Take $k=10$, seed = roll number

$f\bmod(a,b) \rightarrow$ Remainder of $\frac{a}{b}$.

Rejection Algorithm

A) Sampling from truncated distns.

Suppose $X \sim f(x), x \in A$, and sampling from f is known.

To draw iid obs. from the truncated dist. $f(x) \mid x \in B$ where $B \subseteq A$.

Algorithm

1. Draw a dist. x from $f(x)$.
2. If $x \in B$ accept it ~~and~~ as an obs. from $f(x) \mid x \in B$
3. If $x \notin B$ repeat the procedure by taking one more obs. from $f(x)$.

Justification

Suppose $X_n \overset{iid}{\sim} f(x), n \geq 1$.

Define $N = \min\{n \geq 1 : X_n \in B\}$

Then $X_N \sim f(x \mid x \in B)$

Proof: $P(X_N \leq x) = P(X_N \in G), \; G = (-\infty, x]$

$$= \sum_{n=1}^{\infty} P(X_N \in G, N = n)$$

$$\left(\because P(A) = \sum P(A \cap B_i) \right)$$

$$= \sum_{n=1}^{\infty} P(X_1 \notin B, X_2 \notin B, \ldots, X_{n-1} \notin B, X_n \in B, X_n \in G)$$