Application

$$\int_0^1 \sqrt{\tan x}\, dx = E\left\{\sqrt{\tan x} \mid X \sim U(0,1)\right\}$$

$$\rightarrow X_1, \ldots, X_n \overset{iid}{\sim} U(0,1)$$

$$\frac{1}{n}\sum X_i \xrightarrow{P} E(X_1) < \infty$$

$$\therefore \int_0^1 \sqrt{\tan x}\, dx \longrightarrow \frac{1}{n}\sum \sqrt{\tan x_i}$$

Suppose $X_1, X_2, \ldots, X_n$ are iid obs. from some distn. with finite $E(g(X_1))$. Then

$$\frac{1}{n}\sum_{i=1}^{n} g(X_i) \xrightarrow[P]{a.s.} Eg(X_1)$$

$\frac{1}{n}\sum_{i=1}^{n} g(X_i)$ is called the Monte-Carlo estimator of $\theta = E\{g(X_1)\}$ i.e. $\hat{\theta}_M = \frac{1}{n}\sum_{i=1}^{n} g(X_i)$. This is a consistent estimator.

Applications:

A) Evaluation of summary measures:

eg1: $X \sim Bin(4, 0.75)$

Calculate: i) $P(X=2)$ ii) $P(X \geq 1)$ iii) $E(Xe^{5X})$

iv) $E\left(\frac{1}{X^2+1}\right)$

Soln: i) $\theta = P(X=2) = E\{I(X=2)\}$

$$\hat{\theta}_M = \frac{1}{n}\sum_{i=1}^{n} I(X_i = 2)$$

where $X_1, \ldots, X_n \overset{iid}{\sim} Bin(4, 0.75)$

X	0	1	2	3	4	f
X_1						+1
X_2						0
⋮						

§ Find the S.E. of $\hat{\theta}$'s and the