Solving (2.5) with $B(T,T)=0$ gives

$$B(t,T)=\frac{1-e^{-k(T-t)}}{k}. \tag{2.6}$$

Substituting (2.6) into (2.4) gives

$$A(T,T)-A(t,T)=\int_t^T\Big[\theta(u)B(u,T)-\frac{\sigma^2B(u,T)^2}{2}\Big]du. \tag{2.7}$$

Solving (2.7) with $A(T,T)=0$ leads to

$$A(t,T)=\int_t^T\Big[-\theta(u)B(u,T)+\frac{\sigma^2B(u,T)^2}{2}\Big]du. \tag{2.8}$$

Fitting the observed initial forward price $f^*(0,T)$ results

$$f^*(0,T)=-\frac{\partial}{\partial T}\log P^*(0,T)=-A_T(0,T)+B_T(0,T)r_0, \tag{2.9}$$

where A_T, B_T denote the first derivative of A, B with respect to T.
From (2.6) and (2.8) respectively, it is easy to, differentiating with respect to T, get

$$B_T=e^{-k(T-t)},$$

and

$$\begin{aligned}
A_T &= \frac{\partial}{\partial T}\Big(\int_t^T\Big[-\theta(u)B(u,T)+\frac{\sigma^2B(u,T)^2}{2}\Big]du\Big)\\
&= -\theta(T)B(T,T)+\frac{\sigma^2B(T,T)^2}{2}+\int_0^T\big[-\theta(u)B_T(u,T)+\sigma^2B_T(u,T)B(u,T)\big]\,du\\
&= \int_0^T\big[-\theta(u)B_T(u,T)+\sigma^2B_T(u,T)B(u,T)\big]du.
\end{aligned}$$

Hence (2.9) becomes

$$\begin{aligned}
f^*(0,T) &= r_0B_T(0,T)-A_T(0,T)\\
&= r_0e^{-kT}-\frac{\sigma^2}{k}\int_0^T e^{-k(T-u)}(1-e^{-k(T-u)})du+\int_0^T\theta(u)e^{-k(T-u)}du\\
&= r_0e^{-kT}-\frac{\sigma^2}{2k^2}(1-e^{-kT})^2+\int_0^T\theta(u)e^{-k(T-u)}du\\
&= r_0e^{-kT}-\frac{\sigma^2}{2}B(0,T)^2+\int_0^T\theta(u)e^{-k(T-u)}du
\end{aligned}$$

Setting

$$x(T)=:r_0e^{-kT}+\int_0^T\theta(u)e^{-k(T-u)}du,$$