Machine Learning for Finance
March 17, 2023 (updated March 21)
Deadline: March 30, 2023 (11.55pm)

Name: ____________________
Assignment
Teacher ____________________

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1. All answers should be written using a Latex editor and submitted in Canvas as a PDF document.
2. Coding should be done using the statistical software R.
3. Grades will be rescaled to 10 (without rounding, without curving) and reported via Canvas.
4. Clearly explain your solutions. Any R code you use should be added to the report. It is important that the R Code is clearly exlained.
5. Upload a separate R script, with the R code used in your report. Make sure we can run the R code line by line to create the numbers, tables and figures in your report. Non-reproducable work will result in a 0 for the assignment.
6. It is allowed to work in groups of 2. Note that plain copying R code, text or ideas without adding your own interpretation is considered as cheating. This will be reported to the department and will lead to a grade of zero.
7. GOOD LUCK!

Question	Points	Max
Implementation		25
Modeling		25
Results		25
Layout of the report		15
Writing style		10
Total		**100**

1. In this assignment you are asked to investigate how neural networks can be used to hedge a liability containing both financial and actuarial risks.

The setting:
Consider an insurer who sells pension liabilities. A policyholder invests a single amount P into a fund and receives a lump sum at the maturity T, provided the policyholder is still alive. For this example, we assume the maturity 10 years. One unit of the fund value today is worth 1 euro. The value of the fund value at a future time t is random and denoted by $Y(t)$. The lump sum is based on the value of the fund at maturity, but the policyholder can never receive an amount smaller than his initial premium P, i.e. we have that

$$\text{Payoff in case the policyholder is alive:} = \max\left(P, PY(T)\right).$$

In case the policyholder dies before the maturity of the contract, the payout is zero.

The insurer has a portfolio with $N(0) = 10,000$ identical and independent policyholders at time $t = 0$, each paying the same premium $P = 100$. The process $\{N(t) |\ t \in [0, T]\}$ models the number of surviving policyholders at each time t. The liability of the insurer at maturity is denoted by S and

$$S = N(T) \times \max\left(P, PS(T)\right).$$

The aim is to determine a hedging strategy and a valuation for the liability S.

The fund is modeled using a Black & Scholes model:

$$\mathrm{d}Y(t) = Y(t)(\mu \mathrm{d}t + \sigma \mathrm{d}W_1(t)),$$

where W_1 is a standard Brownian motion. We assume $\mu = 0.08$, $\sigma = 0.15$. There is also a risk-free bank account paying the continuously compounded interest rate $r = 0.03$, i.e. 1 euro deposited in this risk-free account grows to e^{rt} at time t.

The probability that a policyholder in the portfolio is still alive at some future time t is modeled as follows. Denote by τ_i the time of death of policyholder i, then the survival probability is given by

$$\mathbb{P}\left[\tau_i > t\right] = \mathbb{E}\left[\mathrm{e}^{-\int_x^{x+t} \lambda(s)\mathrm{d}s}\right],$$

where x is the age of the policyholders, which are assumed to have all the same age when they initiate the contract. In this example, we assume $x = 55$. The process $\{\lambda(t) |\ t \in [0, T]\}$ is a stochastic process modeling the force of mortality of the policyholders. We assume that

$$\mathrm{d}\lambda(t) = c\lambda(t)\mathrm{d}t + \eta \mathrm{d}W_2(t),$$

where W_2 is a standard Brownian model which is independent of the Brownian motion W_1. We assume $\lambda(0) = 0.01, c = 0.0750$ and $\eta = 0.000597$.

Update 21-03-2023: At time t, given a path of the force of mortality, the population size develops over a small time step h according to a binomial distribution with $N(t)$ trials and (random) success probability $p(t)$:

$$N(t+h) \sim \mathrm{Binom}(N(t), p(t)), \quad p(t) = \mathrm{e}^{-\int_t^{t+h} \lambda(s)\mathrm{d}s}.$$

The integral here may be treated as an ordinary Riemann integral for a given path of λ, which allows for the usual discrete approximation approaches.

Your task:
Assume you are a consultant who is hired by the insurance company. You are asked to valuate the pension portfolio described above and provide a risk assessment. You will use neural networks and Monte Carlo simulation to numerically determine an appropriate dynamic hedging strategy.

You are asked to write a report which gives a detailed description of your models, methodology, results and conclusions. Your report should contain the following items:

- Provide an introduction that describes the main problem and most important results. An introduction should be non-technical and should give a brief overview of the key elements in your report.
- Provide a detailed description of your methodology and comment on important assumptions you may need to make. In particular, describe how you account for financial and actuarial risks in your approach.
- Discuss possible shortcomings and improvements of your model.
- Provide tables and figure to illustrate how well the hedging strategy works. The results should be presented in an organized way and each figure/table should be discussed.
- Add a conclusion section where you mention your most important findings and conclusions to the insurance company.

Your report will be send to the insurance company and a variety of employees will study this report. However, different employees may have different backgrounds. For example, some of them will be technically oriented (experts in statistics and machine learning). These readers may want to see the details of your model and will require mathematical rigor in your text, such that they can replicate the results. However, also managers will read your report and will mainly focus on the intuition, the main steps and the conclusion of your report. Based on this input, they may want to make decisions. Therefore it is important to structure your report in an adequate way. Here are some tips you can take into account:

- A report should have a clear structure with a beginning, middle and end. Headings should be meaningful. Variables (X, Y, etc.) and notation should be defined first, before they appear in the text.
- Your report should be pleasant to read. Make sure to introduce and explain the different steps in your analysis. Read your report several times.
- A report should have a meaningful title and the name of the author(s).
- Screen your report for typos.
- Take care of the layout of your document. Each figure and table should have a caption with a meaningful explanation. All figures and tables should be discussed in the main text.
- Take care of the quality of the figures. Packages such as `ggplot` can also be a good tool to create better figures.
- Research your project!