

Point Estimation

3. $X_1, X_2, \dots, X_m \sim N(\theta_1, \sigma_1^2)$
 $X_1, X_2, \dots, X_m \sim N(\theta_2, \sigma_2^2)$
 $X_1, X_2, \dots, X_m \sim N(\theta_3, \sigma_3^2)$ independent

the joint pdf of $X_1, X_2, \dots, X_m, X_1, X_2, \dots, X_m, X_1, X_2, \dots, X_m$

$$f_{X_1, X_2, \dots, X_m, X_1, X_2, \dots, X_m, X_1, X_2, \dots, X_m} = \frac{1}{(2\pi)^{m/2}} e^{-\frac{1}{2\sigma_1^2} \sum_{i=1}^m (x_{1i} - \theta_1)^2} \cdot \frac{1}{(2\pi)^{m/2}} e^{-\frac{1}{2\sigma_2^2} \sum_{i=1}^m (x_{2i} - \theta_2)^2} \cdot \frac{1}{(2\pi)^{m/2}} e^{-\frac{1}{2\sigma_3^2} \sum_{i=1}^m (x_{3i} - \theta_3)^2}$$

$$= \frac{1}{(2\pi)^{\frac{m+m+m}{2}}} e^{-\frac{1}{2} \left[\frac{1}{\sigma_1^2} \sum_{i=1}^m (x_{1i} - \theta_1)^2 + \frac{1}{\sigma_2^2} \sum_{i=1}^m (x_{2i} - \theta_2)^2 + \frac{1}{\sigma_3^2} \sum_{i=1}^m (x_{3i} - \theta_3)^2 \right]}$$

$$= \frac{1}{(2\pi)^{\frac{m+m+m}{2}}} e^{-\frac{1}{2} \left[\sum_{i=1}^m x_{1i}^2 - 2\theta_1 \sum_{i=1}^m x_{1i} + m\theta_1^2 + \sum_{i=1}^m x_{2i}^2 - 2\theta_2 \sum_{i=1}^m x_{2i} + m\theta_2^2 + \sum_{i=1}^m x_{3i}^2 - 2\theta_3 \sum_{i=1}^m x_{3i} + m\theta_3^2 \right]}$$

Let $\tilde{X} = (X_{11}, X_{12}, \dots, X_{1m}, X_{21}, X_{22}, \dots, X_{2m}, X_{31}, X_{32}, \dots, X_{3m})$ and $\tilde{Y} = (Y_1, Y_2, \dots, Y_m, Y_1, Y_2, \dots, Y_m, Y_1, Y_2, \dots, Y_m)$

be two distinct samples. $\bar{\theta} = (\theta_1, \theta_2, \theta_3)$

$$\therefore p_{\bar{\theta}}(\tilde{X}) \cdot p_{\bar{\theta}}(\tilde{Y}) = e^{-\frac{1}{2} \left[\sum_{i=1}^m x_{1i}^2 - 2\theta_1 \sum_{i=1}^m x_{1i} + m\theta_1^2 + \sum_{i=1}^m x_{2i}^2 - 2\theta_2 \sum_{i=1}^m x_{2i} + m\theta_2^2 + \sum_{i=1}^m x_{3i}^2 - 2\theta_3 \sum_{i=1}^m x_{3i} + m\theta_3^2 \right]}$$

$$= \frac{e^{-\frac{1}{2} \left[\sum_{i=1}^m y_{1i}^2 - 2\theta_1 \sum_{i=1}^m y_{1i} + m\theta_1^2 + \sum_{i=1}^m y_{2i}^2 - 2\theta_2 \sum_{i=1}^m y_{2i} + m\theta_2^2 + \sum_{i=1}^m y_{3i}^2 - 2\theta_3 \sum_{i=1}^m y_{3i} + m\theta_3^2 \right]}}{e^{-\frac{1}{2} \left[\sum_{i=1}^m y_{1i}^2 - 2\theta_1 \sum_{i=1}^m y_{1i} + m\theta_1^2 + \sum_{i=1}^m y_{2i}^2 - 2\theta_2 \sum_{i=1}^m y_{2i} + m\theta_2^2 + \sum_{i=1}^m y_{3i}^2 - 2\theta_3 \sum_{i=1}^m y_{3i} + m\theta_3^2 \right]}}$$