Lab Report - Site Percolation on a Square Lattice

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Abstract

This report focuses on the study of site percolation on a square lattice. Using Monte Carlo simulations, we analyze the probability of path formation between the first and last rows, the average size of the maximum cluster, and the distribution of cluster sizes as functions of site occupancy probability p. Visualization includes results from the Burning Method, which labels each site with its propagation number, and the Hoshen-Kopelman algorithm, which identifies clusters with distinct colors.

1 Introduction

Site percolation is a fundamental problem in statistical physics, where each site of a lattice is occupied independently with probability p. The model exhibits a percolation threshold p_c, beyond which a spanning cluster forms. This report addresses the following tasks:

1. Implementation of the site percolation model.
2. Visualization using the Burning Method and Hoshen-Kopelman (HK) algorithm.
3. Analysis of the percolation probability P_{flow} as a function of p.
4. Determination of the average size of the maximum cluster $\langle s_{\max} \rangle$.
5. Investigation of cluster size distributions $n(s, p, L)$.

2 Methodology

2.1 Initialization

Input parameters for the lattice size L, number of trials T, probability range $[p_0, p_k]$, and step size Δp are loaded from a configuration file `perc-ini.txt` for user convenience.

2.2 Monte Carlo Simulation

The simulation process consists of the following steps:

1. Generate a random $L \times L$ lattice where each site is occupied with probability p.
2. Use the Burning Method to determine whether a spanning path connects the first and last rows.
3. Apply the Hoshen-Kopelman algorithm to identify clusters and compute their sizes.
4. Record P_{flow}, $\langle s_{\max} \rangle$, and cluster size distributions for analysis.

2.3 Visualization

1. The Burning Method visualizes each site's contribution to a spanning path by assigning a propagation number. Figure 5 provides sample visualizations.
2. The HK algorithm identifies and colors clusters. Each cluster is assigned a unique color for easy identification. Figure 6 illustrates these results.

3 Results

3.1 Percolation Probability

Figure 1 illustrates the percolation probability P_{flow} as a function of p for $L = 10, 50, 100$. A sharp transition is observed near the critical probability p_c.

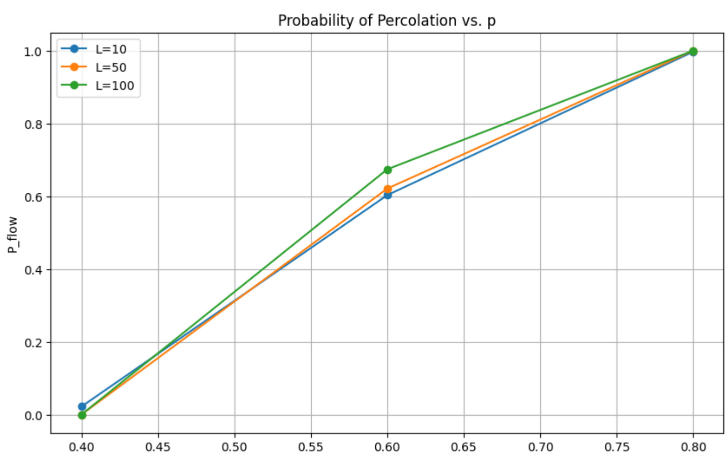


Figure 1: Percolation probability P_{flow} vs. p for $L = 10, 50, 100$.

3.2 Maximum Cluster Size

Figure 2 shows the average size of the maximum cluster $\langle s_{\max} \rangle$ as a function of p. The growth of $\langle s_{\max} \rangle$ is consistent with finite-size scaling near p_c.

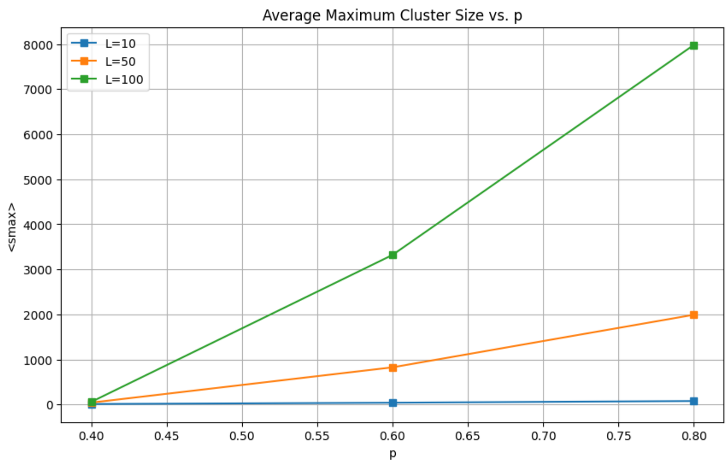


Figure 2: Average maximum cluster size $\langle s_{\max} \rangle$ vs. p for $L = 10, 50, 100$.

3.3 Cluster Size Distribution

Figure 3 depicts the cluster size distribution $n(s, p, L)$ for selected p values, including $p = 0.2, 0.3, 0.4, 0.5, p_c = 0.592746, 0.6, 0.7, 0.8$. Subplots in Figure 4 focus on $p < p_c$, $p = p_c$, and $p > p_c$.

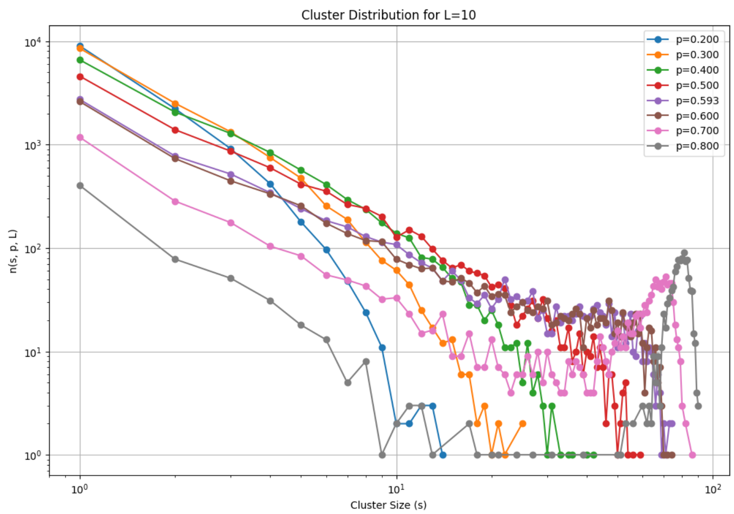


Figure 3: Cluster size distribution $n(s, p, L)$ for $p = 0.2, 0.3, 0.4, 0.5, p_c = 0.592746, 0.6, 0.7, 0.8$.

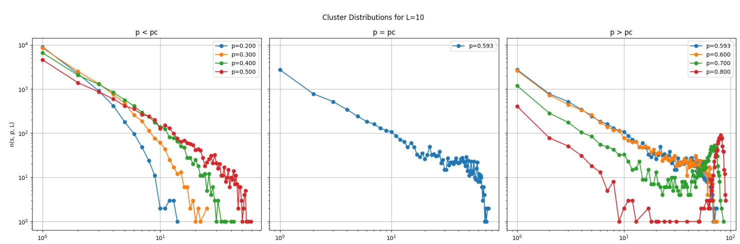
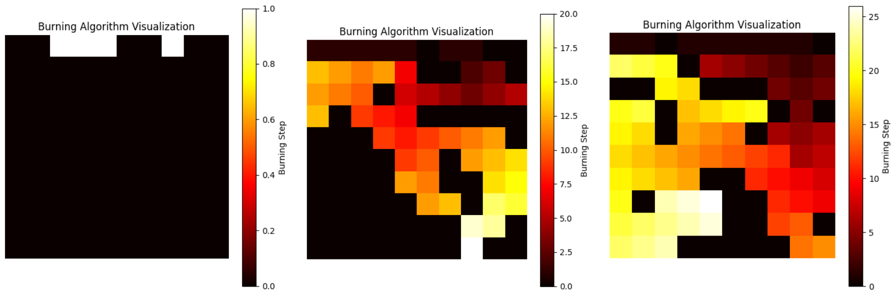


Figure 4: Cluster size distributions for $p < p_c$, $p = p_c$, and $p > p_c$.

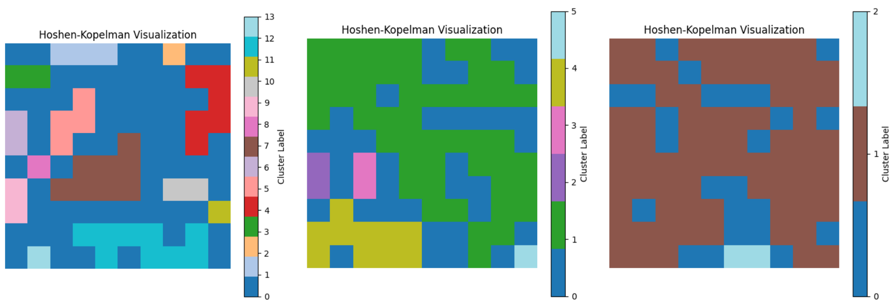
3.4 Burning Algorithm and HK Visualization

Figure 5 visualizes a sample configuration using the Burning Method, where each site's propagation number is displayed. Figure 6 shows a corresponding visualization using the HK algorithm, where clusters are uniquely colored.



(a) P = 0.4 (b) P = 0.6 (c) P = 0.8

Figure 5: Burning Algorithm for L = 10 and p = 0.4, 0.6, 0.8



(a) P = 0.4 (b) P = 0.6 (c) P = 0.8

Figure 6: HK Algorithm for L = 10 and p = 0.4, 0.6, 0.8

4 Discussion

4.1 Percolation Threshold

The results highlight the critical behavior of the percolation model. The sharp increase in P_{flow} near p_c confirms the existence of a phase transition.

4.2 Cluster Size Scaling

The scaling of $\langle s_{\max} \rangle$ and $n(s, p, L)$ is consistent with theoretical predictions, showcasing critical phenomena and finite-size effects.

5 Conclusion

This report demonstrates the implementation of a site percolation model and statistical analysis of its critical behavior. Visualization using the Burning Method and HK algorithm provided insight into cluster formation and connectivity. The study highlights the percolation threshold, scaling of maximum cluster sizes, and the power-law distribution of cluster sizes.

6 Repository

Repository: `Percolation Simulation`
`https://github.com/itillushka/Percolation`