CONSTELLATE

How does ChatGPT work?

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Part 2

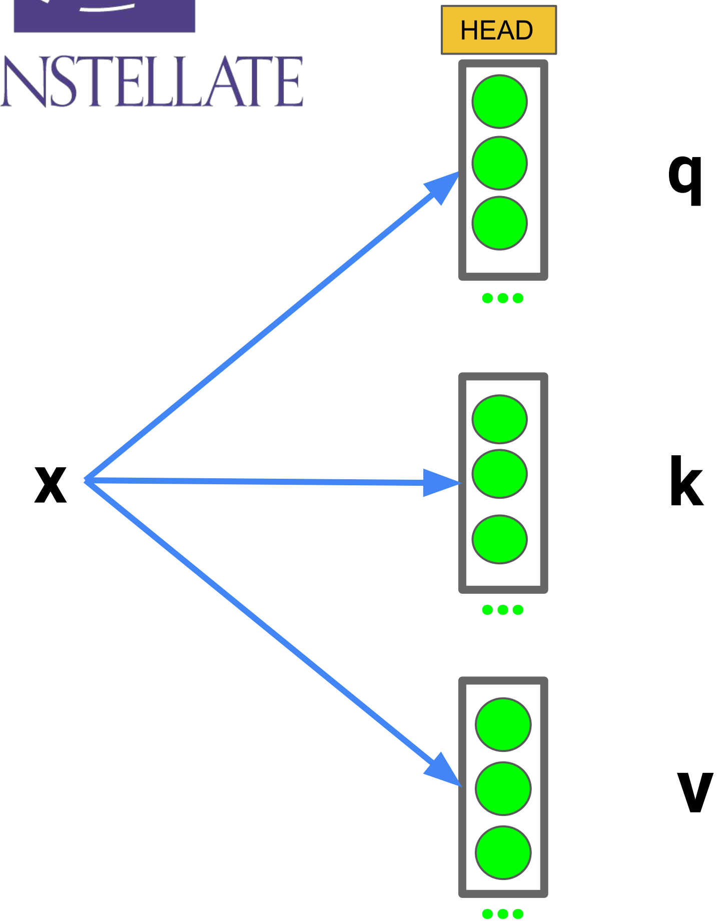
Learning objectives

- Concepts
  - Understand the temporal dimension of language
  - Understand some basic concepts in attention
    - query, key, value, self-attention
  - Understand a head as a representation of a certain relationship between words
- Computation
  - Understand the power of matrix multiplication for parallel computation

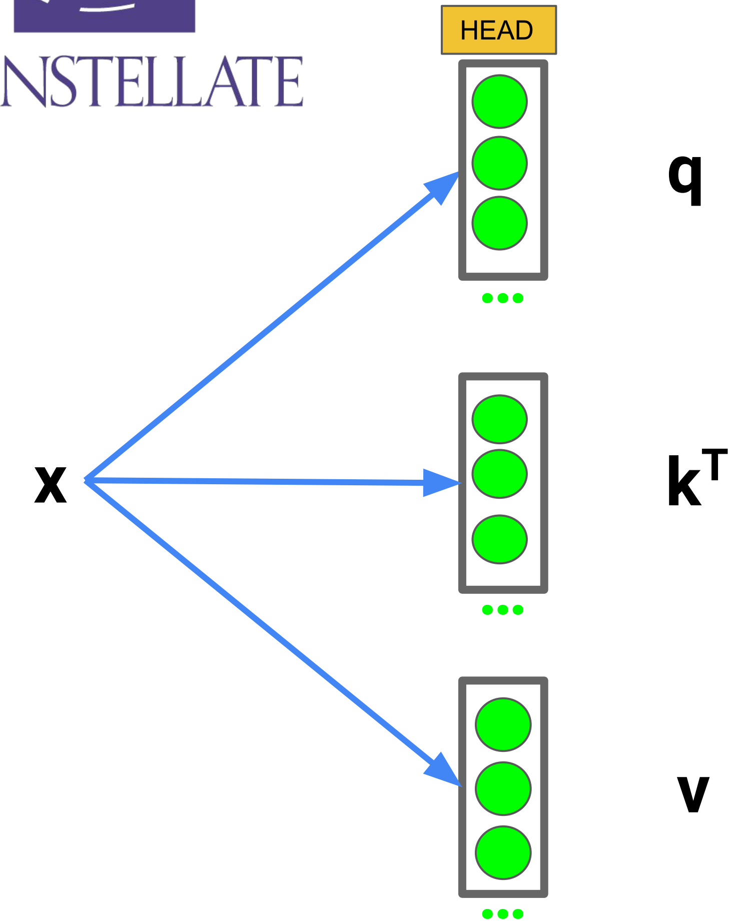
Last session

A single self-attention head

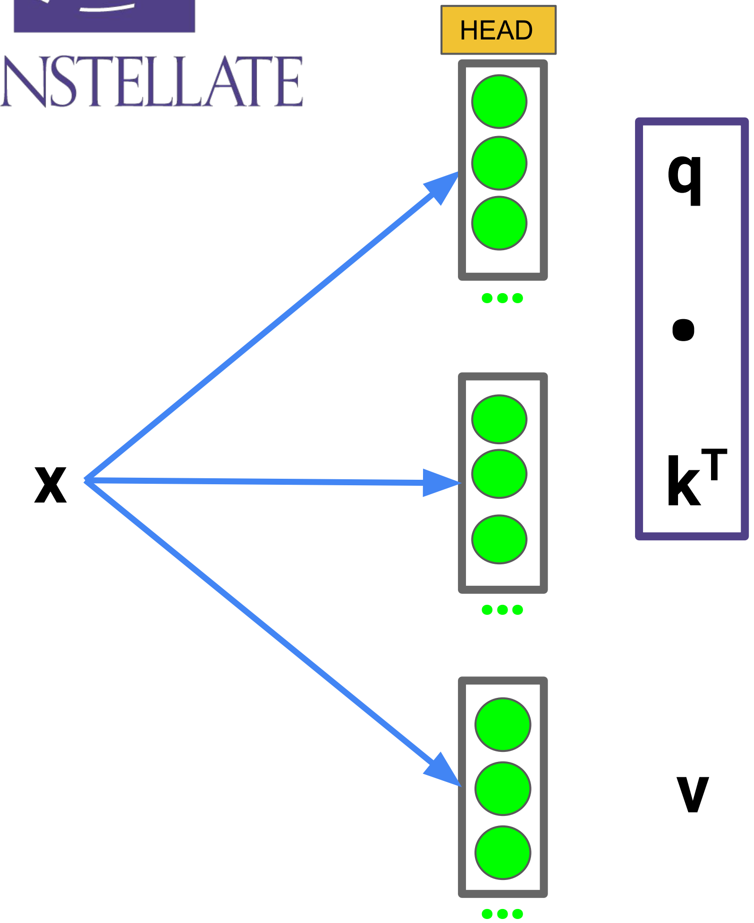
A visual of what we have learned so far



A visual of what we have learned so far

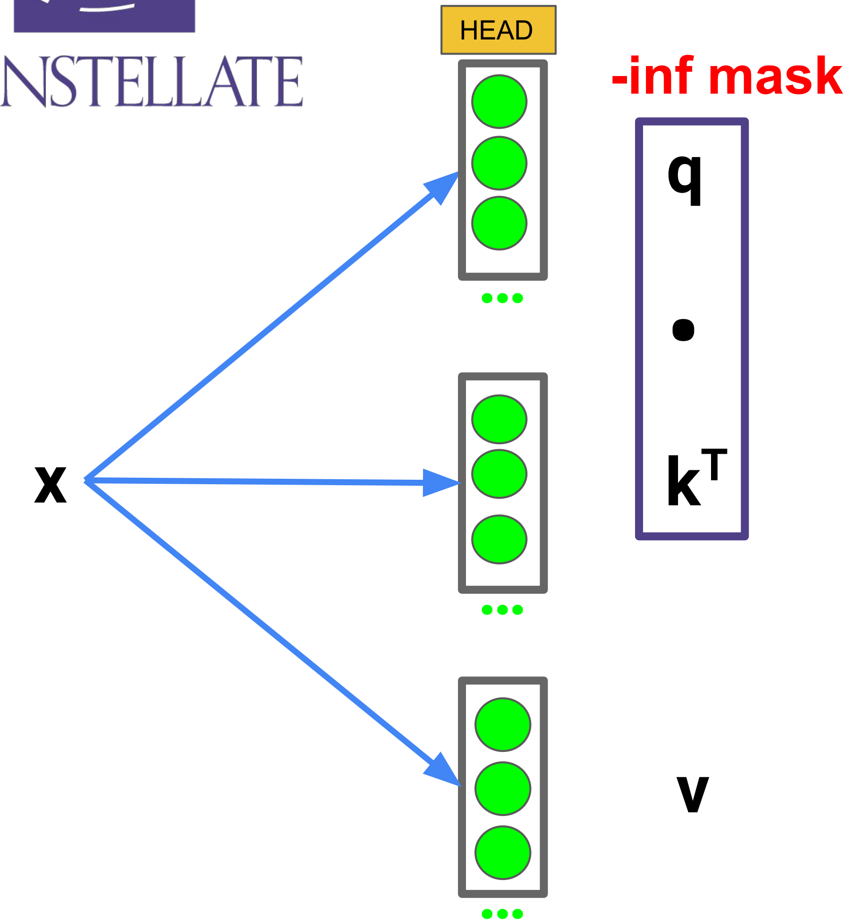


A visual of what we have learned so far



Attention scores

A visual of what we have learned so far



HEAD

-inf mask

$q \bullet k^T$

x

v

A visual of what we have learned so far

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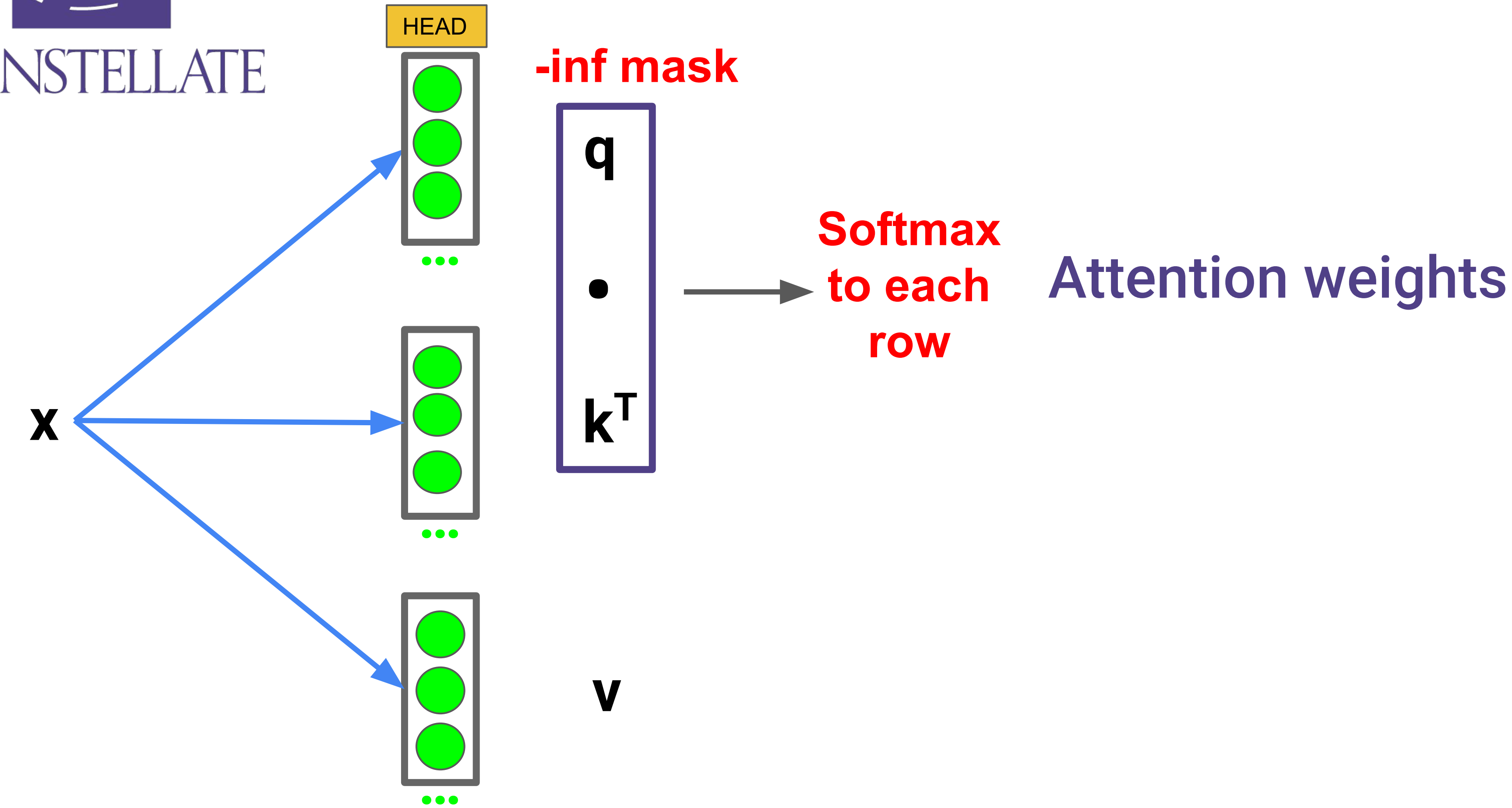


x

HEAD

-inf mask

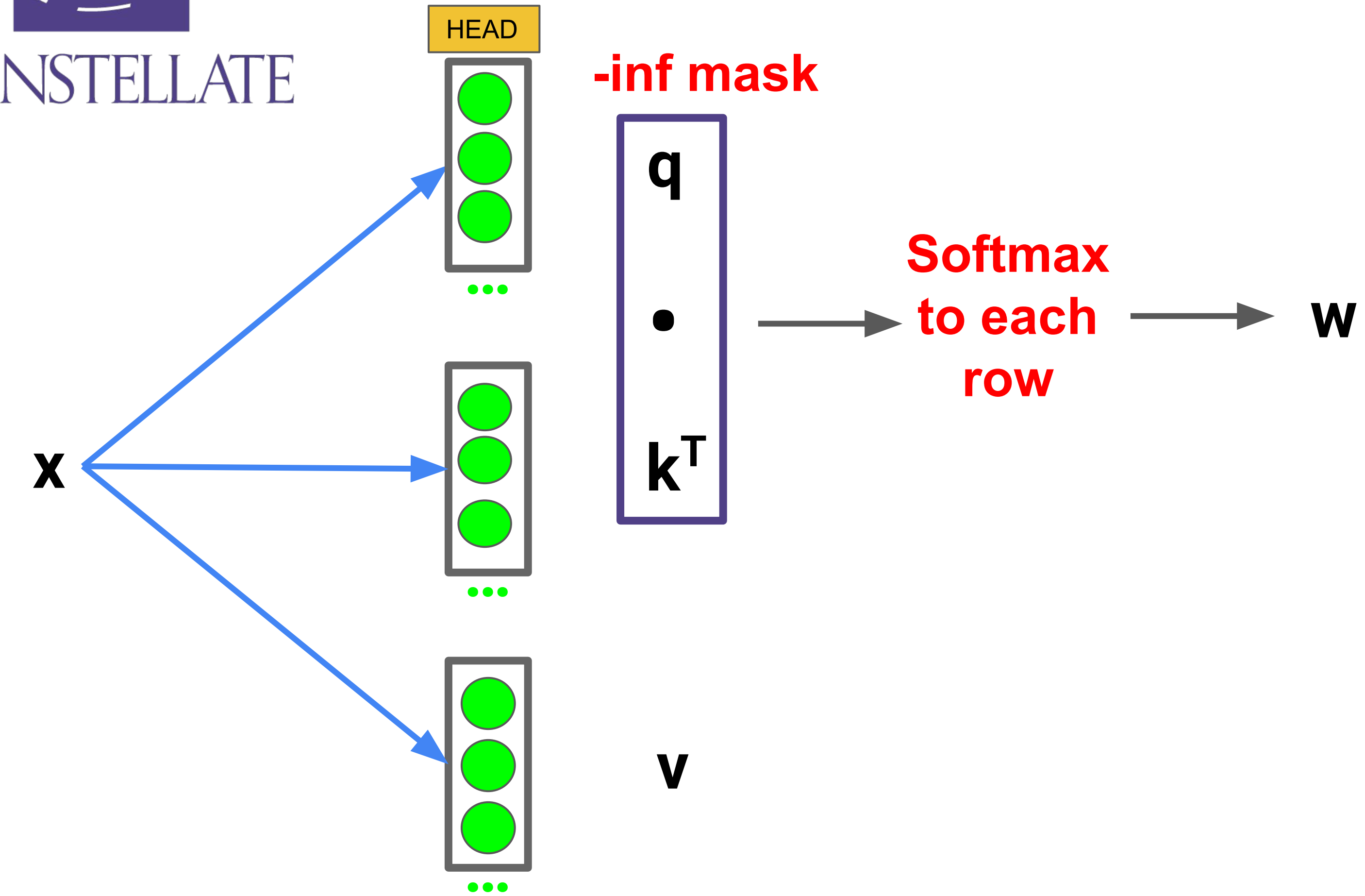
$q \cdot k^T$

v

Softmax to each row

Attention weights

A visual of what we have learned so far



HEAD

-inf mask

q

•

k^T

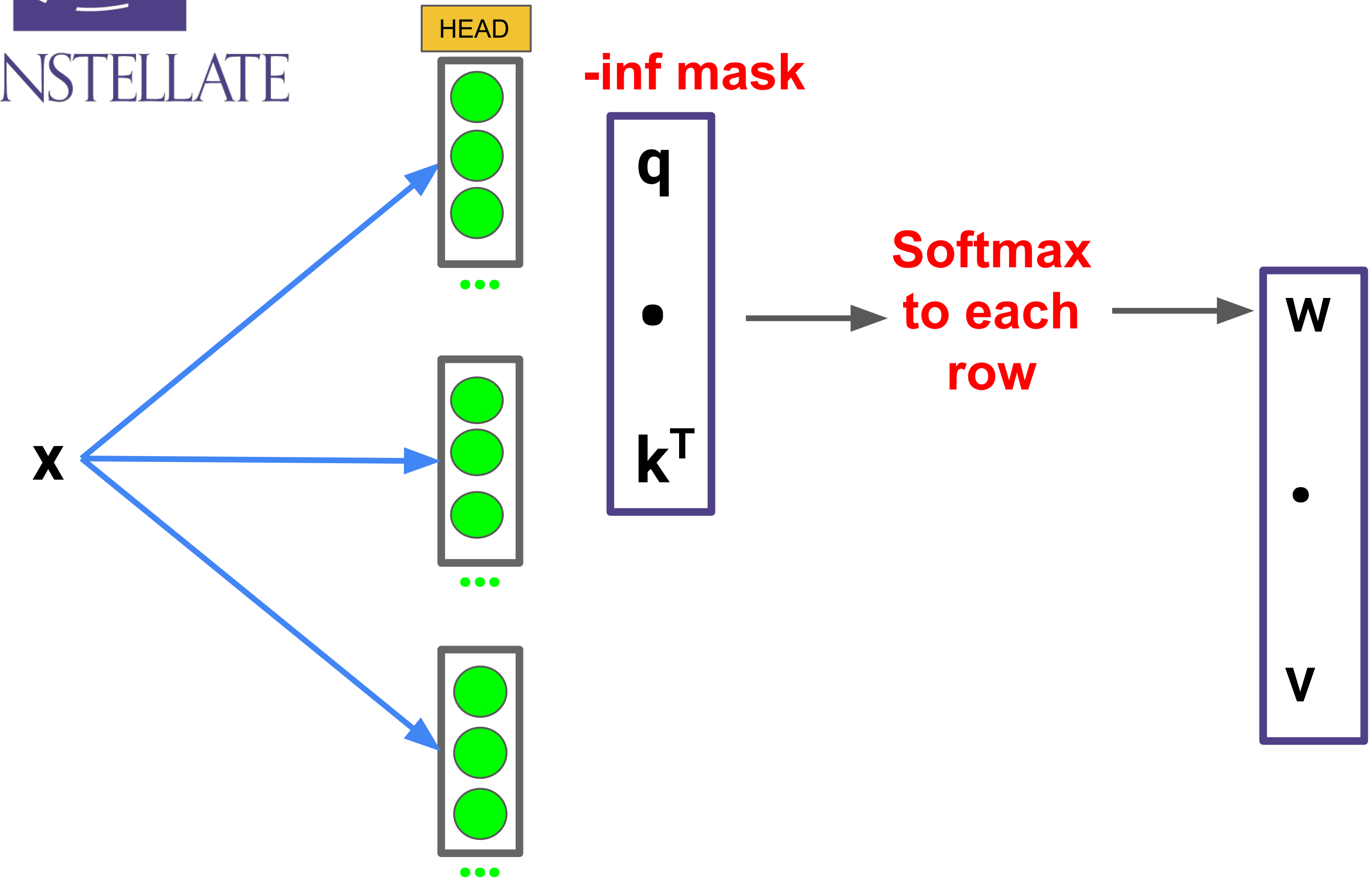
Softmax to each row

w

x

v

A visual of what we have learned so far



-inf mask

Softmax to each row

Any questions?

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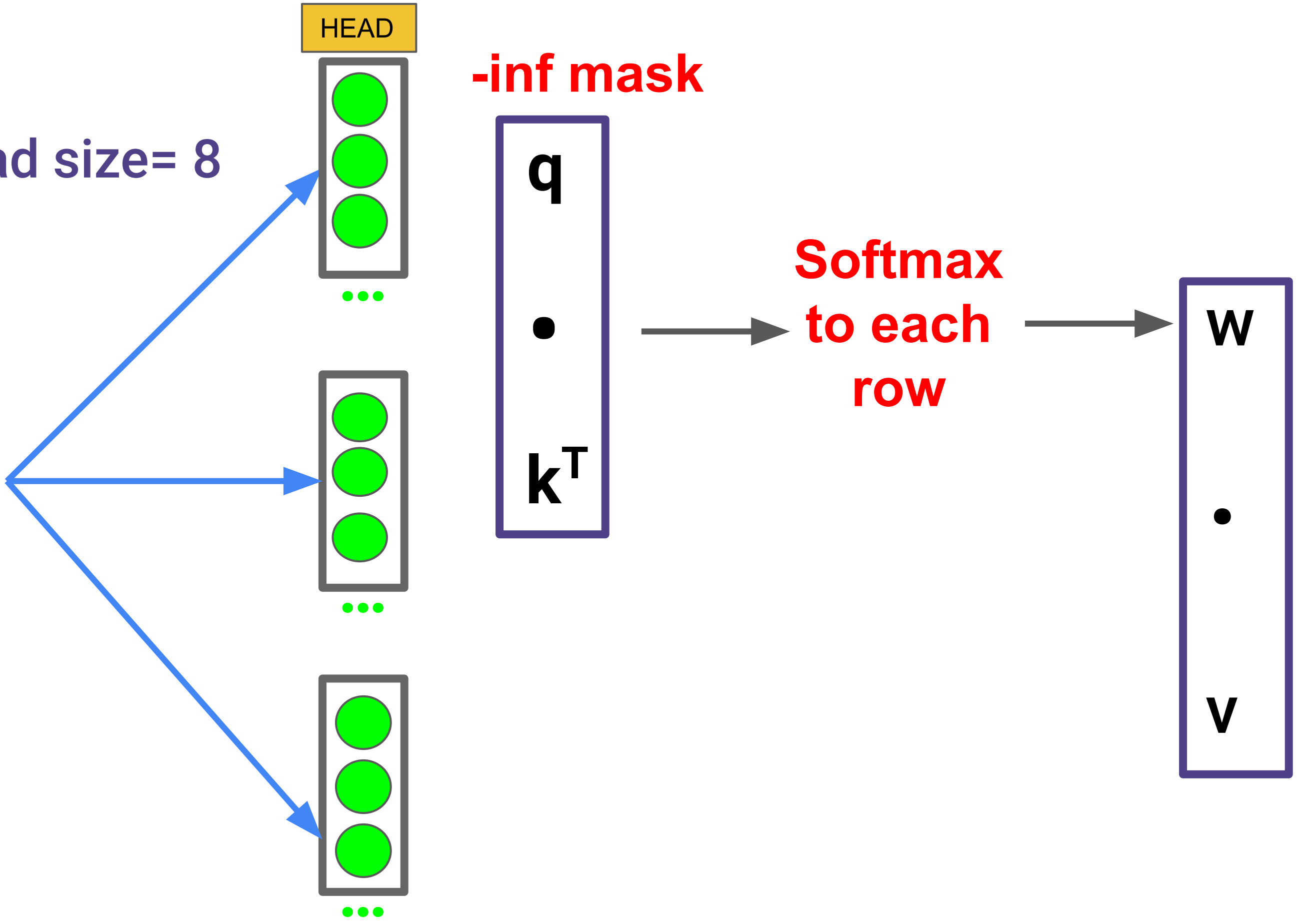
Multi-head attention mechanism

A visual of what we have learned so far

$$\begin{bmatrix} x_1^{t1} & x_2^{t1} & \cdots & x_{32}^{t1} \\ x_1^{t2} & x_2^{t2} & \cdots & x_{32}^{t2} \\ \cdots & \cdots & \cdots & \cdots \\ x_1^{t8} & x_2^{t8} & \cdots & x_{32}^{t8} \end{bmatrix}$$

32 features



Head size= 8

HEAD

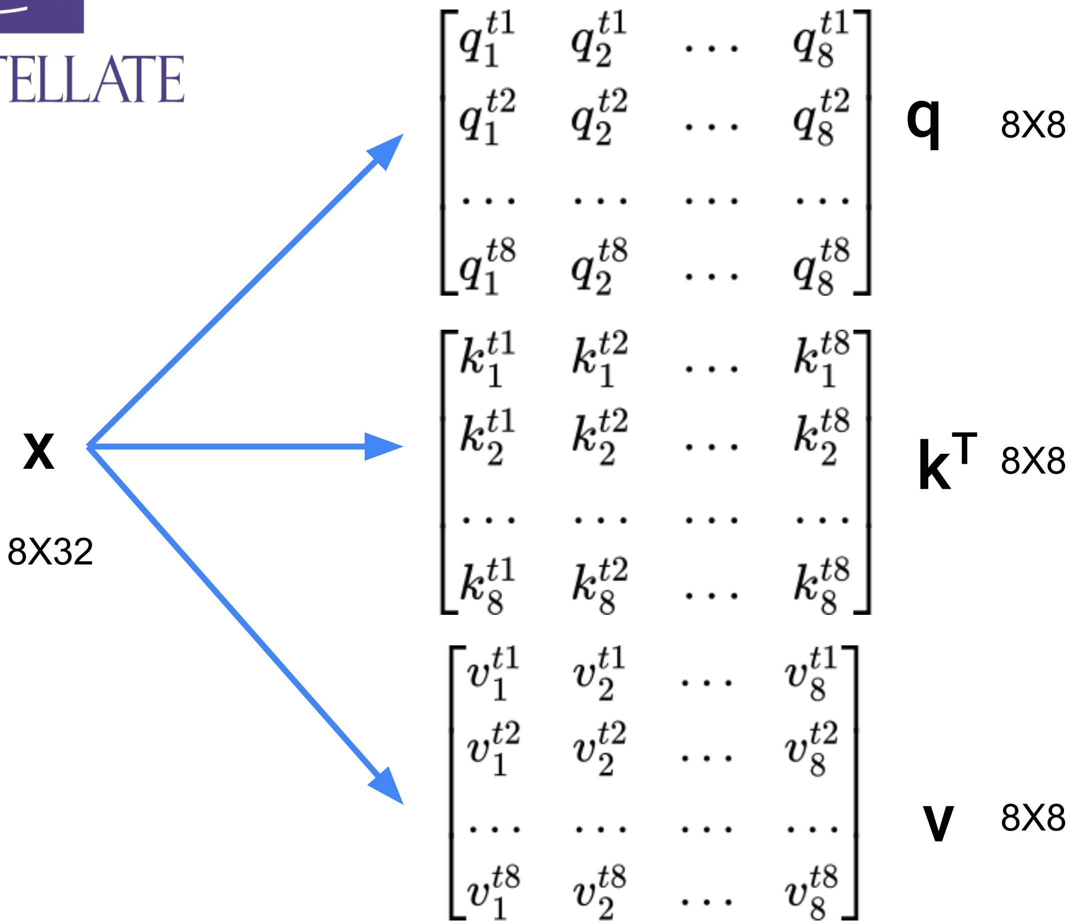
-inf mask

q • k^T

Softmax to each row

w • v

A single head

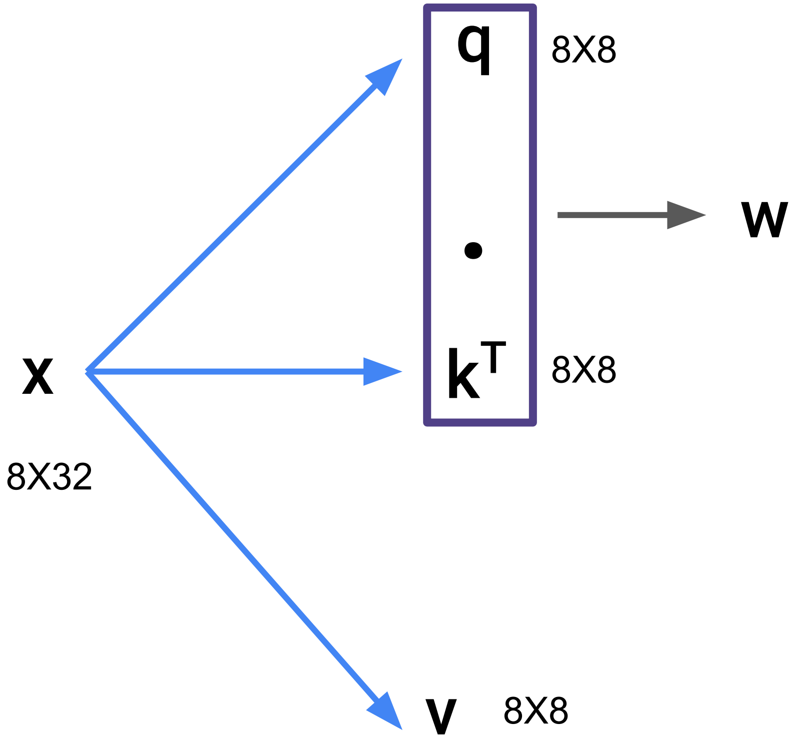


$$\mathbf{q} = \begin{bmatrix} q_1^{t1} & q_2^{t1} & \cdots & q_8^{t1} \\ q_1^{t2} & q_2^{t2} & \cdots & q_8^{t2} \\ \cdots & \cdots & \cdots & \cdots \\ q_1^{t8} & q_2^{t8} & \cdots & q_8^{t8} \end{bmatrix} \quad 8X8$$

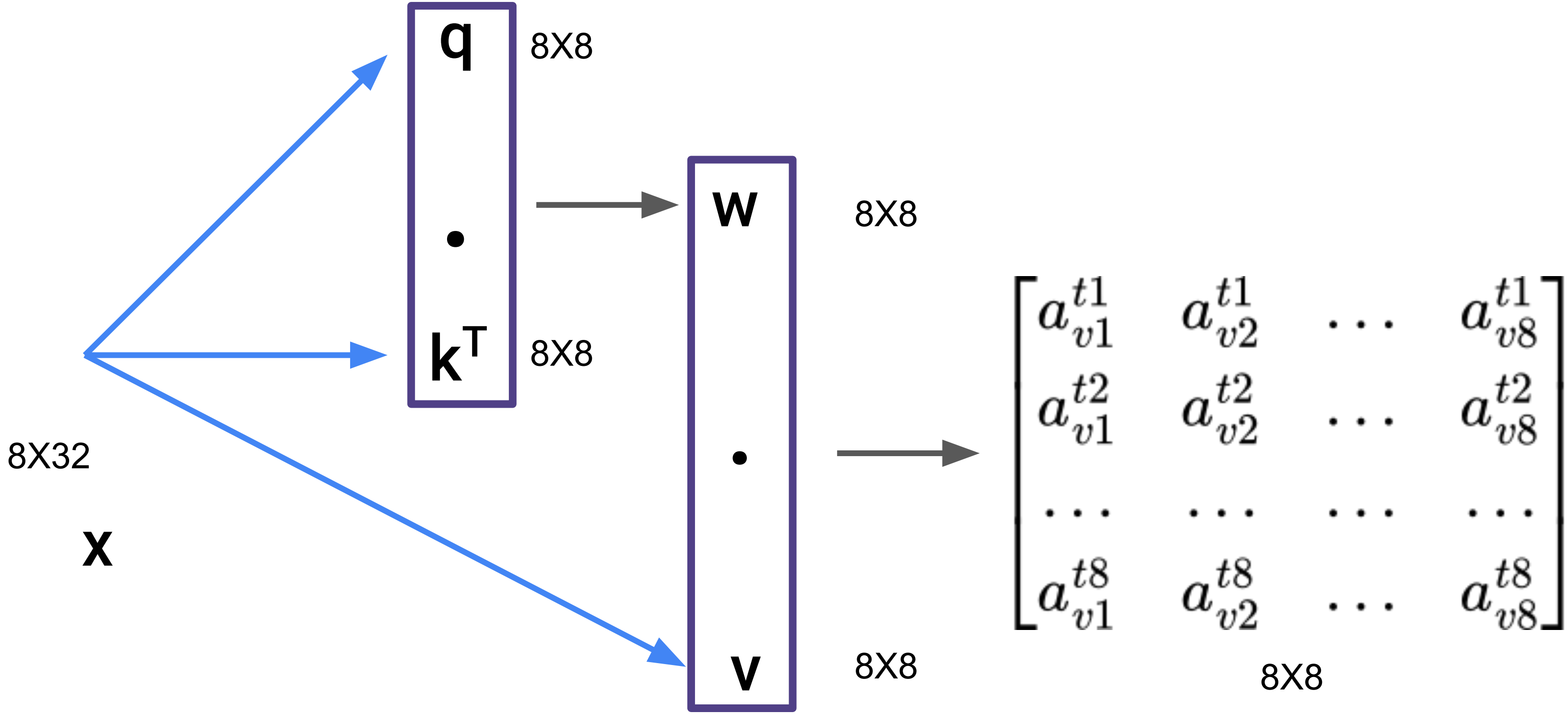
$$\mathbf{k}^T = \begin{bmatrix} k_1^{t1} & k_1^{t2} & \cdots & k_1^{t8} \\ k_2^{t1} & k_2^{t2} & \cdots & k_2^{t8} \\ \cdots & \cdots & \cdots & \cdots \\ k_8^{t1} & k_8^{t2} & \cdots & k_8^{t8} \end{bmatrix} \quad 8X8$$

$$\mathbf{v} = \begin{bmatrix} v_1^{t1} & v_2^{t1} & \cdots & v_8^{t1} \\ v_1^{t2} & v_2^{t2} & \cdots & v_8^{t2} \\ \cdots & \cdots & \cdots & \cdots \\ v_1^{t8} & v_2^{t8} & \cdots & v_8^{t8} \end{bmatrix} \quad 8X8$$

A single head



A single head



$$\begin{bmatrix} a_{v1}^{t1} & a_{v2}^{t1} & \dots & a_{v8}^{t1} \\ a_{v1}^{t2} & a_{v2}^{t2} & \dots & a_{v8}^{t2} \\ \dots & \dots & \dots & \dots \\ a_{v1}^{t8} & a_{v2}^{t8} & \dots & a_{v8}^{t8} \end{bmatrix}$$

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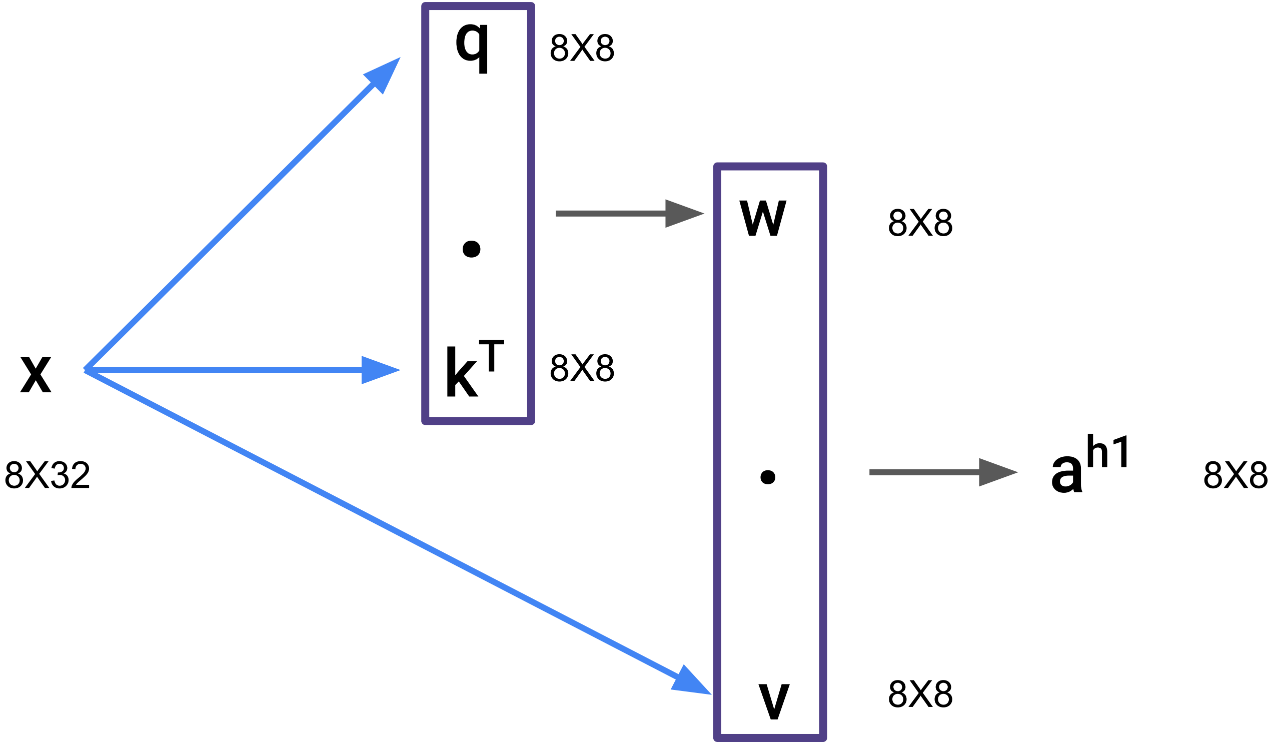
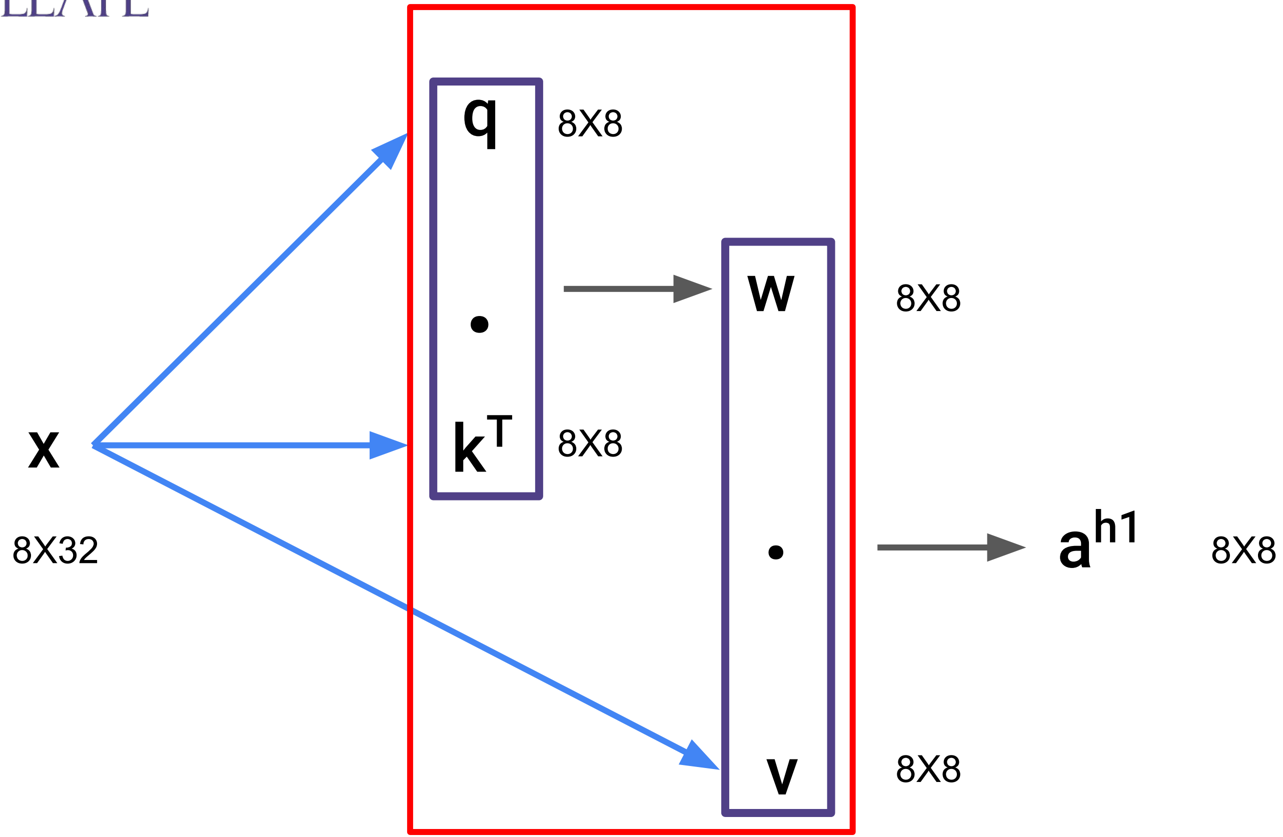
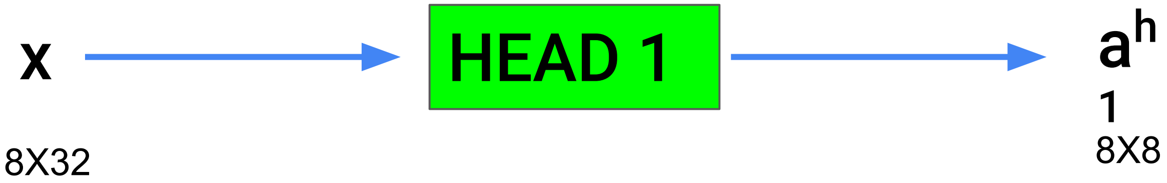
A single head



A single head



A single head



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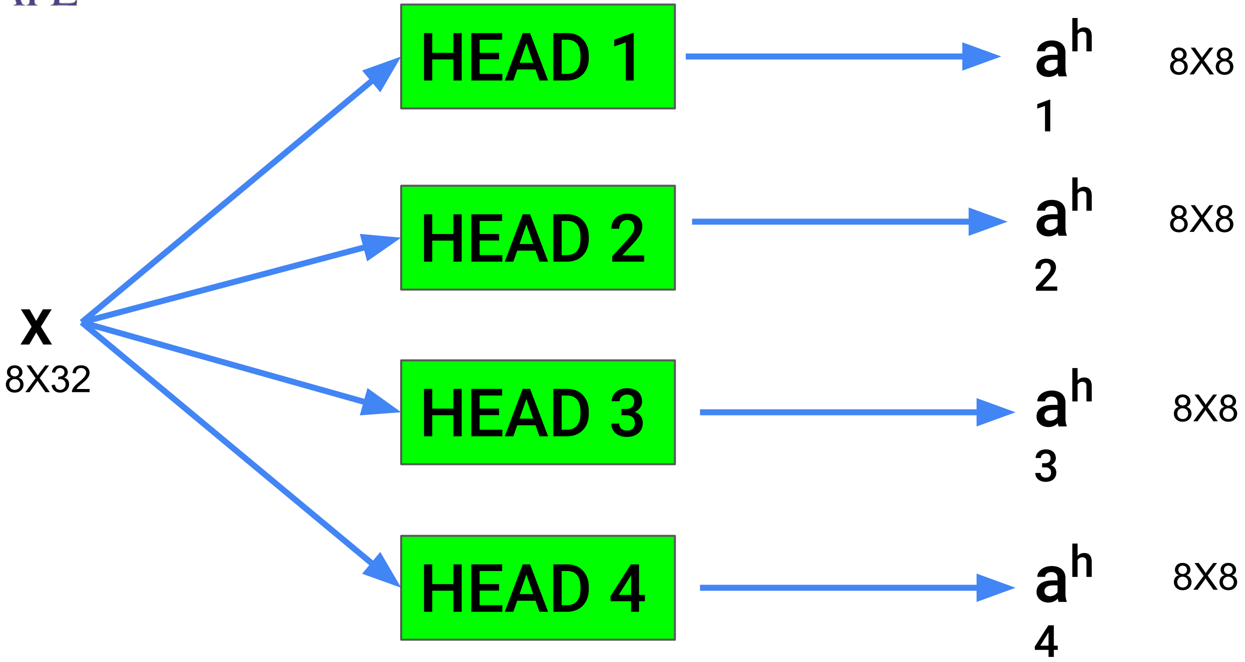
Multi-head

CONSTELLATE

Multi-head



x

8X32

HEAD 1

HEAD 2

HEAD 3

HEAD 4

a^h_1 8X8

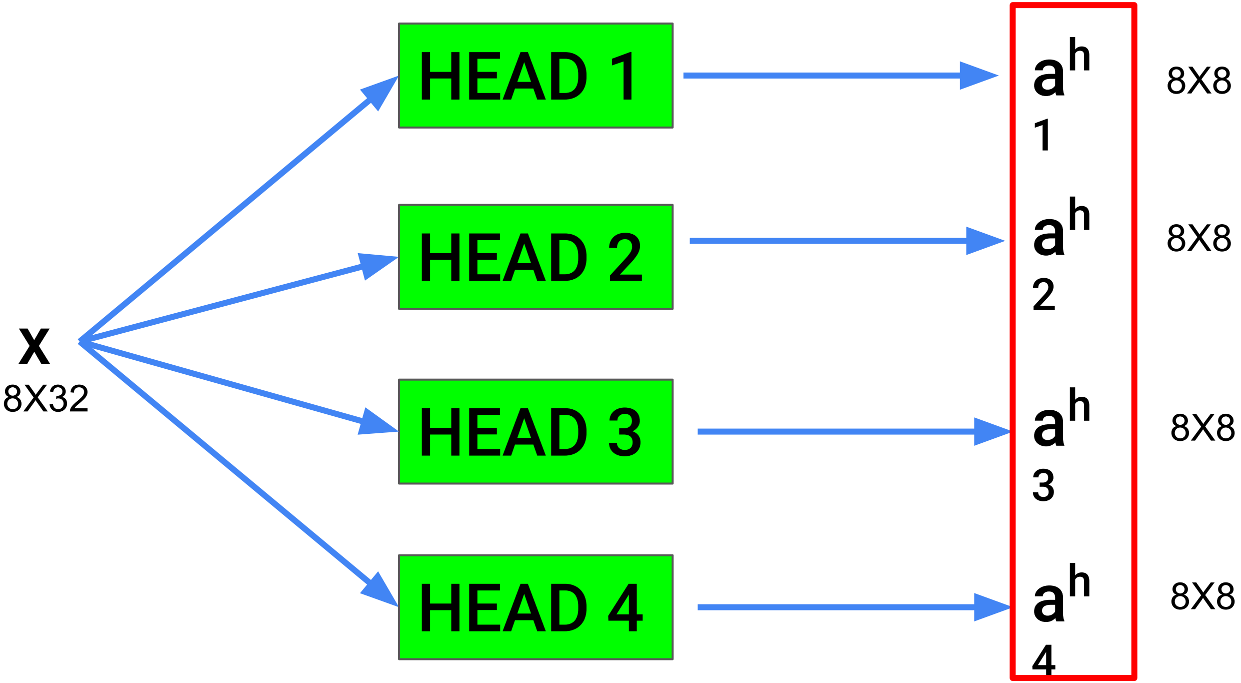
a^h_2 8X8

a^h_3 8X8

a^h_4 8X8

CONSTELLATE

Multi-head



x

8X32

HEAD 1

HEAD 2

HEAD 3

HEAD 4

a^h_1 8X8

a^h_2 8X8

a^h_3 8X8

a^h_4 8X8

Multi-head

$$\begin{bmatrix} a^{h1}_{t1_{v1}} & a^{h1}_{t1_{v2}} & \cdots & a^{h1}_{t1_{v8}} \\ a^{h1}_{t2_{v1}} & a^{h1}_{t2_{v2}} & \cdots & a^{h1}_{t2_{v8}} \\ \cdots & \cdots & \cdots & \cdots \\ a^{h1}_{t8_{v1}} & a^{h1}_{t8_{v2}} & \cdots & a^{h1}_{t8_{v8}} \end{bmatrix}$$

8X8

$$\begin{bmatrix} a^{h2}_{t1_{v1}} & a^{h2}_{t1_{v2}} & \cdots & a^{h2}_{t1_{v8}} \\ a^{h2}_{t2_{v1}} & a^{h2}_{t2_{v2}} & \cdots & a^{h2}_{t2_{v8}} \\ \cdots & \cdots & \cdots & \cdots \\ a^{h2}_{t8_{v1}} & a^{h2}_{t8_{v2}} & \cdots & a^{h2}_{t8_{v8}} \end{bmatrix}$$

8X8

Multi-head

$$\begin{bmatrix} a^{h1}_{t1_{v1}} & a^{h1}_{t1_{v2}} & \cdots & a^{h1}_{t1_{v8}} & a^{h2}_{t1_{v1}} & a^{h2}_{t1_{v2}} & \cdots & a^{h2}_{t1_{v8}} \\ a^{h1}_{t2_{v1}} & a^{h1}_{t2_{v2}} & \cdots & a^{h1}_{t2_{v8}} & a^{h2}_{t2_{v1}} & a^{h2}_{t2_{v2}} & \cdots & a^{h2}_{t2_{v8}} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a^{h1}_{t8_{v1}} & a^{h1}_{t8_{v2}} & \cdots & a^{h1}_{t8_{v8}} & a^{h2}_{t8_{v1}} & a^{h2}_{t8_{v2}} & \cdots & a^{h2}_{t8_{v8}} \end{bmatrix}$$

8X16

Multi-head

8X32

$$\begin{bmatrix}
a^{h1}_{t1_{v1}} & \cdots & a^{h1}_{t1_{v8}} & a^{h2}_{t1_{v1}} & \cdots & a^{h2}_{t1_{v8}} & a^{h3}_{t1_{v1}} & \cdots & a^{h3}_{t1_{v8}} & a^{h4}_{t1_{v1}} & \cdots & a^{h4}_{t1_{v8}} \\
a^{h1}_{t2_{v1}} & \cdots & a^{h1}_{t2_{v8}} & a^{h2}_{t2_{v1}} & \cdots & a^{h2}_{t2_{v8}} & a^{h3}_{t2_{v1}} & \cdots & a^{h3}_{t2_{v8}} & a^{h4}_{t2_{v1}} & \cdots & a^{h4}_{t2_{v8}} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
a^{h1}_{t8_{v1}} & \cdots & a^{h1}_{t8_{v8}} & a^{h2}_{t8_{v1}} & \cdots & a^{h2}_{t8_{v8}} & a^{h3}_{t8_{v1}} & \cdots & a^{h3}_{t8_{v8}} & a^{h4}_{t8_{v1}} & \cdots & a^{h4}_{t8_{v8}}
\end{bmatrix}$$

HEAD 1

HEAD 2

HEAD 3

HEAD 4

Multi-head

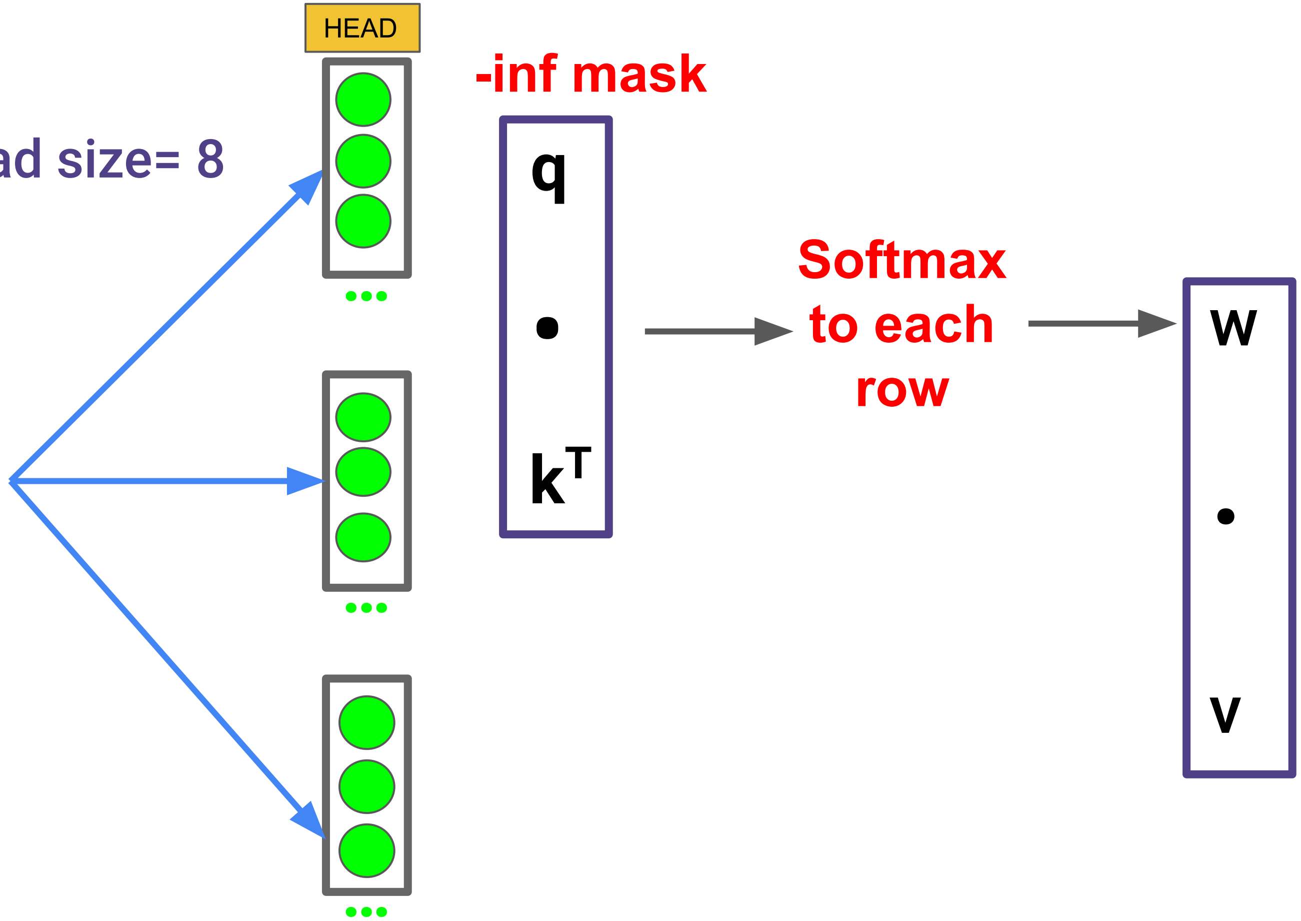
Still remember the shape of the original input?

A visual of what we have learned so far

$$\begin{bmatrix} x_1^{t1} & x_2^{t1} & \cdots & x_{32}^{t1} \\ x_1^{t2} & x_2^{t2} & \cdots & x_{32}^{t2} \\ \cdots & \cdots & \cdots & \cdots \\ x_1^{t8} & x_2^{t8} & \cdots & x_{32}^{t8} \end{bmatrix}$$

8X32

Head size= 8



HEAD

-inf mask

$q \bullet k^T$

Softmax to each row

$w \bullet v$

Multi-head

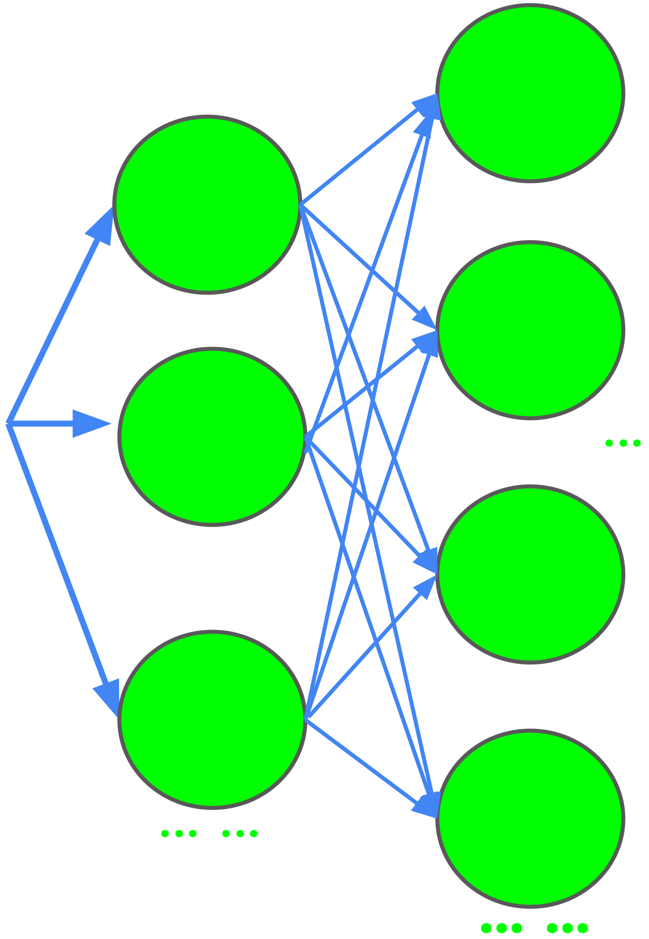
8X32

$$\left[\underbrace{\begin{matrix} a^{h1}_{t1_{v1}} & \cdots & a^{h1}_{t1_{v8}} \\ a^{h1}_{t2_{v1}} & \cdots & a^{h1}_{t2_{v8}} \\ \cdots & \cdots & \cdots \\ a^{h1}_{t8_{v1}} & \cdots & a^{h1}_{t8_{v8}} \end{matrix}}_{\text{HEAD 1}} \quad \underbrace{\begin{matrix} a^{h2}_{t1_{v1}} & \cdots & a^{h2}_{t1_{v8}} \\ a^{h2}_{t2_{v1}} & \cdots & a^{h2}_{t2_{v8}} \\ \cdots & \cdots & \cdots \\ a^{h2}_{t8_{v1}} & \cdots & a^{h2}_{t8_{v8}} \end{matrix}}_{\text{HEAD 2}} \quad \underbrace{\begin{matrix} a^{h3}_{t1_{v1}} & \cdots & a^{h3}_{t1_{v8}} \\ a^{h3}_{t2_{v1}} & \cdots & a^{h3}_{t2_{v8}} \\ \cdots & \cdots & \cdots \\ a^{h3}_{t8_{v1}} & \cdots & a^{h3}_{t8_{v8}} \end{matrix}}_{\text{HEAD 3}} \quad \underbrace{\begin{matrix} a^{h4}_{t1_{v1}} & \cdots & a^{h4}_{t1_{v8}} \\ a^{h4}_{t2_{v1}} & \cdots & a^{h4}_{t2_{v8}} \\ \cdots & \cdots & \cdots \\ a^{h4}_{t8_{v1}} & \cdots & a^{h4}_{t8_{v8}} \end{matrix}}_{\text{HEAD 4}}\right]$$

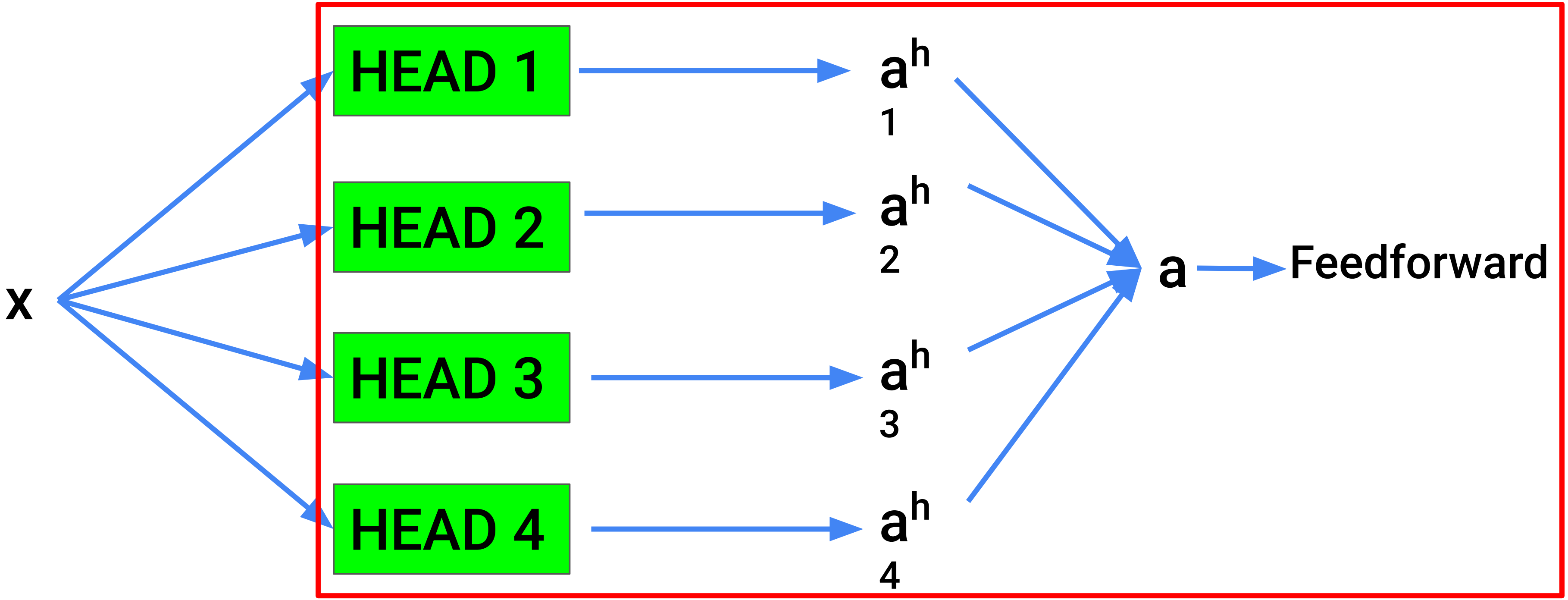
Feedforward

$$\begin{bmatrix}
a^{h1}_{t1_{v1}} & \dots & a^{h1}_{t1_{v8}} & a^{h2}_{t1_{v1}} & \dots & a^{h2}_{t1_{v8}} & a^{h3}_{t1_{v1}} & \dots & a^{h3}_{t1_{v8}} & a^{h4}_{t1_{v1}} & \dots & a^{h4}_{t1_{v8}} \\
a^{h1}_{t2_{v1}} & \dots & a^{h1}_{t2_{v8}} & a^{h2}_{t2_{v1}} & \dots & a^{h2}_{t2_{v8}} & a^{h3}_{t2_{v1}} & \dots & a^{h3}_{t2_{v8}} & a^{h4}_{t2_{v1}} & \dots & a^{h4}_{t2_{v8}} \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
a^{h1}_{t8_{v1}} & \dots & a^{h1}_{t8_{v8}} & a^{h2}_{t8_{v1}} & \dots & a^{h2}_{t8_{v8}} & a^{h3}_{t8_{v1}} & \dots & a^{h3}_{t8_{v8}} & a^{h4}_{t8_{v1}} & \dots & a^{h4}_{t8_{v8}}
\end{bmatrix}$$

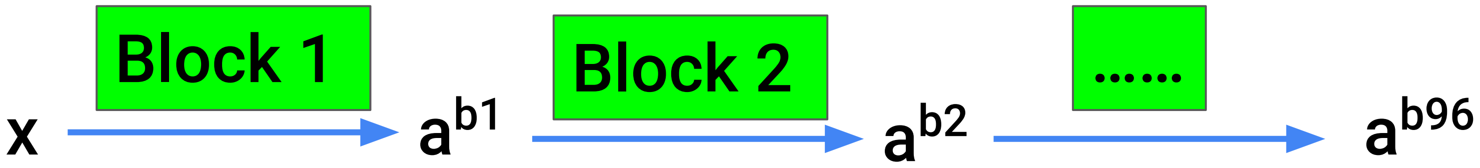
8X32



Attention block



N attention blocks



$$x \xrightarrow{\text{Block 1}} a^{b1} \xrightarrow{\text{Block 2}} a^{b2} \xrightarrow{\text{......}} a^{b96}$$

Any questions?

Positional encoding

Motivation

The order of the words in a sequence matters.

Positional encoding

Even though she did not win the award, she was satisfied.

Even though she did win the award, she was not satisfied.

Positional encoding

$$\begin{bmatrix} x_1^{t1} & x_2^{t1} & \dots & x_{32}^{t1} \\ x_1^{t2} & x_2^{t2} & \dots & x_{32}^{t2} \\ \dots & \dots & \dots & \dots \\ x_1^{t8} & x_2^{t8} & \dots & x_{32}^{t8} \end{bmatrix} + \begin{bmatrix} p_1^{t1} & p_2^{t1} & \dots & p_{32}^{t1} \\ p_1^{t2} & p_2^{t2} & \dots & p_{32}^{t2} \\ \dots & \dots & \dots & \dots \\ p_1^{t8} & p_2^{t8} & \dots & p_{32}^{t8} \end{bmatrix}$$

Positional encoding

$$\begin{bmatrix} x_1^{t1} & x_2^{t1} & \dots & x_{32}^{t1} \\ x_1^{t2} & x_2^{t2} & \dots & x_{32}^{t2} \\ \dots & \dots & \dots & \dots \\ x_1^{t8} & x_2^{t8} & \dots & x_{32}^{t8} \end{bmatrix} + \begin{bmatrix} p_1^{t1} & p_2^{t1} & \dots & p_{32}^{t1} \\ p_1^{t2} & p_2^{t2} & \dots & p_{32}^{t2} \\ \dots & \dots & \dots & \dots \\ p_1^{t8} & p_2^{t8} & \dots & p_{32}^{t8} \end{bmatrix}$$

Positional encoding

$$\begin{bmatrix} x_1^{t1} & x_2^{t1} & \dots & x_{32}^{t1} \\ x_1^{t2} & x_2^{t2} & \dots & x_{32}^{t2} \\ \dots & \dots & \dots & \dots \\ x_1^{t8} & x_2^{t8} & \dots & x_{32}^{t8} \end{bmatrix} + \begin{bmatrix} 0 & 0 & \dots & 0 \\ 1 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots \\ 7 & 7 & \dots & 7 \end{bmatrix}$$

Positional encoding

$$\begin{array}{cccc} i=0 & i=1 & \ldots\ldots & i=32 \\ \left[x_1^{t1}\right. & x_2^{t1} & \ldots & \left.x_{32}^{t1}\right] \end{array}$$

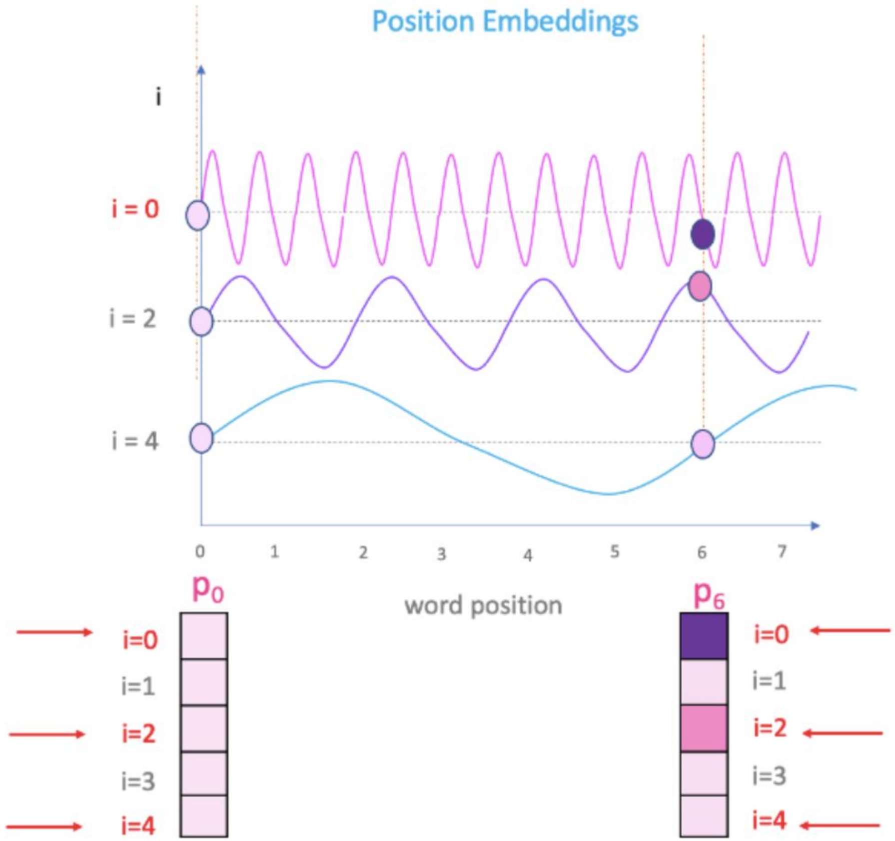
$$PE_{(}pos, 2i) = sin\left(\frac{pos}{10000^{\frac{2i}{d}}}\right)$$

d=32

$$PE_{(}pos, 2i+1) = cos\left(\frac{pos}{10000^{\frac{2i}{d}}}\right)$$

Positional encoding

$$PE_{(pos,2i)} = \sin\left(\frac{pos}{10000^{\frac{2i}{d}}}\right)$$



Positional encoding

Even though she did not win the award, she was satisfied.

Even though she did win the award, she was not satisfied.

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Any questions?

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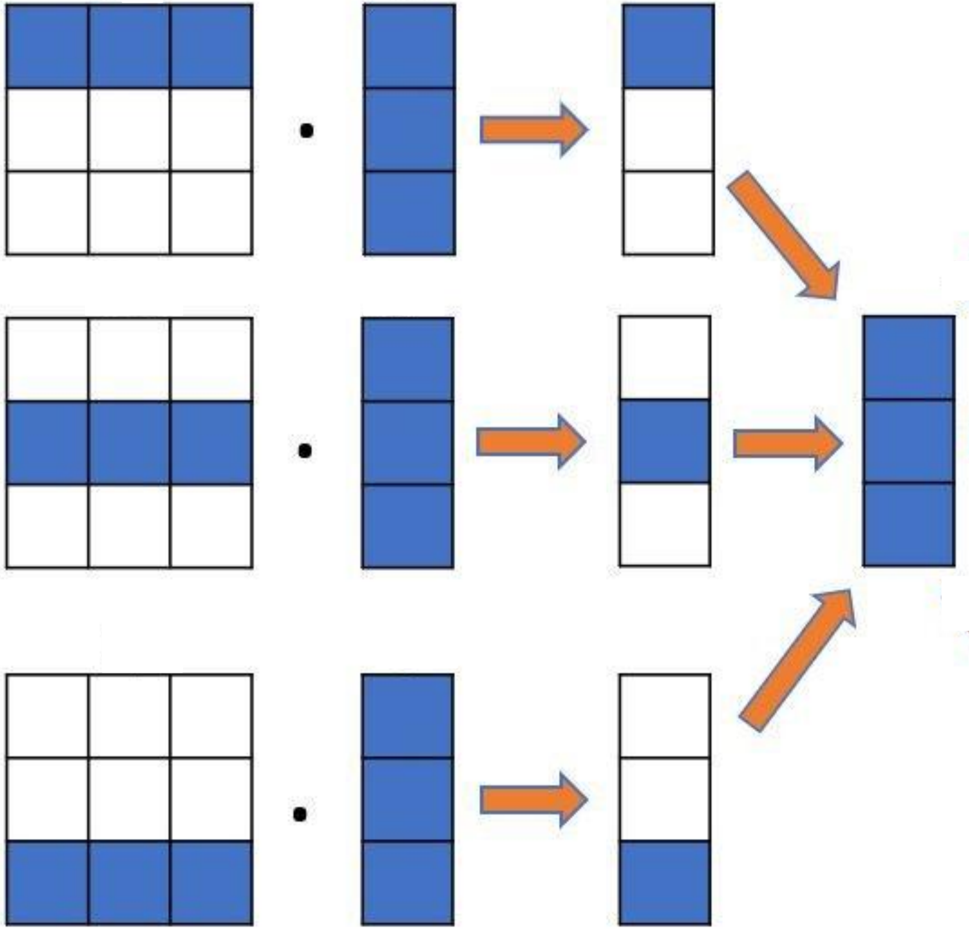
Parallel computation



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Calculate all observations at one time: Matrix multiplication

$$\begin{array}{c|cccc} & & & & \\ \mathbf{x}^{(1)} & x_1^{(1)} & x_2^{(1)} & \dots & x_n^{(1)} \\ \mathbf{x}^{(2)} & x_1^{(2)} & x_2^{(2)} & \dots & x_n^{(2)} \\ & \dots & \dots & \dots & \dots \\ \mathbf{x}^{(m)} & x_1^{(m)} & x_2^{(m)} & \dots & x_n^{(m)} \end{array} \bullet \begin{array}{cccc} \mathbf{W}^{c}_{1} & \mathbf{W}^{c}_{2} & & \mathbf{W}^{cK} \\ \hline w_1^{c1} & w_1^{c2} & \dots & w_1^{cK} \\ w_2^{c1} & w_2^{c2} & \dots & w_2^{cK} \\ \dots & \dots & \dots & \dots \\ w_n^{c1} & w_n^{c2} & \dots & w_n^{cK} \end{array} + \begin{bmatrix} \boxed{b^{c1}} & b^{c2} & \dots & b^{cK} \end{bmatrix}$$

$$= \begin{bmatrix} \boxed{\mathbf{w}^{c1}\mathbf{x}^{(1)} + b^{c1}} & \mathbf{w}^{c2}\mathbf{x}^{(1)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(1)} + b^{cK} \\ \mathbf{w}^{c1}\mathbf{x}^{(2)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(2)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(2)} + b^{cK} \\ \dots & \dots & \dots & \dots \\ \mathbf{w}^{c1}\mathbf{x}^{(m)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(m)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(m)} + b^{cK} \end{bmatrix}$$

Calculate all observations at one time: Matrix multiplication

$$
\begin{array}{c}
\mathbf{x}^{(1)} \\ \mathbf{x}^{(2)} \\ \\ \mathbf{x}^{(m)}
\end{array}
\begin{bmatrix}
x_1^{(1)} & x_2^{(1)} & \dots & x_n^{(1)} \\
x_1^{(2)} & x_2^{(2)} & \dots & x_n^{(2)} \\
\dots & \dots & \dots & \dots \\
x_1^{(m)} & x_2^{(m)} & \dots & x_n^{(m)}
\end{bmatrix}
\bullet
\begin{array}{cccc}
\mathbf{w}^{\mathrm{c}}_{1} & \mathbf{w}^{\mathrm{c}}_{2} & & \mathbf{w}^{\mathrm{cK}} \\
\left[\begin{matrix}
w_1^{c1} & w_1^{c2} & \dots & w_1^{cK} \\
w_2^{c1} & w_2^{c2} & \dots & w_2^{cK} \\
\dots & \dots & \dots & \dots \\
w_n^{c1} & w_n^{c2} & \dots & w_n^{cK}
\end{matrix}\right]
\end{array}
+ \begin{bmatrix} b^{c1} & b^{c2} & \dots & \boxed{b^{cK}} \end{bmatrix}
$$

$$
= \begin{bmatrix}
\mathbf{w}^{c1}\mathbf{x}^{(1)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(1)} + b^{c2} & \dots & \boxed{\mathbf{w}^{cK}\mathbf{x}^{(1)} + b^{cK}} \\
\mathbf{w}^{c1}\mathbf{x}^{(2)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(2)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(2)} + b^{cK} \\
\dots & \dots & \dots & \dots \\
\mathbf{w}^{c1}\mathbf{x}^{(m)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(m)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(m)} + b^{cK}
\end{bmatrix}
$$

Calculate all observations at one time: Matrix multiplication

$$
\begin{array}{r}
\mathbf{x}^{(1)} \\[1em] \mathbf{x}^{(2)} \\[1em] \\[1em] \mathbf{x}^{(m)}
\end{array}
\begin{bmatrix}
x_1^{(1)} & x_2^{(1)} & \dots & x_n^{(1)} \\
x_1^{(2)} & x_2^{(2)} & \dots & x_n^{(2)} \\
\dots & \dots & \dots & \dots \\
x_1^{(m)} & x_2^{(m)} & \dots & x_n^{(m)}
\end{bmatrix}
\bullet
\begin{array}{c}
\begin{array}{cccc}
\mathbf{w}^{c}_{1} & \mathbf{w}^{c}_{2} & & \mathbf{w}^{cK}
\end{array} \\
\begin{bmatrix}
w_1^{c1} & w_1^{c2} & \dots & w_1^{cK} \\
w_2^{c1} & w_2^{c2} & \dots & w_2^{cK} \\
\dots & \dots & \dots & \dots \\
w_n^{c1} & w_n^{c2} & \dots & w_n^{cK}
\end{bmatrix}
\end{array}
+
\begin{bmatrix}
\boxed{b^{c1}} & b^{c2} & \dots & b^{cK}
\end{bmatrix}
$$

$$
=
\begin{bmatrix}
\mathbf{w}^{c1}\mathbf{x}^{(1)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(1)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(1)} + b^{cK} \\
\boxed{\mathbf{w}^{c1}\mathbf{x}^{(2)} + b^{c1}} & \mathbf{w}^{c2}\mathbf{x}^{(2)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(2)} + b^{cK} \\
\dots & \dots & \dots & \dots \\
\mathbf{w}^{c1}\mathbf{x}^{(m)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(m)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(m)} + b^{cK}
\end{bmatrix}
$$

CONSTELLATE

Calculate all observations at one time: Matrix multiplication

$$\begin{array}{c}\mathbf{x}^{(1)}\\ \mathbf{x}^{(2)}\\ \\ \mathbf{x}^{(m)}\end{array}\begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \dots & x_n^{(1)} \\ \boxed{x_1^{(2)} \quad x_2^{(2)} \quad \dots \quad x_n^{(2)}} & & & \\ \dots & \dots & \dots & \dots \\ x_1^{(m)} & x_2^{(m)} & \dots & x_n^{(m)} \end{bmatrix} \bullet \begin{array}{c}\begin{array}{cccc}\mathbf{w}^{c}_{1} & \mathbf{w}^{c}_{2} & & \mathbf{w}^{cK}\end{array}\\ \begin{bmatrix} w_1^{c1} & w_1^{c2} & \dots & w_1^{cK} \\ w_2^{c1} & w_2^{c2} & \dots & w_2^{cK} \\ \dots & \dots & \dots & \dots \\ w_n^{c1} & w_n^{c2} & \dots & w_n^{cK} \end{bmatrix}\end{array} + \begin{bmatrix} b^{c1} & b^{c2} & \dots & \boxed{b^{cK}} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{w}^{c1}\mathbf{x}^{(1)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(1)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(1)} + b^{cK} \\ \mathbf{w}^{c1}\mathbf{x}^{(2)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(2)} + b^{c2} & \dots & \boxed{\mathbf{w}^{cK}\mathbf{x}^{(2)} + b^{cK}} \\ \dots & \dots & \dots & \dots \\ \mathbf{w}^{c1}\mathbf{x}^{(m)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(m)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(m)} + b^{cK} \end{bmatrix}$$

Calculate all observations at one time: Matrix multiplication

$$\begin{array}{c|cccc} & & & & \\ \mathbf{x}^{(1)} & x_1^{(1)} & x_2^{(1)} & \dots & x_n^{(1)} \\ \mathbf{x}^{(2)} & x_1^{(2)} & x_2^{(2)} & \dots & x_n^{(2)} \\ & \dots & \dots & \dots & \dots \\ \mathbf{x}^{(m)} & x_1^{(m)} & x_2^{(m)} & \dots & x_n^{(m)} \end{array} \bullet \begin{array}{cccc} \mathbf{w}^{c}_{1} & \mathbf{w}^{c}_{2} & & \mathbf{w}^{cK} \\ \hline w_1^{c1} & w_1^{c2} & \dots & w_1^{cK} \\ w_2^{c1} & w_2^{c2} & \dots & w_2^{cK} \\ \dots & \dots & \dots & \dots \\ w_n^{c1} & w_n^{c2} & \dots & w_n^{cK} \end{array} + \begin{bmatrix} \boxed{b^{c1}} & b^{c2} & \dots & b^{cK} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{w}^{c1}\mathbf{x}^{(1)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(1)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(1)} + b^{cK} \\ \mathbf{w}^{c1}\mathbf{x}^{(2)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(2)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(2)} + b^{cK} \\ \dots & \dots & \dots & \dots \\ \boxed{\mathbf{w}^{c1}\mathbf{x}^{(m)} + b^{c1}} & \mathbf{w}^{c2}\mathbf{x}^{(m)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(m)} + b^{cK} \end{bmatrix}$$

Calculate all observations at one time: Matrix multiplication

$$\begin{array}{c} \mathbf{x}^{(1)} \\ \mathbf{x}^{(2)} \\ \\ \mathbf{x}^{(m)} \end{array} \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \dots & x_n^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \dots & x_n^{(2)} \\ \dots & \dots & \dots & \dots \\ \boxed{x_1^{(m)}} & \boxed{x_2^{(m)}} & \boxed{\dots} & \boxed{x_n^{(m)}} \end{bmatrix} \bullet \begin{array}{cccc} \mathbf{w}^{c}_{1} & \mathbf{w}^{c}_{2} & & \mathbf{w}^{cK} \\ \multicolumn{4}{c}{\begin{bmatrix} w_1^{c1} & w_1^{c2} & \dots & \boxed{w_1^{cK}} \\ w_2^{c1} & w_2^{c2} & \dots & \boxed{w_2^{cK}} \\ \dots & \dots & \dots & \boxed{\dots} \\ w_n^{c1} & w_n^{c2} & \dots & \boxed{w_n^{cK}} \end{bmatrix}} \end{array} + \begin{bmatrix} b^{c1} & b^{c2} & \dots & \boxed{b^{cK}} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{w}^{c1}\mathbf{x}^{(1)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(1)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(1)} + b^{cK} \\ \mathbf{w}^{c1}\mathbf{x}^{(2)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(2)} + b^{c2} & \dots & \mathbf{w}^{cK}\mathbf{x}^{(2)} + b^{cK} \\ \dots & \dots & \dots & \dots \\ \mathbf{w}^{c1}\mathbf{x}^{(m)} + b^{c1} & \mathbf{w}^{c2}\mathbf{x}^{(m)} + b^{c2} & \dots & \boxed{\mathbf{w}^{cK}\mathbf{x}^{(m)} + b^{cK}} \end{bmatrix}$$

References

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