2) $f = \text{Beta}_I(a,b)$

$U_1, U_2, \ldots \sim U(0,1)$

$$\frac{\sum_{i=1}^{a} \log U_i}{\sum_{i=1}^{a+b} \log U_i} \sim \text{Beta}_I(a,b), \quad a, b \text{ are integers.}$$

Optimality of SPRT

SPRT for testing $H_0: \theta = \theta_0$ vs. $H_1: \theta = \theta_1$ minimises $E_0(N)$ among the tests (fixed/sequential) of strength (α, β) under both H_0 & H_1.

Lemma: (A property of mgf)

Let Z be a random variable with

a) $P(Z>0) > 0$ & $P(Z<0) > 0$

b) $E(Z) \neq 0$

& c) $M(t) = E(e^{tZ})$ exists.

then there exists a unique $t_0 (\neq 0)$ such that $M(t_0) = 1$.

Note:

If $E(Z) = 0$ then $t_0 = 0$

> 0 then $t_0 < 0$

< 0 then $t_0 > 0$

Result 5 (Fundamental identity of sequential Analysis)

Let $Z_i, i \geq 1$ be a sequence of iid r.v.s with $P(Z_1 = 0)$ and finite mgf $M(t) = E(e^{tZ_i})$. If $N = \min\{n : \sum_{i=1}^{n} Z_i$