$\phi(x-\mu)$

$$= E\left\{ I(x>c)\, e^{\left(-\mu^2/2 + \mu x\right)} \,\middle|\, x \sim N(\mu,1) \right\}$$

$$\hat{\theta}_{IS} = \frac{1}{n}\sum_{i=1}^{n} I(x_i>c)\, e^{-\frac{\mu^2}{2}+\mu x}, \quad x_1, \ldots, x_n \overset{iid}{\sim} N(\mu,1)$$

Take $\mu = c$.

eg: 1. $$\int_0^{\pi/2} \sin^5 x \cos^2 x \, dx \to \frac{\pi}{2}\cdot\frac{1}{n}\sum_{i=1}^{n} \sin^5 x_i \cos^2 x_i \,\Big|\, x \sim U\left(0,\frac{\pi}{2}\right)$$

2. $$\int_{-2}^{2} \left\{ |x-1| + |x| \right\} dx \to 4\cdot\frac{1}{n}\sum_{i=1}^{n} |x_i-1| + |x_i| \,\Big|\, x \sim U(-2,2)$$

3. $$\int_0^1 x(\tan^{-1}x)^2 \, dx \to \frac{1}{n}\sum_{i=1}^{n} x_i(\tan^{-1}x_i)^2 \,\Big|\, x \sim U(0,1)$$

4. $$\int_0^3 \sqrt{\sin x}\, dx \to 3\cdot\frac{1}{n}\sum_{i=1}^{n} \sqrt{\sin x_i} \,\Big|\, x \sim U(0,3)$$

5. $$\int_1^\infty e^{-x} x^3 \, dx = \int_0^\infty I(x>1)\, e^{-x} x^3 \, dx$$

$$= \int_0^\infty \frac{I(x>1)\, e^{-x} x^3}{e^{-x}} \cdot e^{-x} \, dx \quad \text{or} \quad \int_1^\infty \frac{e^{-x} x^3 e^{-(x-1)}}{e^{-(x-1)}} \, dx$$

$$= E\left\{ I(x>1) x^3 \,\middle|\, x \sim Exp(1) \right\} \to \frac{1}{n}\sum_{i=1}^{n} I(x_i>1)\, x_i^3 \,\Big|\, x \sim Exp(1)$$

6. $$\int_1^\infty \frac{\log x}{x^3}\, dx = \frac{1}{2}\int_1^\infty \log x \cdot \frac{2}{x^3}\, dx$$

$$= \frac{1}{2} E\left(\log x \,\middle|\, x \sim Pareto(\cdot) \right) \to \frac{1}{2}\cdot\frac{1}{n}\sum_{i=1}^{n} \log x_i \,\Big|$$