy £1)]</u>

User 1 is a bank sponsor who funds in **£**. "User 2" is actually multiple users in the market, so the impact for User 2 is not symmetrical to the impact for User 1.

At time 0:

User 1		**User 2**
Buys a call on $\$1.3$, K=£1 at $\$x^0$		Sells a call on $\$1.3$, K=£1 at $\$x0$ Posts $\$1.3$ collateral *(alternatively, buys a discount bond)*
Buys a discount bond on $\$1.3$ at (£1 - $x^{\wedge 0}$)		Sells a discount bond on $\$1.3$ at (£1 - $x^{\wedge 0}$) Posts £1 collateral *(alternatively, buys a call)*
[standard forward mkt convention sell $\$F_0$ / buy £1] Agrees to buy £($1.3/F_0$) against $\$1.3$ at time T Posts call option and discount bond as collateral		Agrees to sell £($1.3/F_0$) against $\$1.3$ at time T Posts £($1.3/F_0$) as collateral Alternatively, could post: call on £1 + discount bond on £1 + [£($1.3/F_0$) - £1] ➔ could be \$ funded MM (equation 2)
User 1 spends £[1 - ($x^{\wedge 0} - x^0$)] ie. "lends" £[1 - ($x^{\wedge 0} - x^0$)]		

At time T:

User 1 is repaid £($1.3/F_0$)

- User 1 receives \$1.3 for the long call and long discount bond
- User 1 receives £($1.3/F_0$) and pays $\$1.3$
- Net, User 1 receives £($1.3/F_0$) from escrow

Lending rate equation 4b (standard forward market quoting convention (sell $\$F_0$ / buy £1) **$[1 - (x^{\wedge 0} - x^0)] * (1 + r_{£coll\$}) = 1.3/F_0$** User 1 will quote the forward at $\$F_0$ based on its hurdle rate for $r_{£coll\$}$, which in turn is dependent on its deposit funding rate and return on equity hurdle. ***$r_{£coll\$} = r_\$ + [(1.3/F_0 - 1) / 1]$*** *TBD if this identity holds in crypto markets*

Lending rate equation 4c (£-$) (without forward contract)

User 1 is a bank sponsor who funds in **£**. "User 2" is actually multiple users in the market, so the impact for User 2 is not symmetrical to the impact for User 1.

At time 0:

User 1		**User 2**
Sells £1 / buys $1.3 spot	⟺	Buys $1.3 / sells £1 spot
Buys a call on £1, K=1.3 at x	⟺	Sells a call on $1.3, K=£1 at £$x^0$ Posts $1.3 as collateral *(alternatively, buys a discount bond)*
Buys a discount bond on £1 at (S_0 - x^)	⟺	Sells a discount bond on $1.3 at (£1 – y^{*0}) Posts £1 as collateral *(alternatively, buys a call)*
⇓		
User 1 spends £[1 - (x^ - x)*1.3] ie. "lends" £[1 - (x^ - x)*1.3]		

At time T:

User 1 is repaid £1

- User 1 receives £1 from escrow for the long call and long discount bond

Lending rate equation 4c (without forward contract) **$[1 - (x^\wedge - x)*1.3] * (1 + r_{£coll\$}) = 1$** User 1 will only put on this package of trades when $r_{£coll\$}$ exceeds its hurdle rate (as a minimum, x^ > x)

Lending rate equation 5 ($-$ Box-Spread)

User 1 is a bank sponsor who funds in **$**. “User 2” is actually multiple users in the market, so the impact for User 2 is not symmetrical to the impact for User 1.

Multiple ways to describe the trade:

- User 1 buys box-spread (buys bull-call-spread and bear-put-spread)
- User 1 buys synthetic at lower strike, sells synthetic at upper strike
- User 1 locks in strike differential pay-out at maturity

At time 0:

User 1		**User 2**
Buys a Call on £1, $K=S_L$ at x^{CL}	⟷	Sells a Call on £1, $K=S_L$ at x^{CL} Posts £1 collateral
Sell a Call on £1, $K=S_U$ at x^{CU} Posts Call ($K=S_L$) as collateral	⟷	Buys a Call on £1, $K=S_U$ at x^{CU}
Buys a Put on £1, $K=S_U$ at x^{PU}	⟷	Sells a Put on £1, $K=S_U$ at x^{PU} Posts $\$S_U$ collateral
Sells a Put on £1, $K=S_L$ at x^{PL} Posts Put ($K=S_U$) as collateral	⟷	Buys a Put on £1, $K=S_L$ at x^{PL}
↓		
User 1 spends $[(x^{CL} - x^{CU}) + (x^{PU} - x^{PL})]$ ie. “lends” $[(x^{CL} - x^{PL}) - (x^{CU} - x^{PU})]$		

At time T:

User 1:

- User 1 receives the Strike differential $(S_U - S_L)$

Lending rate equation 5

$[(x^{CL} - x^{PL}) - (x^{CU} - x^{PU})] * (1 + r_\$) = S_U - S_L$

User 1 will only put on this package of trades when $r_\$$ exceeds it’s hurdle rate